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Concerning the Stability of Shock Waves

by S. P. D'iakov

Translated by Boyd Cary

University of Maryland
College Park, Maryland

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Concerning the Stability of Shock Waves

S. P. D'îakov

Investigating the question of the stability of shock waves in arbitrary media.

We shall examine a plane stationary shock wave of arbitrary amplitude which is not a permanent property of the medium. We shall assume only the fulfillment of the following necessary conditions:

$$S > S_0; \quad v_0 > c_0; \quad v < c. \tag{1}$$

In all fairness, we shall not need, generally speaking, the theorems of Tsemplena; our investigation will in this way also include weak shock, waves of rarefaction.

We shall direct the y axis normal to the wave; let the direction of the velocity be in the positive direction of the y axis.

For a study of the question of stability, we shall assume that the surface of the discontinuity undergoes a weak perturbation, so that the equation of the perturbation of the surface has the following form:

$$\eta = \eta_0 e^{i(kx - \omega t)}. \tag{2}$$

This "ripple" at the surface of the discontinuity causes a perturbation of the flow for $y < 0$ (the flow in the region $y > 0$ does not undergo a perturbation on the strength of the condition $v_0 > c_0$). We denote by $\delta S, \delta p, \delta V, \delta V$ small changes of entropy, pressure velocity, and of the specific volume for a perturbation. Substituting into the hydrodynamical equations the perturbed values of the quantities and arranging by means of a linear approximation through small perturbations, we get the following system of equations:

$$\frac{\partial \delta s}{\partial t} + v \frac{\partial \delta s}{\partial y} = 0 \qquad (3)$$

(eq. of entropy)

$$\frac{\partial \delta v}{\partial t} + v \frac{\partial \delta V}{\partial y} + V \nabla \delta p = 0 \qquad (4)$$

(eq. of Euler),

$$\frac{\partial \delta p}{\partial t} + v \frac{\partial \delta p}{\partial y} + \rho c^2 \operatorname{div} \delta V = 0 \qquad (5)$$

(eq. of continuity).

We seek a solution of this system, in which all the quantities are proportional to a multiple of the sort exp. $\{i(Kx + ly - \omega t)\}$. Then from eqs. 3-5 we get [1]

$$(vl - \omega)\delta s = 0; \qquad (6)$$

$$(vl - \omega)\delta v_x + VK\delta p = 0; \qquad (7)$$

FOOTNOTE 1: Here and everywhere in the future by the quantities $\delta s, \delta v$ etc. we shall use only the coefficients for the perturbation; the exponential multiplier will be assumed to be omitted.

$$(vl - \omega)\delta v_y + Vl\delta p = 0; \qquad (8)$$

$$(vl - \omega)\delta p + \rho c^2 (K\delta v_x + l\delta v_y) = 0. \qquad (9)$$

On the basis of eq. (6) we conclude that two types of solution are possible (all quantities pertaining to the solutions of these two types will be designated by indices 1 and 2). The solution of the first type corresponds to $v l_1 - \omega = 0$ and $\delta s^{(1)}$ is different from zero. Then from (7) and (8) we get $\delta p^{(1)} = 0$ and from (9)

$$k \delta v_x^{(1)} + l_1 \delta v_y^{(1)} = 0 \tag{10}$$

The quantity $\delta V^{(1)}$ is in addition different from zero and connected with $\delta s^{(1)}$ by means of the relation $\delta V^{(1)} = \left(\frac{\partial V}{\partial s}\right)_p \delta s^{(1)}$ This solution, an "entropy vortex" wave is, evidently, simply the transfer by the moving gases of the perturbed entropy and of the rotational velocity.

