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OPERATIONAL DEFINITIONS OF MECHANICAL MOBILITY OF MOTOR VEHICLES

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BY
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OPERA ONAL DEFINITIONS OF MECHANICAL MOBILITY OF MOTOR VEHICLES

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M. G. BEKKER

ORDNANCE CORPS
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ORDNANCE TANK-AUTOMOTIVE COMMAND

ACKNOWLEDGEMENT

Appendix I was computed and written by Mr. T. A. Davis, Graduate Student University of Michigan. Appendix II was prepared at the Planning Research Corporation of Los Angeles, California by Messrs. L. P. Holliday, W. J. Dixon, C. H. Gordon and C. P. Siska.

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ABSTRACT

Before broader concepts of Tactical Mobility are more satisfactorily defined, a definition of a narrower concept of Mechanical Mobility must be established within the realm of applied mechanics. This also is needed to guide the engineering progress in the development of more mobile vehicles, and in a physical evaluation of soil-vehicle systems.

A procedure is proposed ~~To this end it has been proposed~~ to optimize mechanical performances of vehicles within the given spectra of terrain conditions by using operations research techniques. Such an optimized value of soil-vehicle system has been proposed as a definition of mechanical mobility within that system.

The proposed procedure leads directly to the establishment of mathematical models of mobility within the given system, which in turn means that lengthy and costly proving ground techniques requiring prior development of full pledged vehicles may be substituted with much faster and cheaper computing techniques for mobility evaluation of vehicle concepts ~~"in statu nascendi."~~

This enables one to evaluate mobilities of all the conceivable soil-vehicle systems pertaining to the given project, which presently is physically impossible because of cost and time involved in building and testing experimental models. Thus considerable rationalization and economy of research and development policies may be expected in Land Locomotion when using the proposed method of mobility definition.

To foster this approach further refinement of presently available principles of the mechanics of land locomotion must be pursued at an accelerated rate.

OPERATIONAL DEFINITIONS
OF MECHANICAL MOBILITY OF MOTOR VEHICLES

PROBLEM

The problem is to provide a working method for establishing a practical definition of vehicle mobility, particularly in off-the-road operation, with the ultimate purpose of using it as a quantitative yardstick of ENGINEERING PROGRESS in VEHICLE DESIGN and SOIL-VEHICLE SYSTEMS EVALUATION.

BACKGROUND

Both the vehicle user and designer look for more mobility. While both agree that more mobile land vehicles are imperative today, they often cannot agree what mobility means.

It appears that the difficulty stems from a duality of viewpoints represented by both sides. Mobility in the current military-technical parliance embodies not only engineering but also tactical values (1, 2). A rationalization of such a concept has been extremely difficult as many attempts to do so have demonstrated (3,4,5,6,7).

These attempts have further complicated the problem, since they have stressed the user's aspect of mobility as defined by subjective and empirical factors influenced by the experience of each writer (4,8,9). The scientific viewpoint which would guide engineering progress by means of the basic principles of mechanics, or a general theory of land locomotion, appears to be lagging as indicated by an almost complete lack of pertinent literature.

Without minimizing the necessity of satisfying the mobility requirements stemming from practical users experience, one must agree that a compromise made predominantly for that purpose does not cont in the seeds of a complete solution of the problem. To the contrary, it tends to freeze the conventional and the traditional which may be clearly seen in the current development trends of military motor vehicles.

This seems to indicate that an objective and rigorous definition of physical mobility should be introduced with the proper emphasis in order to establish a clear-cut engineering aspect of the problem within any broader definition of tactical mobility.

It is believed that without a prior definition of what may be called the narrow concept of MECHANICAL MOBILITY, no such broader and more general concept of TACTICAL MOBILITY may ever be expressed in a more satisfactory way than it is possible today. In addition, it appears quite certain that purely engineering progress in vehicle design and evaluation cannot be planned without a strict formulation of mobility concept based solely on t e mechanics of the soil-vehicle relatioship.

THE SCOPE AND THE NATURE OF THE PROPOSED MOBILITY CONCEPT

Whenever mobility is defined in such terms as a success against "purely numerical superiority," for instance, its meaning becomes of unlimited scope. It may include a general class of values pertaining to the morale, training decision, human behavior, etc., besides a quantitative class of engineering values related to traction, flotation, thrust, fuel economy, maneuverability, etc. To start with the solution of such a broad pro' em, as mentioned before, only the second aspect of the general mobility concept should be analyzed first. Thus the considered definition of Mechanical Mobility will forego the problems pertaining to the qualities of the ART of war (9) and concentrate solely on the quantities related to the PHYSICS of locomotion (11). Accordingly, the contemplated definition of mobility will be expressible in terms of a directly measurable system of values: pound, foot, second, or in units compounded of these values. It will be measurable in terms of vehicle performance or in composite terms of various performances determined on the physico-geometrical background of terrain-vehicle systems. Since the cost in dollars per pound, per foot, and per second is a logical consequence of this type of evaluation, the monetary value also may be introduced.

The values of mobility, however, as stated above would have only a limited degree of generalization if the unavoidable variations of terrain, particularly in off-the-road operation, are not included. Since the choice of routes and the influence of geological and climatological factors is of statistical nature, the frequency and, or the probability of their occurrence based on observational data may consititue another value needed in any long range study or more general assessment of mobility and design.

Mobility criteria established under these assumptions will not necessarily aim at a selective elimination of less successful types of vehicles, for instance, until only "the best" or "[illegible]" vehicle is adopted, but they may aim at resolving such questions as "how many" vehicles of type I, type II, type III, etc, are needed in order to perform the given task, in the given area, with the maximum of over-all efficiency.

Only this approach may fully assess the gains and losses whenever a single "universal" type locomotion is postulated. This procedure cannot now be fully used in vehicle evaluation because the various empirical indices of mobility have not been based on the mechanics of soil-vehicle relationships and do not allow the construction of mathematical models which can be evaluated quickly on electronic computers.

The proposed concept of mechanical mobility must embrace all of the necessary kinds of locomotive performances which leads to a definition of mobility based not on a single value but on a number of performance values such as speed, thrust, acceleration, weight, load, fuel consumption, range of action, obstacle crossing, towing power, buoyancy, form, size, etc. To arrive at a cumulative value or values of various performances specified above, the process of optimization as applied in OPERATIONS RESEARCH will be used.

The optimization may be performed in an indefinite number of ways depending on the importance of factors singled out when defining the cumulative values of mobility. There may be NO SINGLE VALUE OF MOBILITY but an infinite number of values. The choice between possible definitions is based NOT on the criterion of TRUENESS, BUT solely on the basis of USEFULNESS of the given definition in the accomplishment of the given task.

This situation is not unusual. It is the only way in which all the evaluations may be conducted. For instance, soils may be characterized from geological, pedological, or civil engineering viewpoint. Each of these evaluations embraces only those values which are useful in pursuing the activity within the given area and foregoes all the others. Accordingly, in this paper, physical and geometrical soil values pertaining to locomotion will be the only ones used.

In a broad sense, the proposed method of defining mobility is not new. It has even been applied at a number of occasions(12). The main objective of this paper, therefore, is only a formalization of the method in the light of the latest developments in land locomotion mechanics rather than a fostering of a new line of approach. Although these new developments are in the state of "infancy," it is hoped that they constitute a radical step in the rationalization of progress as they attempt to make it less dependent on qualitative "indices" and "factors" of unspecified dimensions.

It thus may be stressed that in the realm of physico-geometrical relationships between soils and a vehicle there can be NO SINGLE FORMULA for mobility. There is, however, a possibility of the establishment of a METHOD by means of which a desired definition of mobility may be arrived at with the purpose of accomplishing the particular task in design and performance evaluation.

A METHOD OF FORMULATING A DEFINITION OF MECHANICAL MOBILITY

In accordance with the foregoing remarks, the whole problem may now be presented as follows: To arrive at a definition of mechanical mobility which can serve as an evaluation of a specific aspect of a problem, one must combine performance values in a strictly defined way. Accordingly, the first step is the formulation of those values. Some of them, such as speed, pay-load, thrust, flotation, etc., were mentioned before. Others may be added whenever necessary. Since all of them depend on terrain properties, it is necessary to express them in terms of vehicle-terrain relationships.

Assume that there are a number of vehicles I, II, III, etc., and a number of terrains B_1, B_2, B_3, etc. The latter are expected to represent a typical cross section of the terrain under consideration and have been selected and defined in accordance with the methods of sampling techniques and the mechanics of land locomotion (11).

By using test data obtained at the proving grounds which represent the same terrain distribution or by applying theoretical analysis, it is possible to establish numbers pertaining to each type of performance. For instance, one may find that vehicle I will develop speed $(V_{B1})_I$, in terrain B_1, $(V_{B2})_I$, in terrain B_2, etc. Vehicle II will cruise at speeds $(V_{B1})_{II}$, $(V_{B2})_{II}$, etc. These values may be tabulated in what may be called the SPEED MATRIX as shown below:

Terrain / Vehicle	B_1	B_2	B_3
I	$(V_{B1})_I$	$(V_{B2})_I$	$(V_{B3})_I$
II	$(V_{B1})_{II}$	$(V_{B2})_{II}$	$(V_{B3})_{II}$
III	$(V_{B1})_{III}$	$(V_{B2})_{III}$	$(V_{B3})_{III}$

In a similar way, other matrices such as those of payload, fuel consumption, gradeability, flotation, fordability, range of action, time of maintenance, cost, etc., may be established.

All these conceivable matrices, taken together, represent, in accordance with the main premise of this paper, a "parametric" form of the definition of Mechanical Mobility. These matrices may be optimized using standard operation research techniques into a single over-all solution.

In an oversimplified and rather trivial case, for instance, the following may illustrate the problem. Assume that average fuel consumption of vehicles I, II and III in a specific terrain B may be expressed by numbers quoted in the following Fuel Economy Matrix:

Terrain / Vehicle	B_1	B_2	B_3
I	6	10	15
II	10	5	8
III	7	8	10

Which vehicles should be selected for an exclusive operation in the particular terrain in order to minimize the total fuel consumption in the whole area?

Assume that the distances travelled in each terrain, B_1, B_2, and B_3, remain unchanged between vehicles. Then the sought optimum will take place when the sum of particular consumptions is a minimum. Taking the minima shown in circles on the matrix, it will be found that vehicles I, II and II operating in B_1, B_2, and B_3 respectively produce the minimum fuel consumption of 19 units while any other selection of vehicle types would produce a greater fuel consumption up to a maximum of 35 units. Thus in this particular case, only the first two types of vehicles would be selected and the third eliminated.

As a further illustration of the similar procedure take the following matrices of fuel consumption (f), speed (v), payload (p):

f-matrix

Terrain / Vehicle	B_1	B_2	B_3
I	15	20	25
II	12	15	25
III	8	17	15

v-matrix

	B_1	B_2	B_3
I	20	15	10
II	15	15	18
III	10	17	20

p-matrix

	B_1	B_2	B_3
I	2	2	1
II	3	3	2
III	4	4	3

Assume that for each terrain B_1, B_2, and B_3 only one of vehicle types I, II and III will be selected, and that the cargo will be re-loaded to another vehicle upon arriving at the terrain border point. This may be an acceptable and economic solution if distances travelled in each terrain are sufficiently large and if the unloading and re-loading of the cargo may be performed in a quick way by container type pick-up chassis equipped with hoists and quick acting fasteners. If the time lost for switching from one vehicle to another is neglected, then the total of $3^3 = 27$ combinations must be considered. Assuming for the sake of simplicity that distances travelled in each terrain are equal, it will be obtained:

B_1	B_2	B_3	f	v	p	vp	vp/f
I	I	I	60	13.8	1	13.8	0.23
I	I	II	60	17.4	2	34.8	0.58
I	I	III	50	18.0	2	36.0	0.72
I	II	I	55	13.8	1	13.8	0.25
I	III	I	57	14.4	1	14.4	0.25
II	I	I	57	12.9	1	12.9	0.23
III	I	I	53	11.2	1	11.2	0.21
II	II	II	52	15.9	2	31.8	0.61
II	II	I	52	12.9	1	12.9	0.25
II	II	III	42	16.4	3	49.2	1.17
II	I	II	55	15.9	2	31.8	0.58
II	III	II	54	16.6	2	33.2	0.61
I	II	II	55	17.4	2	34.8	0.63
III	II	II	48	13.5	2	27.0	0.56
III	III	III	40	14.4	3	43.2	1.08
III	III	I	50	11.6	1	11.6	0.23
III	III	II	50	14.0	2	28.0	0.56
III	I	III	43	13.8	2	27.6	0.64
III	II	III	38	13.8	3	41.4	1.09
I	III	III	47	18.9	2	37.8	0.80
II	III	III	44	17.1	3	51.3	1.16
I	II	III	45	18.0	2	36.0	0.80
II	III	I	54	13.3	1	13.3	0.25
III	II	I	45	11.3	1	11.3	0.25
III	I	II	53	13.5	2	27.0	0.51
I	III	II	57	18.3	2	36.6	0.64
II	I	III	47	16.4	2	32.8	0.70

vp represents the payload delivery rate and vp/f is the payload delivery rate per quantity of fuel consumed which should be maximised.

In the above optimisation, it was assumed that each vehicle carries the minimum payload as restricted by the less favorable terrain - vehicle type combination. For instance, payloads $(p_{B1})_I$, $(p_{B2})_{III}$, and $(p_{B3})_{III}$ amount to 2, 4 and 3 respectively. The minimum of 2 has been selected accordingly for an over-all operation and is shown in line I-III-III, under the column p.

It results from that example that the optimum delivery of cargo per unit of time will be provided by the combination of vehicles II-III-III ($vp = 51.3$) while the most economic operation which will deliver the maximum pv/f payload per [illegible] of fuel burned is the combination II-II-III ($v \times p/f = 1.17$). The best speed belongs to the combination of I-III-III ($v = 18.9$).

The final choice of "most mobile" vehicle depends on what is more important - fuel economy, delivery rate, or speed of operation. A compromise can be made easily when using the summary matrix as previously discussed and assuming operational values other than those considered in the definition of mechanical mobility.

In a similar way the obstacle crossing ability, for instance, may be included in the mobility definition by establishing a matrix of maximum widths of obstacles which may be crossed in the given terrain by the given vehicle. If to this matrix, the frequency of occurrence of these obstacles or the probability of their encountering is added, then one may choose the "most mobile" vehicles based on the optimum payload, for example, delivered per unit of time and per unit of fuel consumed while considering the probability that only the minimum percentage of vehicles will never arrive to the destination because they will be held up by too wide ditches, rivers, or streams.

The examples quoted illustrate that many criteria may be chosen [illegible] over-all definition of mobility. They also illustrate the RELATIVE MERITS OF MOBILITY which depend on the performed optimisation. In addition, they demonstrate that the meaning of these merits make sense ONLY within VEHICLE-TERRAIN SYSTEM under consideration.

A still broader scope of defining vehicle mobility within the terrain-vehicle system may be seen in a case when the probability of occurrence of changes in terrain conditions due to the geological climate or other variations are considered. Examples of such procedure are shown in Appendix I in two examples of mobility evaluation in which one case is based on Time, or Speed Criterion, while the other pertains to the Cost of moving certain payload under the assumed terrain conditions.

If the distances travelled vary and/or are subject to specific selections of routes, other criteria which may involve statistical analysis must be introduced. Such criteria have been admirably described in a paper by R. H. Petersen (13) but are beyond the scope of this report.

The discussed cases were simple. In more complicated problems the procedure will be more involved. The operations research approach must be used in making the final decision. It is beyond the normal activities of the design or test engineer as now commonly assumed.

EMPIRICAL AND THEORETICAL ESTABLISHMENT OF MATRICES OF PERFORMANCE

General

As stated before, the first task in mobility evaluation is the establishment of matrices of performance within the assumed soil-vehicle systems. To this end two techniques may be used: 1) all the pertinent values may be measured on a controlled proving ground built in accordance with established sampling techniques in order to represent the given typical area, or 2) the values may be calculated with a certain degree of accuracy from equations established by the mechanics of land locomotion.

In the first case, techniques for the necessary measurements are available. Seemingly not available, however, are proving grounds truly representative of terrain conditions within the complete span of climatic changes typical of of the given geographical area. Moreover, the existing proving grounds are beyond the control of the engineer as the timing of tests and atmospheric changes are difficult to coordinate with research programs.

This problem suggests the necessity for construction of artificial courses for vehicle testing which would provide the necessary minimum number of various terrain conditions under strict controls; so any desired analog of terrain can be made available as required within the span of critical surface conditions.

