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AN EXPERIMENTAL INVESTIGATION ON PLASTIC STRESS-STRAIN RELATIONS

by

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An Experimental Investigation on Plastic Stress-Strain Relations*

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INTRODUCTION

This paper describes experiments dealing with some current problems of the theory of plasticity. One of these problems is whether corners in the yield surface do exist; and, if they exist, to find conditions under which they appear. Experiments by Drucker and Stockton[1], Phillips[2], and Naghdi[3], indicate that corners in the yield surface do exist. Although Drucker[1] and Phillips[2] have shown that corners appear occasionally, Naghdi[3] has given examples of tests the results of which can be interpreted as indicating a regular appearance of corners.

In Naghdi's tests the loading paths are such that the direction of the increment of the stress vector changes considerably and abruptly at regular intervals. The material used in Naghdi's tests possessed considerable initial anisotropy, and the loading machine was of the straining type. Finally, the corners in his results appear in terms of negative plastic strain-increments.

Negative plastic strain-increments are quite unorthodox except when they are the result of recovery phenomena. For this reason the author decided to make a few tests with another, this time isotropic, material and a deadweight type of machine. In these tests the path of loading shows again at regular intervals very abrupt changes of the direction of the increment of the stress vector. The results of these tests can be interpreted as indicating the regular appearance of corners in the yield surface; but this time the corners are very obtuse and no negative plastic strain-increments are associated with these corners.

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A different interpretation of these results is that the yield surface depends slightly on the path of loading. When this last interpretation is adopted the regular appearance of corners is not necessary for explaining the results of these tests.

When corners do not appear the plastic-strain increment vector is normal to the yield surface, and from its direction it is possible to find whether the yield surface is the surface of constant octahedral shearing stress. In the experiments reported in this paper it has been found that the yield surface is very nearly the surface of constant octahedral shearing stress. This is equivalent to saying that the flow rule of the simple incremental theory of plasticity is valid with good approximation. Indeed, the flow rule is identical to the normality between the increment of the plastic-strain vector and the yield surface, and the constant octahedral shearing stress surface is the one which is used in the simple incremental theory of plasticity[4]. Therefore, the results of the present experiments extend to the loading paths used here the validity of the results obtained in previous tests[2,5].

When creep occurs, the yield surface remains stationary while the creep strains develop, provided the yield condition depends on the stresses only and not on the strains. Thus, it is expected that the plastic-strain increment vector will retain the same direction it had when only instantaneous strains were present. Our experiments indicate with good approximation that such is the case.

EXPERIMENTAL DETAILS

These experiments have been made on a tension-torsion combined stress machine shown in Fig. 1. A diagrammatic sketch of this machine is shown in Fig. 2.

The specimen g is attached to the frame by means of a collar f, machined hanger blocks b, c, and d, and a support plate e which has a spherical point support. At the lower end of the specimen there is a collar f on which the torsion wheel h is threaded. The vertical load platform n is suspended from the torsion wheel by means of a universal joint m. The weight pans k for the torsion load are suspended from the torque wheel by means of flexible cables i and the essentially frictionless pulleys j.

The block b is welded to the frame. The block c hangs from the block d. The support plate e rests on the block c, and the block d rests on the spherical point support of the plate e. The collar f hangs from the block d, and the specimen is threaded into f. Vertical lines--not shown in the figure--have been marked on f, d, and c, so that when these lines fall on the same vertical the axis of the specimen passes exactly through the spherical point support. Also, the different parts have been machined in such a manner that the axis of the vertical load and of the torque load coincide with the axis of the specimen.

As the specimen is loaded in torsion the lower collar f rotates with the two plates h and n. The upper collar f rotates by as much as the block d can rotate about the spherical point support until block d is stopped by block c; the amount of this rotation is very small. The vertical load platform rotates with the lower end of the specimen. Thus it is not necessary to introduce ball bearings for the vertical loading. The axial and the torque loads are measured directly by the dead weight loads applied on the vertical load platform and on the weight pans for the torque loads. A separate instrumentation for the measurement of the stresses is not necessary.

The specimen is shown in Fig. 3. The wall thickness of each specimen has been measured in 20 places to an accuracy of 0.0001 in. by a special instrument described in reference 6. For most tubes the variations of wall thickness from the average value were not as small as desired. Indeed, because of the difficulties in machining these tubes, it has not been possible to achieve always a satisfactory concentricity of the outer and inner boundaries of the cross section. Due to this lack of concentricity, the following maximum variations in inches of the wall thickness have been found: ± 0.0020 in tests G-1, G-3; ± 0.0040 in tests G-4, G-7; ± 0.0060 in tests G-5, G-6; ± 0.0130 in test G-2. The specimen in test G-2 has been used despite the substantial lack of concentricity, in order to see whehter this lack of concentricity, together with corrective measures explained below, would have much influence on the results; no influence has been observed.

The wall thickness to diameter ratio was 1:16. Despite this rather small ratio, no difficulties due to buckling did appear--probably because the axial tensile load acted as a preventive to buckling.

