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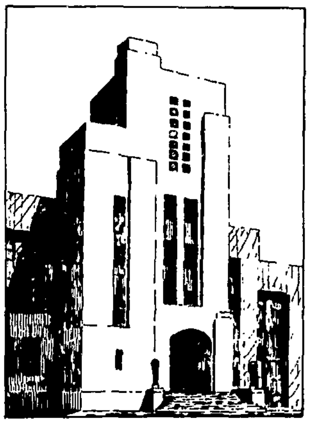
AN APPROXIMATE METHOD OF OBTAINING STRESS IN A PROPELLER BLADE

by

William B. Morgan



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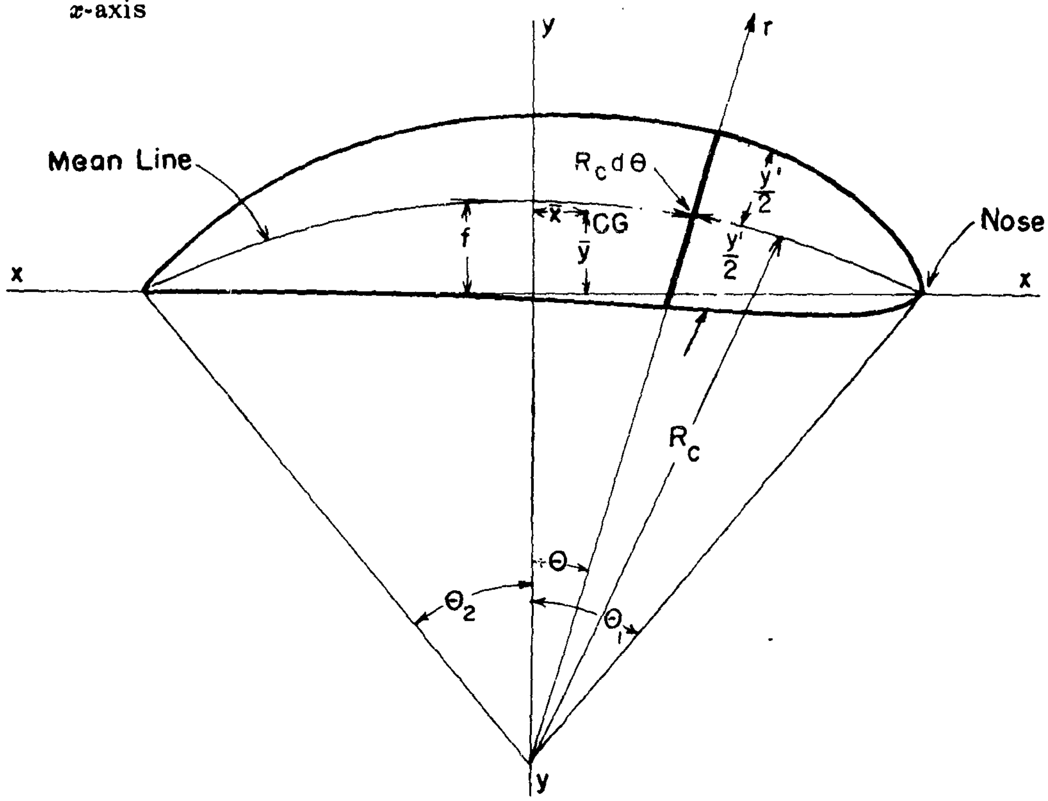
Errata for Report 919

Page

iv	Reads	Z	Number of blades
	Should read	z	Number of blades

4 The lower arrow indicating $y'/2$ should be at the lower heavy line instead of the x-axis



8 Reads $x_3 = x_1 -$ --------) $\sin \frac{0.06387\ S}{R_c}$

Should read $x_3 = x_1 -$ --------) $\sin \frac{0.06387\ s}{R_c}$

8 Reads $y_3 = (\quad) \cos \frac{0.06387\ S}{R_c} +$ -------

8 Should read $y_3 = (\quad) \cos \frac{0.06387\ s}{R_c} +$ -------

12 Reads $y_1 = (\quad)$

Should read $y_1 = -(\quad)$

15 Reads $MQ_b =$ -----------

Should read $M_{Q_b} =$ -----------

17 Reads $= 4.958$

Should read $= 4.958\ \mathrm{ft}^4$

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NOTATION

A	Area of section
a	Coefficient of area
b	Coefficient of I_{x_0}
c	Coefficient of I_{y_0}
c_L	Lift coefficient
c_{T_i}	Thrust coefficient for nonviscous flow
D	Diameter of propeller
f	Maximum ordinate of mean line (camber line)
I_{x_0}	Moment of inertia about an axis parallel to the x-axis and passing through the center of gravity
I_{xx}	Moment of inertia about the x-axis
I_{y_0}	Moment of inertia about an axis parallel to the y-axis and passing through the center of gravity
I_{yy}	Moment of inertia about the y-axis
l	Length of blade section
M_{Q_b}	Bending moment from torque per blade
M_{T_b}	Bending moment from thrust per blade
M_{x_0}	Bending moment about abscissa with reference to the center of gravity
M_{y_0}	Bending moment about ordinate with reference to the center of gravity
P	Pitch at any radius
Q_b	Torque per blade
Q_i	Torque for nonviscous flow
R	Radius of propeller
R_c	Radius of mean line
r	Variable radius
r_0	Radius of any propeller blade section
s	Length of mean line
t	Maximum thickness of section
T_b	Thrust per blade
T_i	Thrust for nonviscous flow
V_a	Speed of advance of propeller

x' Distance along mean line

$\bar{x}$ Abscissa of center of gravity with reference to midpoint of section

x_1 Abscissa of nose with reference to axis through center of gravity

x_2 Abscissa of tail with reference to axis through center of gravity

x_3 Abscissa of point of maximum thickness with reference to axis through center of gravity

$y'/2$ Ordinate of the section measured perpendicular to the mean line

$\bar{y}$ Ordinate of center of gravity with reference to nose-tail line

y_1 Ordinate of nose with reference to axis through the center of gravity

y_2 Ordinate of back with reference to axis through the center of gravity

y_3 Ordinate of point of maximum thickness with reference to axis through the center of gravity

z Number of blades

β_i Hydrodynamic pitch angle

ϵ Drag-lift ratio

θ Angle of arc of the mean line

ρ Density

ϕ $\mathrm{Tan}^{-1}\left(\frac{P/D}{\pi x}\right)$ = Corrected hydrodynamic pitch angle

ABSTRACT

An approximate method of obtaining stress in a propeller blade from simple beam theory is presented. The method is designed to minimize the work required for calculating the geometric properties of the blade section. An example is given with this method applied to a modern thickness form.

