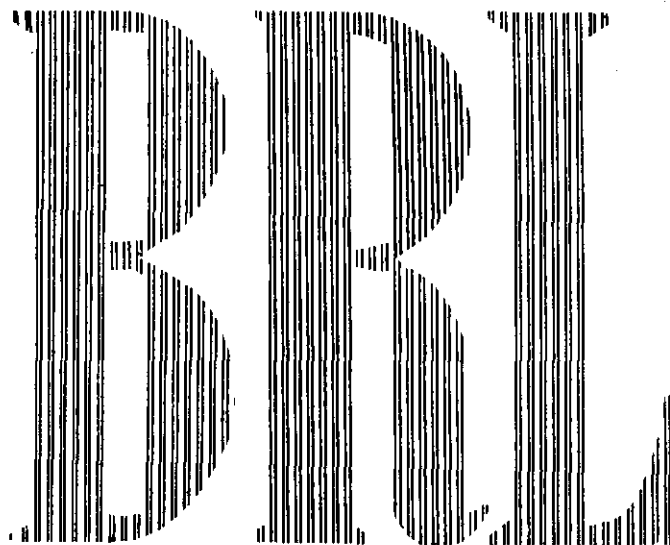
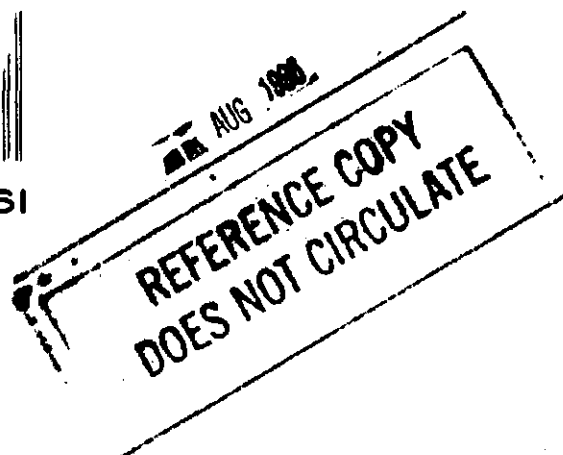




MEMORANDUM REPORT NO. 1161

AUGUST 1958



CHRONOGRAPH ERROR IN DRAG MEASUREMENTS

G. D. KAHL

F. D. BENNETT

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STINFO BRANCH
BRL, APG, MD. 21005

DEPARTMENT OF THE ARMY PROJECT NO. 5803-03-001
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT NO. TB3-0108

BALLISTIC RESEARCH LABORATORIES



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GDKahl/FDBennett/sc
Aberdeen Proving Ground, Md.
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CHRONOGRAPH ERROR IN DRAG MEASUREMENTS

ABSTRACT

The precision of the least squares method of fitting time-distance data from an observed trajectory is considered for the case where time is assumed to be a polynomial in distance. A relation between the variances of time and distance errors, assumed in a previous work, is derived. With this relation and a model of digital chronographs, the variance of the timing error is found for selected chronograph arrangements; in a favorable arrangement, this variance is only one-twelfth as large as the squared counter resolving time. A comparison of time and distance errors is made.

I. INTRODUCTION

The direct free-flight technique of determining the drag properties of a missile requires measurements of the time-distance history of a model along a trajectory. The distance measurements, usually obtained from flash photographs of the missile and a fiducial marker, are combined with simultaneous time observations, which are ordinarily obtained from digital chronographs coordinated with the flash pictures. These observations are used in a least squares procedure to evaluate the drag properties of the model. The accuracy of the method has been analyzed by Karpov¹, with special attention paid to the spatial arrangement of the observation stations. The effect of the pure timing error inherent in digital type chronographs has not been as well clarified; and from the following discussion, the size of this error appears to be considerably smaller than the counter resolving time, which has been frequently used as a measure of the timing error. In conventional digital chronographs, the counter resolving time is necessarily finite, and the complexity and expense of the chronograph increase rapidly as the resolving time is decreased. An appreciation of the magnitude of timing error is necessary for fixing the specifications of the chronograph needed for data of a given accuracy.

