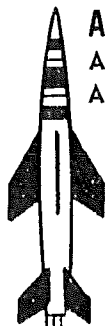


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Ex. 4

**EXPERIMENTAL DETERMINATION OF JET SPREADING  
FROM SUPERSONIC NOZZLES AT HIGH ALTITUDES**

By

E. K. Latvala and T. P. Anderson

ETF, ARO, Inc.

January 1959

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EXPERIMENTAL DETERMINATION OF JET SPREADING  
FROM SUPERSONIC NOZZLES AT HIGH ALTITUDES

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January 1959

ARO Project No. 100813

Contract No. AF 40(600)-700 S/A 13(59-1)

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## ABSTRACT

This report describes the results of an experimental investigation of the jet spreading of underexpanded supersonic jets. A  $13^\circ$  conical nozzle with an area ratio of 5 was used. Unheated air was used as the working fluid. Ratios of nozzle exit pressure to ambient pressure ranging from 9 to 623 were obtained. It was found that the initial portion of the jet boundaries could be approximated by circular arcs having the same radii for the range of pressure ratios tested. Existing theories predict the spreading of the jet quite well for the region of examination covered by these tests.

## NOMENCLATURE (see Fig. 2)

|            |                                             |
|------------|---------------------------------------------|
| A          | Nozzle flow area                            |
| D          | Diameter of nozzle exit                     |
| K          | Constant                                    |
| P          | Pressure                                    |
| R          | Radius of jet boundary                      |
| $\delta$   | Jet spreading angle measured at nozzle exit |
| $\gamma$   | Ratio of specific heats                     |
| $\theta_n$ | Nozzle semi-divergence angle                |

## SUBSCRIPTS

|   |                                                                |
|---|----------------------------------------------------------------|
| c | Static conditions surrounding the nozzle exit, i.e., test cell |
| j | Static conditions in the jet at the nozzle exit                |
| o | Stagnation conditions                                          |

## SUPERSCRIP TS

|   |               |
|---|---------------|
| * | Nozzle throat |
|---|---------------|

## INTRODUCTION

Recent testing of missiles at extremely high altitudes has indicated that the vernier rocket exhaust jets may spread enough to produce serious erosion and heating of missile surfaces. A number of theoretical studies of this problem have been conducted which predict that, for extreme cases of underexpanded jets, the jet spreading angle will exceed  $90^\circ$  and some back flow will occur (Refs. 1 and 2). However, because of the complexity of the calculations, these theoretical results are available only for particular cases.

The flow fields associated with supersonic nozzles exhausting to low ambient pressures are difficult to determine. The solutions to the general partial differential equations describing this flow cannot be expressed in a closed mathematical form. However, since these equations are hyperbolic, the characteristic curves in supersonic flow will be real, and, therefore, the method of characteristics may be used to determine an exact numerical solution to any particular problem. This solution is based upon the assumption of a perfect gas with constant properties. A further restriction is that the method of characteristics may be applied only in the region between shock waves where the fluid properties are continuous.

Experimental results of jet spreading tests have not been previously reported in the literature except for cases of relatively low ratios of nozzle exit pressure to ambient pressure. The study reported here was conducted at the Engine Test Facility, Arnold Engineering Development Center (ETF-AEDC) to provide an experimental verification of theoretical predictions for high pressure ratios. This initial phase was conducted with unheated air ( $\gamma = 1.4$ ) and with only one nozzle. Investigations using other area ratio nozzles and gases having different specific heats are currently underway.

## APPARATUS

The nozzle used in the experiments reported here was tested in the T5C-R1 test cell (Fig. 1) of the ETF. This test cell consists of a stub of 24-in. ducting connected to the exhaust side of the T5C test cell. The air to the nozzle

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\*Manuscript released by authors December 1958.



is furnished from three large air storage bottles charged to 2200 psi from a high pressure air supply. Pipe of 1-1/2-in. diameter connects the bottles to the nozzle inlet chamber. The nozzle exhausts through the T5C-R1 test cell into the T5C exhaust system. Plant exhausters are connected in a series configuration which can reduce the test cell pressure to as low as 5 mm Hg; a further drop to approximately 1 mm Hg is provided by the ejector effect of the jet.

Figure 2 is a drawing of the nozzle showing the physical dimensions and indicating the locations of important pressure orifices. The nozzle has an expansion ratio of 5.00 and an exit Mach number of 3.17 (calculated on the assumption of one dimensional isentropic flow and  $\gamma = 1.4$ ). The divergent section of the nozzle is conical, having a semi-divergence angle of  $13^\circ$  and a throat diameter of 0.500 in.

A stagnation pressure probe was located just upstream of the nozzle inlet, and this pressure was read on a 0-to-1500 psia Bourdon-tube gage. Pressures on a plate mounted below the nozzle (Fig. 2) were measured with a 10-tube manometer bank filled with Dow-Corning No. 200 fluid. The vertical position of the plate was adjustable, and the plate could be tilted. A 0-to-500 mm Hg absolute pressure gage was connected to the pressure tap on the plate at the point nearest the nozzle exit (see Fig. 2). Another gage was connected to the cell wall behind the nozzle exit. The boundary of the jet was recorded photographically with a double-lens, single-pass schlieren and a 4 by 5-in. camera.

