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TABLES FOR ONE-SIDED STATISTICAL TOLERANCE LIMITS

BY

GERALD J. LIEBERMAN

TECHNICAL REPORT NO. 34

NOVEMBER 1, 1957

PREPARED UNDER CONTRACT N6onr-25126
(NR-042-002)

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FOR
OFFICE OF NAVAL RESEARCH

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TABLES FOR ONE-SIDED STATISTICAL TOLERANCE LIMITS

by

Gerald J. Lieberman

1. INTRODUCTION.

The quality of manufactured product is often specified by setting a range, the bounds of which are called tolerance limits. These limits have the property that a certain percentage, $100(1-\alpha)\%$, of the product may be expected to fall within these limits. If the quality of the items is normally distributed with known mean, μ , and known standard deviation, σ , two-sided tolerance limits are formed by adding to and subtracting from μ the quantity $K_{\alpha/2} \sigma$, where $K_{\alpha/2}$ is the normal deviate corresponding to $\alpha/2$. An upper one-sided tolerance limit is given by $\mu + K_{\alpha} \sigma$ and a lower one-sided tolerance limit is given by $\mu - K_{\alpha} \sigma$. Unfortunately, in practice, the values μ and σ are rarely known and estimates of these values, $\bar{x}$ and s , respectively, are used. However, where before it could be stated that 95% of a manufactured product lies below $\mu + 1.645 \sigma$, the upper one-sided tolerance limit, this statement cannot be extended to the limit $\bar{x} + 1.645 s$. The quantities $\bar{x}$ and s are random variables, and hence the limit depends upon the particular outcome of the sample. Different samples will lead to different limits. How close these limits are to $\mu + 1.645 \sigma$ depends upon how good the estimates are.

It is evident then that the fraction of the items included between

- a. $[\bar{x} - K_{\alpha/2}s, \bar{x} + K_{\alpha/2}s]$ for two-sided tolerance limits,
- b. $[-\infty, \bar{x} + K_{\alpha}s]$ for an upper one-sided tolerance limit,
- c. $[\bar{x} - K_{\alpha}s, \infty]$ for a lower one-sided tolerance limit

will not always contain a specified proportion $1 - \alpha$ of the manufactured items. However, it is possible to determine constants K such that in a large series of samples from a normal distribution, a fixed proportion γ of the intervals

- a. $[\bar{x} - Ks, \bar{x} + Ks]$ for two-sided tolerance limits,
- b. $[-\infty, \bar{x} + Ks]$ for an upper one-sided tolerance limit, and
- c. $[\bar{x} - Ks, \infty]$ for a lower one-sided tolerance limit

will include $100(1 - \alpha)\%$ or more of the distribution. The finite limits of these intervals are known as statistical tolerance limits. Thus, an upper one-sided statistical tolerance limit is given by $\bar{x} + Ks$ and has the property that the probability is equal to a preassigned value γ that the interval $[-\infty, \bar{x} + Ks]$ includes at least a specified proportion $1 - \alpha$ of the distribution. Note that in most practical situations γ is usually a large number close to 1. Statistical tolerance intervals should not be confused with confidence intervals for a parameter of the distribution. Confidence limits for the mean of a normal distribution are such that in a given fraction, say .95, of the samples from which they are computed, the interval bounded by the limits will include the true mean of the distribution. For confidence interval estimation, .95 is also called the confidence coefficient.

Many extensive tables of factors K for two-sided tolerance limits have been computed. A. H. Bowker presents such a table in [1] and very recently A. Lieberman issued a set of tables [2] which are an extension of Bowker's tables in that they can be used when s is computed from sample sizes other than n . Resnikoff [3] and Mitra [4] computed values of K when σ is estimated by the average range of subgroups. Tables of factors K for one-sided tolerance limits have never been computed and it is the purpose of this paper to present such tables.

2. TABLES OF ONE-SIDED TOLERANCE LIMITS.

Factors K such that $\bar{x} + Ks$ and $\bar{x} - Ks$ are an upper one-sided tolerance limit and a lower one-sided tolerance limit, respectively, for a normal universe are given in Table 1 for $n = 3, 4, \dots, 25, 30, 35, 40, 45, 50$; for $\gamma = 0.75, 0.90, 0.95, 0.99$; and for $\alpha = 0.25, 0.10, 0.05, 0.01, 0.001$. $\bar{x}$ is the sample mean ($\bar{x} = (x_1 + x_2 + \dots + x_n)/n$) and s is the sample standard deviation

$$s = \sqrt{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2 / (n-1)}.$$

For n greater than 50, the values of K can be obtained using the asymptotic normality property of $\bar{x} + Ks$. With this approximation, K is given by

$$K = \frac{K_\alpha + \sqrt{K_\alpha^2 - ab}}{a}$$

where

$$a = 1 - \frac{K_\gamma^2}{2(n-1)}$$

$$b = K_\alpha^2 - \frac{K_\gamma^2}{n}$$

and K_α and K_γ (which equals $-K_{1-\gamma}$ so that $K_\gamma^2 = K_{1-\gamma}^2$) are the normal deviates corresponding to α and γ respectively, i.e.