The strains have been measured by means of SR-4 strain gages. Measurements by the author[7] as well as published results in the literature[1] show that the accuracy of these gages is satisfactory for the range of strains encountered in these tests (to $3\frac{1}{2}\%$). The strains have been measured in three directions: vertical, horizontal, and 45°. To counteract the lack of concentricity, three gages connected in series have been used for measuring the strain in each direction; these three gages have been placed 120° apart around the tube. For each direction a separate strain-gage indicator was used, so that the readings were taken simultaneously for all three directions. In tests G-1, G-3, and G-5, however, only one indicator was used, which was connected to a switching unit; therefore, in these three tests the readings were taken consecutively.

In tests G-2, G-4, G-6, and G-7 the strains were measured half a minute after application of the load increment and then at 2 minute intervals until a sufficient number of creep readings had been taken--usually three readings, but sometimes as many as eight readings. In tests G-1, G-3, and G-5 the strains were read as soon as consecutive readings could be taken without occurrence of any appreciable increment of strain during the time interval needed for switching from the first to the last direction.

The material used in the tests was commercial pure aluminum in the annealed condition (2S-O). The conventional stress-strain diagram in tension for the material is shown in Fig. 4, in which the results of 4 tests with four different tubular specimens are shown. Of the 7 tests reported in this paper, test G-3 has been made with a used tube, while the other 6 tests were made with virgin tubes. The specimens have been machined from tubes of 1" outer diameter and of 0.125" wall thickness. Microscopic study of the material showed that the grains were equiaxed in the transverse direction. A very small amount of preferred orientation was visible in the longitudinal direction. The average size of the grains is 0.002 inches.

RESULTS OF THE TESTS

The tests described in this paper are of two types. In the first four tests G-1 to G-4, Figs. 5 and 6, the specimens are loaded first in tension, and then in torsion while the tensile load is being kept constant; afterwards, the loading process consists in either increasing σ while decreasing τ, or in increasing τ while decreasing σ. The reversals from increasing σ to decreasing σ, and vice versa, have been made very sharp; the octahedral shearing stress, however, is continuously increasing so that there is always loading. In the last three tests G-5 to G-7, Figs. 7 and 8, the specimens are loaded first in torsion, and then in tension while the shearing stress is being kept constant; afterwards the loading process is the same as in the first four tests.

In Figs. 5 to 8 with coordinates $(\sigma, \sqrt{3}\tau)$ the loading increment is represented by the segment between two consecutive points in the loading path. To each loading increment there corresponds another segment which starts at about the middle of the loading segment and is directed away from the origin. This segment is the plastic-strain increment vector corresponding to the loading increment. It is drawn in a $(\sqrt{3}d\epsilon'', d\gamma'')$ coordinate system superposed on the $(\sigma, \sqrt{3}\tau)$ system*. In Figs. 5 to 8, only the slopes of the plastic-strain increment vectors are shown. The plastic-strain increment vectors have been drawn beginning from a stage in the loading process in which the plastic-strain increments were large enough so that the slopes could be determined accurately. In some cases when the plastic-strain increments are not large enough for the accurate determination of the slope we combined the plastic-strain increments of two or even three load increments so that a more accurate determination of the slope could be made. This is the case, for example, with a portion of the path of G-1.

In combined tension and torsion the octahedral shearing stress is

$$\tau_0 = \frac{\sqrt{2}}{3}\left(\sigma^2 + 3\tau^2\right)^{1/2}$$

i.e., in Figs. 5 to 8, it is equal to $\sqrt{2}/3$ times the distance of the point representing the state of stress from the origin. The yield surface of constant

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*We remark that $d\epsilon''$ is the plastic axial strain increment and $d\gamma''$ is the plastic shearing strain increment.

τ_0 is represented by a circle with the origin as a center.

The requirement of normality between yield surface and plastic-strain increment vectors is expressed by the proportionality

$$d\varepsilon''/d\gamma'' = [\partial f/\partial\sigma]/[\partial f/\partial\tau] \qquad (1)$$

where $f = 0$ is the equation of the yield surface. When

$$f = (\sigma^2 + 3\tau^2)^{1/2}$$

equation (1) gives

$$d\varepsilon''/d\gamma'' = \sigma/3\tau$$

from which

$$\sqrt{3}\, d\varepsilon''/d\gamma'' = \sigma/\sqrt{3}\tau.$$

Hence, the normality is preserved when we use the coordinates $(\sigma, \sqrt{3}\tau)$ and $(\sqrt{3}d\varepsilon'', d\gamma'')$. Therefore, if the yield surface is the constant τ_0 surface, the plastic-strain increment vectors will be directed radially from the origin.

From Figs. 5 to 8 it is seen that as a first approximation the plastic-strain increment vectors are directed radially from the origin. Hence the yield surface is very nearly the surface of constant τ_0, and the flow rule of the simple incremental theory of plasticity is valid as a first approximation. A closer study of Figs. 5 to 8 reveals that the plastic-strain increment vectors deviate slightly from the radial direction, and as a general rule the deviation is in the direction of the loading increment. This behavior is most pronounced in tests G-4 and G-7, but it is apparent in the other tests also. Obviously each time the direction of the loading increment vector changes abruptly*,

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* In our tests the direction of the path of loading changed each time by an amount sometimes exceeding 320°.

the deviation of the plastic-strain increment vector changes sign. As mentioned previously, this phenomenon has been observed by Naghdi[3], but in combination with the appearance of negative plastic strain-increments; it has been attributed by him to the existence of corners in the yield surface. In our tests the phenomenon is shown to appear without negative plastic strain-increments. It can be explained by assuming that corners do appear; these corners, however, would then be very obtuse. A much simpler explanation would be that the yield surface depends on the path of loading and rotates slightly when the direction of the path of loading changes by a substantial angle very abruptly. With this explanation we do not need to assume that corners appear every time the direction of the path of loading changes abruptly by a substantial angle. Corners, however, do appear occasionally, as shown from nearly all the tests. These corners are indicated when the plastic-strain increment vector changes its direction from one step to the next in an abrupt way, while the loading increments keep their directions constant.