## PROCEDURE

After the exhausters were turned on, major leaks were sealed, and the test cell pressure was stabilized at a minimum value (approximately 5 mm Hg). Individual runs were then made at nozzle inlet pressures from 50 to 1500 psia. The air supply was controlled manually and could be set and maintained at any pressure level by opening or closing a 1-1/2-in. needle valve. Once the flow reached a steady-state condition, a photograph was taken.

The nozzle exit pressure,  $P_j$ , was calculated from the stagnation pressure,  $P_o$ , on the basis of one dimensional isentropic flow without boundary layer correction. The minimum test cell pressure,  $P_c$ , obtained in these tests, was approximately 1 mm Hg; the maximum obtainable pressure ratio,  $P_j/P_c$ , was 623.

## RESULTS AND DISCUSSION

## CURVATURE OF INITIAL PORTION OF JET BOUNDARY

Typical schlieren photographs obtained during the investigation are shown in Fig. 3. A careful examination of a large number of similar photographs indicated that the initial portion of the jet boundary from the nozzle lip to the point of maximum jet diameter can be approximated by a circular arc. For the particular nozzle used in these tests the radius of the circular arc boundaries was found to be invariant with pressure ratio. A schlieren photograph with the circular arc inked in is shown in Fig. 3a.

A representative number of schlieren photographs was used to determine the circular arc boundaries shown in Fig. 4. The equation of these circular arcs can be expressed in general form as

$$\delta = \theta_n + K \frac{R}{D_j} \log P_j/P_c \quad (1)$$

For the nozzle used in these experiments  $\theta_n = 13^\circ$  and  $D_j = 1.12$  in.;  $R$  and  $K$  were determined from the experimental data:

$$R = 10.78 \text{ in.} \quad K = 2.228$$

Equation (1) plots as a straight line on a semi-log graph (Fig. 5). The solid line represents the equation; the actual experimental results are indicated by small circles.

It should be noted that the maximum pressure ratio,  $P_j/P_c$ , for these tests was approximately 623, and, therefore, Eq. (1) is actually applicable only to the experimental range of pressure ratios and the particular nozzle and gas tested. However, it is expected that in its general form the equation will be valid for higher pressure ratios,  $P_j/P_c$ , for other area-ratio nozzles,  $A/A^*$ , and for different ratios of specific heat,  $\gamma$ . The jet spreading increases with a decrease in  $\gamma$ ; hence, the radius,  $R$ , would be expected to increase with a decrease in  $\gamma$ . Also, for a constant  $\gamma$  for the same pressure ratio, the jet spreading would be greater for smaller area ratio nozzles, and the value of  $R$  would, therefore, decrease with an increase in  $A/A^*$ .

Additional testing is planned to determine the values of the constants in Eq. (1) for different ratios of specific heats and for area ratios of 1.5 to 25.

Presently available methods of calculating jet boundaries (e.g. Refs. 1 through 3) are not easily applied to any specific case because of the lengthy computations that are required. A relatively simple method of predicting the shape of the initial portion of jet boundaries that would be sufficiently accurate for most engineering purposes would be desirable. The results of the tests reported here indicate that such a simple method may be possible.

## COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

### Method of Characteristics

A comprehensive theoretical analysis of the flow fields of axially symmetric supersonic nozzles has been reported by Love (Ref. 3). The problem was set for machine computation, and the flow fields were calculated for a wide range of parameters. Conical nozzles with semi-divergence angles of 5, 10, 15, and 20° were investigated; area ratios corresponding to Mach numbers of 1.5, 2.0, 2.5, and 3.0 were considered; and calculations were made for gases with gammas of 1.115, 1.200, 1.300, 1.400, and 1.667. The limiting factor in the application of the data of Ref. 3 to the study reported here is the range of pressure ratios (only 1 to 10) which is far below the maximum experimental values obtained in the ETF tests. However, these data have been extrapolated to fit the case of the ETF nozzle, and the results of this extrapolation are shown in Fig. 5. No theoretical justification exists for this extreme extrapolation, but it fits the experimental data surprisingly well.

Both Refs. 1 and 2 contain calculations for conical nozzles for high ratios of nozzle exit pressure to ambient pressure. The data of Ref. 2 cannot be compared directly since, for these tests, the jet Mach number was 2.50; for the ETF tests it was 3.17. However, in Ref. 1 boundaries are shown for conical nozzles with jet Mach numbers of 2, 3, 4, and 5 and gammas of 1.15, 1.25, 1.35, and 1.67. The jet boundary for a semi-cone angle of 15° and a gamma of 1.35 (from Ref. 1) is superimposed on a schlieren photograph (Fig. 6) obtained during the ETF tests at the same pressure ratio. The experimental and theoretical boundaries are found to agree fairly well.