INTRODUCTION

The stress calculation usually applied to ship propellers was developed by D.W. Taylor.[1] The basic assumptions are that the simple beam theory is applicable to a propeller blade and that thrust and torque depend linearly on the radius. Usually the maximum stress is determined for only one section of the blade close to the hub. The thickness of the sections is then assumed to vary linearly with the distance from the hub.

As to the first assumption, a closer approximation to the stresses can be obtained if the propeller blade is considered as a cantilever plate[2] rather than as a simple beam. However, the time required for determining the stresses of a cantilever plate appears to be excessive for the present application. Therefore, the assumption of the simple beam theory is retained in the following consideration. As to the dependence of thrust and torque on the radius, the circulation theory, developed since Taylor's work, makes possible the calculation of thrust and torque distributions on a propeller blade. Assuming these distributions to be known, the bending moments and stresses at any section can be calculated; thus the minimum thickness of a section at any radius may be determined.

The problem then becomes one of obtaining the different geometric properties of the blade sections. Muckle[3] has developed a simple method of obtaining the center of gravity and moments of inertia for ogival sections. This method enables the designer to obtain minimum thickness for these sections without numerical integration for the geometric properties of the section. It is important that the blade thickness be kept at a minimum for efficiency and cavitation considerations; therefore, there is a need for a method of obtaining the geometric properties of sections of any shape quickly and accurately. In principle, these properties can be derived by means of numerical integration; however, this is time consuming.

An approximate method is developed in this report for determining the stress in a propeller blade and an example is given of its use. A method for determining the geometric properties of a blade section (similar to that given in Reference 3) is given for the TMB EPH thickness form superimposed on several mean lines. By introducing correction factors, this has been applied to other thickness forms in use today, i.e., NACA 16 and NACA 65A superimposed on the same mean lines.

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[1]References are listed on page 18.

STRESSES FROM BEAM THEORY

The stresses in a propeller blade are caused by thrust, torque, and centrifugal force. The most important stresses are the bending stresses due to the thrust and torque. Bending stress due to centrifugal force requires consideration when the blades are raked. The other stresses, that is, shear caused by thrust and torque and tension caused by centrifugal force, are usually negligible.

For this particular development, only the bending stresses caused by thrust and torque will be considered (Figure 1). For the determination of stress caused by centrifugal force see References 1 and 3.

Assuming that the thrust and torque distribution (dT_b/dr and dQ_b/dr) on the propeller blade are known, the bending moment at radius r_0 caused by the thrust is given as:

$$M_{T_b} = \int_{r_0}^{R} (r - r_0) \frac{dT_b}{dr} dr$$

and that by the torque as:

$$M_{Q_b} = \int_{r_0}^{R} \left(\frac{r - r_0}{r}\right) \frac{dQ_b}{dr} dr$$

where R is the radius of the propeller and r is a variable radius.

The bending moments are resolved into two moments: M_{x_0} about an axis parallel to the nose-tail line of the section and the other M_{y_0} perpendicular to this line. Both of these axes x_0 and y_0 pass through the center of gravity (centroid) of the blade section (Figure 2). When the pitch angle ϕ and the moments M_{T_b} and M_{Q_b} are known, the bending moments about the x_0- and y_0-axes are given as follows:

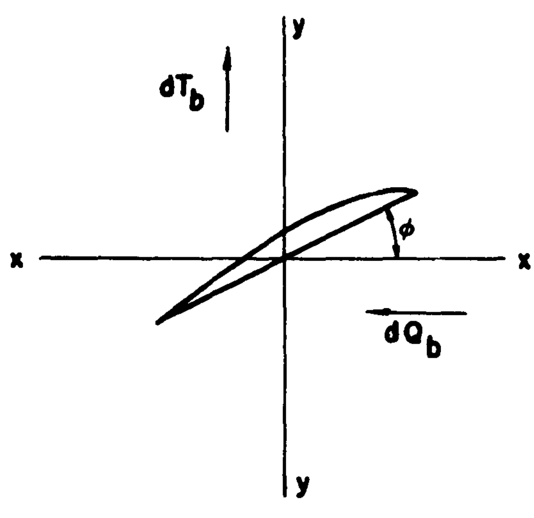


Figure 1 - Forces on a Blade Section

$$M_{x_0} = M_{T_b} \cos \phi + M_{Q_b} \sin \phi$$

$$M_{y_0} = M_{T_b} \sin \phi - M_{Q_b} \cos \phi$$

The positive sense of each of these moments is indicated in Figure 2.

If I_{x_0} and I_{y_0} are the moments of inertia about the x_0-and y_0-axes, the stresses in the blade are given by the following relationships:

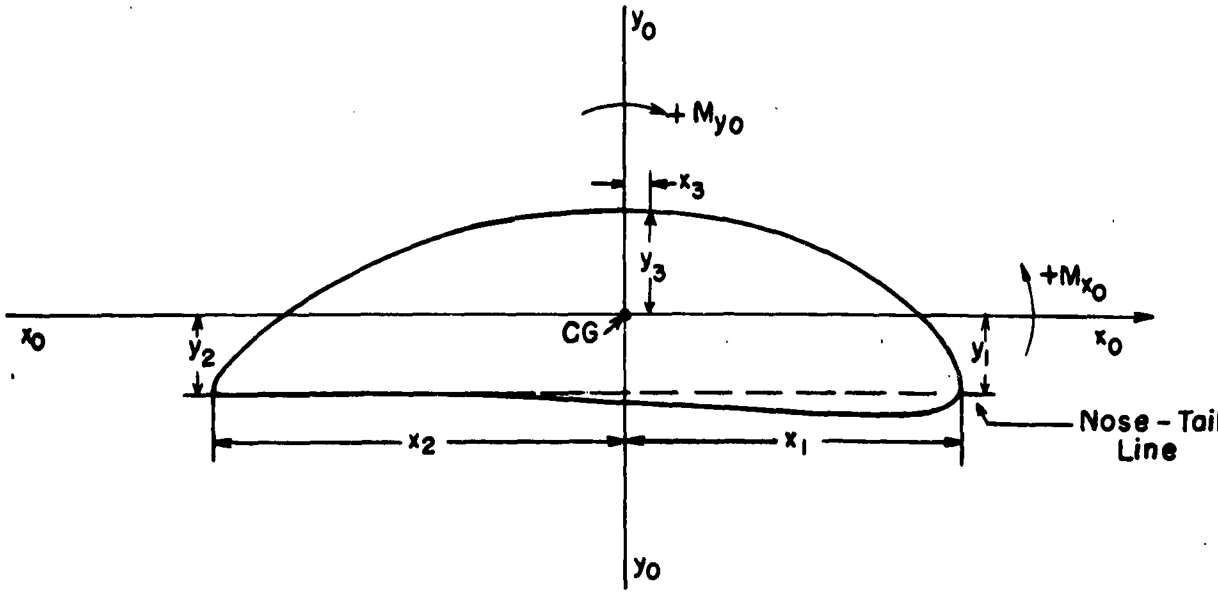


Figure 2 - Geometric Properties of a Blade Section

$$\text{Stress in leading edge} = -\frac{y_1 M_{x_0}}{I_{x_0}} - \frac{x_1 M_{y_0}}{I_{y_0}}$$

$$\text{Stress in trailing edge} = -\frac{y_2 M_{x_0}}{I_{x_0}} - \frac{x_2 M_{y_0}}{I_{y_0}}$$

$$\text{Stress on back at point of maximum thickness} = -\frac{y_3 M_{x_0}}{I_{x_0}} - \frac{x_3 M_{y_0}}{I_{y_0}}$$

A positive stress denotes tension and a negative stress denotes compression.

As shown in Figure 2, x_1, x_2, and x_3 are abscissas of the nose, tail, and point of maximum thickness, respectively. Likewise, y_1, y_2, and y_3 are ordinates of the nose, tail, and point of maximum thickness. It should be noted that the x- and y-coordinates must be used with their proper signs in order to obtain the correct signs for the stresses from the above formulas.

QUANTITIES ARISING FROM THE GEOMETRY OF THE SECTION

Once the bending moments are derived, the problem becomes one of obtaining the geometric properties of the blade sections. These properties are evaluated by solving the following integrals between their proper limits in which the x-axis is the nose-tail line and the

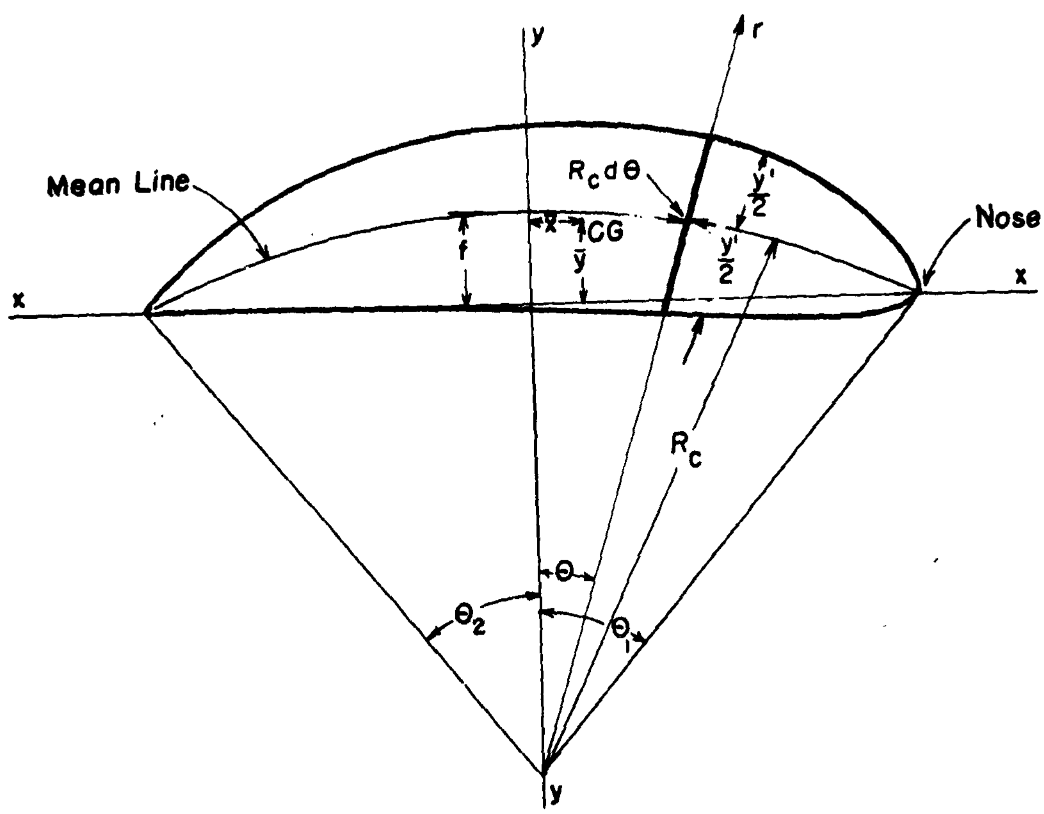


Figure 3 - Coordinates of Blade Section

y-axis is a line perpendicular to and passing through the midpoint of the nose-tail line (Figure 3).