Although negative plastic strain-increments did not appear in combination with the deviation of the plastic-strain increment vector mentioned above, in test G-2 one negative plastic strain did appear.

As mentioned previously, creep strains have been recorded in tests G-2, G-4, G-6, and G-7. In Fig. 5 the two dashed pastic-strain increment vectors represent the slopes of the plastic strain increment vectors including creep, for two particular instances. The difference in the slope between the dashed vector and the solid one represents the deviation due to additional plastic strain produced because of creep. It is seen that in the one instance this deviation is negligible, and this case is representative of most of our measurements. In the other instance the deviation is not negligible, but it is not too large; this case represents the largest deviation observed in these tests.

We conclude therefore that the direction of the increment of the plastic strain vector for both plastic deformation and primary creep is approximately the one predicted by the simple incremental theory of plasticity.

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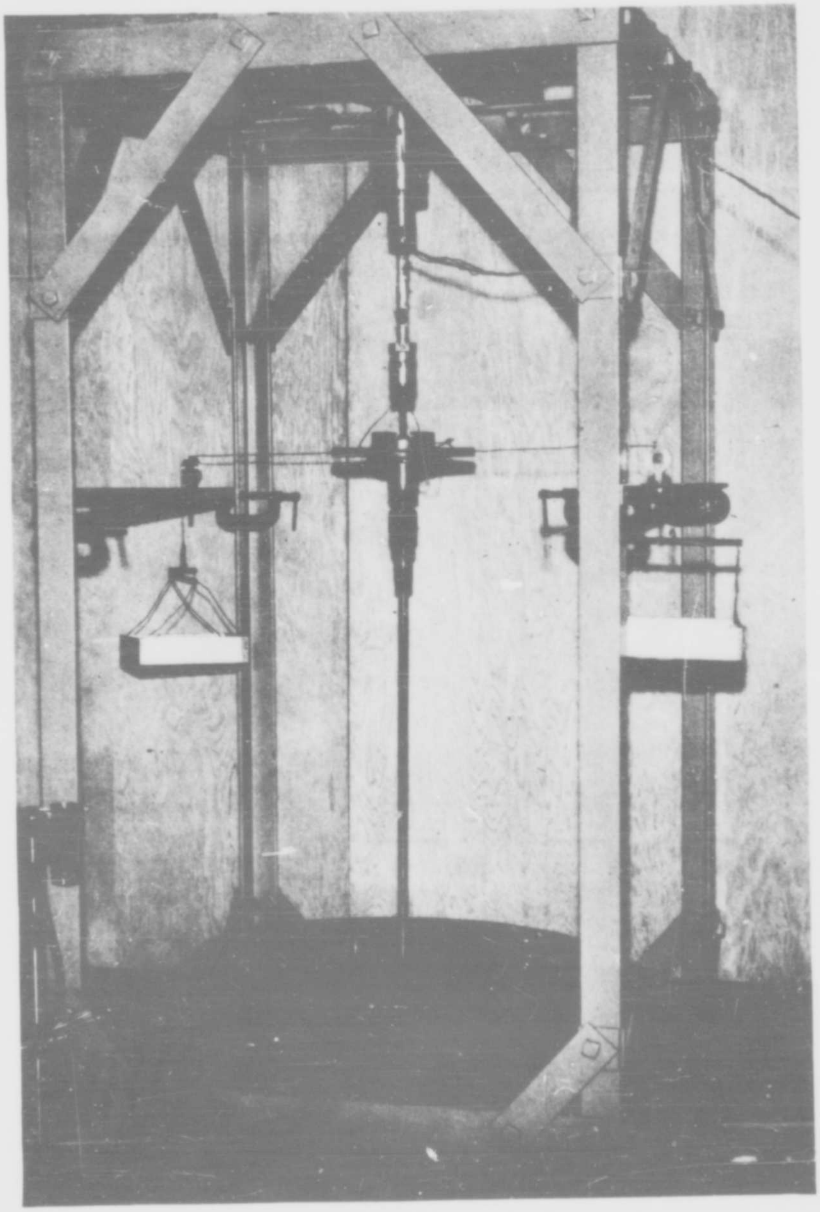