$$\int_{K_\alpha}^{\infty} e^{-\frac{1}{2}z^2} dz = \alpha \quad \text{and} \quad \int_{K_\gamma}^{\infty} e^{-\frac{1}{2}z^2} dz = \gamma$$

3. EXAMPLE.

A manufacturer of light bulbs would like to specify a single lower limit above which he can be assured, with probability 0.95, that at least 99% of his production will lie. A sample of 30 bulbs is taken and the sample mean and sample standard deviation are found to be 987.2 and 5.963 respectively. A value of $K = 3.064$ corresponding to $n = 30$, $\gamma = 0.95$ and $\alpha = 0.01$ is obtained from Table 1. The required lower tolerance limit is given by $\bar{x} - Ks = 987.2 - (3.064)(5.963) = 968.9$.

4. CONSTRUCTION OF THE TABLES.

For the case of a normal distribution with unknown parameters μ and σ , exact one-sided tolerance limits may be computed by the following procedure. Let $t_{n-1, \sqrt{n} K_\alpha}$ be a non-central t statistic with $n-1$ degrees of freedom and non-centrality parameter $\sqrt{n} K_\alpha$. K_α is defined by

$$\frac{1}{\sqrt{2\pi}} \int_{K_\alpha}^{\infty} e^{-z^2} dz = \alpha.$$

Determine t_0 such that $P(t_{n-1, \sqrt{n} K_\alpha} > t_0) = 1 - \gamma$. Let $K = t_0 / \sqrt{n}$. Then if $\bar{x}$ and s are the sample mean and the sample standard deviation, respectively, based on a sample $x_1, x_2, \dots, x_n$ the

$$\bar{x} = \frac{\sum x_i}{n} \quad \text{and} \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} ,$$

$\bar{x} + Ks$ and $\bar{x} - Ks$ are upper one-sided tolerance limit and a lower one-sided tolerance limit, respectively. Thus, the probability is γ that at least a proportion $1-\alpha$ of the distribution will be less than $\bar{x} + Ks$ (or greater than $\bar{x} - Ks$).

The value t_0 may be obtained from tables of the distribution of the non-central t statistic. Such tables are those of Johnson & Welch [5] and Resnikoff and Lieberman [6]. The latter tables are perhaps more convenient for this application, and were used in the computation of Table 1.

TABLE 1.

Tolerance Factors for Normal Distributions

Factors K such that the probability is γ that at least a proportion $1-\alpha$ of the distribution will be less than $\bar{x} + Ks$ (or greater than $\bar{x} - Ks$), where $\bar{x}$ and s are estimates of the mean and the standard deviation computed from a sample of size n .

