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STUDY OF PHASE ERROR AND TOLERANCE EFFECTS IN MICROWAVE REFLECTORS

HANDBOOK

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By

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STUDY OF PHASE-ERROR AND TOLERANCE EFFECTS IN MICROWAVE ANTENNAS

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Approved by:

David K. Cheng
Project Director

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PREFACE

This handbook is a compilation of the theoretical results which have been obtained in the course of the study of the phase-error and tolerance effects in microwave antennas under Contract No. AF 30(602)-924 with the Rome Air Development Center. The material is divided into four major sections. In Section I, the radiation characteristics for a rectangular aperture with phase deviations of the linear, step, staircase, quadratic, and cubic types are presented. For each type of phase deviation, computation is carried out for two different amplitude illumination functions. They are: (1) Uniform illumination, which is analytically simple and serves as a convenient reference; and (2) $\cos^2(0.375\pi x)$ illumination, which gives a desirable side-lobe level of approximately -26 db. Section II presents the radiation characteristics for a circular aperture with phase deviations of the linear, quadratic, and cubic types. In all figures the radiation pattern for the respective amplitude illumination function with no phase error is shown for ready comparison. Important characteristics for the various radiation patterns are also presented in table form. In order to calculate the far-zone radiation patterns, it is necessary to correlate the mechanical deviations in a reflector with the phase and amplitude distributions in an aperture plane. Analytical formulas for determining the aperture phase and amplitude distributions for an arbitrary reflector with a point source are given in Sections III and IV respectively. Special cases for reflectors with constant, linear, and exponential deviations are computed and the distribution curves plotted. Derivations of the formulas are not included in this handbook. Details of the theoretical aspects of the phase-error and tolerance problem and results of the experimental investigation conducted in connection with this study are presented in the final report, S.U.R.I. Report No. EE276-564F, for this contract.

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I. THE RECTANGULAR APERTURE

A. INTRODUCTION

Radiation patterns and their important characteristics for a rectangular aperture are presented in this chapter for phase deviations of various types. Two kinds of amplitude illumination are considered, namely, a uniform illumination and a $\cos^2(0.375\pi x)$ illumination. The uniform illumination case is simple analytically and serves as a convenient reference for comparison purposes. However, it is of little practical interest because the side-lobe levels of its radiation pattern are too high; the first side lobes are only 13.27 db down from the main lobe. The $\cos^2(0.375\pi x)$ type of amplitude function is chosen to give a side-lobe level of approximately -26 db which is considered desirable in practice.

There are a number of amplitude illumination functions which, with proper taper, give first side lobes that are more than 26 db down from the main-lobe level. Most of the functions (Gable, Cosine-squared, and Gaussian), however, show larger side lobes beyond the first pair. This is an undersirable feature. The cosine-cubed function $\cos^3(\frac{\pi x}{2})$ with 11-db taper will give first side lobes of about 26 db down and higher-order side lobes of decreasing magnitude; but this function introduces extra complication in subsequent integrating processes. It is therefore necessary to look for a simple amplitude function that will give 26-db first side lobes and higher-order side lobes of decreasing magnitude.

The function $\cos^2 nx$ with $n = 0.375\pi$ as shown in Fig. 1 satisfies these requirements. The amplitude is 0.1465 at the edges as compared with 1 at the center of the aperture. In other words, the taper is 16.68 db.

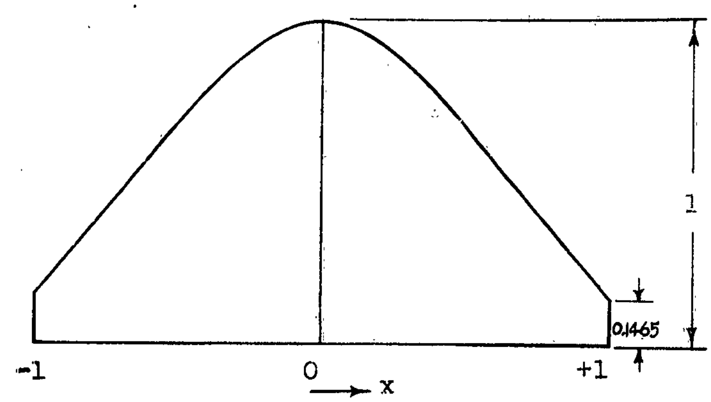

Fig. 1 - $\mathrm{Cos}^2 nx$ Amplitude Function, $n = 0.375\pi$

With appropriate approximations[1] the normalized radiation-pattern function in a principal plane for a rectangular aperture with a separable field distribution can be reduced to

$$g(u) = \int_{-1}^{1} f(x)\, e^{jux}\, dx$$

where $f(x)$ is the aperture-field distribution function; x is linear dimension normalized with respect to the half-width $a/2$ of the aperture; and $u = (\pi a/\lambda) \sin\theta$, θ being the azimuth angle. Antenna patterns are graphical plots of the radiation pattern function $|g(u)|$. The important characteristics are:

(a) Beam shift - The direction at which the diffraction field is maximum. Both the direction and the amount of shift from the z-axis are of interest.

(b) Magnitude of the maximum diffraction field - The maximum value of $|g(u)|$, from which the gain of the antenna system may be calculated.

[1]For details, see D. K. Cheng and R. E. Gildersleeve, Report No. EE276-564F Final Report for Contract No. AF 30(602)-924, Syracuse University Research Institute; 31 January 1956.

(c) <u>Beamwidth</u> - The widths of the main lobe at the 3-db and the 10-db levels (referring to the maximum value in (b)); they define the sharpness of the main lobe.

(d) <u>Side-lobe levels</u> - The db-levels of side lobes referred to the main lobe; they have to satisfy certain minimum requirements in practice. Generally speaking, the levels of the first side lobes are the most important, but those of other side lobes should not be disregarded. The latter may sometimes rise higher than the first pair of side lobes.

(e) <u>Gain factor</u> - The gain factor is defined as the ratio of the maximum gain of the given antenna with the given aperture field distribution (G_m) to that of an antenna of the same aperture area but with a uniform amplitude illumination and a constant phase (G_o). The case of uniform amplitude illumination and constant phase is chosen as a convenient reference because it is known that it gives the highest maximum gain:

$$G_o = \frac{4\pi A}{\lambda^2}$$

where A is the area of the aperture. In terms of ratio of powers, the gain factor can be written as

$$G.F. = \frac{G_m}{G_o} = \frac{\lambda^2}{A} \cdot \frac{P_{max.}}{P_{total}}$$

It is easy to show that for rectangular apertures the above expression reduces to

$$G.F. = \frac{1}{2} \cdot \frac{\left(\max. \left|\int_{-1}^{1} f(x)\, e^{jux}\, dx\right|\right)^2}{\int_{-1}^{1} \left| f(x) \right|^2 dx}$$

The integral in the denominator can be readily calculated when the amplitude illumination function is given, but the maximum value of the integral in the numerator cannot be conveniently determined analytically. The latter value can, however, be read off from the graphical plot.

Aside from characteristics (a) to (e), the general shape of the pattern is also of interest. Important characteristics of the radiation patterns for rectangular apertures with constant phase and the two types of amplitude illumination considered are tabulated in Table 1.

Table 1

Radiation-Pattern Characteristics with Constant Aperture Phase (Rectangular Aperture)

	$f(x) = 1$	$f(x) = \cos^2(0.375\pi x)$
Max. $\lvert g(u)\rvert$	2.000	1.300
3-db Beamwidth	0.91π	1.14π
10-db Beamwidth	1.48π	1.96π
1st. Side-Lobe Level	-13.27db	-25.90db
Gain Factor	1.000	0.848

In following sections, the radiation patterns for certain special types of aperture phase deviation are presented graphically. In each case, the pattern for the corresponding amplitude illumination without phase deviation is sketched in for comparison. The important characteristics are collected and tabulated in Tables 2 to 5.

B. UNIFORM ILLUMINATION

The normalized radiation patterns for a rectangular aperture with uniform amplitude illumination and phase deviations of the following types are plotted in this section. (Phase deviations occur in the region $1/2 \leq |x| \leq 1$ only and ψ represents the maximum error at the edge of the aperture.)

1. Linear Phase Deviation
   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 2
   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 3
   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 4
2. Step Phase Deviation
   a. Phase advance on one end ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 5
   b. Antisymmetrical deviation ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 6
   c. Symmetrical phase advance ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 7
3. Staircase Phase Deviation (m = No. of steps)
   a. Phase advance on one end ($\psi = 90^\circ$; m = 4 and 6) - Fig. 8
   b. Antisymmetrical deviation ($\psi = 90^\circ$; m = 4 and 6) - Fig. 9
   c. Symmetrical phase advance ($\psi = 90^\circ$; m = 4 and 6) - Fig. 10
4. Quadratic Phase Deviation
   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 11
   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 12
   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 13
5. Cubic Phase Deviation
   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 14
   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 15
   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 16

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   c. Symmetrical phase advance ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 7

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   c. Symmetrical phase advance ($\psi = 90^\circ$; $m = 4$ and 6) - Fig. 10

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   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 13

5. Cubic Phase Deviation
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   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 15
   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 16

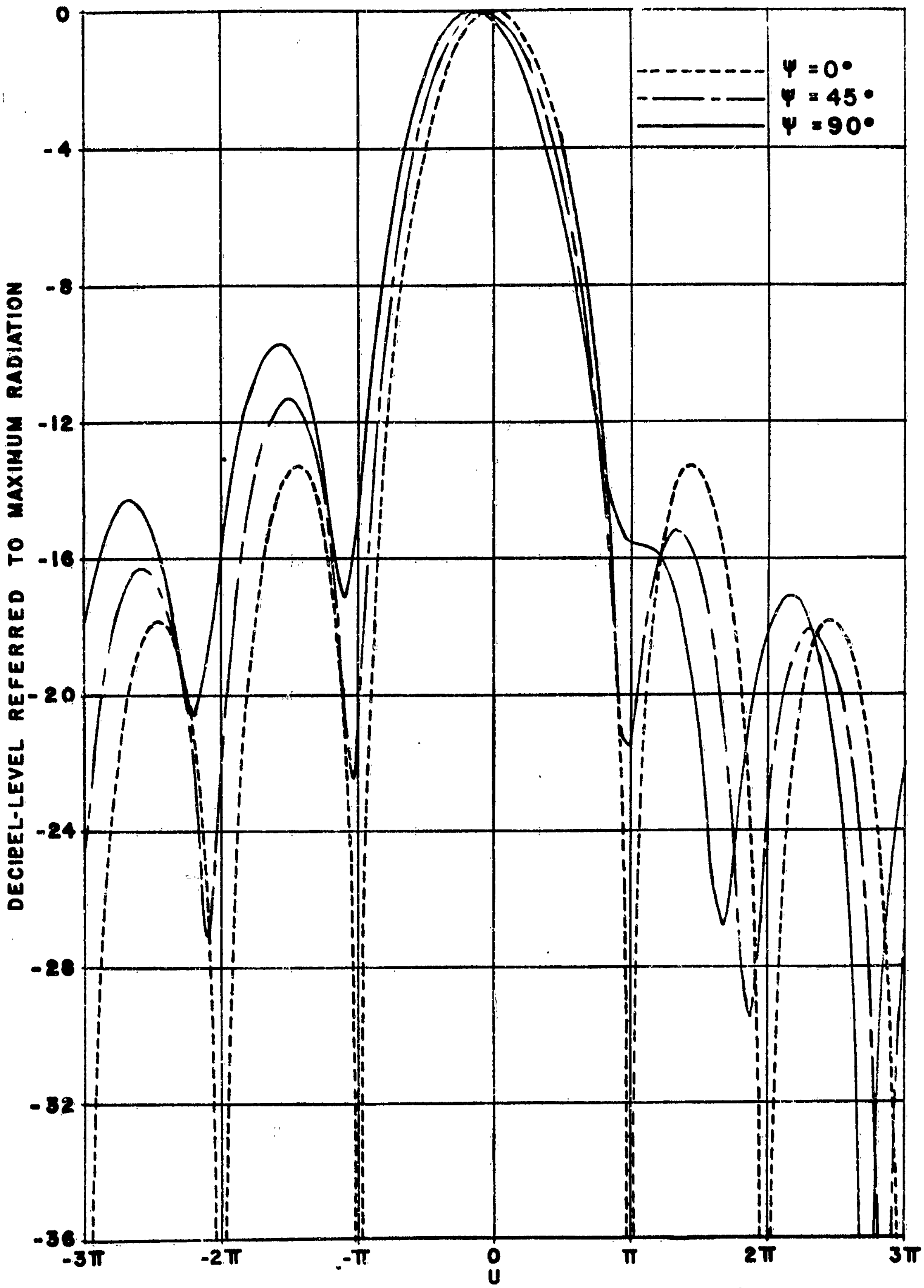
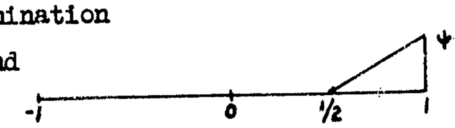


Fig. 2 - Radiation Patterns: Uniform Illumination and Linear Phase Advance on One End (Rectangular Aperture)



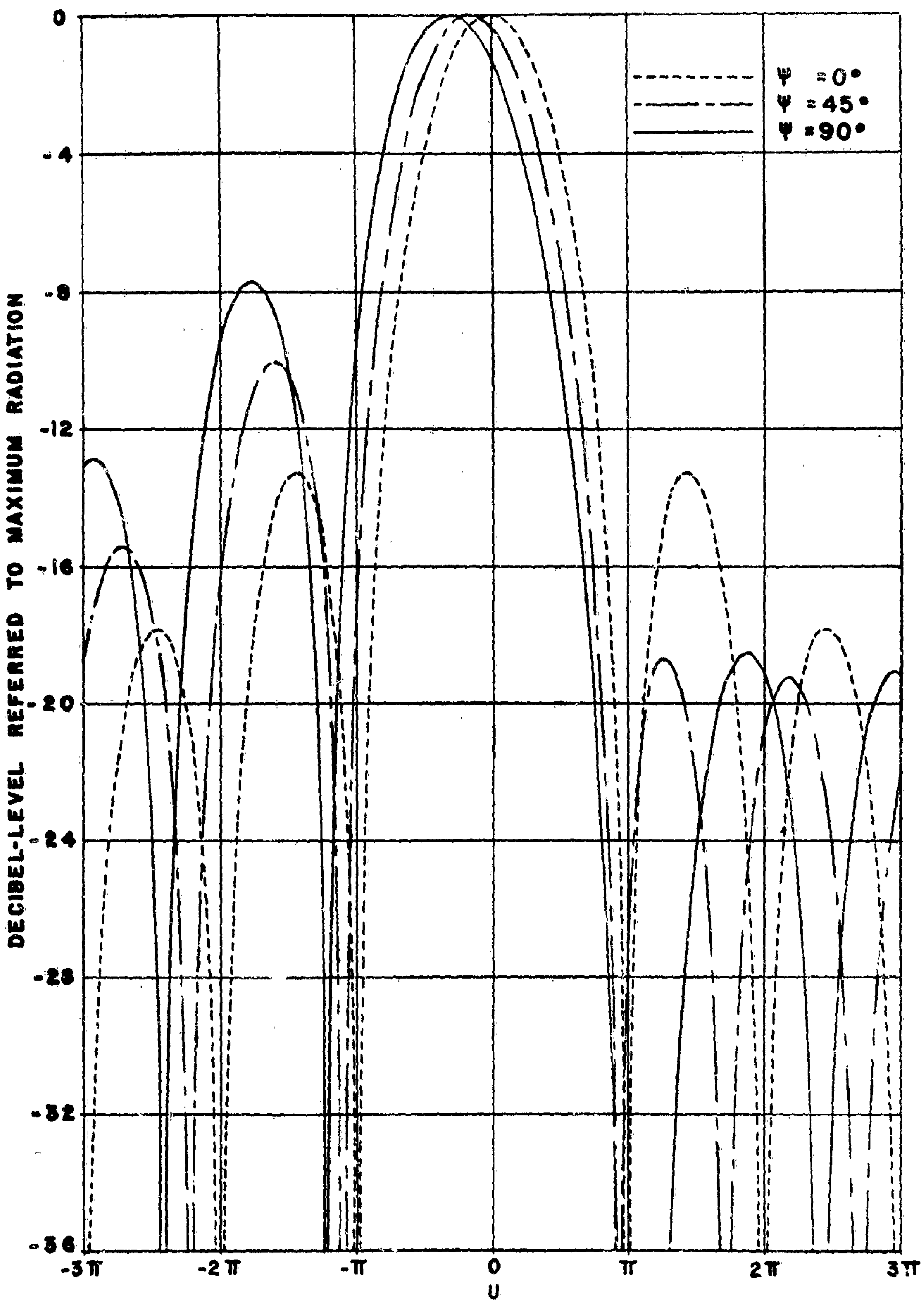
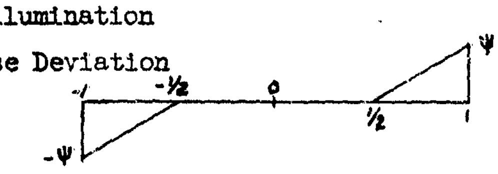


Fig. 3 - Radiation Patterns: Uniform Illumination
and Antisymmetrical Linear Phase Deviation
(Rectangular Aperture)



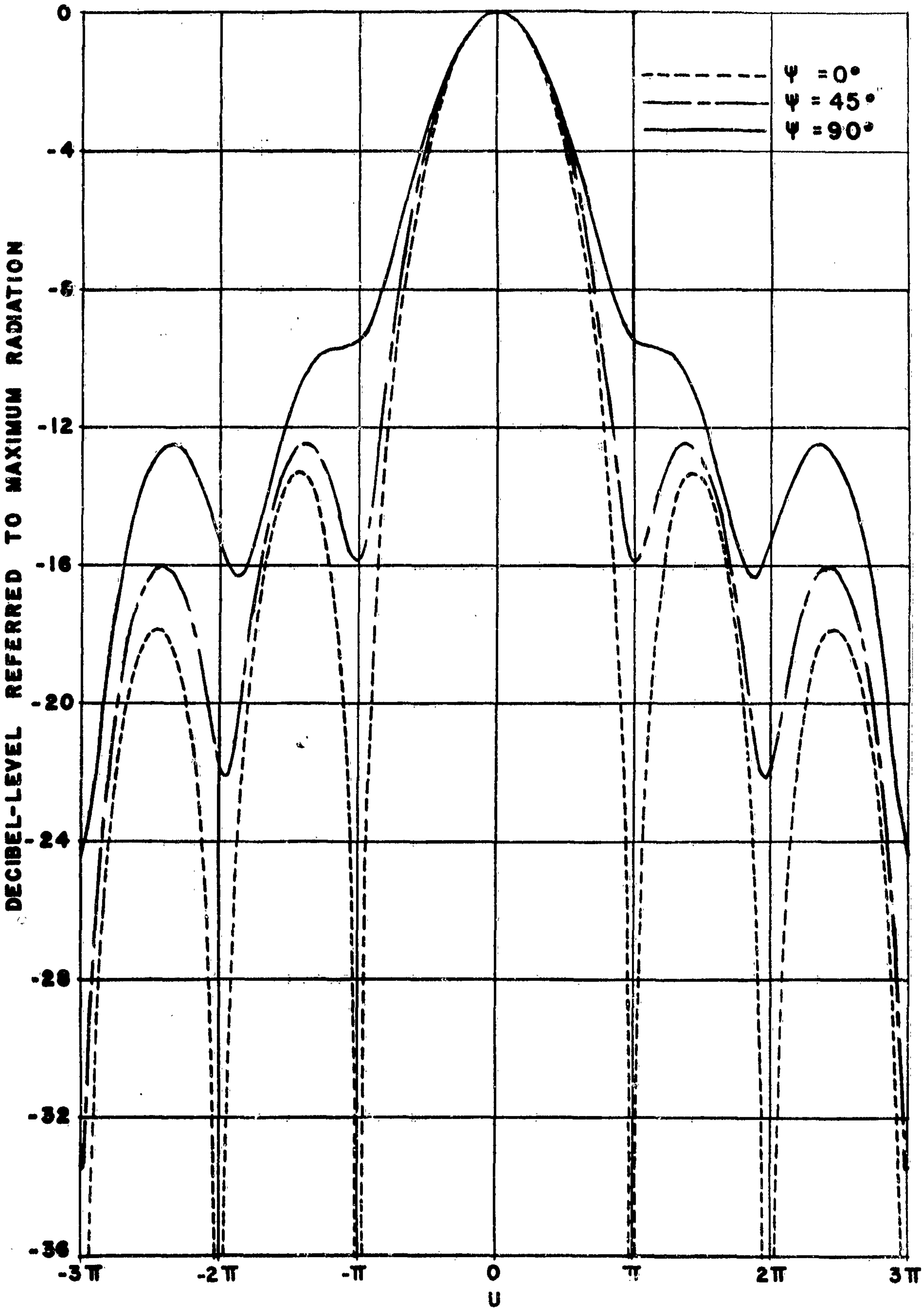


Fig. 4 - Radiation Patterns: Uniform Illumination and Symmetrical Linear Phase Advance (Rectangular Aperture)

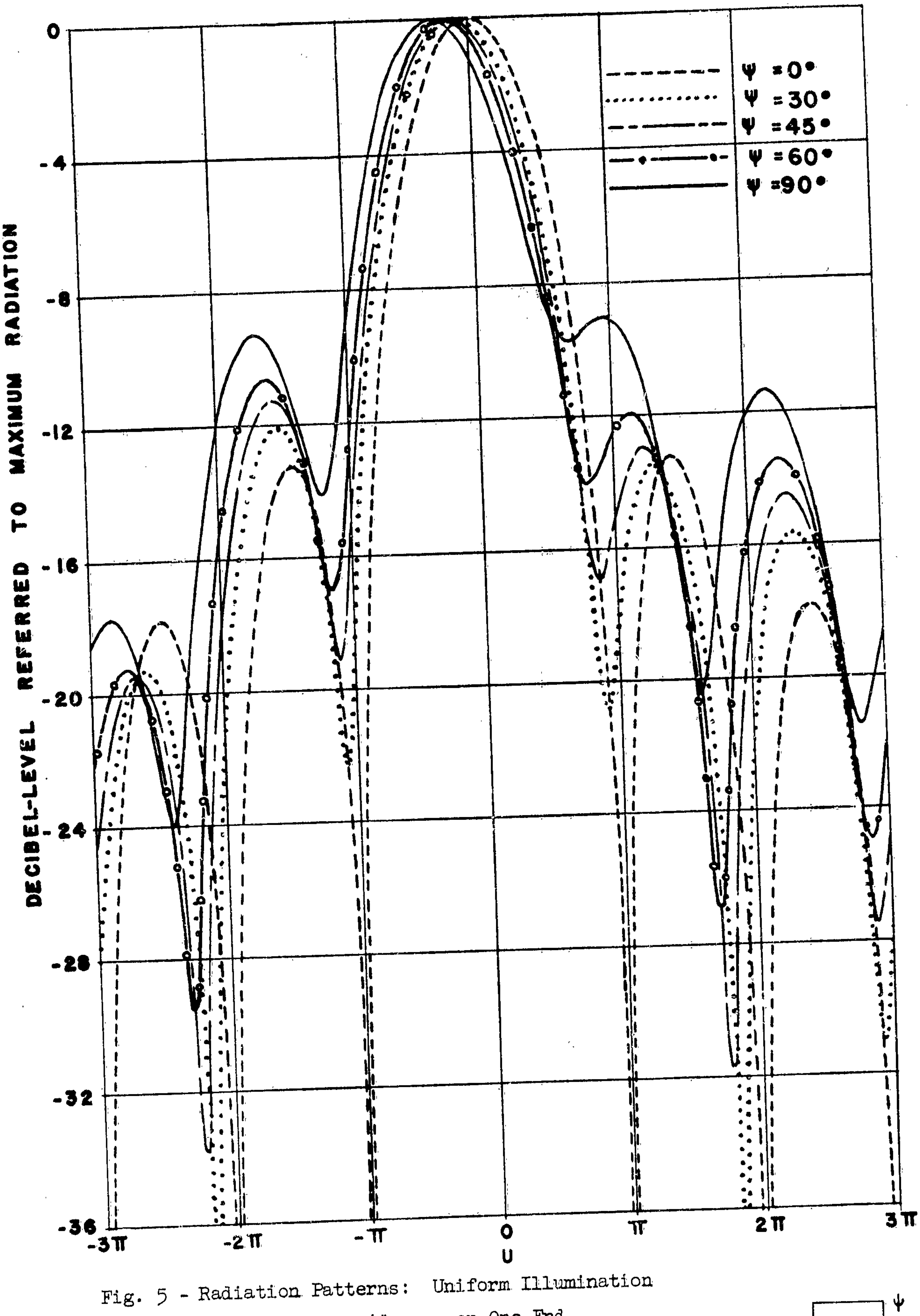
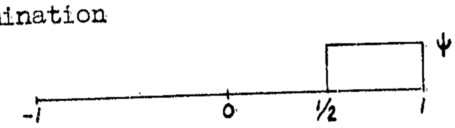


Fig. 5 - Radiation Patterns: Uniform Illumination and Step Phase Advance on One End (Rectangular Aperture)