In the solution of the second type according to (6) $v l_2 - \omega \neq 0$ and $\delta s^{(2)} = 0$. Multiplying eq. (7) by k , eq. (8) by l_2 adding them and eliminating $\delta v_x^{(2)}$ and $\delta v_y^{(2)}$ in the resulting equation by means of (9), we get the following relation between l_2 and ω :

$$(\omega - v l_2)^2 = c^2 (k^2 + l_2^2). \tag{11}$$

To relate the quantity $\delta V^{(2)}$ with $\delta p^{(2)}$ the desired two eqs. are chosen from eqs. (7) - (9) for instance,

$$(v l_2 - \omega) \delta v_x^{(2)} + V k \delta p^{(2)} = 0; \tag{12}$$

$$(v l_2 - \omega) \delta v_y^{(2)} + V l_2 \delta p^{(2)} = 0. \tag{13}$$

Moreover, on strength of the condition $\delta s^{(2)} = 0$, we have,

$$\delta p^{(2)} = -(c^2/V^2) \delta V^{(2)} \tag{14}$$

This is a solution of the type of a sound wave and the relation (11) has to relate the frequencies with the wave vector by means of a calculation of the Dappler effect. The general solution of the equation is given by a linear combination of both types of solutions. The necessary combination of the solutions has to satisfy the boundary conditions at the discontinuity, which are obtained by an analysis of the conditions at the disturbed shock wave. We shall write out here the relations at the disturbed discontinuity and we shall use them for satisfaction (of the boundary conditions). We shall introduce orthogonal vectors by the tangent t and the normal n to the perturbed discontinuity; it is easily seen that the components of these vectors to a just approximation equal $\vec{t}(l, ik\eta)$ and $\vec{n}(-ik\eta, 1)$ The condition of equality of the tangential component of the velocity at the perturbed discontinuity has the form:

$$\vec{V_0}\vec{t} = (\vec{V} + \delta\vec{V})\vec{t}$$

where $\delta\vec{V} = \delta\vec{V}^{(1)} + \delta\vec{V}^{(2)}$ is the desired combination. With accuracy up to the first order of magnitude this condition is rewritten in the following way:

$$\delta v_x^{(1)} + \delta v_x^{(2)} = ik(v_0 - v)\eta. \qquad (15)$$

In an analogous way the condition for the jump of the normal velocity is:

$$\vec{V_0}\vec{n} - (\vec{V} + \delta\vec{V})\vec{n} = \sqrt{(p - p_0 + \delta p)(V_0 - V - \delta V)}$$

after analysis by a small perturbation it takes the form:

$$\delta v_y^{(1)} + \delta v_y^{(2)} = \frac{v - v_0}{2}\left[\frac{\delta p^{(2)}}{p - p_0} - \frac{\delta V^{(1)} + \delta V^{(2)}}{V_0 - V}\right] \qquad (16)$$

(here we used the fact that $\delta p^{(1)}=0$)

Now it is necessary to write down the condition which is applied at the perturbed discontinuity by means of the Hugoniot adiabatic. The Hugoniot adiabatic is represented as some functional dependence $p=p(V)$ which is determined by the thermodynamic characteristics of the medium, in which the shock wave is propagated. Therefore the connection between δp and δV is established by the formula

$$\delta p^{(2)}=\left(\frac{\partial p}{\partial V}\right)_H\left(\delta V^{(1)}+\delta V^{(2)}\right), \tag{17}$$

where the index H stands for that derivative taken along the Hugoniot adiabatic.

Finally, the last relation is gotten from the expression for the velocity of the discontinuity, as a function of the jumps in p and V. Having denoted by D the normal velocity of the points on the profile of the perturbed wave we write down this condition in the following manner:

$$\left[(\vec{V}_0-\vec{D})\vec{n}\right]^2=\vec{V}_0^2\,\frac{p-p_0+\delta p}{V_0-V-\delta V}.$$

In the first approximation, it is necessary to compute that $\vec{D}$ is directed along the y axis; then $D\approx -i\omega\eta$ (correction on that, that $\vec{D}$ has a component along the x axis, which will be of the second order of smallness.). Analyzing our formula moreover which was taken from the expression for D, we get the desired condition

$$\frac{2i\omega}{v_0}\eta=\frac{\delta p^{(2)}}{p-p_0}+\frac{\delta V^{(1)}+\delta V^{(2)}}{V_0+V}. \tag{18}$$

Formulas (10), (12) - (18) give a system of eight equations in eight unknowns $\delta p^{(2)}, \delta V^{(1)}, \delta V^{(2)}, \delta v^{(1)}, \delta v^{(2)}, \omega$.

