

JUN 10 1964

NAVORD REPORT

5719

FEB 10 1963

JUN 9 1982

HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED TO SUPERSONIC AIR FLOW

PROPERTY OF U. S. AIR FORCE
ALDC LIBRARY
AF 40(600)-700

1 SEPTEMBER 1957



This document is not to be released for public release
its distribution is unlimited. Per TAB 74-21
Dtd 11 Oct, 1974

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

AEDC TECHNICAL LIBRARY



Aerodynamics Research Report 16

HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED
TO SUPERSONIC AIR FLOW

Prepared by:

E. C. Brooke

ABSTRACT: Heat-transfer rates to a 5 cm diameter dry ice sphere for Mach numbers 1.86, 2.87, and 4.25 were determined from ablation experiments. Results are compared with heat-transfer rates to a non-evaporative model subjected to the same flow conditions. Heat-transfer rates to the subliming model calculated from mass ablation rates were found to be two to three times as great as heat-transfer rates to non-evaporative models when only convection was considered. This report shows that heat conduction effects internal to the dry ice model are significant, especially at the highest Mach number tested, where the greatest change in environmental pressure occurred. Other factors contributing to the large heat-transfer rates in the subliming models are: (1) change in model shape during blow; (2) increase in surface roughness during blow; and (3) loss of mass without heat transfer through fragmentation.

This document is classified "Confidential" for public release
its distribution is unlimited. Per TAB 74-21
Dtd 11 OCT, 1974.

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

NAVORD Report 5719

1 September 1957

Missiles flown at high speeds encounter severe aerodynamic heating which may cause serious damage to the vehicle. The use of ablating materials as a scheme to reduce these high temperatures is receiving particular attention. For this reason, investigations concerning the behavior of an ablating substance, dry ice, under the conditions of supersonic flow, have been conducted at the U. S. Naval Ordnance Laboratory under Task Number NO 502-825/51014/01. This report utilizes experimental mass transfer data, obtained by K. H. Gruenewald in the NOL wind tunnel, for the consideration of heat transferred from air to a solid carbon-dioxide sphere subjected to supersonic flow.

The author is indebted to Dr. K. H. Gruenewald and Mr. I. Korobkin for their assistance in the preparation of this report.

W. W. WILBOURNE
Captain, USN
Commander

R. KENNETH LOBB
By direction

NAVORD Report 5719

CONTENTS

Introduction	1
Convective Heat Transfer to Dry Ice Spheres	1
Heat Transfer to a Non-Evaporative Hemisphere.	4
Results and Discussion	4
Conclusions	7
References	9
Appendix A	10

ILLUSTRATIONS

- Figure 1. Surface temperature of sublimation versus angular position on a dry ice sphere
- Figure 2. Heat of sublimation for solid CO₂
- Figure 3. Heat of sublimation over a dry ice sphere in supersonic flow
- Figure 4. Dry ice sphere, before and after blowing

SYMBOLS

A	= surface area involved in heat-transfer process
a	= radius of sphere
B	= constant
c	= specific heat
D	= diameter of sphere
h	= convective heat transfer
H	= heat of sublimation
$\bar{H}$	= average heat of sublimation
$J_n(\alpha r)$	= Bessel function
k	= thermal conductivity
K	= thermal diffusivity = $k/\rho c$
m	= mass
m_i	= mass at beginning of blow
m_f	= mass at end of blow
p	= static pressure
p_{∞}	= free-stream static pressure
$P_n(\mu)$	= Legendre function of the first kind (zonal harmonic)
q'	= dynamic pressure
q	= time rate of heat transfer per unit area
Q	= total time rate of heat transfer
r	= distance from center of sphere
t	= time
T	= temperature
T_1	= constant (initial temperature of dry ice sphere)
T_e	= equilibrium temperature

NAVORD Report 5719

T_w	= surface temperature
u	= steady-state temperature over sphere surface
w	= variable temperature within sphere
v	= $u + w$ = total temperature
x	= distance from stagnation point in direction of flow
α_n	= root of $J_{n+1/2}(\alpha r)$
B	= gradient of local velocity evaluated at stagnation point
θ	= angular position on sphere measured away from stagnation point
ρ	= density
ν	= kinematic viscosity

HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED TO SUPERSONIC AIR FLOW

INTRODUCTION

1. Missiles which are to fly at supersonic and hypersonic speeds will encounter severe aerodynamic heating. Various schemes have been proposed for coping with the heating problem and the principle of surface ablation has received particular attention. Rather than attempting to prevent deterioration of the missile surface due to heating, it may be advantageous to allow ablation to occur on the surface in order to protect the remainder of the vehicle. Such an approach seems quite feasible because of the vast amounts of energy which materials can absorb when undergoing a change of phase. In effect, melting, evaporation, and sublimation represent constant temperature heat sinks. For these reasons investigations concerning the behavior of an ablating substance under the conditions of supersonic flow have been conducted at the Naval Ordnance Laboratory.

2. Results of experimental research at NOL pertaining to the drag and rate of sublimation of dry ice models subjected to supersonic flow have been published by Gruenewald, reference (a). The present report utilizes Gruenewald's data for the consideration of heat transferred from the air to the subliming, solid carbon-dioxide and includes the effects of conduction within the models. These results are compared with the heat transfer obtained using non-evaporative models.

Convective Heat Transfer to Dry Ice Spheres

3. Assuming that only convective heat-transfer effects are present, the time rate of heat transfer from the air to a dry ice model can be determined from ablation considerations by the following equation:

$$Q = \frac{\Delta m}{\Delta t} \cdot \bar{H} \quad (1)$$

where Q = total rate of heat transfer to the sphere;
 $\Delta m/\Delta t$ = rate of ablation; and $\bar{H}$ = average latent heat of sublimation over the sphere.

4. Ablation rates per square centimeter of cross-sectional area for 5 cm diameter dry ice spheres as determined from Gruenewald's data are given below:

<u>Mach Number</u>	<u>Ablation Rate - gram/sec cm²</u>
1.86	0.0448
2.87	0.0185
4.25	0.00829

The average latent heat of sublimation can be expressed as a function of mass by

$$\bar{H}(m) = \frac{1}{m_f - m_i} \int_{m_i}^{m_f} H(m) dm \quad (2)$$

where $H(m)$ is the latent heat associated with each ablated particle. Therefore the integral in Equation (2) represents the total heat of ablation.

