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RESEARCH MEMORANDUM

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AIR BATTLE THEORY:
STATISTICAL SURVIVAL ANALYSIS FOR CLOSE CONTROLLED INTERCEPTORS VERSUS BOMBERS

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I. SUMMARY

A simplified air battle model has been constructed in statistical terms, based upon the assumption that continuous close control of each interceptor can be maintained throughout the battle. The bomber and interceptor losses resulting from this analysis are believed to represent practical upper limits to the attrition rates for air battle.

II. INTRODUCTION

Simplified statistical air battle models for use in attrition studies can be developed from several basic bomber survival probability expressions, depending upon the assumptions made regarding the distribution of interceptor attacks per bomber. The distribution assumed herein represents that obtainable through very effective and continuous close control of each interceptor. Close control implies directing the interceptor through the required flight pattern to bring it into approximate firing position, leaving to the pilot only the responsibility for making final aiming corrections. Upon completion of firing, the interceptor is directed into the approach pattern for a new attack upon a bomber preselected by the air combat controller. This preselection enables the interceptor attacks to be uniformly spread over the bomber formation, thus obtaining very effective interceptor utilization. For example, if 47 interceptors are deployed against 32 bombers, this model pictures two interceptors directed against each of 15 bombers and one each against the remaining 17.

This uniform distribution obtained by close control is in contrast to the more random distribution resulting from broadcast control, whereby each interceptor is provided with frequent bomber stream location plots, from which he must seek out a target and determine his own attack pattern. Consequently, under broadcast control, some bombers may bear the brunt of the attacks while others in the formation may get through unmolested. Other factors being equal, it is believed the picture of close controlled interception presented here represents the optimum interceptor effectiveness and provides an upper limit to the bomber attrition rate for air battle.

Other factors, for which insufficient information presently exists to enable their implicit inclusion in this preliminary model, include:

(a) The effect of bomber formation in overall survival probabilities.

(b) The effect of bomber location in the formation upon individual survival probabilities.

(c) The effect of interceptor tactics upon both the probable bomber and interceptor losses. (For example, fewer interceptor losses should be expected if they attack simultaneously rather than singly).

(d) Difficulty interceptors might have in penetrating a tight bomber formation to attack interior bombers.

(b) Arrival rate of interceptors into combat, which will affect the interceptor-bomber ratio.

(f) Reduction of effectiveness of close control in interceptor attack allocation per bomber because of finite time lag in the assessment of bomber damage.

(g) Effect of cumulative damage from successive attacks upon kill probability.

As a mathematical simplification of the problem, full use will be made of the stage or attack wave concept of the air battle as introduced in D-627. This concept considers successive attacks by each interceptor as occurring in successive stages, each stage of combat being carried out by the survivors of the previous stage. The number of stages or attack waves is small, being limited by the fuel or ammunition supply of the interceptors.

III. NOTATION

M = initial number of bombers

N = initial number of interceptors encountered

M_i = number of bombers surviving i th stage

N_i = number of interceptors surviving i th stage

R_i = N_i/M_i = interceptor/bomber ratio surviving i th stage

$\bar{R}_i$ = $[R_i]$ = greatest integer in R_i

S = probability (constant) that a bomber will survive an attack by one interceptor

P_i = probability that a bomber will survive i th stage of interceptor attacks

σ = probability (constant) that a bomber will survive an attack long enough to launch its allocated ammunition against the attacking interceptor.

s = probability (constant) that an interceptor will survive an attack long enough to launch its allocated ammunition against the bomber

p = probability (constant) that an interceptor will survive an attack

c = probability that an attack will not fail due to personnel or equipment errors, RCM, etc.

IV. Case A - Interceptor Losses Neglected

The simplest version to analyze occurs when the interceptor losses are neglected. For constant bomber survival probability per attack (S), the optimum distribution of interceptors for the first stage follows the schedule:

$\overline{R} + 1$ interceptors attacking each of $N - M\overline{R}$ bombers

$\overline{R}$ interceptors attacking each of $M + M\overline{R} - N$ bombers

Because of finite time lag in the assessment or bomber damage, particularly under all weather operation, these $\overline{R}$ of $\overline{R} + 1$ attacks upon a certain bomber must be considered as occurring simultaneously. In this sense, simultaneous attacks imply multiple attacks on the same bomber which overlap in time such that a second attack is launched before the bomber damage from the first attack can be ascertained. The combined probability that a given bomber will survive $\overline{R}$ simultaneous attacks is then the product of the survival probabilities for each attack. Consequently, with no interceptor losses, the expected number of bombers surviving the first stage or attack wave is given by the expression

$$M_1 = (N - M\overline{R})\, S^{\overline{R}+1} + (M + M\overline{R} - N)\, S^{\overline{R}} \qquad \text{Eq. (1)}$$

while the mean probability that a bomber will survive this first stage is represented by

$$P_1 \equiv \frac{M_1}{M} = (R - \overline{R})\, S^{\overline{R}+1} + (1 + \overline{R} - R)\, S^{\overline{R}} \qquad \text{Eq. (2)}$$

When $R < 1$, so that $\overline{R} = 0$, the survival probability can be simplified to the form

$$P_1 = RS + 1 - R = 1 - R\,(1 - S) \qquad \text{Eq. (3)}$$

In the event R is an integer, so that $R - \overline{R} = 0$, the expression reduces to

$$P_1 = S^R \qquad \text{Eq. (4)}$$

V. Case B - Interceptor Losses Considered

The inclusion of interceptor losses in the analysis has the effect of reducing the effective number of attacks per bomber. The probability that $\overline{R} - n$ attacks will be carried to completion out of $\overline{R}$ attacks launched against a given bomber is

$$f(\bar{R}, n) = \binom{\bar{R}}{n} s^{\bar{R} - n} (1 - s)^n \qquad \text{Eq. (5)}$$

The joint probability that $\bar{R} - n$ attacks will be completed out of $\bar{R}$ launched and the bomber survives all these is given by

$$g(\bar{R}, n) = S^{\bar{R} - n} \binom{\bar{R}}{n} s^{\bar{R} - n} (1 - s)^n \qquad \text{Eq. (6)}$$

The probability that the bomber survives the $\bar{R}$ attacks launched is

$$h(\bar{R}) = \sum_{n=0}^{\bar{R}} \binom{\bar{R}}{n} (Ss)^{\bar{R} - n} (1 - s)^n$$

$$= \left\{1 - s(1 - S)\right\}^{\bar{R}} \qquad \text{Eq. (7)}$$

The expected number of bombers surviving all attacks in the first stage considering interceptor losses becomes

$$M_1 = (M + M\bar{R} - N)\left\{1 - s(1 - S)\right\}^{\bar{R}} + (N - M\bar{R})\left\{1 - s(1 - S)\right\}^{\bar{R} + 1} \qquad \text{Eq. (8)}$$

The corresponding mean survival probability for a bomber is

$$P_1 = (1 + \bar{R} - R)\left\{1 - s(1 - S)\right\}^{\bar{R}} + (R - \bar{R})\left\{1 - s(1 - S)\right\}^{\bar{R} + 1} \qquad \text{Eq. (9)}$$

Similarly, in computing the interceptors chances for survival, consideration should be given to the occasions when the bomber is destroyed early in the attack before it can launch its allocated defensive ammunition, thereby improving the interceptors survival probability. Neglecting at this time the second order effect of bomber firepower dilution during overlapping attacks, the expected number of interceptors surviving the first stage of combat is given by

$$N_1 = N\left\{1 - \sigma(1 - p)\right\} \qquad \text{Eq. (10)}$$

In like manner, the expected survivors of the second stage of combat become

$$M_2 = M_1 (1 + \bar{R}_1 - R_1)\left\{1 - s(1 - S)\right\}^{\bar{R}_1} + M_1 (R_1 - \bar{R}_1)\left\{1 - s(1 - S)\right\}^{\bar{R}_1 + 1} \qquad \text{Eq. (11)}$$

$$N_2 = N_1 \left\{ 1 - \sigma (1 - p) \right\} = N \left\{ 1 - \sigma (1 - p) \right\}^2 \qquad \text{Eq. (12)}$$

This stepwise manner of determining survivors from each stage is continued for as many stages as the interceptors can enter before withdrawing for lack of fuel or ammunition.