Having eliminated all of the hydrodynamical variable, it is possible to obtain the characteristic equation for the frequency ω the investigation of which permits denoting the criterion of stability of the shock wave. If we eliminate $\delta \vec{V}^{(1)} + \delta \vec{V}^{(2)}$ from eq. (18) with the help of (17); we shall have

$$\frac{\delta p^{(2)}}{p - p_0}\left[1 + \frac{1}{j^2}\left(\frac{\partial p}{\partial V}\right)_H\right] = \frac{2 i \omega}{v_0 j^2}\left(\frac{\partial p}{\partial V}\right)_H \eta , \qquad (19)$$

where $j = \frac{v_0}{V_0} = \frac{v}{V}$ is the flow of the substance across the shock.

Further, eq. (16), after a preliminary elimination of $\delta V^{(1)} + \delta V^{(2)}$ by means of eq. (18) we multiply by ω/v, and eq. (15) by K and add. On the strength of (10), (12) and (13) we shall have

$$(v - v_0)\left(K^2 + \frac{\omega^2}{v v_0}\right)(v l_2 - \omega) i \eta = V\left(K^2 + \frac{\omega^2}{v^2}\right)\delta p^{(2)} \qquad (20)$$

Having multiplied eqs. (19) and (20) we get

$$\frac{2\omega v}{v_0}\left(K^2 + \frac{\omega^2}{v^2}\right) = \left(\frac{\omega^2}{v v_0} + K^2\right)(\omega - v l_2)\left[1 + j^2\left(\frac{\partial V}{\partial p}\right)_H\right] \qquad (21)$$

This is the characteristic equation of frequencies; it is necessary to understand l_2 here as a function of ω which is determined by eq. (11).

2. <u>The Investigation of the Stability Shock Waves.</u>

In the future it is convenient to transform eq. (21). Namely, we shall introduce the angle θ between the wave vector(K , l_2) and the normal to the discontinuity. Let

$$l_2 = \frac{\Omega}{c}\cos\theta, \quad K = \frac{\Omega}{c}\sin\theta \qquad (22)$$

where Ω for which the relation

$$\Omega^2 = c^2(k^2 + l_2^2), \tag{23}$$

evidently holds is not the same as the frequency of sound in a system of coordinates moving together with the substance behind the shock wave. Further, according to eq. (11) we have

$$\omega = \Omega + v l_2 = \Omega\left(1 + \frac{v}{c}\cos\theta\right) \tag{24}$$

Substituting (22) and (24) into (21), we get a quadratic eq. with regard to $\cos\theta$

$$\frac{v^2}{c^2}\left[\frac{4}{1+j^2(\partial V/\partial p)_H} + \frac{v_0}{v} - 1\right]\cos^2\theta +$$
$$+\frac{2v}{c}\left[\frac{3+(v^2/c^2)}{1+j^2(\partial V/\partial p)_H} - 1\right]\cos\theta + \frac{2[1+(v^2/c^2)]}{1+j^2(\partial V/\partial p)_H} - \left(1 + \frac{v v_0}{c^2}\right) = 0, \tag{25}$$

the investigation of which is considerably eas'er than that of the original equation with the additional condition (11).

The quantities l_2 and ω are expressed in terms of $\cos\theta$ by the equations:

$$l_2 = k\,\mathrm{ctg}\,\theta, \quad \omega = ck\left(\frac{1+(v/c)\cos\theta}{\sin\theta}\right) \tag{26}$$

The usual condition of instability is contained in the existence of a solution growing exponentially with time, whereupon the size of the perturbation exponentially decays at infinity as $y \to \infty$; for this imaginary part, generally speaking, the complex expression for the frequency ω and the y-component of the wave vector l_2 must be positive, i.e.

$$\mathrm{Im}(\omega) > 0; \quad \mathrm{Im}(l_2) > 0. \tag{27}$$

The non-fulfillment of even only one of these conditions implies the stability of the discontinuity. We note that by this the quantities θ and Ω are complex and $\cos\theta$ is complex or real accordingly as the absolute value is larger than unity.