Area

$$A = \iint dy\, dx$$

Horizontal distance to center of gravity

$$\bar{x} = \frac{\iint x\, dydx}{A}$$

Vertical distance to center of gravity

$$\bar{y} = \frac{\iint y\, dydx}{A}$$

Moment of inertia about x-axis

$$I_{xx} = \iint y^2\, dydx$$

Moment of inertia about y-axis

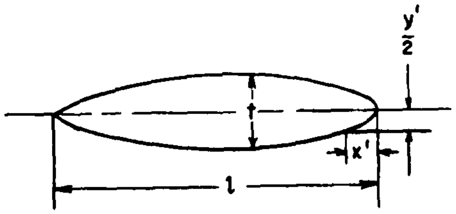
$$I_{yy} = \iint x^2\, dydx$$

These integrals are evalutated for several section forms.

TABLE 1

Offsets for TMB EPH Sections

x'/l	$\frac{y'/2}{t}$	x'/l	$\frac{y'/2}{t}$
0	0	0.500	0.4946
0.005	0.0756	0.550	0.4830
0.010	0.1064	0.600	0.4647
0.020	0.1498	0.650	0.4399
0.040	0.2092	0.700	0.4085
0.070	0.2717	0.750	0.3705
0.100	0.3186	0.800	0.3260
0.150	0.3774	0.850	0.2749
0.200	0.4204	0.900	0.2170
0.250	0.4522	0.930	0.1778
0.300	0.4750	0.950	0.1480
0.350	0.4902	0.970	0.1129
0.400	0.4983	0.990	0.0642
0.43613	0.5000	1.000	0
0.450	0.4997		



Nose: $0 \leqq x'/l \leqq 0.4361302$

$$\left(1-\frac{x'/l}{0.4361302}\right)^2+4\left(\frac{y'/2}{t}\right)^2=1$$

Body: $0.4361302 \leqq x'/l \leqq 0.8722604$

$$\frac{1}{2}\left(\frac{x'/l}{0.4361302}-1\right)^2+2\left(\frac{y'/2}{t}\right)=1$$

Tail: $0.8722604 \leqq x'/l \leqq 1$

$$\left(1+\frac{1-x'/l}{0.3083906}\right)^2-16\left(\frac{y'/2}{t}\right)^2=1$$

The radius of the nose is $0.5732\ t^2/l$, the radius of the tail $0.2027\ t^2/l$

EPH - THICKNESS FORM, CIRCULAR ARC MEAN LINE

The usual method of approach is to solve the preceding integrals by the use of Simpson's Rule. A direct solution can be made if the section chosen can be expressed mathematically. The TMB EPH thickness form (see Table 1) is formed of parts of an ellipse, a parabola, and a hyperbola; therefore, it can be expressed mathematically. Likewise, a circular arc mean line can be represented by a simple mathematical expression. For this reason, the foregoing integrals were solved using the EPH section in combination with a circular arc mean line.

The method of solution was first to transform the formulas both for the three parts of the EPH section and for the geometric properties into polar coordinates r and θ (Figure 3).

Thus, using polar coordinates, the formulas are expressed as follows, where $y'/2$ is the ordinate of the section measured perpendicularly to the mean line:

$$A = \int_{\theta_1}^{\theta_2} \int_{(R_c - y'/2)}^{(R_c + y'/2)} r\,dr\,d\theta = \int_{\theta_1}^{\theta_2} y' R_c\,d\theta \qquad [1]$$

$$\bar{x} = \frac{\int_{\theta_1}^{\theta_2} \int_{(R_c - y'/2)}^{(R_c + y'/2)} r^2 \sin\theta\,dr\,d\theta}{A} = \frac{\int_{\theta_1}^{\theta_2} R_c^2 \sin\theta\, y' \left(1 + \frac{1}{12}\frac{y'^2}{R_c^2}\right) d\theta}{A}$$

$$\doteq \frac{\int_{\theta_1}^{\theta_2} R_c^2 \sin^2\theta\, y'\,d\theta}{A}, \text{ when } t/l \leqq 0.21 \text{ and } f/l \leqq 0.05 \qquad [2]$$

$$\bar{y} = \frac{\int_{\theta_1}^{\theta_2} \int_{(R_c - y'/2)}^{(R_c + y'/2)} r[r\cos\theta - (R_c - f)]\,dr\,d\theta}{A} \qquad [3]$$

$$I_{xx} = \int_{\theta_1}^{\theta_2} \int_{(R_c - y'/2)}^{(R_c + y'/2)} r[r\cos\theta - (R_c - f)]^2\,dr\,d\theta \qquad [4]$$

$$I_{yy} = \int_{\theta_1}^{\theta_2} \int_{(R_c - y'/2)}^{(R_c + y'/2)} r^3 \sin^2\theta\,dr\,d\theta = \int_{\theta_1}^{\theta_2} R_c^3 \sin^2\theta\, y' \left(1 + \frac{1}{4}\frac{y'^2}{R_c^2}\right) d\theta$$

$$\doteq \int_{\theta_1}^{\theta_2} R_c^3 \sin^2\theta\, y'\,d\theta, \text{ when } t/l \leqq 0.21 \text{ and } f/l \leqq 0.05 \qquad [5]$$

Within the range: $0.06387 \frac{s}{R_c} \leqq \theta \leqq 0.5 \frac{s}{R_c}$

$$y' = \frac{t}{0.43613}\sqrt{0.18613 + 0.12774\frac{R_c\theta}{s} - \frac{R_c^2\theta^2}{s^2}} \qquad [6]$$

Within the range: $-0.37226 \frac{s}{R_c} \leqq \theta \leqq 0.06387 \frac{s}{R_c}$

$$y' = 2.62868\, t\left(0.37634 + 0.12774\frac{R_c\theta}{s} - \frac{R_c^2\theta^2}{s^2}\right) \qquad [7]$$

Within the range: $-0.5 \frac{s}{R_c} \leqq \theta \leqq -0.37226 \frac{s}{R_c}$

$$y' = \frac{t}{0.61678}\sqrt{0.55839 + 1.61678\frac{R_c\theta}{s} + \frac{R_c^2\theta^2}{s^2}} \qquad [8]$$

In these equations, R_c is the radius, f is the maximum height, and s is the length of the mean line and l is the length of the nose-tail line.