A study of this problem currently is being pursued by the Land Locomotion Research Branch of the Research and Development Division of the Ordnance Tank-Automotive Command. Tests performed by means of "miniature proving grounds" (Figures 1 and 2) and artificial "soils"(14) clearly indicate that in order to have a full picture of the vehicle performance within the whole spectrum of terrain changes in the given area, it is necessary to reproduce from three to five soil conditions. Only when testing vehicles within that spectrum, a complete relative order of merit may be derived and proper generalization of tests performed made possible. This is shown in Figure 3 graphically. Vehicles 1, 2, and 3 perform in a different way in different conditions over the same terrain. Thus what may be the best in proving Ground A may be the worst in proving Ground B or the same in proving Ground C. However, when strictly defining soils and their ralative place in the spectrum of soil changes, it is possible to obtain a generalised picture of performance shown by a complete curve

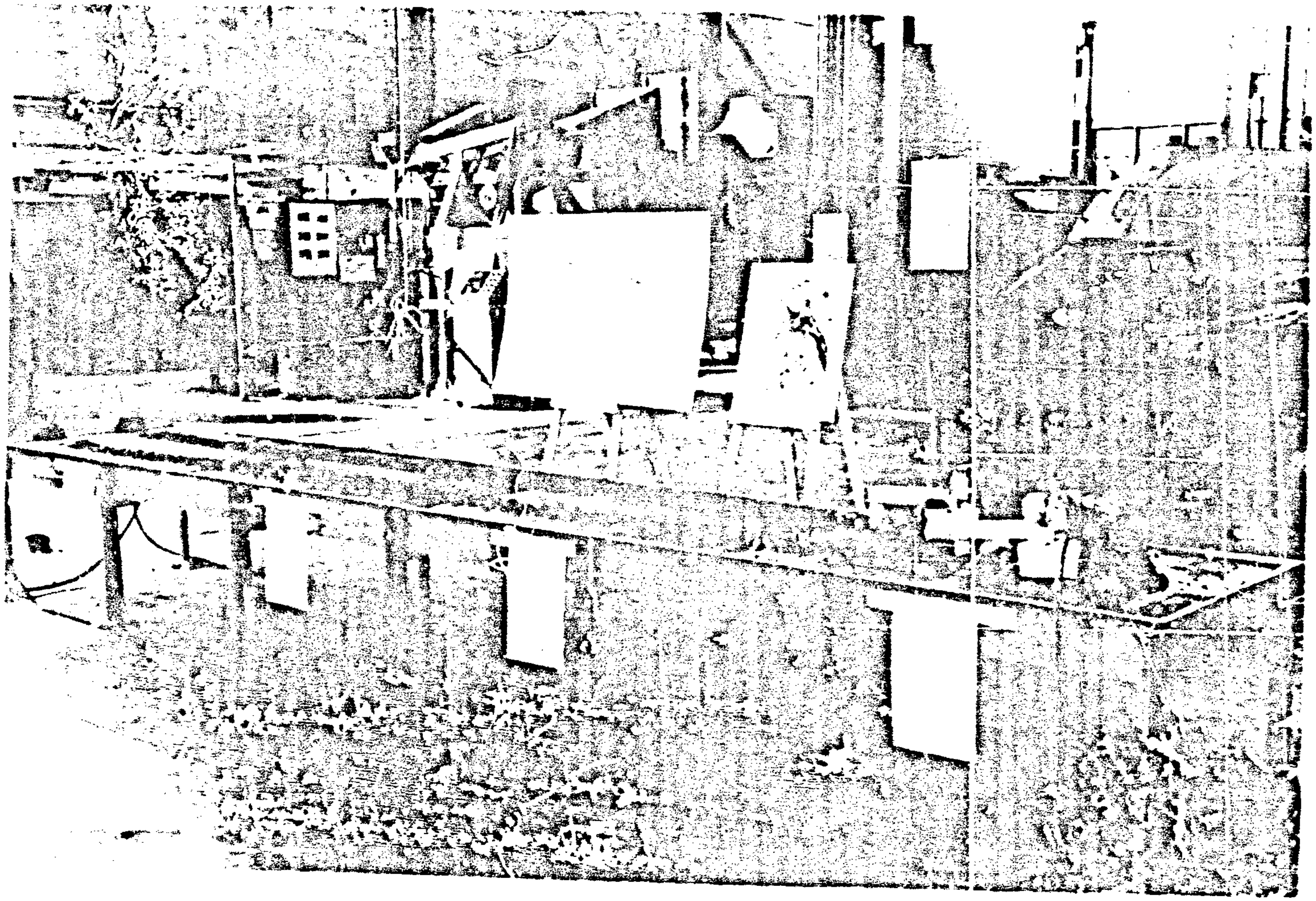


FIGURE 1

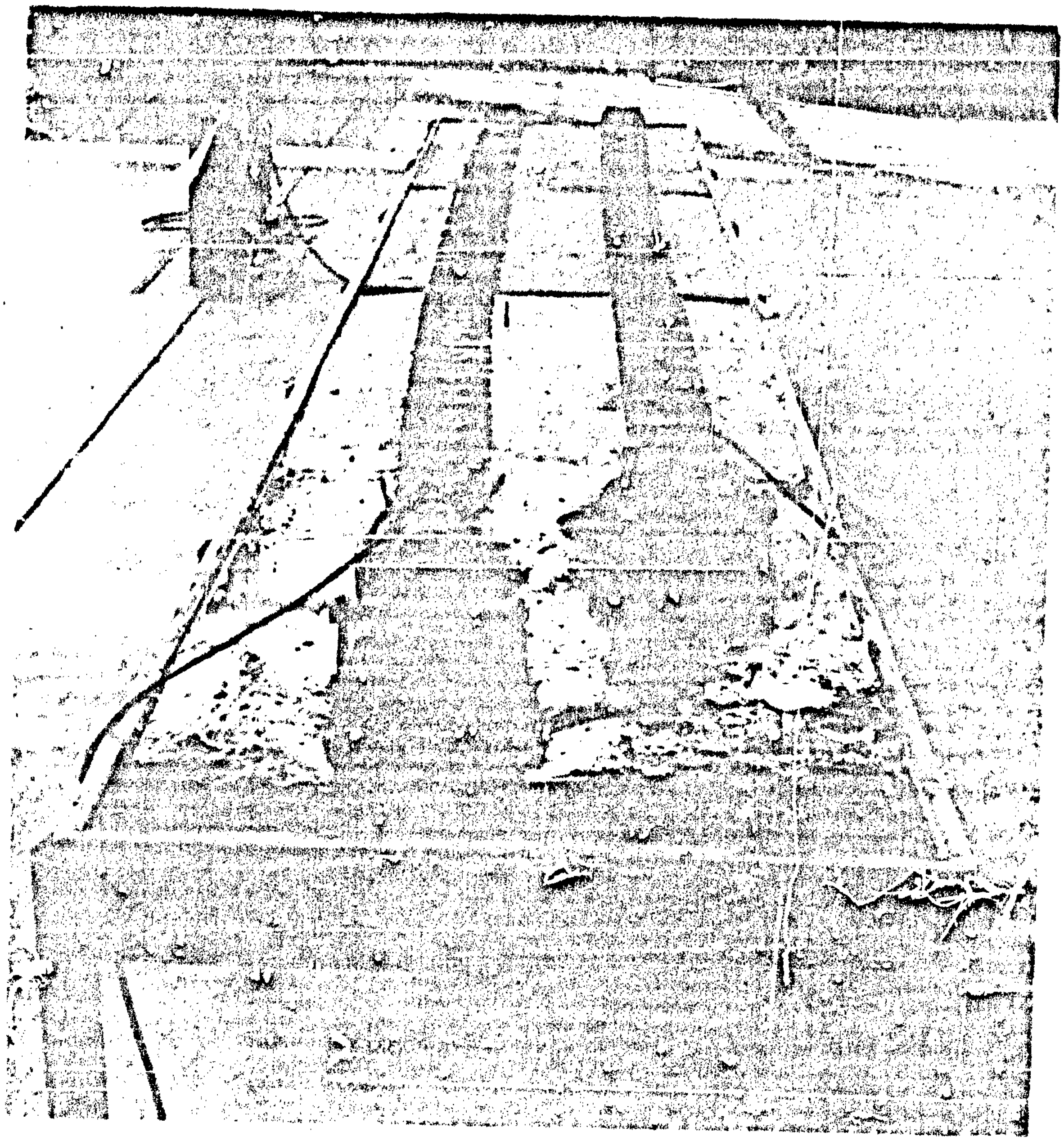


FIGURE 2

OVER-ALL VEHICLE MOBILITY FOR FULL SOIL VALUE RANGE *VS.* LIMITED PROVING GROUND TESTS

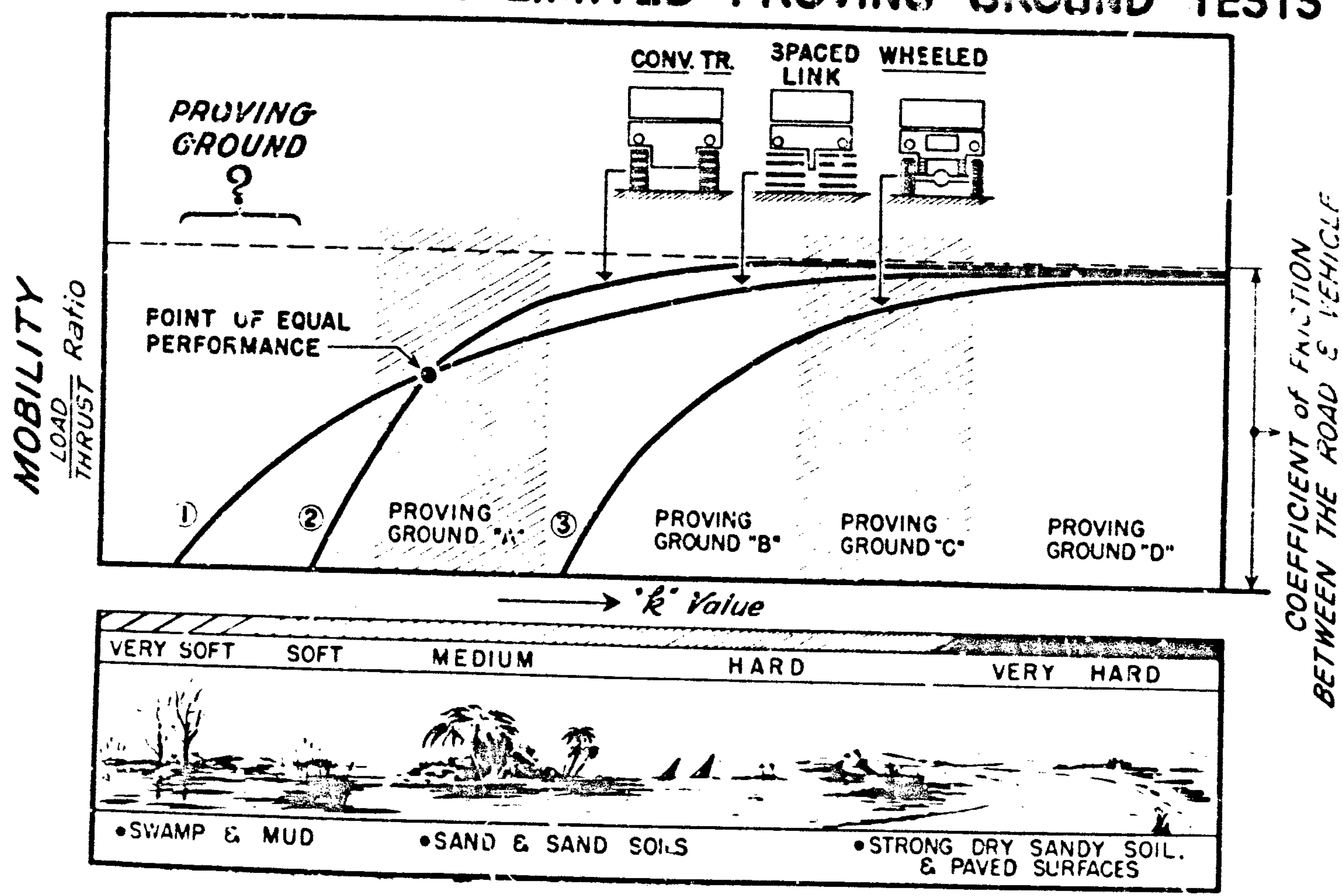


FIGURE 3

of performances under the considered conditions. Such a picture is not available today because of vehicle testing under unspecified conditions which cannot be properly located within the environment spectrum.

Investigations performed indicate that in order to reproduce a terrain and its conditions and to allocate the proper space in the discussed spectrum, one must identify pertinent soil properties. The Land Locomotion Research Laboratory has developed a soil value system which is being used to solve a great variety of design and performance evaluation problems.

The development of this system also has enabled the Laboratory to originate a general approach to the mechanics of land locomotion which has led to the establishment of a number of equations. These equations enable the researcher to compute performance matrices when the proving ground data is not available (11,15). Although the equations in question must be improved, they present a fair order of approximation and provide general mathematical models of various phenomena pertaining to locomotion with encouraging degrees of insight and generality.

A Soil Value System

As described in detail in (11) and (15), the Land Locomotion soil value system is composed of the following measures:

Strength Values	Deformation Values	
	Sinkage	**Slippage**
Friction ϕ	Moduli: k_c k_ϕ	Exponents: K_1, K_2
Cohesion c	Exponent, n	

Measurement of these values by means of existing equipment (Figures 4 and 5) enable one to define strictly the physical characteristics of soil under

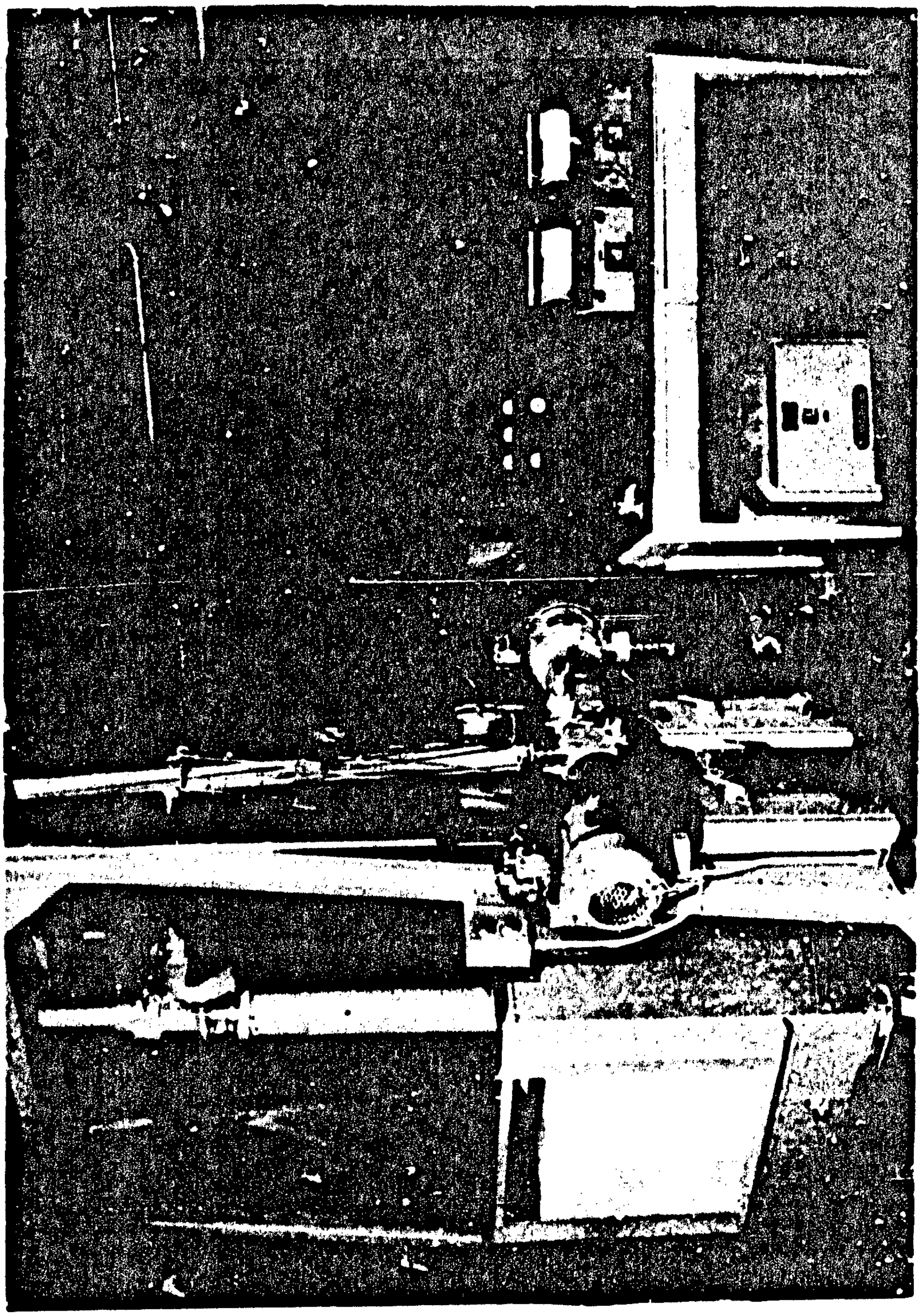


FIGURE 4

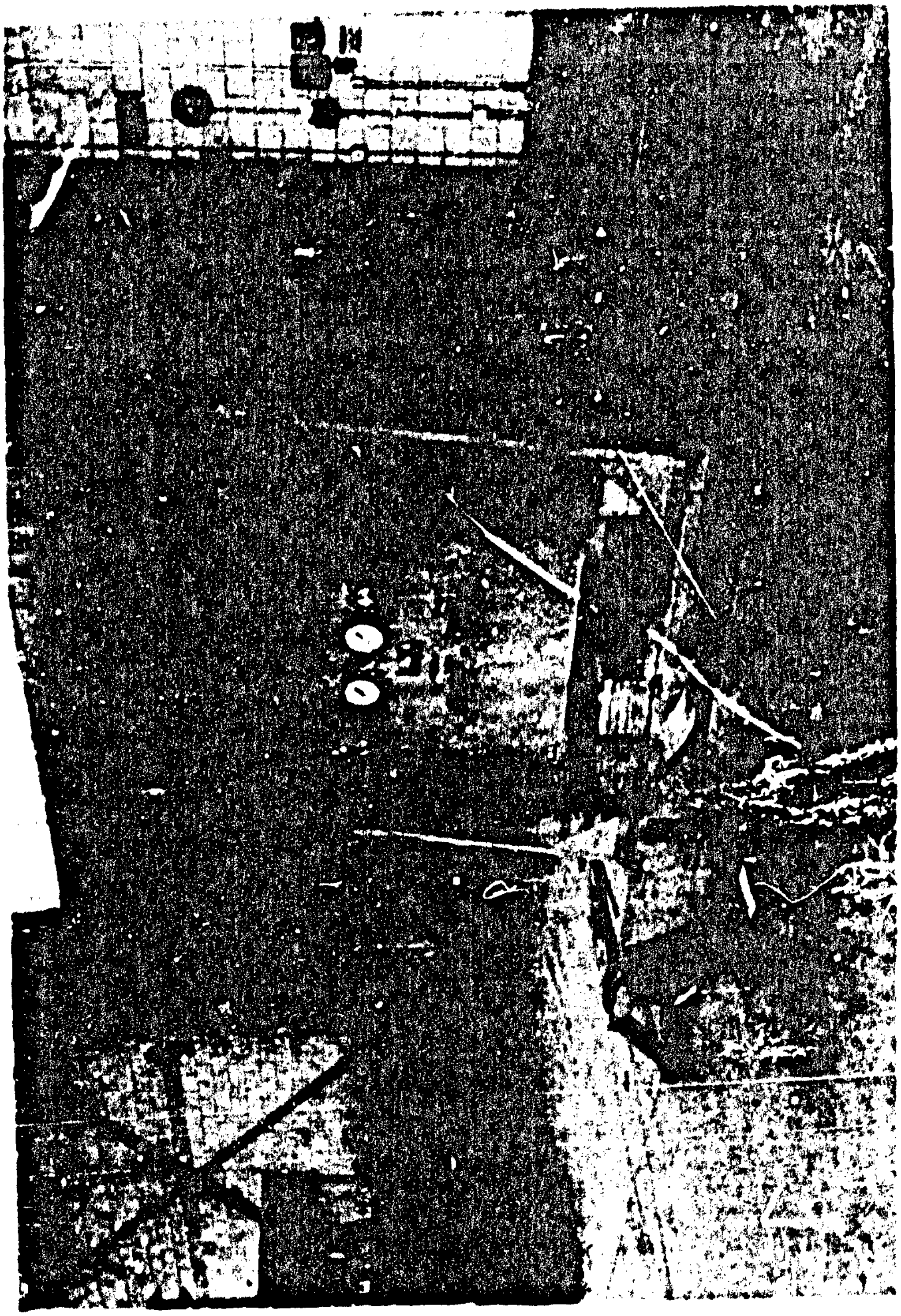


FIGURE 5

any moisture condition or snow at any temperature. With high moisture contents, the soil no longer exhibits plastic behavior. Viscosity then becomes the pertinent mud parameter. Figure 6 shows an example of the changes in soil consistency described in terms of k_c, k_ϕ, n, c and ϕ-values by an addition of 3% moisture content while Figures 7, 8 and 9 show typical characteristics of a snow cover in Northern Michigan described in similar terms. With these values it is possible not only to reproduce the desired soil condition by using artificial masses(14), but also to establish with a reasonable accuracy any desired equations which determine vehicle performance or design parameters.