For the future, we shall convince ourselves that besides the exponential growth and exponential decay with time of the solution the system permits in certain cases solutions with real ω and l_2 ; we shall discuss the meaning of such solutions below and immediately proceed to examine the case of complex ω and l_2 satisfying the conditions (27). We shall introduce the quantity[2]

$$\xi = \rho e^{i\phi} \equiv \mathrm{ctg}\,(\theta/2). \tag{28}$$

Then, as is easily seen $\mathrm{Im}\, l_2 = k\,\mathrm{Im}\,\mathrm{ctg}\,\theta = \frac{k}{2}\left(\rho + \frac{1}{\rho}\right)\sin\phi;$

$$\mathrm{Im}\,\omega = ck\,\mathrm{Im}\,\frac{1+(v/c)\cos\theta}{\sin\theta} = \frac{ck}{2}\left[\left(1+\frac{v}{c}\right)\rho - \left(1-\frac{v}{c}\right)\frac{1}{\rho}\right]\sin\phi \tag{29}$$

The first condition of (27) implies $\sin\phi > 0$, i.e. $0 < \phi < \pi$ (30)

the second condition gives

$$\rho^2 > \frac{1-(v/c)}{1+(v/c)}. \tag{31}$$

In eq. (25) it is expedient to replace the variable $\cos\theta$ by another variable

$$\chi = \left(\frac{1+\cos\theta}{1-\cos\theta}\right)\left(\frac{1+(v/c)}{1-(v/c)}\right)$$

after which the equation takes the form:

$$\left(1-\frac{v^2}{c^2}\right)\left[\frac{2(1+(v/c))}{1+j^2(\partial V/\partial p)_H} - 1\right]\chi^2 + 2\left\{\left(1-\frac{v^2}{c^2}\right)\left[\frac{1-j^2(\partial V/\partial p)_H}{1+j^2(\partial V/\partial p)_H}\right] - \frac{2vv_0}{c^2}\right\}\chi +$$

$$+\left(1-\frac{v^2}{c^2}\right)\left[\frac{2(1-(v/c))}{1+j^2(\partial V/\partial p)_H} - 1\right] = 0. \tag{32}$$

Condition (31) then implies that χ must be in absolute value larger than unity, and $\cos\theta$ is complex or real accordingly as the absolute

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2 The above method of investigation of the characteristic equations was pointed out by academician, L. D. Landau.

value is larger than unity.

It is known, that if x_1 , and x_2 are essential roots of the quadratic equation with real coefficients

$$ax^2 + bx + c = 0,$$

then at $|a| > |c|,\ |b| < |a+c|,\ |x_1|,\ |x_2| < 1;$

at $|a| < |c|;\ |b| < |a+c|,\ |x_1|,\ |x_2| > 1;$

at $|a| \gtrless |c|,\ |b| > |a+c|,\ |x_1| < 1,\ |x_2| > 1,$

whereupon in the last case on the strength of the reality of the coefficients, x_1 and x_2 must be real. The interesting case for us is when at least one of the roots of eq. (32) is in absolute value larger than unity, i.e., when the second or third of the conditions enumerated above occurs. It is easy to see that the second of these conditions can never be fulfilled. Actually, the condition $|a| < |c|$ for eq. (32) has the form

$$j^2 \left(\frac{\partial V}{\partial p}\right)_H > 1,$$

and the condition $|b| < |a+c|$

$$\frac{v v_0}{c^2} < \left(1 - \frac{v^2}{c^2}\right) \frac{1 - j^2 (\partial V/\partial p)_H}{1 + j^2 (\partial V/\partial p)_H},$$

evidently contradicts it.