The next step involves substituting Equations [6], [7], and [8] into Equations [1], [2], [3], [4], and [5]. It will be noted that when these substitutions are made, integrands of the form $\sin {}^n\theta\sqrt{f(\theta)}$ and $\cos {}^n\theta\sqrt{f(\theta)}$ are obtained. An approximate solution for these integrals can be obtained by expanding $\sin\theta$ and $\cos\theta$. Assuming $\sin\theta = \theta - \theta^3/6$ and $\cos\theta = 1 - \theta^2/2 + \theta^4/24$, a very close approximation will be obtained since the camber ratio f/l is normally less than 0.05. For this value of f/l, $\sin\theta$ and $\cos\theta$ are correct to four places.

This integration is performed separately for each of the three parts of the blade section. By adding the results of these separate integrations, the geometric properties of the whole section are obtained as follows:

$$A = 0.746\, st$$

$$\bar{x} = 0.027\, s + 0.00528\frac{s^3}{R_c^2}$$

$$\bar{y} = f - 0.0256\frac{s^2}{R_c} + 0.06\frac{t^2}{R_c}$$

$$I_{xx} = 0.04487\, s\, t^3 + 0.08975\frac{s\, t^3 f}{R_c} + 0.001625\frac{s^3t^3}{R_c^2}$$

$$+ 0.746\, stf^2 - 0.04468\frac{s^3tf}{R_c} + 0.00188\frac{s^5t}{R_c^2}$$

$$I_{yy} = 0.04469\ s^3t - 0.012943\ \frac{s^5t}{R_c^2}$$

For use in the stress formula, the moments of inertia must be taken about the center of gravity and x_1, x_2, x_3, y_1, y_2, and y_3 must be obtained from $\bar{x}$ and $\bar{y}$ (Figure 2). Thus the following equations result:

$$A = 0.746\ st$$

$$x_1 = 0.5l - 0.027s - 0.00528\ s^3/R_c^2$$

$$x_2 = x_1 - l$$

$$x_3 = x_1 - 0.5\,l + (0.5\,t + R_c)\sin\frac{0.06387\ s}{R_c}$$

$$y_1 = 0.0256\,\frac{s^2}{R_c} - 0.060\,\frac{t^2}{R_c} - f$$

$$y_2 = y_1$$

$$y_3 = (0.5\,t + R_c)\cos\frac{0.06387\ s}{R_c} + f - R_c + y_1$$

$$I_{x_0} = I_{xx} - A\,\bar{y}^2 = \left[0.04487\ t^2 + 0.00391\ \frac{s^2t^2}{R_c^2} - 0.00645\,\frac{s^2f}{R_c} + 0.00139\,\frac{s^4}{R_c^2} - 0.00269\,\frac{t^4}{R_c^2}\right] s\,t$$

$$I_{y_0} = I_{yy} - A\,\bar{x}^2 = \left[0.04415 - 0.01298\,\frac{s^2}{R_c^2} - 0.000021\,\frac{s^4}{R_c^4}\right] s^3t$$

Since from geometry

$$s = \frac{1}{3}\left(4\sqrt{1 + 4(f/l)^2}\ -1\right) l \quad \text{(for small cambers),}$$

$$R_c = \left[\frac{1 + 4(f/l)^2}{8\ f/l}\right] l$$

and noting that $t = (t/l)\,l$ and $f = (f/l)l$, the equations can be written in terms of the section length, thickness ratio, and camber ratio.

These equations are cumbersome, but their use can be simplified by limiting the thickness ratio to the maximum value of 0.21 and the camber ratio to 0.05. The equations are then evaluated for a number of different thickness and camber ratios. By plotting the resulting values, the formulas may then be reduced to the following forms:

$A = (a\ t/l)\ l^2 \qquad a$ from Figure 4

$x_1 = (0.47325 - 0.026\ f/l)\ l$

$x_2 = x_1 - l$

$x_3 = x_1 - 0.4361\ l + 0.17 \left(\frac{f}{l}\right)^2 l$

$y_1 = -(0.114\ t/l + 0.79)\ (f/l)\ l$

$y_2 = y_1$

$y_3 = 0.5\ t + 0.984\ f + y_1$

$I_{x_0} = [b\ (f/l)^2 + 0.04487\ (t/l)^3]\ l^4 \quad b$ from Figure 5

$I_{y_0} = (c\ t/l)\ l^4 \quad c$ from Figure 6

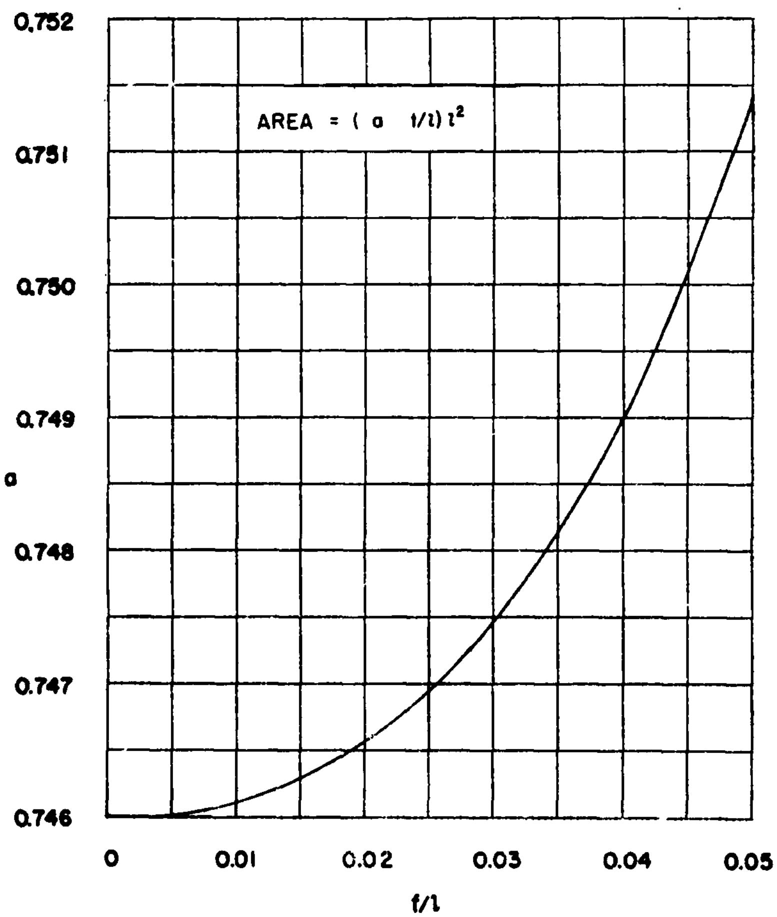


Figure 4 - Curve for Obtaining Area of a Blade Section

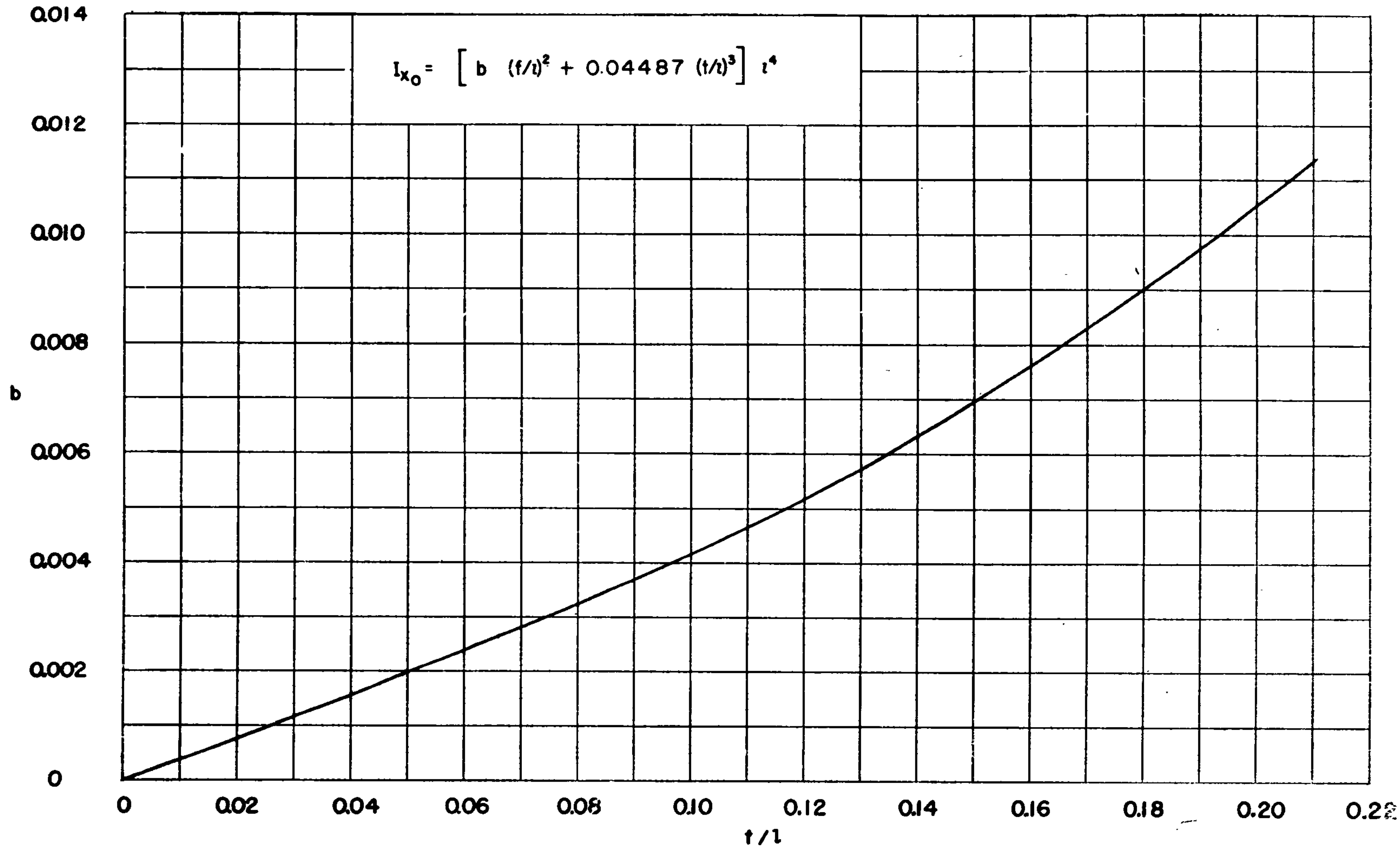


Figure 5 - Curve for Obtaining Moment of Inertia (I_{x_0}) of Blade Section

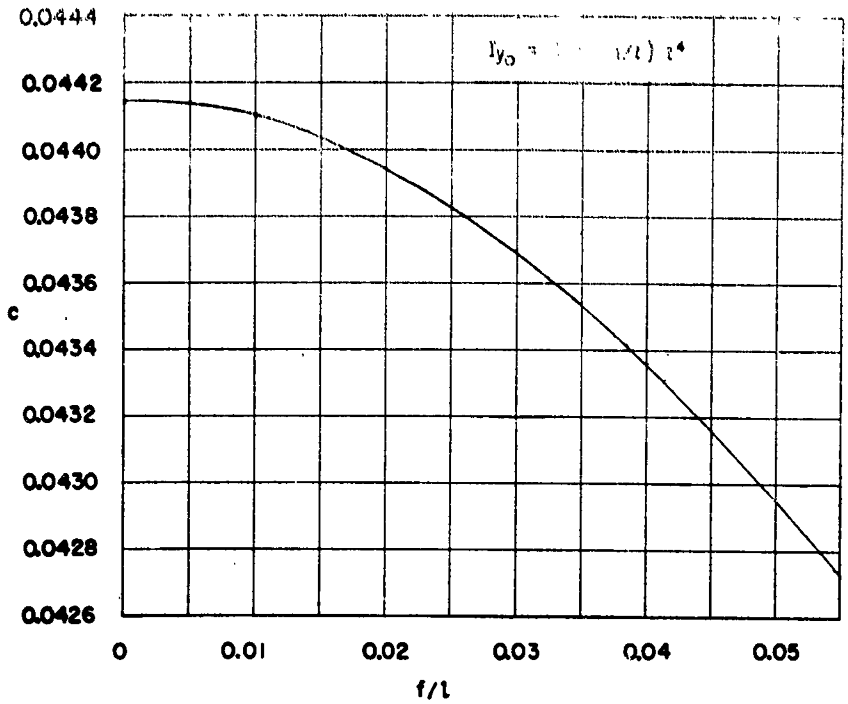


Figure 6 - Curve for Obtaining Moment of Inertia (I_{y_0}) of Blade Section

The quantities obtained for the EPH, circular arc section have been numerically compared with the respective quantities for the EPH thickness form combined with a NACA $a = 1.0$ and $a = 0.8$ mean line. Within the limits of accuracy of integration, the different mean lines had no effect on the numerical results obtained.