n	α	$\gamma = 0.75$					$\gamma = 0.90$				
		0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001
3	1.464	2.501	3.152	4.396	5.805	2.602	4.258	5.310	7.340	9.651	
4	1.256	2.134	2.680	3.726	4.910	1.972	3.187	3.957	5.437	7.128	
5	1.152	1.961	2.453	3.421	4.507	1.698	2.742	3.400	4.666	6.112	
6	1.087	1.860	2.336	3.243	4.273	1.540	2.494	3.091	4.242	5.556	
7	1.043	1.791	2.250	3.126	4.118	1.435	2.333	2.894	3.972	5.201	
8	1.010	1.740	2.190	3.042	4.008	1.360	2.219	2.755	3.783	4.955	
9	0.984	1.702	2.141	2.977	3.924	1.302	2.133	2.649	3.641	4.772	
10	0.964	1.671	2.103	2.927	3.858	1.257	2.065	2.568	3.532	4.629	
11	0.947	1.646	2.073	2.885	3.804	1.219	2.012	2.503	3.444	4.515	
12	0.933	1.624	2.048	2.851	3.760	1.188	1.966	2.448	3.371	4.420	
13	0.919	1.606	2.026	2.822	3.722	1.162	1.928	2.403	3.310	4.341	
14	0.909	1.591	2.007	2.796	3.690	1.139	1.895	2.363	3.257	4.274	
15	0.899	1.577	1.991	2.776	3.661	1.119	1.866	2.329	3.212	4.215	
16	0.891	1.566	1.977	2.756	3.637	1.101	1.842	2.299	3.172	4.164	
17	0.883	1.554	1.964	2.739	3.615	1.085	1.820	2.272	3.136	4.118	
18	0.876	1.544	1.951	2.723	3.595	1.071	1.800	2.249	3.106	4.078	
19	0.870	1.536	1.942	2.710	3.577	1.058	1.781	2.228	3.078	4.041	
20	0.865	1.528	1.933	2.697	3.561	1.046	1.765	2.208	3.052	4.009	
21	0.859	1.520	1.923	2.686	3.545	1.035	1.750	2.190	3.028	3.979	
22	0.854	1.514	1.916	2.675	3.532	1.025	1.736	2.174	3.007	3.952	
23	0.849	1.508	1.907	2.665	3.520	1.016	1.724	2.159	2.987	3.927	
24	0.845	1.502	1.901	2.656	3.509	1.007	1.712	2.145	2.969	3.904	
25	0.842	1.496	1.895	2.647	3.497	0.999	1.702	2.132	2.952	3.882	
30	0.825	1.475	1.869	2.613	3.454	0.966	1.657	2.080	2.884	3.794	
35	0.812	1.458	1.849	2.588	3.421	0.942	1.623	2.041	2.833	3.730	
40	0.803	1.445	1.834	2.568	3.395	0.923	1.598	2.010	2.793	3.679	
45	0.795	1.435	1.821	2.552	3.375	0.908	1.577	1.986	2.762	3.638	
50	0.788	1.426	1.811	2.538	3.358	0.894	1.560	1.965	2.735	3.604	

TABLE 1.

Tolerance Factors for Normal Distributions (continued).

n	α	$\gamma = 0.95$					$\gamma = 0.99$				
		0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001
3		3.804	3.150	7.655	10.552	13.857					
4		2.619	4.163	5.145	7.042	9.215					
5		2.149	3.407	4.202	5.741	7.501					
6		1.895	3.006	3.707	5.062	6.612	2.849	4.408	5.409	7.334	9.540
7		1.732	2.755	3.399	4.641	6.061	2.490	3.856	4.730	6.411	8.348
8		1.617	2.582	3.188	4.353	5.686	2.252	3.496	4.287	5.811	7.566
9		1.532	2.454	3.031	4.143	5.414	2.085	3.242	3.971	5.389	7.014
10		1.465	2.355	2.911	3.981	5.203	1.954	3.048	3.739	5.075	6.603
11		1.411	2.275	2.815	3.852	5.036	1.854	2.897	3.557	4.828	6.284
12		1.366	2.210	2.736	3.747	4.900	1.771	2.773	3.410	4.633	6.032
13		1.329	2.155	2.670	3.659	4.787	1.702	2.677	3.290	4.472	5.826
14		1.296	2.108	2.614	3.585	4.690	1.645	2.592	3.189	4.336	5.651
15		1.268	2.068	2.566	3.520	4.607	1.596	2.521	3.102	4.224	5.507
16		1.242	2.032	2.523	3.463	4.534	1.553	2.458	3.028	4.124	5.374
17		1.220	2.001	2.486	3.415	4.471	1.514	2.405	2.962	4.038	5.268
18		1.200	1.974	2.453	3.370	4.415	1.481	2.357	2.906	3.961	5.167
19		1.183	1.949	2.423	3.331	4.364	1.450	2.315	2.855	3.893	5.078
20		1.167	1.926	2.396	3.295	4.319	1.424	2.275	2.807	3.832	5.003
21		1.152	1.905	2.371	3.262	4.276	1.397	2.241	2.768	3.776	4.932
22		1.138	1.887	2.350	3.233	4.238	1.376	2.208	2.729	3.727	4.866
23		1.126	1.869	2.329	3.206	4.204	1.355	2.179	2.693	3.680	4.806
24		1.114	1.853	2.309	3.181	4.171	1.336	2.154	2.663	3.638	4.755
25		1.103	1.838	2.292	3.158	4.143	1.319	2.129	2.632	3.601	4.706
30		1.059	1.778	2.220	3.064	4.022	1.249	2.029	2.516	3.446	4.508
35		1.025	1.732	2.166	2.991	3.934	1.195	1.957	2.431	3.334	4.364
40		0.999	1.697	2.126	2.941	3.866	1.154	1.902	2.365	3.250	4.255
45		0.978	1.669	2.092	2.897	3.811	1.122	1.857	2.313	3.181	4.168
50		0.961	1.646	2.065	2.863	3.766	1.096	1.821	2.296	3.124	4.096

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