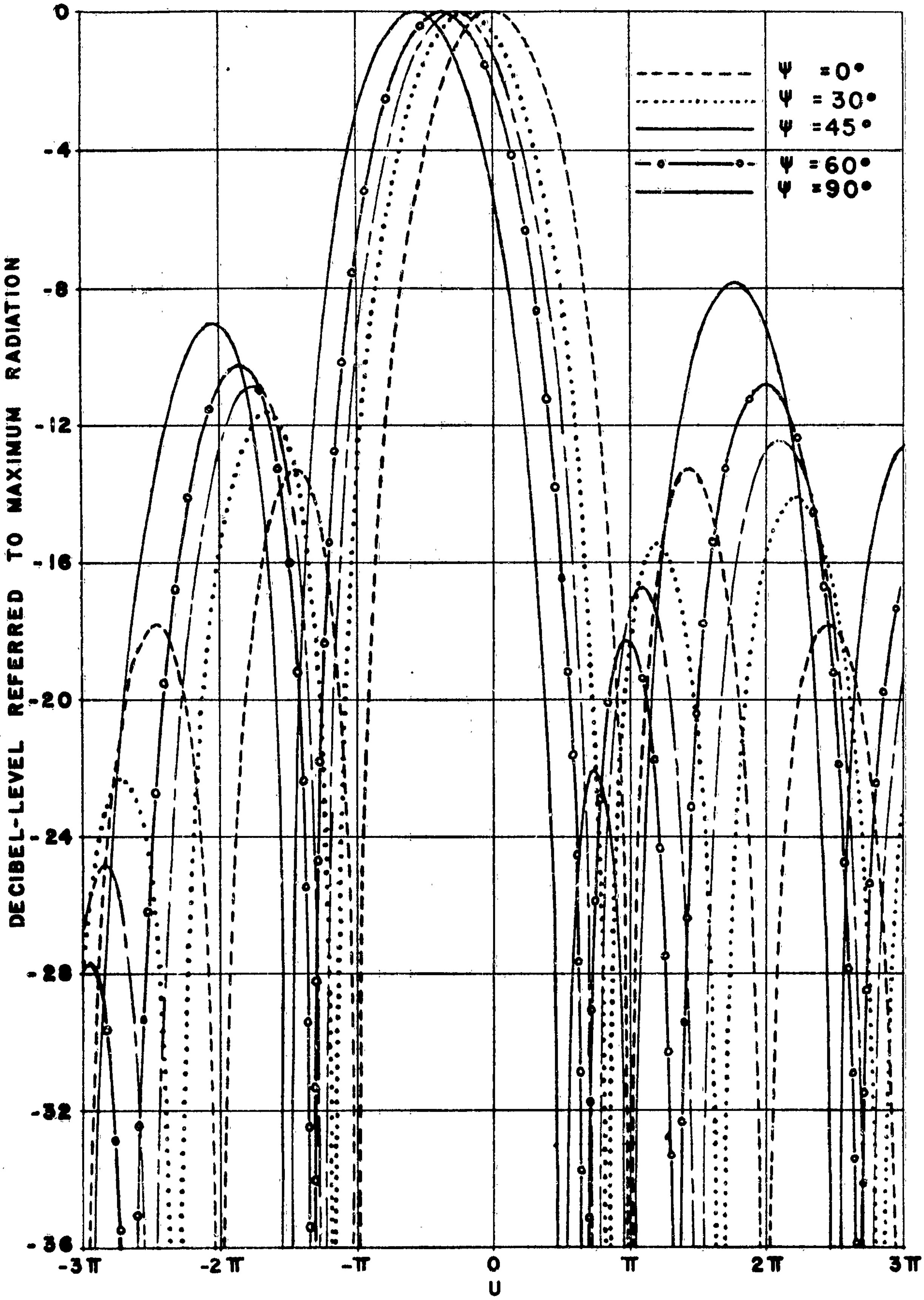
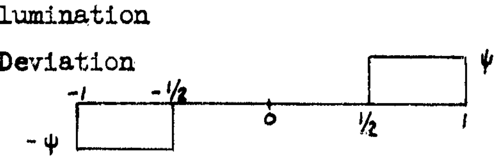


Fig. 6 - Radiation Patterns: Uniform Illumination
and Antisymmetrical Step Phase Deviation
(Rectangular Aperture)



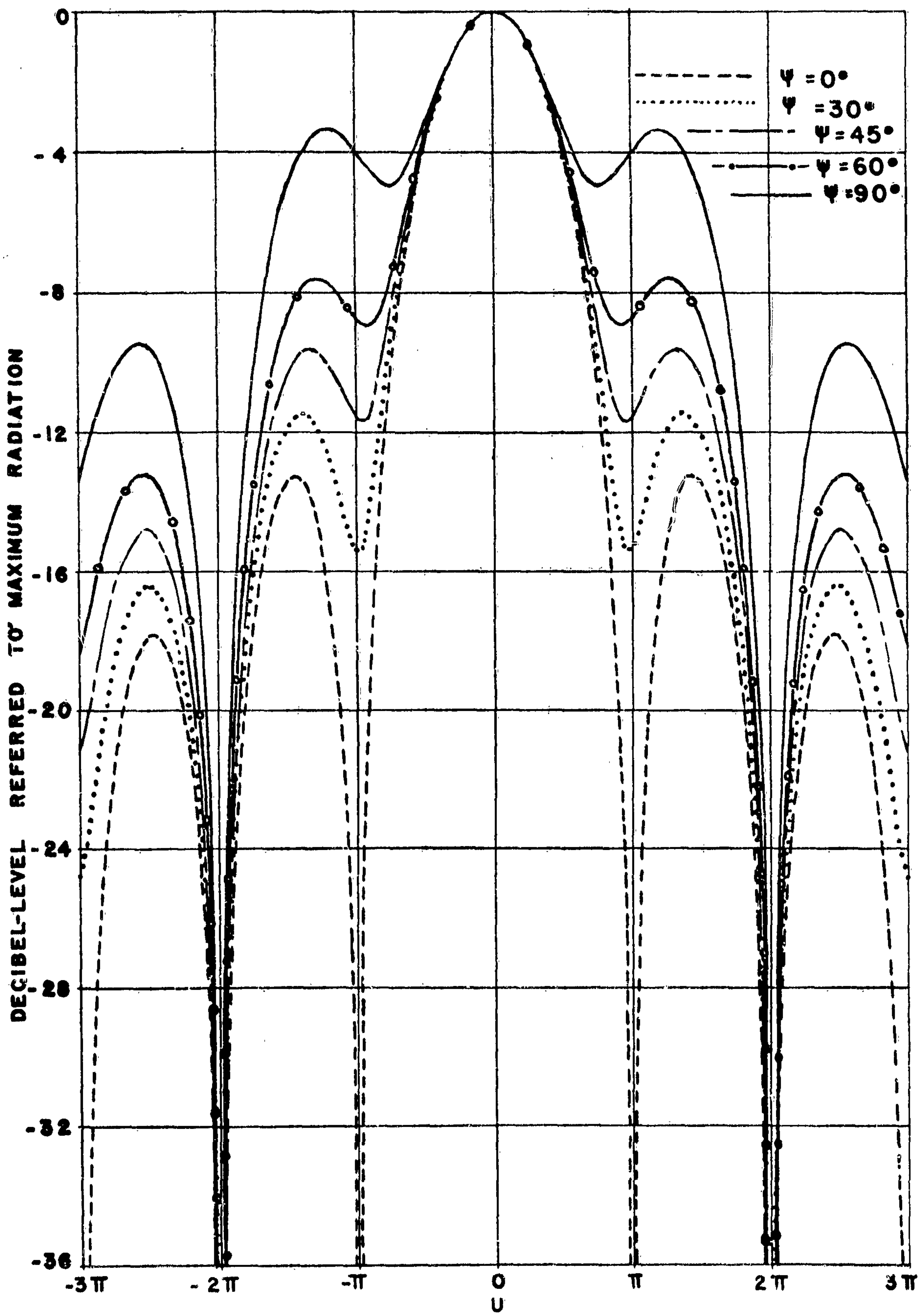
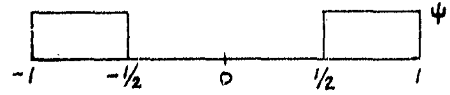


Fig. 7 - Radiation Patterns: Uniform Illumination and Symmetrical Step Phase Advance (Rectangular Aperture)



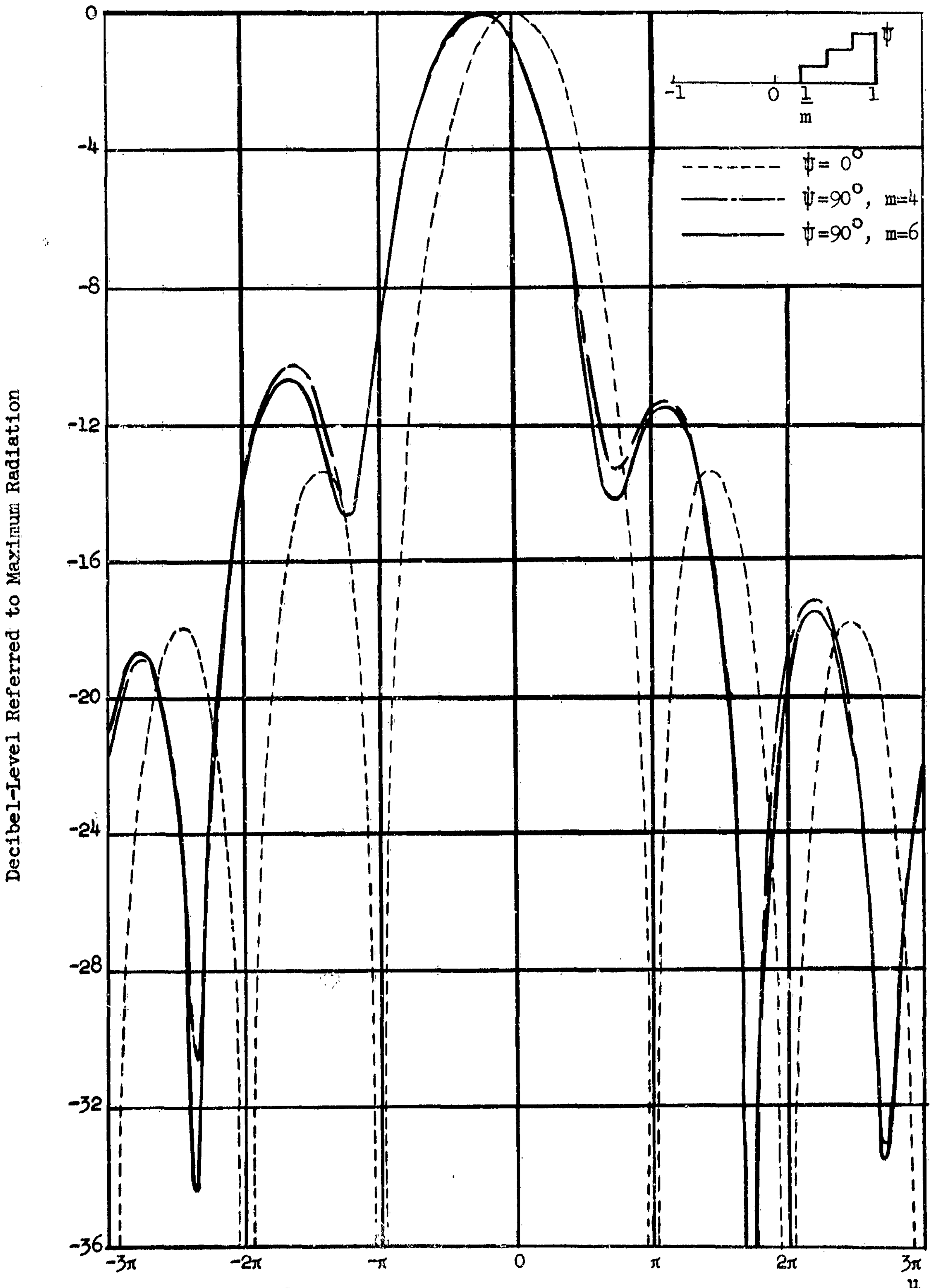


Fig. 8 - Radiation Patterns: Uniform Illumination and Staircase Phase Deviation on One End (Rectangular Aperture)

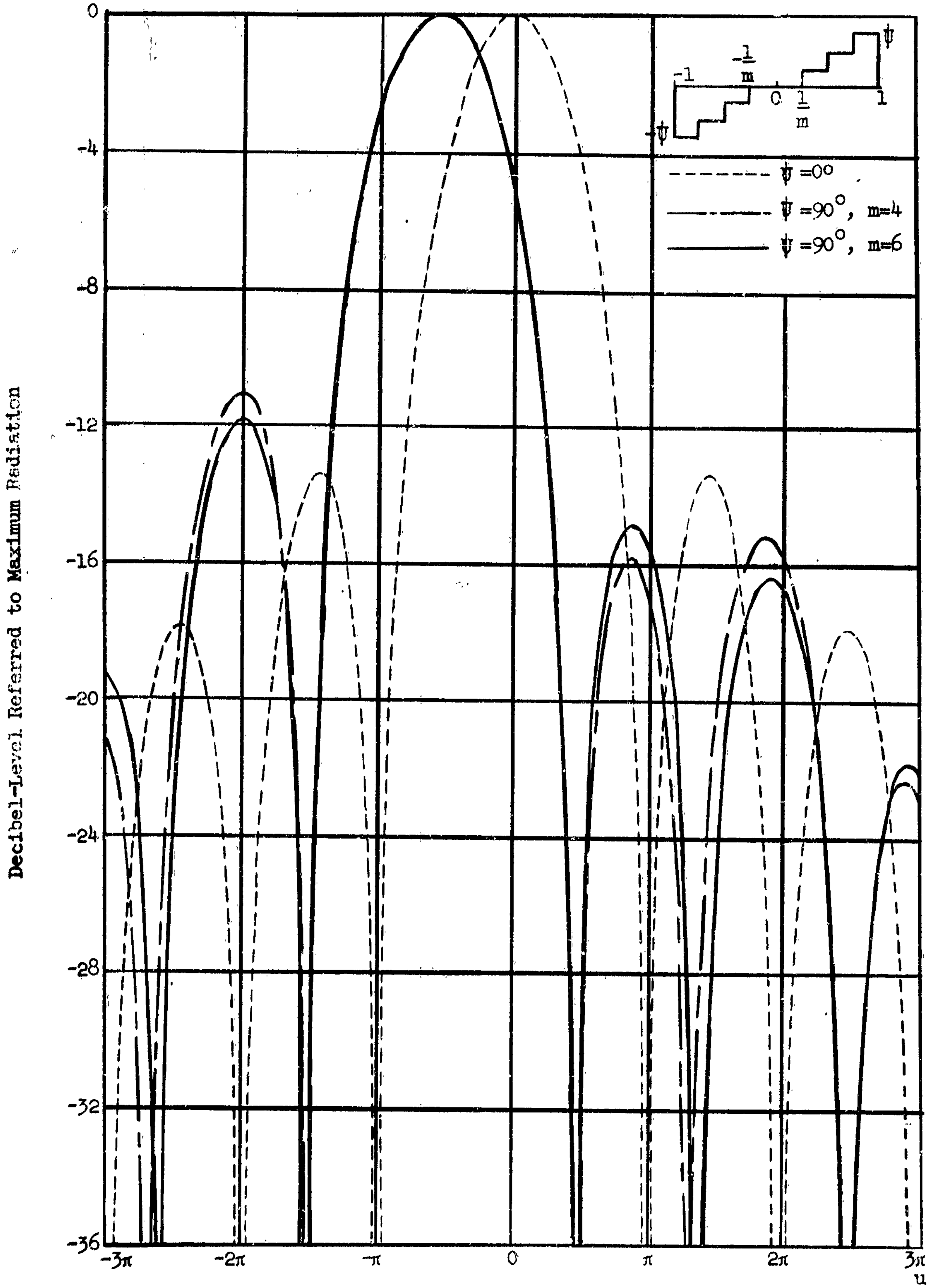


Fig. 9 - Radiation Patterns: Uniform Illumination and Antisymmetrical Staircase Phase Deviation (Rectangular Aperture)

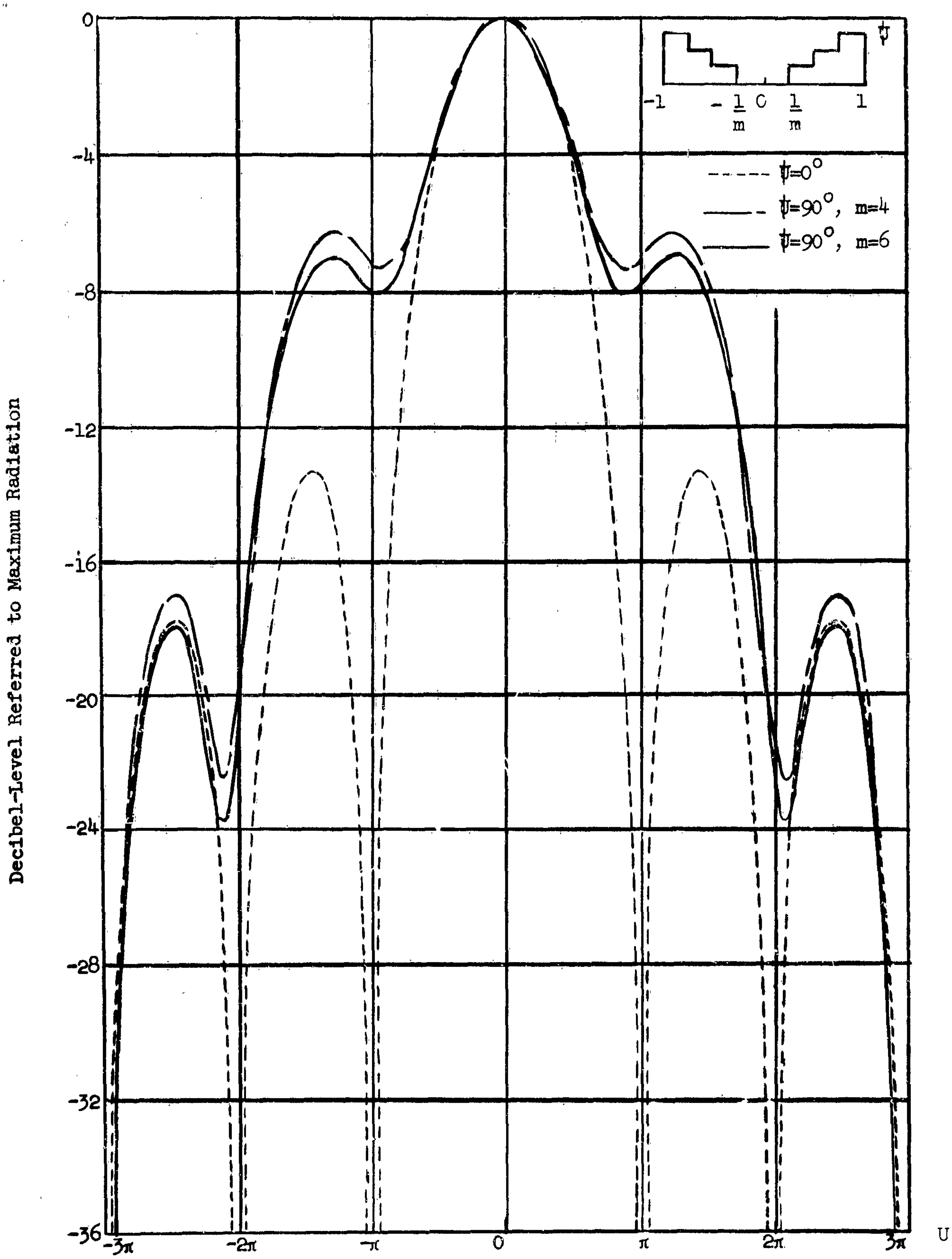


Fig. 10 - Radiation Patterns: Uniform Illumination and Symmetrical Staircase Phase Deviation (Rectangular Aperture)

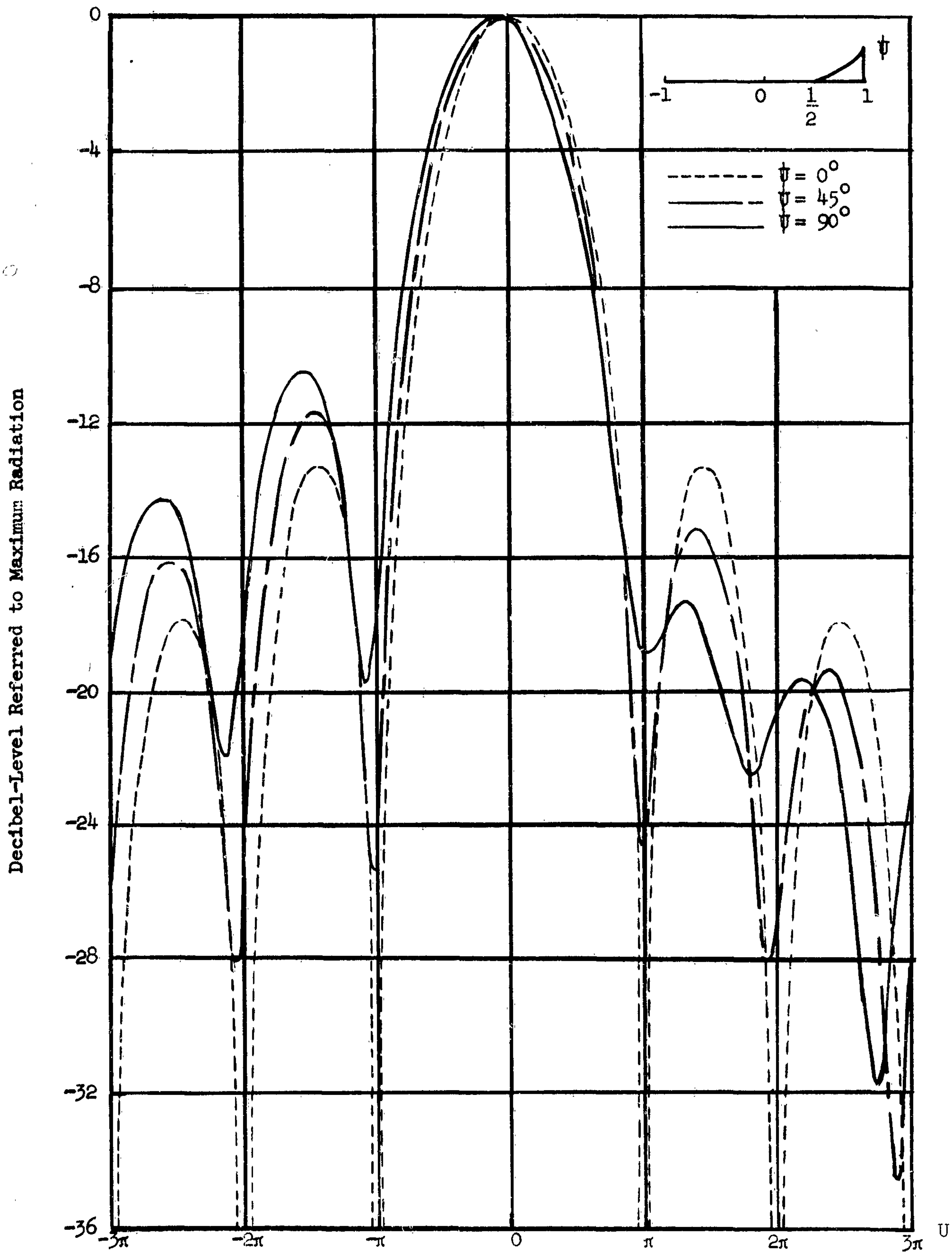


Fig. 11 - Radiation Patterns: Uniform Illumination and Quadratic Phase Advance on One End (Rectangular Aperture)

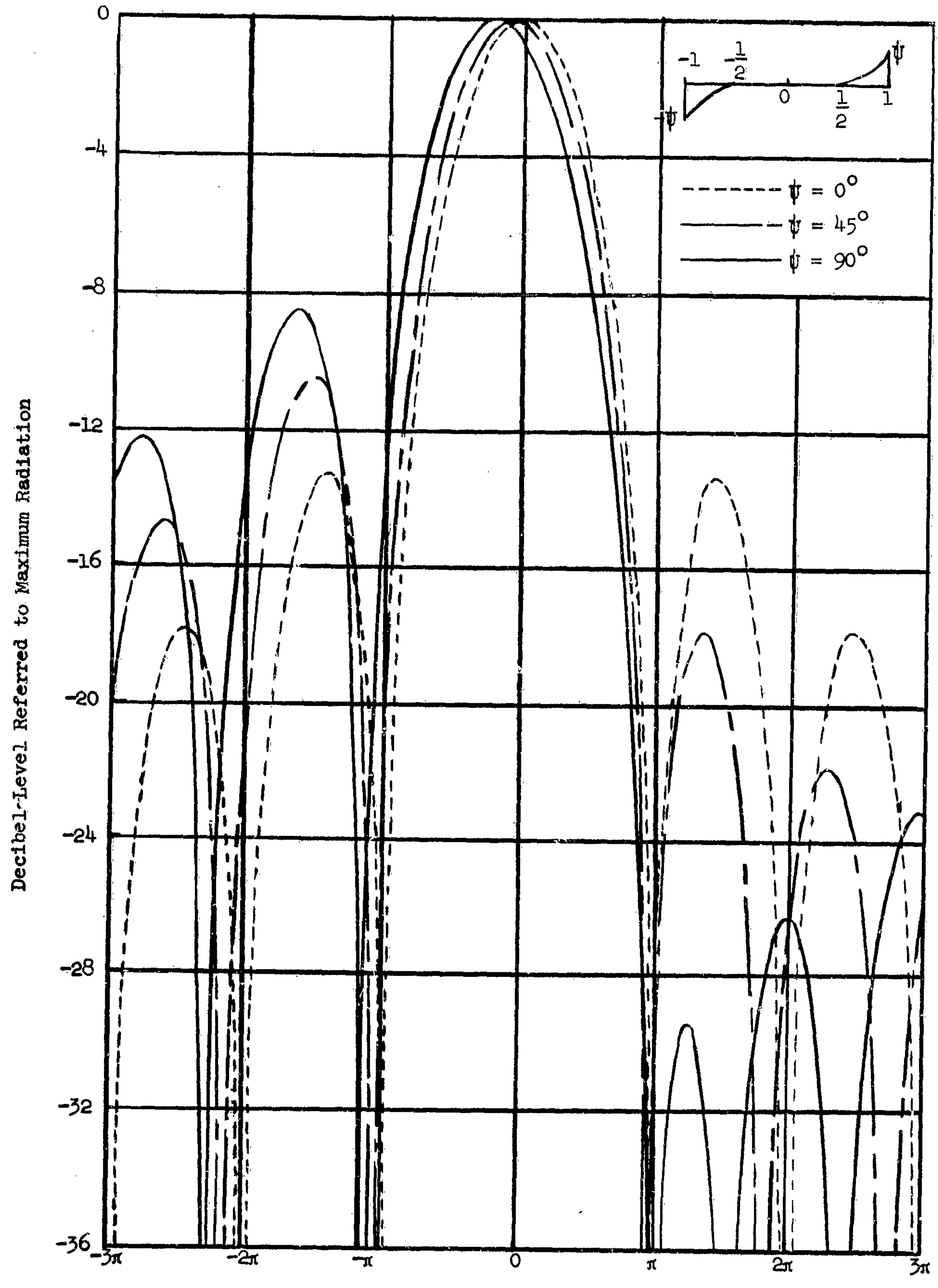


Fig. 12 - Radiation Patterns: Uniform Illumination and Antisymmetrical Quadratic Phase Deviation (Rectangular Aperture)

