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ENGINEERING EXPERIMENT STATION
of the Georgia Institute of Technology
Atlanta, Georgia

FINAL TECHNICAL REPORT

PROJECT NO. A-241-2

REFLECTION AND TRANSMISSION OF MICROWAVES BY THIN METAL FILMS

by

VERNON CRAWFORD

30 JUNE 1956

Department of the Navy
Office of Naval Research
Contract No. NOnr-991(02)

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I. ABSTRACT

The reflection and transmission coefficients of metal films are functions of film thickness. An investigation is carried out to determine (a) whether a measurement of the coefficients at microwave frequencies can be expected to yield accurate values of the thickness; (b) whether films of known thickness can be useful as waveguide components. The analysis shows that at thicknesses greater than ~10A, for metal films on glass substrates, the reflection coefficient is not a sufficiently sensitive thickness indicator, and the transmission coefficient is too small to permit accurate measurements. At thicknesses less than 10A the electrical parameters are unknown, and such small dimensions are not accurately reproducible.

II. INTRODUCTION

The Engineering Experiment Station of the Georgia Institute of Technology has been engaged for a number of years in a study of the physical properties of thin metal films. In any such study one of the parameters of particular interest is the film thickness. For the thin (100A - 10,000A) evaporated or sputtered films which are frequently used in these researches the thickness measurement presents a number of problems. Two methods of thickness determination are in use at present. In the first the weight of the metal deposited on a substrate is determined by means of a microbalance, and certain assumptions are made about the manner in which the metal is distributed; in the second method one[1] of the common interferometric techniques is used.

The first method suffers from some obvious deficiencies. A film is usually deposited on a glass microscope slide, the weight of which is many times that of the film. This severely limits the precision of the determination. The assumptions that the constitution of the film is uniform and that its density is the same as that for the metal in bulk represent departures from reality.

The interferometric method is more direct and is free from the aforementioned objections. Unfortunately, however, it results in the permanent alteration of the film, requiring as it does that a channel be cut in the film and the whole be overlaid with a highly reflecting coating.

A precise, non-destructive method of measuring film thickness is greatly to be desired. It occurred to the author that the effects which a metal film would have on the microwaves in a waveguide might be sufficiently dependent on the film thickness to afford such a method. If the film were placed transverse to the axis of the guide the incident radiation would be partially reflected, partially absorbed and partially transmitted. Also the phases of the waves would be modified by the film. If these effects should turn out to be sufficiently dependent on film thickness then not only would the research achieve its goal of thickness measurement, but also would demonstrate the utility of metal films of known thickness as waveguide components.

With the above considerations as an incentive the necessary calculations were made to determine the magnitude of the effects. It was quickly apparent that, for metal films at least, the method could be applicable only if they were of such extreme thinness ($<$ 10A) that (a) their electromagnetic parameters are for the most part unknown, (b) they are not of current research interest, (c) they could not be reproduced with sufficient precision to make them useful waveguide components. In view of the negative nature of the findings no experimental work was conducted. In this report the calculations are presented and general expressions are derived for the reflection and transmission coefficients (including phase) for a film on a substrate of finite thickness (the dual-layer-film problem).

III. GENERAL THEORETICAL CONSIDERATIONS

The theory of the propagation of electromagnetic waves in material media is well presented in many places[2,3]. Only an outline will be given here sufficient to establish the notation and the nomenclature.

On the assumption that all of the field quantities involved are simple harmonic functions of the time, and that the free-charge density is zero everywhere, Maxwell's electro-magnetic field equations may be combined to yield the following one dimensional wave equation:

$$\frac{\partial^2 (E_s)_y}{\partial x^2} = k^2 (E_s)_y \quad . \tag{1}$$

This represents the very special case of a wave propagated in the x-directions with its electric vector polarized in the x y plane. The subscript 's' denotes the fact that only the spatial components of the electric field are being considered, the time factor, $e^{j\omega t}$, having been divided out, and the subscript 'y' denotes the particular spatial component; 'k' is the propagation constant and is related to the conductivity, σ, the permittivity ϵ, the permeability, μ, and the angular frequency, ω, as follows:

$$k^2 = - \mu \epsilon \omega^2 + j \omega \mu \sigma \quad . \tag{2}$$

Since k is in general complex it is conveniently represented as the sum of its real and imaginary parts:

$$k = \alpha + j \beta \quad . \tag{3}$$

It follows from (2) and (3) that

$$\alpha = \omega \left[\mu \epsilon/2 \left(\sqrt{1 + \sigma^2/(\omega^2 \epsilon^2)} - 1 \right) \right]^{1/2} \quad , \tag{4}$$

$$\beta = \omega \left[\mu\, \epsilon/2 \left(\sqrt{1 + \sigma^2/(\omega^2\, \epsilon^2)} + 1 \right) \right]^{1/2} . \qquad (5)$$

There are two limiting cases of particular interest. The first pertains when $\sigma^2/(\omega^2\, \epsilon^2)$ is much larger than unity. This case includes all reasonably good conductors for all frequencies up to some very high limiting value. For this case

$$\alpha = \beta = \sqrt{\omega\, \sigma\, \mu/2} . \qquad (6)$$

The second case is that for which $\sigma^2/(\omega^2\, \epsilon^2)$ is much less than unity and includes most dielectrics at frequencies greater than some limiting value which is on the order of 10^5/sec. For this case

$$\alpha = (\sigma/2)\sqrt{\mu/\epsilon} , \qquad (7)$$

and

$$\beta = \omega\sqrt{\mu\epsilon} . \qquad (8)$$

In any case, the solution of the wave equation (1) is

$$(E_s)_y = E_m\, e^{-kx} + E_n\, e^{+kx} , \qquad (9)$$

where E_m and E_n are constants. Now, since $E_y = (E_s)\, e^{j\omega t}$, and $k = \alpha + j\,\beta$, Equation (9) becomes

$$E_y = E_m\, e^{-\alpha x}\, e^{j(\omega t - \beta x)} + E_n\, e^{+\alpha x}\, e^{j(\omega t + \beta x)} . \qquad (10)$$

The first term on the right hand side of (10) represents a wave with a phase constant β, attenuated by the constant α, traveling in the positive x-direction. The second term differs from the first only in its amplitude, E_n, and direction of propagation, which is in the negative x-direction.

The magnetic field, H_z, associated with the wave may be deduced from Maxwell's equations. It is related to the electric field, E_y, in a manner determined by the four parameters σ, ϵ, μ and ω. It is convenient to represent the quantity E_y/H_z, the "impedance" of the medium, by the symbol ς. The unit of ς is readily seen to be the ohm. A simple analysis shows that the impedance of any medium is given by

$$\varsigma = \mu\, \omega\, (\beta + j\, \alpha)\Big/(\alpha^2 + \beta^2) \quad . \tag{11}$$

When an electromagnetic wave impinges on an interface separating two media, some of the energy is reflected and the remainder is transmitted. In general, a phase change occurs also. To compute the amplitude of the reflected and transmitted waves one must know, in addition to the impedances of the media, both the angle of incidence and the state of polarization of the incident wave. A particularly simple state of affairs exists when the wave is incident in a direction normal to the interface, because then its state of polarization is immaterial. This is the situation envisioned in the present research where the film was to have been placed normal to the axis of the waveguide. For this case the following relations hold:

$$r_{jk} = (\varsigma_k - \varsigma_j)\Big/(\varsigma_k + \varsigma_j) \quad , \tag{12}$$

$$t_{jk} = 2\varsigma_k\Big/(\varsigma_k + \varsigma_j) \quad , \tag{13}$$

where r_{jk} represents the ratio of the amplitude of the reflected wave to that of the incident wave when incidence occurs from medium j onto the interface separating j from k, and where t_{jk} is the transmitted amplitude ratio. These quantities will be referred to as the reflection and transmission coefficient respectively. They are, in general, complex quantitites and therefore contain the information concerning the amount of phase shift at reflection and transmission.

