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PERFORMANCE CALCULATION OF AERODYNE SYSTEMS IN CRUISING FLIGHT

March 1957

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A. M. LIPPISCH

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SYMBOLS

L	= lift	lb	S	= cross-section area	ft^2
W	= weight	lb	V	= velocity	ft/sec
D	= drag	lb	ρ	= density	$lb \cdot sec^2/ft^4$
P	= power	ft lb/sec	m_s	= mass per unit time	lb·sec/ft

The indices o designate values concerning the outer flow process.
The indices 1 designate values taken inside the flow channel.
The indices 2 designate values of the internal flow at the exit of the body.

COEFFICIENTS

$$K_L = \frac{L}{\rho/2 \; V_o^2 \cdot S_2} = \text{lift coefficient}$$

$$K_D = \frac{D}{\rho/2 \; V_o^2 \cdot S_2} = \text{drag coefficient}$$

$$K_{D_o} = \frac{D_{paras}}{\rho/2 \; V_o^2 \cdot S_2} = \text{parasitic drag coefficient}$$

$$K_P = \frac{\eta_i \cdot P}{\rho/2 \; V_o^3 \cdot S_2} = \text{net power coefficient}$$

η_{mech} = mechanical or ideal efficiency of propulsion

η_i = internal efficiency of the propulsive flow process

$S_o/S_2 = \phi$ = area ratio

$V_2/V_o = \lambda$ = velocity ratio

γ = angle of deflection

ABSTRACT

This report contains the derivation of the basic equations characterizing the flight performance of an internal flow system as used on an aerodyne aircraft.

The investigation shows that the influence of the external flow process on the performance of such aircraft systems is considerable.

The performance numbers derived from this investigation show that the aerodyne concept can be developed into an efficient aircraft with favorable range and endurance flight characteristics.

GENERAL DESCRIPTION OF THE FLOW PROCESS AND DERIVATION OF THE DIMENSIONLESS COEFFICIENTS

We consider a flow process around an elongated body which is propelled by an internal flow process.

In the following derivation we suppose the whole process - internal and external - to be within the subsonic range so that incompressible flow can be assumed.

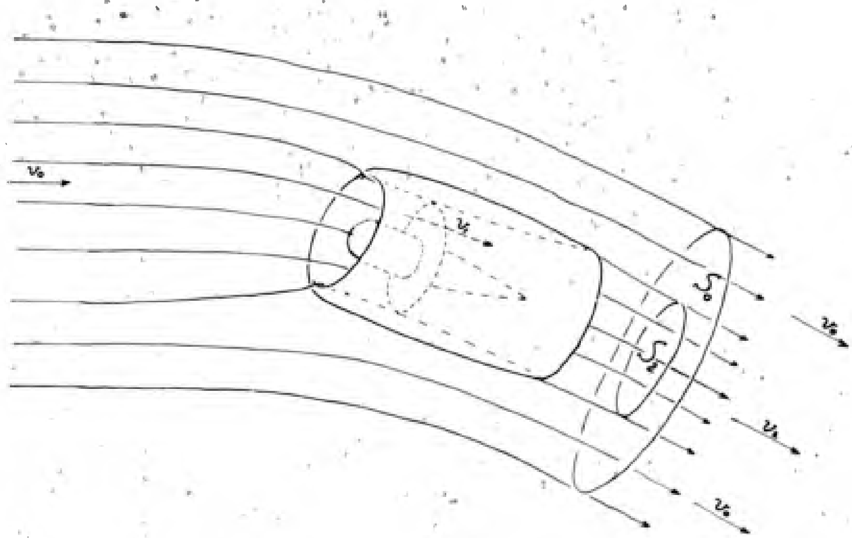

Figure 1. Illustration of the Process

The process is illustrated by figure 1. A ducted body flies through the air with the velocity V_0 along a straight horizontal path. Inside the body is a propulsion system which propels the air which goes through this system from the initial velocity V_0 to a velocity V_2 at the exit of this body. The angle between the horizontal and the direction of the exhausting stream is γ.

Due to this internal flow process and the influence of the outer shape of the body, the outer flow is also deflected by the body. Theoretically, this influence on the outer flow field extends toward infinity.

Since the outer flow produces lift, a circulation around the body must exist which causes the generation of a vortex field behind the body. This vortex field induces the outer flow and causes a deflection of the flow lines outside of the body. But the exhausting jet stream has also an influence on the outer flow, since a vortex sheet is generated along the boundary of the stream. This discontinuity surface consists of an infinite row of ring vortices which decrease in vorticity with increasing distance from the rear nozzle. The outer flow goes into the mixing zone of the jet stream and obtains, therefore, finally the same flow direction as the jet stream itself.

Figure 2 and figure 3 show a two-dimensional flow picture of such process taken in the two-dimensional smoke tunnel of the Collins Aeronautical Research Laboratory. Figure 2 shows the flow without the propulsion of the inner flow, while figure 3 shows the same arrangement with propelling the jet stream. The influence of this jet stream on the outer flow should be observed, and we see also that the action of the inner flow stabilizes the flow over the outer surface of the body. This particular phenomenon is of major interest in regard to the magnitude of the parasitic drag of such configuration. It explains that the parasitic drag of a ducted body will be diminished by the stabilizing action of the jet stream.