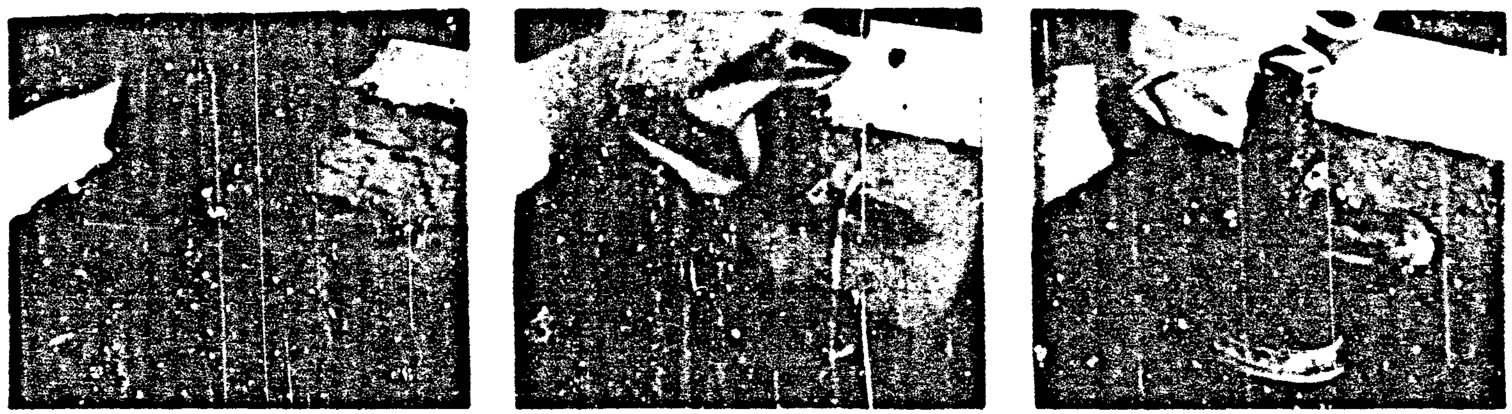
Equations of Performance

The development of applied mechanics of land locomotion is in the state of infancy. Nevertheless, a number of equations so far developed seem to indicate unlimited potentialities of this approach and produce more general answers than empirical methods. Detailed derivations and bases of these equations of vehicle performance have been shown elsewhere (11, 16, 17). In this paper only a general discussion of limitations and practical validity will be given.

Sinkage z of tracks may be expressed by means of the modified Bernstein formula assuming a uniform load distribution and rigid type suspension.

$$z = \left(\frac{p}{k_c/b + k_\phi} \right)^{1/n} \qquad \ldots\ldots\ldots\ 1$$

SANDY LOAM



	M. C. - A	M. C. - B	M. C. - C
MOISTURE CONTENT	22%	20%	19%
ϕ	36°	38°	36°
C	0.25	0.53	0.60
K_ϕ	2.2	7.0	9.0
K_c	2.5	18	20
η	0.18	0.17	0.16

FIGURE 6

Army - OTAC - Detroit

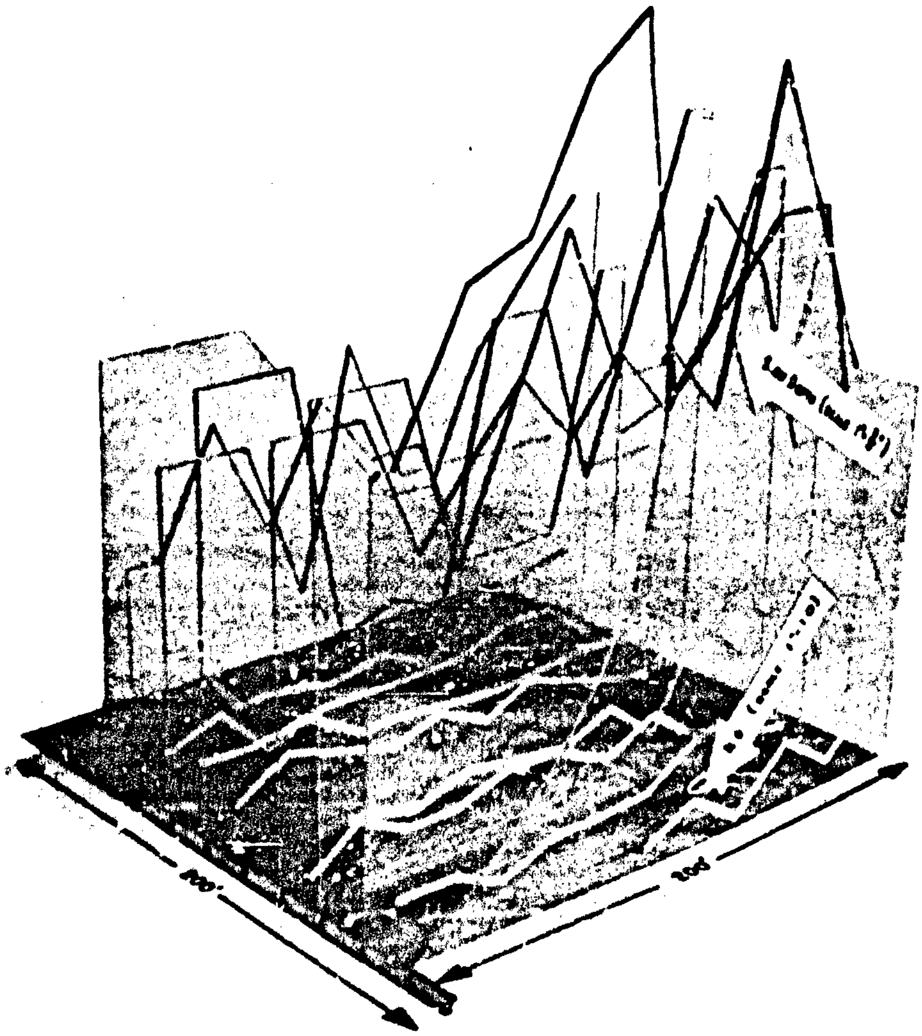


FIGURE 7

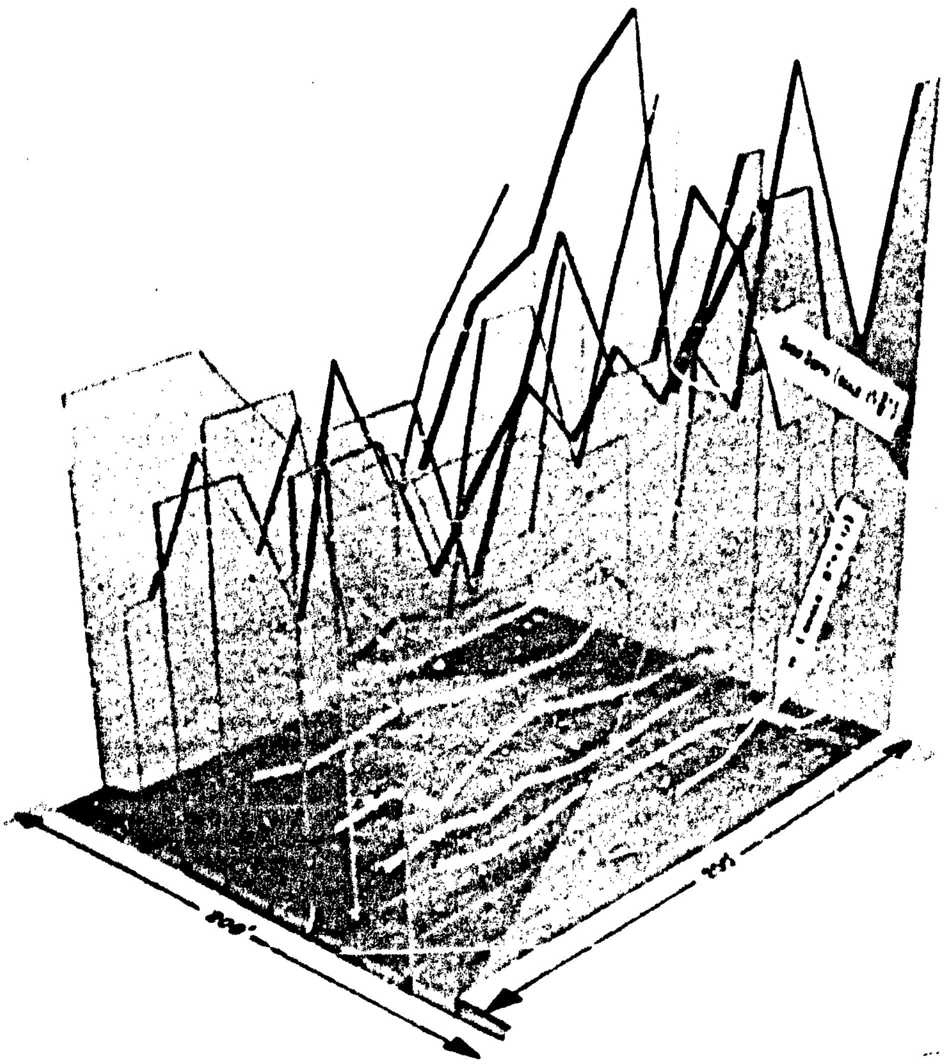


FIGURE 8

Army - OTAC - Detroit

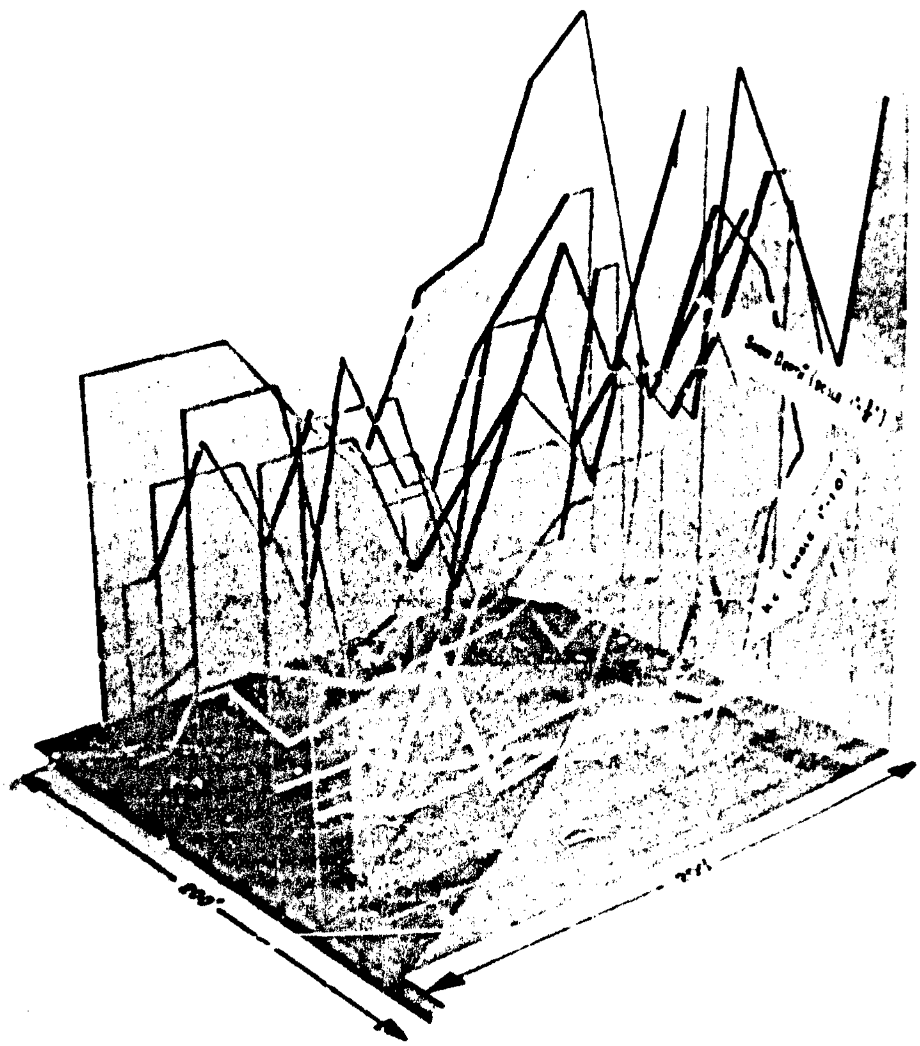


FIGURE 9

Army - OTAC - Detroit

Where, p is the "ground pressure" and b is the width (smaller dimension) of the loading area. If vehicle weight W is used then

$$z = \left(\frac{W}{2\ell (k_c + bk_\phi)} \right)^{1/n} \quad \ldots\ldots\ldots\ldots 2$$

where ℓ is the length (larger dimension of the loading area). The accuracy of this equation has been checked repeatedly and is quite satisfactory. Any error depends on variation of soil data k_c, k_ϕ, and n rather than on other factors involved.

To obtain good correlation between the experiment and computation, the frequency distribution of soil data over the measured area is necessary and the selection of a mean value is advisable.

Sinkage of wheels. When considering a rigid wheel, a fairly accurate prediction of z may be obtained from the following equation:

$$Z = \left[\frac{3W}{(k_c + bk_\phi)(3-n)\sqrt{D}} \right]^{2/(2n+1)} \quad \ldots\ldots\ldots\ldots 3$$

This equation also applies in a first approximation to conventional pneumatic tires and to soils whose bearing capacity p expressed by the formula:

$$p = 1.3c\, N_c + 0.6\, \gamma_r\, N_\gamma < pt \quad \ldots\ldots\ldots 4$$

if the safe ground pressure or bearing capacity, p is smaller than tire inflation pressure p_t. In such soils even a low pressure tire will behave practically like a rigid wheel. In equation 4, N_c and N_γ are bearing capacity factors. Their values may be found for given soils in references (11, 12). γ is the specific weight of soil which may be assumed, in most cases, as equal to 0.05 lb/cu. in.; r is the radius of the ground contact area which is assumed, in the case of conventional tires, to be almost circular in shape. If the tire is narrow and large in diameter, the contact area will be rather elliptical and the bearing capacity of such an area should be expressed by equation:

$$p = cN_c + 0.5\gamma bN_\gamma \qquad \ldots\ldots\ldots\ldots 5$$

where b is the width (smaller axis of the ellipse) of the print.

If p is greater than p_t, then equations 1 and 2 give a better approximation of tire sinkage. However, utmost caution and good judgment in sinkage evaluation is recommended because inflation pressures which are close to the bearing capacity of the ground present a rather wide band and the expected values may be obtained by interpolation between results obtained by equations 1 and 2.

More general and possible more accurate methods of sinkage evaluation of pneumatic tires are under the development by the Land Locomotion Research Branch, and it is hoped that the unavoidable degree of arbitrariness in the interpolation will be soon eliminated.

Sinkage of flat uniformly loaded footing resting on a thin layer of plastic soil supported by a firm stratum may be evaluated from equation:

$$h = \frac{s}{2} \quad \frac{b\ell^2}{W} \qquad \ldots\ldots\ldots\ldots 6$$

where s is the shearing strength of the layer equal to cohesion c, and h is the compressed thickness of the layer which will support load W resting on a strip b inches wide and ℓ inches long.

The sinkage of a wheel or track requires work for soil compaction. This work results in a so called compaction resistance which is one portion of the general resistance in motion.

Compaction Resistance of a flat, rigid, and uniformly loaded ground contact area of a track or low-pressure pneumatic tire may be approximately expressed by:

$$R_c = \frac{1}{(n+1)(k_c + bk_\phi)^{1/n}} \left(\frac{W}{\ell}\right)^{(n+1)/n} \qquad \ldots\ldots 7$$

Compaction resistance of a rigid wheel is expressed by formula:

$$R_c = \frac{1}{(n+1)(k_c + bk_\phi)^{1/(2n+1)}} \left[\frac{3W}{(3-n)\sqrt{D}}\right]^{(2n+2)/(2n+1)} \qquad \ldots\ldots 8$$

For pneumatic tires applied to the bordering conditions of p approximately equal to p_t, resistances must be evaluated in accordance with previous remarks related to sinkage.