IV. REFLECTION FROM, AND TRANSMISSION THROUGH, A SINGLE FILM

Consider the film of thickness d_m of medium m which separates two semi-infinite media ℓ and n (Figure 1). If a wave of electromagnetic radiation is incident from medium ℓ onto the ℓ-m interface both reflection and transmission take place. The transmitted wave is divided at the m-n interface, the reflected portion being divided again at the m-ℓ interface, and so on <u>ad infinitum</u>. The result is that the wave in medium ℓ which is reflected from the film is the summation of an infinite number of components, and the same can be said of the resultant wave transmitted through the film in medium n.

The multiple reflections which occur in a film are utilized in certain types of optical equipment such as the Fabry-Perot interferometer and the Lummer-Gehrke interferometer[4] to give large values of spectral resolution, consequently the basic theory of the interference of multiple-reflected beams is well known and has been presented in a number of places of which the cited references[4,5] are representative. It may be shown[5] that if the incident wave has unit amplitude the amplitudes of the resultant reflected wave, $R_{\ell m}$, and the resultant transmitted wave, $T_{\ell mn}$, are given by:

$$R_{\ell mn} = r_{\ell m} + \frac{t_{\ell m}\, t_{m\ell}\, r_{mn}\, e^{j\,2k_m d_m}}{1 - r_{m\ell}\, r_{mn}\, e^{j\,2k_m d_m}} \tag{14}$$

$$T_{\ell mn} = \frac{t_{\ell m}\, t_{mn}\, e^{j\,k_m d_m}}{1 - r_{m\ell}\, r_{mn}\, e^{j\,2k_m d_m}} \,. \tag{15}$$

V. THE DUAL-LAYER FILM

In practice the films with which one has to deal are frequently so thin that they must be backed by a substrate for mechanical support. This is invariably the case for evaporated and sputtered films which are built up from zero thickness by deposition. The microscope slide is a commonly used substrate for deposited metal films.

The problem of reflection from, and transmission through, a film then is complicated by the presence of a substrate, for one has actually to deal with a dual-layer film comprising deposited film and substrate.

In Figure 1 all of the transmitted waves and all of the reflected waves except "a" may be regarded as having been produced by the first ray (of amplitude $t_{\ell m}$) transmitted by the ℓ-m interface. Let us define a "reduced reflection-coefficient," $X_{\ell mn}$, and a "reduced transmission-coefficient," $Y_{\ell mn}$, as follows:

$$X_{\ell mn} = \frac{R_{\ell mn} - r_m}{t_{\ell m}} , \tag{16}$$

$$Y_{\ell mn} = \frac{T_{\ell mn}}{t_{\ell m}} . \tag{17}$$

Figure 2 shows, in a schematic form, the various reflections and refractions taking place in a dual-film of media 1 and 2, thickness d_1 and d_2, respectively, immersed in medium 0. In practice medium 0 is air, medium 1 the deposited film, and medium 2 the substrate.

The incident wave of unit amplitude, labeled 1, is partially reflected (r_{01}) and partially transmitted (t_{01}). The transmitted portion gives rise to an infinite number of waves, of summed amplitude $t_{01}\ X_{012}$, back into medium 0, and another infinite set, of summed amplitude $t_{01}\ Y_{012}$, through to medium 2. The first of these sets has escaped the film. The second, on the other hand, because of the multiple reflections in medium 2, gives rise to a set of waves of summed amplitude $t_{01}\ Y_{012}\ Y_{120}$ which has escaped as a