II. PRECISION OF THE LEAST SQUARES METHOD

II-1. Polynomial Fitting

It is assumed that the time, t , when the missile is located at a particular distance, Z , along the trajectory is related to the distance by an algebraic polynomial of degree q , so that

$$t = \sum_{j=0}^q a_j z^j \quad (1)$$

The a_j are the coefficients of the polynomial and typify the motion. The derivative of t with respect to z represents the reciprocal of the missile velocity, V , along the trajectory, thus,

$$\frac{dt}{dz} = 1/V = \sum_{j=0}^q j a_j z^{j-1} \quad (2)$$

From the free-flight experiment, discrete pairs of values of time and distance, t_k and z_k , are obtained at N observation stations. The number N is usually arranged to be larger than $q + 1$, the number of unknown coefficients in Eq(1), in order that a least squares technique be applicable.

We define the k th residual, R_k , to be the distance parallel to the t axis from the "best" curve to the k th data point; or,

$$R_k = t_k - \sum_{j=0}^q a_j z_k^j \quad (3)$$

Fitting for the smallest value of the sum of the squared residuals in the conventional manner requires that the a_j satisfy the normal equations

$$[Z]\bar{\alpha} = \bar{\tau} \quad (4)$$

Here Z is a $q + 1$ square, symmetric matrix, with

$$[Z]_{ij} = \sum_{k=1}^N z_k^{i+j} \quad , \quad (5)$$

and $\bar{\alpha}$ and $\bar{\tau}$ are column vectors each of $q + 1$ components, as follows:

$$[\bar{\alpha}]_i = a_i \quad , \quad (6)$$

and

$$[\bar{\tau}]_i = \sum_{k=1}^N t_k z_k^i \quad , \quad (7)$$

where i and j range from 0 to q . A special case where the distance data is symmetric about a central origin is notable because of the vanishing of the particular elements of the Z matrix where $i + j$ is odd.

II-2. Effect of Time Origin

The addition of an arbitrary constant b_0 to each t_k affects only the value of a_0 in the $\bar{\alpha}$ vector. This result is seen by observing that the addition of a vector $\bar{\beta}$ (whose first component is b_0 and all other components are zero) to the $\bar{\alpha}$ vector makes the product $Z(\bar{\alpha} + \bar{\beta})$ equal to $\bar{\tau} + \bar{\tau}_0$, where the components of $\bar{\tau}_0$ are just those of the first column of the Z matrix multiplied by b_0 ; but $\bar{\tau} + \bar{\tau}_0$ also results if the value b_0 is added to each t_k in the $\bar{\tau}$ vector. This fact is not unexpected (since it merely corresponds to a simple change of time origin which will not affect the intrinsic properties of the polynomial) and proves useful in the later discussion.

II-3. Error of Fit

To investigate errors in the a_i due to errors in t_k and z_k , it is observed that, although the z_k values may be in error, the least squares fit is made with residuals in the t direction only. However, the contribution of the distance errors should be included when evaluating the precision measure for the $\bar{\alpha}$ components. Since $\bar{\alpha}$ is a function of both the z_k and the t_k , we have:

$$d\bar{\alpha} = \sum_{k=1}^N \frac{\partial \bar{\alpha}}{\partial t_k} dt_k + \sum_{k=1}^N \frac{\partial \bar{\alpha}}{\partial z_k} dz_k \quad (8)$$

The variance of $d\bar{\alpha}$ is found from the sum of the variances of the two right hand members of Eq(8), since the error distribution in time and distance are independent. A limiting upper bound for $d\bar{\alpha}$ can be found, providing bounds of the dt_k and dz_k are known.

II-4 Relation Between Time and Distance Errors

To further simplify matters, an approximate relation exists between $\frac{\partial \bar{\alpha}}{\partial z_k}$ and $\frac{\partial \bar{\alpha}}{\partial t_k}$. Partial differentiation of Eq(4) yields the two expressions (after transposing and multiplying by Z^{-1}):

$$\frac{\partial \bar{\alpha}}{\partial t_k} = Z^{-1} \frac{\partial \bar{\tau}}{\partial t_k} \quad (9)$$

and

$$\frac{\partial \bar{\alpha}}{\partial z_k} = Z^{-1} \left(\frac{\partial \bar{\tau}}{\partial z_k} - \frac{\partial Z}{\partial z_k} \bar{\alpha} \right) \quad (10)$$