The vortex system of the whole process is therefore quite complex and will depend on the body configuration. A generalized vortex configuration - like the horseshoe vortex - as used to derive the induced flow field on wings could not be used in this case.

But we can simplify the analysis of such flow process in the same way as it is often used to derive the relations of the induced flow on wings. The mass flow in different directions induced by the vortex system can be replaced by a mass flow with uniform direction and

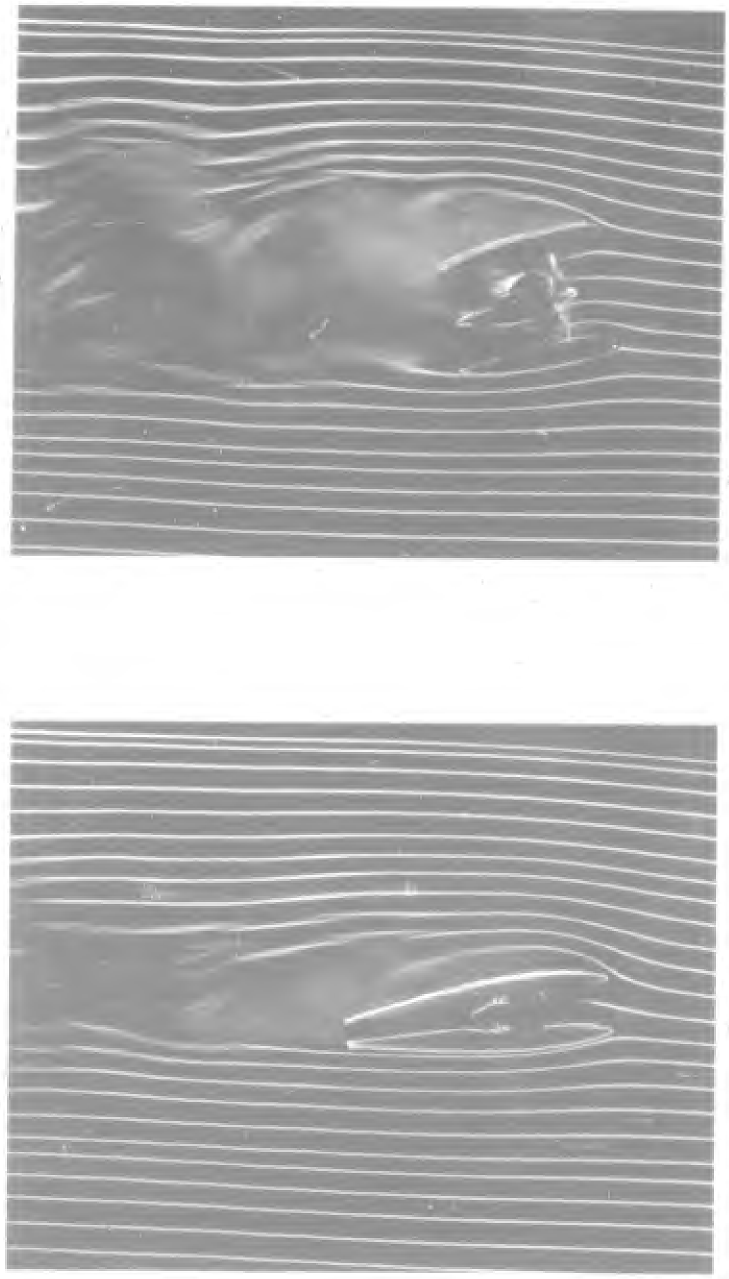

Figure 2. Flow Without Propulsion

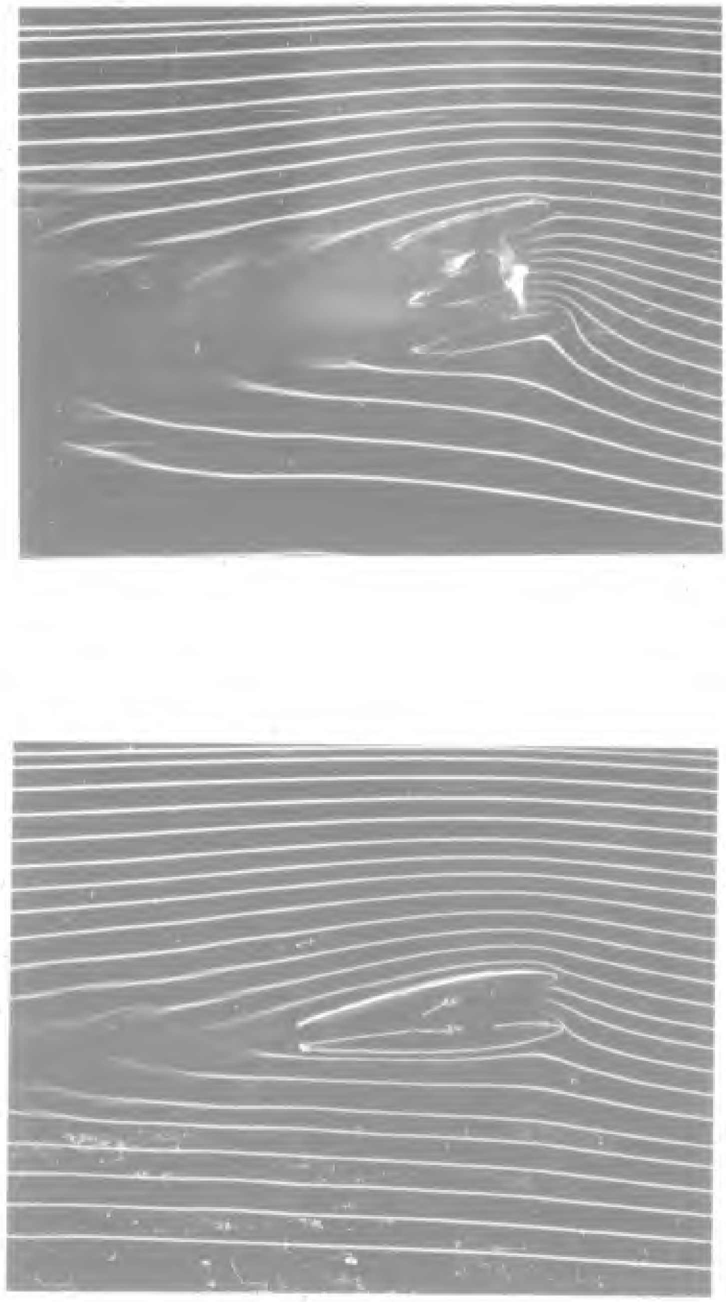

Figure 3. Flow With Propulsion

velocity along a cylindrical tube around the wing or the body. (Ref 1 and 2). The mass which is then used in this simplified analysis is often called the virtual mass. The magnitude of this mass depends on the shape of the body and the magnitude of the jet stream itself.

The ratio of the cross section through this outer virtual mass and the cross section of the jet stream is a function of the body size and shape and might also depend on the velocity ratio between the jet stream and the initial velocity. Since this velocity ratio for cruising flight conditions is nearly one, the velocity ratio will influence the mass ratio only at conditions of low speed flight, which we will not consider in this report.

According to the flow conditions explained above we suppose that the outer flow has the same deflection angle "γ" as the jet stream and is included in a tube around the body with the cross section S_o. The velocity of this outer flow will not change since there is no change in the total head of the outer flow behind the body. We can therefore derive the lift and drag forces acting on the body from the momentum change of the inner and outer mass flow.

$$L = \rho S_2 V_2^2 \sin\gamma + \rho S_o V_o^2 \sin\gamma$$

$$D = \rho S_2 V_2 (V_o - V_2 \cos\gamma) + \rho S_o V_o^2 (1 - \cos\gamma) + D_{paras.}$$

The corresponding lift and drag coefficients are therefore:

$$K_L = 2 \sin\gamma \left((V_2 / V_o)^2 + S_o / S_2 \right)$$

$$K_D = 2 V_2 / V_o \, (1 - V_2 / V_o \cos\gamma) + 2 S_o / S_2 \, (1 - \cos\gamma) + K_{D_o}$$