In addition to compaction resistance which is in most cases the main portion of total motion resistance, the bulldozing resistance may also be considered. The present equation, based on passive earth pressure, is not quite satisfactory as it contains a number of over-simplifying assumptions(11). This is particularly true with reference to the rigid wheel or pneumatic tire. However, for the sake of comparison, the following equation may be used in an estimate of bulldozing resistance:

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin \alpha \cos \phi} \left[2 z c K_c + \gamma z^2 K_\gamma \right] + \frac{\pi \gamma t^2 (90 - \phi)}{540} +$$

$$c \pi t^2 + c t^2 \tan\left(45 + \frac{\phi}{2}\right) \qquad \ldots\ldots\ldots 9$$

where

$$\left.\begin{aligned} K_c &= (N_c - \tan \phi) \cos^2 \phi \\ K_\gamma &= \left(\frac{2 N_\gamma}{\tan \phi} + 1 \right) \cos^2 \phi \end{aligned}\right\} \qquad \ldots\ldots\ldots 10$$

α is the "angle of approach" of the track or wheel and t may be determined from:

$$t = z \tan^2\left(45 + \frac{\phi}{2}\right) \qquad \ldots\ldots\ldots 11$$

The "angle of approach" of a wheel or tire is normally assumed as the angle of slope of a line connecting the lowest sunken point of wheel circumference with the point made by the intersection of the wheel circumference with the ground surface. z is the sinkage evaluated by means of equations 3 or 4.

Although the R_c and R_b values as defined above may not express all the resistance encountered (11) experience so far gained indicated that they offer a fair picture of vehicle capability. As this picture appears to be more rational and comprehensive than the empirical indices previously tried, it has been often used in vehicle evaluation.

Drag R_D of wheels and tracks operating in a half fluid mud resting on a hard bottom can be determined from equation:

$$R_d = C_d \left(\rho \frac{v^2}{2} A_2\right) \qquad \ldots\ldots\ldots\ldots\ 12$$

where C_d is drag coefficient, ρ density of mud, v speed and A the wetted area (19).

The maximum net thrust available in the ground is expressed by Coulomb's equation containing a correction for the action of spuds or tread:

$$H_{max} = bl c\left(1 + \frac{2h}{b}\right) + W \tan \phi \left\{1 + 0.64\left[\left(\frac{h}{b}\right)\cot^{-1}\left(\frac{h}{b}\right)\right]\right\} \quad \ldots\ 13$$

where, as above, b is the smaller and l is the larger dimension of the ground contact area assumed, in a first approximation, as a rectangle. h is the height of the spud or tread, and W is the load resting upon the area under consideration.

This equation is quite accurate: numerous field and laboratory tests have shown that it tends to give values approximately 5%-10% lower than those measured. The above is explained by the lack of correction for grouser spacing which might complicate equation 13 beyond the range of its usefulness.

Equation 13 applies to both tracks and wheels. The values thus obtained determine only maximum thrust available in the ground under assumed conditions at an optimum slippage. To obtain thrust at any desired slippage which may occur in vehicle operation, another formula is needed within the desired order of approximation(11):

$$H = \frac{2b(c + p \tan \phi)}{K_1 i_0 Y_{max}} \left[\frac{e^{(-K_2 + \sqrt{K_2^2 - 1})K_1 i_0 \ell} - 1}{-K_2 + \sqrt{K_2^2 - 1}} - \frac{e^{(-K_2 - \sqrt{K_2^2 - 1})K_1 i_0 \ell} + 1}{-K_2 - \sqrt{K_2^2 - 1}} \right] \qquad \ldots\ldots\ldots 14$$

where K_1 and K_2 are slippage parameters; i_0 is slippage in %. Y_{max} is the maximum of the function:

$$y = e^{(-K_2 + \sqrt{K_2^2 - 1})K_1 i_0 \ell} - e^{(-K_2 - \sqrt{K_2^2 - 1})K_1 i_0 \ell}$$

and p is the "ground pressure" which is assumed to be uniformly distributed. However, a graphical method developed by Weiss (20) enables one to determine H as a function of slippage i_0 which may be used for uniform or non-uniform load distribution.

Tests performed in snow with the purpose of predicting drawbar pull versus slip of a number of vehicles have shown quite satisfactory results at low sinkages (20). For high sinkage, the Drawbar pull, DP, may be determined if from H-values, equations 13, 14, the motion resistance, R, equations 7, 8, 9, is subtracted:

$$DP = H - (R_c + R_b) \quad \ldots\ldots\ldots\ldots\ldots 15$$

Thus the "coefficient of traction," DP/W, corresponding in concept to the drag/lift ratio, generally accepted as one of the broadest exponents of vehicular performance, may be expressed in the following form:

$$\frac{T}{W} = \left(\frac{c}{p} + \tan\phi\right) - \frac{b^{1/n}}{l}\left\{\frac{l^{1/n}}{(n+1)^{/n}}\left(\frac{p}{k_c + bk_\phi}\right)^{1/n} + \left(\frac{p}{k_c + bk_\phi}\right)^{1/n}\right.$$

$$\left. \times \frac{\sin(\alpha + \phi)}{2p\sin\alpha\cos\phi}\left|2cK_c + \gamma K_\gamma\left(\frac{pb}{k_c + bk_\phi}\right)^{1/n}\right]\right\}$$

$$\ldots\ldots\ldots 16$$

In equation 16, the R_b value of equation 9 has been introduced without its last three members as the error thus allowed appears to be smaller than the over-all accuracy of the proposed solution. That solution compares fairly well with results obtained experimentally for tracks. In the case of wheels, it is definitely less accurate on account of the phenomena discussed in connection with equations 1, 3, 4, 7, and 8.

The change in performance of a wheel following the rut of the proceeding wheel also has not been considered. Utmost care must be given to the evaluation of soil bearing capacity in order to determine whether a tire behaves like a "track" or a rigid wheel.

Typical curve of DP/W for various single tires and three types of soil consistencies illustrated by photographs located close to the corresponding $k = (k_c/b + k_\phi)$ values is shown in Figure 10. Curve for two complete vehicles is shown in Figure 11. From graphs of this type, any DP/W performance figure may be correlated with the given terrain in a DP/W matrix.

Similar curves may be computed for the whole vehicle, if the loads W acting upon driven and driving axles are known. Idling wheels will then produce only resistance (R_t/W) while the propelling wheels will supply net thrust $(DP/W)_d = \left[(H - R)/W\right]_d$. Hence the total DP/W value will be:

$$\frac{DP}{W} = \frac{1}{W}\left[\left(H - R\right)_d - R_t\right] \quad \ldots\ldots\ldots 17$$

The main problem in such computations is to know the k_c', k_ϕ' and n' values WHICH EXIST IN THE RUT MADE BY front wheels or tracks after they cross a virgin ground characterized by k_c, k_ϕ, and n values. Changes in soil taking place under such circumstances are being investigated by the Land Locomotion Research Branch, OTAC. However, it has been found that for the purpose of pure comparison, an assumption of all the wheels crossing the undeformed soil also produces quite reliable results.

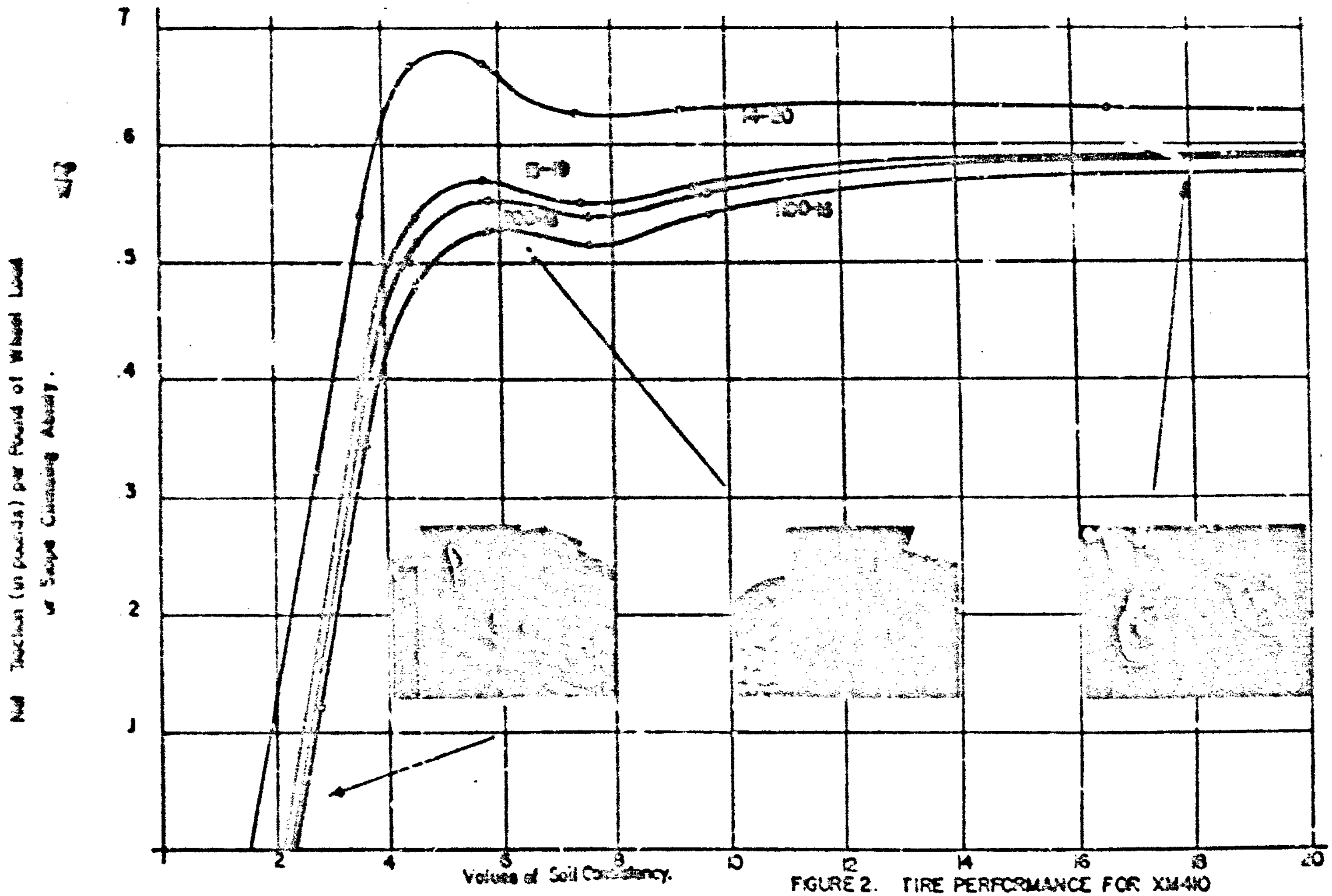
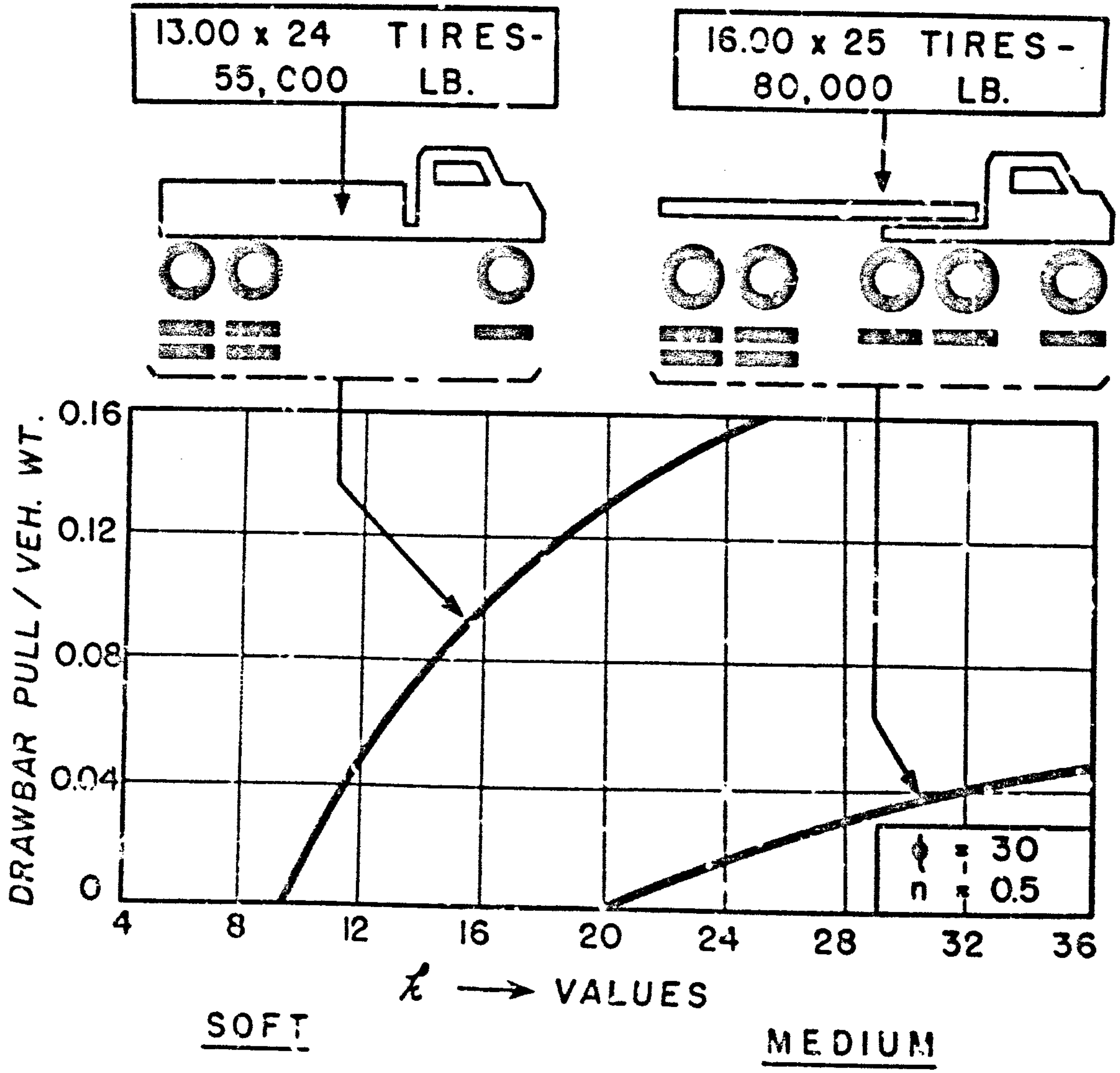


FIGURE 2. TIRE PERFORMANCE FOR XM-410
8x8 TRUCK UNDER VARYING SOIL CONSISTANCIES.

FIGURE 10



13.00 x 24 TIRES-
55,000 LB.
16.00 x 25 TIRES-
80,000 LB.
DRAWBAR PULL / VEH. WT.
0.16
0.12
0.08
0.04
0
4
8
12
16
20
24
28
32
36
ϕ = 30
n = 0.5
k ⟶ VALUES
SOFT
MEDIUM

FIGURE 11

Speed matrix may be established if the motion resistance of the given terrain is known from equations 7, 8, and 9. Assuming that engine power is N, transmission losses η, the maximum speed developed will be:

$$v_{max} = \frac{N\eta}{R} \quad \ldots\ldots\ldots\ldots 18$$

However, if speed is determined by the throttling of the engine with the purpose of avoiding excessive vibrations over a rough terrain and not by the maximum of resistance, then the determination of such speeds must be performed in accordance with orthodox automotive engineering procedures described in reference 11 or through the simulation of vehicle vibrations by means of an analog computer assuming certain criteria of ride "comfort"(11). To this end, the geometry of the ground surface and its energy spectrum (21) must be known. The Land Locomotion Research Laboratory is engaged in a study of this problem (22). Typical graphs relating pitch and bounce amplitude and speed to the particular terrain "wave" is given in Figures 12 and 13 as computed in reference 22. Upon determining the speed-resistance matrix, a fuel consumption matrix may be established for the given terrain following well established procedures described in reference 11. Knowing the distances and tank capacity, the range of action matrix also may be automatically defined.