The appendix gives an example of the use of the above equations.

NACA 16 THICKNESS FORM, $a = 0.8$ MEAN LINE

The NACA 16 thickness form[4] has an elliptical nose similar to the EPH section but with a thinner tail. Using this thickness form with a NACA $a = 0.8$ mean line, the geometric properties were obtained numerically for various thickness and camber ratios. From these results, correction factors were determined for the formulas developed previously. Thus, the equations become:

$$A = 0.986\,(a\,t/l)\,l^2 \quad a \text{ from Figure 4}$$

$$x_1 = (0.4838 - 0.026\,f/l)\,l$$

$$x_2 = x_1 - l$$

$$x_3 = x_1 - 0.5\,l$$

$$y_1 = -(0.113\,t/l + 0.782)\,(f/l)\,l$$

$$y_2 = y_1$$

$$y_3 = (0.5t + f) + y_1$$

$$I_{x_0} = 0.9925\,[b(f/l)^2 + 0.04487(t/l)^3\,]\,l^4 \qquad b \text{ from Figure 5}$$

$$I_{y_0} = 0.946\,(c\;t/l)l^4 \quad c \text{ from Figure 6}$$

When the quantities obtained for this section (NACA 16, $a = 0.8$) are compared numerically with those obtained for NACA 16, circular arc and $a = 1.0$ sections, it can be shown that the different mean lines have no effect on the numerical results.

NACA 65A THICKNESS FORM, $a = 0.8$ (MODIFIED) MEAN LINE.

This section is a modified form of the NACA 65 section. A mean line, NACA $a = 0.8$ (modified), was designed for this form.[5] The NACA 65A thickness form is desirable for ship propellers since the nose of the section is more blunt than an ellipse, and the tail is thicker than that of the NACA 65 section.

Using the same method as followed with the NACA 16, $a = 0.8$ section, the formulas for the geometric properties are:

$$A = 0.903\,(a\;t/l)\,l^2 \qquad a \text{ from Figure 4}$$

$$x_1 = (0.4467 - 0.026\,f/l)\,l$$

$$x_2 = x_1 - l$$

$$x_3 = x_1 - 0.4\,l + 0.26\left(\frac{f}{l}\right)^2 l$$

$$y_1 = -\,(0.115\;t/l + 0.798)\,(f/l)\,l$$

$$y_2 = y_1$$

$$y_3 = 0.5\,t + 0.96\,f + y_1$$

$$I_{x_0} = 0.864\,[b\,(f/l)^2 + 0.04487\,(t/l)^3\,]l^4 \quad b \text{ from Figure 5}$$

$$I_{y_0} = 0.789\,(c\;t/l)\,l^4 \quad c \text{ from Figure 6}$$

As with the EPH and NACA 16 sections, the various mean lines have little effect on the numerical results.

SUMMARY

1. It has been assumed that the simple beam theory can be applied to a propeller blade. Certain approximations can also be made to facilitate integration when determining the

geometric properties of the section.

2. The formulas for the geometric properties of the EPH section are obtained in simplified form by limiting the thickness ratio to 0.21 and the camber ratio to 0.05. These formulas apply to the EPH section when used with the circular arc mean line. These same formulas may also be used with the mean lines NACA $a = 1.0$ and $a = 0.8$ with sufficiently accurate results.

3. Formulas for use with the NACA 16 and NACA 65A thickness forms are obtained by applying correction factors to the formulas for the EPH section. The resulting equations are applicable when these sections are used in conjunction with the circular arc, NACA $a = 1.0$, NACA $a = 0.8$, or NACA $a = 0.8$ (modified) mean lines.

4. If desired, correction factors may be obtained for a number of other sections by a method similar to that used here.

APPENDIX

SAMPLE STRESS CALCULATION

The following calculations apply to a propeller blade with EPH sections and circular arc mean lines. It will be assumed that the distributions of the thrust coefficient $(dC_{T_i})/dx$, lift coefficient c_L, and hydrodynamic pitch angle β_i are known from propeller theory. The particulars for the propeller are assumed as follows:

Diameter = 20 ft
Speed of advance = 50 fps
Number of blades = 6
Length of section = 5.0 ft
Camber ratio = 0.013
Thickness ratio = 0.18
Pitch at 0.2R = 26.12 ft
Pitch angle at 0.2R = 64.31 deg
Density/2 = $\rho/2 \doteq 1$ slug/ft^3

r/R	$\frac{dC_{T_i}}{dx}$	c_L	$\tan \beta_i$
0.2	0.122	0.222	1.982
0.3	0.310	0.266	1.305
0.4	0.533	0.242	0.963
0.5	0.732	0.194	0.754
0.6	0.850	0.154	0.612
0.7	0.876	0.123	0.519
0.8	0.781	0.099	0.432
0.9	0.559	0.077	0.372
1.0	0	0	0.324

The moments of thrust and torque will be obtained from the following derivations and by the use of Simpson's Rule:

For the moment of thrust M_{T_b}

$$dT_i = \frac{\rho}{2} R^2 \pi V_a^2 \, dC_{T_i}$$

$$dT_b = \frac{1 - \epsilon \tan \beta_i}{z} \, dT_i, \quad \epsilon \doteq \frac{0.008}{c_L}$$

$$dT_b = \frac{\rho}{2} R^2 \pi V_a^2 \frac{1}{z}(1 - \epsilon \tan \beta_i) \, dC_{T_i}$$