With the symbols for $V_2 / V_o = \lambda$ and $S_o / S_2 = \phi$ we have

$$K_L = 2 \sin\gamma (\lambda^2 + \phi) \qquad (1)$$

$$K_D = 2 \left[\lambda (1 - \lambda \cos\gamma) + \phi (1 - \cos\gamma) \right] + K_{D_o} \qquad (2)$$

Now we have to derive the power which is required to produce the jet stream. The outer flow has the pressure P_o and the velocity V_o. It is sucked through the intake nozzle toward the propelling agent - which might here be thought to be a ducted propeller - and has in front of this propulsion the velocity V_1 and the pressure P_1. Behind the propulsion the pressure is increased to $P_1 + \Delta P$ while the velocity is still V_1. The flow might then be accelerated or decelerated and leaves the internal flow system with the velocity V_2 at ambient pressure P_o.

The inflow is described by:

$$P_o + \rho/2 \; V_o^2 = P_1 + \rho/2 \; V_1^2$$

The outflow is described by:

$$P_1 + \Delta P + \rho/2 \; V_1^2 = P_o + \rho/2 \; V_2^2$$

The pressure differential is therefore:

$$\Delta P = \rho/2 \; (V_2^2 - V_o^2) \tag{3}$$

The net power required for this flow process is the increase of kinetic energy per unit time. It is therefore

$$P_{net} = \rho/2 \; S_2 \; V_2 \; (V_2^2 - V_o^2)$$

The dimensionless power coefficient is expressed by

$$K_P = \lambda \, (\lambda^2 - 1) \tag{4}$$

The actual power is larger than this value since any internal flow process causes certain losses. These losses will account for an internal efficiency factor η_i; and the total power is therefore

$$P_{Tot} = \rho/2 \; V_o^3 \cdot S_2 \cdot \frac{K_P}{\eta_i} \tag{5}$$

The ideal or mechanical efficiency of the propulsion process is included in the value of K_P due to the derivation as described above.

If we compare the power requirements of an Aerodyne and a conventional aircraft we have to keep in mind that K_P is equivalent to C_D/η_{mech}.

In any steady horizontal flight condition the drag must be zero. The expression for the drag force contains the terms which are derived from the propulsive forces of the jet stream as well as the induced and parasitic drag terms.

Without the flow through the body the whole process would change completely and we can not separate thrust from drag as it is usually done with winged aircraft.