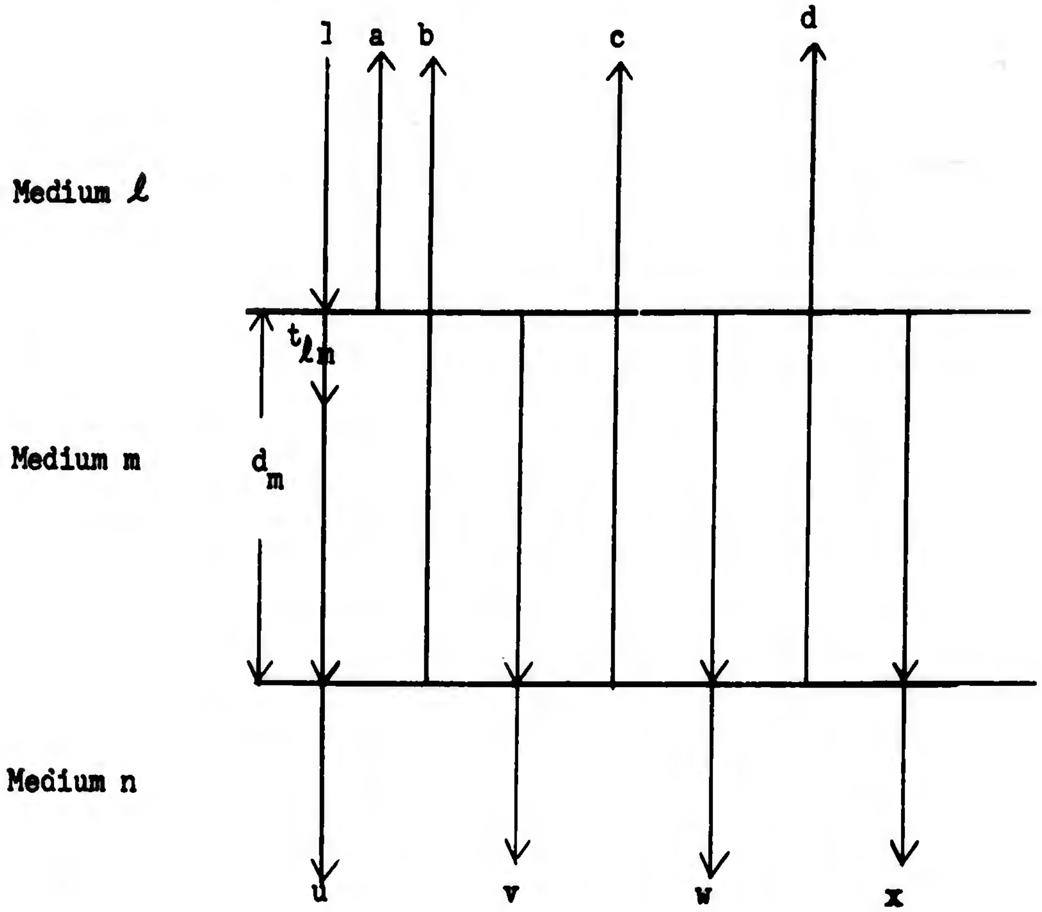

Figure 1. Schematic Representation of Multiple Reflections in a Film.

The arrows represent the direction of propagation of waves of infinite extent in space and time. They are offset in the diagram merely to make the representation more convenient. The arrow labeled 1 represents the incident wave. The total reflected amplitude $R_{\ell mn}$ is obtained from the infinite sum

$$R_{\ell mn} = a + b + c + d + \ldots\ldots \quad ,$$

and the total transmitted amplitude, T, from

$$T_{\ell mn} = u + v + w + x + \ldots\ldots \quad ,$$

also an infinite sum.

