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1 OF 1

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NOTE ON STATIC PERFORMANCE OF SHROUDED PROPELLERS

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NOTE ON STATIC PERFORMANCE OF SHROUDED PROPELLERS

Momentum theory, based on the assumption of incompressible flow and constant jet velocity throughout the slipstream, predicts a relationship between the static thrust and the ideal power of an air-moving propulsive device. As applied to shrouded propellers it may be combined with Bernoulli's equation to produce the relationship between propeller thrust and shroud thrust, and states that $T_{prop} = T_{shroud}$. By altering the distribution of velocity across a radius it is possible to change the ratio of propeller thrust to shroud thrust. This may be done by changing the blade shape, the shroud and hub contours, or the axial position of the propeller within the shroud. Such an adjustment of the thrust ratio also changes the static performance of the entire unit. One contribution to this change can be dealt with in a preliminary fashion by a generalization of the momentum theory.

Consider a stationary device which ingests air and exhausts it in a parallel stream of arbitrary cross section to produce thrust. For the moment let the velocity of the stream be uniform, so that for any element of cross-sectional area ΔA of the exhaust, the thrust produced is

$$\Delta T_o = \Delta \dot{M} V$$
$$= \rho \Delta A V^2$$

Now take any two elementary areas of the exhaust cross section, and require that the velocity through each be changed by a small but finite amount ΔV, with the total mass flow through the two cross sections unchanged. The mass flow through one of the areas will then be $\rho \Delta A(V+\Delta V)$ and that through the other, $\rho \Delta A(V-\Delta V)$. The thrust produced by the flow through the two areas together will be

$$\Delta T_1 + \Delta T_2 = \rho \Delta A(V+\Delta V)^2 + \rho \Delta A(V-\Delta V)^2$$
$$= 2\rho \Delta A(V^2+\Delta V^2)$$

Comparing this thrust with the original total thrust $2\Delta T_o$ gives

$$\frac{\Delta T_1 + \Delta T_2}{2\Delta T_o} = \frac{2\rho \Delta A(V^2+\Delta V^2)}{2\rho \Delta A V^2} = 1 + \frac{\Delta V^2}{V^2}$$

The ideal power, or the kinetic energy lost to the exhaust, will be

$$\Delta P_1 + \Delta P_2 = \frac{\rho}{2}\Delta A(V+\Delta V)^3 + \frac{\rho}{2}\Delta A(V-\Delta V)^3$$

$$= \rho\Delta A(V^3+3V\Delta V^2)$$

Comparing this with the original total power $2\Delta P_o$ gives

$$\frac{\Delta P_1+\Delta P_2}{2\Delta P_o} = \frac{\rho\Delta A(V^3+3V\Delta V^2)}{2(\frac{\rho}{2}\Delta AV^3)} = 1 + \frac{3\Delta V^2}{V^2}$$

Therefore any inequality in the exhaust velocities at any two points in the exhaust stream at constant mass flow will result in proportionately greater increase in the power expended than in the thrust developed. To see what happens if thrusts instead of mass flows are equated, the notation of the above can be retained and a thrust T_3 defined equal to $\Delta T_1 + \Delta T_2$:

$$2\Delta T_3 = 2\rho\Delta AV_3{}^2 = 2\rho\Delta A(V^2+\Delta V^2)$$

then

$$V_3{}^2 = V^2+\Delta V^2$$

and

$$\frac{2\Delta P_3}{\Delta P_1+\Delta P_2} = \frac{(V^2+\Delta V^2)^{3/2}}{(V^3+3V\Delta V^2)}$$

By squaring both sides and simplifying, a ratio of the squares of the total powers is obtained which can be inspected for $V = 1$ and ΔV between zero and 1, wherefrom it will appear that $2\Delta P_3$ will always be less than $\Delta P_1+\Delta P_2$.

The stationary shrouded propeller is a device such as discussed above. Its static thrust is the summation of the thrusts contributed by annular areas of exhaust stream $dA = 2\pi rdr$, or

$$T = \int_0^R \rho(2\pi rdr)v^2$$

$$= 2\pi\rho\int_0^R v^2rdr$$

Similarly the energy lost in the exhaust is

$$P = \rho\pi \int_0^R v^3 r\,dr$$

The commonly used assumption of constant exhaust velocity throughout the exhaust stream (V = const.) gives

$$T_o = \rho(\pi R^2)\,V^2$$

and

$$P_o = \frac{\rho}{2}(\pi R^2)\,V^3$$

The thrust per horsepower is

$$\frac{T_o}{P_o} = \frac{2}{V}$$

Now let the exhaust have instead a linear radial variation of velocity from, say, zero at the center to the tip radius R,

$$V = kr$$

If the thrust of this unit is equal to that of the first,

$$T_1 = T_o$$

$$\rho(\pi R^2)V^2 = 2\pi\rho \int_0^R k^2 r^3 dr$$

whence

$$k = \sqrt{2}\,\frac{V}{R}$$

and the ratio of the slipstream energies -

$$\frac{P_1}{P_o} = \frac{2\int_0^R \left(\frac{\sqrt{2}V}{R}\right)^3 r^4 dr}{R^2V^3}$$

$$= 1.13$$

Thus a propulsive unit with the radial exhaust velocity gradient as specified would expend 13% more power to produce the same thrust, than would one whose exhaust velocity is constant across the slipstream area. Note that while nothing is said about the ratio of shroud thrust to propeller thrust, the consequence of nonuniformity of the exhaust velocity, however produced, is shown to be deleterious. Similar reasoning can be applied to the operation of the shrouded propeller at forward speed.