To derive the performance and to investigate different configurations we have to compute the ratio between the lift and the power coefficient instead of the L/D term which characterizes the performance of conventional aircraft.

It is easy to see that the term K_L/K_P is equivalent to L/D since it is:

$$\frac{K_L}{K_P} = \frac{L \cdot V_o}{P_{net}}$$

and since P/V_o corresponds to the drag of conventional aircraft it is

$$\frac{K_L}{K_P} \sim \eta_{mech} \frac{L}{D} \qquad \text{(conventional aircraft)}$$

It is quite important to keep this in mind and to realize that a system as used on the aerodyne, which produces lift and propulsion from a primary process, has different characteristics. Instead of comparing lift and drag and determining a certain glide ratio, it is necessary here to compare lift and power and to derive from the dimensionless lift and power coefficients certain expressions which indicate the performance of such aircraft.

PERFORMANCE CALCULATION IN CRUISING FLIGHT

We have now the three basic relations for lift, drag and power of a body propelled by an internal flow system.

The dimensionless coefficients are:

$$K_L = 2 \sin \gamma \, (\lambda^2 + \phi) \tag{1}$$

$$K_D = 2 \lambda \, (1 - \lambda \cos \gamma) + 2 \phi \, (1 - \cos \gamma) + K_{D_o} \tag{2}$$

$$K_P = \lambda \, (\lambda^2 - 1) \tag{3}$$

For the conditions of horizontal flight, the drag coefficient must be zero. It is therefore:

$$2 \lambda \, (\lambda \cos \gamma - 1) = 2 \phi \, (1 - \cos \gamma) + K_{D_o} \tag{6}$$

From this condition we can calculate for instance the value of λ assuming certain values for ϕ, γ and K_{D_o}.

Then we are able to calculate lift and power coefficient and we can draw a performance diagram for this special case. Figure 4 illustrates such an example. The upper part of the figure 4 shows the function between the lift coefficient K_L and the power coefficient K_P. Such representation is similar to a polar diagram of a conventional aircraft except that instead of K_P you could use C_D/η_{mech}.

The lower part is similar to the performance diagram of an airplane where the power required is plotted as function of the speed. The diagram shown here represents such a chart in a dimensionless form. The abscissa shows the value of power required related to the power required in hovering flight. The ordinate is a dimensionless velocity term. The tangent on the curve from the origin indicates the condition of the most economical flight. The optimum cruising flight condition is marked by the optimum value of K_L/K_P which correspond to $\eta_{mech} \cdot C_L/C_D$ for a conventional airplane.

We see that for reasonable values of K_{D_0} and $S_0/S_2 = \phi$ the deflection angle (γ) is quite small and the velocity ratio is only slightly larger than one.

To simplify the derivation of the performance in cruising flight, we can therefore assume that the deflection angle is small so that

$$\sin\gamma \approx \gamma \qquad (7)$$

$$\cos\gamma \approx 1 - \gamma^2/2$$

and for the velocity ratio we can write

$$\lambda = 1 + \epsilon \qquad (8)$$

where ϵ is a small number.

Neglecting the second and third order terms of ϵ and using the above simplification for $\sin\gamma$ and $\cos\gamma$ we arrive at:

$$K_L = 2\gamma(2\epsilon + 1 + \phi) \qquad (1a)$$

$$K_D = -2\epsilon(1 - \gamma^2) + \gamma^2(1 + \phi) + K_{D_0} \qquad (2a)$$

$$K_P = 2\epsilon \qquad (4a)$$

The condition of horizontal steady flight gives

$$2\epsilon = \frac{\gamma^2(1 + \phi) + K_{D_0}}{1 - \gamma^2}$$

The ratio of K_L/K_P can then be calculated as:

$$\frac{K_L}{K_P} = \frac{2\gamma(1 + \phi + K_{D_0})}{\gamma^2(1 + \phi) + K_{D_0}}$$