Fig. 1

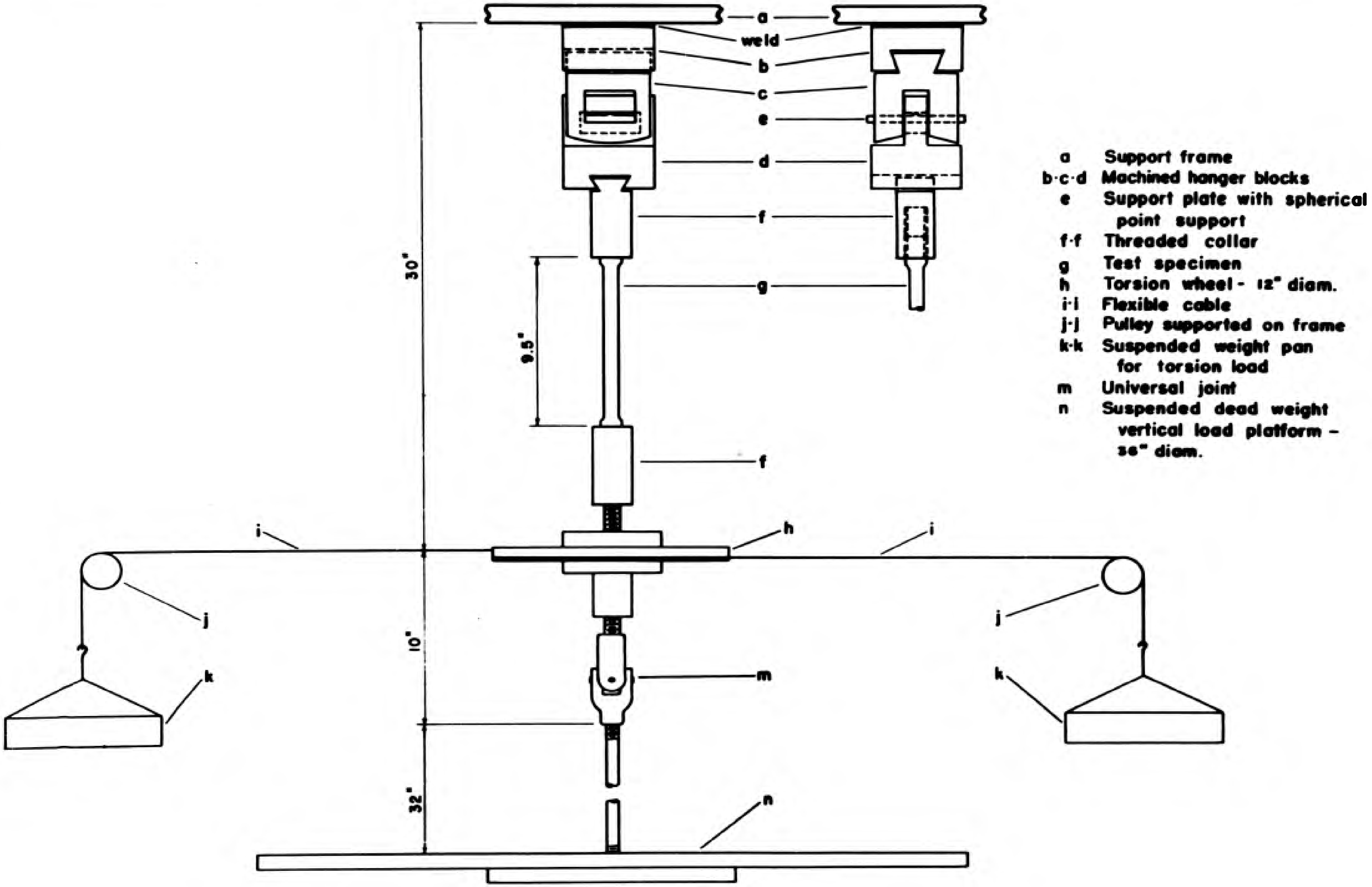
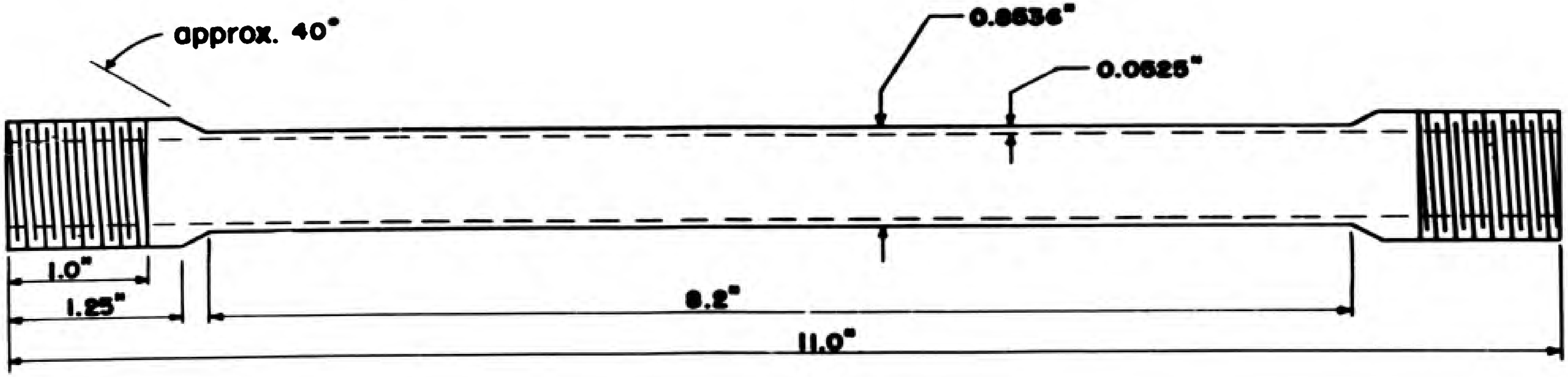