5. The local heats of sublimation depend upon the temperatures of the particular particles of mass involved, but these temperatures as well as other physical properties of the ablating substance are difficult to measure. Also it is not known how the ablating process influences these properties. However, the temperature at which sublimation occurs is a function of the vapor pressure of CO_2 . If the concentration of CO_2 vapor in the vicinity of the sphere were 100 percent, then the vapor pressure would be identical to the static pressure on the sphere. It has been assumed for the purposes of this report that the CO_2 vapor pressures and the model static pressures are identical; therefore, it is necessary to know the static pressure distribution around the sphere for the Mach numbers being considered. These pressure distribution data have been obtained from the work of Korobkin, reference (b). Sublimation temperatures corresponding to the static pressures on the upstream hemisphere were determined from the Handbook of Physics and Chemistry, reference (c), and are shown in Figure 1. Undoubtedly, the true sublimation temperatures are lower than the values used in this analysis. For instance, if the vapor pressures of CO_2 were 80 percent of the static pressure, the surface temperature would be lowered by approximately 3°K at the stagnation point and by 1°K at $\theta = 90^\circ$. Therefore, the results may be regarded as a limiting condition.

6. The calculated sublimation temperatures can be approximated by the empirical equation:

$$T = A_0 + A_1 \cos \theta \quad (3)$$

where A_0 and A_1 are constants for a given Mach number and θ is the angle shown in Figure 1.

7. Figure 2 is a plot by Gruenewald (unpublished) of the latent heat of sublimation of CO_2 as a function of temperature. For $163^\circ\text{K} \leq T \leq 193^\circ\text{K}$ theoretical values of H were computed from the equation,

$$H = 6.440 + 1.73 \times 10^{-3}T - 1.78 \times 10^{-5}T^2 \quad (4)$$

given by Kelly (reference d). Plank and Kuprianoff, reference (e), give theoretical values of H for $140^\circ\text{K} \leq T \leq 195^\circ\text{K}$. Using a Clausius-Claperon modification for an imperfect gas, other values of H are given for $173^\circ\text{K} \leq T \leq 216^\circ\text{K}$. Experimental points obtained by Eucker and Donath, Maass and Barnes, and Kuenen and Robson are given in reference (e). Since these points fall between the computed theoretical values, they have been taken as the basis for the solid curve in Figure 2 and used in the heat-transfer analysis. Combining the information of Figures 1 and 2, the variation in the local heat of sublimation around the hemisphere was determined and is shown in Figure 3.

8. Gruenewald, reference (a), reported only gross mass losses on the ablating sphere, and did not indicate the spherical distribution of these losses. Therefore, it is impossible to determine $H(m)$ as specified by Equation (2). Fortunately, the variation of H along the sphere is small, Figure 3, and it is safe to assume that $\bar{H}$ can be determined for each Mach number on a purely geometrical basis, i.e.,

$$\bar{H} \approx \frac{1}{A} \int_0^A H dA \quad (5)$$

Furthermore, Gruenewald has indicated that no visible changes occurred on the rear of the sphere nor in the model diameter normal to the flow during his wind tunnel tests. All of the ablation appeared to be restricted to the upstream half of the sphere. Therefore, the following values of $\bar{H}$ were determined from Equation (5) for the front hemisphere.

Average Heat of Sublimation for a Hemisphere

<u>Mach Number</u>	<u>$\bar{H}$(cal/gram)</u>
1.86	138.9
2.87	140.6
4.25	142.2

All heat-transfer calculations using Equation (1) were performed for only the upstream hemisphere.

Heat Transfer to a Non-Evaporative Hemisphere

9. Because only the upstream hemisphere of the dry ice models appeared to be subject to sublimation, a direct comparison between the heat-transfer results of the present analysis and previously published work on heat transfer to non-ablating hemispheres can be made.

10. The time rate of heat transfer to a non-evaporative body is obtained from the equation

$$Q = h A (T_e - T_w) \quad (6)$$

Where h is the heat-transfer coefficient; A the surface area involved in the heat-transfer process; T_e the equilibrium temperature; and T_w the surface temperature.

11. Wall temperatures were taken to be the same as those of the dry ice spheres and equilibrium temperatures were computed using values of T_e/T_o as given by Korobkin, reference (b).

12. Since local flow properties for the various Mach numbers are known from the experimental test conditions, reported in reference (a), laminar heat-transfer coefficients for non-evaporative hemispheres at each Mach number were obtained from

$$F(\theta) = \frac{\frac{hD}{k}}{\left(\frac{8D^2}{\gamma}\right)^{1/2}} \quad (7)$$

given by Korobkin, reference (b). Heat-transfer rates were then calculated from Equation (6).

RESULTS AND DISCUSSION

13. The over-all heat-transfer rates for both 5 cm diameter dry ice hemispheres as determined from Equation (1) and for the corresponding non-evaporative hemisphere in laminar flow are given in the following table:

Over-all Heat-Transfer Rates for Dry Ice
and for Non-Evaporative Hemispheres

<u>Mach Number</u>	<u>Over-all Q in Kcal/hr*</u>	
	<u>Dry Ice</u>	<u>Non-Evaporative</u>
1.86	440	129
2.87	184	86
4.25	83	40

As can be seen from the above table, the amount of heat transferred to the dry ice spheres was found to be two to three times as great as the heat-transfer calculated for non-evaporative bodies. Calculations which have been performed for transpiration cooling by Smith, reference (f), and others indicate that injection and diffusion effects tend to reduce the heat transfer, not to increase it. A possible explanation of the discrepancy is the effect of conduction within the sphere.

14. Prior to testing in the wind tunnel, the sphere is uniformly at the sublimation temperature corresponding to atmospheric pressure. In the tunnel, sphere surface pressures are less than atmospheric and sublimation will occur at a lower surface temperature. Consequently, heat will be transferred from the high temperature interior of the sphere to the exterior causing additional sublimation. In order to make an estimate of the heat conduction effects, a solid dry ice sphere with an initial temperature, T_1 and a surface temperature distribution $F(\theta)$, Equation (3) was examined.