From the definition of $\bar{\tau}$, the i th component of the column vector $\frac{\partial \bar{\tau}}{\partial t_k}$ is found directly to be

$$\left[\frac{\partial \bar{\tau}}{\partial t_k} \right]_i = z_k^i \quad (11)$$

Differentiating the Z matrix with respect to z_k yields a similar square symmetric matrix with elements given by

$$\left[\frac{\partial Z}{\partial z_k} \right]_{ij} = (i+j) z_k^{i+j-1} \quad (12)$$

and it is clear that the $(0,0)$ element is zero for any z_k . Then $\frac{\partial Z}{\partial z_k} \bar{\alpha}$ is clearly a column vector with component value

$$\left[\frac{\partial Z}{\partial z_k} \bar{\alpha} \right]_i = \sum_{j=0}^q (i+j) z_k^{i+j-1} a_j \quad (13)$$

Moreover, $\frac{\partial \bar{\tau}}{\partial z_k}$ is a column vector whose i th component is

$$\left[\frac{\partial \bar{\tau}}{\partial z_k} \right]_i = i t_k z_k^{i-1} \quad (14)$$

By rearranging and combining terms of (13) and (14), the component value of $\left(\frac{\partial \bar{\tau}}{\partial z_k} - \frac{\partial Z}{\partial z_k} \bar{\alpha} \right)$ becomes

$$\left[\frac{\partial \bar{\tau}}{\partial z_k} - \frac{\partial Z}{\partial z_k} \bar{\alpha} \right]_i = i z_k^{i-1} \left\{ t_k - \sum_{j=0}^q z_k^j a_j \right\} - z_k^i \sum_{j=0}^q (j) z_k^{j-1} a_j \quad (15)$$

This simplifies to

$$\left[\frac{\partial \bar{\tau}}{\partial z_k} - \frac{\partial Z}{\partial z_k} \bar{\alpha} \right]_i = i z_k^{i-1} R_k - z_k^i / V_k \quad (16)$$

upon making use of Eqns. (2) and (3), and letting V_k be the missile velocity at z_k . Since R_k ordinarily represents a very small time interval, while z_k/V_k represents approximately the time interval for the missile to traverse the distance between the k th station and the origin, the R_k terms may be neglected. Then the i th component becomes, approximately, $-z_k^i/V_k$; but the column vector with these components is just $(-\frac{\partial \bar{\tau}}{\partial t_k}/V_k)$, by Eq(11). Therefore, a good approximation to (10) is

$$\frac{\partial \bar{\alpha}}{\partial z_k} \approx -Z^{-1} \left(\frac{\partial \bar{\tau}}{\partial t_k} \right) V_k^{-1} \quad (17)$$

Making use of (9) and (17), Eq(8) becomes:

$$d\bar{\alpha} = \sum_{k=1}^N Z^{-1} \frac{\partial \bar{\tau}}{\partial z_k} (dt_k - dz_k/V_k) \quad (18)$$

Eq(18) demonstrates that an "equivalent timing error" can be constructed from the pure time error and the pure distance error; this assumption was made in reference [1], and from the derivation of (18) appears to be amply justified.

An expression for the variance of the elements of $d\bar{\alpha}$ follows from (18), and, assuming corresponding conditions, yields results identical with those in [1]. Letting $\sigma^2(x)$ represent the variance of x , we have

$$\sigma^2(d\bar{\alpha}_i) = \sum_{k=1}^N \left[Z^{-1} \frac{\partial \bar{\tau}}{\partial z_k} \right]_i^2 \left\{ \sigma^2(dt_k) + \sigma^2(dz_k/V_k) \right\} \quad (19)$$

This equation (in different notation) is used in reference [1] to discuss the effect of the spatial distribution of observation stations on errors in the $\bar{\alpha}$ elements, and an equation similar to (18) is employed to estimate upper bounding errors for a five station range in reference [2].

From (19), the variance of the observation errors is seen to be the sum of the variances of both chronograph and distance errors. We shall now determine the variance of the pure time error to be expected from digital type chronographs, in order to compare it with the distance error.