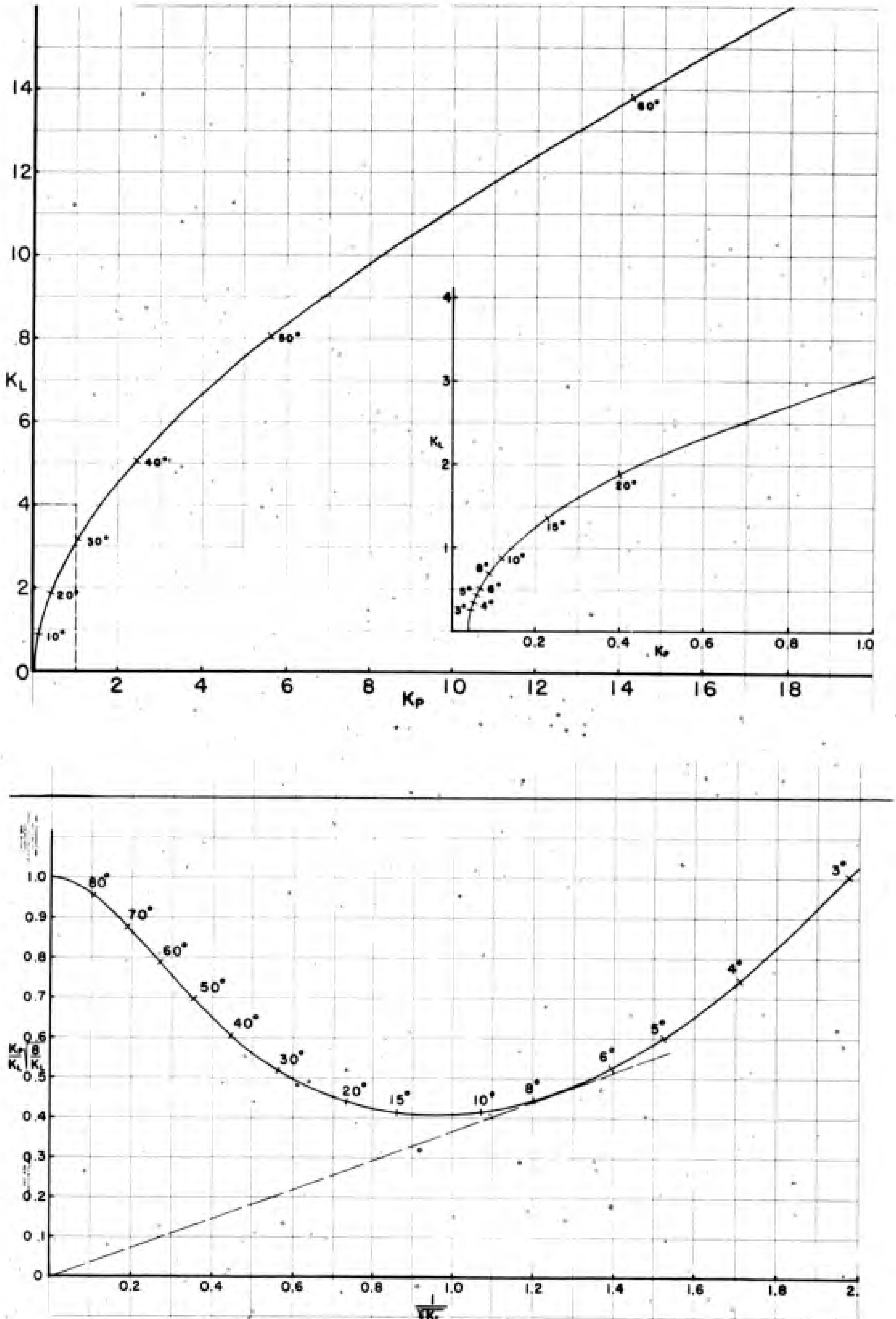

Figure 4. Performance Diagram Calculated for $K_{D_0} = 0.04; \phi = 1.4$

If we derive the optimum value of this ratio we find that

$$\gamma_{opt} = \sqrt{\frac{K_{D_o}}{1 + \phi}} \tag{9}$$

and

$$\frac{K_L}{K_P}_{opt} = \sqrt{\frac{1+\phi}{K_{D_o}}} + \sqrt{\frac{K_{D_o}}{1 + \phi}} . \tag{10}$$

This can also be written as:

$$\frac{K_L}{K_P}_{opt} = \frac{\frac{1+\phi}{K_{D_o}} + 1}{\sqrt{\frac{1 + \phi}{K_{D_o}}}} \tag{10a}$$

It is well known that for the condition of hovering flight the following relation must be fulfilled:

$$\frac{W}{\eta \cdot P} \sqrt{\frac{W}{4 \rho S_2}} = 1$$

This expression transformed into a relation between lift and power coefficient is:

$$\frac{K_L}{K_P} \sqrt{\frac{K_L}{8}} = 1 \qquad \text{(hovering)}$$

If we calculate the same expression for cruising flight conditions, the value of the expression

$$\frac{K_L}{K_P} \sqrt{\frac{K_L}{8}} \quad \text{cruise}$$

is the ratio between the power in hovering flight to the power required for cruising under the assumption that the jet stream areas are the same and both flight conditions are compared at the same air density. Otherwise the number has to be corrected with the term

$$\sqrt{\frac{(\rho S_2)\ \text{cruise}}{(\rho S_2)\ \text{hover}}}$$

For the optimum conditions of cruising flight we obtain:

$$\frac{K_L}{K_P}\sqrt{\frac{K_L}{8}}_{\text{cruise}} = \frac{\sqrt{1+\phi}}{2}\left(\frac{1+\dfrac{K_D}{1+\phi}}{\sqrt[4]{\dfrac{K_{D_o}}{1+\phi}}}\sqrt{\frac{1+\dfrac{K_{D_o}}{1+\phi}}{1-\dfrac{K_{D_o}}{1+\phi}}}\right) \qquad (11)$$

Since $K_D / 1 + \phi$ is usually a very small number we can write approximately

$$\frac{K_L}{K_P}\sqrt{\frac{K_L}{8}}_{\text{cruise}} \approx \frac{1}{2}\sqrt[4]{\frac{(1+\phi)^3}{K_{D_o}}}\left(1 + 2\frac{K_{D_o}}{1+\phi}\right) \qquad (11a)$$

This value does not represent the lowest power required for horizontal flight. The derivation of the minimum power can also be done using the above simplications. As shown later, this minimum power value can be calculated approximately from

$$\frac{K_L}{K_P}\sqrt{\frac{K_L}{8}} \approx \frac{3}{4}\sqrt[4]{\frac{(1+\phi)^3}{K_{D_o}}} \qquad \text{(minimum power required)} \qquad (12)$$

The example calculated in figure 4 shows that the point for optimum cruising conditions required more power than the minimum value which would be used for endurance and climbing flight.