15. The surface conditions were considered time independent. The total temperature was taken to be equal to $v = u + w$, where u is the solution of the steady temperature problem; and w is the solution of the heat flow through a sphere with a given initial temperature, reference (g). From Appendix A, Equation (11), the total temperature is

$$v = A_0 + \frac{A_1 r}{a} \cos \theta + \sum_{\alpha_{01}} \left[A_{0\alpha} e^{-K\alpha_{01}^2 t} (\alpha_{01} r)^{-1/2} J_{1/2}(\alpha_{01} r) \right] + \sum_{\alpha_{11}} \left[A_{1\alpha} e^{-K\alpha_{11}^2 t} (\alpha_{11} r)^{-1/2} J_{3/2}(\alpha_{11} r) \cos \theta \right]$$

*Values are the average taken from a number of blows.

Taking $t = 40$ seconds (the average length of blow in wind tunnel for dry ice tests), $\rho = 1.52 \text{ g/cm}^3$ (determined from dry ice used in tests), $C = 0.31 \text{ cal/g}^\circ\text{C}$ (reference h) and $k = .00108 \text{ cal/cm}^\circ\text{C sec}$, (reference i), and $a = 2.5 \text{ cms}$, the heat transfer at the surface was determined from the equations

$$q = -k \left(\frac{dv}{dr} \right)_{r=a} \text{ and } Q = \int_0^A q dA$$

The results are included in the table below.

Heat Transfer by Conduction in 5 cm Diameter
Dry Ice Hemisphere

Mach Number	1.86	2.87	4.25
Difference between over-all heat-transfer rates for dry ice and non-evaporative hemispheres	311	98	43
Difference in total heat transfer to dry ice and non-evaporative hemispheres over a period of 40 sec.	3.456	1.088	0.4776
Computed heat transfer by conduction in dry ice hemisphere in 40 sec.	0.05557	0.1066	0.1614
Percentage of total heat transfer difference accounted for by conduction	1.61	9.80	33.80

The conduction effects are more pronounced at the higher Mach numbers at which wind tunnel pressures and temperatures are lowest. The conduction calculation has ignored the effects at the rear of the sphere since it was assumed initially that no ablation occurred in that region. Actually, the base of the sphere is subjected to the lowest temperatures and conduction effects are probably greatest in that region. An upper limit to the total conduction to the entire sphere for a period of 40 seconds has been calculated as:

M	Kcal	Percent of difference in heat transfer between ablating an non-evaporative model
1.86	0.2003	5.8
2.84	0.3436	31.6
4.25	0.4625	96.8

16. In all cases, the dry ice model has been assumed spherical in shape for heat-transfer computations. However, at the lower Mach numbers there is considerable change in the contour of the model during a blow. This change from a spherical front to a more cone-like or to an irregular front would cause an increase in the heat-transfer rates. Also, probably due to the inhomogeneties of the dry ice material, the surface becomes quite rough and again the heat-transfer rates would tend to increase.

17. Another possible explanation of the apparently high heat transfer to the ablating sphere is mechanical failure of the model material. At the lower Mach numbers, strong dynamic forces acting upon the dry ice sphere caused loss of mass through surface fragmentation with particles being swept downstream. However, as the Mach number increased, the amount of such visible erosion decreased. Thus, mechanical erosion effects are most important where conduction effects are minimum.

CONCLUSIONS

18. The determination of heat-transfer rates to subliming dry ice spheres as simply calculated from mass ablation rates yields results which are higher than can be justified by convective considerations. This may be accounted for by: (1) increase in local heat-transfer rates due to surface roughness; (2) increase in heat transfer due to change in model shape; (3) loss of mass without heat transfer through fragmentation; (4) lack of knowledge concerning properties of solid CO₂; (5) lack of knowledge concerning vapor pressure around model; and (6) internal heat conduction. Heat conduction effects internal to

the model are significant, especially at the highest Mach number tested, when the greatest change in environmental pressure occurred.

19. Before genuinely conclusive results can be obtained on ablating bodies in supersonic flow, either methods must be devised for obviating the phenomena of mechanical erosion and internal conduction or these effects must be accounted for accurately.

REFERENCES

- (a) Gruenewald, Karl H., "Drag and Evaporation Rate of Dry Ice Models in Supersonic Air Flow," NAVORD Report 2954, 21 Sept 1953
- (b) Korobkin, I., "Laminar Heat-Transfer Characteristics of a Hemisphere for Mach Number Range 1.9 to 4.9," NAVORD Report 3841, 10 Oct 1954
- (c) Hodgman, C. D., Handbook of Chemistry and Physics, 34th edition, page 1987, 1952-1953
- (d) Kelly, K. K., Contributions to the data on theoretical metallurgy III. "The Free Energies of Vaporization and Vapor Pressures of Inorganic Substances," Bulletin 383, U. S. Dept. of Interior, Bureau of Mines, 1935
- (e) Plank, R., and Kuprianoff, J., "Die Thermischen Eigenschaften der Kohlensaure im Gasformigen, Flussigen und Festen Zustand," 1929, Beihefte zur Zeitschrift fur die gesamte Kalte-Industrie, Reihe 1, Heft 1
- (f) Smith, J. W., "Effect of Diffusion Fields on the Laminar Boundary Layer," Jour. of Aero. Sci., Vol. 21, No. 3, pp 154-164, March 1954
- (g) Carslaw and Jaeger, "Conduction of Heat in Solids (page 212) University Press, Oxford, Second Impression, 1948
- (h) National Bureau of Standards, Circular Letter LC-763, Oct 1944
- (i) Zeitschrift fur die gesamte Dalte-Industrie, pp 109-118, June 1938

APPENDIX A

Heat Conduction Through a Solid Carbon-Dioxide Sphere

1. Considering a solid sphere and assuming that surface conditions do not vary with time,

$$\frac{\partial v}{\partial t} = k \nabla^2 v \text{ through the solid}$$

$$v = T_1 \text{ initially}$$

$$v = F(\theta) \text{ at the surface}$$

Let

$$v = u + w \quad (A1)$$

where u is a function of θ only and satisfies $\nabla^2 u = 0$ through the solid; $u = F(\theta)$ at the surface; and where w is a function of r , θ and t such that $\partial w / \partial t = K \nabla^2 w$ through the solid; $w = [g(r, \theta) - u]$ initially; and $w = 0$ at the surface. Then u is the solution of the steady temperature problem

$$\sum_{n=0}^{\infty} A_n \left(\frac{r}{a} \right)^n P_n(\mu) \quad (2)$$

reference (g). The steady temperature distribution over the surface of the front half of a dry ice sphere satisfied the relation $F(\theta) = A_0 + A_1 \cos \theta$ where A_0 and A_1 are constant for a given Mach number, see page 6 of text and Figure 3. So that its solution from Equation (2) is

$$u = A_0 P_0(\mu) + A_1 \left(\frac{r}{a} \right) P_1(\mu) \quad (3)$$

since $\mu = \cos \theta$, $P_0(\mu) = 1$ and $P_1(\mu) = \cos \theta$ and γ has values of 0 and 1 only.