a
weld
b
c
e
d
f
g
f
h
i
i
j
j
k
k
m
n
30"
9.5"
10"
32"
a Support frame
b·c·d Machined hanger blocks
e Support plate with spherical point support
f·f Threaded collar
g Test specimen
h Torsion wheel - 12" diam.
i·i Flexible cable
j·j Pulley supported on frame
k·k Suspended weight pan for torsion load
m Universal joint
n Suspended dead weight vertical load platform - 36" diam.

Fig.2



Tube No. G-4

Fig.3

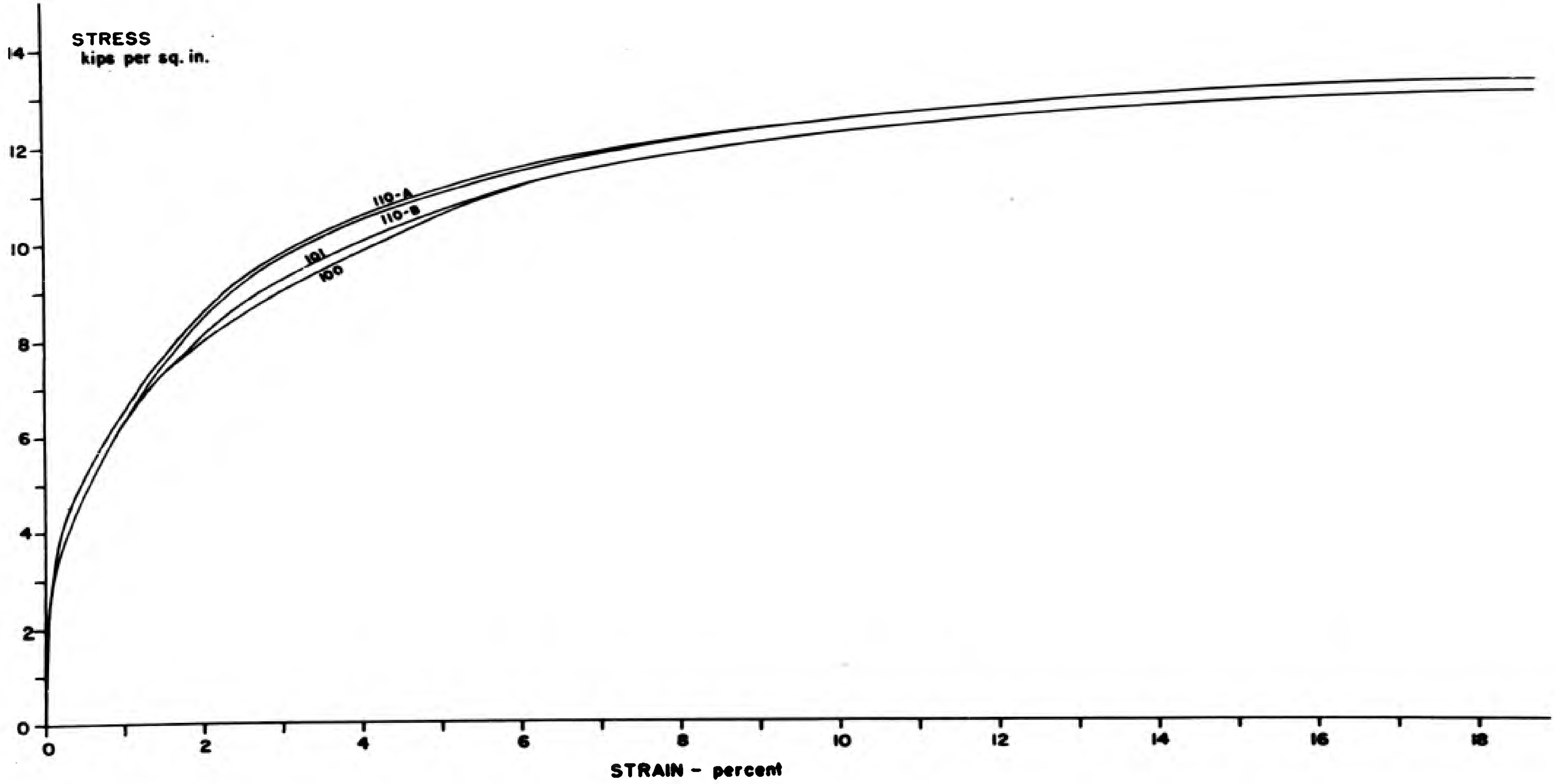
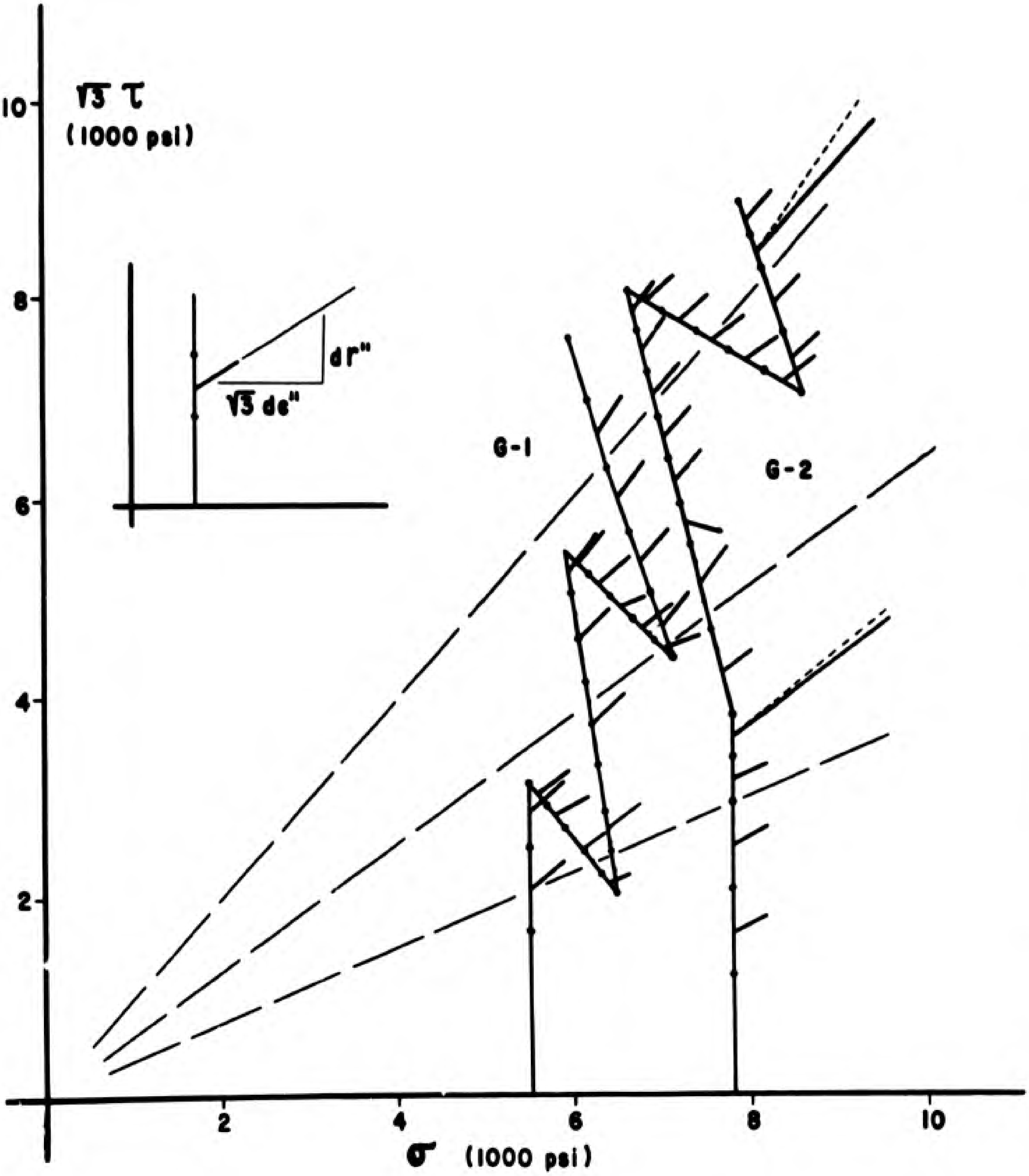