The values for

$$\frac{K_L}{K_P}_{\text{opt}} = \frac{W \cdot V_o}{\eta_1 \cdot P}$$

as well as the values for

$$\frac{K_L}{K_P}\sqrt{\frac{K_L}{8}}_{\text{cruise}} = \frac{W}{\eta_1 \cdot P}\sqrt{\frac{W}{4\rho S_2}}$$

are plotted in the diagram of figure 5 as functions of K_{D_0} and ϕ.

It might be of interest to obtain some information about the magnitude of these two parameters for actual flight conditions.

The parasitic drag coefficient is referred to the initial velocity and the cross section of the exhausting jet stream. We can assume that the area S_2 is of the same order as the cross section through the body. A streamlined body has a parasitic drag coefficient referred to its cross section of

$$C_D = 0.03 - 0.06$$

Since the losses of the internal flow are to be included in the internal efficiency value, the parasitic drag coefficient is related to the friction drag of the outer wetted area. Since the jet stream at the rear of the body stabilizes this surface flow and eliminates the flow around the rear end of the body where most of the parasitic drag due to incomplete pressure recovery is generated, the parasitic drag of a ducted body should be lower than the values for a solid body of this size.

We should also keep in mind that boundary layer control can well be established on such body without great technical effort. From such considerations we can expect quite low values for K_{D_0} under cruising conditions.

Theoretically a ducted elongated body of a circular cross section would have a cross section ratio $S_0/S_2 = \phi = 1$. Tests of the lift of symmetrical fuselages with a diamond-shaped cross section have shown that larger lift coefficients than those of a circular body can be obtained.

Low-speed wind tunnel tests which were carried out by this laboratory have shown a considerable increase of the ϕ value for a circular shroud with short fins along its horizontal axis so that similar conditions as on the diamond shaped body would be present.