### Prandtl-Meyer Expansion Theory

Another possible approach to the prediction of jet spreading is the use of the two-dimensional Prandtl-Meyer expansion theory. Since the Mach number at the nozzle exit will be 3.17 for all nozzle inlet pressures (assuming one-

dimensional isentropic flow), the Prandtl-Meyer expansion angle can be calculated as a function of pressure ratio,  $P_j/P_c$ . When  $13^\circ$  (the nozzle divergence angle) is added to this value, the result is a prediction of the jet spreading angle. The Prandtl-Meyer expansion angle is shown in Fig. 5 for the experimental range of pressure ratios. (See Ref. 4 for method of calculating the Prandtl-Meyer angle.) Poor agreement with experimental results is evident at higher values of pressure ratio.

The Prandtl-Meyer analysis is based on the assumption of isentropic flow in which (1) all flow properties are uniform at any cross section and the Mach lines are straight and (2) for given initial conditions, the magnitude of the velocity at any point depends only on the flow direction. Since the test nozzle obviously does not fulfill these conditions, some deviation between the predicted and actual angles is to be expected. At the higher values of pressure ratio, the assumption of one-dimensional isentropic expansion in the nozzle would be expected to be valid only for the flow in the central core portion of the jet at the nozzle exit. Near the nozzle walls the Mach number would be lower, and the pressure would be higher than calculated, and, therefore, the jet would expand more than is predicted by the Prandtl-Meyer theory, as it does.

#### JET IMPINGING ON ADJACENT SURFACES

Typical schlieren photographs of the jet impinging on a flat plate located below the nozzle are shown in Figs. 7a and b and 8a and b. In Fig. 7 the plate was parallel to and was located 2 in. below the nozzle centerline; in Fig. 8, the plate was tilted  $13^\circ$  from parallel about a point 2 in. below the nozzle centerline at the nozzle exit. For these photographs the schlieren was focused in the region of the jet impingement; the remainder of the schlieren window (jet) was covered so as to minimize light wave refractions. Pressure distribution on the plate (corresponding to Figs. 7a and b) are shown in Fig. 7c; pressure distribution corresponding to Figs. 8a and b are shown in Fig. 8c. Pressure distributions with the plate located 3 in. and 5 in. below the nozzle centerline (out of view of the schlieren system) are shown in Fig. 9. The pressure ratio,  $P/P_c$ , refers to the ratio of the pressure,  $P$ , at a particular plate location to the pressure in the test chamber,  $P_c$ .

The jet impinging on a plate at high pressure ratios,  $P_j/P_c$ , exhibits the characteristics of supersonic flow in a compression corner. From the schlieren photograph (Fig. 7b) and the corresponding pressure distribution figure (Fig. 7c),

it is evident that the jet under the conditions of this test was unable to negotiate the abrupt change in direction and the attendant pressure rise without separation of the boundary layer. A finite length is required for reattachment, after which the boundary layer increases in thickness in the usual manner.