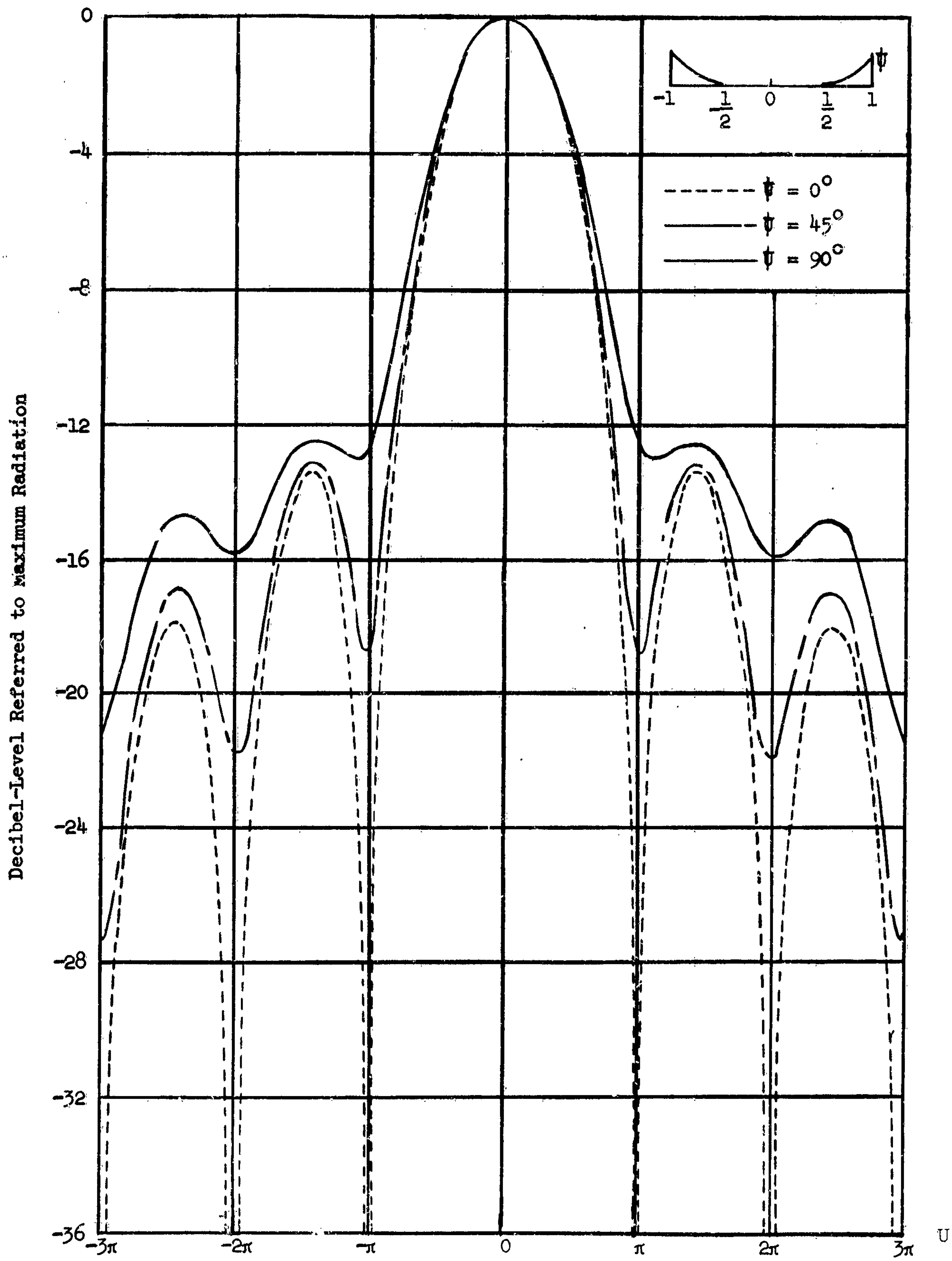


Fig. 13 - Radiation Patterns: Uniform Illumination and Symmetrical Quadratic Phase Advance (Rectangular Aperture)

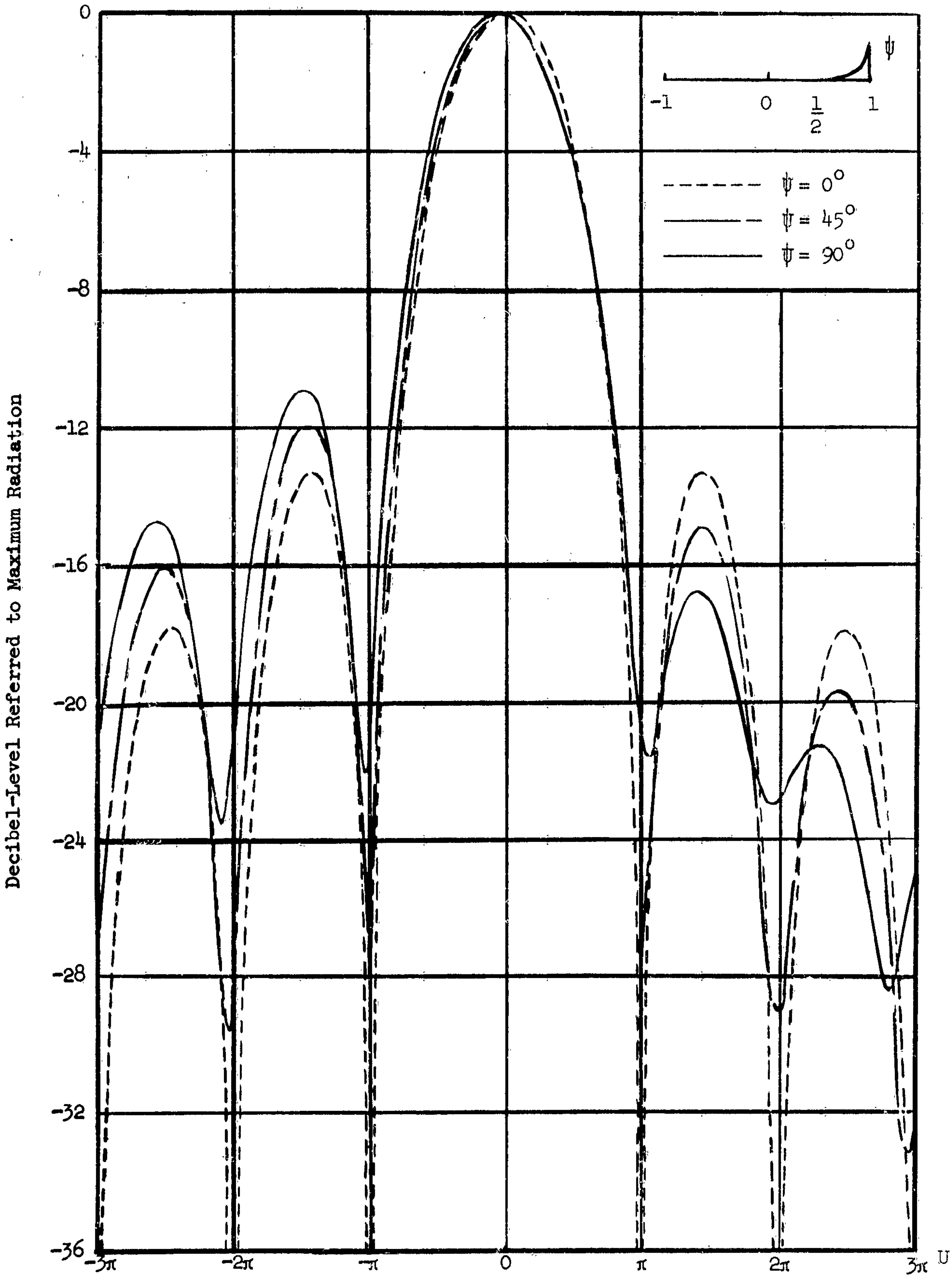


Fig. 14 - Radiation Patterns: Uniform Illumination and Cubic Phase Advance on One End (Rectangular Aperture)

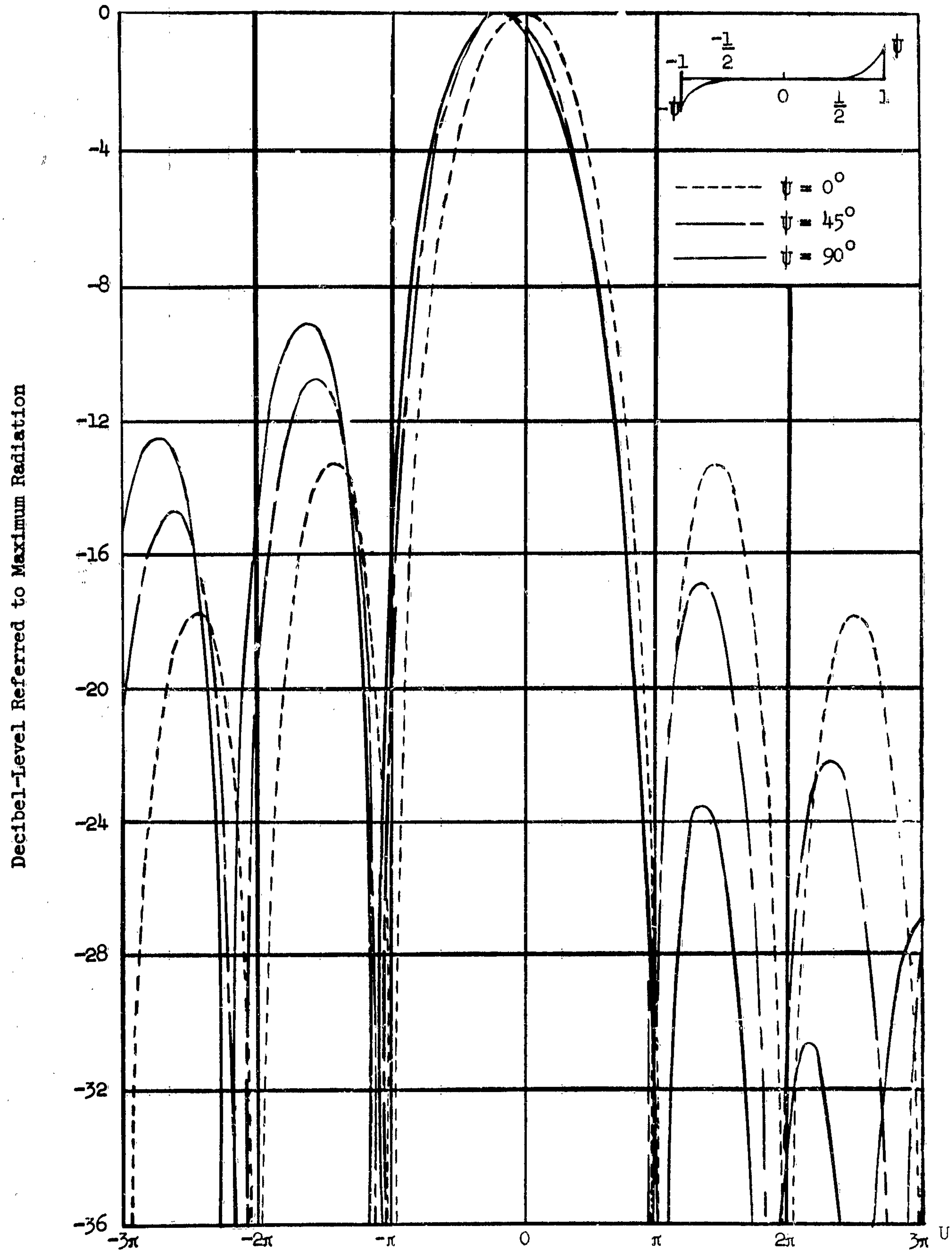


Fig. 15 - Radiation Patterns: Uniform Illumination and Antisymmetrical Cubic Phase Deviation (Rectangular Aperture)

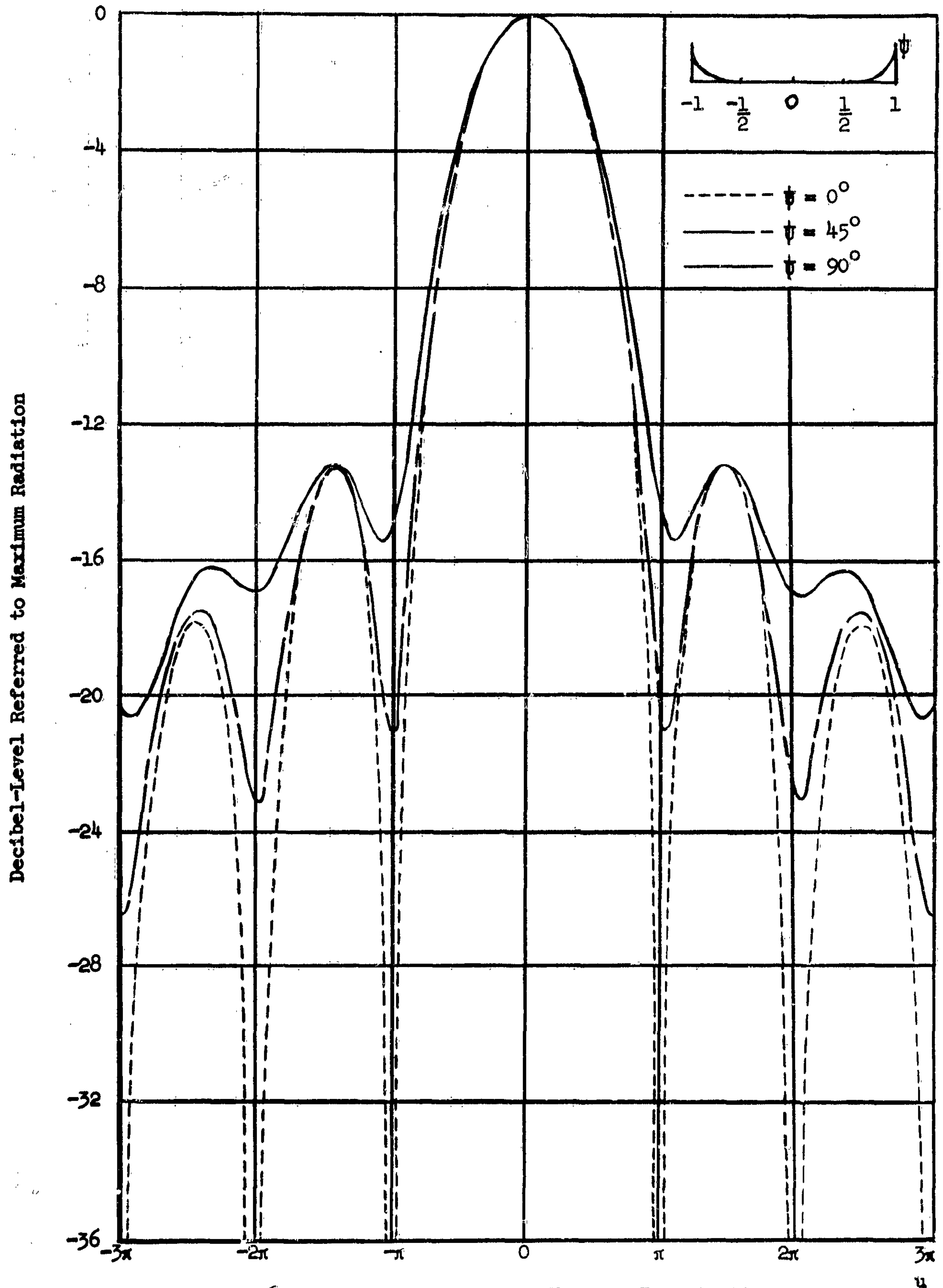


Fig. 16 - Radiation Patterns: Uniform Illumination and Symmetrical Cubic Phase Advance (Rectangular Aperture)

C. Cos^2(0.375πx) ILLUMINATION

The normalized radiation patterns for a rectangular aperture with $\cos^2(0.375\pi x)$ amplitude illumination and phase deviations of the following types are plotted in this section. (Phase deviations occur in the region $1/2 \leq |x| \leq 1$ only; and ψ represents the maximum error at the edge of the aperture.)

1. Linear Phase Deviation

   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 17

   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 18

   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 19

2. Step Phase Deviation

   a. Phase advance on one end ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 20

   b. Antisymmetrical deviation ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 21

   c. Symmetrical phase advance ($\psi = 30^\circ$, 45°, 60° and 90°) - Fig. 22

3. Staircase Phase Deviation (m = No. of steps)

   a. Phase advance on one end ($\psi = 90^\circ$; $m = 4$ and 6) - Fig. 23

   b. Antisymmetrical deviation ($\psi = 90^\circ$; $m = 4$ and 6) - Fig. 24

   c. Symmetrical phase advance ($\psi = 90^\circ$; $m = 4$ and 6) - Fig. 25

4. Quadratic Phase Deviation

   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 26

   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 27

   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 28

5. Cubic Phase Deviation

   a. Phase advance on one end ($\psi = 45^\circ$ and 90°) - Fig. 29

   b. Antisymmetrical deviation ($\psi = 45^\circ$ and 90°) - Fig. 30

   c. Symmetrical phase advance ($\psi = 45^\circ$ and 90°) - Fig. 31

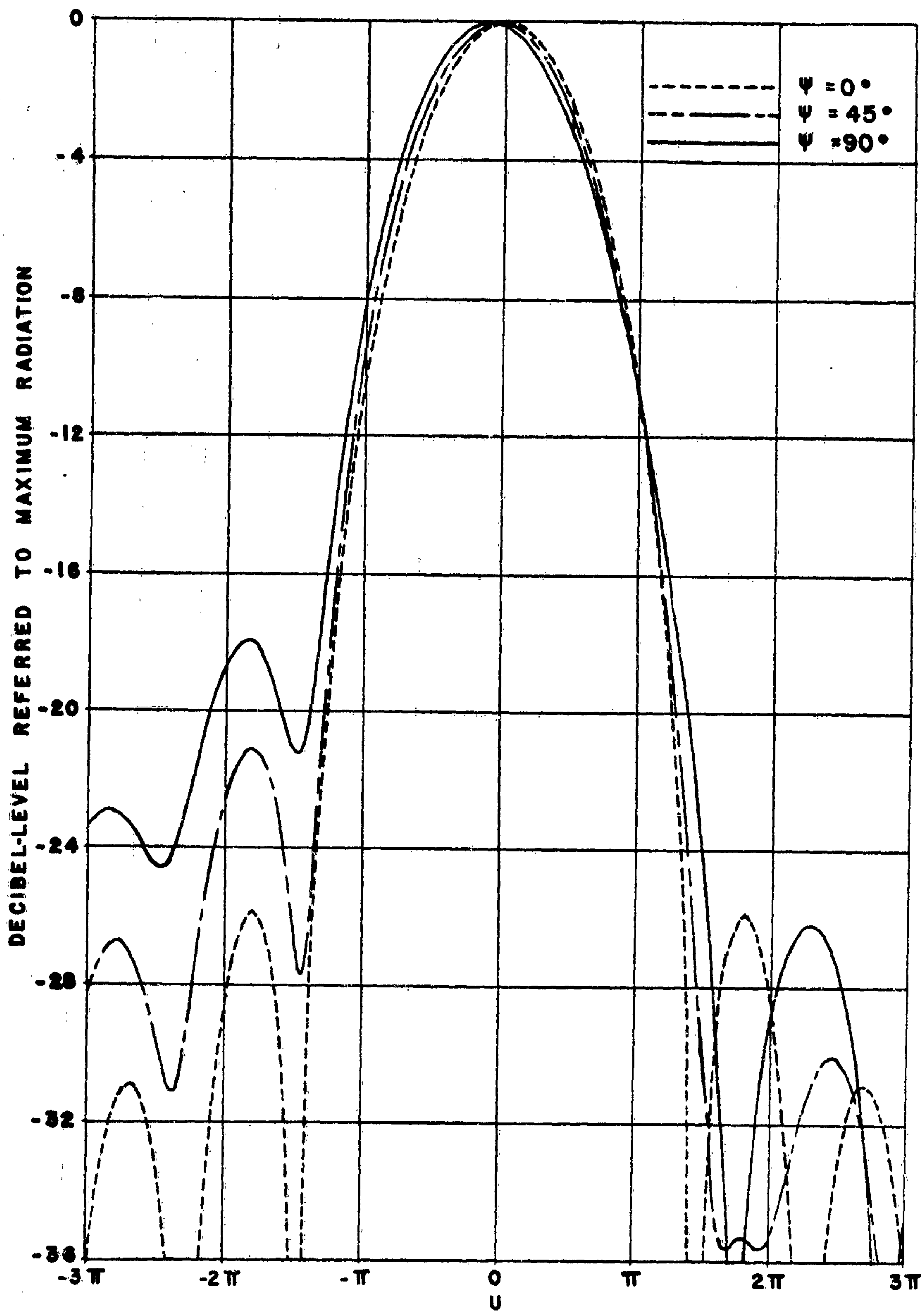
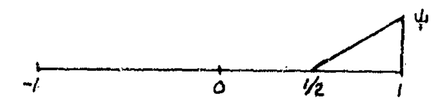


Fig. 17 - Radiation Patterns: $\text{Cos}^2(0.375\pi x)$ Illumination

and Linear Phase Advance On One End

(Rectangular Aperture)



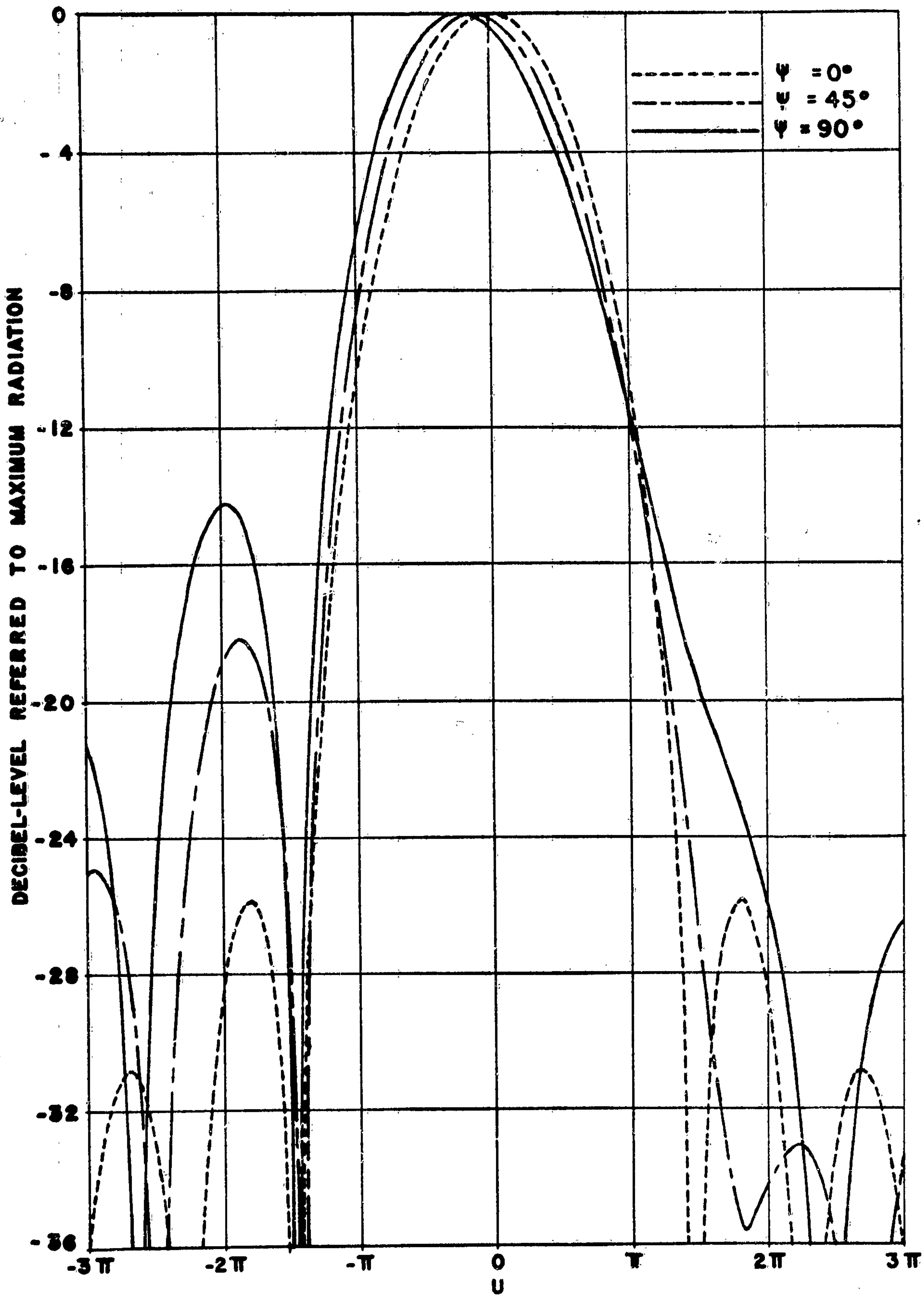
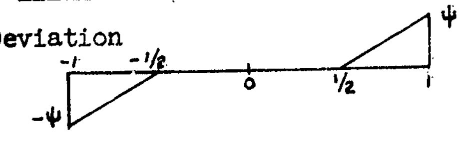


Fig. 18 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination

and Antisymmetrical Linear Phase Deviation

(Rectangular Aperture)



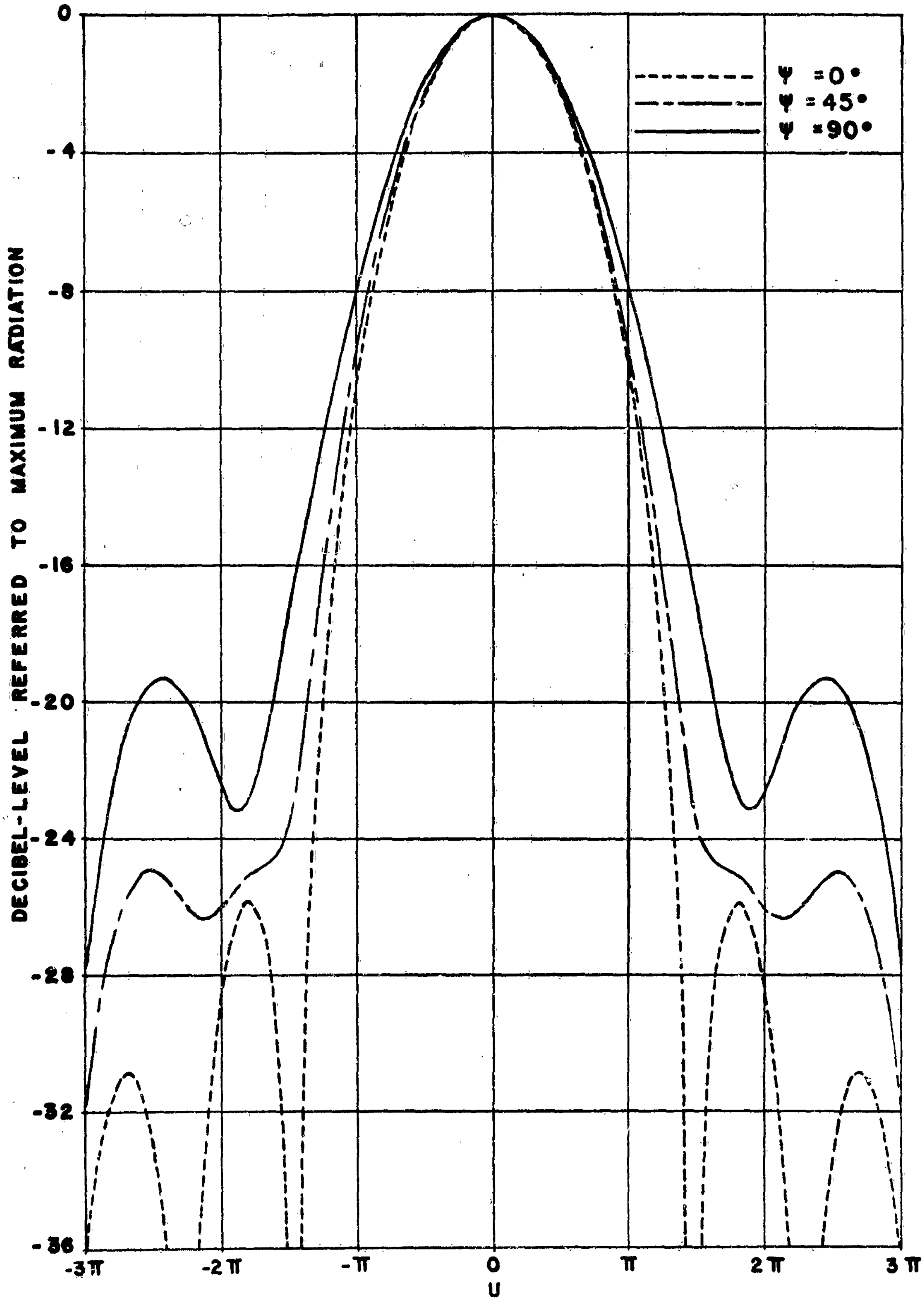


Fig. 19 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination and Symmetrical Linear Phase Advance (Rectangular Aperture)