III. CHRONOGRAPH ERROR

III-1. Model Chronographs

To find a mathematical expression for the chronograph error, we consider model chronographs with properties which correspond generally to those of actual chronographs. We assume:

- (a) All chronographs used have identical properties;
- (b) The driving signal from the oscillator to each counter is periodic, with period ϵ ;
- (c) The driving signal is admitted to the digital counter by a gate circuit controlled by identical start and stop signals; however, the effective response time of the chronograph to the start signal is allowed to differ from that of the stop signal.

III-2. Time Observations

The oscillator pulses provide the time scale by dividing time into adjacent intervals each of duration ϵ . The leading edge of each interval (that part of the interval at smallest time) is numbered consecutively. In addition to the ϵ interval, we consider two other time intervals, α and β , which depend on the response of the chronograph to start and stop signals, respectively. These times are made definite by the following definitions:

- (a) If the start signal begins at a time t_B within the interval given by $m\epsilon + \alpha < t_B \leq (m+1)\epsilon + \alpha$, (20)

then the $m+1$ interval causes the First count to be recorded in the counter;

- (b) If the gate closing signal begins at a time t_E within the interval given by

$$n\epsilon + \beta < t_E \leq (n+1)\epsilon + \beta, \quad (21)$$

then the n th interval is the last one recorded in the counter.

We consider all possible paired values of time at which the start and stop signals can occur within the above limits to give a counter reading (in ϵ units) of $n-m$. From (20) and (21), the maximum time interval is $[(n+1)\epsilon + \beta] - [m\epsilon + \alpha]$, while the minimum is $[n\epsilon + \beta] - [(m+1)\epsilon + \alpha]$.

The difference between the maximum and minimum intervals is 2ϵ , and the mean interval is $(n+m)\epsilon - (\alpha-\beta)$. The quantity $\gamma (= \alpha-\beta)$ is a time interval representing the differing response of the chronograph to start and stop signals, and may be either positive, negative, or zero. In an actual chronograph the magnitude of γ is probably much smaller than ϵ ; but we retain it in our analysis for the sake of generality.

The reading C_k on the k th counter is thus seen to represent a true time interval, $\Delta t_k = t_E - t_B$ only to within a starting error, δ_{1k} , a stopping error δ_{2k} , and γ as follows:

$$\Delta t_k = C_k - \gamma + \delta_{1k} + \delta_{2k} \quad (22)$$

The errors δ_{jk} can each have any value between $\pm \epsilon/2$ with the same probability; the distribution for this type of function is therefore rectangular, with a variance, $\sigma^2(\delta_{jk})$, given by

$$\sigma^2(\delta_{jk}) = \epsilon^2/12 \quad (23)$$

The time data used in the column vector $\bar{\tau}$ of Eq(4) are the counter readings converted to suitable units. The error incurred by using the counter readings in lieu of the "true" times will now be examined. It is found that the arrangement and coupling of the counters in the counter bank affects the size of the error in the important elements of $\bar{\alpha}$.

III-3. Arrangement of Counters; N Counters

It is necessary to distinguish between two methods of coupling the counters. If all counters are driven by one frequency source, they are said to be "phased"; if they are driven by separate unphased sources of the same frequency, the counters are said to be "unphased".

The first arrangement considered is one with N counters to indicate time values at the N observation stations; all counters receive a simultaneous start signal prior to the instant of the first observation,

and are stopped in sequence with the corresponding distance observations at each station. We assume the "true" time origin to be that of the first observation, and the time at the k th station is t'_k , where

$$t'_k = C_k - C_1 + (\delta_{1k} + \delta_{2k}) - (\delta_{11} + \delta_{21}) \quad (24)$$

The difference in the readings of the first and the k th counters represents the true time to within the indicated errors. With this choice of time origin there appears to be no error at the first station, although the timing error at any other station could be as large as 2ϵ . However, we can take advantage of the results of paragraph II-2, and use a new time scale, t , where $t = t' + \delta_{11} + \delta_{21}$, to give

$$t_k = C_k - C_1 + \delta_{1k} + \delta_{2k} \quad (25)$$

Now $C_k - C_1$ represents the "true" time to within the error $(\delta_{1k} + \delta_{2k}) \leq \epsilon$ for every station including the first.