In this way only one and moreover the real root of eq. (32) can lie outside the unit circle of the plane for the complex transformation

But the fulfillment of the condition $|x| > 1$ is still insufficient for instability; namely, it is necessary in addition to demand, as it was pointed out above, that $\cos\theta$ was complex or real, accordingly as the absolute value was larger than unity. A complex $\cos\theta$ is not possible on the strength of the reality of x ; further, in the relation with the expression for x at $x > 1$, $|\cos\theta| < 1$ and for $x < -1$, $|\cos\theta| > 1$. Thus, the additional condition, guaranteeing instability, is included in the requirement that one root of eq. (32) was less than -1. Comparing signs of the quadratic polynomial (32) at the values $\chi = \pm 1$, $\pm \infty$ it is easily found that the condition of instability in substance is

$$\left(\frac{\partial V}{\partial p}\right)_H < -\frac{1}{j^2} \tag{33}$$

$$\left(\frac{\partial V}{\partial p}\right)_H > \frac{1}{j^2}\left(1 + 2\frac{v}{c}\right). \tag{34}$$

Up to now we examined the case of complex ω and l_2 ; now we shall consider the case of real ω and l_2 , when the solution describes actual sound and entropy waves being propagated. A sound in a solution of such a type may be both "coming", i.e., incident on the discontinuity from without opposite to the flow, and "going", i.e., which is radiated by the discontinuity. The first type of solution, to which corresponds obvious, $l_2 < 0$ and $\cos\theta < 0$, does not offer interest and does not have any relation to the problem of stability; the task in this case reduces to the finding of such a connection between the frequency and the wave vector of the sound wave, incident from without and "catching up" to the shock wave, upon the fulfillment of which a reflected sound wave is absent.

The case of spontaneous radiation of the sound by the discontinuity to which corresponds $l_2 > 0$ and $\cos\theta > 0$ arouses much interest. Employing the well known theory of Sturm to eq. (25), it is easy to find the condition, for which the root $\cos\theta$ lies between zero and one.

$$\frac{1}{j^2}\,\frac{1 + 2(v^2/c^2) - (v v_0/c^2)}{1 + (v v_0/c^2)} < \left(\frac{\partial V}{\partial p}\right)_H < \frac{1}{j^2}\left(1 + 2\frac{v}{c}\right). \tag{35}$$

It is not entirely clear what implies the physical possibility of the spontaneous radiation of sound by the discontinuity; it is extremely probable that this attests very much to the instability of the discontinuity of the original type.

It follows that the investigation of stability in this case in the second approximation in principal should permit the clarification of the question, but the task in such a scheme encounters gross mathematical difficulties.

The results of the investigation of the stability are summarized in the table reproduced below

Table

Condition	Result
$\left(\frac{\partial V}{\partial p}\right)_H < -\frac{1}{j^2}$	absolute instability
$-\frac{1}{j^2} < \left(\frac{\partial V}{\partial p}\right)_H < \frac{1}{j^2}\,\frac{1+2(v^2/c^2)-(v v_0/c^2)}{1+(v v_0/c^2)}$	stability
$\frac{1}{j^2}\,\frac{1+2(v^2/c^2)-(v v_0/c^2)}{1+(v v_0/c^2)} < \left(\frac{\partial V}{\partial p}\right)_H < \frac{1}{j^2}\left(1+2\frac{v}{c}\right)$	Possibility of the spontaneous radiation of sound
$\left(\frac{\partial V}{\partial p}\right)_H > \frac{1}{j^2}\left(1+2\frac{v}{c}\right)$	absolute instability

3. Discussion of the results

Turning now to the discussion of the question concerning the possibility of the actual realization of instability we shall not pretend to an exhaustive survey of the types of Hugoniot adiabatics for which the condition of instability is fulfilled; we shall limit ourselves at most to an examination of the simple aspects of the adiabatics. We shall examine initially the conditions of literal instability (33) and (34), for which a solution exists which increases exponentially with time. The first condition, indicating that the slope of the tangent to the Hugoniot adiabatic is negative and by coefficient is smaller than the slope of Michel' son's line, is realized perhaps only in the case when the derivative $\left(\frac{\partial p}{\partial V}\right)_H$ is negative at the beginning portion of the adiabatic near the point p_0, V_0, passing through infinity, it changes sign to positive and then, passing through zero, becomes negative again. In Figs. 1 and 2 we schematically represented the shape of the Hugoniot adiabatics for waves of compression and expansion, respectively, for which this condition is perhaps fulfilled. At the region ab the condition of instability (33) is fulfilled.