Introducing the nondimensional radius $x = r/R$, one obtains:

$$dM_{T_b} = \frac{\rho}{2} R^3 \pi V_a^{\,2} \frac{1}{z} (1 - \epsilon \tan \beta_i)(x - x_0)\, dC_{T_i}$$

$$M_{T_b} = \frac{\rho}{2} R^3 \pi V_a^{\,2} \frac{1}{z} \int_{x_0}^{1} (x - x_0)(1 - \epsilon \tan \beta_i) \frac{dC_{T_i}}{dx}\, dx$$

For the moment of torque M_{Q_b}

$$dQ_i = r \tan \beta_i \, d\, T_i$$

$$dQ_b = \frac{1}{z}\left(1 + \frac{\epsilon}{\tan \beta_i}\right) dQ_i$$

$$dM_{Q_b} = \left(\frac{r - r_0}{r}\right) d_{Q_b}$$

$$= \frac{1}{z}\left(1 + \frac{\epsilon}{\tan \beta_i}\right)(r - r_0) \tan \beta_i \, dT_i$$

Introducing the nondimensional radius one obtains:

$$dM_{Q_b} = \frac{\rho}{2} R^3 \pi V_a^{\,2} \frac{1}{z} (x - x_0)(\tan \beta_i + \epsilon)\, dC_{T_i}$$

$$M_{Q_b} = \frac{\rho}{2} R^3 \pi V_a^{\,2} \frac{1}{z} \int_{x_0}^{1} (x - x_0)(\tan \beta_i + \epsilon) \frac{dC_{T_i}}{dx}\, dx$$

The constant for M_{T_b} and M_{Q_b} is

$$\frac{\rho}{2} R^3 \pi V_a^{\,2} \frac{1}{z} = 1{,}809{,}000 \text{ ft-lb}$$

Further:

r/R	ϵ	$\epsilon \tan \beta_i$	$1 - \epsilon \tan \beta_i$	$(\tan \beta_i + \epsilon)$
0.2	0.036	0.0714	0.929	2.018
0.3	0.030	0.0392	0.961	1.335
0.4	0.033	0.0318	0.968	0.996
0.5	0.041	0.0309	0.969	0.795
0.6	0.052	0.0318	0.968	0.664
0.7	0.065	0.0337	0.966	0.584
0.8	0.081	0.0350	0.965	0.513
0.9	0.104	0.0387	0.961	0.476
1.0	∞	-	-	-

The following calculations are restricted to the hub radius $x_0 = 0.2$. M_{T_b}, for this radius, follows from Simpson's Rule:

	1	2	3	4	5
r/R	$\frac{dC_{T_i}}{dx}(1-\epsilon \tan\beta_i)$	$(x-x_0)$	Col. 1×2	Simpson's Multipliers	$f(M_{T_b})$
0.2	0.113	0	0	1	0
0.3	0.298	0.1	0.0298	4	0.1192
0.4	0.516	0.2	0.1032	2	0.2064
0.5	0.709	0.3	0.2127	4	0.8508
0.6	0.823	0.4	0.3292	2	0.6584
0.7	0.846	0.5	0.4230	4	1.6920
0.8	0.759	0.6	0.4554	2	0.9108
0.9	0.537	0.7	0.3759	4	1.5036
1.0	0	0.8	0	1	0
					$\Sigma = 5.9412$

$$M_{T_b} = \frac{1,309,000}{3}\,\Delta x\Sigma = \frac{1,309,000 \cdot 0.1 \cdot 5.9412}{3} = 259,230 \text{ ft-lb}$$

M_{Q_b} is obtained by the same method as applied to M_{T_b}.

$$M_{Q_b} = \frac{1,309,000 \cdot \Delta x \cdot \Sigma}{3} = 164,950 \text{ ft-lb}$$

The bending moments about the center of gravity are given as follows:

$$M_{x_0} = M_{T_b} \cos\phi + M_{Q_b} \sin\phi$$

$$\sin\phi = 0.901$$

$$\cos\phi = 0.434$$

$$M_{x_0} = 259,230 \cdot 0.434 + 164,950 \cdot 0.901 = 261,130 \text{ ft-lb}$$

$$M_{y_0} = M_{T_b} \sin\phi - M_{Q_b} \cos\phi$$

$$= 259,230 \cdot 0.901 - 164,950 \cdot 0.434 = 161,980 \text{ ft-lb}$$

The stress on the back of the blade at point of maximum thickness is given by:

$$-\frac{y_3 M_{x_0}}{I_{x_0}} - \frac{x_3 M_{y_0}}{I_{y_0}}$$

The distances from the center of gravity to the point of maximum thickness, and the moments of inertia are given as follows (see page 9)

$$x_3 = \left(0.47325 - 0.026\frac{f}{l}\right) l - 0.4361\, l + 0.17 \left(\frac{f}{l}\right)^2 l$$

$$= 0.184 \text{ ft}$$

$$y_3 = 0.5\, t + 0.984\, f - \quad 0.114\frac{t}{l} + 0.79 \quad \left(\frac{f}{l}\right) l$$

$$= 0.462 \text{ ft}$$

$$I_{x_0} = [b\,(f/l)^2 + 0.04487\,(t/l)^3]\, l^4 \quad b \text{ from Figure 5}$$

$$= [0.009(f/l)^2 + 0.04487\,(t/l)^3]\, l^4$$

$$= 0.1645 \text{ ft}^4$$

$$I_{y_0} = (c\ t/l)\, l^4 = (0.04407\ t/l)\, l^4 \quad c \text{ from Figure 6}$$

$$= 4.958 \text{ ft}^4$$

The stress on the back at the point of maximum thickness is thus:

$$-\frac{0.462 \cdot 261{,}130}{0.1645} - \frac{0.184 \cdot 161{,}980}{4.980}$$

$$= -739{,}400 \text{ psf}$$

or 5135 psi compressive stress.

In like manner, the stresses at the nose and tail have been calculated and are 44.3 psi tension and 1,179 psi tension, respectively.

For complete propeller stress analysis, these calculations must be made at a number of radii.

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