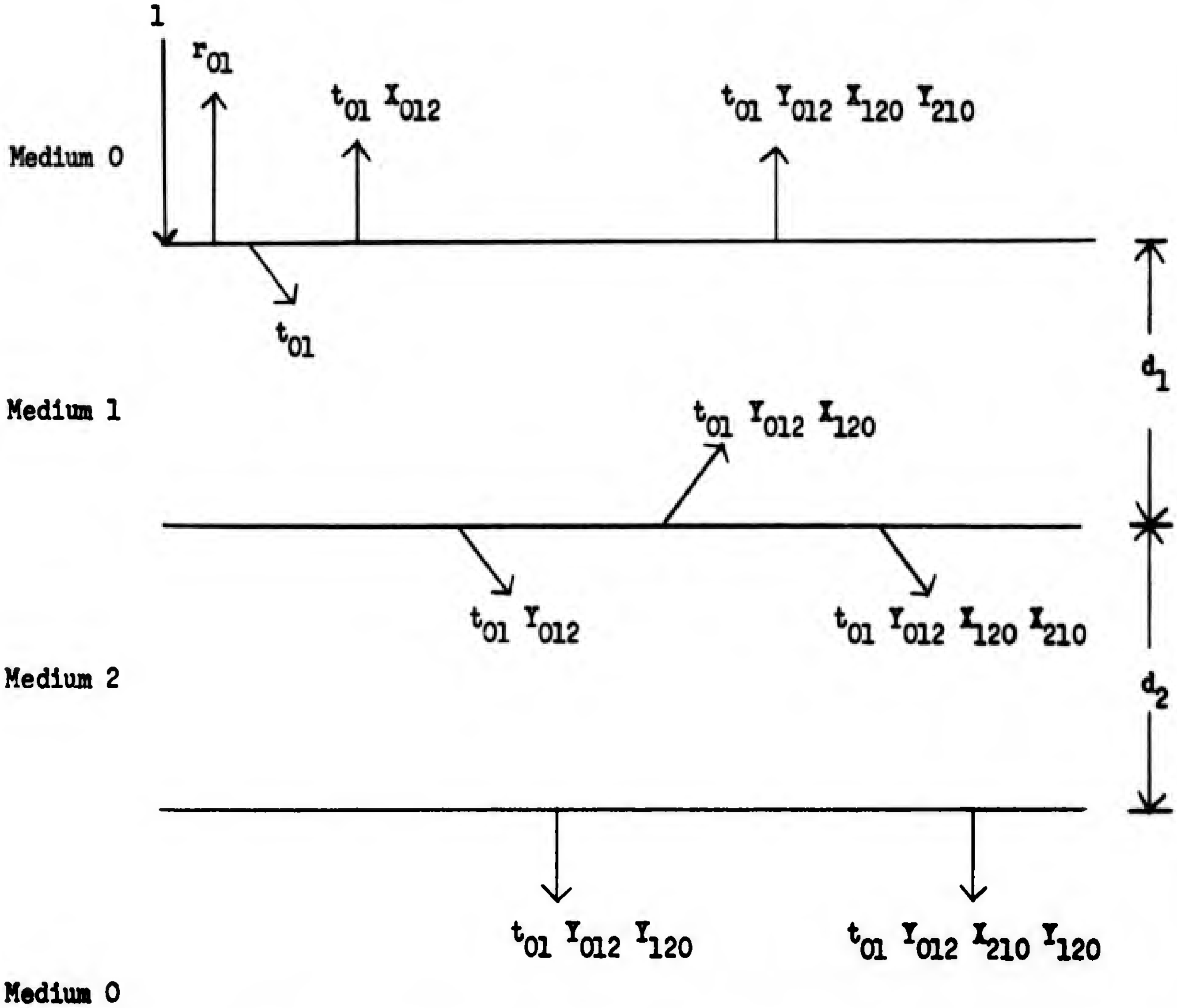

Figure 2. Schematic Representation of Reflection and Refraction by a Dual-Layer Film.