$$g(r, \theta) = T_1 - u = (T_1 - A_0) P_0(\mu) - A_1 \left(\frac{r}{a} \right) P_1(\mu) \quad (4)$$

w is the solution of the problem of variable temperatures with zero surface temperature, reference (g).

$$W = \sum_n \sum_{\alpha_{ni}} e^{-K\alpha_{ni}^2 t} (\alpha_{ni} r)^{-\frac{1}{2}} J_{n+\frac{1}{2}}(\alpha_{ni} r) P_n(u) A_n \quad (5)$$

where α_{ni} is a root of $J_{n+1/2}(\alpha_{ni} a)$. Since n has values of 0 and 1 there are two sets of α 's and Equation (5) can be expanded as

$$W = \sum_{\alpha_{0i}} e^{-K\alpha_{0i}^2 t} (\alpha_{0i} r)^{-\frac{1}{2}} J_{\frac{1}{2}}(\alpha_{0i} r) A_{0\alpha} + \sum_{\alpha_{1i}} e^{-K\alpha_{1i}^2 t} (\alpha_{1i} r)^{-\frac{1}{2}} J_{\frac{3}{2}}(\alpha_{1i} r) P_1(u) A_{1\alpha} \quad (6)$$

where

$$A_{0\alpha} = \left\{ a^2 \alpha_0^{-\frac{1}{2}} [J'_{\frac{1}{2}}(\alpha_0 r)]^2 \right\}^{-1} \int_0^a r^{\frac{3}{2}} J_{\frac{1}{2}}(\alpha_0 r) dr \int_{-1}^1 g(\theta, r) P_0(u) du \quad (7)$$

$$A_{1\alpha} = \left\{ a^2 \alpha_1^{-\frac{1}{2}} [J'_{\frac{3}{2}}(\alpha_1 r)]^2 \right\}^{-1} \int_0^a r^{\frac{3}{2}} J_{\frac{3}{2}}(\alpha_1 r) dr \int_{-1}^1 g(\theta, r) P_1(u) du \quad (8)$$

$$A_{0\alpha} = \frac{2(T_1 - A_0) J_{\frac{3}{2}}(\alpha_0 a)}{(\alpha_0 a)^{\frac{1}{2}} [J'_{\frac{1}{2}}(\alpha_0 a)]^2} \quad (9)$$

$$A_{1\alpha} = \frac{-2 A_1 J_{\frac{5}{2}}(\alpha_1 a)}{(\alpha_1 a)^{\frac{1}{2}} [J'_{\frac{3}{2}}(\alpha_1 a)]^2} \quad (10)$$

Adding Equations (3) and (6) give

$$v = u + W = A_0 + \frac{A_1}{a} (\cos \theta) r + \sum_{\alpha_{0i}} \left[A_{0\alpha} e^{-K\alpha_{0i}^2 t} (\alpha_{0i} r)^{-\frac{1}{2}} J_{\frac{1}{2}}(\alpha_{0i} r) \right] + \sum_{\alpha_{1i}} \left[A_{1\alpha} e^{-K\alpha_{1i}^2 t} (\alpha_{1i} r)^{-\frac{1}{2}} J_{\frac{3}{2}}(\alpha_{1i} r) \cos \theta \right] \quad (11)$$

The heat transferred at the sphere surface, ($r=a$), can be expressed by

$$q = -k \left(\frac{dr}{dr} \right)_{r=a} = -k \left[B_1 \cos \theta - \sum_{\alpha_{oi}} B_{\alpha} e^{-K \alpha_{oi}^2 t} - \sum_{\alpha_{ii}} B_{i\alpha} e^{-K \alpha_{ii}^2 t} \cos \theta \right] \quad (12)$$

where $B_1 = \frac{A_1}{a}$, $B_{\alpha} = A_{\alpha o} \left(\frac{\alpha_o}{a} \right)^{1/2} J_{3/2}(\alpha_o a)$ and $B_{i\alpha} = A_{i\alpha} \left(\frac{\alpha_i}{a} \right)^{1/2} J_{5/2}(\alpha_i a)$

For a hemisphere,

$$Q = \int_0^A q dA = \int_0^{\pi/2} 2\pi a^2 q \sin \theta d\theta \quad \text{so that} \quad (13)$$

$$Q = -k\pi a^2 \left[B_1 - 2 \sum_{\alpha_{oi}} B_{\alpha} e^{-K \alpha_{oi}^2 t} - \sum_{\alpha_{ii}} B_{i\alpha} e^{-K \alpha_{ii}^2 t} \right]$$

or

$$Q = -k\pi a \left[A_1 - 4(T_1 - A_o) \sum_{\alpha_{oi}} e^{-K \alpha_{oi}^2 t} + 2A_1 \sum_{\alpha_{ii}} e^{-K \alpha_{ii}^2 t} \right] \quad (14)$$

Heat conducted through a hemisphere in t_1 seconds can be obtained by integrating Q with respect to t through the interval 0 to t_1

$$\int_0^{t_1} Q dt = -k\pi a \left\{ A_1 t_1 - 4(T_1 - A_o) \sum_{\alpha_{oi}} \left(\frac{1 - e^{-K \alpha_{oi}^2 t_1}}{K \alpha_{oi}^2} \right) + 2A_1 \sum_{\alpha_{ii}} \left(\frac{1 - e^{-K \alpha_{ii}^2 t_1}}{K \alpha_{ii}^2} \right) \right\} \quad (15)$$