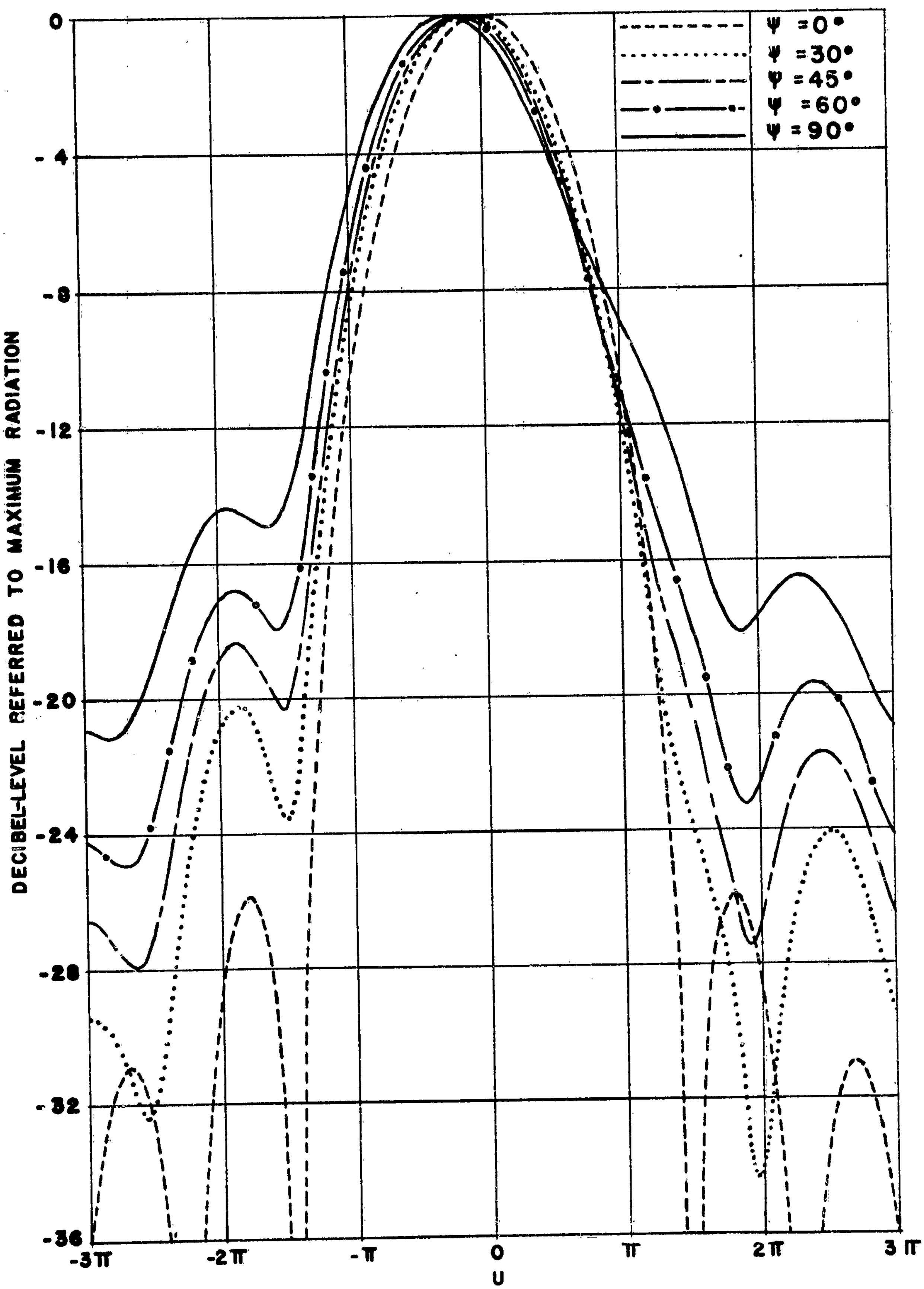
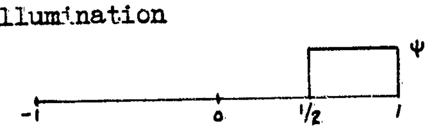


Fig. 20 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination
and Step Phase Advance on One End
(Rectangular Aperture)



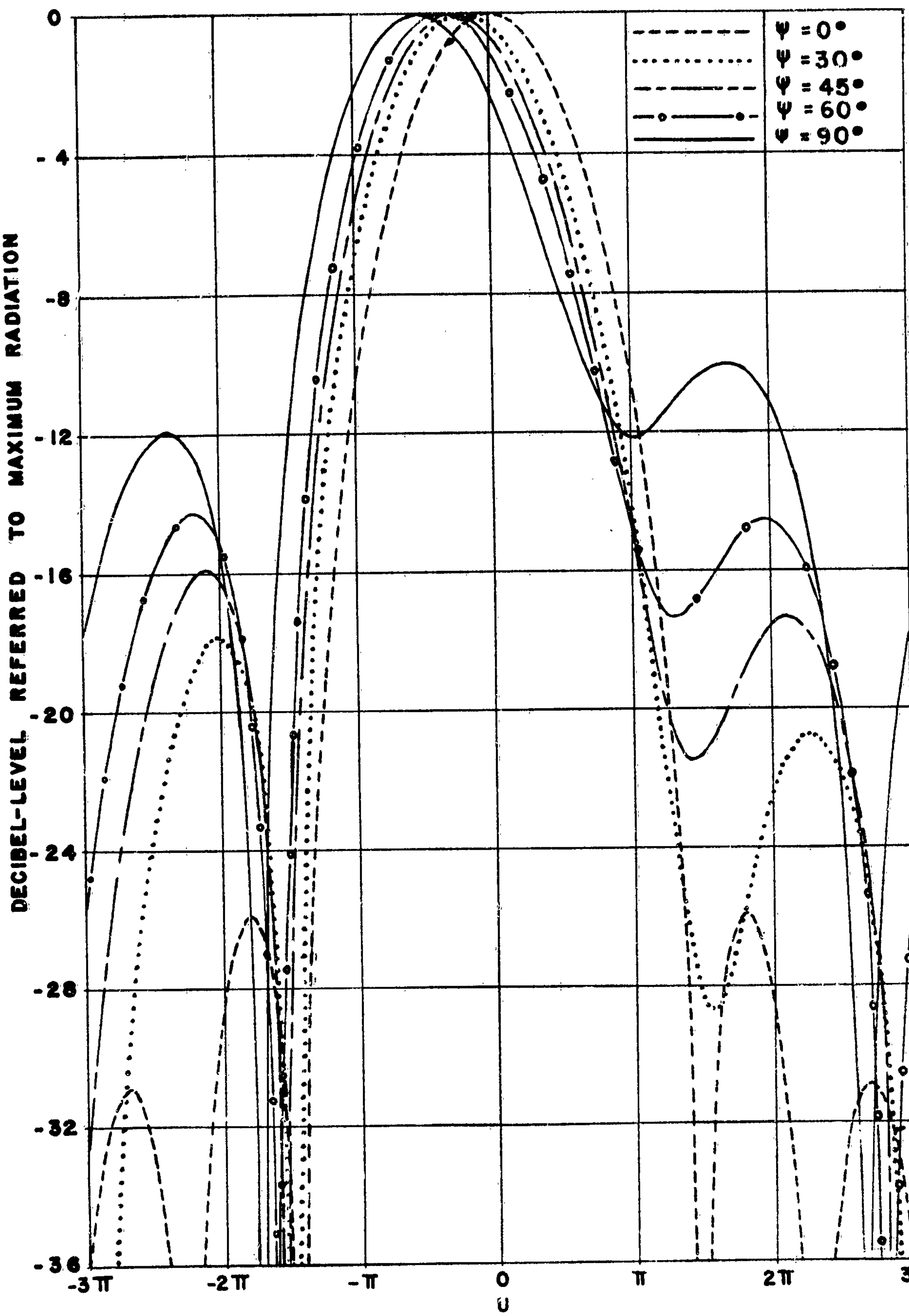
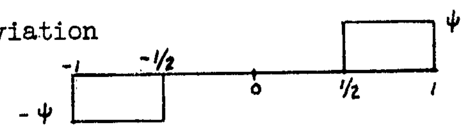


Fig. 21 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination and Antisymmetrical Step Phase Deviation (Rectangular Aperture)



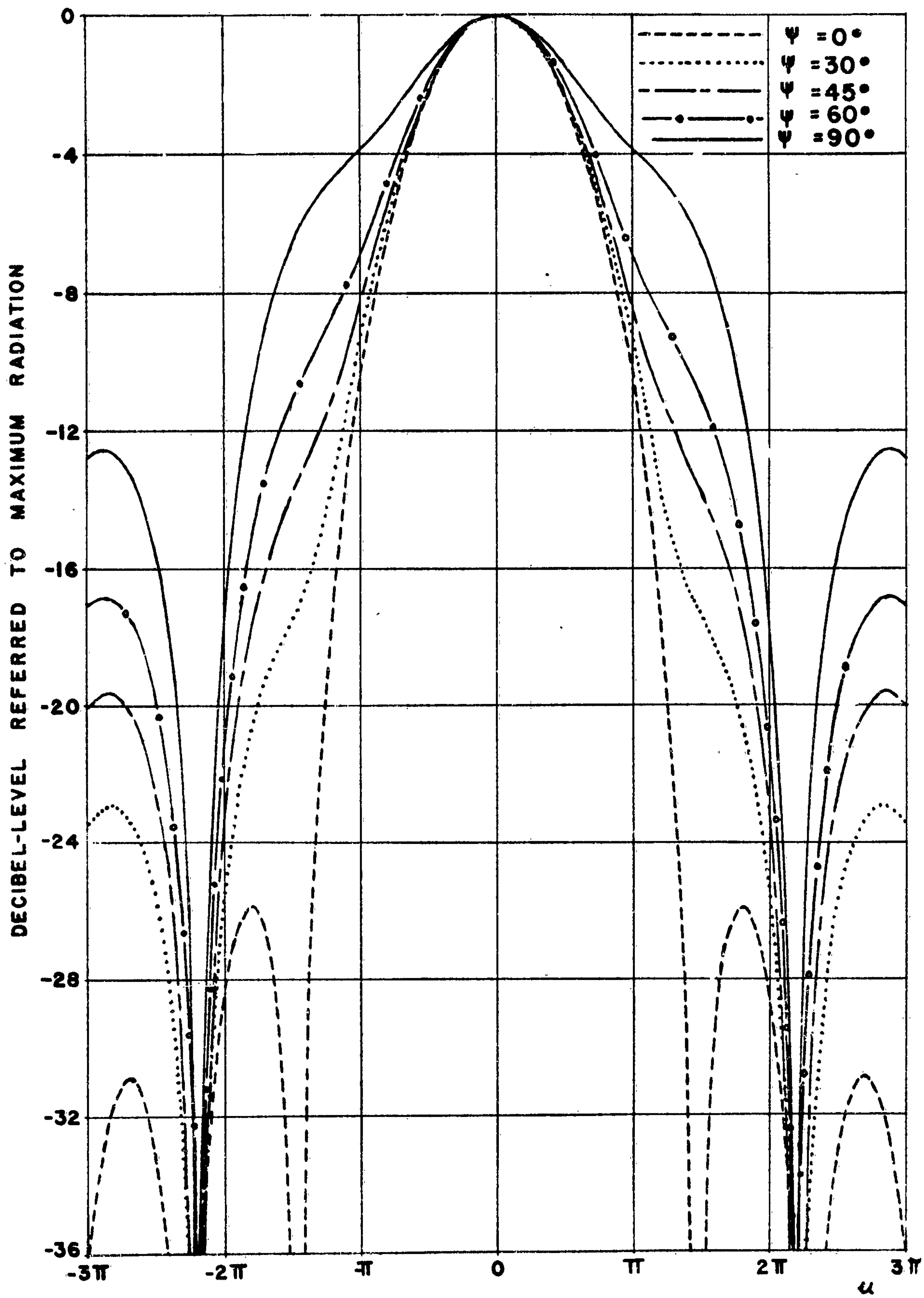
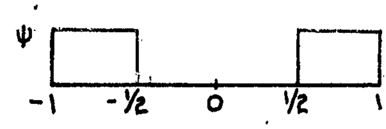


Fig. 22 - Radiation Patterns: $\text{Cos}^2\,(0.375\pi x)$ Illumination and Symmetrical Phase Advance (Rectangular Aperture)



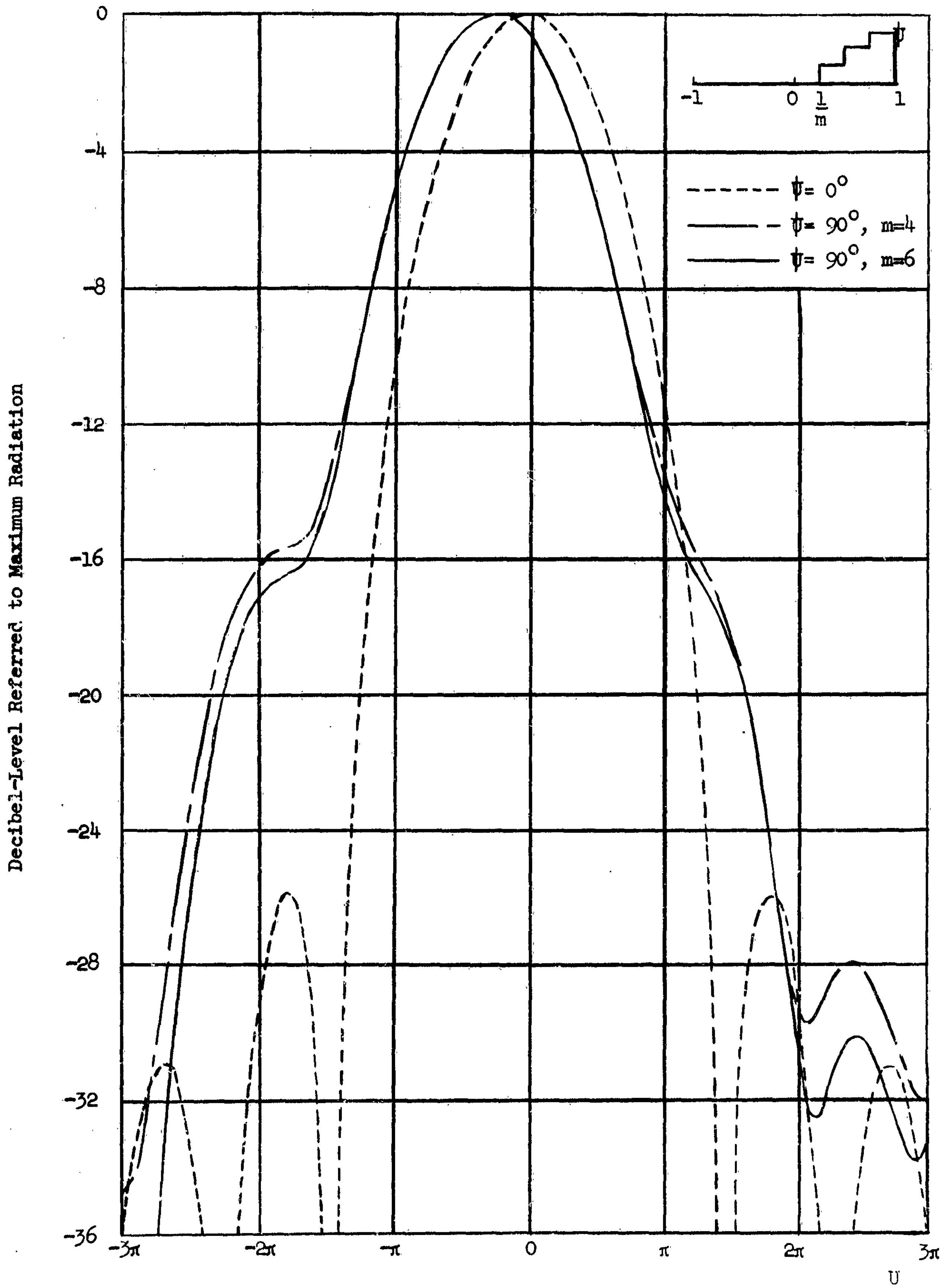


Fig. 23 - Radiation Patterns: $\cos^2 (0.375\pi x)$ Illumination and Staircase Phase Advance on One End (Rectangular Aperture)

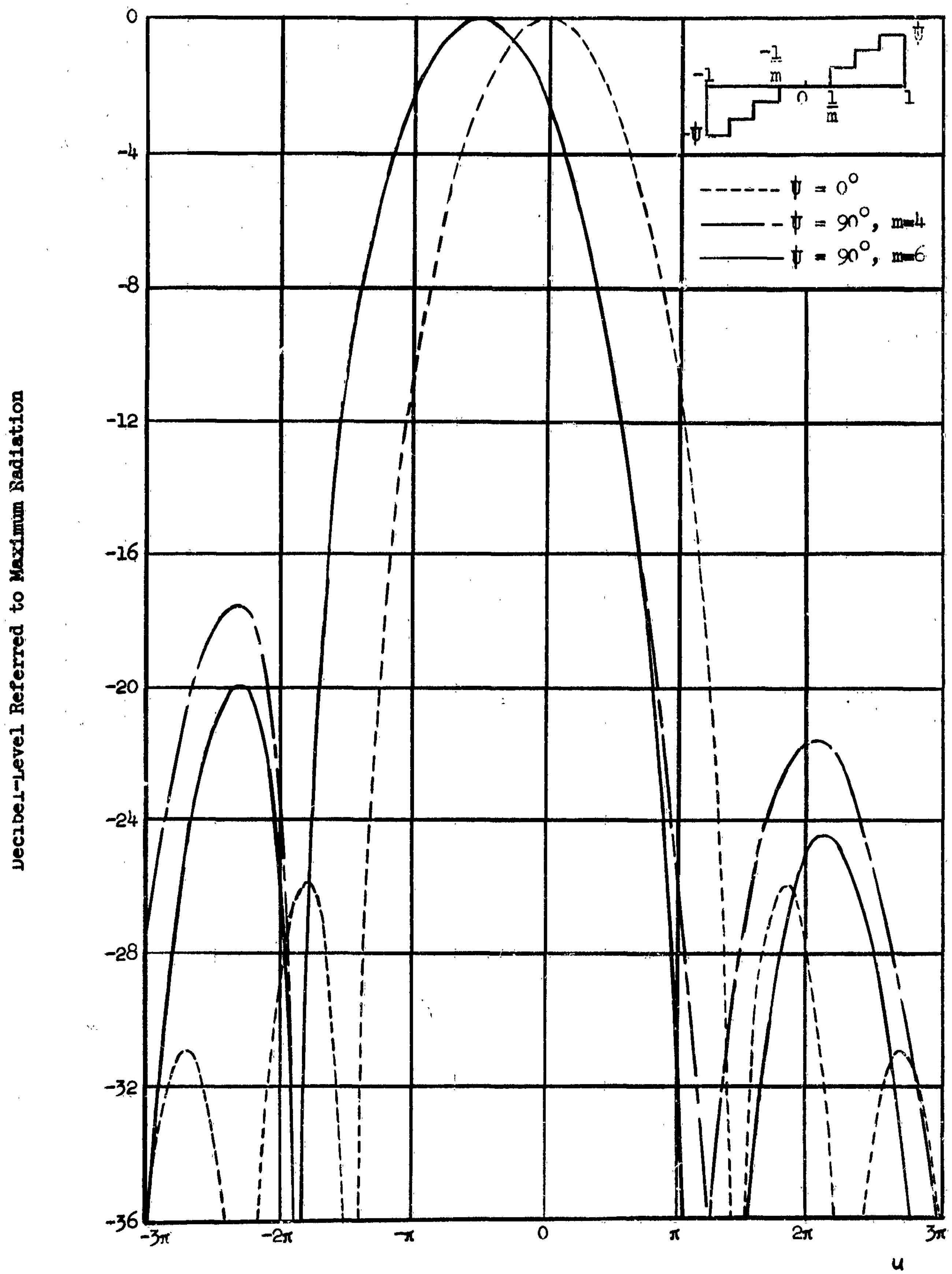


Fig. 24 - Radiation Patterns: Cos^2 (0.375πx) Illumination and Antisymmetrical Staircase Phase Deviation (Rectangular Aperture)

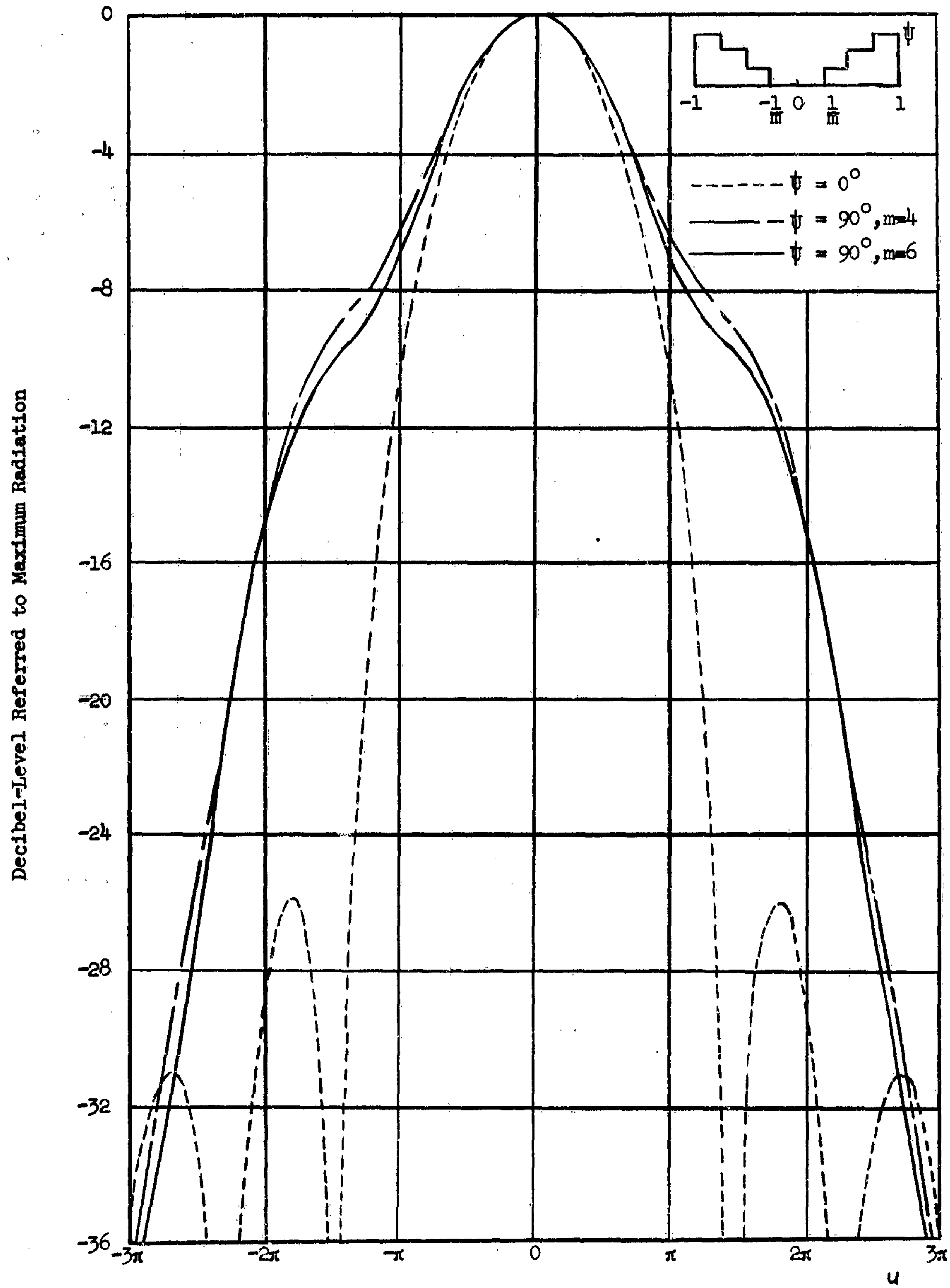


Fig. 25 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination and Symmetrical Staircase Phase Advance (Rectangular Aperture)