Condition (34), being less rigid so that it does not demand the inversion of $\left(\frac{\partial p}{\partial V}\right)_H$ at zero is fulfilled in the positive region for the derivative $\left(\frac{\partial p}{\partial V}\right)_H$ and demands only that the slope of the adiabatic may be sufficiently small.

In Figs. 1 and 2 this condition is surely fulfilled at the sections ca and bd, immediately adjoining to ab, and noticeably extends the region of instability. Condition (34) may also be fulfilled for the adiabatics schematically shown in Figs. 3 and 4, for which a region of the type ab is generally absent; for this it is only necessary that the slope may be sufficiently small in the vicinity of the point of the bend (section cd).

The examination of the "bending" adiabatics, having a point of tangency with use of Michel' son's lines, does not arouse interest in connection with the problem of stability, so that the section, lying at the point of tangency, is not realized in practice on the strength of the violation of the last of the conditions (1).

We shall now examine the case of the spontaneous radiation of sound. The condition for the possibility of the spontaneous radiation of sound (35) is less rigid than condition (34). The presence on the adiabatic of a section of type cd, at which condition (34) is fulfilled automatically guarantees the existence of sections adjoining cd on both sides (ec and df on the figs.), for which condition (35) is satisfied. Thus the condition for the spontaneous radiation of sound extends the region of stability still further.

Condition (35) may be fulfilled at the bow (curve) of the adiabatic, where $\left(\frac{\partial p}{\partial V}\right)_H > 0$, and by the absence of a section of type cd (fig. 5 and 6); furthermore, this condition is perhaps satisfied in principal also in the region where $\left(\frac{\partial p}{\partial V}\right)_H < 0$ if at this (region)

$$c^2 < v v_0 - 2v^2$$

Therefore, the point f on Figs. 3 and 5 lies perhaps in principal arbitrarily high on the adiabatic, and the point e lies perhaps lower than that point of the adiabatic, where the tangent is vertical to it. Furthermore, a section of type ef may exist on an adiabatic of the usual shape, where the derivative $\left(\frac{\partial p}{\partial V}\right)_H$ is everywhere negative. (Figs. 7, 8).

It is of interest to raise the question concerning the possibility of experimental observation of the instability of shock waves. In all probability, it is extremely difficult to indicate substances with Hugoniot adiabatics of the type represented in Figs. 1 - 2. Considerably more physical cases are shown in Figs. 3, 5 and 7. Adiabatics of the shape given in Figs. 3,5 must be observed in diatomic and polyatomic gases, dissociating in the presence of an elevation of temperature; an adiabatic of the type shown in Fig. 7 appears conventional. The numerical calculations for adiabatics of these types, permitting an investigation of the possibility of fulfillment of the conditions of instability could be extremely useful.

Conclusions

1. The problem of the stability of shock waves in an arbitrary medium was examined.
2. Conditions for the instability of shock waves are found which if satisfied result in an exponential growth of the disturbances with time.
3. The original phenomenon which was discovered is the spontaneous radiation of sound by the shock wave and conditions which determined the possibility of its occurrence.
4. Examples are given of the shapes of Hugoniot adiabatics in which it is possible for the conditions of instability and for the possibility of the spontaneous radiation of sound to be fulfilled. It was shown in which media the fulfillment of these conditions is to be expected..

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Institute of Physical Problems
Academy of Sciences USSR

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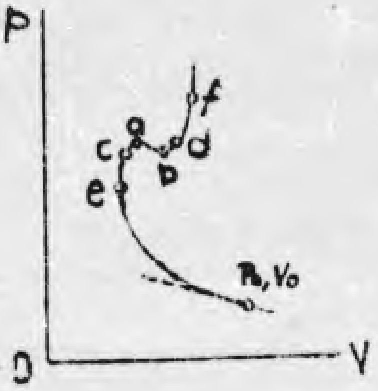