Considering a whole sphere

$$Q = \int_0^{\pi} 2\pi a^2 q \sin \theta d\theta = 4k\pi a^2 \sum_{\alpha_{oi}} B_{\alpha} e^{-K \alpha_{oi}^2 t} \quad (16)$$

or

$$Q = 8k\pi a (T_1 - A_o) \sum_{\alpha_{oi}} e^{-K \alpha_{oi}^2 t}$$

and for a period of t_1 seconds

$$\int_0^{t_1} Q dt = 8k\pi a (T_1 - A_o) \sum_{\alpha_{oi}} \left(\frac{1 - e^{-K \alpha_{oi}^2 t_1}}{K \alpha_{oi}^2} \right) \quad (17)$$

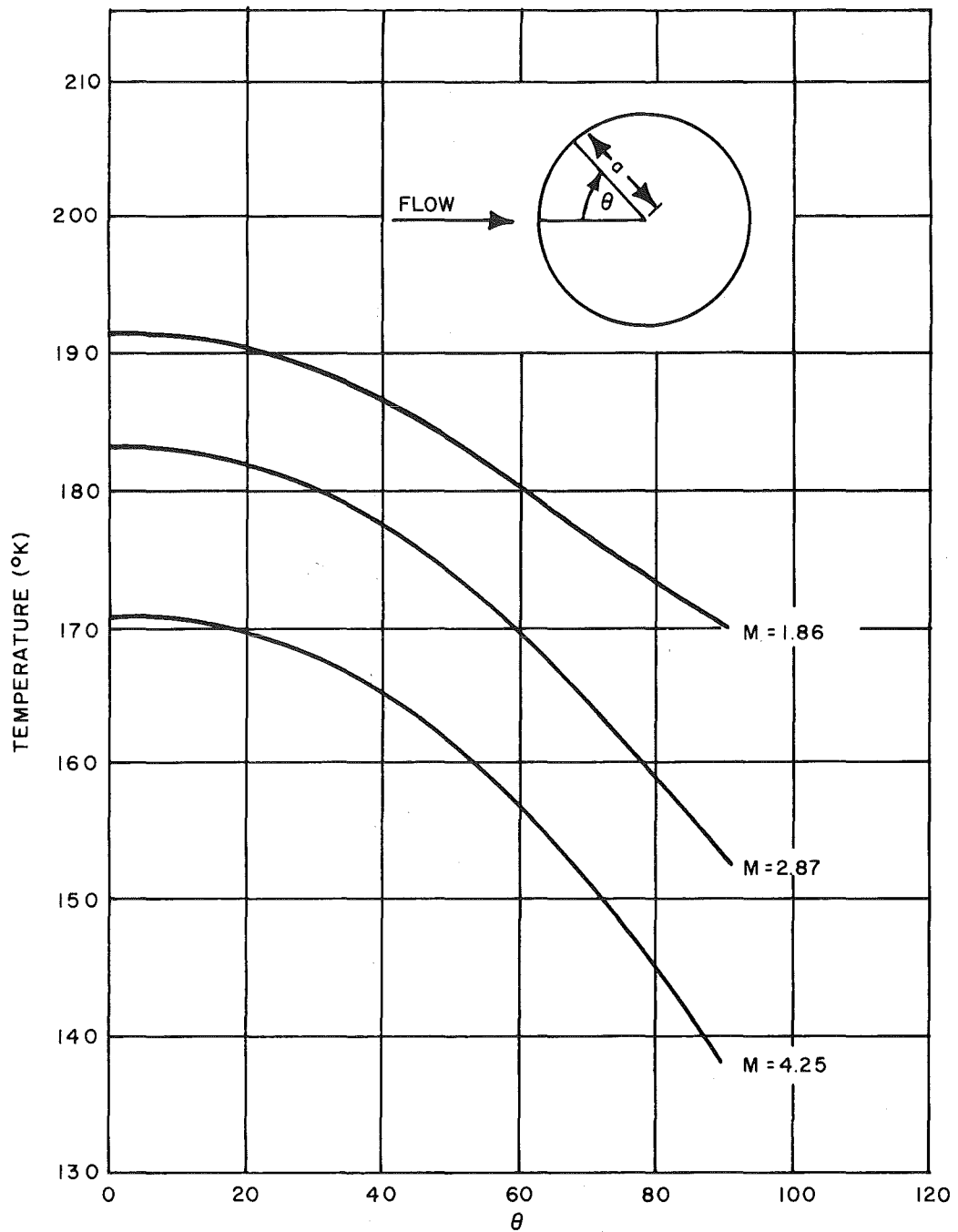
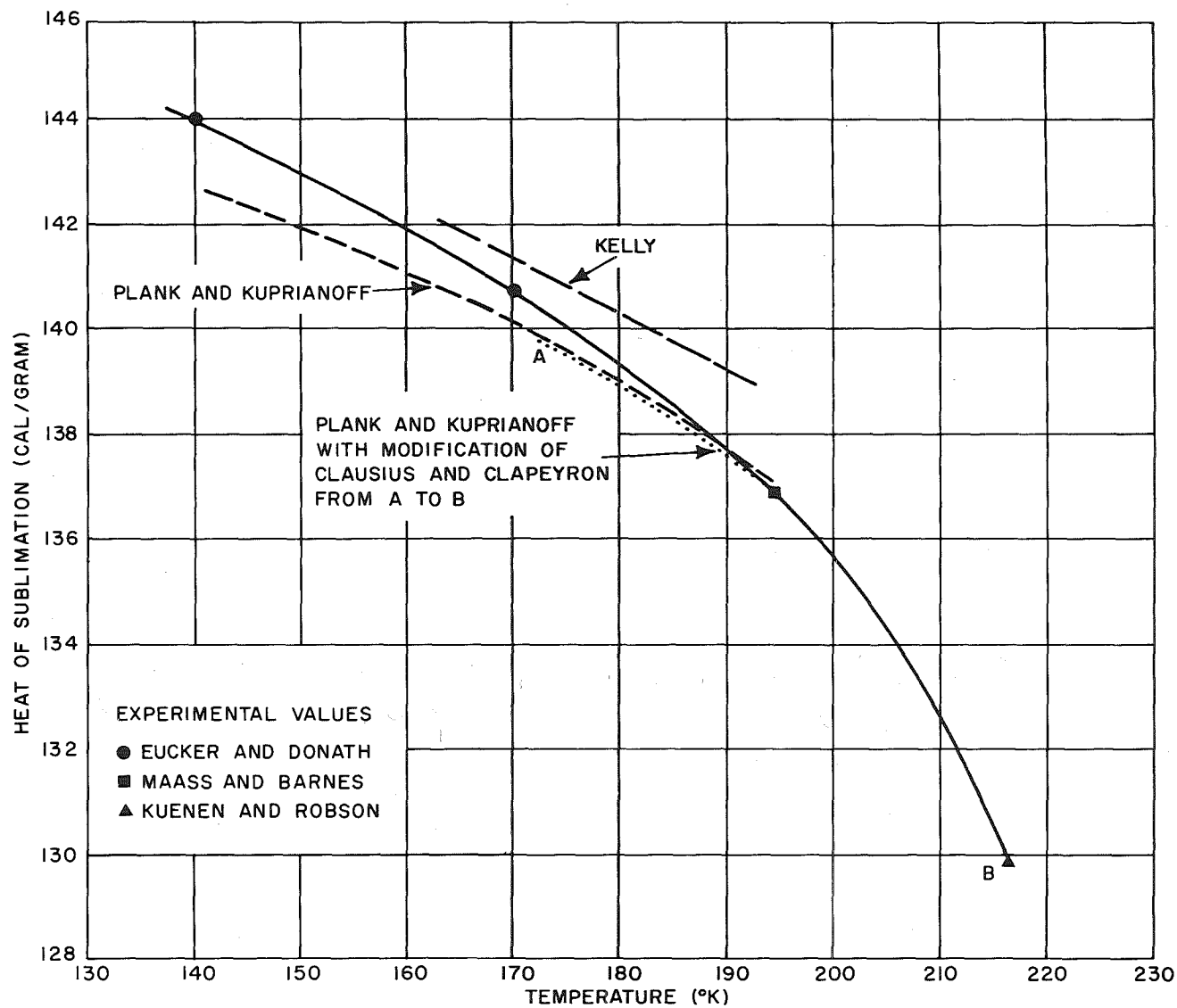