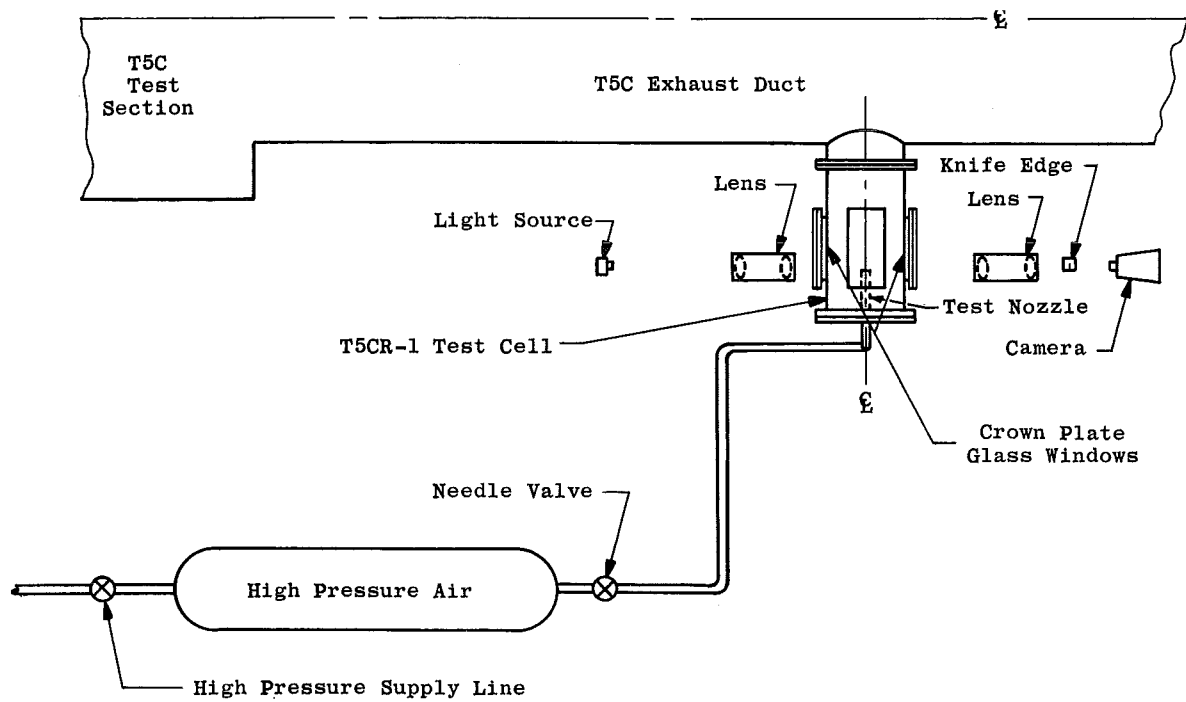
### CONCLUSIONS

As a result of the experiments at ETF to determine the jet spreading characteristics of supersonic jets at high altitudes, the following conclusions may be reached.

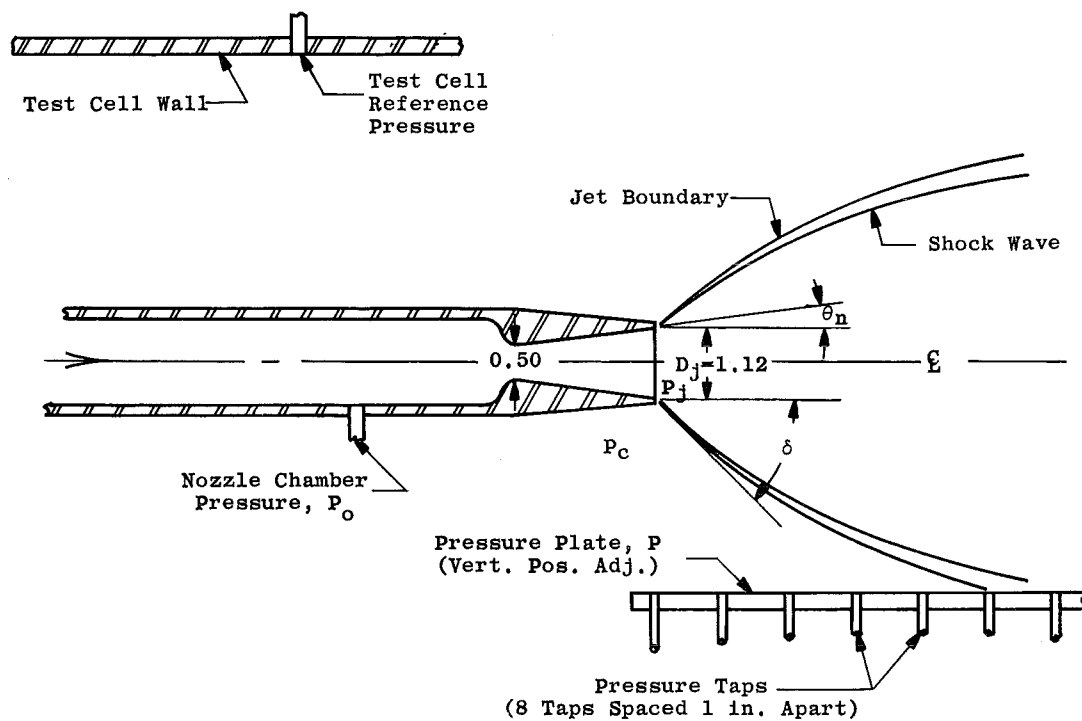
1. The initial portion of the jet boundary from the nozzle exit to the point of maximum jet diameter for the range of pressure ratios tested can be approximated by a circular arc. The radii of the circular arc boundaries were practically the same for the range of pressure ratios tested.
2. A relatively simple method of predicting the shape of the initial portion of jet boundaries may be possible. Additional experimental testing will be required to determine whether the simple relationship for jet spreading will hold for other area ratio nozzles, other specific heat ratios, and for higher pressure ratios.
3. An estimate of the initial jet spreading angle based on the assumption of two dimensional Prandtl-Meyer expansion theory is reasonable for pressure ratios below 40 for the nozzle tested, but the error increases rapidly for higher values of pressure ratio.
4. Existing theories based on the method of characteristics predict the spread of a supersonic jet quite well, at least for the initial portion of the jet boundary.