If the counters are "unphased", the variance of the timing error $\sigma^2 (\delta_{1k} + \delta_{2k})$ at each station is $\epsilon^2/6$. But if the counters are phased, δ_{1k} is constant independent of k . Again moving the "true" time origin, in this instance to δ_{1k} , we can replace (25) by

$$t'_k = C_k - C_1 + \delta_{2k} \quad ; \quad (26)$$

the variance of the time error is now $\epsilon^2/12$. The use of phased counters has thus halved the variance for this counter arrangement.

III-4. N-1 Counters

An alternative counter arrangement makes use of $N-1$ counters for N observation stations; all counters are started simultaneously with the first observation and stopped in sequence at the succeeding stations. The counters are numbered from 2 to N . Assuming the origin of the "true" time t'_{1k} to coincide with the first observation, the time of the k th observation is

$$t_k^i = C_k - \gamma + \delta_{1k} + \delta_{2k}, \quad (k \geq 2); \quad (27)$$

with

$$t_1^i = 0.$$

For unphased counters, $C_k - \gamma$ represents the "true" time at all stations, save the first, to within the error $(\delta_{1k} + \delta_{2k})$; there is no timing error at the first station. Thus, comparing the unphased counter arrangements, it appears that slightly better data is obtained with the N-1 arrangement than with the N arrangement because there is no timing error at the first station when N-1 counters are used. For this accuracy to be obtained, it is essential that the value of γ be known.

If phased counters are used in the N-1 arrangement, δ_{1k} ($= \delta_{11}$) has the same value for any k ; redefining the time origin by $t = t^i - \delta_{1k}$ enables us to replace (27) by

$$t_k = C_k - \gamma + \delta_{2k} \quad (k \geq 2) \quad (28)$$

and

$$t_1 = -\delta_{11}$$

Then using $C_k - \gamma$ for the time at each station except the first, where the value 0 is used, implies that there is an error with magnitude between the values $\pm \epsilon/2$ at every station; the error variance is $\epsilon^2/12$. These errors are comparable to those of the N phased counter arrangement.

Although the discussion of errors implies that data from N-1 counters is at least as good as that from N counters, the N-1 arrangement has serious practical disadvantages. Aside from the fact that the value of γ must be known, we note that a failure to start the N-1 counters at the first station results in the loss of time data at every station. For this reason alone the additional counter may be worth-while.

IV. COMPARISON OF TIME AND DISTANCE ERRORS

The effective observation error is made up of both time and distance error according to (19). Since these errors are of parallel importance, their comparative sizes are useful in deciding on the chronograph resolution time needed. For example, a bank of phased 1.6 megacycle chronographs ($\epsilon = .625 \mu \text{ sec}$) yields a timing error variance of $\epsilon^2/12$; assuming the r.m.s. distance error to be 10^{-3} feet at each station, the separate contributions of time and distance error to the total error are the same at a missile velocity of 5540 ft./sec. At lower velocities the distance error is the larger one.

Similarly, to make the contributions the same at a missile velocity of 10,000 ft./sec. using 10 megacycle phased counters, the distance error should be reduced to .0035 inches. Considering the difficulties of measuring distances on the order of 50 feet to within an error smaller than this makes it appear that little advantage is gained by using counters with smaller resolving times unless ultra-high velocities are contemplated.

Since the reproducibility of the experiments depends also on physical quantities other than the time-distance history (such as the fluid medium and the similarity of models), errors in these other quantities should be considered in efforts to make measurements of greater precision.



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2. F. D. Bennett and J. M. Bartos, "Bounds on Errors in K_D Caused by Random Errors of Measurement in a Five Station Range", BRLM Report No. 780.

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G. D. Kahl & F. D. Bennett

ERLM 1161

DA Proj 5B03-03-001, ORD Proj TB3-0108
UNCLASSIFIED Report

The precision of the least squares method of fitting time-distance data from an observed trajectory is considered for the case where time is assumed to be a polynomial in distance. A relation between the variances of time and distance errors, assumed in a previous work, is derived. With this relation and a model of digital chronographs, the variance of the timing error is found for selected chronograph arrangements; in a favorable arrangement, this variance is only one-twelfth as large as the squared counter resolving time. A comparison of time and distance errors is made.

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