The question of obstacle performance does not present difficulty once the geometry of obstacles is known and their distribution assumed (11,23). Definitions and passability of penetrable obstacles are usually expressed in terms of dimensions pertaining to obstacle geometry. It

COMPARISON OF "GOER VEHICLES"—(SPRUNG / UNSPRUNG)

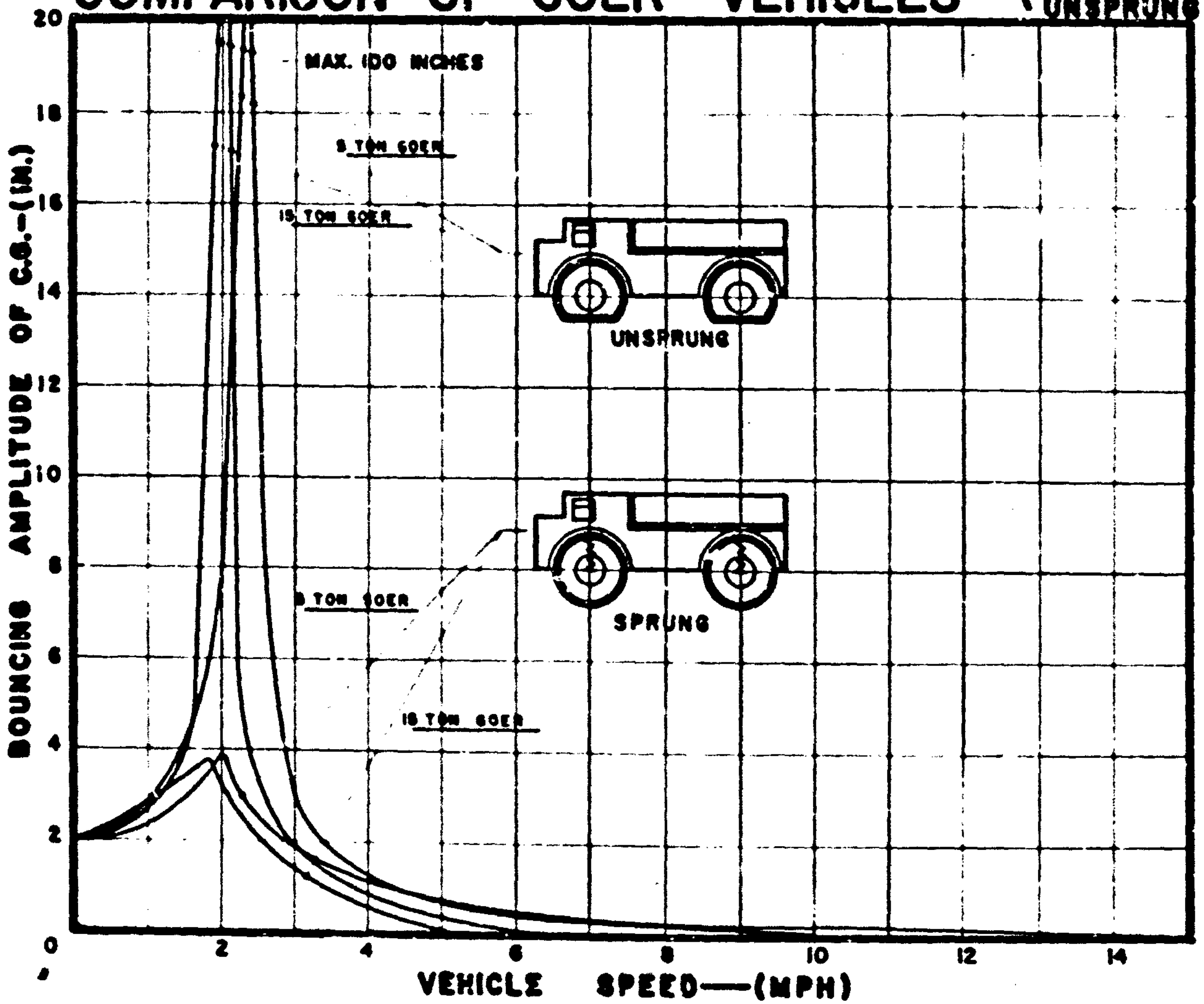


FIGURE 12

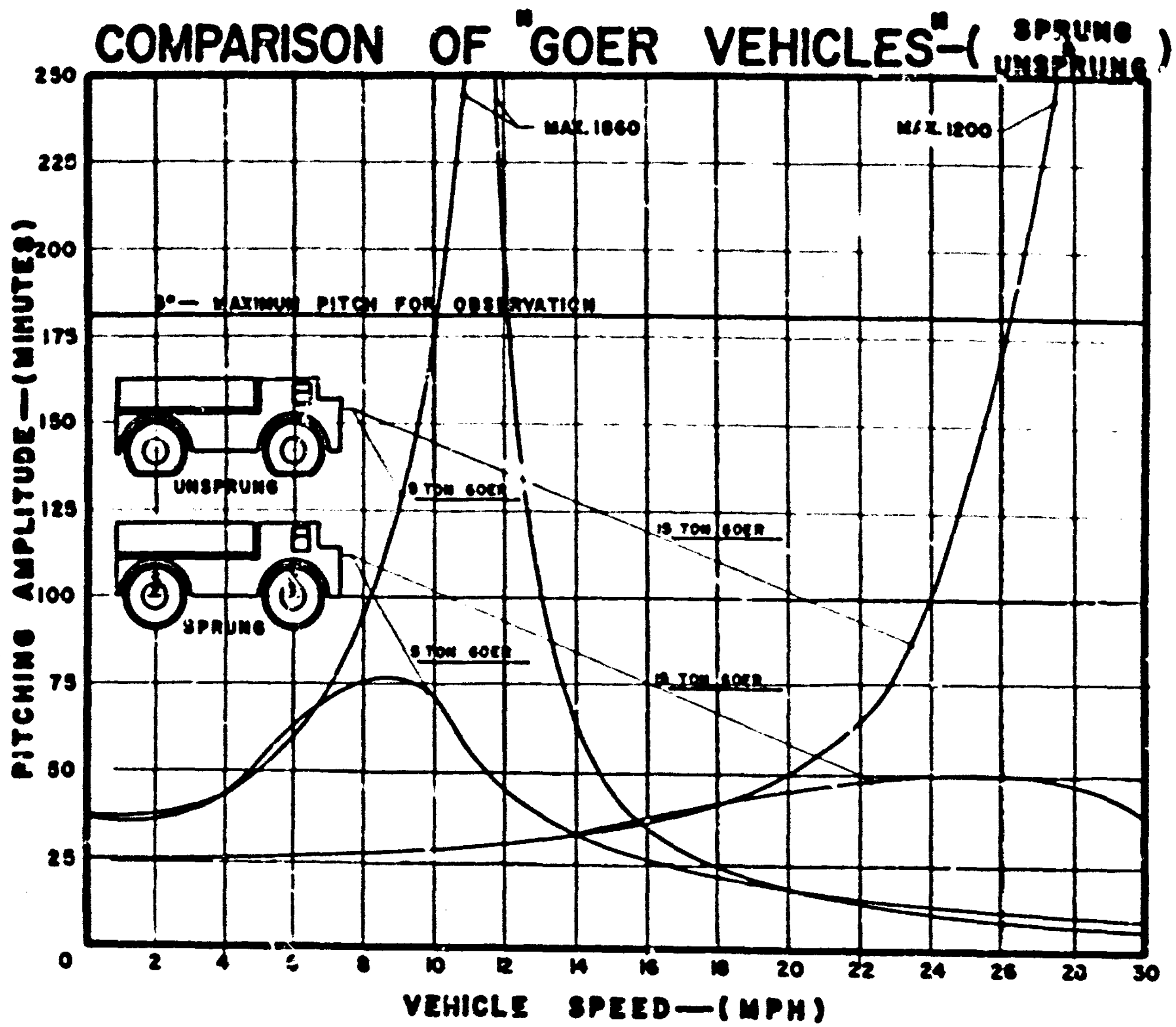


FIGURE 13

Army - OTAC - Detroit

may be mentioned that negotiable slope matrix is given by DP/W matric which in principle expresses the tangents of slopes accessible to the vehicle. Impenetrable obstacles which will affect average speeds because of the necessity of by-passing them have been discussed in reference 24.

The discussed method of performance evaluation is based on the knowledge of physico-geometrical terrain properties and enables one to determine any type of composite performance, for instance, fuel consumption per ton mile, momentum of load x speed (cargo delivery rate), actual fuel consumption in no-refueling area when fuel has to be carried in a convoy, cost per ton mile, etc., as was demonstrated before.

Samples of this type of evaluations are given in Appendix I, in a general form, and in specific numerical examples. Appendix II gives an example of another operational evaluation of mobility of a hypothetical family of rigid wheels with a number of parameters changing within wide limits in various soil conditions ranging from very strong to very loose.

It is apparent that this type of mobility evaluation requires enormous amounts of computations. To this end, electronic computers are of irreplaceable value because even in the cases of most complex terrain characteristics, value matrices may be programmed with relative ease and may be obtained quickly while changing weather parameters, for instance (which is immediately fed into computers by appropriate c, ϕ, k_c, k_ϕ, n, K_1, K_2 values). Similar changes in vehicle loads or geometry may be introduced.

Tests performed by the Land Locomotion Research Branch with the assistance of the Computer Section of the Ordnance Tank-Automotive Command encourages one to hope that upon further developing the land locomotion mechanics and exploring the world soils much testing and experimentation with full size vehicles will be eliminated. Savings in time and money would be enormous. Faster, cheaper and more flexible "proving grounds" programmed in a high speed computer may definitely replace the present test tracks to a large extent.

This is one of the great potentialities of the proposed method. The simulation of environmental and vehicle conditions electronically has been used extensively in aeronautical and naval engineering. It appears only a matter of time that the same will be used in land locomotion. To this end, however, more rapid progress in a systematic study of the mechanics of soil-vehicle relationship is needed.

CONCLUSIONS

1. Mechanical Mobility of a motor vehicle may be defined as a product of the operational optimization of performance values within the physico-geometrical content of the soil-vehicle system.

2. Such an optimization can be conducted in a number of ways depending on the type of answers sought.

3. There is no single true definition of mobility but an infinite number of useful definitions.

4. A single method for the determination of such definitions can be established and must be adopted in order to elimate the present ambiguities in mobility and design evaluation.

5. The usefulness of that method is warranted by the established principles of applied mechanics and operations research techniques.

6. The method is based on soil values measurable in physico-geometrical terms, and on vehicle performance matrices.

7. When determining the matrices of performance on the proving grounds, the latter must be modernized and adapted to the new requirements.

8. When using theoretical methods and mathematical models as illustrated in this study, there is no end to the possible improvement of the generality and accuracy of procedures discussed.

9. This in result demands a continuous development of the mechanics of land locomotion.

10. The full development of this mechanics based on experimentally verified facts will lead ultimately to the more extensive determination of vehicle mobility by means of electronic computers.

11. This will bring enormous savings by limiting the full size proving ground testing and/or by eliminating the absolute necessity of having the "hardware" manufactured before its preliminary value can be assessed.

12. Thus the discussed concept of mobility will make possible the economic study of new unusual ideas whose development today cannot be authorized because of unproven merits which could be hitherto discovered only by costly experiments.

13. The method illustrated here also enables one to study whole families of concepts before any particular idea is singled out for development.

RECOMMENDATIONS:

1. It is recommended that the development of land locomotion mechanics and the introduction of operations research techniques at the design evaluation level be hastened. In particular it is recommended that:

2. A single soil-value system based on stress-strain measurements in slippage and sinkage be adopted without delay.

3. Terrain measurements and soil cataloging based on these measurements be started immediately, and

4. New concepts be first evaluated theoretically by using the proposed mobility definitions before development programs are established.

REFERENCES

1. Colby, Elbridge "Masters of Mobil Warfare" Princeton University Press, Princeton 1943

2. Weeks, Sir Ronald M. "Organization and Equipment for War" Cambridge University Press, Cambridge 1950

3. Heigl, F. "Taschenbuch der Tanks" J. F. Lehman Publishing House, Munich 1926-1938

4. Jarret, G. B. "The Combat Efficiency of Tanks" Armored Cavalry Journal, March-April 1950

5. Michelet, L.C. "An Analysis of the Characteristics of Combat Vehicles" Armament Technology, September-October 1950

6. Blagden, W. M. "Mobility and Mobigriety" Proc. Soil Mechanics Symposium Ministry of Supply, London 1948

7. "A proposed Definition of Tactical Mobility" Res. Study 2 CONARC, May 1951

8. "A Summary of Trafficability Studies Through 1955" Waterways Experimental Station, U.S. Corps of Engineers. Vicksburg, Mississippi, December 1956

9. McKee, G. L. & Rigg, R. T. Lt Cols "Goer for Ground Mobility" Army, April 1958

10. Hittle, T. D. "Jomini and His Summary of the Art of War" Military Service, Harrisburg, Pennsylvania 1947

11. Bekker, M. G. "Theory of Land Locomotion-The Mechanics of Vehicle Mobility" University of Michigan Press, Ann Arbor, Michigan 1956

12. Ehrlich, I. R. Capt "Evaluation of Tires for the XM410 8x8 2½ Ton Truck." Land Locomotion Research Branch R&D Division OTAC, Report No. 26, January 1958.

13. Petersen, R. H. "A Mathematical Model of the Manner in which Routes for Tanks Are Chosen Across Terrain BRL MR No. 1095 Aberdeen P.G., August 1957

14. Hanamoto, B "Artificial Soils for Laboratory Studies in Land Locomotion " Land Locomotion Research Branch R&D Division OTAC, Report No. 20 1958

15. Bekker, M.G. "Terrain Evaluation in Automotive Off-the-Road Operations" Land Locomotion Research Branch R&D Division OTAC, Report No. 13 1957

16. Bekker, M.G. "A Practical Outline of the Mechanics of Automotive Land Locomotion Seminar Notes, Detroit Arsenal 1956.

17. Bekker, M.G. "An Introduction to Research on Vehicle Mobility" First Printing Dept of National Defense Ottawa, Canada, 1948; Second, Aberdeen Proving Ground U.S. OrD Corps., 1953; Third, Ord. Tank-Automov Command 1958

18. Terzaghi, K. "Theoretical Soil Mechanics" John Wiley & Sons New York 1945

19. Hegedus, E. "Drag Coefficient in Locomotion Over Viscous Soils" Part I. Land Locomotion Research Branch R&D Division OTAC, Report No. 25 1958

20. Harrison, Wm. "Study of Snow Values Related to Vehicle Performance" Land Locomotion Research Branch R&D Division OTAC Report No. 23 1958

21. Bekker, M.G. "Latest Developments in Off-the-Road Locomotion" Journal of the Franklin Institute Vol 263 No. 5, May 1957

22. Ehrlich, I.R. Capt "Power Spectrum of Terrain" Land Locomotion Research Branch R&D Division OTAC, Report No. 21 1958

23. Bekker, M.G. & Hettig G.P. "Obstacle Performance of Wheeled Vehicles" Land Locomotion Research Branch R&D Division OTAC, Report No. 29 1958

24. Brooks, F.C. "Effect of Impenetrable Obstacles on Vehicle Operational Speed" Land Locomotion Research Branch R&D Division OTAC, Report No. 28 1958

APPENDIX I

GENERAL

In deciding which type of vehicle is best in transporting a payload W, from a number of available types, it is necessary to know the criterion on which best is based. In one instance speed may be the criterion. i.e., the best vehicle is the one that will transport the payload W, from its present location O_1 to a new location O_2, in the least amount of time. In other instances conservation of fuel or cost may be the criterion. In still others the certainty that all of the payload will arrive intact may be the criterion. i.e., the vehicle that offers the greatest chance for survival of the trip would be best. Almost any standard may be the basis for deciding which type of vehicle is best.

The abilities of the different types of vehicles, relative to any criterion, vary with the terrain and the trafficability of the soil encountered. In the following discussion we will assume that the trafficability varies primarily with weather conditions. And it is therefore possible to determine the best vehicle to use in transporting a given load over a designated distance, if we know the performance values of the available vehicles for the various terrains and weather conditions that are found in the area.

The following notation will be used throughout the discussion:

D : The distance to be traveled between locations O_1 and O_2.

W : The weight of the payload to be transported.

j : The jth type of vehicle, $j = 1, \ldots, M$

y_j : The yth vehicle of type j, $y = 1$

e_j : The original cost of the jth vehicle.

f_j : The cost per gallon of fuel for the jth vehicle.

w_j : The weight per gallon of fuel for the jth vehicle.

A : The total area of terrain to be covered.

B_{ik} : The ith terrain under climatic conditions k, $i = 1 \ldots, N$ and $k = 1 \ldots, K$.

a_i : The area of the ith terrain.

P_{ik} : The probability of finding the kth climatic condition in the ith terrain.

V_{ikj}: The speed of the jth vehicle in land condition ik.

R_{ikj}: The range of the jth vehicle in land condition ik.

H_j : The gasoline tank capacity of the jth vehicle.

W_{ikj}: The payload of the jth vehicle in land condition ik.

L_{ikj}: The life expectancy of the jth vehicle in land condition ik.

m_{ikj}: The miles per gallon of fuel of the jth vehicle in land condition ik.

$g_j(x)$: The maintainence cost function as a function of hours traveled.

x_{ik} : The number of hours traveled in land condition ik.

X_j : The total number of hours running time for the jth vehicle.

X_{o_y} : The age of vehicle y_j in running time at the location O_1.

q_j : The maintainence time required per 100 hours running time for the jth vehicle.

Q_j : The time required for a major overhaul of the jth vehicle.

t_{f_j} : The time required for refueling the jth vehicle.

N_{f_j} : The number of refueling stops required for the jth vehicle to traverse the distance D.

W_{f_j} : The weight of the required extra fuel per vehicle to be carried by the jth vehicle.

d_{ik} : The number of miles of land condition ik encountered.

G_j : The total number of gallons of fuel required for the jth vehicle to traverse the distance D

F_j : The total fuel cost for the jth vehicle.

C_j : The cost per vehicle to move the jth vehicle from location O_1 to O_2.

J : The number of vehicles of type j required to transport the payload W.

TC_j : The cost to transport the payload W over the distance D by the jth vehicle.

T_j : The delivery time for the jth vehicle.

