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SEPTEMBER 1958

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THE DERIVATION OF RANGE DISPERSION
PARAMETERS FROM RANGE FIRING DATA

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ABERDEEN PROVING GROUND, MARYLAND

BALLISTIC RESEARCH LABORATORIES

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SEPTEMBER 1958

**THE DERIVATION OF RANGE DISPERSION PARAMETERS
FROM RANGE FIRING DATA**

Charles T. Odom

**Department of the Army Project No. 5B0307003
Ordnance Research and Development Project No. TB3-0430**

ABERDEEN PROVING GROUND, MARYLAND

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CTOdom/wec
Aberdeen Proving Ground, Md.
September 1958

THE DERIVATION OF RANGE DISPERSION PARAMETERS
FROM RANGE FIRING DATA

ABSTRACT

Among the by-products of artillery range firings are the components of range dispersion. Since the principal parameters influencing the range are known to be muzzle velocity, drag, and angle of departure it was assumed that dispersion in these parameters would also influence range dispersion. The report discusses this assumption and the methods used to derive the dispersion in these parameters from range firing data.

Among the easily derived by-products of artillery range firings, conducted primarily to determine the drag (or ballistic coefficient) of a given ammunition lot, are the components of the range dispersion. The derivation of the components is described in this report. The users of artillery firing tables have always enjoyed the benefit of tabulated probable error data useful in the solution of fire problems. Prior to the employment of the method described in this report, firing table probable errors were based on a non-analytic graphical representation of the probable errors measured in the range firing. Although the graphical representation furnished data of some usefulness, it did not contribute much to the understanding of dispersion, nor did it lend itself to easy mechanization for high speed computer firing table solutions.

Spurred by the advent and capability of high speed computers a re-examination of the firing table probable error problem was made. Although several forms of fitting expressions were tried none proved adequate except the familiar statistical expression for the components of variance, which says that the total variance is equal to the sum of its components.

Under otherwise constant conditions it was known that those parameters which influence the range of a shell are muzzle velocity, drag, and angle of departure. It was felt that since these parameters influence the range, dispersion in these parameters would also influence the range dispersion. (This argument assumes the rounds would be fired over a sufficiently short time interval so that meteorological conditions would be constant and not affect the range dispersion.) Therefore, the expression for components of variance was written as follows:

$$1. \quad \sigma_x^2 = \sigma_v^2 \left(\frac{\delta x}{\delta v} \right)^2 + \sigma_c^2 \left(\frac{\delta x}{\delta c} \right)^2 + \sigma_\phi^2 \left(\frac{\delta x}{\delta \phi} \right)^2$$

Since probable error, PE, is related to standard deviation, σ , by a constant factor the above expression could be written in terms of probable errors as follows:

$$2. \text{PE}_x^2 = \text{PE}_v^2 \left(\frac{\delta x}{\delta v} \right)^2 + \text{PE}_c^2 \left(\frac{\delta x}{\delta c} \right)^2 + \text{PE}_\phi^2 \left(\frac{\delta x}{\delta \phi} \right)^2$$

where

PE_x = probable error in range to impact.

PE_v = probable error in muzzle velocity.

PE_c = probable error in ballistic coefficient or drag.

PE_ϕ = probable error in angle of departure.

$\frac{\delta x}{\delta v}$ = first derivative of range with respect to muzzle velocity.

$\frac{\delta x}{\delta c}$ = first derivative of range with respect to ballistic coefficient.

$\frac{\delta x}{\delta \phi}$ = first derivative of range with respect to angle of departure.

It was assumed that the terms on the right were independent of each other and that the PE_v , PE_c and PE_ϕ were constant for a particular charge. It was obvious that the success of this expression as a means of treating probable errors analytically depended upon the quality of the assumptions. PE_v could be obtained directly from the muzzle velocity measurements taken during the range firing and was found to be constant for each charge except for normal random variations. PE_c and PE_ϕ would have to be inferred from the range firing data and the quality of the assumptions, that PE_c and PE_ϕ were constant and that the three terms on the right were independent, tested by examining the residual PE_x 's.

The three derivatives appearing in the terms on the right could be computed and were available in the firing tables so it was possible to apply a least squares procedure to the range data and to infer simultaneous values of PE_c and PE_ϕ which minimized the sum of squares of the residual probable errors in range.

The assumption of independence might be questioned since both muzzle velocity and ballistic coefficient are affected by variations in projectile weight. A fourth term can be introduced on the right which removes the dependence on weight between muzzle velocity and ballistic coefficient. This term is

written as follows:

$$PE_w^2 \left(\frac{\delta x}{\delta c} \right) \frac{c}{w} \left[\left(\frac{\delta x}{\delta c} \right) \left(\frac{c}{w} \right) + 2 \left(\frac{\delta x}{\delta v} \right) \frac{nv}{w} \right]$$

In this term, w is the projectile weight, c is the ballistic coefficient, n is an empirical constant defined by the relation

$$\frac{\Delta v}{v} = -n \frac{\Delta w}{w},$$

which gives the change in velocity for a given change in projectile weight. PE_w can be obtained directly from observations of the weight of each round. The dependence can also be removed by correcting each measured velocity to what it would have been if each round were of the same weight, and then compute PE_v of the corrected velocities. Experience, however, has shown that the amount of dependence usually present is small and it is only under unusual circumstances that either of the above procedures must be employed.

The method of analysis described here has been applied to a number of sets of range firing data and has been found to give satisfactory results. Since the residual probable errors in range are reasonable one can conclude that, except for the small weight dependence which can usually be ignored, the three terms on the right are independent of each other and that PE_c and PE_ϕ are constant within a particular charge.

In addition to analyzing the range probable error of range firing data and deriving the range dispersion parameters, expression 2 can be used to estimate the range probable errors of new types of ammunition. Estimates of the dispersion parameters can be obtained from other sources and substituted into expression 2 to obtain estimates of range probable errors.

Charles T. Odom

CHARLES T. ODOM