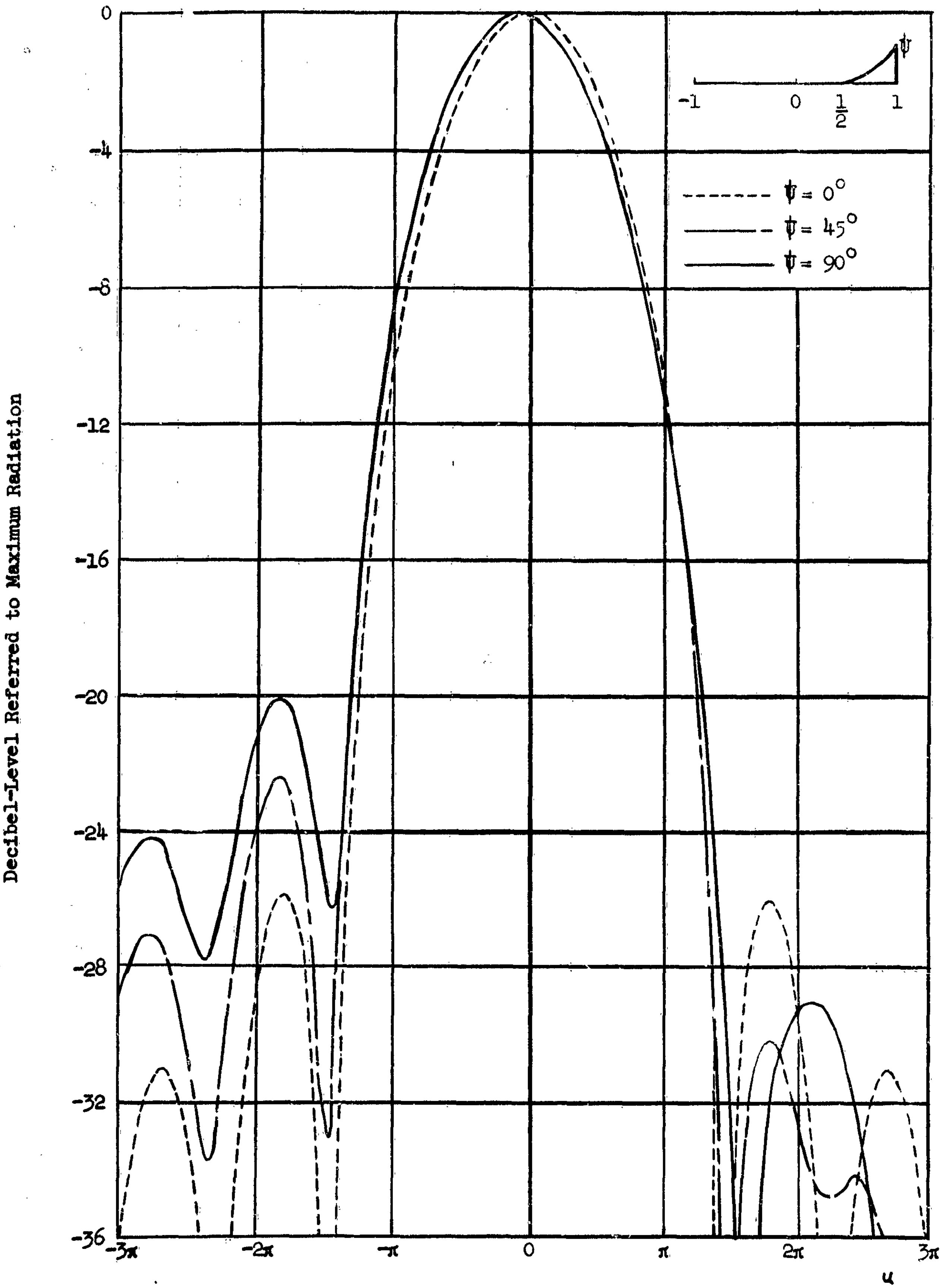


Fig. 26. - Radiation Patterns: Cos^2 (0.375πx) Illumination and Quadratic Phase Advance on One End (Rectangular Aperture)

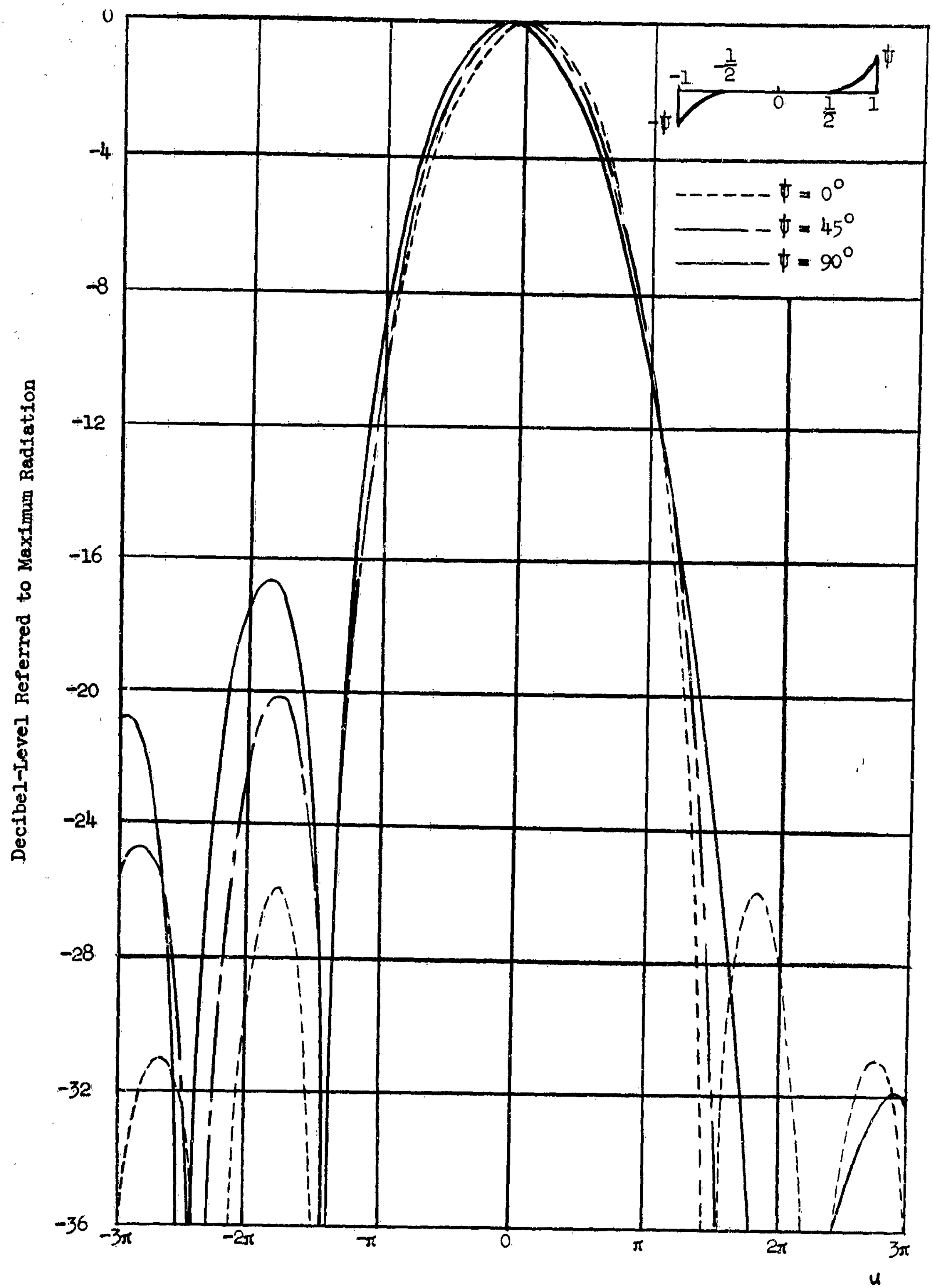


Fig. 27 - Radiation Patterns: Cos ^{2} (0.375πx) Illumination and Antisymmetrical Quadratic Phase Deviation (Rectangular Aperture)

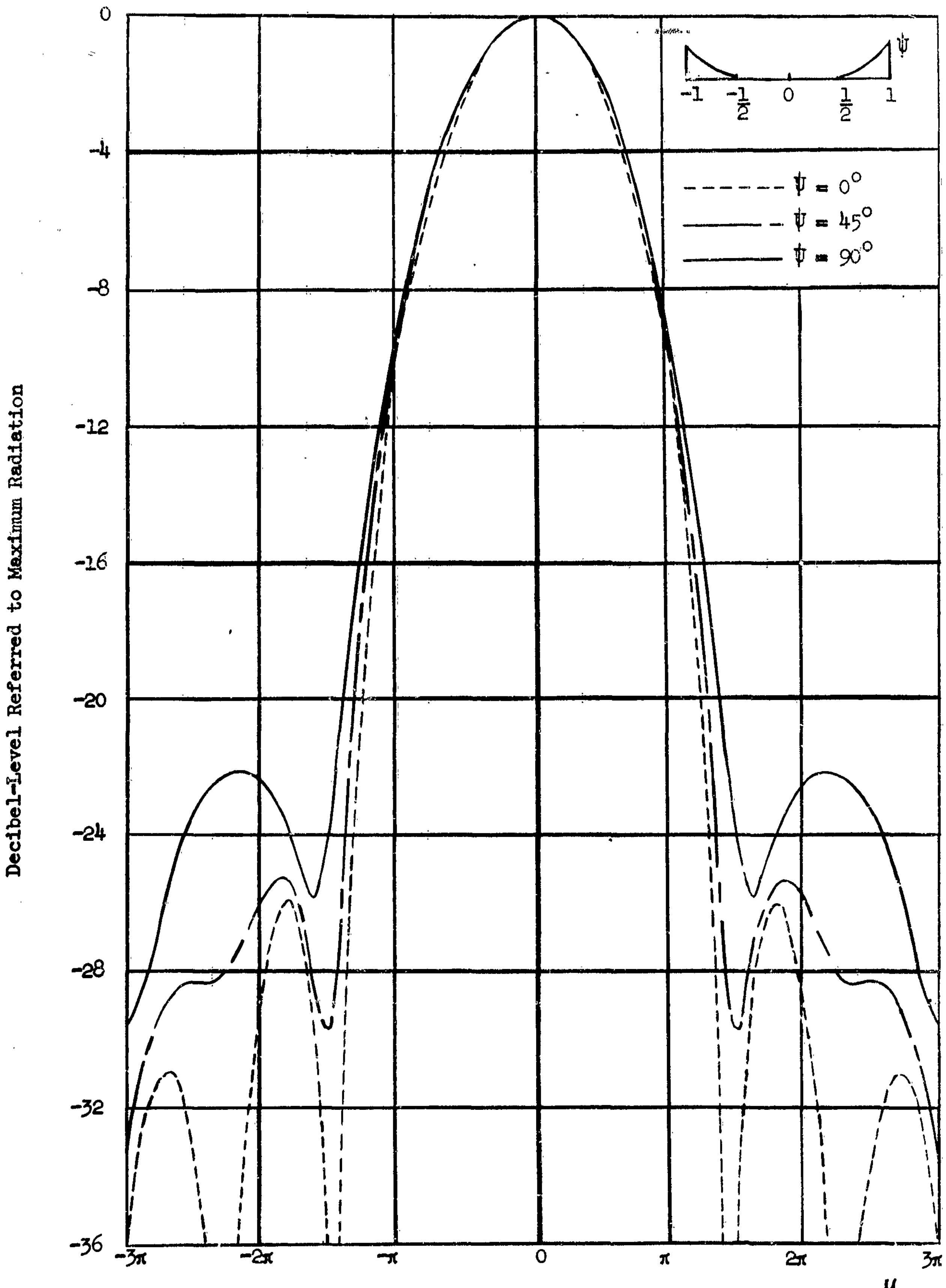


Fig. 28 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination and Symmetrical Quadratic Phase Advance (Rectangular Aperture)

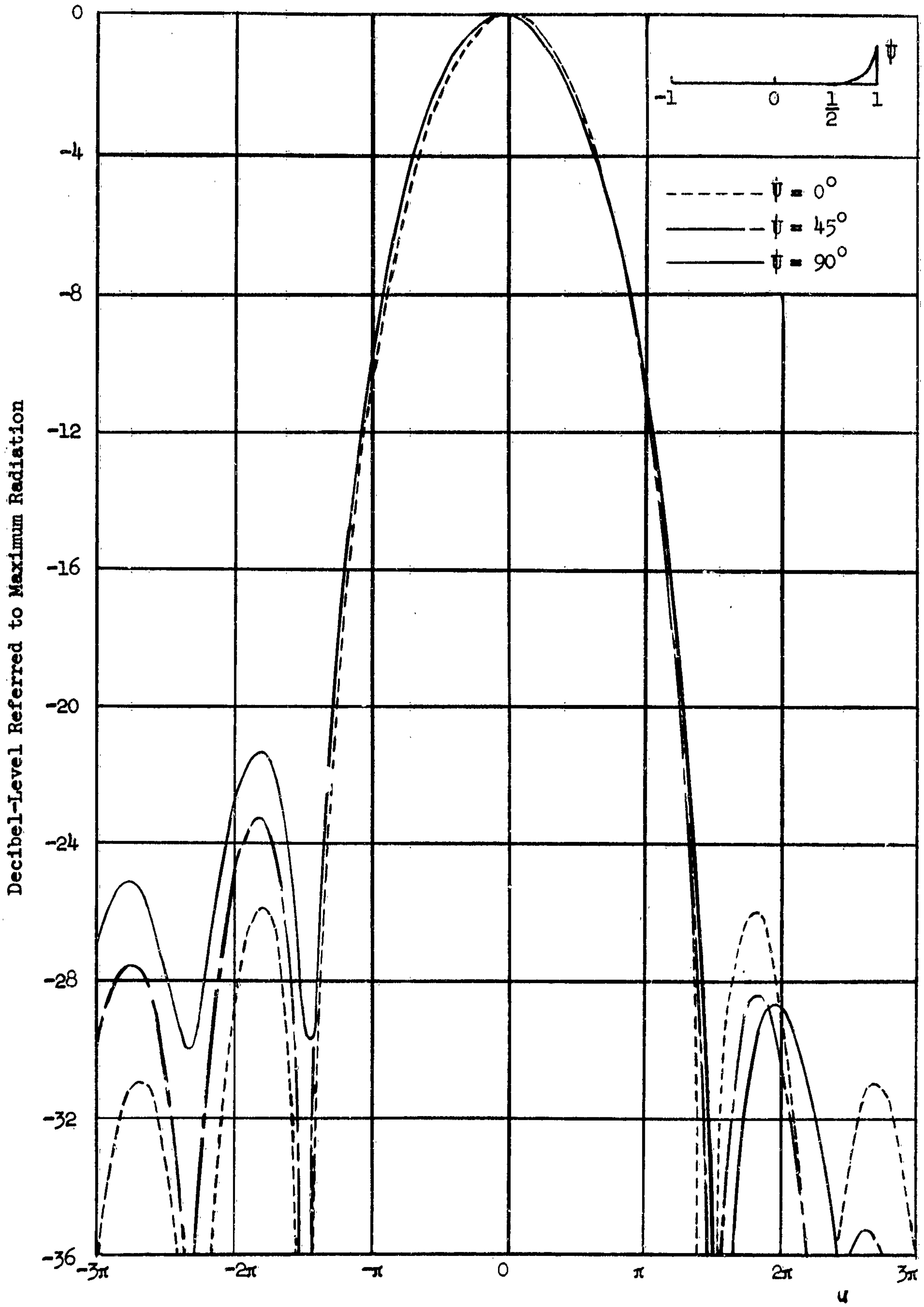


Fig. 29 - Radiation Patterns: $\cos^2(0.375\pi x)$ Illumination and Cubic Phase Advance on One End (Rectangular Aperture)

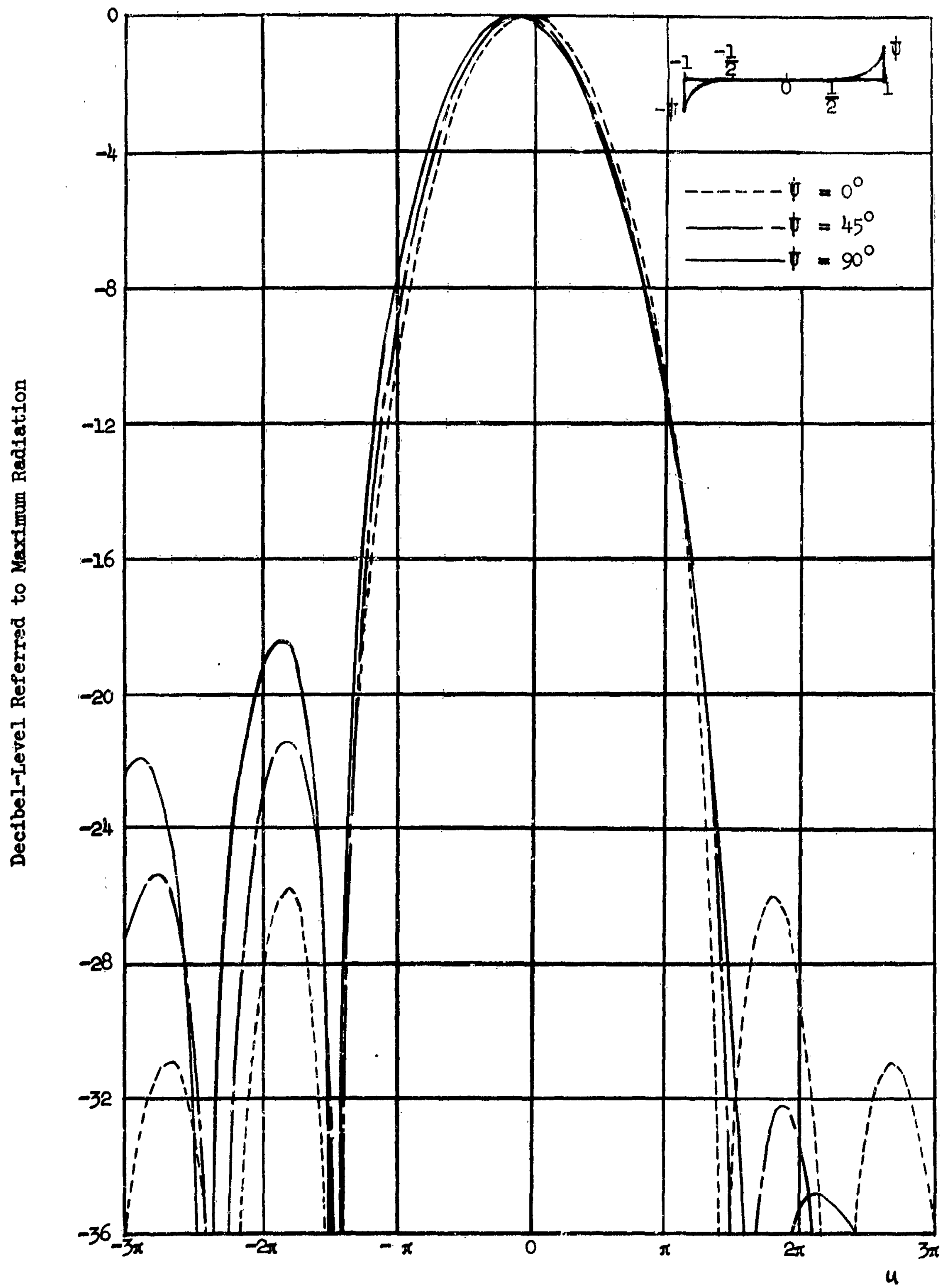


Fig. 30 - Radiation Patterns: $\cos^2$ (0.375πx) Illumination and Antisymmetrical Cubic Phase Deviation (Rectangular Aperture)

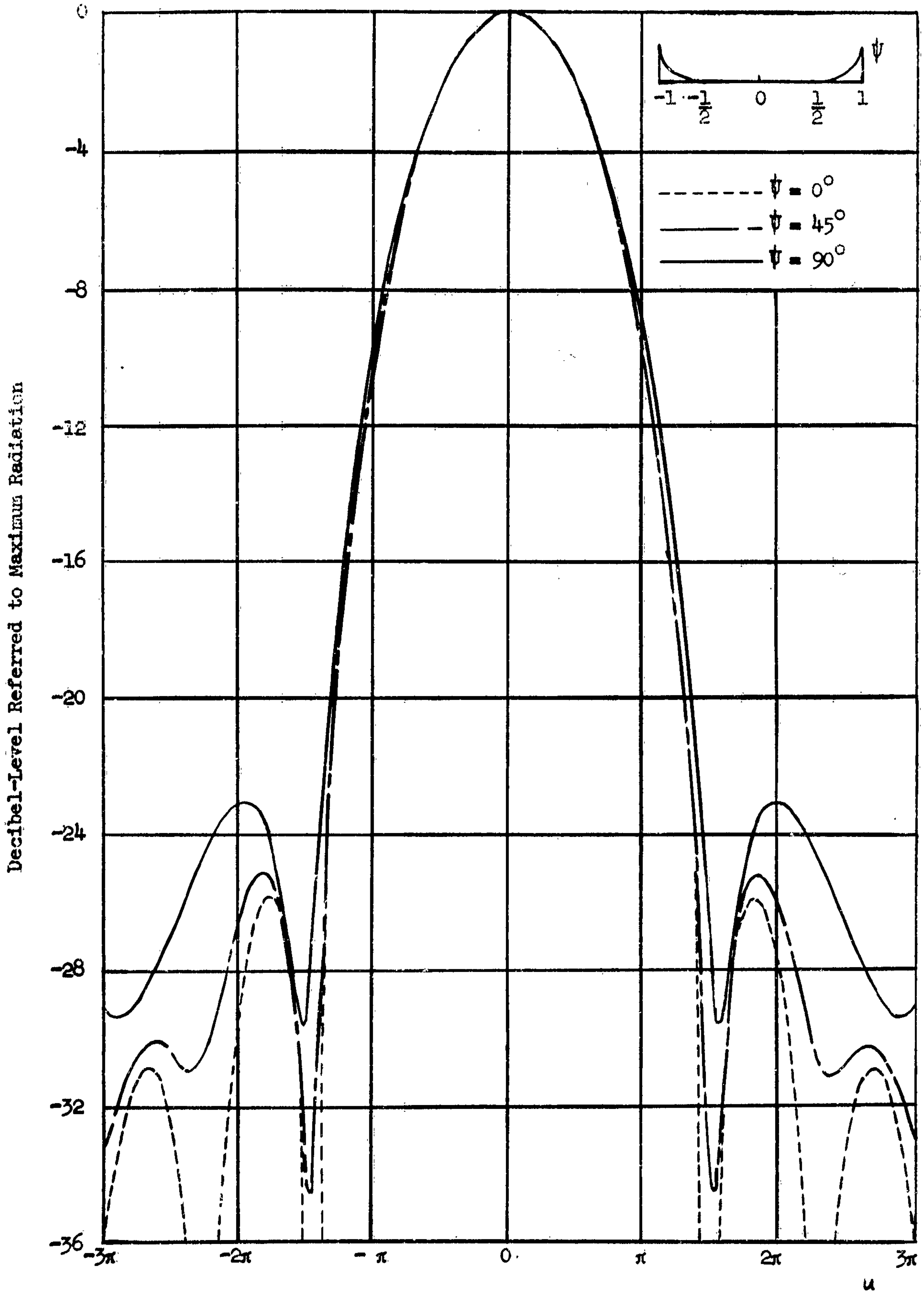
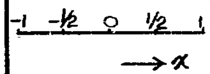
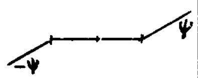
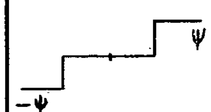
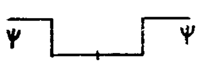


Fig. 31 - Radiation Patterns: $\cos^2$ (0.375πx) Illumination and Symmetrical Cubic Phase Advance (Rectangular Aperture)

D. TABULATION OF RESULTS

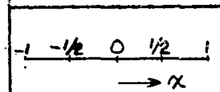
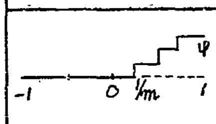
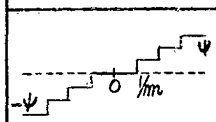
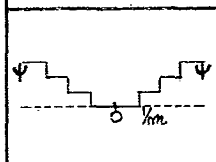
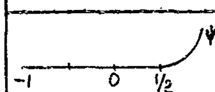
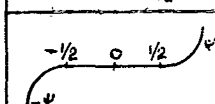
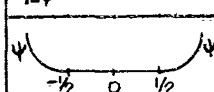
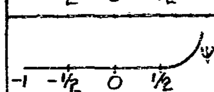
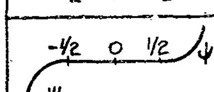
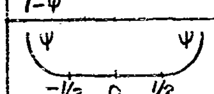
The important characteristics of the radiation patterns for a rectangular aperture with the different types of phase deviations considered are tabulated in Tables 2 to 5. Tables 2 and 3 are for the uniform-illumination case; and Tables 4 and 5 are for an amplitude illumination of the $\cos^2(0.375\pi x)$ type. Corresponding figure numbers are also given for easy reference.

Table 2 - Radiation-Pattern Characteristics: Uniform Amplitude Illumination (Rectangular Aperture)

Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\|g(u)\|$	Main-Lobe Beamwidth, Δu		Levels of first side lobes (-db)		Gain Factor G.F.	Fig. No.	Remarks (See footnotes)
				-3 db	-10 db	Left(u<0)	Right(u>0)			
	None	0	2.000	0.91π	1.48π	13.27	13.27	1.000	Shown in all following figures	
	45^o 90^o	-0.08π -0.16π	1.978 1.916	0.89π 0.90π	1.49π 1.56π	11.37 9.70	15.19 17.10	0.978 0.918	2	(a)
	45^o 90^o	-0.16π -0.30π	1.979 1.913	0.89π 0.89π	1.48π 1.51π	10.04 7.74	18.71 41.30	0.979 0.915	3	(b)18.45(u>0)
	45^o 90^o	0 0	1.937 1.757	0.90π 0.97π	1.54π 2.62π	12.46 12.49	12.46 12.49	0.938 0.772	4	(a)
	30^o 45^o 60^o 90^o	-0.09π -0.14π -0.20π -0.28π	1.978 1.950 1.912 1.804	0.89π 0.89π 0.89π 0.89π	1.48π 1.50π 1.54π 2.19π	11.98 11.28 10.64 9.31	13.52 12.93 11.98 9.02	0.978 0.951 0.914 0.814	5	(b)11.39(u>0)
	30^o 45^o 60^o 90^o	-0.19π -0.28π -0.38π -0.57π	1.979 1.952 1.916 1.814	0.88π 0.88π 0.89π 0.87π	1.47π 1.47π 1.46π 1.45π	11.59 10.84 10.19 9.05	15.43 16.75 18.15 22.11	0.979 0.953 0.918 0.823	6	(b)14.03(u>0) (b)12.43(u>0) (b)10.81(u>0) (b) 7.84(u>0)
	30^o 45^o 60^o 90^o	0 0 0 0	1.932 1.848 1.732 1.414	0.89π 0.90π 0.90π 0.96π	1.52π 1.61π 3.18π 3.47π	11.45 9.58 7.57 3.38	11.45 9.58 7.57 3.38	0.933 0.854 0.750 0.500	7	(c)13.19 (c) 9.43

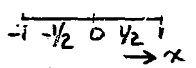
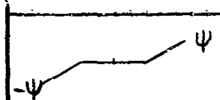
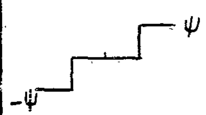
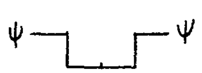
Remarks:
(a) Shoulders appear on the main lobe.
(b) Second side lobe higher than the first side lobe; figure is level of second side lobe in -db.
(c) 10-db beamwidth includes first minima and first side lobes; figure is level of second side lobe in -db.