FIG.1 SURFACE TEMPERATURE OF SUBLIMATION
VS. ANGULAR POSITION ON DRY ICE SPHERE

FIG.2 HEAT OF SUBLIMATION FOR SOLID CO₂

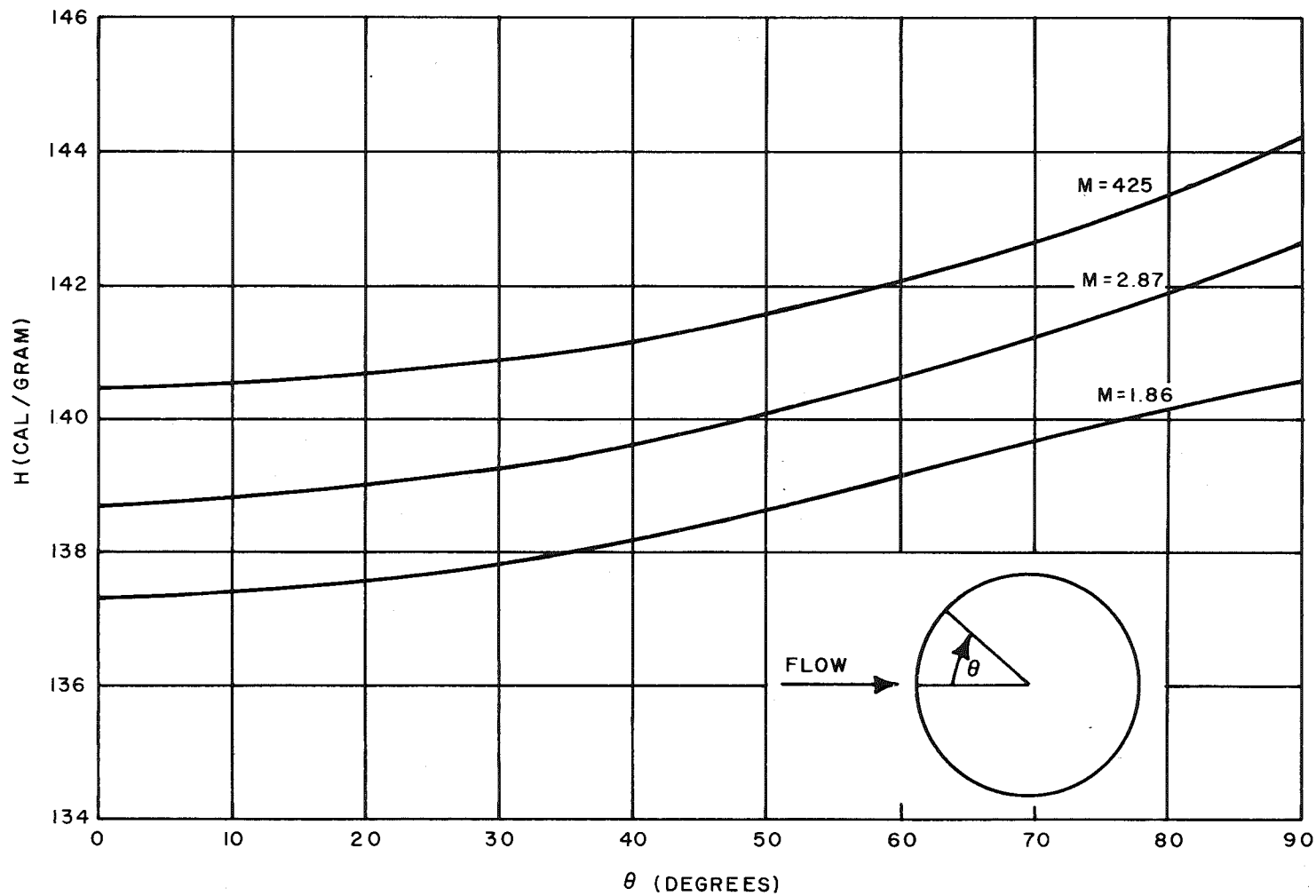


FIG.3 HEAT OF SUBLIMATION OVER A
DRY ICE SPHERE IN SUPERSONIC FLOW

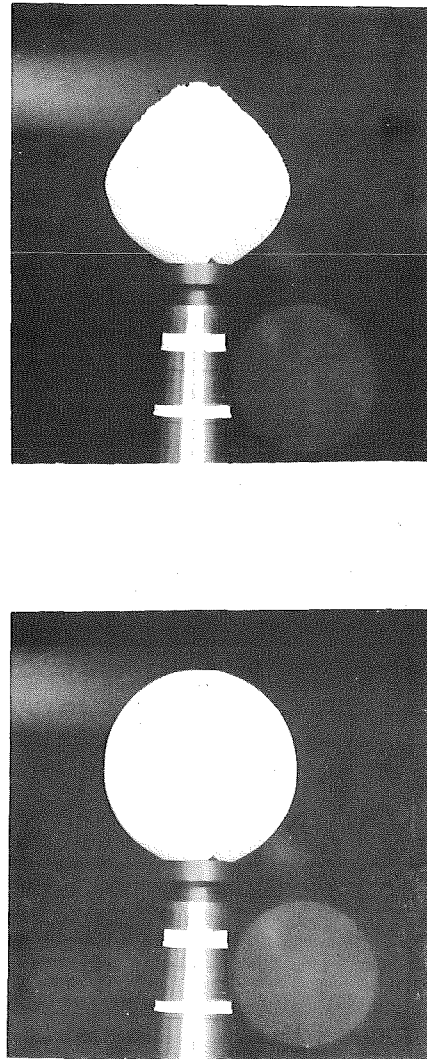


FIG. 4 DRY ICE SPHERE, BEFORE AND AFTER BLOWING
 $M=1.86$

AERODYNAMICS DEPARTMENT
EXTERNAL DISTRIBUTION LIST (A1)

<u>No. of Copies</u>		<u>No. of Copies</u>	
	Chief, Bureau of Ordnance		NACA
	Department of the Navy		Langley Aeronautical Laboratory
	Washington 25, D. C.		Langley Field, Virginia
1	Attn: Ad3	3	Attn: Librarian
1	Attn: Ree	1	Attn: C. H. McLellan
1	Attn: ReO3	1	Attn: J. J. Stack
1	Attn: ReS1-e	1	Attn: Adolf Busemann
1	Attn: SP2722	1	Attn: Comp. Res. Div.
		1	Attn: Theoretical Aerodynamics Div.
	Office of Naval Research		NACA
	Room 2709, T-3		Ames Aeronautical Laboratory
	Washington 25, D. C.		Moffett Field, California
1	Attn: Head, Mechanics Br.	1	Attn: Librarian
	Director, DTMB		NACA
	Aerodynamics Laboratory		Lewis Flight Propulsion Lab.
	Washington 7, D. C.		21000 Brookpark Road
1	Attn: Library		Cleveland 11, Ohio
	Officer in Charge, NPG	1	Attn: Librarian
	Dahlgren, Virginia	1	Attn: Chief, Propulsion
1	Attn: Library		Aerodynamics Div.
	Commander, U. S. NOTS		NACA
	China Lake, California		1512 H Street, N. W.
1	Attn: Technical Library		Washington 25, D. C.
1	Attn: Code 503	1	Attn: Chief, Division of
1	Attn: Code 406		Research Information
	Director, NRL		Office of the Assistant
	Washington 25, D. C.		Secretary of Defense (R and D)
1	Attn: Code 2027		Room 3E1065, The Pentagon
12	Commanding Officer		Washington 25, D. C.
	Office of Naval Research	1	Attn: Technical Library
	Branch Office		Research and Development Board
	Box 39, Navy 100		Room 3D1041, The Pentagon