Fig. 1

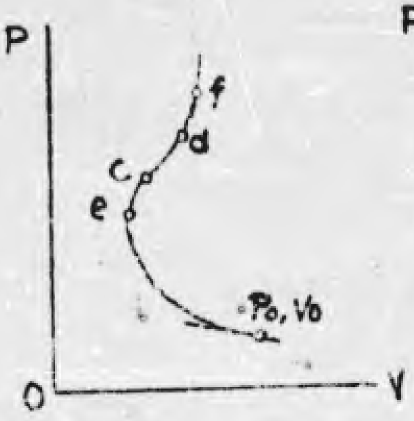

Fig. 3

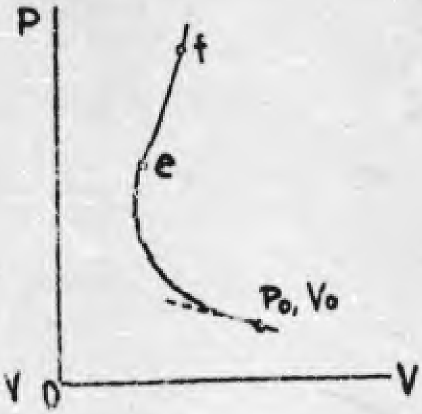

Fig 5

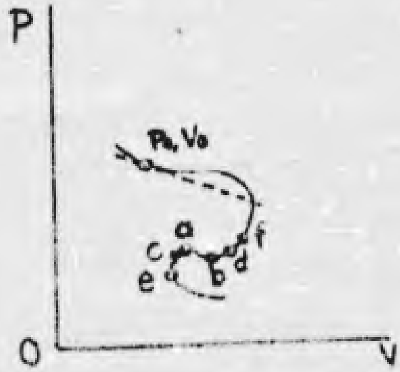

Fig. 2

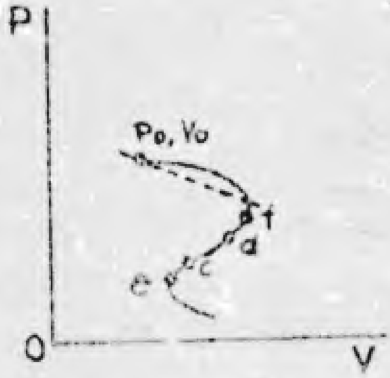

Fig. 4

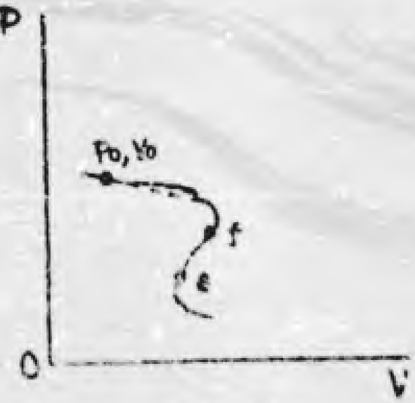

Fig 6

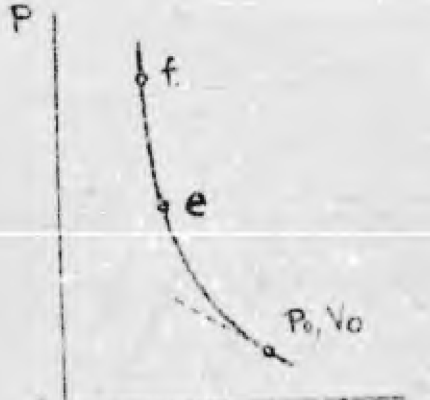

Fig 7

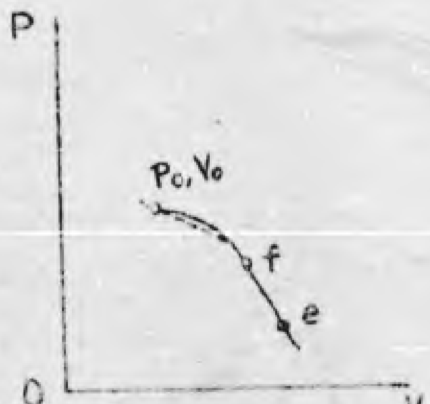

Fig 8