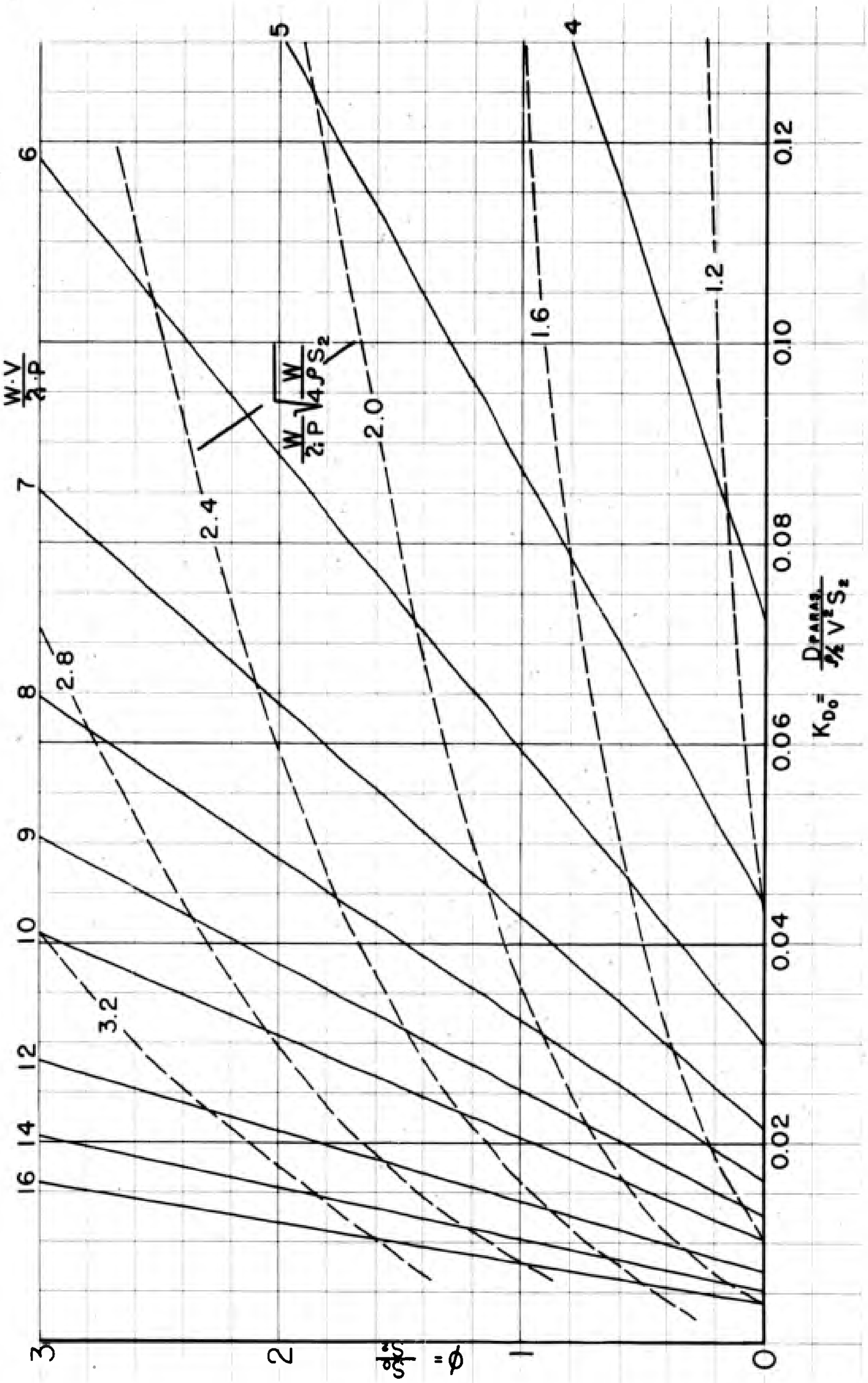

Figure 5. Performance Chart for Aerodyne in Cruising Flight

The explanation of this effect is quite obvious. If an elongated body shall produce lift, a circulation around such body must take place to generate the lift. Since the body has a limited span, the circulation must leave the body along two "tip vortices", the strength of these vortices being proportional to the lift on the body.

In the case of a circular body the surface flow toward the rear under the condition of an angle of attack toward the flow direction tends to go around the body from the bottom to the top, which causes even a negative lift force at the rear part of the body. The generation of the tip vortices is almost extinguished at the lower angles of attack.

If we now attach a rim along the horizontal center line of the body, the flow around this edge generates two tip vortices on both sides of such body, which on the other hand find their equivalent in a considerable increase of the lift of such body.

It is therefore possible to increase the circulation of the outer flow by such changes in the outer shape of the body and to arrive at values of ϕ which are larger than one.

Evaluation of test on a shrouded propeller with small fins on the sides of the circular shroud gave values of ϕ in the range of 1.5 - 2.0.

A proper design will therefore give ϕ values between 1 and 2.

With these values of K_{D_o} and ϕ the performance numbers for the Aerodyne aircraft type are well within the range of conventional aircraft types.

It is also of interest to mention that the angle of deflection for an Aerodyne with good efficiency is quite small. The relation between γ and K_L/K_P is illustrated in figure 6.

It is therefore not necessary to use deflector vanes in the outlet of the jet stream in cruising flight since such small deflections can well be produced by a slightly cambered diffuser behind the propulsion system.

The internal efficiency for the cruising flight condition will therefore be as high as measured on shrouded propellers.

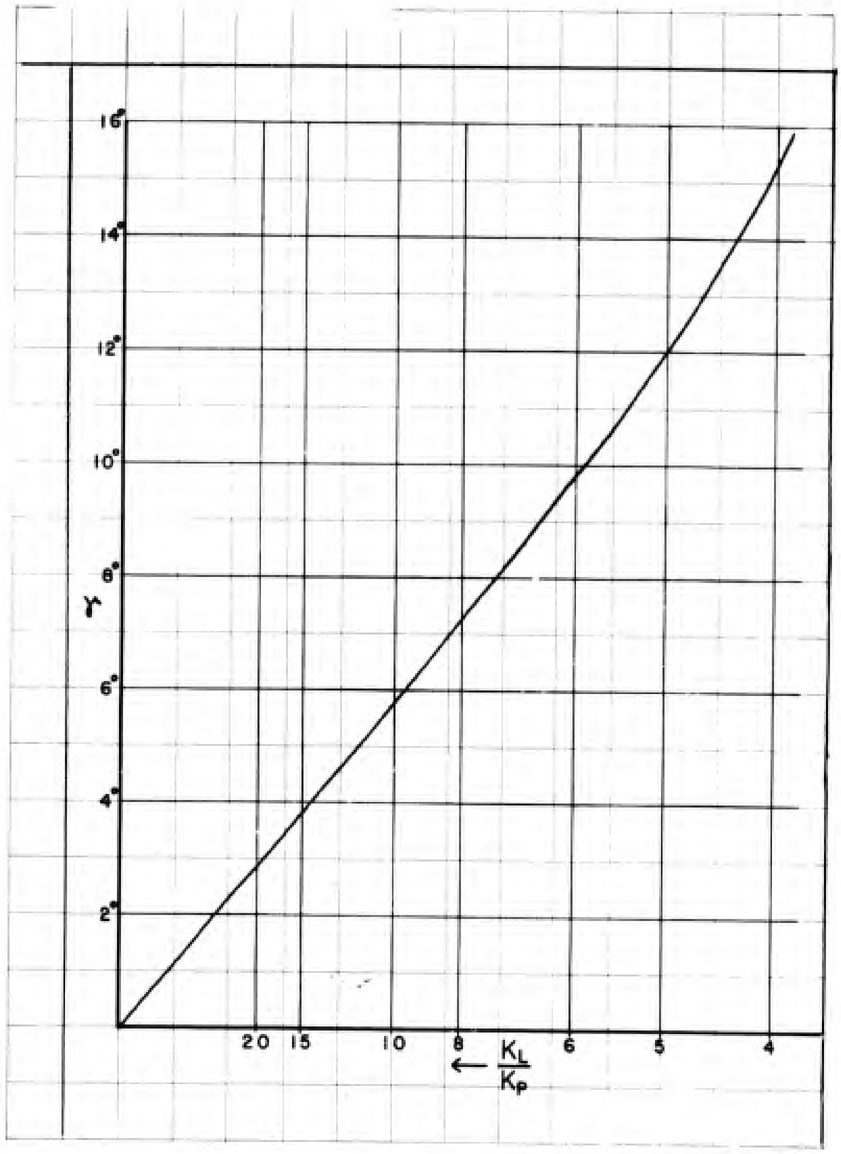

Figure 6. Deflection Angle for Cruising Flight Conditions

COMPARISON OF THE AERODYNE CONCEPT WITH THE CONVENTIONAL WINGED AIRCRAFT

It might be of interest to use the expressions we have derived above for a comparison of the aerodynamic characteristics of the aerodyne and the aeroplane.