	Fleet Post Office		Washington 25, D. C.
	New York, N. Y.	2	Attn: Library
	NACA		ASTIA
	High Speed Flight Station	10	Document Service Center
	Box 273		Knott Building
	Edwards Air Force Base,		Dayton 2, Ohio
	California		
1	Attn: W. C. Williams		

No. of
Copies

No. of
Copies

1 Commander, NAMTC
Point Mugu, California
Attn: Technical Library

1 Commanding General
Aberdeen Proving Ground, Md.
Attn: Technical Info. Br.

1 Attn: Ballistics Res. Lab.

3 Chief, BuAer
Washington 25, D. C.
Attn: Document Library

1 Director of Intelligence
Headquarters, USAF
Washington 25, D. C.
Attn: AFOIN-3B

2 Commander, WADC
Wright-Patterson AF Base
Ohio
Attn: WCOSI-3

1 Attn: WCLSW-5

3 Attn: WCRRD

1 Washington AF Development
Field Office
Room 3816, Main Navy Bldg.
Washington 25, D. C.
Attn: Maj. H. W. Keller

1 Air Force Ballistic Missile Div.
HQ Air Research & Development
Command
P. O. Box 262
Inglewood, California
Attn: WDTLAR

1 Chief, AFSWP
Washington 25, D. C.
Attn: Document Library

1 Commanding General
Arnold Engineering Development
Center
Tullahoma, Tennessee
Attn: Technical Library

5 Attn: AEKS

1 Commanding Officer, DOFL
Washington 25, D. C.
Attn: Library
Room 211, Bldg. 92

1 Commanding General
Army Ballistic Missile Agency
Huntsville, Alabama
Attn: ORDAB-DA

1 Attn: Dr. E. Geissler

1 Attn: Mr. T. Reed

1 Attn: Mr. H. Paul

1 Attn: Mr. W. Dahm

1 Commanding General
Redstone Arsenal
Huntsville, Alabama
Attn: Mr. N. Shapiro,
ORDDW-MRF

1 Office, Chief of Ordnance
Department of the Army
Washington 25, D. C.
Attn: ORDTU

2 APL/JHU (C/Nord 7386)
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Tech. Reports Group

1 Attn: Mr. E. Bonney

1 Attn: Mr. D. Fox

1 Attn: Dr. F. Hill
Via: INSORD

AERODYNAMICS DEPARTMENT
EXTERNAL DISTRIBUTION LIST (A2)