The following performance data is given:

Distance to be traveled: D

Weight to be transported: W

Original Cost: e_j

Vehicle	I	II	...	j	...	M
Cost	e_I	e_{II}	...	e_j	...	e_M

Fuel Cost per Gallon: f_j

Vehicle	I	II	...	j	...	M
Cost	f_I	f_{II}	...	f_j	...	f_M

Total Area of Terrain: A

Area of Terrain B_i : a_i

Terrain	B_1	B_2	...	B_j	...	B_N
Area	a_1	a_2	...	a_j	..	a_N

Probability of Finding Land Conditions ik : p_{ik}

Climate / Terrain	1	2	...	k	...	K
B_1	p_{11}	p_{12}	...	p_{1k}	...	p_{1K}
B_2	p_{21}	p_{22}	...	p_{2k}	...	p_{2K}
⋮	⋮	⋮		⋮		⋮
B_i	p_{i1}	p_{i2}	...	p_{ik}	...	p_{iK}
⋮	⋮	⋮		⋮		⋮
B_N	p_{N1}	p_{N2}	...	p_{Nk}	...	p_{NK}

Speed in Miles per Hour: V_{ikj}

Vehicle / Conditions	I	II	...	j	...	M
B_{11}	V_{111}	V_{112}	...	V_{11j}	...	V_{11M}
B_{12}	V_{121}	V_{122}	...	V_{12j}	...	V_{12M}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{1k}	V_{1k1}	V_{1k2}	...	V_{1kj}	...	V_{1kM}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{1K}	V_{1K1}	V_{1K2}	...	V_{1Kj}	...	V_{1KM}
B_{21}	V_{211}	V_{212}	...	V_{21j}	...	V_{21M}
B_{22}	V_{221}	V_{222}	...	V_{22j}	...	V_{22M}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{2K}	V_{2k1}	V_{2k2}	...	V_{2kj}	...	V_{2kM}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{2K}	V_{2K1}	V_{2K2}	...	V_{2Kj}	...	V_{2KM}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{i1}	V_{i11}	V_{i12}	...	V_{i1j}	...	V_{i1M}
B_{i2}	V_{i21}	V_{i22}	...	V_{i2j}	...	V_{i2M}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{ik}	V_{ik1}	V_{ik2}	...	V_{ikj}	...	V_{ikM}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{iK}	V_{iK1}	V_{iK2}	...	V_{iKj}	...	V_{iKM}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
B_{N1}	V_{N11}	V_{N12}	...	V_{N1j}	...	V_{N1M}

Vehicle/Conditions

B_{N2}	V_{N21}	V_{N22}	...	V_{N2j}	...	V_{N2M}
⋮	⋮	⋮		⋮		⋮
B_{Nk}	V_{Nk1}	V_{Nk2}	...	V_{Nkj}	...	V_{NkM}
⋮	⋮	⋮		⋮		⋮
B_{NK}	V_{NK1}	V_{NK2}	...	V_{NKj}	...	V_{NKM}

Range in Miles: R_{1kj}

Vehicle / Condition	I	II	...	j	...	M
B_{1k}	R_{1k1}	R_{1k2}	...	R_{1kj}	...	R_{1kM}
B_{2k}	R_{2k1}	R_{2k2}	...	R_{2kj}	...	R_{2kM}
⋮	⋮	⋮		⋮		⋮
B_{ik}	R_{ik1}	R_{ik2}	...	R_{ikj}	...	R_{ikM}
⋮	⋮	⋮		⋮		⋮
B_{Nk}	R_{Nk1}	R_{Nk2}	...	R_{Nkj}	...	R_{NkM}

$k = 1, 2, \ldots, K.$

Gasoline Tank Capacity: H_j

Vehicle	I	II	...	j	...	M
Capacity	H_I	H_{II}	...	H_j	...	H_M

$k = 1, 2, \ldots, K.$

Payload in Pounds: W_{ikj}

Vehicle / Conditions	I	II	...	J	...	M
B_{1k}	W_{1k1}	W_{1k2}	...	W_{1kJ}	...	W_{1kM}
B_{2k}	W_{2k1}	W_{2k2}	...	W_{2kJ}	...	W_{2kM}
⋮	⋮	⋮	.	⋮		⋮
B_{ik}	W_{ik1}	W_{ik2}	...	W_{ikJ}	...	W_{ikM}
⋮	⋮	⋮		⋮		⋮
B_{Nk}	W_{Nk1}	W_{Nk2}	...	W_{NkJ}	...	W_{NkM}

$k = 1, 2, \ldots, K.$

Life Expectancy of the Vehicle in Miles, L_{ikj}

Vehicle / Life	I	II	...	J	...	M
B_{1k}	L_{1k1}	L_{1k2}	...	L_{1kJ}	...	L_{1kM}
B_{2k}	L_{2k1}	L_{2k2}	...	L_{2kJ}	...	L_{2kM}
⋮	⋮	⋮		⋮		⋮
B_{ik}	L_{ik1}	L_{ik2}	...	L_{ikJ}	...	L_{ikM}
⋮	⋮	⋮		⋮		⋮
B_{Nk}	L_{Nk1}	L_{Nk2}	...	L_{NkJ}	...	L_{NkM}

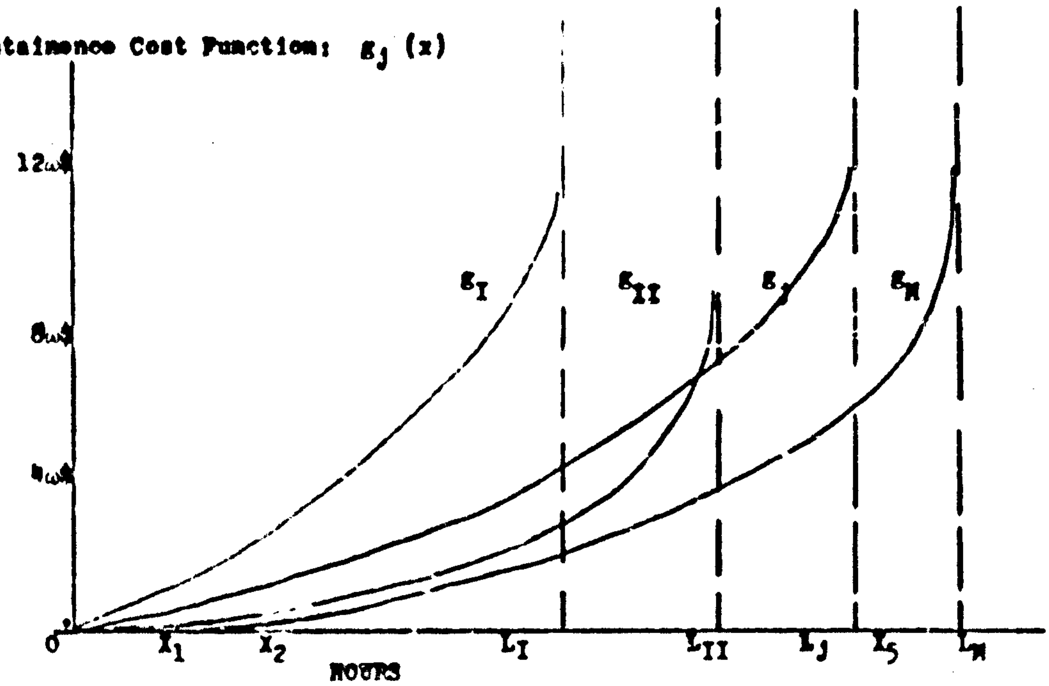
$k = 1, 2, \ldots K.$

Miles per Gallon: m_{ikj}

Vehicle \ Conditions	I	II	...	j	...	N
B_{1k}	m_{1k1}	m_{1k2}	...	m_{1kj}	...	m_{1kN}
B_{2k}	m_{2k1}	m_{2k2}	...	m_{2kj}	...	m_{2kN}
⋮	⋮	⋮		⋮		⋮
B_{ik}	m_{ik1}	m_{ik2}	...	m_{ikj}	...	m_{ikN}
⋮	⋮	⋮		⋮		⋮
B_{Nk}	m_{Nk1}	m_{Nk2}	...	m_{Nkj}	...	m_{NkN}

$k = 1, 2, \ldots, K.$

Maintainence Cost Function: $g_j(x)$



$g_j(x)$ = The cos. of maintaining running condition for the first x hours.

Maintainence Time per 100 Hours of Running Time: q_j

Vehicle	I	II	...	j	...	M
Hours	q_I	q_{II}	...	q_j	...	q_M

Time Required to Refuel: t_r

Vehicle	I	II	...	j	...	M
Hours	t_I	t_{II}	...	t_j	...	t_M

MOBILITY ACCORDING TO TIME CRITERION

Let time first be used as a criterion for choosing the most "mobile vehicle. It is assumed that there is an unlimited number of vehicles of each type.

The time required to traverse the distance D, between locations O_1 and O_2 over the area A, is dependent upon:

(1) the running time,

(2) the refueling time, and

(3) the maintainence time.

The key to deciding which vehicle is best is the number of miles of each land condition d_{ik} encountered in the distance D between locations O_1 and O_2 .

$$d_{ik} = \frac{D\, a_i\, p_{ik}}{A}$$

Once these distances d_{ik} are obtained, the time follows directly from them.

(1) The running time X_j is found by dividing the distance to be traveled

in each terrain d_{ik} by the number of miles per hour V_{ikj} averaged by the jth vehicle in that land condition.

$$x_{ikj} = \frac{D\, a_i\, p_{ik}}{A} \cdot \frac{1}{V_{ikj}} = \text{hours}$$

This gives the number of hours of running time x_{ikj} required to traverse each type of land condition. The summation of all such land conditions gives the total running time X_j required to traverse the distance D.

$$X_j = \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ikj} = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}} = \text{hours}$$

(2) To find the refueling time divide the distance to be traveled in each land condition d_{ik} by the range in each terrain R_{ikj}.

$$\frac{D\, a_i\, p_{ik}}{A} \cdot \frac{1}{R_{ikj}} = \text{tanks}$$

This gives the number of tanks of fuel consumed in each terrain. The summation of all such land conditions gives the number of tanks of fuel $N + 1$ necessary to traverse the distance D.

$$N + 1 = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, R_{ikj}} = \text{tanks of fuel}$$

Since the vehicle starts from the location O_1 with the gasoline tank filled, the number of refueling stops is N. Therefore the refueling time is

$$t_{f_j}N = (t_{f_j}) \sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, R_{ikj}} - 1 \qquad = \text{hours.}$$

Where t_{f_j} is the time required for each refueling of the jth vehicle.

(3) To find the maintainence time, divide the maintence time per 100 hours of running time q_j by 100 hours, and multiply the result by the number of hours X_j required to traverse the distance D.

$$\frac{q_j}{100} \cdot \frac{X_j}{1} = \frac{q_j}{100} \sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, V_{ikj}} = \text{hours.}^{1}$$

The sum of the preceding three amounts of time gives the total time T_j required to transport the payload W between the location O_1 and O_2 by means of the jth type vehicle.

$$T_j = \sum^{N} \sum^{K} \frac{D\, a_i \; P_{ik}}{A\, V_{ikj}} + (t_{f_j}) \left(\sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, R_{ikj}} - 1 \right) +$$

$$+ \frac{q_j}{100} \sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, V_{ikj}}$$

Or by combining the first and the last term we get:

$$T_j = \left(1 + \frac{q_j}{100}\right) \sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, V_{ikj}} + (t_{f_j}) \sum_{i=I}^{N} \sum_{k=I}^{K} \frac{D\, a_i \; P_{ik}}{A\, R_{ikj}} - 1$$

1 See Annex I

The minimum of the T_j for $j = 1, \ldots, M$, gives the best type of vehicle to use in transporting the payload W a distance D, through the area A.

MOBILITY ACCORDING TO COST CRITERION

There are three major costs involved in operating a vehicle:

(1) The cost of the fuel,

(2) The depreciation cost, and

(3) The maintenance cost.

As in the case where speed was the criterion, the distances d_{ik}

$$d_{ik} = \frac{D_{ai} \quad p_{ik}}{A}$$

encountered in each land condition between O_1 and O_2 , are the basis for calculating the cost.

(1) The fuel cost per vehicle is found by dividing the distance to be traveled in each terrain d_{ik} by the number of miles per gallon m_{ik} attained by the jth vehicle in that terrain.

$$\frac{D_{ai} \quad p_{ik}}{A} \cdot \frac{1}{m_{ikj}} = \text{gallons}$$

This gives the number of gallons of fuel required to traverse each type of land condition. The summation of all such land conditions gives the total number of gallons G_j required to traverse the distance D between locations O_1 and O_2.

$$G_j = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D_{ai} \quad p_{ik}}{A \quad m_{ikj}} = \text{gallons}$$

Then the cost of the fuel F_j is

$$F_j = (f_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D a_i \quad P_{ik}}{A \; m_{ikj}} = \$.$$

Where f_j is the cost of the fuel per gallon.

(2) To find the depreciation cost per vehicle, divide the distance to be traversed in each terrain d_{ik} by the life expectancy L_{ikj} of the vehicle.

$$\frac{D \; a_i \; P_{ik}}{A} \cdot \frac{1}{L_{ikj}} = \text{Life expectancies}$$

This gives the fraction of the vehicle life required to traverse each type of land conditions. The summation of all such land conditions gives the total fraction of the vehicle life expectancy required to traverse the designated distance D.

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D \; a_i \; P_{ik}}{A \; L_{ikj}} = \text{Total fraction of the life expectancy}$$

Then the cost due to depreciation is

$$(e_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D \; a_i \; P_{ik}}{A \; L_{ikj}} = \$.$$

Where e_j is the original cost of the jth vehicle.

(3) To find the maintainence cost, it is necessary first to find the running time X_j required to cross the distance D. The division of the distance to be traveled in each land condition

d_{ik} by the number of miles per hour V_{ikj} averaged by the jth vehicle in that land condition, gives the number of hours of running time x_{ikj} required to traverse each type of land condition.

$$x_{ikj} = \frac{D\, a_i\, p_{ik}}{A} \cdot \frac{1}{V_{ikj}} = \text{hours}$$

The summation of all such land conditions gives the total running time X_j required to traverse the distance D.

$$X_j = \sum_{i=1}^{N} \sum_{k=1}^{M} x_{ikj} = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}} = \text{hours.}$$

Now let us assume that the ages X_{o_y} , $y = 1, \ldots ,$ in running time at location O_1 of the type j vehicles are evenly distributed between 0 hours and $L_j - X_j$ hours.[2] Where L_j is the life expectancy of the jth type vehicle. (We want to exclude any vehicle that will reach an age equal to its life expectancy during the trip. i.e. $X_{o_y} + X_j$ should be less than L_j for all Vehicles.) Then the average maintainence cost per hour of running time is

$$\frac{g\,(L_j)}{L_j}$$

Where $g(L_j)$ is the cost of maintaining a vehicle for its entire life expectancy. The maintainence cost, then is just the average cost per hour

$$\frac{g(L_j)}{L_j}$$

2 See Annex II.

times the number of hours X_j required to traverse the distance D.

$$\frac{g(L_j)}{L_j} \quad (X_j) = \frac{g\ (L_j)}{L_j} \quad \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\ a_i\ \ p_{ik}}{A\ V_{ikj}} = \$.$$

The sum of the preceding three costs gives the total cost C_j of moving one of the type j vehicles from location O_1 to location O_2.

$$C_j = (f_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\ a_i\ \ p_{ik}}{A\ m_{ikj}} + (e_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\ a_i\ \ p_{ik}}{A\ L_{ikj}} +$$

$$+ \frac{g(L_j)}{L_j} \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\ a_i\ \ p_{ik}}{A\ V_{ikj}}$$

J is the number of vehicles of type j required to transport the payload W.

$$J = \frac{W}{\min w_i - W_{f_j}}$$

Where $\min w_i$ is the payload that the jth vehicle can carry in the most restricted land condition. And W_{f_j} is the weight of the extra fuel that each vehicle of the jth type must carry.

$$W_{f_j} = (G_j - H_j)\,\omega$$

Where G_j is the number of gallons needed to traverse the distance D, H_j is the fuel tank capacity, and ω is the weight of the fuel per gallon.

Therefore the total cost TC_j of moving a payload W a distance D, between locations O_1 and O_2 is the cost of moving one vehicle from O_1 to O_2 times the number of vehicles J needed to carry the payload W.

$$TC_j = C_j J =$$

$$= \left(f_j \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, m_{ikj}} + e_j \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, L_{ikj}} + \right.$$

$$\left. + \frac{g(L_j)}{L_j} \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, V_{ikj}} \right) \left(\frac{W}{\min w_i - W_{f_j}} \right)$$

The minimum of the TC_j for $j = 1, \ldots, M$, gives the best type of vehicle to use in transporting the payload W a distance D, through the area A.

ANNEX I.

1. In general this expression for the maintainence time will hold only for journeys with running time X_j less than 100 hours. For running times greater than 100 hours, we must add a term that allows for a major overhauling of each vehicle every 100 hours. Therefore for X_j greater than 100 hours, the maintainence time is

$$\frac{q_j}{100} \cdot \frac{X_j}{1} + \frac{Q_j}{100} \cdot \frac{X_j}{1} = \text{hours.}$$

Where Q_j is the time needed for a major overhauling of the vehicle. By combining terms we get the maintainence time to be

$$\frac{X_j}{100}(q_j + Q_j) = \left(\frac{q_j + Q_j}{100}\right) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, V_{ikj}} = \text{hours.}$$

2. To be completely correct here, it would be necessary to compute the maintainence cost for each vehicle y_j and then to take the sum of all vehicles.

If X_{o_y} is the age of vehicle y_j in running time at location O_1, then the maintainence cost for y_j is

$$g(X_{o_y} + X_j) - g(X_{o_y}) = \$$$

The summation of all the individual costs gives the total maintainence cost associated with crossing the distance D.

$$\sum_{y=1}^{J} \left(g(X_{o_y} + X_j) - g(X_{o_y}) \right) = \$$$

Where J is the number of vehicles of type j required to transport the payload W.

$$J = \frac{W}{\max w_j - W_{f_j}}$$

It is felt that in most cases the ages X_{o_y} of the vehicles will closely approximate an even distribution between 0 hours and $L_j - X_j$ hours. The assumption is made to facilitate computation. And because in many instances the individual vehicle histories may not be available.

If the ages of the vehicles are known approximately, appropriate modifications can be made. Consider the case where all the vehicles of type j are new, i.e.

$$X_{o_y} = 0 \qquad y = 1, \ldots, J.$$

then the average cost per hour of running time w.

$$\frac{g(X_j)}{X_j}.$$

From this point on the procedu.. is the same as in o.. original assumption.

NUMERICAL EXAMPLES

To illustrate the procedure, let us consider a hypothetical case. Let the objective be the moving of a payload, weighing 96,000 pounds, from its present location O_1 to a new location O_2 500 miles away. Let the area covered be a strip ten miles wide from O_1 to O_2. And suppose that there are four types of terrain encountered in the area:

B_1, a hard smooth surface with fair drainage,

B_2, a soft smooth surface with fair drainage,

B_3, a hard medium rough surface with good drainage,

and B_4, a hard rough surface with good drainage.

The percentage, of the total area, in each of the four terrains encountered is based on trafficability maps prepared in advance. For the sake of the illustration, we will assume that each type of terrain has three degrees of trafficability which vary with the moisture content. And the moisture content probabilities are rough estimates based on the number of wet and dry months in a year.

We will also assume that we have two types of vehicles from which to choose; one of them a tracked vehicle and the other one a wheeled vehicle. The performance data for these fictitious vehicles are loosely patterned after a Cargo Vehicle M 76, tracked vehicle, and a 2 1/2 ton cargo truck.

First let us determine which of these will require the least amount of time to transport the payload from location O_1 to O_2. And second determine which will cost the least to transport the payload from O_1 to O_2.