Table 3 - Radiation-Pattern Characteristics: Uniform Amplitude Illumination (Rectangular Aperture)

Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\lvert g(u)\rvert$	Main-Lobe Beamwidth, Δu -3db	-10 db	Levels of first side lobes (-db) Left(u<0)	Right(u>0)	Gain Factor G.F.	Fig. No.	Remarks (See footnotes)
	None	0	2.000	0.91π	1.48π	13.27	13.27	1.000		Shown in all following figures
	90°	-0.26π	1.908	0.89π	1.54π	10.22	11.25	0.911	8	m=4
	90°	-0.26π	1.926	0.90π	1.53π	10.67	11.48	0.927	8	m=6
	90°	-0.53π	1.969	0.88π	1.47π	11.06	15.83	0.968	9	m=4 (b)15.27(u>0)
	90°	-1.02π	1.987	0.88π	1.48π	11.83	14.91	0.986	9	m=6
	90°	0	1.672	0.92π	3.38π	6.35	6.35	0.699	10	m=4 (a)17.12 (2nd. S.L.)
	90°	0	1.724	0.90π	3.31π	7.02	7.02	0.743	10	m=6 (a)17.92 (2nd S.L.)
	45° 90°	-0.05π -0.10π	1.983 1.986	0.90π 0.90π	1.50π 1.54π	11.71 10.45	15.13 17.32	0.983 0.985	11	Quadratic deviation
	45° 90°	-0.12π -0.21π	1.977 1.915	0.90π 0.90π	1.48π 1.52π	10.45 8.47	17.92 29.30	0.977 0.917	12	Quadratic deviation (c)23.02(u>0)
	45° 90°	0 0	1.956 1.833	0.90π 0.96π	1.51π 1.70π	13.10 12.46	13.10 12.46	0.956 0.840	13	Quadratic deviation
	45° 90°	-0.05π -0.08π	1.986 1.946	0.88π 0.90π	1.48π 1.52π	11.95 10.86	14.85 16.70	0.985 0.946	14	Cubic deviation
	45° 90°	-0.09π -0.16π	1.979 1.920	0.90π 0.92π	1.50π 1.53π	10.85 9.08	16.90 23.46	0.979 0.922	15	Cubic deviation
	45° 90°	0 0	1.966 1.872	0.89π 0.94π	1.51π 1.63π	13.22 13.15	13.22 13.15	0.966 0.876	16	Cubic deviation

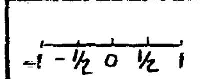
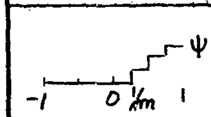
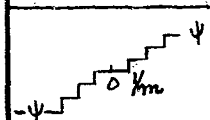
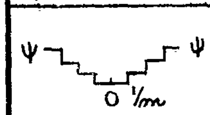
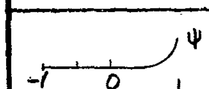
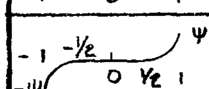
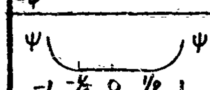
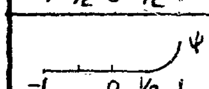
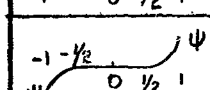
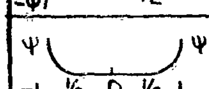
(a) Shoulders appear on the main lobe. (b) Second side lobe higher than first side lobe; figure is level of second side lobe in -db. (c) Third side lobe higher than first and second; figure is level of third side lobe in -db.

Table 4 - Radiation-Pattern Characteristics: $\cos^2(0.375\pi x)$ Amplitude Illumination (Rectangular Aperture)

Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\lvert g(u)\rvert$	Main-Lobe Beamwidth, Δu		Levels of first side lobes (-db)		Gain Factor G.F.	Fig. No.	Remarks (See footnotes)
				-3 db	-10 db	Left(u<0)	Right(u>0)			
	None	0	1.300	1.14π	1.96π	25.90	25.90	0.848	Shown in all following figures	
	45° 90°	-0.05π -0.12π	1.293 1.270	1.14π 1.17π	1.98π 2.05π	21.06 17.94	35.40 26.10	0.838 0.809	17	(b)30.05 (b)25.39
	45° 90°	-0.12π -0.23π	1.290 1.260	1.14π 1.17π	1.98π 2.05π	18.19 14.21	33.05 26.34	0.835 0.796	18	(b)31.16 (a)
	45° 90°	0 0	1.278 1.220	1.16π 1.24π	2.02π 2.25π	24.91 19.30	24.91 19.30	0.819 0.746	19	(a)
	30° 45° 60° 90°	-0.09π -0.12π -0.17π -0.25π	1.288 1.273 1.247 1.183	1.14π 1.14π 1.16π 1.21π	1.98π 2.04π 2.09π 2.38π	20.23 18.27 16.80 14.42	24.09 21.64 19.67 16.50	0.832 0.813 0.780 0.702	20	(a) (a) (a) (a)
	30° 45° 60° 90°	-0.17π -0.26π -0.35π -0.52π	1.284 1.267 1.239 1.166	1.14π 1.14π 1.14π 1.14π	1.97π 1.97π 2.00π 2.09π	17.85 15.90 14.28 11.94	20.75 17.35 14.57 10.03	0.827 0.805 0.770 0.682	21	
	30° 45° 60° 90°	0 0 0 0	1.262 1.222 1.152 0.981	1.15π 1.17π 1.25π 1.62π	2.06π 2.25π 2.77π 3.50π	22.93 19.64 16.90 12.53	22.93 19.64 16.90 12.53	0.799 0.749 0.666 0.483	22	(a) (a) (a) (a)

Remarks: (a) Shoulders appear on the main lobe.
(b) Second side lobe higher than the first side lobe; figure is level of second side lobe in -db.

Table 5 - Radiation-Pattern Characteristics: $\cos^2(0.375\pi x)$ Amplitude Illumination (Rectangular Aperture)

Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\lvert g(u)\rvert$	Main-Lobe Beamwidth, Δu		Levels of first side lobes (-db)		Gain Factor G.F.	Fig. No.	Remarks (See footnotes)
				-3db	-10db	Left(u<0)	Right(u>0)			
	None	0	1.300	1.14π	1.96π	25.90	25.90	0.848	Shown in all following figures	
	90°	-0.25π	1.251	1.15π	2.07π	> 36	28.00	0.785	23	m=4 (a)
	90°	-0.25π	1.261	1.15π	2.07π	> 36	30.23	0.797	23	m=6 (a)
	90°	-0.51π	1.278	1.12π	1.96π	17.50	21.55	0.819	24	m=4
	90°	-0.50π	1.291	1.07π	1.95π	19.92	24.51	0.836	24	m=6
	90°	0	1.132	1.28π	3.10π	-	-	0.643	25	m=4 (a)
	90°	0	1.159	1.23π	2.99π	-	-	0.674	25	m=6 (a)
	45° 90°	-0.03π -0.07π	1.295 1.282	1.14π 1.15π	1.97π 2.00π	22.36 20.10	30.05 29.10	0.841 0.825	26	Quadratic deviation
	45° 90°	-0.06π -0.13π	1.293 1.272	1.14π 1.32π	1.97π 2.02π	20.13 16.68	- 31.85	0.840 0.811	27	Quadratic deviation
	45° 90°	0 0	1.288 1.256	1.15π 1.20π	1.99π 2.09π	25.22 24.02	25.22 24.02	0.832 0.791	28	Quadratic deviation
	45° 90°	-0.02π -0.05π	1.296 1.287	1.14π 1.16π	1.97π 1.98π	23.38 21.37	28.40 28.69	0.842 0.830	29	Cubic deviation
	45° 90°	-0.05π -0.10π	1.294 1.278	1.15π 1.17π	1.97π 2.03π	21.43 18.42	32.18 34.71	0.841 0.819	30	Cubic deviation
	45° 90°	0 0	1.292 1.270	1.15π 1.18π	1.98π 2.05π	25.20 23.07	25.20 23.07	0.838 0.809	31	Cubic deviation

(a) Shoulders appear on the main lobe.

II. THE CIRCULAR APERTURE

A. INTRODUCTION

With appropriate approximations[1] the normalized radiation-pattern function for a circular aperture with a symmetrical field distribution can be reduced to

$$g(u) = \int_0^1 f(\rho)\, J_0\,(u\rho)\, \rho\, d\rho$$

where $f(\rho)$ is the circularly symmetrical aperture field distribution function; ρ is radial dimension normalized with respect to the radius $d/2$ of the aperture; and $u = (\pi d/\lambda) \sin \theta$, θ being the azimuth angle. The gain factor is expressible as

$$G.F. = 2\, \frac{\left(\max. \left| \int_0^1 f(\rho)\, J_0\,(u\rho)\, \rho\, d\rho \right| \right)^2}{\int_0^1 \left| f(\rho) \right|^2 \rho\, d\rho}$$

Important characteristics of the radiation patterns for circular apertures with constant phase and uniform and $\cos^2(0.375\pi\rho)$ types of amplitude illumination are tabulated in Table 6.

[1]For details, see D. K. Cheng and R. E. Gildersleeve, Report No. EE276-564F Final Report for Contract No. AF 30(602)-924, Syracuse University Research Institute; 31 Jaunary 1956.

Table 6

Radiation-Pattern Characteristics with Constant Aperture Phase (Circular Aperture)

	$f(\rho) = 1$	$f(\rho) = \cos^2(0.375\pi\rho)$
Max. $\lvert g(u) \rvert$	0.5000	0.2463
3-db Beamwidth	1.02π	1.24π
10-db Beamwidth	1.73π	2.13π
1st. Side-Lobe Level	-17.58 db	-28.00 db
Gain Factor	1.000	0.800

In following sections, the radiation patterns for certain special types of aperture phase deviation are presented graphically. In each case, the pattern for the corresponding amplitude illumination without phase deviation is sketched in for comparison. The important characteristics are collected and tabulated in Tables 7 and 8.

B. UNIFORM ILLUMINATION

The normalized radiation patterns for a circular aperture with uniform amplitude illumination and circularly symmetrical phase deviations of the following types are plotted in this section. (Phase deviations occur in the region $1/2 \leq \rho \leq 1$ only; and ψ represents the maximum error at the edge of the aperture.)

1. Linear Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 32
2. Quadratic Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 33
3. Cubic Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 34

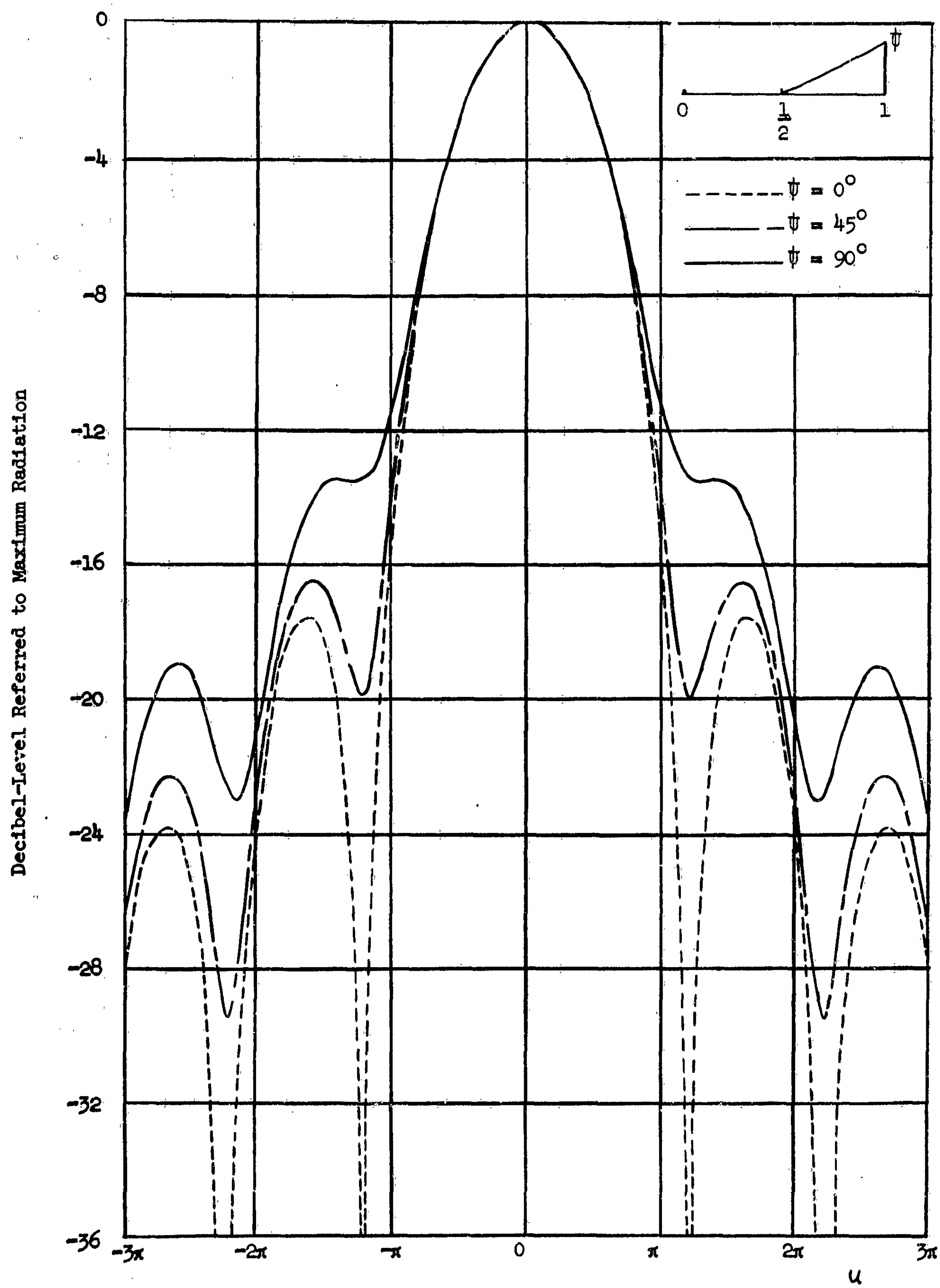


Fig. 32 - Radiation Patterns: Uniform Illumination and Linear Phase Deviation (Circular Aperture)

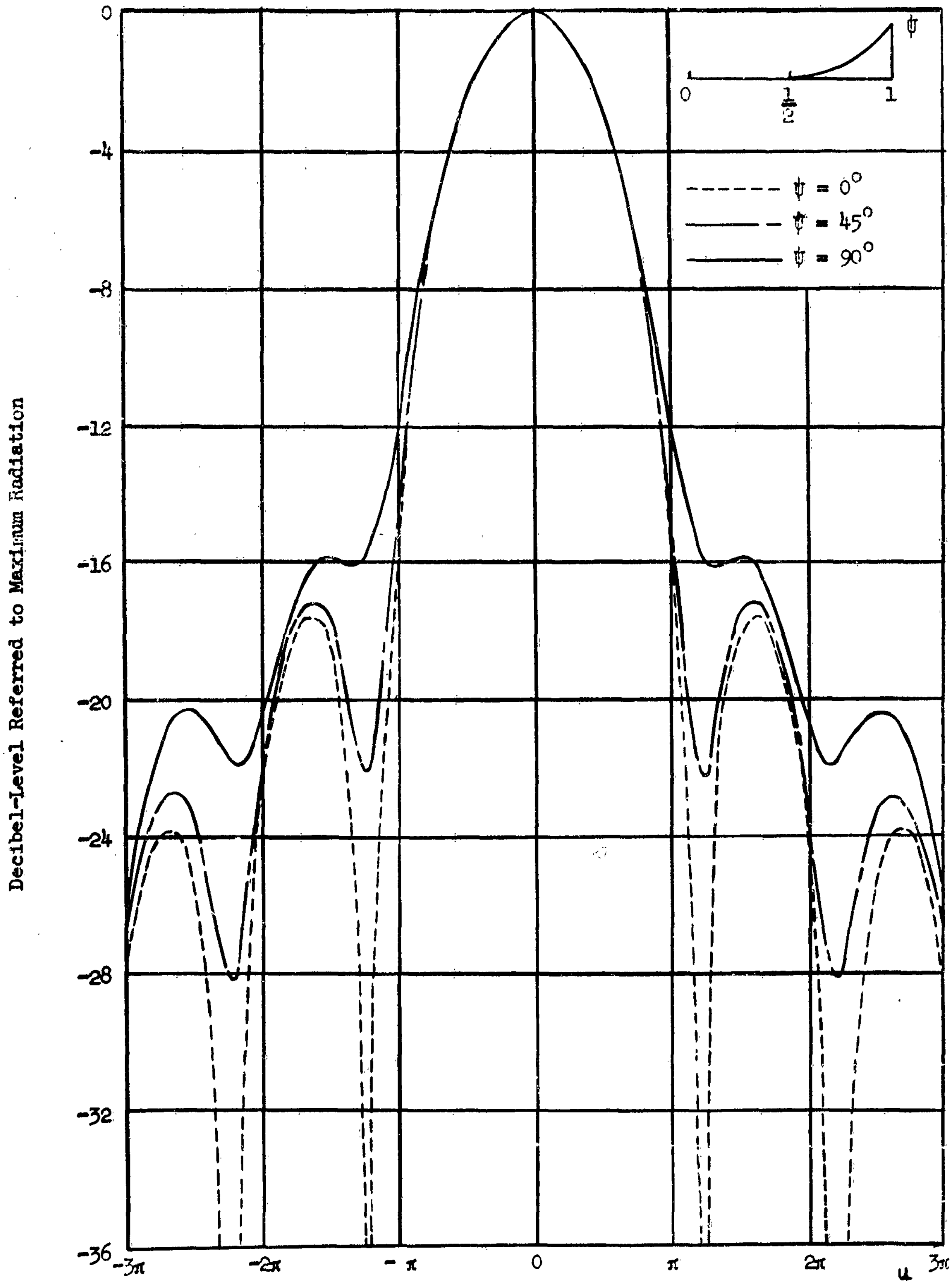


Fig. 33 - Radiation Patterns: Uniform Illumination and Quadratic Phase Deviation (Circular Aperture)

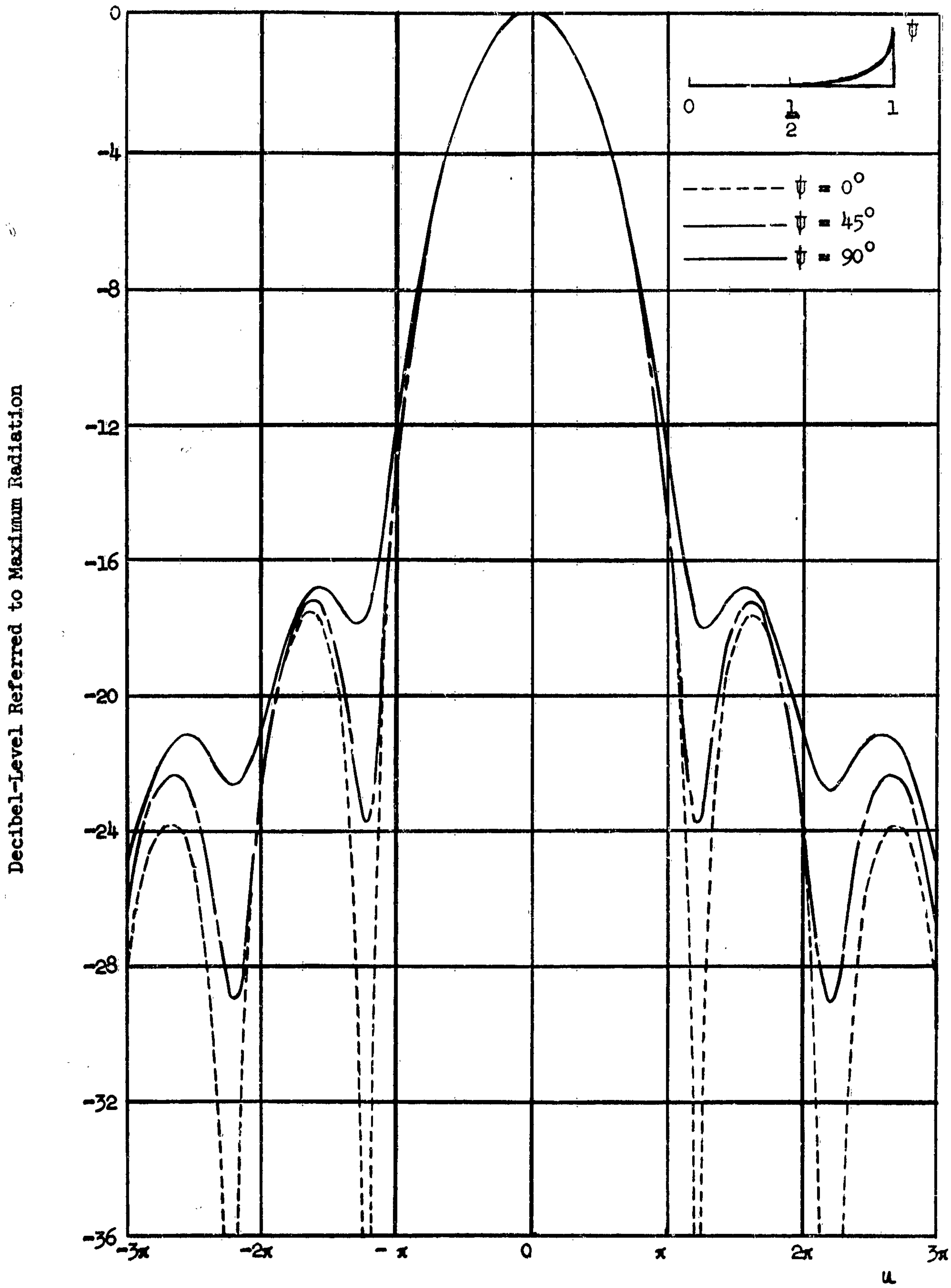


Fig. 34 - Radiation Patterns: Uniform Illumination and Cubic Phase Deviation (Circular Aperture)

C. $\text{Cos}^2(0.375\pi\rho)$ ILLUMINATION

The normalized radiation patterns for a circular aperture with $\cos^2(0.375\pi\rho)$ amplitude illumination and circularly symmetrical phase deviations of the following types are plotted in this section. (Phase deviations occur in the region $1/2 \leq \rho \leq 1$ only; and ψ represents the maximum error at the edge of aperture.)

1. Linear Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 35
2. Quadratic Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 36
3. Cubic Phase Deviation ($\psi = 45^\circ$ and 90°) - Fig. 37

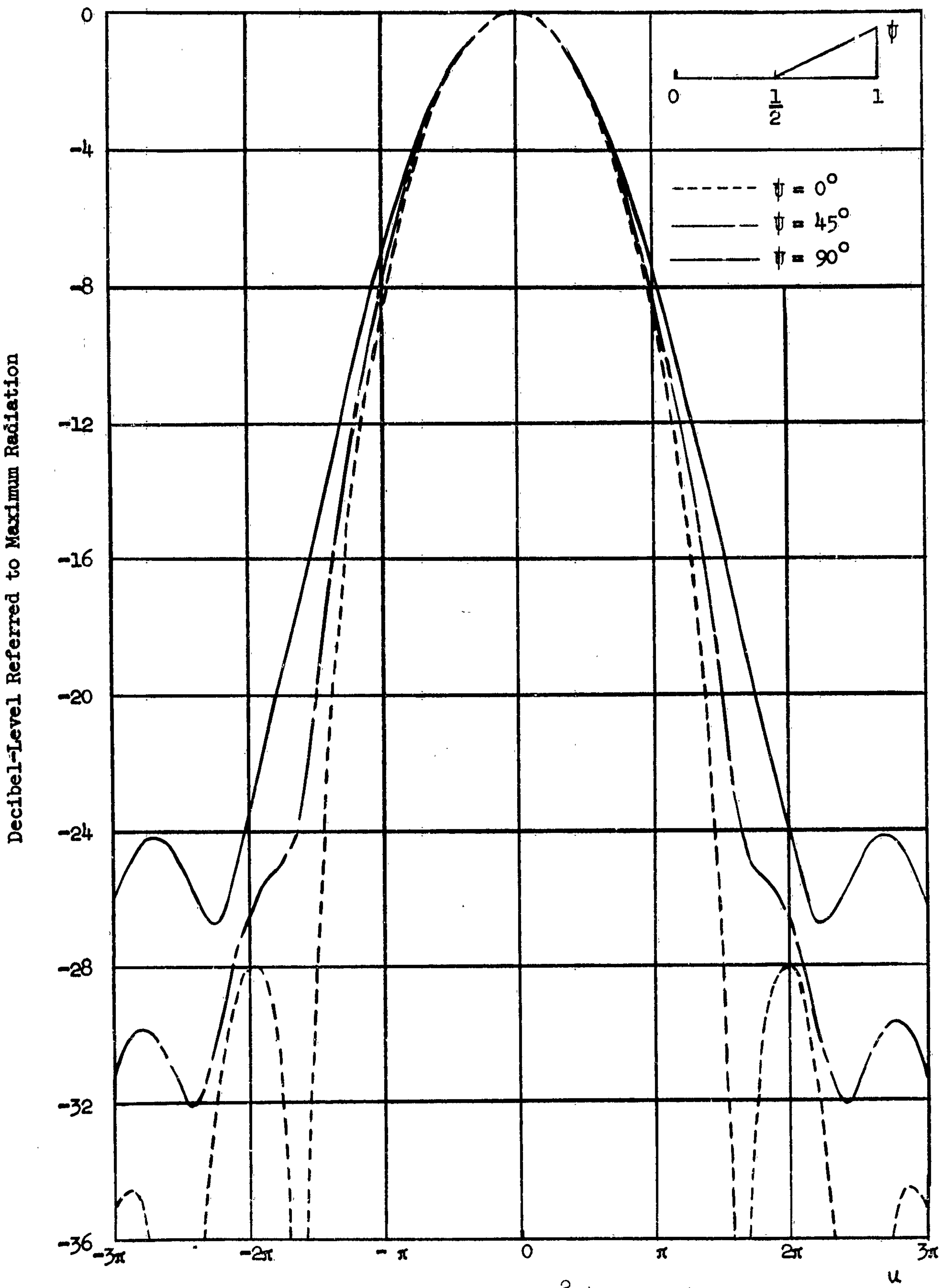


Fig. 35 - Radiation Patterns: $\text{Cos}^2\ (0.375\ \pi\rho)$ Illumination and Linear Phase Deviation (Circular Aperture)