If we derive the power coefficient K_P as a function of K_L and K_{D_0} using the linearized form for cruising flight we find:

$$K_P = \frac{K_L^2}{4(1 + \phi + K_P)} + K_{D_0}$$

Since the value of K_P is quite small in the cruising flight range compared with $(1 + \phi)$ we can neglect K_P in the denominator of the first term and write:

$$K_P \approx \frac{K_L^2}{4(1 + \phi)} + K_{D_0} \qquad (13)$$

The optimum value of K_L / K_P derived from this simplified expression is:

$$K_L / K_{P\,opt} \approx \sqrt{\frac{1 + \phi}{K_{D_0}}}$$

Since the second term in (10) can well be neglected for lower K_{D_0} values the above relation gives a conservative estimate of $K_L / K_{P\,opt}$. In a similar way we can find the minimum of $K_L / K_P^{1.5}$ and derive in this way the expression (12).

The relation (13) resembles the equation for the drag coefficient of an airplane which can be given as:

$$C_D = \frac{C_L^2}{\pi(AR)} + C_{d_0}$$

AR is the effective aspect ratio number of the wing. If we want to compare both systems we have to refer the coefficients in both expression to equivalent areas in both cases. Such area would be the cross section of the deflected air mass.

In the case of the aerodyne this cross section is the sum of the cross sections of the inner and the outer flow

$$S_{Tot} = S_o + S_2$$

or

$$S_{Tot} = S_2 (1 + \phi)$$

The coefficients referred to this total area shall be designated as K_L', K_D', and K_P'.

In the case of the aeroplane - that means a conventional winged aircraft - the cross section of the "virtual" mass is the circle over the span of the wing. Therefore the reference area is

$$S_{Tot} = \pi/4 \; b^2$$

We will designate the coefficients for the aeroplane referred to this area as C_L', C_D', and C_{D_o}'.

It is not very difficult to find that the drag coefficient is then represented by

$$C_D' = \frac{C_L'^2}{4} + C_{d_o}' \qquad \text{(Aeroplane)}$$

The transformation of the coefficients to the total area of the mass flow in the case of the aerodyne gives:

$$K_P' = \frac{K_L'^2}{4} + K_{D_o}' \qquad \text{(Aerodyne)}$$

Since the drag coefficient of an airplane is similar to the power coefficient of an aerodyne, except that the latter includes the mechanical - or ideal - efficiency of the propulsion system, the expressions are the same in both cases. The question of comparing both systems turns out to be a comparison of the parasitic drag coefficients mainly.

An investigation of this parameter shows that in the range of higher subsonic speed the wetted surface of an aerodyne can be made smaller than the wetted area of an airplane. This conclusion is mainly drawn from the fact that the aerodyne can fly without forward speed and its layout in size of the body containing the internal flow system is not affected by a consideration of landing or take-off speed limitations.

Since the term $\frac{W \cdot V}{\eta \cdot P}$ has its physical limits, the power loading (W/P) at higher cruising speeds (V) must be reduced. But with such low power loadings the size of an internal flow system for hovering flight will become quite small so that the wetted area of an aerodyne reduces below values of an equivalent conventional aircraft.

If we want to evaluate different aircraft configurations, it is necessary to find first a common term for such comparison. Several publications on this matter did base such comparison on terms of the gliding angle available with power-off condition. It is obvious that the aerodyne concept will not have any comparative number under such conditions. Actually, the airplane of today does not make use any more of the power-off gliding ability since its speed even in gliding is of such magnitude that landing without power on an unprepared field is not possible without a severe damage.

A uniform qualitative analysis of different systems can therefore only be worked out properly if similar flight conditions for all systems are taken into consideration.

A fully objective study shows then very clearly that the range and endurance flight performance of the aerodyne concept is certainly not inferior to the aeroplane configuration, especially in the ranges of higher cruising speeds which have to be considered if future aircraft design shall be evaluated.

We might also mention here that the internal flow system of the aerodyne design for supersonic flight offers the possibility to approach the shape of the body of minimum resistance.

CONCLUSION

The equation for the flight of an aerodyne system was derived and a linearized solution of these expressions for the case of cruising flight was given. The optimum conditions for this flight attitude were derived and it was shown how such expression can be used for a comparison between the aerodyne concept and the conventional winged airplane.

REFERENCES

1. Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen

   1. Lieferung (Dr. L. Prandtl).
      Paragraph 3, pages 35-36.

2. Zeitschrift für Flugwissenschaften Sept. 1956 Heft 9

   G. Schulz: Der Kraftangriff bei Strahltriebwerken und ihren Verkleidungen.
   Page 286.

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