## REFERENCES

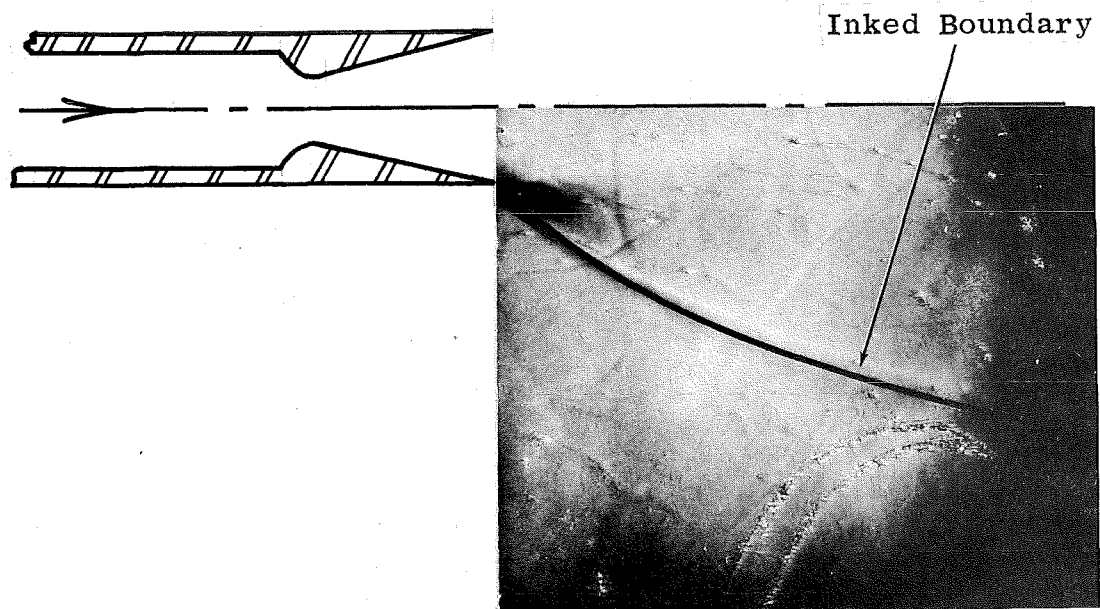
1. Wang, C. J. and Peterson, J. B. "Spreading of Supersonic Jets from Axially Symmetric Nozzles." American Rocket Society preprint No. 462-57, June 1957.
2. Love, E. S. and Lee, L. P. "Shape of Initial Portion of Supersonic Axisymmetric Free Jets at Large Jet Pressure Ratios." NACA TN-4195, January 1958.
3. Love, E. S., Woodling, M. J., and Lee, L. P. "Boundaries of Supersonic Axisymmetric Free Jets." NACA RM L56G18, October 1956.
4. Ames Research Staff. "Equations, Tables and Charts for Compressible Flow." NACA TR 1135, 1953.



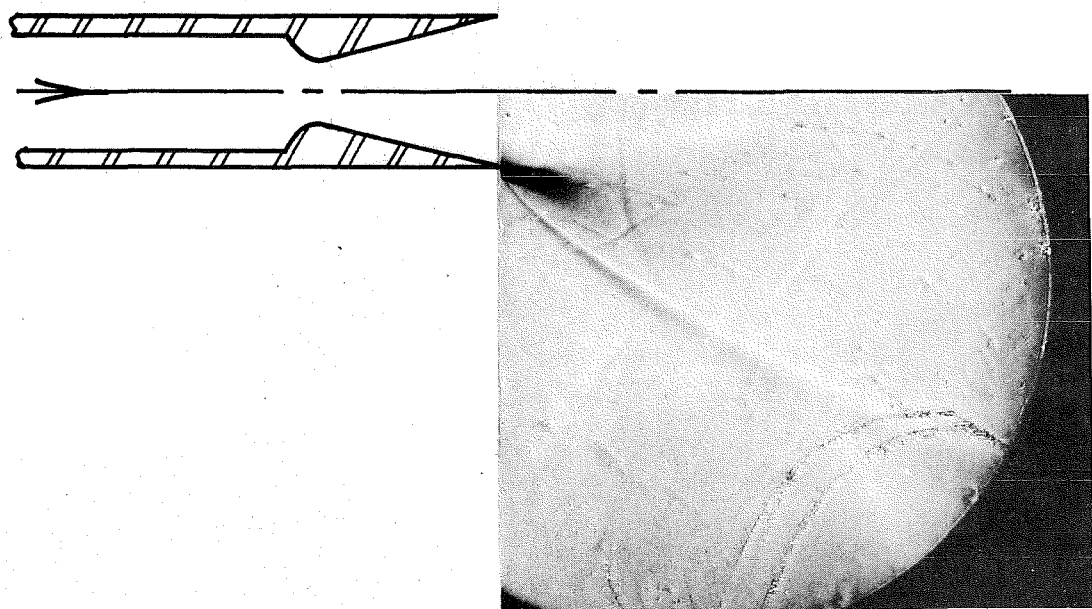
**Fig. 1. Schematic of T5CR-1 Test Area**