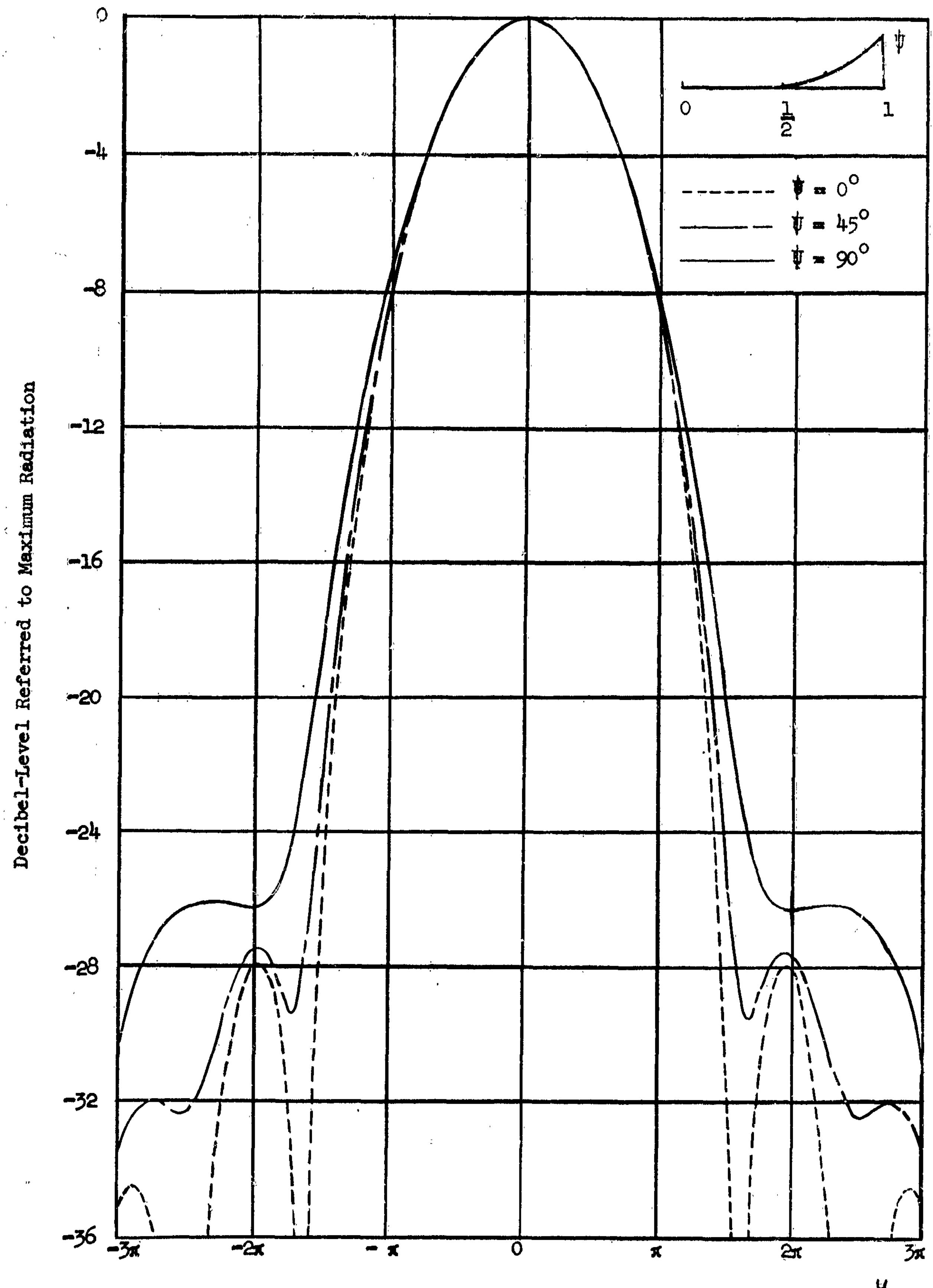


Fig. 36 - Radiation Patterns: $\text{Cos}^2\ (0.375\pi\rho)$ Illumination and Quadratic Phase Deviation (Circular Aperture)

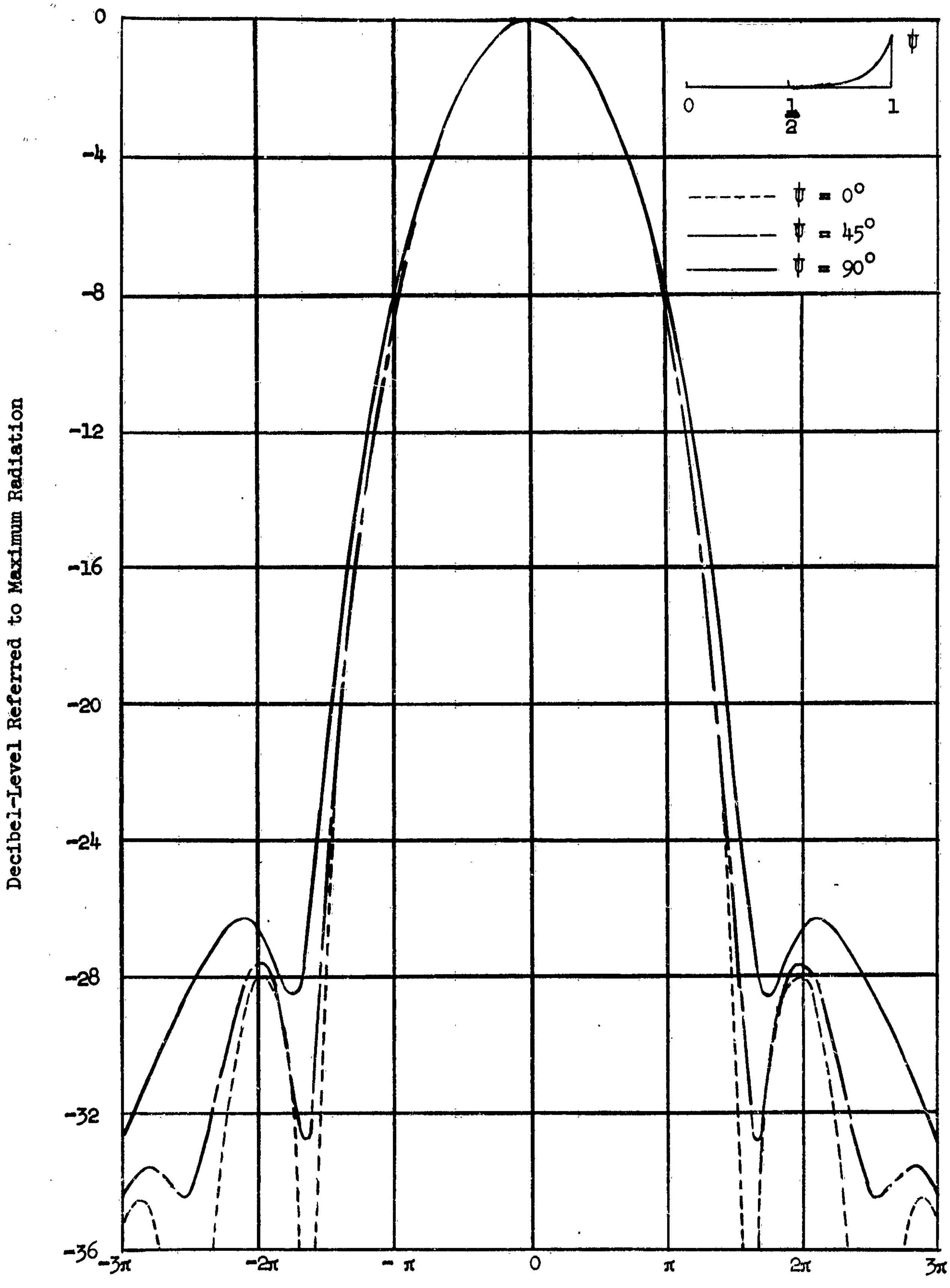


Fig. 37 - Radiation Patterns: $\text{Cos}^2\,(0.375\pi\rho)$ Illumination and Cubic Phase Deviation (Circular Aperture)

D. TABULATION OF RESULTS

The important characteristics of the radiation patterns for a circular aperture with the different types of phase deviations considered are tabulated in Tables 7 and 8. Table 7 is for the uniform-illumination case; and Table 8 is for an amplitude illumination of the $\cos^2(0.375\pi\rho)$ type. Corresponding figure numbers are also given for easy reference.

Table 7 - Radiation-Pattern Characteristics: Uniform Amplitude Illumination (Circular Aperture)

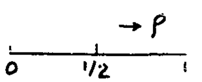
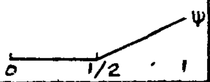
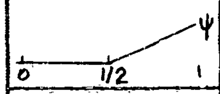
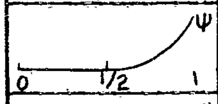
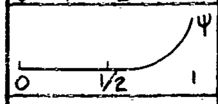
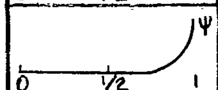
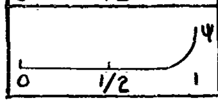
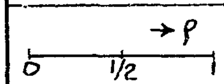
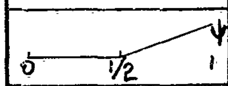
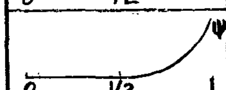
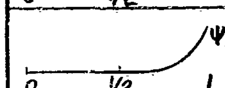
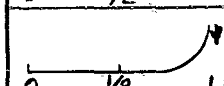
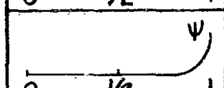
Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\lvert g(u)\rvert$	Main-Lobe Beamwidth, Δu		Levels of first side lobes (-db)		Gain Factor G.F.	Fig. No.	Remarks
				-3db	-10db	Left(u<0)	Right(u>0)			
	None	0	0.5000	1.02π	1.73π	17.58	17.58	1.000	Shown in all following figures	
	45°	0	0.4819	1.03π	1.76π	16.56	16.56	0.929	32	Linear
	90°	0	0.4299	1.05π	1.86π	13.47	13.47	0.739	32	Linear
	45°	0	0.4850	1.04π	1.76π	17.22	17.22	0.941	33	Quadratic
	90°	0	0.4420	1.07π	1.85π	15.87	15.87	0.781	33	Quadratic
	45°	0	0.4787	1.03π	1.74π	17.37	17.37	0.917	34	Cubic
	90°	0	0.4430	1.06π	1.82π	16.87	16.87	0.785	34	Cubic

Table 8 - Radiation-Pattern Characteristics: $\cos^2(0.375\pi\rho)$ Amplitude Illumination (Circular Aperture)

Type of Phase Deviation	Max. Error ψ	Beam Shift u_m	Max. $\lvert g(u) \rvert$	Main-Lobe Beamwidth, Δu		Levels of first side lobes (-db)		Gain Factor G.F.	Fig. No.	Remarks
				-3db	-10db	Left(u<0)	Right(u>0)			
	None	0	0.2463	1.24π	2.13π	28.00	28.00	0.800		Shown in all following figures
	45^o	0	0.2395	1.25π	2.18π	29.74	29.74	0.757	35	Linear (a)
	90^o	0	0.2202	1.29π	2.35π	24.18	24.18	0.640	35	Linear
	45^o	0	0.2419	1.25π	2.16π	27.50	27.50	0.772	36	Quadratic
	90^o	0	0.2295	1.29π	2.26π	26.08	26.08	0.695	36	Quadratic
	45^o	0	0.2430	1.25π	2.15π	27.63	27.63	0.779	37	Cubic
	90^o	0	0.2340	1.28π	2.27π	26.25	26.25	0.722	37	Cubic

(a) Shoulders appear on main lobe.

III. DETERMINATION OF APERTURE PHASE DISTRIBUTION

A. INTRODUCTION

The phase distribution in an aperture plane of an arbitrary reflector due to a point source can be determined by a vector wavefront method. This method is discussed in detail in the final report of this contract. Using the following notations:

- $\bar{A}$ = Source vector or vector representing the incident wavefront
- $\bar{R}$ = Vector representing the reflector surface
- $\bar{B}_i$ = Vector representing the imaginary wavefront at zero optical path-length
- $\bar{B}$ = Vector representing the reflected wavefront
- $\hat{n}$ = Unit normal at the reflector surface
- $\hat{m}$ = Unit normal at the reflected wavefront
- W = Optical path-length from the source to an aperture plane located at a distance d from the source. All distances here are normalized with respect to the focal length f.

it can be proved that the deviation in path-length referred to the center of the aperture is

$$\delta = (2 + d) - \frac{1}{\hat{m} \cdot \hat{k}} (1 + d - \bar{B}_i \cdot \hat{k})$$

where

$$\bar{B}_i = \bar{A} + 2 \hat{n} \left[\hat{n} \cdot (\bar{R} - \bar{A})\right]$$

and $\hat{k}$ is a unit vector in the direction of the reflector axis. The corresponding phase deviation is then $(2\pi f/\lambda)\delta$.

It is also necessary to express the coordinates of a point in the aperture plane (x_o, y_o, z_o) in terms of those for the corresponding point (x,y,z) on the reflector. This can be done by first finding the coordinates of the point (x_1, y_1, z_1) by the above formula for $\bar{B}_i$ and recognizing that

$$x_o = x_1 + \frac{\hat{m} \cdot \hat{i}}{\hat{m} \cdot \hat{k}} (1 + d - z_1)$$

$$y_o = y_1 + \frac{\hat{m} \cdot \hat{j}}{\hat{m} \cdot \hat{k}} (1 + d - z_1)$$

$$z_o = 1 + d$$

The vector $\bar{R}$ representing the reflector surface may be given in parametric form in terms of two parameters s and t. Given any set of values s and t, the path-length deviation δ at the corresponding point in the aperture plane can be found.

A more direct formula for δ, which circumvents the necessity of finding the reflected wavefront $\bar{B}_i$ first, is[1]

$$\delta = 2\left\{ |\bar{A}| + \frac{d}{2} + |\bar{R}-\bar{A}| \; \frac{(\hat{n} \cdot \hat{k})[\hat{n} \cdot (\bar{R}-\bar{A})] - d/2}{(\bar{R}-\bar{A}) \cdot \hat{k} - 2\,(\hat{n} \cdot \hat{k})[\hat{n} \cdot (\bar{R}-\bar{A})]} \right\}$$

The above equation applies when the primary source lies on the reflector axis. If the source is at the focus, the normalized $|\bar{A}|$ would be unity. A positive δ indicates a phase advance and a negative δ represents a phase lag with respect to the center of the aperture.

In the following section, aperture phase distributions for paraboloidal reflectors with several special types of circularly symmetrical mechanical deviations are presented graphically; analytical expressions are given in the Final Report.

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[1] D. K. Cheng and P. Grusauskas, "Determination of Aperture Phase Errors in Microwave Reflectors," Journal of Franklin Institute, Vol. 260, p. 99; August 1955.

B. SPECIAL CASES

Using the formulas listed in the preceding section, the phase distribution curves in an aperture plane which is at a distance from the apex of the reflector equal to twice the focal length are computed and plotted for several special types of reflector deviation. In all cases, the reflector deviation is assumed to be circularly symmetrical so that it is possible to set t = 0 and examine the relationship in the xz-plane.

d = Normalized distance of aperture plane from focal point = 1

D = Normalized diameter of reflector = 2

1. Constant Reflector Deviation - Fig. 38

$$\bar{R} = s\hat{i} + \left[\frac{s^2}{4} + p\, U(s - s_a)\right]\hat{k}$$

where $U(s - s_a)$ is a unit step function which equals 0 for $s < s_a$ and 1 for $s \geq s_a$.

Cases computed: $p = +0.01$, $s_a = 0.5$

$p = -0.01$, $s_a = 0.5$

2. Linear Reflector Deviation - Fig. 39

$$\bar{R} = s\hat{i} + \left[\frac{s^2}{4} + p\,(s - s_a)\, U(s - s_a)\right]\hat{k}$$

Cases computed: $p = +0.05$, $s_a = 0.59$

$p = -0.04$, $s_a = 0.43$

3. Exponential Reflector Deviation - Fig. 40

$$\bar{R} = s\hat{i} + \left\{\frac{s^2}{4} + \left[1 - e^{-p(s-s_a)U(s-s_a)}\right]\right\}\hat{k}$$

Cases Computed: $p = +0.05$, $s_a = 0.59$

$p = -0.025$, $s_a = 0.45$

The values of p and s_a in each case were chosen such that the resulting phase deviation curve would approximate the distribution function used in the theoretical radiation-pattern analysis.

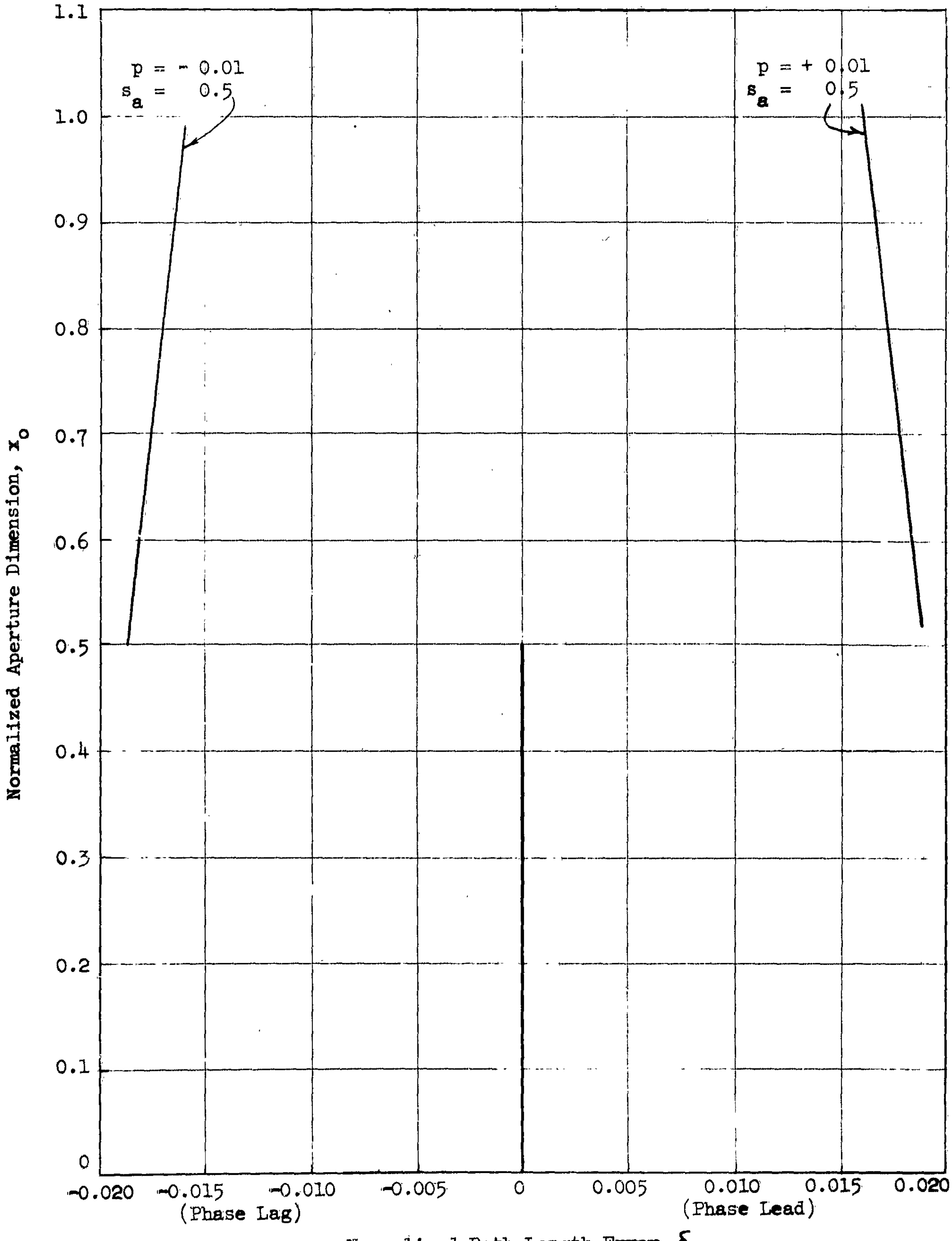

Fig. 38 - Aperture Phase Distribution: Symmetrical Constant Reflector Deviation
(d = 1.0)

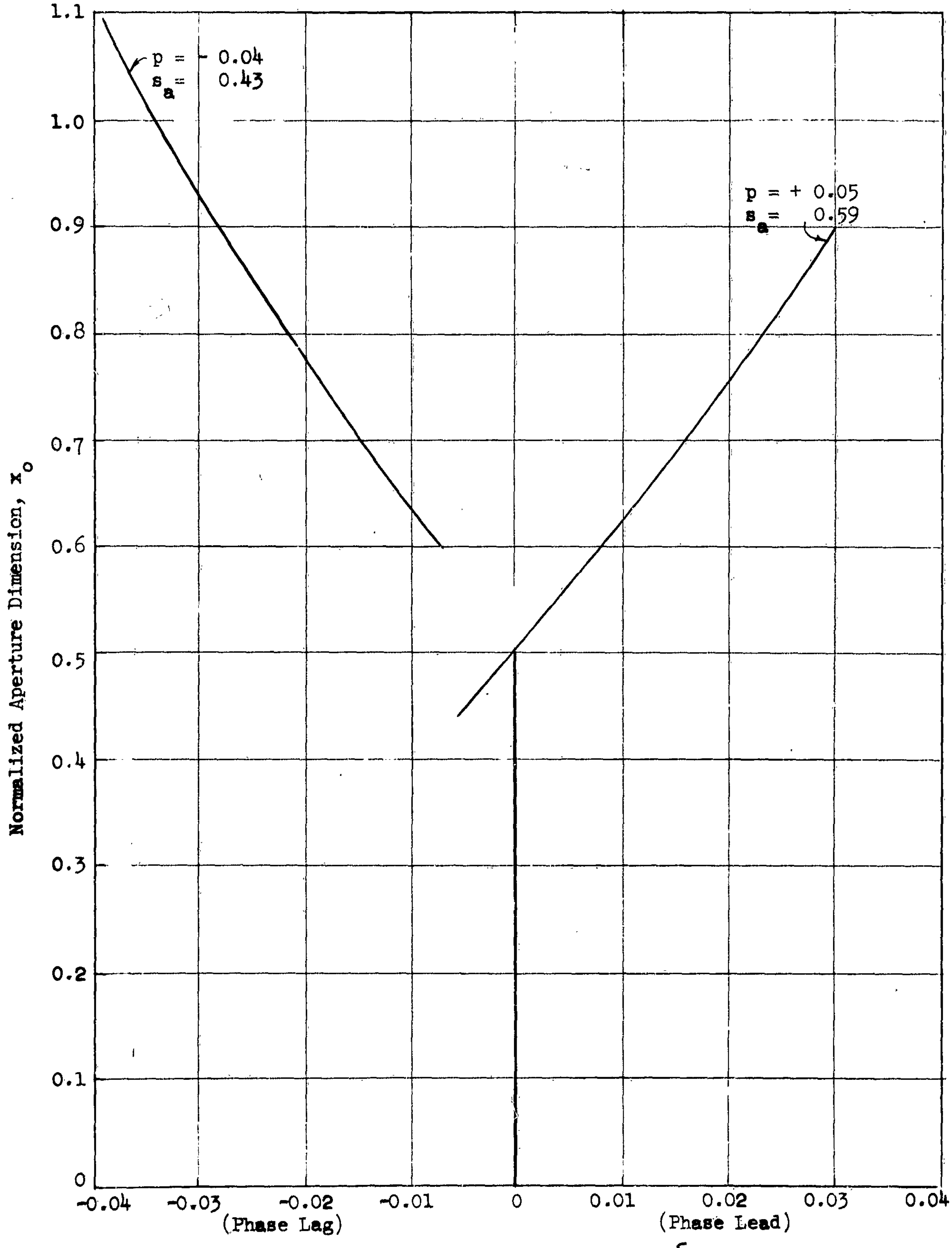

Fig. 39 - Aperture Phase Distribution: Symmetrical Linear Reflector Deviation
(d = 1.0)

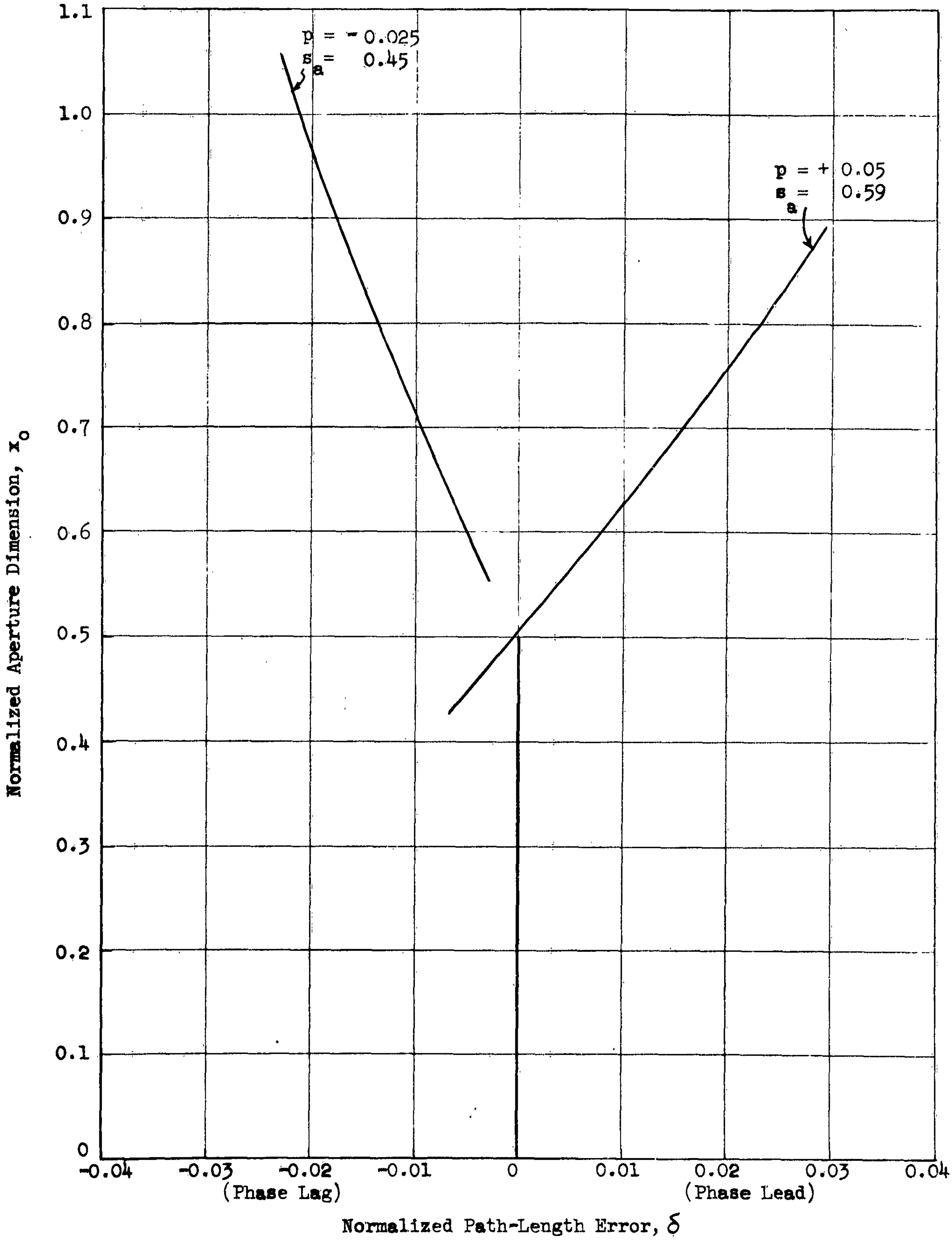

Fig. 40 - Aperture Phase Distribution: Symmetrical Exponential Reflector Deviation (d = 1.0)

IV. DETERMINATION OF APERTURE AMPLITUDE DISTRIBUTION

A. INTRODUCTION

Three approximation techniques are available for determining the amplitude distribution of the field in an aperture plane located at a given distance from a reflector of an arbitrary shape, namely, the geometrical optics method, the current distribution method and the aperture-field method. All three methods assume that the radii of curvature of the reflector and of the incident wavefront be large compared with the wavelength so that Snell's laws of reflection apply. When secondary effects such as the reaction of the reflector on the primary source are not of primary concern, deviations from geometrical propagation of the scattered field due to diffraction are small. The deviations become smaller as the wavelength gets shorter or as the dimensions of the reflector aperture become larger in comparison.