In the following data we will denote the tracked vehicle by I and the wheeled vehicle by II.

The following performance data is given:

Original cost: e_j

Vehicle	I	II
Cost	\$8,000	\$4,000

Fuel cost per gallon: f_j

Vehicle	I	II
Cost	\$.35	\$.35

Fuel weight per gallon: j

Vehicle	I	II
Weight	5.3 lb	5.3 lb

Percentage, of the total area, in each type of terrain: $\frac{a_j}{A}$

Terrain	Percentage
B	3.8%
B	16.9%
B	55.6%
B	23.7%

Probability of finding land condition ik : P_{ik}

Terrain \ Climate	Dry (1)	Intermediate (2)	Wet (3)
B_1	.35	.25	.40
B_2	.25	.25	.50
B_3	.33	.32	.35
B_4	.34	.32	.34

Speed in miles per hour: V_{ikj}

Terrain \ Vehicle	I	II
B_{11}	30	50
B_{12}	25	50
B_{13}	22	40
B_{21}	20	10
B_{22}	18	5
B_{23}	15	3
B_{31}	15	8
B_{32}	12	5
B_{33}	10	5
B_{41}	10	5
B_{42}	10	5
B_{43}	8	5

Range in miles: R_{ikj}

Vehicle / Terrain	I	II
B_{11}	200	350
B_{12}	160	350
B_{13}	135	280
B_{21}	200	175
B_{22}	180	87.5
B_{23}	150	52.5
B_{31}	160	262.5
B_{32}	130	161
B_{33}	110	161
B_{41}	135	105
B_{42}	135	105
B_{43}	105	105

Gasoline tank capacity: H_j

Vehicle	I	II
Capacity	50 gal.	35 gal.

Payload in pounds: W_{ikj}

Vehicle / Terrain	I	II
B_{11}	3000 lb	5000 lb
B_{12}	2500 lb	5000 lb
B_{13}	2000 lb	4000 lb
B_{21}	3000 lb	5000 lb
B_{22}	2700 lb	2500 lb
B_{23}	2250 lb	1500 lb min
B_{31}	2000 lb	2500 lb
B_{32}	1600 lb	1560 lb
B_{33}	1500 lb min	1560 lb
B_{41}	2000 lb	2500 lb
B_{42}	2000 lb	2500 lb
B_{43}	1600 lb	2500 lb

Life expectancy of the vehicle in miles: L_{ikj}

Vehicle / Terrain	I	II
B_{11}	30,000	50,000
B_{12}	25,000	50,000
B_{13}	22,000	40,000
B_{21}	20,000	10,000
B_{22}	18,000	5,000
B_{23}	15,000	3,000
B_{31}	15,000	8,000
B_{32}	12,000	5,000
B_{33}	10,000	5,000
B_{41}	10,000	5,000
B_{42}	10,000	5,000
B_{43}	8,000	5,000

Miles per gallon: m_{ikj}

Vehicle / Terrain	I	II
B_{11}	4.0	10.0
B_{12}	3.2	10.0
B_{13}	2.7	8.0
B_{21}	4.0	5.0
B_{22}	3.6	2.5
B_{23}	3.0	1.5
B_{31}	3.2	7.5
B_{32}	2.6	4.6
B_{33}	2.2	4.6
B_{41}	2.7	4.6
B_{42}	2.7	3.0
B_{43}	2.1	3.0

Maintainence cost for the life of the vehicle: $g_j(L_j)$

L_j = 1,000 Hours, for $j = 1, 2.$

Vehicle	I	II
Cost	$ 1,500	$ 1,000

Maintainence time per 100 hours of running time: q_j

Vehicle	I	II
Hours	10	3

Time required to refuel: t_j

Vehicle	I	II
Hours	.5	.25

The time required for the jth vehicle to transport the payload a distance D is given by:

$$T_j = \left(1 + \frac{q_j}{100} \right) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, V_{ikj}} +$$

$$+ (t_{r_j}) \left(\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, R_{ikj}} - 1 \right).$$

Therefore we must first find

$$\frac{D\, a_i\, P_{ik}}{A}$$

then we must obtain

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, V_{ikj}} \quad \text{and} \quad \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, P_{ik}}{A\, R_{ikj}}$$

for $i = 1, 2, 3, 4.$, $k = 1, 2, 3.$, and j = I, II. Once they are obtained, the time required for the trip for vehicles I and II follows quickly.

Terrain	$\frac{a_i P_{ik}}{A}$	$\frac{D a_i P_{ik}}{A}$
B_{11}	.0133	6.7
B_{12}	.0095	4.8
B_{13}	.0152	7.6
B_{21}	.0422	21.1
B_{22}	.0422	21.1
B_{23}	.0845	42.2
B_{31}	.1835	91.7
B_{32}	.1779	89.0
B_{33}	.1946	97.3
B_{41}	.0806	40.3
B_{42}	.0759	37.9
B_{43}	.0806	40.3
$\sum_{i=1}^{4} \sum_{k=1}^{3}$	1.0000	500.00

$$\frac{D\, a_i\, P_{ikl}}{A\, R_{ik}}$$

Terrain \ Vehicle	I	II
B_{11}	.033	.019
B_{12}	.030	.014
B_{13}	.056	.027
B_{21}	.106	.121
B_{22}	.117	.241
B_{23}	.282	.805
B_{31}	.573	.349
B_{32}	.684	.553
B_{33}	.885	.604
B_{41}	.298	.384
B_{42}	.281	.361
B_{43}	.384	.384
$\sum_{i=1}^{4} \sum_{k=1}^{3}$	3.729	3.862

$= N + 1$

$$x_{ik} = \frac{D\, a_i\, P_{ik}}{A\, V_{ik}} = \text{hours:}$$

Vehicle / Terrain	I	II
B_{11}	.222	.133
B_{12}	.190	.095
B_{13}	.345	.190
B_{21}	1.056	2.113
B_{22}	1.174	4.225
B_{23}	2.817	14.085
B_{31}	6.116	11.468
B_{32}	7.413	17.792
B_{33}	9.730	19.460
B_{41}	4.029	8.058
B_{42}	3.792	7.584
B_{43}	5.036	8.058
$\sum_{i=1}^{4} \sum_{k=1}^{3}$	41.920	93.258

= hours

$$\left(1+\frac{q_i}{100}\right)\sum_{i=1}^{4}\sum_{k=1}^{3}\frac{D\,a_i\,P_{ik}}{A\,V_{ikj}}\ :$$

Vehicle	I	II
$(1+\frac{q_j}{100})X_j$	$(1+\frac{1}{10})(41.92)$	$(1+\frac{3}{100})(93.26)$
Hours	46.11	96.05

$$(t_{f_j})\left(\sum_{i=1}^{4}\sum_{k=1}^{3}\frac{D\,a_i\,P_{ik}}{A\,R_{ikj}}-1\right)\ :$$

Vehicle	I	II
$(t_{f_j})N$	.5(3)	.25(3)
Hours	1.5	.75

$$T_j = \left(1 + \frac{q_j}{100}\right) \sum^{4} \sum^{3} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}} +$$

$$(t_{f_j}) \left(\sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\, p_{ik}}{A\, R_{ikj}} - 1 \right):$$

Vehicle	I	II
T_j (hours)	47.61	96.81

Therefore in our illustration, when speed is the criterion, the tracked vehicle is the best one to use.

The cost, per vehicle, involved in transporting the payload a distance D by the jth type vehicle is given by:

$$C_j = (f_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, m_{ikj}} \quad (e_j) \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, L_{ikj}}$$

$$+ \frac{g(L_j)}{L_j} \sum_{i\;1}^{N} \sum_{k\;1}^{K} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}}$$

Therefore in addition to

$$\frac{D\, a_i\, p_{ik}}{A}$$

and

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}}$$

which we obtained in calculating the time, we must also obtain

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, m_{ikj}}$$

and

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \frac{D\, a_i\, p_{ik}}{A\, L_{ikj}}$$

for $i = 1, 2, 3, 4,\ k = 1, 2, 3,$ and $j = I, II.$

$$\frac{D\, a_i\, p_{ik}}{A\, m_{ikj}} :$$

Vehicle / Terrain	I	II
B_{11}	1.663	.665
B_{12}	1.484	.475
B_{13}	2.815	.950
B_{21}	5.281	4.225
B_{22}	5.868	[illegible].450
B_{23}	14.083	28.167
B_{31}	28.669	12.232
B_{32}	34.215	19.339
B_{33}	44.227	21.152
B_{41}	14.922	13.430
B_{42}	14.045	12.640
B_{43}	19.186	13.430
$\sum_{i=1}^{4} \sum_{k=1}^{3}$	186.458	135.135

= gallons

In our illustration we have assumed that the vehicle life expectancy depends only on the running time. That is, the life expectancy is independent of the land condition being traversed. Therefore the running time X_j divided by the life expectancy L_j gives the fraction of the vehicle life required to travel the distance D. Hence we can replace

$$\sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\ a_i\ P_{ik}}{\lambda\ L_{ikj}} \quad \text{by} \quad \frac{X_j}{L_j} \ .$$ [3]

Fraction of the life expectancy:

Vehicle	I	II
$\frac{X_j}{L_j}$	$\frac{41.92}{1000.00}$	$\frac{93.258}{1000.000}$
Fraction	.041920	.093258

3 See Annex III.

$$(f_j) \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\ P_{ik}}{\lambda\, m_{ikj}} :$$

Vehicle	I	II
$(f_j)G$	.35(186.46)	.35(135.16)
Cost	$ 65.26	$ 47.31

$$(e_j) \frac{X_j}{L_j} :$$

Vehicle	I	II
$(e_j) \frac{X_j}{L_j}$	8000(.04192)	4000(.09326)
Cost	$ 335.36	$ 373.03

$$\frac{g(L_j)}{L_j} \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\ P_{ik}}{A\, V_{ikj}} :$$

Vehicle	I	II
$\frac{g(L_j)}{L_j}$	1.5(41.92)	1(93.26)
Cost	$ 62.88	$ 93.26

$$C_j = (f_j) \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\, P_{ik}}{m_{ikj}} + (e_j) \frac{X_j}{L_j} + \frac{g(L_j)}{L_j} \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\, P_{ik}}{A\, V_{ikj}} ;$$

Vehicle	I	II
C_j	\$ 463.50	\$ 513.60

The number of vehicles of the jth type required to transport the payload W is given by:

$$J = \frac{W}{\min w_i - W_{f_j}}$$

$$W_{f_j} = (G_j - H_j)\omega_j ;$$

Vehicle	I	II
$(G_j - H_j)\omega_j$	(186.46 - 50)5.3	(135.16 - 35)5.3
W_{f_j}	723.24 lb.	530.85 lb.

$$J = \frac{W}{\min w_i - W_{f_j}} ;$$

Vehicle	I	II
$\frac{W}{\min w_i - W_{f_j}}$	$\frac{96{,}000}{1500 - 723.24}$	$\frac{96{,}000}{1500 - 530.85}$
$\frac{W}{\min w_i - W_{f_j}}$	123.7	99.1
J	124	100

$$TC_j = C_j J = \left((\ell_j) \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\, p_{ik}}{A\, m_{ikj}} + (e_j) \frac{X_j}{L_j} + \right.$$

$$\left. + \frac{g(L_j)}{L_j} \sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\, a_i\, p_{ik}}{A\, V_{ikj}} \right) \left(\frac{W}{\min w_i - W_{f_j}} \right):$$

Vehicle	I	II
$C_j J$	(463.50)124	(513.60)100
TC_j	$ 57,474	$ 51,360

Therefore is our illustration, when cost is the criterion, the wheeled vehicle is the best one to use.

Annex III.

3. In general we would have to obtain

$$\sum_{i=1}^{4} \sum_{k=1}^{3} \frac{D\ a_i\ P_{ik}}{A\ L_{ikj}}$$

by finding

$$\frac{D\ a_i\ P_{ik}}{A\ L_{ikj}}$$

for each land condition, and then summing them all up.

$\frac{D\, a_i\, p_{ik}}{A\, L_{ikj}}$:

Vehicle / Terrain	I	II
B_{11}	.00022	.00013
B_{12}	.00019	.00010
B_{13}	.00034	.00019
B_{21}	.00106	.00211
B_{22}	.00117	.00422
B_{23}	.00282	.01408
B_{31}	.00612	.01147
B_{32}	.00741	.01779
B_{33}	.00973	.01946
B_{41}	.00403	.00806
B_{42}	.00379	.00758
B_{43}	.00504	.00806
$\sum_{i=1}^{4}\sum_{k=1}^{3}$	.04192	.09326

= fraction

It is easy to extend the information thus obtained in calculating the time and c6st required to transport the payload a distance D to the following:

(1) The amount of fuel needed to carry out the operation is G.

(2) The average operational speed is

$$\frac{T_j}{D}$$

for the jth type vehicle.

(3) The average running speed maintained by the jth type vehicle is

$$\frac{X_j}{D}$$

(4) The delivery rate (i.e. the tons per hour) for the jth type of vehicle is

$$\frac{W}{T_j}$$

(5) The fuel consumption per ton mile is

$$\frac{G_j}{WD}$$

for the jth type vehicle.

These are only a few of the values that follow immediately from the considered example.

APPENDIX II

SYMBOLS

W	load on wheel, including wheel weight	
R	rolling resistance	R_c due to compaction R_b due to bulldozing R_d due to lateral drag
D	wheel diameter	
b	wheel width	
a	length of rectangular contact area: $a = D \sin a$	
Z	sinkage	
p	ground pressure	
k	sinkage coefficient	$p = kZ^n$
n	sinkage exponent	$p = kZ^n$
k_c	cohesive modulus of deformation	$k = \frac{k_c}{b} k_\phi$
k_ϕ	frictional modulus of deformation	$k = \frac{k_c}{b} k_\phi$
H	pulling force or tractive effort	
DP	drawbar pull: $DP = H - R$	
A	contact area	
c	cohesion, psi	
ϕ	friction angle	
γ	soil density, lb/cu. in.	
N_c, N_γ	"Terzaghi constants," function of ϕ	
r	radius of circular contact area: $r = \frac{1}{2}(b+a)$ for $\frac{a}{b} \leq 2$	
K_c, K_ϕ	functions of ϕ	

MOBILITY OF A FAMILY OF RIGID WHEELS IN A SPECIFIC VARIETY OF SOILS

The problem is to evaluate DP/W in various soils for various size wheels and various loads as specified in the text. In conformity with the previously outlined theoretical establishment of matrices let some of the pertinent procedure be repeated for the sake of clarity.

1. Contact Area and Angle of Approach

The contact area is assumed to be the area determined by the intersection of the wheel with the plane of the surface. This area has a length, s, and a width, b, the wheel width. Hence,

$$A = sb$$

The ground pressure, p, is assumed to be applied to this area, so that

$$A = W/P$$

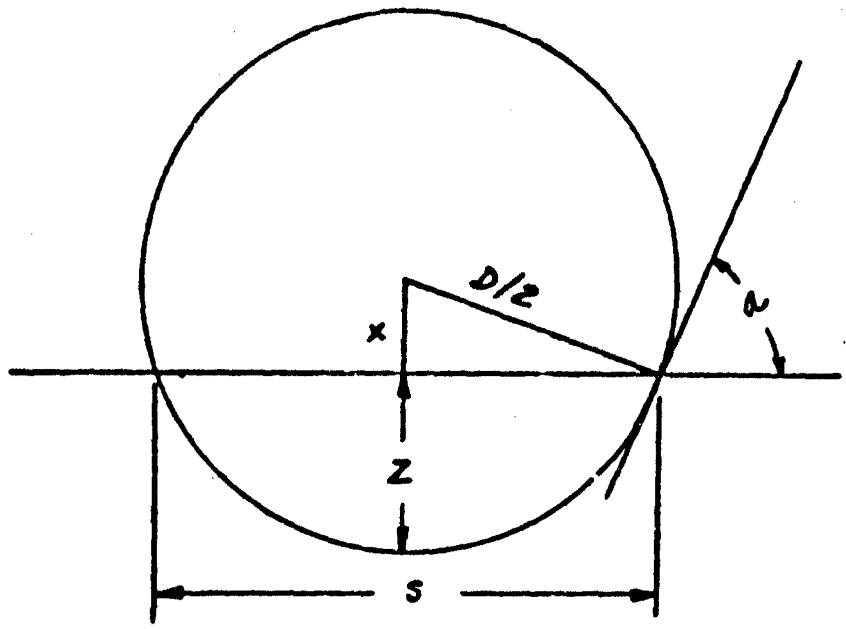
The angle of approach of the wheel is assumed to be the angle between the surface and a line tangent to the wheel at the surface, as shown in Exhibit B-1. Accordingly,

$$a = \cos^{-1} (1 - 2Z/D)$$

$$s = D \sin a$$

EXHIBIT B-1

Assumed angle of approach and contact length



$$x = D/2 - Z$$

$$\sin (90 - a) = \frac{D/2 - Z}{D/2} = \cos a$$

$$a = \cos^{-1} \left(1 - \frac{2Z}{D}\right)$$

$$s = D \cos (90 - a) = D \sin a$$

2. Reasonable Wheel Loading

Although only rigid wheels are being considered in this analysis, some attention should be given to the magnitude of the load assumed for a wheel of given dimensions. A quick method of estimating this maximum load was obtained by plotting some data obtained from the Goodyear Tire and Rubber Company (see Exhibit B-2). The product of maximum tire width and diameter was used as an index of load capacity and the following approximation was obtained.

$$W_{max} = 597 \left(\frac{Db}{100}\right)^{1.26} \qquad (5)$$

The reader may wish to eliminate certain combinations of wheel diameter, width and loading on the basis of equation (5); however, this was not done in the tabulated results.