**Fig. 2. Test Nozzle Installation**



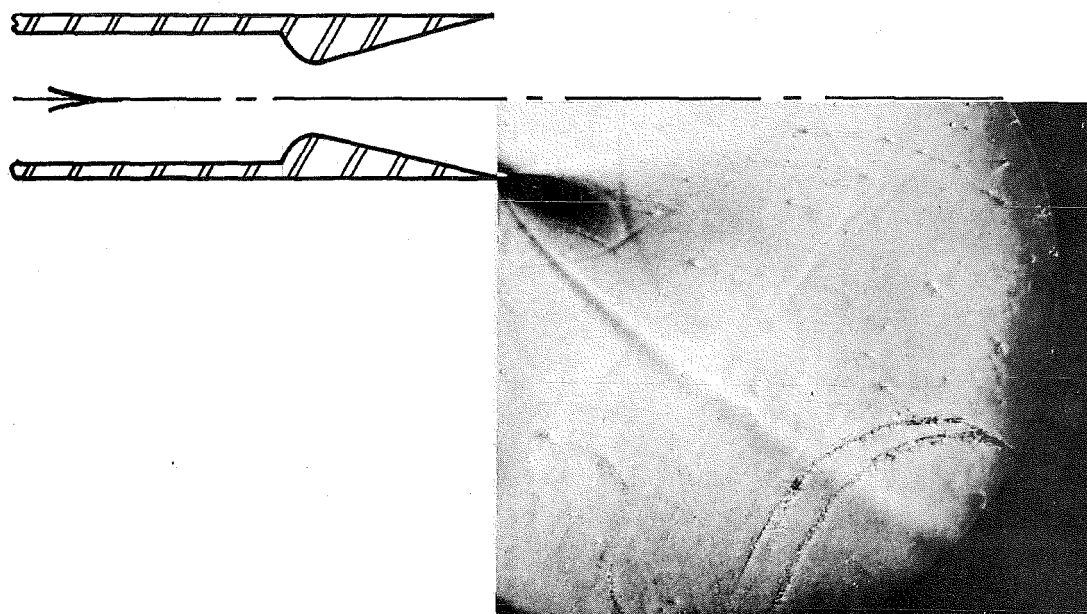
a.  $P_1/P_c = 13.83$



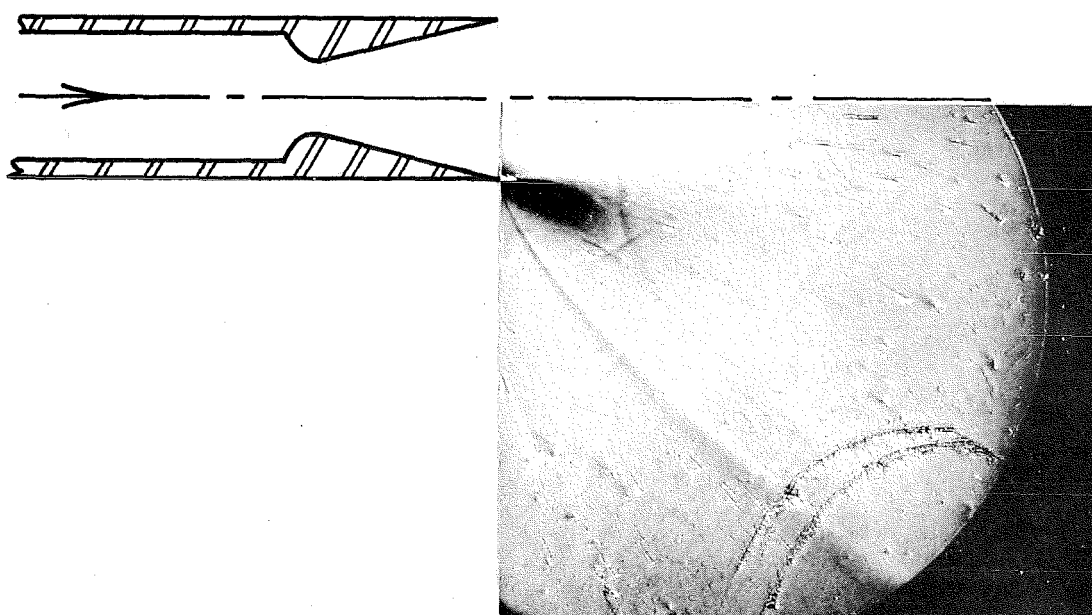
b.  $P_1/P_c = 30.6$

**Fig. 3. Schlieren Photographs of Initial Portion of Jet