Each arrow except 1, r_{01}, and t_{01} represents the resultant of an infinite number of waves. Each slanting arrow generates the two which follow it to the right. The total reflected wave is the resultant of all the vertical arrows emerging from the 0-1 interface; the total transmitted wave is the resultant of all the vertical arrows emerging from the 2-0 interface.

part of the total transmitted wave, and a set of summed amplitude $t_{01} Y_{012} X_{120}$ which is still in medium 1. This process is continued ad infinitum and results in a doubly infinite set of both reflected and transmitted waves. The indicated summations yield

$$R_{0120} = r_{01} + t_{01} \ X_{012} + Y_{012} X_{120} Y_{210} (1 + X_{120} X_{210} + X_{120}^2 X_{210}^2 + \ldots) \ , \tag{18}$$

$$T_{0120} = t_{01} Y_{012} X_{120} (1 + X_{120} X_{210} + X_{120}^2 X_{210}^2 + \ldots) \ . \tag{19}$$

The infinite sum in the above equations is finite. Let it be represented by Z. Then

$$Z = \frac{1}{1 - X_{120} X_{210}} \ , \tag{20}$$

$$R_{0120} = r_{01} + t_{01} (X_{012} + Y_{012} X_{120} Y_{210} Z) \ , \tag{21}$$

and

$$T_{0120} = t_{01} Y_{012} X_{120} Z \ . \tag{22}$$

Equations (21) and (22) are general expressions for the amplitudes of the waves reflected and transmitted by a dual-layer film immersed in a homogeneous medium of infinite extent if the incident wave has unit amplitude and is incident normal to the film.

The values of R_{0120} and T_{0120} were calculated for a dual-layer film consisting of aluminum deposited on a glass microscope slide. The calculations were carried out for four different thicknesses of aluminum. A frequency of 10^{10} cycles per second was assumed. The glass was considered to have zero conductivity, a dielectric constant of 5, and to be 1.2 mm thick. The calculated values of R_{0120} and T_{0120} are given in Table I. The value used for ζ_0 was 376.6 ohms, the impedance of free space. In a waveguide application, a somewhat different value, depending on the ratio of wavelength to guide dimension, would be used.

TABLE I

Reflection and Transmission Coefficients from a Dual-Layer Film Consisting of a Thickness d_1 of Aluminum (Conductivity 3.54×10^7 $\text{ohm}^{-1}\ \text{m}^{-1}$) on a Glass Plate of Thickness 1.2 mm, Zero Conductivity, and Dielectric Constant 5; Frequency = 10^{10} Cycles/Second.

d_1 (Angstroms)	R_{0210}*	T_{0120}*
0	0	1
5	$-0.934\underline{/-0.382}$	$0.195\underline{/1.18}$
10	$-0.973\underline{/-0.178}$	$0.121\underline{/1.33}$
100	$-0.9998\underline{/-0.015}$	$0.011\underline{/1.55}$
1000	~ -1	~ 0

* (Magnitude$\underline{/\text{Phase Angle in Radians}}$)

VI. CONCLUSIONS

The few computations carried out for Table I show that for aluminum films 100A or more thick the reflection coefficient is so large and changing so slowly with thickness that it cannot be used as a sensitive thickness indicator. Films thinner than 100A are not of much interest since they cannot be reproduced accurately and certainly for values of thickness less thar 10A, where R_{0120} begins to vary appreciably with d_1, the electrical properties of the film are unknown and cannot be assumed to be those of the metal in bulk. The transmission coefficient is a somewhat more sensitive indicator of the film thickness, but at a thickness of only 100A the transmitted power is 40 db below the incident, making the method unattractive at best.

The situation would be improved if either the frequency of the wave or the conductivity of the film were decreased. The frequency cannot be made much less than 10^{10} c.p.s., however, without getting below the range of frequencies carried by available waveguides. On the other hand, the conductivity of many materials is much less than that of aluminum, and a thin film of semi-conductor, if such were available, could serve the purpose of introducing a desired amount of attenuation and phase shift. Rough calculations show that materials with conductivities in the range of 10^2 - 10^4 ohm^{-1} m^{-1} could be advantageously used in the form of deposited films of several hundreds of angstroms thickness to produce measurable attenuation and phase shift.

After this investigation was completed the attention of the author was called to some papers by E. A. Lewis and J. P. Casey[6,7,8], who show that a metal grid is superior to a metal film as a waveguide component.

Respectfully submitted,

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Approved by:

J E Boyd

J. E. Boyd, Associate Director
Engineering Experiment Station

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