<u>No. of Copies</u>		<u>No. of Copies</u>	
	Superintendent	1	CONVAIR
	U. S. Naval Postgraduate		A Division of General Dynamics
	School		Corporation
	Monterey, California		Fort Worth, Texas
1	Attn: Tech. Rpts. Section		United Aircraft Corporation
	Library		400 Main Street
	National Bureau of Standards		East Hartford 8, Connecticut
	Washington 25, D. C.	1	Attn: Chief Librarian
1	Attn: Chief, Fluid Mechanics	2	Attn: Mr. W. Kuhrt,
	Section		Research Dept.
	University of Minnesota	1	Attn: Mr. J. G. Lee
	Rosemount Research Laboratories	1	Attn: Mr. W. E. Powers
	Rosemount, Minnesota		Hughes Aircraft Company
1	Attn: Tech. Library		Florence Avenue at Teale Street
			Culver City, California
		1	Attn: Mr. D. H. Johnson
			R and D Tech. Library
1	Director		
	Air University Library	1	McDonnell Aircraft Corporation
	Maxwell AF Base, Alabama		P. O. Box 516
			St. Louis 3, Missouri
	Douglas Aircraft Company, Inc.		Lockheed Aircraft Corporation
	Santa Monica Division		Lockheed Missile Systems Div.
	3000 Ocean Park Boulevard		Sunnyvale, California
	Santa Monica, California	1	Attn: Dr. L. H. Wilson
1	Attn: Chief Missiles Engineer	1	Attn: Mr. W. E. Brandt
1	Attn: Aerodynamics Section	4	Attn: Mr. M. Tucker
		1	Attn: Mr. B. W. March
1	CONVAIR	1	Attn: Mr. W. J. Fleming, Jr.
	A Division of General Dynamics		The Martin Company
	Corporation		Baltimore 3, Maryland
	Daingerfield, Texas	1	Attn: Library
		1	Attn: Chief Aerodynamicist
	CONVAIR		North American Aviation, Inc.
	A Division of General Dynamics		Aerophysics Laboratory
	Corporation		Downing, California
	3595 Frontier Street		Attn: Dr. E. R. Van Driest
	San Diego, California		
1	Attn: Mr. M. Sibulkin	1	BAR
1	Attn: Asst. to the Dir. of		Aerojet-General Corporation
	Scientific Research		6352 N. Irwindale Avenue
			Azusa, California
	Republic Aviation Corporation		
	Farmingdale, New York	1	Boeing Airplane Company
1	Attn: Technical Library		Seattle, Washington

No. of
Copies

No. of
Copies

1	RAND Corporation 1700 Main Street Santa Monica, California Attn: Lib., USAF Project RAND	2	Chance-Vought Aircraft, Inc. Dallas, Texas Attn: Librarian
1	Arnold Research Organization, Inc. Tullahoma, Tennessee Attn: Tech. Library	1	Ramo-Woolridge Corporation Guided Missiles Research Div. Los Angeles 45, California Attn: Dr. G. Solomon
1	Attn: Chief, Propulsion Wind Tunnel		Cornell Aeronautical Lab., Inc. 4455 Genesee Street Buffalo 21, New York
1	Attn: Dr. J. L. Potter	1	Attn: Librarian
	General Electric Company Missiles and Ordnance Systems Dept. 3198 Chestnut Street Philadelphia, Pennsylvania	1	Attn: J. Logan, Jr.
2	Attn: Larry Chasen Mgr. Library	1	Defense Research Laboratory The University of Texas P. O. Box 8029 Austin 12, Texas Attn: Assistant Director
1	Attn: Mr. R. Kirby		Ohio State University Columbus 10, Ohio
1	Attn: Dr. N. Ness	1	Attn: Security Officer
1	Attn: Dr. J. Farber	1	Attn: Aerodynamics Lab.
1	Attn: Dr. G. Sutton	1	Attn: Mr. J. Lee
1	Attn: Dr. W. Carlson	1	Attn: Chairman, Dept. of Aeronautical Engineering
1	Attn: Dr. H. Lew		
	Eastman Kodak Company Navy Ordnance Division 50 West Main Street Rochester 14, New York		CIT Pasadena, California
2	Attn: W. B. Forman	1	Attn: Guggenheim Aeronautical Lab., Aeronautics Library
3	The AVCO Manufacturing Corp. Research Laboratories 2385 Revere Beach Parkway Everett 49, Massachusetts	1	Attn: Jet Propulsion Lab.
		1	Attn: Dr. H. Liepmann
		1	Attn: Dr. L. Lees
		1	Attn: Dr. D. Coles
1	AER, Incorporated 871 East Washington Street Pasadena, California	1	Attn: Mr. A. Roshko
		1	Attn: Mr. S. Dhawan
	Armour Research Foundation 10 West 35th Street Chicago 16, Illinois	1	Case Institute of Technology Cleveland 6, Ohio Attn: G. Kuerti
2	Attn: Dept. M		

No. of
Copies

Massachusetts Institute of
Technology
Cambridge 39, Massachusetts
1 Attn: Prof. J. Kaye
1 Attn: Prof. M. Finston
1 Attn: Mr. J. Baron
1 Attn: Mr. M. Sweeney, Jr.

New York University
45 Fourth Avenue
New York 3, New York
1 Attn: Prof. R. Courant
1 Attn: Prof. H. Ludloff

Polytechnic Institute of
Brooklyn
527 Atlantic Avenue
Freeport, New York
1 Attn: Dr. A. Ferri
1 Attn: Dr. M. Bloom
1 Attn: Dr. P. Libby

Brown University
Division of Engineering
Providence, Rhode Island
1 Attn: Prof. R. Probststein
1 Attn: Prof. C. Lin

University of Minnesota
Minneapolis 14, Minnesota
1 Attn: Dr. E. R. G. Eckert
1 Attn: Dr. J. Hartnett
1 Attn: Heat Transfer Lab.
1 Attn: Tech. Library

Gruen Laboratories
c/o Polytechnic Institute of
Brooklyn
527 Atlantic Avenue
Freeport, New York
1 Attn: Mr. Walter Daskin

Rensselaer Polytechnic Institute
Troy, New York
1 Attn: Dept. of Aeronautical
Engineering

No. of
Copies

Princeton University
James Forrestal Research
Center
Gas Dynamics Laboratory
1 Attn: Prof. S. Bogdonoff

Institute for Fluid Dynamics
and Applied Mathematics
University of Maryland
College Park, Maryland
2 Attn: Director
1 Attn: Dr. J. Burgers

University of Michigan
Ann Arbor, Michigan
1 Attn: Dr. A. Kueth

1 Applied Mathematics and
Statistics Laboratory
Stanford University
Stanford, California

Cornell University
Graduate School of Aero. Engr.
Ithaca, New York
1 Prof. W. R. Sears

The Johns Hopkins University
Charles and 34th Streets
Baltimore, Maryland
1 Attn: Dr. F. H. Clauser

University of California
Berkeley 4, California
1 Attn: G. Maslach
1 Attn: Dr. S. Schaaf