Boundary**



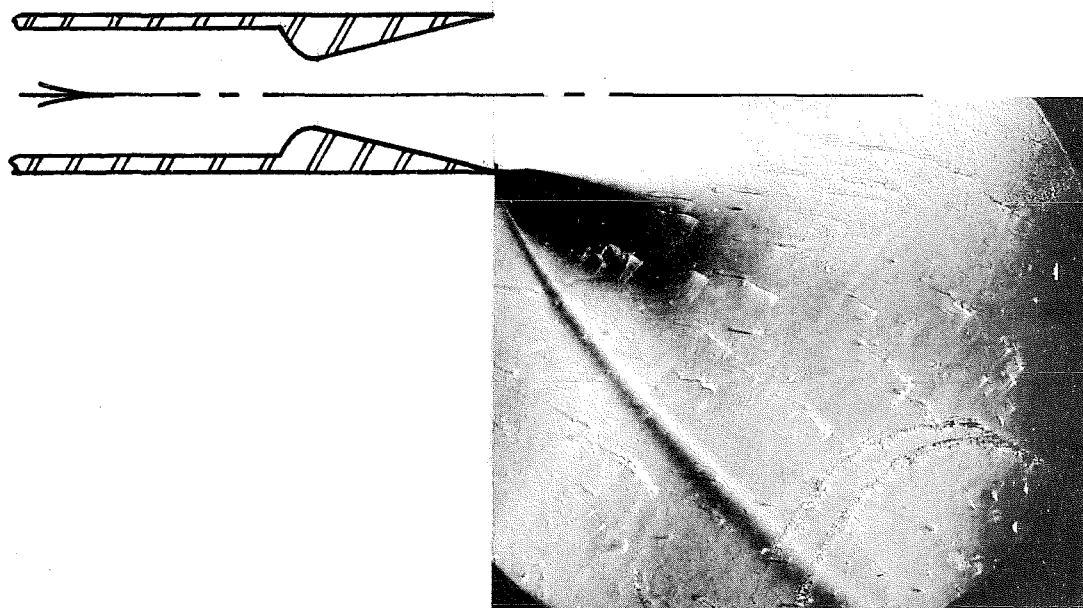


c.  $P_i/P_c = 61.9$

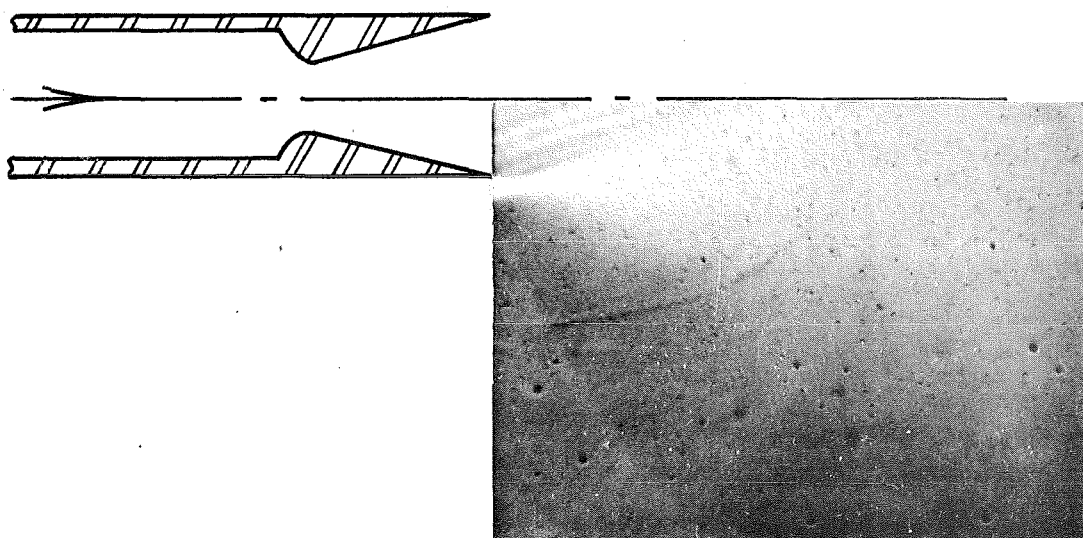


d.  $P_i/P_c = 121.44$

Fig. 3. Continued

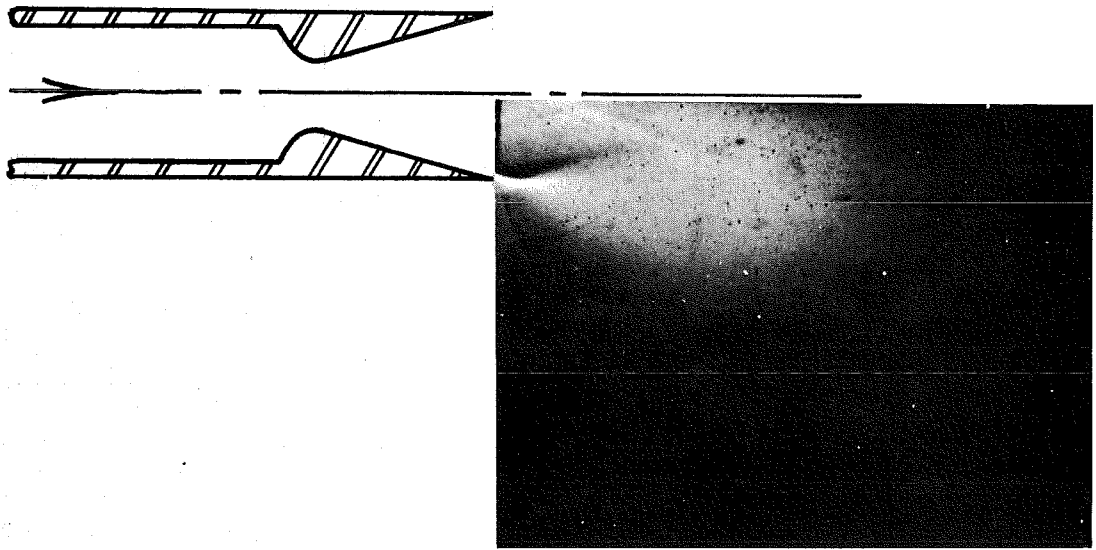


e.  $P_t/P_c = 174.0$