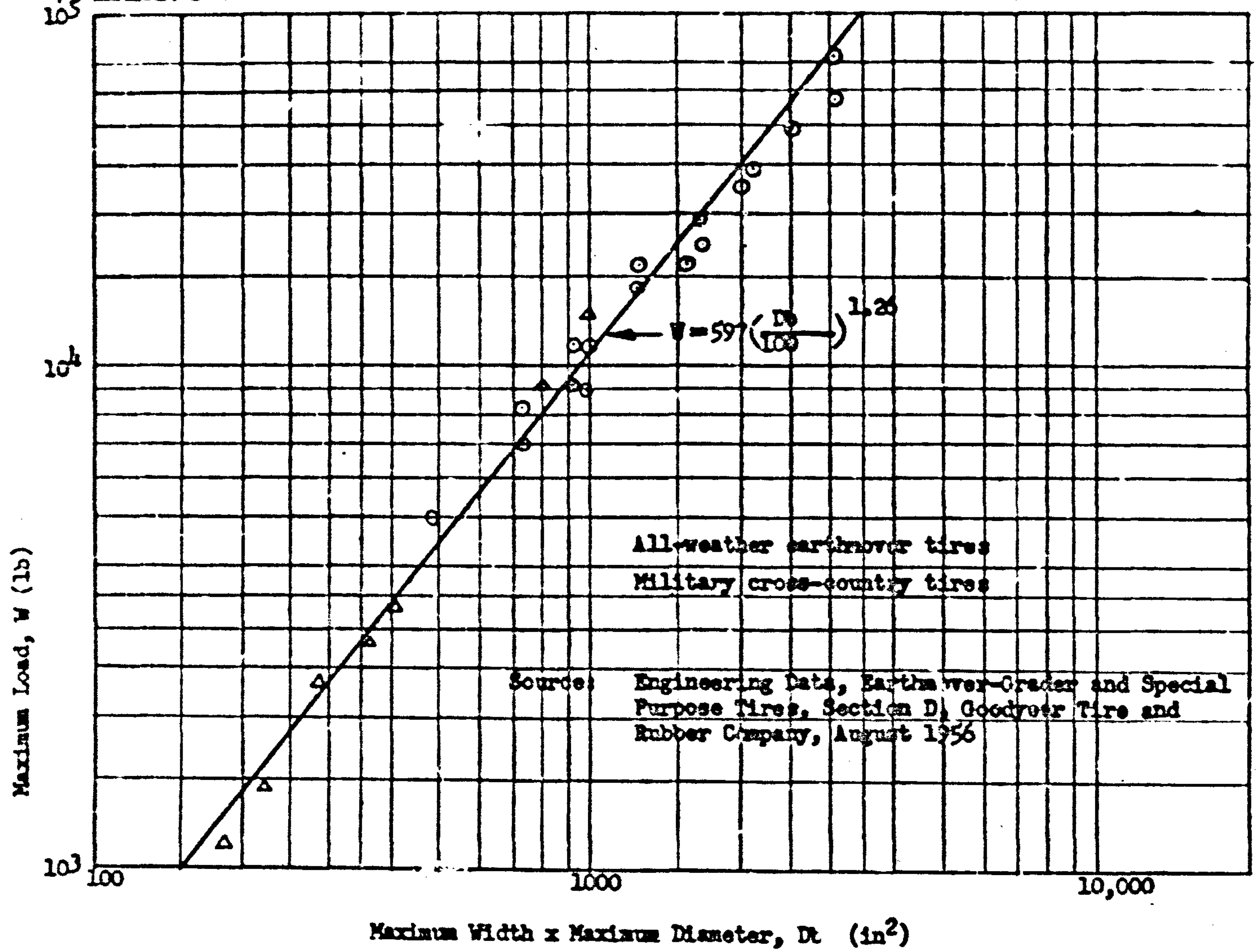
3. Computing Procedure

The complete procedure used for computing the drawbar pull of a rigid wheel is outlined below, along with values of inputs used in the simple calculations.

a. Inputs and Values Used

Inputs	Values Used
k	5, 15, 25
n	0.5, 1, 1.5
c	0, 0.5, 5
ϕ	0°, 10°, 30°
D (for W = 1000)	20, 50, 100

Exhibit B-2 Maximum Tire Load As A Function Of Width-Diameter Product



D (for $W = 3000, 5000$)	50, 80, 100
b	10
γ	.06

b. Procedure

(1) $$Z = \left[\frac{3W}{kb(3-n)\ \gamma D}\right]^{\frac{2}{2n+1}}$$

(2) $$a = \cos^{-1}\left(1 - \frac{2Z}{D}\right)$$

(3) $$s = D \sin a$$

(4) $$A = sb$$

(5) $$R_c = \frac{\left(\frac{3W}{\sqrt{D}\gamma}\right)^{\frac{2n+2}{2n+1}}}{(3-n)^{\frac{2n+2}{2n+1}}(n+1)(kb)^{\frac{1}{2n+1}}}$$

(6) $$K_c = (N_c - \tan \phi) \cos^2 \phi$$

(7) $$K_\gamma = \left(\frac{2N_\gamma}{\tan \phi} + 1\right) \cos^2 \phi$$

N_c, N_γ are given for each value of ϕ

(8) $$t = Z \tan^2 (45 - \phi/2)$$

(9) $$R_b = \frac{b \sin (a + \phi)}{2 \sin a \cos \phi}\left(2ZcK_c + \gamma Z^2 K_\gamma\right.$$

$$\left. + \frac{\pi \gamma t^3 (90 - \phi)}{540} + \frac{c\pi t^2}{180} + ct^2 \tan(45+\phi/2)\right)$$

(10) $$H = Ac + W \tan \phi$$

(11) $DP = H - R_c - R_b$

The equations for steps (1) and (5) - (11) previously discussed. The equation for R_b was simplified when it was discovered that the third term contained a trigonometric expression equal to unity:

$$\sqrt{1 + \tan^2 (45 + \phi/2)} \cos (45 + \phi/2) = 1$$

4. Performance Criteria

In addition to drawbar pull, several other performance criteria are of interest.

a. Drawbar Pull per Unit of Contact Area

This is a measure of the efficiency with which the contact area is being used. It is obtained from the ratio DP/A.

b. Drawbar Pull per Unit of Load

This is a measure of the slope-climbing ability of a wheel. It is of interest to see how DP/W varies with wheel diameter and with approximate wheel volume (diameter squared).

c. Relative Range of a Wheel

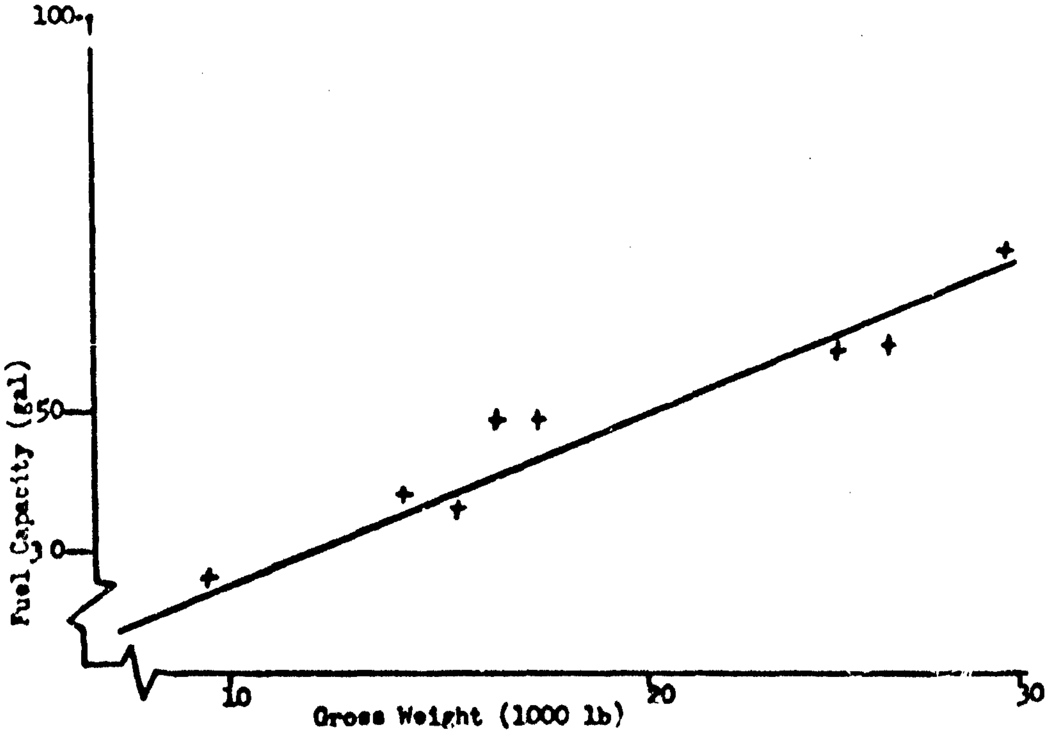
"The larger a truck, the larger its fuel tank." This is verified by the present family of military cargo trucks, as shown by Exhibit B-9. Fuel capacity can be assumed proportional to gross weight.

Fuel consumption (gal/mi) is proportional to rolling resistance. Since range is equal to fuel capacity divided by fuel consumption, the "relative range" of a wheel can be approximated by the ratio of load to rolling resistance (fuel capacity per wheel assumed proportional to load). Hence,

$$\text{Relative wheel range} = \frac{W}{R_c + R_b} \tag{6}$$

EXHIBIT B-3

Fuel capacity of military cargo trucks as a function of gross weight



This criterion only holds for families of vehicles which have fuel capacities proportional to their gross weight.

5. Results and Conclusions

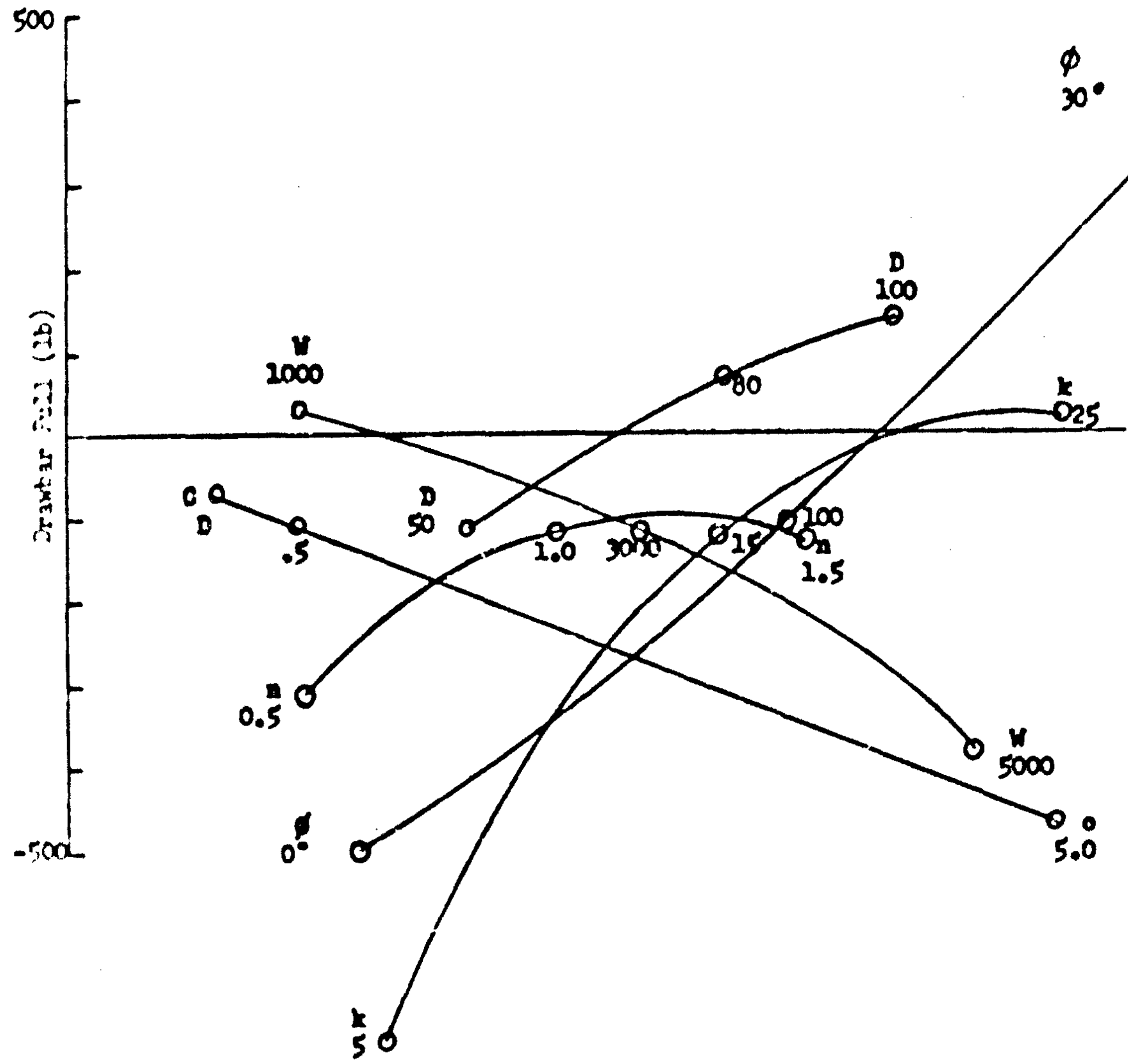
The procedure outlined in section 3b was used in making some sample calculations, covering the values of inputs given in section 3a. Several thousand cases would be necessary in order to draw complete sets of curves. Due to the preliminary nature of this investigation, the computing was held to a total of 576 cases, which was sufficient to show trends and the relative importance of the parameters k, n, c, ϕ, D and W.

Because of the number of parameters involved, a great ... matrices would be required to show all possible relationships between parameters. The particular trend desired can be readily obtained by extracting the appropriate numbers from computed values. As an example, the effect of varying each parameter separately while holding the other parameters at mid-range values is shown in Exhibit B-5. Negative values of drawbar pull are included to show the trends throughout the ranges of values considered.

No definite conclusions can be drawn from Exhibit B-5 because it shows only one possible set of relationships. However, the trends indicated can be followed up in more detail by reference to the tabulated data. For example, the advantage of low wheel loading seems to hold for all types of soil and for all wheel diameters. Increasing wheel diameter (at constant width) shows improvement of performance

EXHIBIT B-5

The effect on drawbar pull of a rigid wheel of varying each parameter separately while holding all other parameters at intermediate values



in all cases. This illustrates the role of particular dimensions of the wheel in its over all mobility.

LIST OF PUBLICATIONS OF THE LAND LOCOMOTION
RESEARCH BRANCH, RES & DEV DIVISION, OTAC
DETROIT ARSENAL, CENTER LINE, MICHIGAN

A. REPORTS

NO.	TITLE
1	Minutes of the First Meeting of the Scientific Advisory Committee (Tech. Memo. M-01)
2	Preliminary Study of Snow Values Related to Vehicle Performance (Tech. Memo. M-02)
3	An Investigation of Spades for Recovery Vehicles (Tech. Memo. M-03)
4	Techniques for the Evaluation of Track and Road Wheel Design (Tech. Memo. M-04)
5*	A Definition of the Engineering Concept of Mobility (Tech. Memo. M-05)
6*	Present State of Off-the-Road Locomotion and Its Future (Tech. Note M-06)
7*	Variable Pitch Hydrofoil Wheel (Tech. Note M-07)
8*	A Study of Air Flow Effect on the Holding Power of Vacuum Devices in Soils (Tech Note M-08)
9*	Goals, Methods and Activities of the Land Locomotion Research Laboratory (Tech Note M-09)
10*	Shear and Sinkage Tests in Local Snows (Tech. Note M-10)
11*	Soil Measurement at the Ordnance Depot, Port Clinton, Ohio (Tech. Note M-11)
12*	Preliminary Study of Synthetic Soils for Vehicle Mobility Investigation (Tech. Note M-12)
13	Terrain Evaluation in Automotive Off-the-Road Operations
14	Application of a Variable Pitch Propeller as a Booster of Lift and Thrust for Amphibian Vehicles

NO.	TITLE
15	Mobility on Land; Challenge and Invitation
16	Minutes of the Second Meeting of the Scientific Advisory Committee
17*	Preliminary Evaluation of Mobility Aspects of the GOER Concept
18	An Analysis of New Techniques for the Estimation of Footing Sinkage in Soils
19	An Investigation of Gein-Anchoring Spades Under the Action of Impact Loads
20	Artificial Soils for Laboratory Studies in Land Locomotion
21*	Power Spectrum of Terrain
22	An Introduction to Research on Vehicle Mobility
23	Study of Snow Values Related to Vehicle Performance
24*	A Practical Application of the Theoretical Mechanics of Land Locomotion: The Prediction of Vehicle Performance
25	Drag Doefficients in Locomotion Over Viscous Soils (Wheel) Part I
26*	Evaluation of Tires for the XM410 8x8, 2-1/2 Ton Truck
27	Effect of Water Content on "B" Values of Soil
28	Effect of Impennetrable Obstacles on Vehicle Operational Speed
29	Obstacle Performance of Wheeled Vehicles
30*	Role of Land Locomotion Research in the Development of Motor Vehicles
31	Performance and Design of Crawler Tractors
32	Application of a Paddle Track as a Booster of Lift and Thrust
33	Determination of Soil Sinkage by Rigid Wheel Sinkage
34*	A Concept of an Open Track, and Its Performance
35	Estimation of Sinkage in Off the Road Locomotion

NO.	TITLE
36	Methods of Obtaining "LL" Soil Values
37*	Comparison of a Plastic Soil Between Two Plates
38*	Comparison of Low and High Profile Tire Performance
39*	Soil Testing at Ft. Knox
40	Operational Definition of Mechanical Mobility of Motor Vehicles

*NOTE: Reports marked with an asterisk (*) are working papers, published in a small number of copies for limited distribution.

B. GENERAL PUBLICATIONS

NO.	TITLE
a)	Research Report No. 1
b)	Research Report No. 2
c)	Research Report No. 3
d)	Research Report No. 4
e)	A Practical Outline of the Mechanics of Automotive Land Locomotion (Seminar Notes Presently Out of Print, New Edition Under Preparation.)
f)	Interservice Vehicle Mobility Symposium, Held at Stevens Institute of Technology, Hoboken, New Jersey, 18-20 April 1955:

Volume I Minutes, Abstracts and Discussions
Volume II Papers