Based upon the geometrical optics approach, the following expression relating the electric field intensities $|E_r|$ and $|E_b|$ at corresponding points on the reflector and in space separated by a distance b is obtained:

$$|E_b| = |E_r| \frac{R_o}{b} \left| \frac{\rho_\xi \rho_\eta \cos i}{\left(\frac{R_o}{b}\right)^2 \rho_\xi \rho_\eta \cos i - 2\frac{R_o}{b}\left\{R_o(\rho_\xi \sin^2\theta_1 + \rho_\eta \sin^2\theta_2) + \rho_\xi \rho_\eta \cos i\right\} + \left\{(\rho_\xi \rho_\eta + 4R_o^2)\cos i + 2R_o(\rho_\xi \sin^2\theta_1 + \rho_\eta \sin^2\theta_2)\right\}} \right|^{1/2}$$

where ρ_ξ, ρ_η = Principal radii of curvature of the reflector at a point P under consideration.

θ_1, θ_2 = Angles made by the line joining point P and the primary source with the principal axes of the reflector at P.

R_o = Distance from the primary source to P.

i = Angle of incidence.

This expression can be written as

$$|E_b| = |E_r| \frac{R_o}{b} D_A$$

where D_A will be called the amplitude distribution factor.

In the following section, amplitude distribution factors in an aperture plane of paraboloidal reflectors with several special types of circularly symmetrical mechanical deviations are presented graphically. The mechanical deviations have been chosen to be of the same types and of the same magnitudes as those used for aperture phase calculations.

B. SPECIAL CASES

Using the formula of the preceding section, the amplitude distribution factors in an aperture plane which is at a distance from the apex of the reflector equal to twice the focal length are computed and plotted for the same types of reflector deviation as listed in Section III-B. In all cases, the reflector deviation is assumed to be circularly symmetrical.

$$d = 1 \quad , \quad D = 2$$

1. Constant Reflector Deviation - Figs. 41 and 42

   Cases computed: $p = +0.01$, $s_a = 0.5$ (Fig. 41)

   $p = -0.01$, $s_a = 0.5$ (Fig. 42)

2. Linear Reflector Deviation - Figs. 43 and 44

   Cases computed: $p = +0.05$, $s_a = 0.59$ (Fig. 43)

   $p = -0.04$, $s_a = 0.43$ (Fig. 44)

3. Exponential Reflector Deviation - Figs. 45 and 46

   Cases computed: $p = +0.05$, $s_a = 0.59$ (Fig. 45)

   $p = -0.025$, $s_a = 0.45$ (Fig. 46)

It is noted that for some types of reflector deviation ($p > 0$ for constant type, $p < 0$ for linear and exponential types) the aperture phase in the deviated region lags and the amplitude distribution factor is for the most part greater than unity; there also exists a region of no radiation. For other types of reflector deviation ($p < 0$ for constant type, $p > 0$ for linear and exponential types) the aperture phase in the deviated region leads and the amplitude distribution factor is for the most part less than unity; there exists a region of overlapping rays. Combination of the contributions from the deviated and undeviated portions of the reflector in the overlapping region should take both amplitude and phase relationships into consideration.

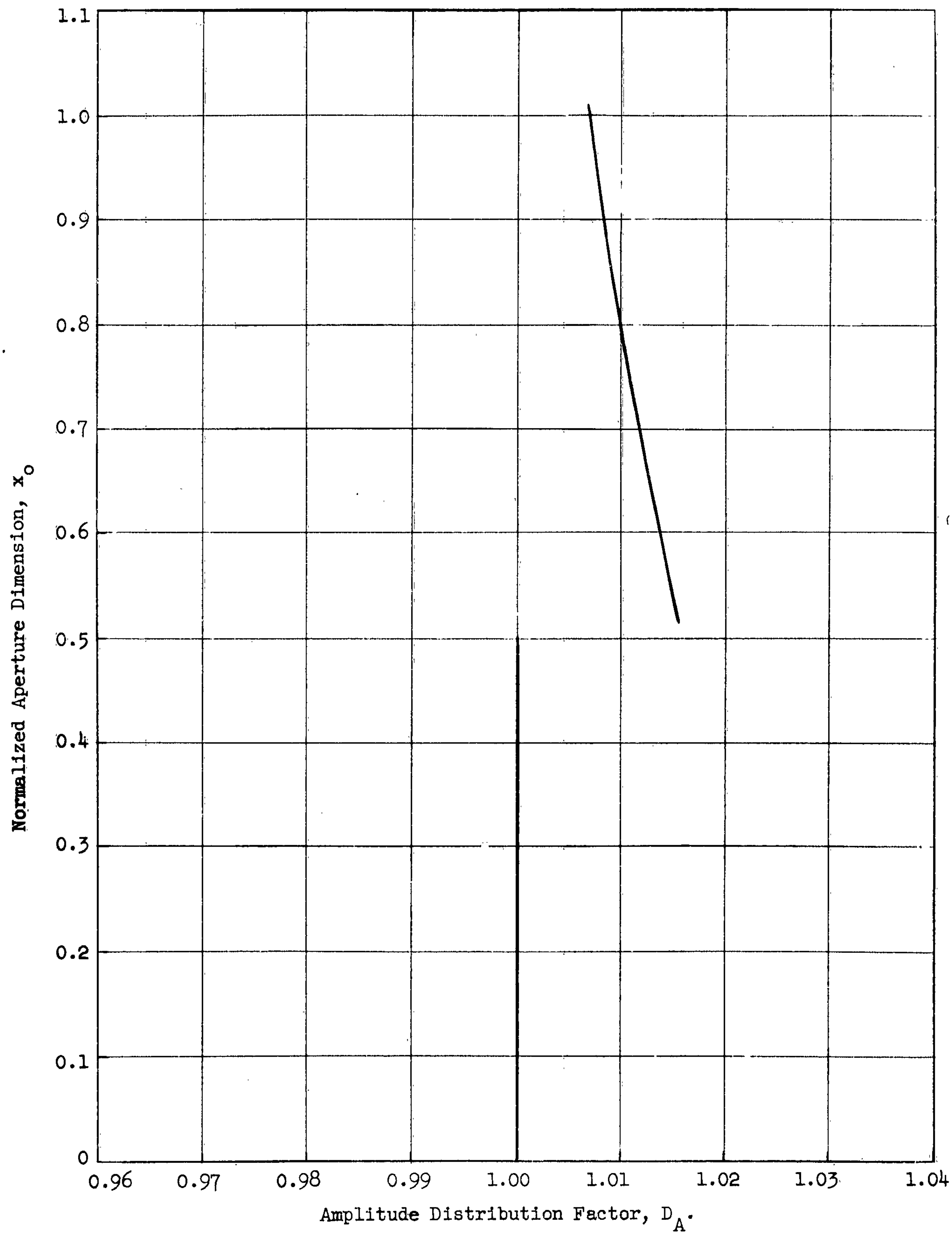

Fig. 41 - Amplitude Distribution Factor: Symmetrical Constant Reflector Deviation ($d = 1.0$, $p = + 0.01$, $s_a = 0.5$)

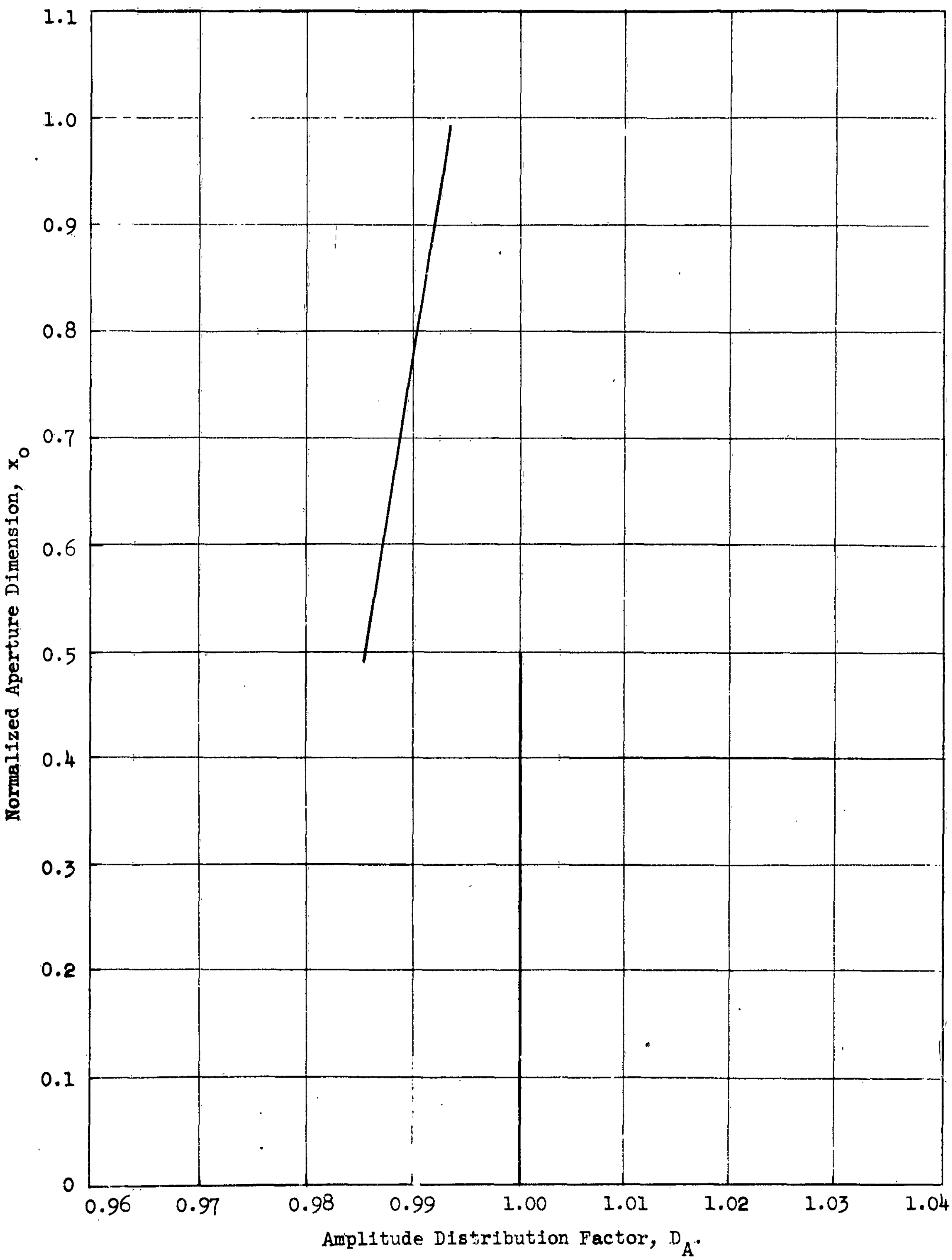

Fig. 42 - Amplitude Distribution Factor: Symmetrical Constant Reflector Deviation ($d = 1.0$, $p = -0.01$, $s_a = 0.5$)

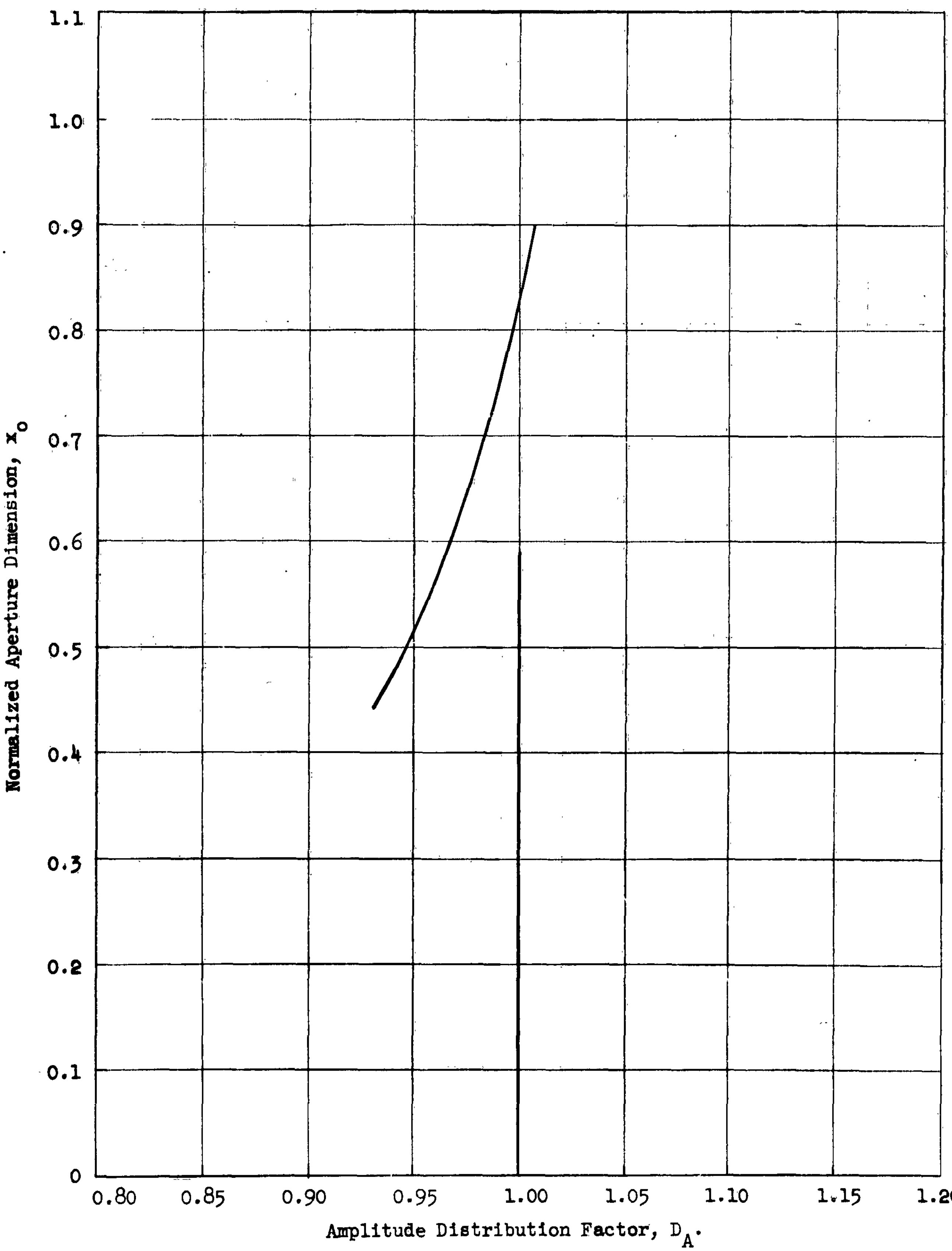

Fig. 43 - **Amplitude Distribution Factor: Symmetrical Linear Reflector Deviation** ($d = 1.0$, $p = + 0.05$, $r_a = 0.59$)

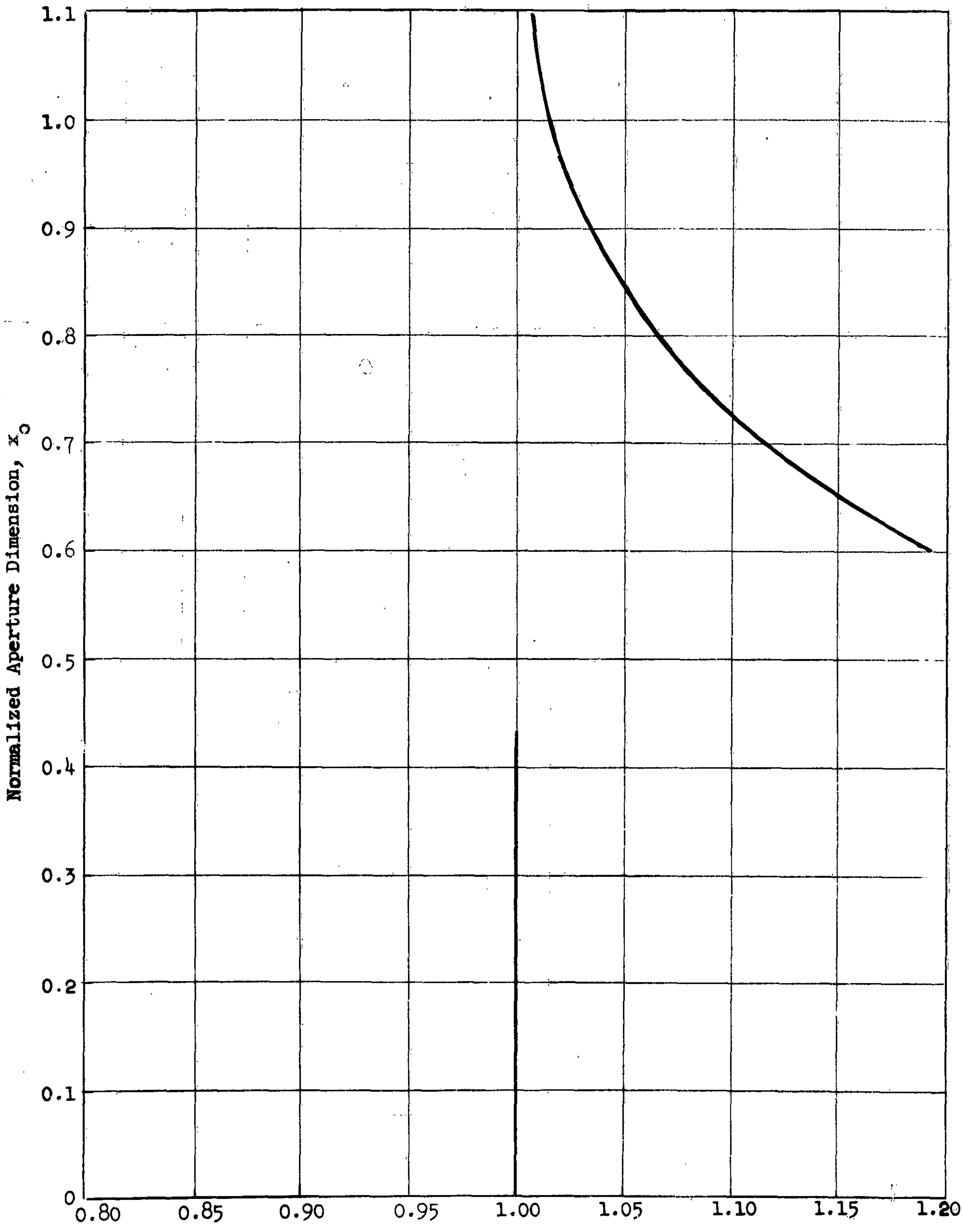

Fig. 44 - Amplitude Distribution Factor: Symmetrical Linear Reflector Deviation ($d = 1.0$, $p = -0.04$, $r_a = 0.43$)

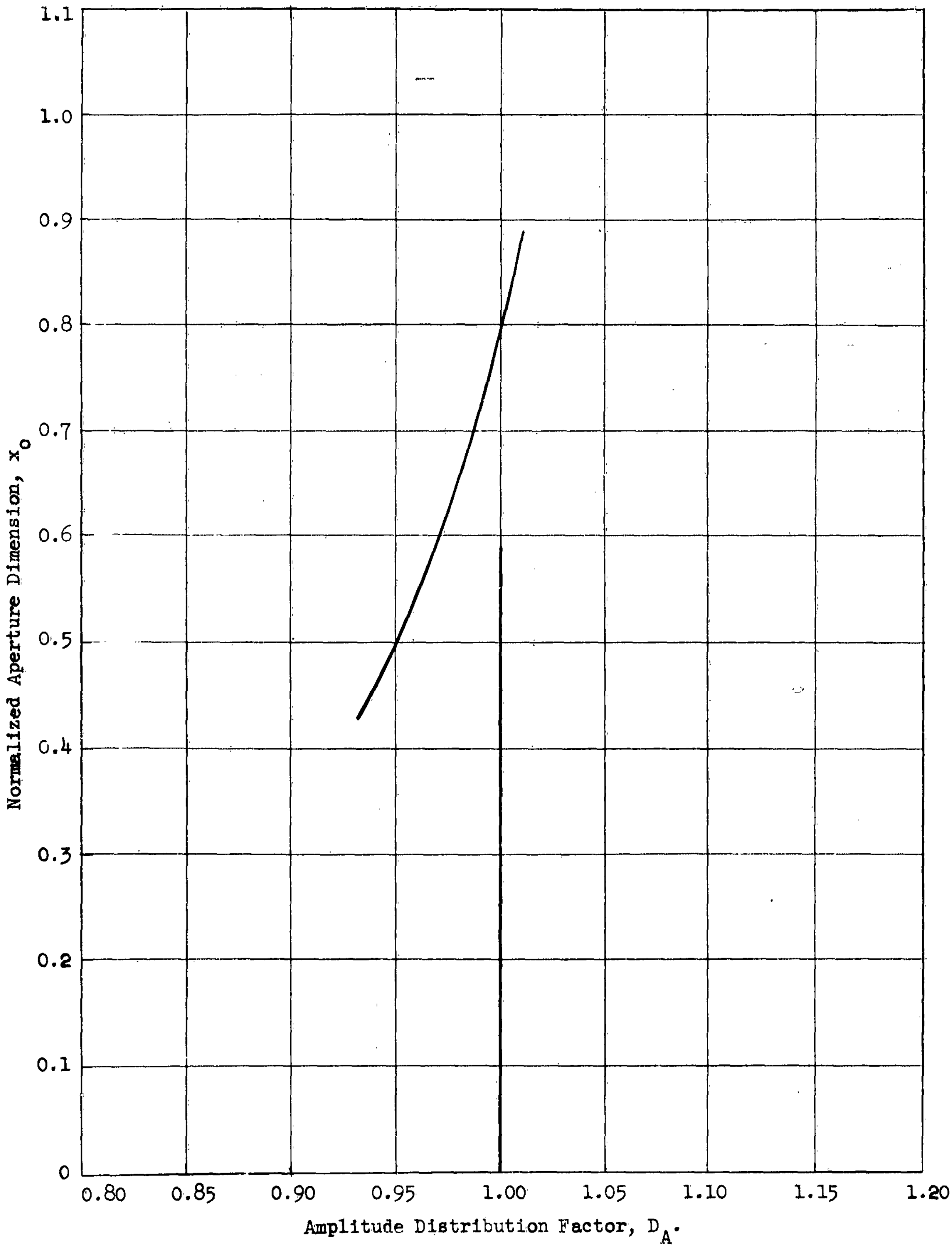

Fig. 45 - Amplitude Distribution Factor: Symmetrical Exponential Reflector Deviation ($d = 1.0$, $p = + 0.05$, $r_a = 0.59$)

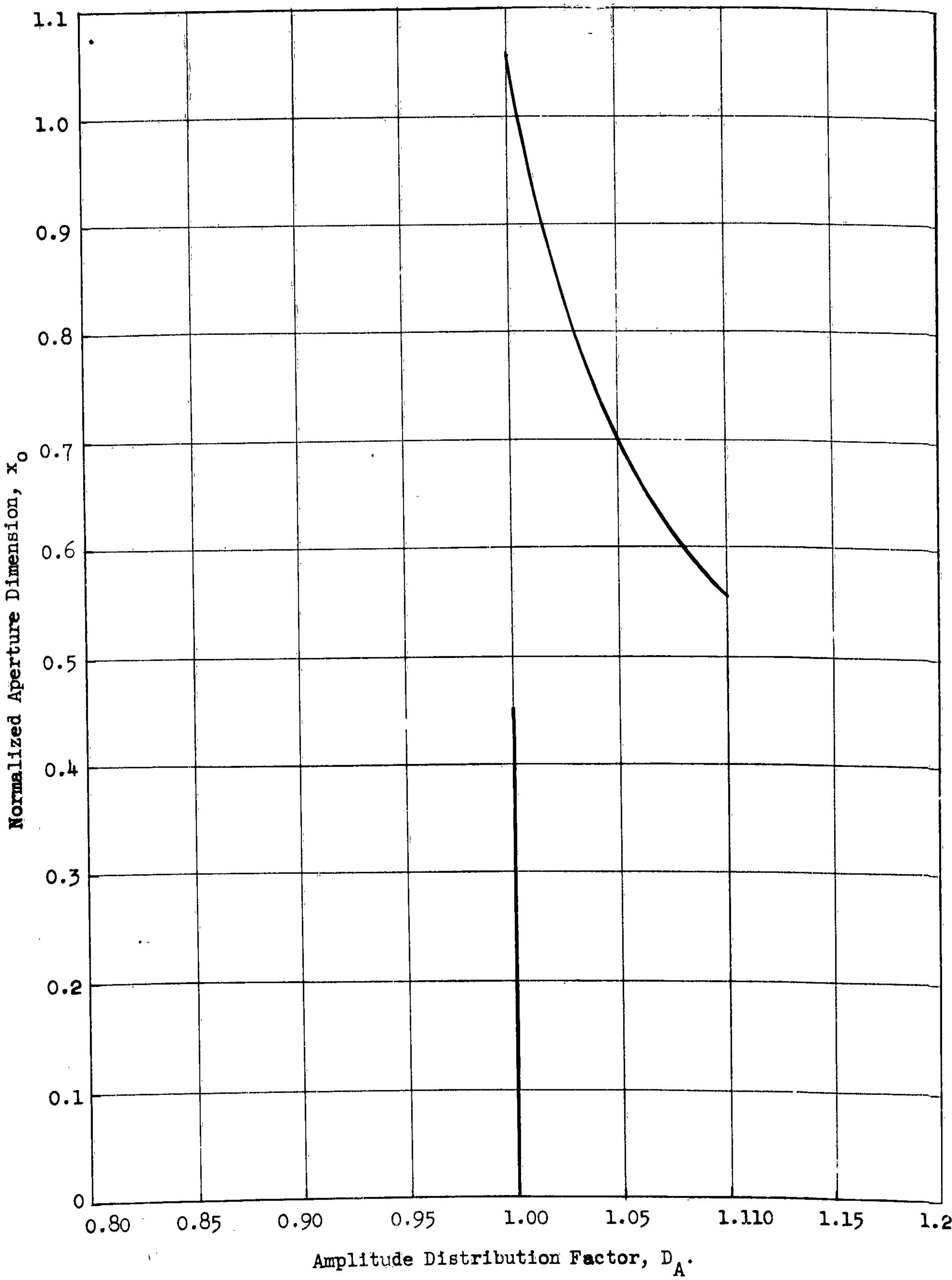

Fig. 46 - Amplitude Distribution Factor: Symmetrical Exponential Reflector Deviation ($d = 1.0$, $p = -0.025$, $r_a = 0.45$)