STRESS
kips per sq. in.
14
12
10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
18
STRAIN - percent
110-A
110-B
101
100

Fig.4



$\sqrt{3}\,\tau$
(1000 psi)
10
8
6
4
2
2
4
6
8
10
σ (1000 psi)
$d\gamma''$
$\sqrt{3}\,d\epsilon''$
G-1
G-2

Fig.5

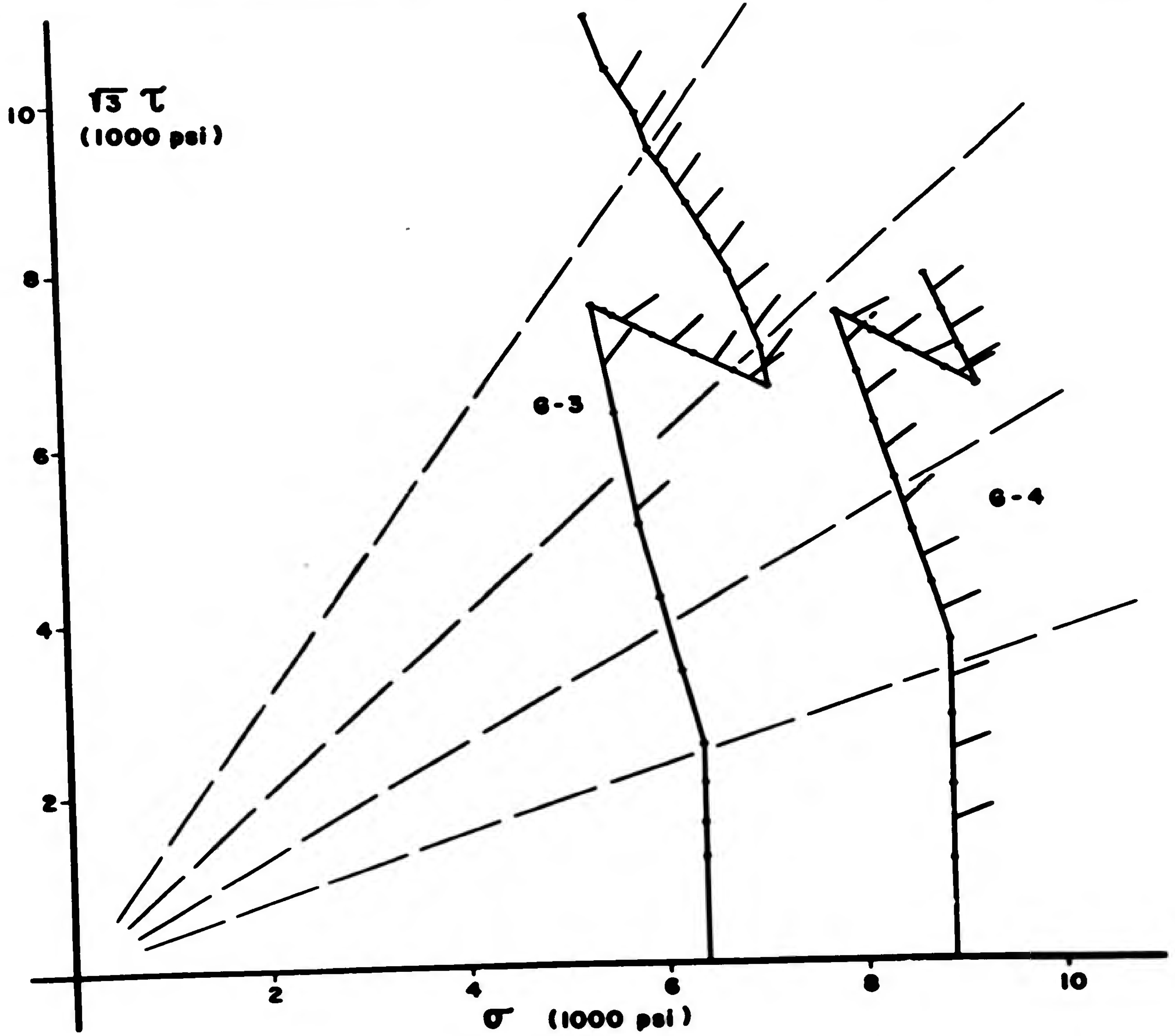


Fig.6

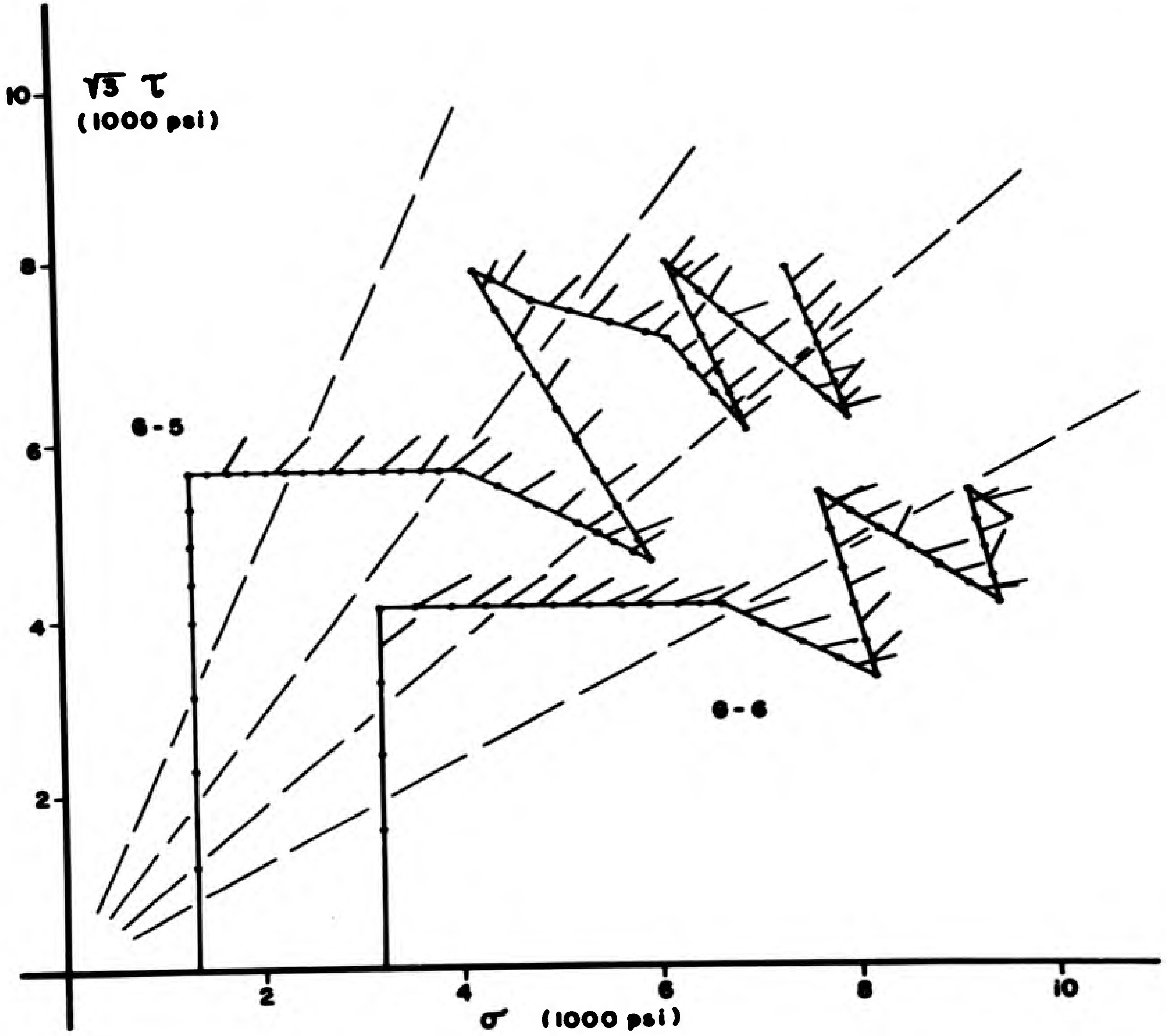


Fig.7

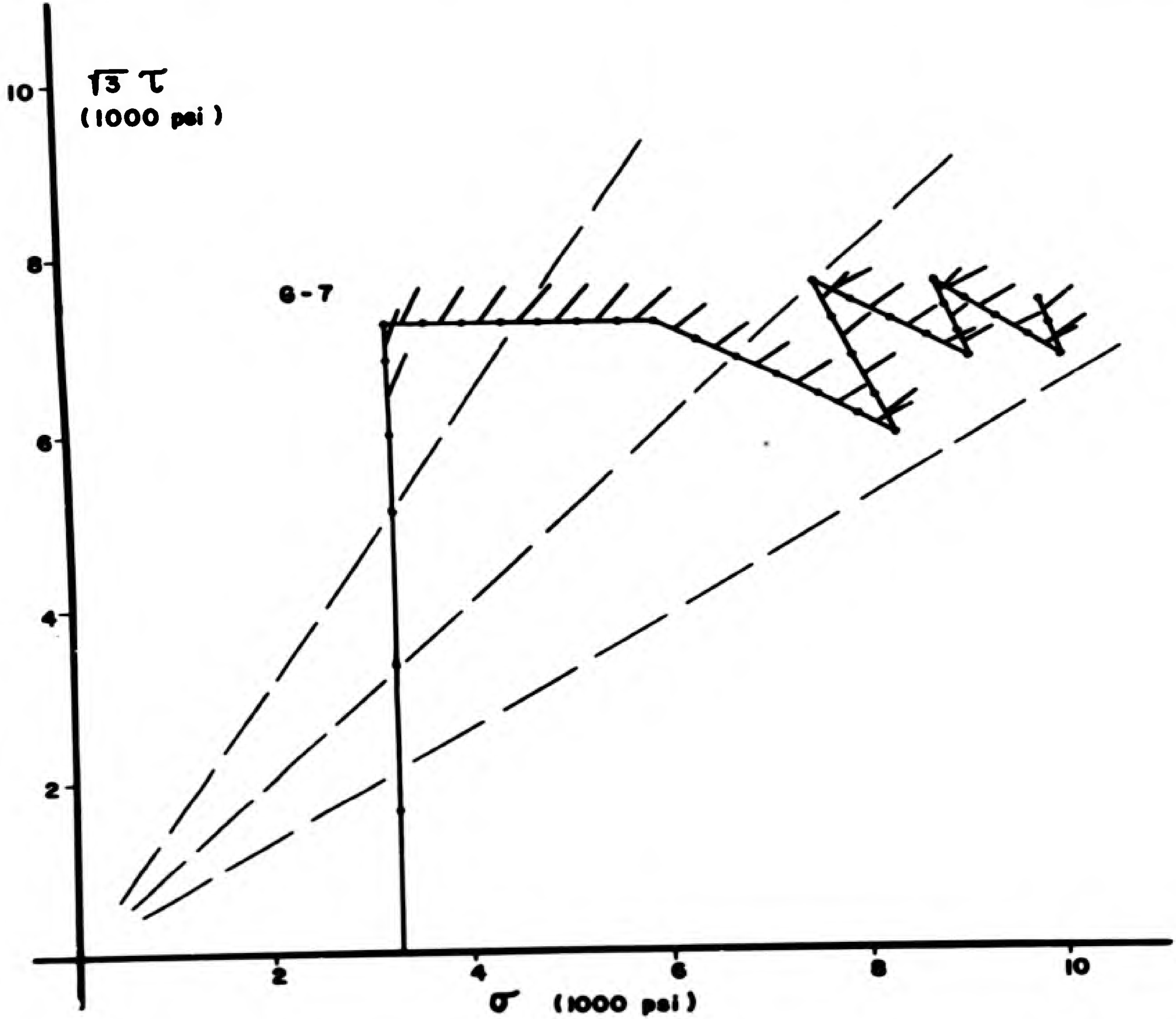


Fig.8

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