VI. Case C - Interceptor Errors Considered

Allowances can also be made in this air combat picture for incompleted attacks due to personnel errors, radar countermeasures or other causes which do not result in destruction of the interceptor. The interceptors failing an attack in this manner are still available to engage in the next attack stage and are not lost to the defense. It is assumed that complete equipment failure rendering an interceptor useless for combat will be discovered prior to the first attack stage, consequently these interceptors will be deleted from the available interceptor force and given no consideration in the battle picture. The probability that $\overline{R} - n$ attacks will be completed out of $\overline{R}$ launched and that the bomber survives is

$$j(\overline{R}, n, c) = \binom{\overline{R}}{n} (sc)^{\overline{R} - n} (1 - sc)^n \, S^{\overline{R} - n} \qquad \text{Eq. (13)}$$

The probability that a bomber survives all attacks completed out of $\overline{R}$ launched is given by

$$K(\overline{R}, c) = \sum_{n = o}^{\overline{R}} \binom{\overline{R}}{n} (S\,sc)^{\overline{R} - n} (1 - sc)^n$$

$$= \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R}} \qquad \text{Eq. (14)}$$

The expected number of bombers surviving the first attack stage is expressed by

$$M_1 = (M + M\overline{R} - N) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R}} + (N - M\overline{R}) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R} + 1} \qquad \text{Eq. (15)}$$

The mean survival probability for the bombers can be written as

$$P_1 = (1 + \overline{R} - R) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R}} + (R - \overline{R}) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R} + 1} \qquad \text{Eq. (16)}$$

The expected number of interceptors surviving the first stage is the sum of those completing the attack without being destroyed and those making an incomplete attack due to errors or RCM, as given by the form

$$N_1 = Nc \left\{ 1 - \sigma (1 - \quad) \right\} + N (1 - c) = N \quad 1 - c\sigma (1 - p) \qquad \text{Eq. (17)}$$

Continuing, the aircraft surviving the second attack are obtained from

$$M_2 = M_1 \quad (1 - \overline{R}_1 - R_1) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R}_1} + M_1\,(R_1 - \overline{R}_1) \left\{ 1 - sc\,(1 - S) \right\}^{\overline{R}_1 + 1} \qquad \text{Eq. (18)}$$

$$N_2 = N_1 \left\{ 1 - c\sigma (1 - p) \right\} = N \left\{ 1 - c\sigma (1 - p) \right\}^2 \qquad \text{Eq. (18)}$$

VII. Case D - Approximation to Case C for $R > 1$

For the occasions when $R > 1$ (i.e., more interceptors than bombers), approximations to Eqs. (15) and (16) can be given as

$$M_1 = M \left\{ 1 - sc\,(1 - S) \right\}^R \qquad \text{Eq. (20)}$$

$$P_1 = \left\{ 1 - sc\,(1 - S) \right\}^R \qquad \text{Eq. (21)}$$

These expressions have no physical interpretation except in the rare cases where R is an integer; essentially they represent the exponential approximations to the polygonal curves of mean survival probability as a function of R. In most cases, survivors predicted by Eq. (20) are reasonably close to those given by the more rigorous form expressed in Eq. (15). A further approximation leads to an expression for the expected survivors of the i th stage of combat:

$$M_i = M \left\{ 1 - sc\,(1 - S) \right\}^{iR} \qquad \text{Eq. (22)}$$

The corresponding interceptor survivors can be found from the extension of Eq. (19)

$$N_i = N \left\{ 1 - c\sigma (1 - p) \right\}^i \qquad \text{Eq. (23)}$$

VIII. Numerical Example

To illustrate the effects of the various parameters involved, a typical example is presented in Table I for the following assumed quantities:

$M = 100$	$S = 0.6$	$s = 0.4$
$N = 340$	$\sigma = 0.7$	$p = 0.3$

TABLE I

Expected Survivors from Close Controlled Air Battle

for

$M = 100$ $S = 0.6$ $s = 0.4$
$N = 340$ $\sigma = 0.7$ $p = 0.3$

Stage	Case A		Case B		Case C (c=.8)		Case C (c=.5)		Case D (c=.8)		Case D (c=.5)	
	M_i	N_i	M_i	N_i	M_i	N_i	M_i	N_i	M_i	N_i	M_i	N_i
0	100	340	100	340	100	340	100	340	100	340	100	340
1	18	340	55	173	63	207	75	257	63	207	75	257
2	1	340	32	88	40	126	56	194	39	126	57	194
3	0	340	20	45	26	77	42	147	25	77	43	147
4	0	340	14	23	17	47	31	111	16	47	32	111
5	0	340	11	12	12	28	23	84	10	28	24	84