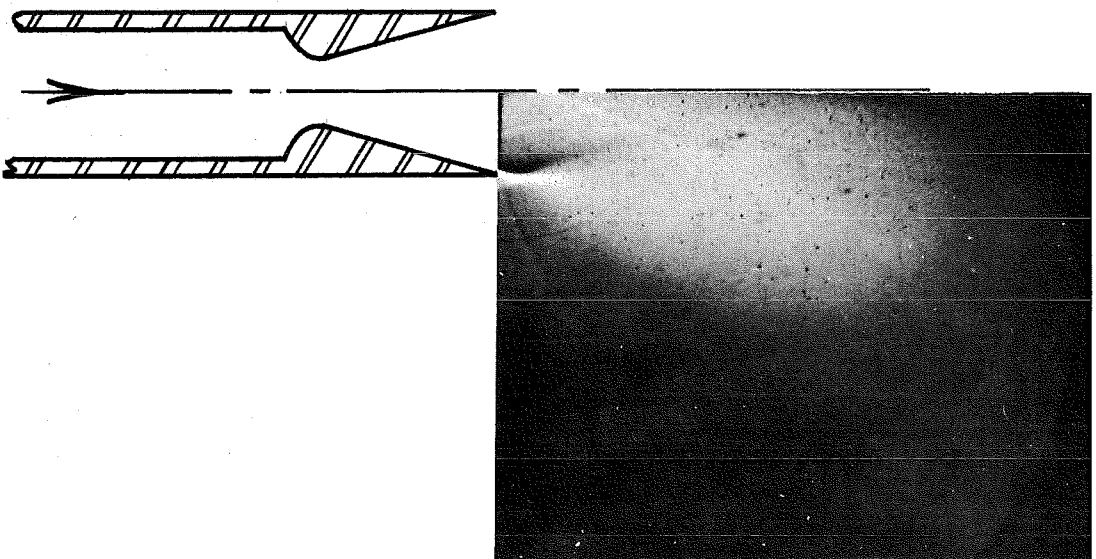


f.  $P_t/P_c = 324.0$

Fig. 3. Continued

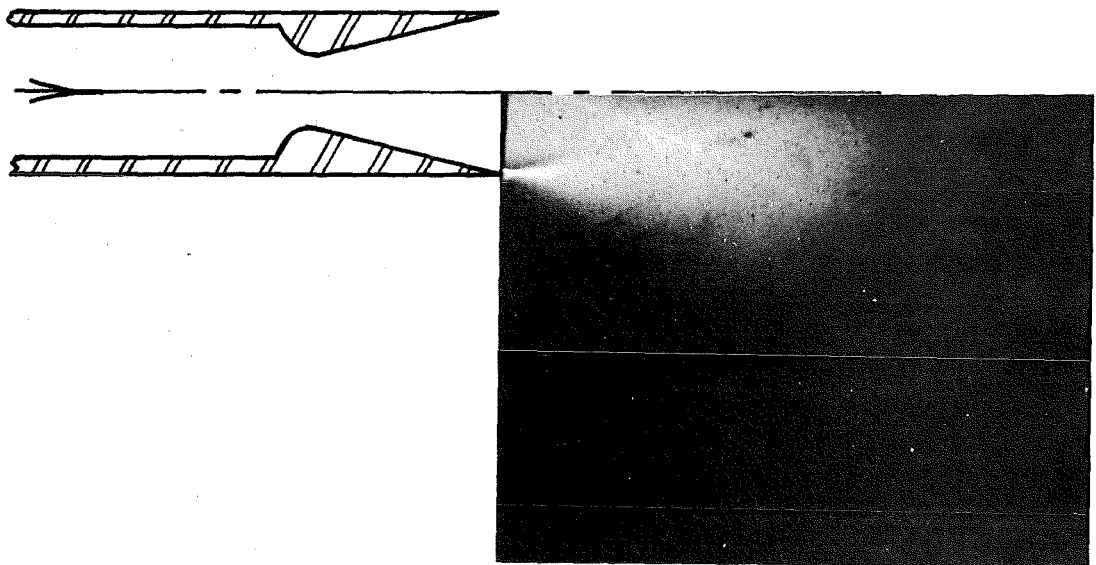


g.  $P_i/P_c = 433.0$



h.  $P_i/P_c = 530.0$

Fig. 3. Continued



i.  $P_1/P_c = 623.0$

**Fig. 3. Concluded**

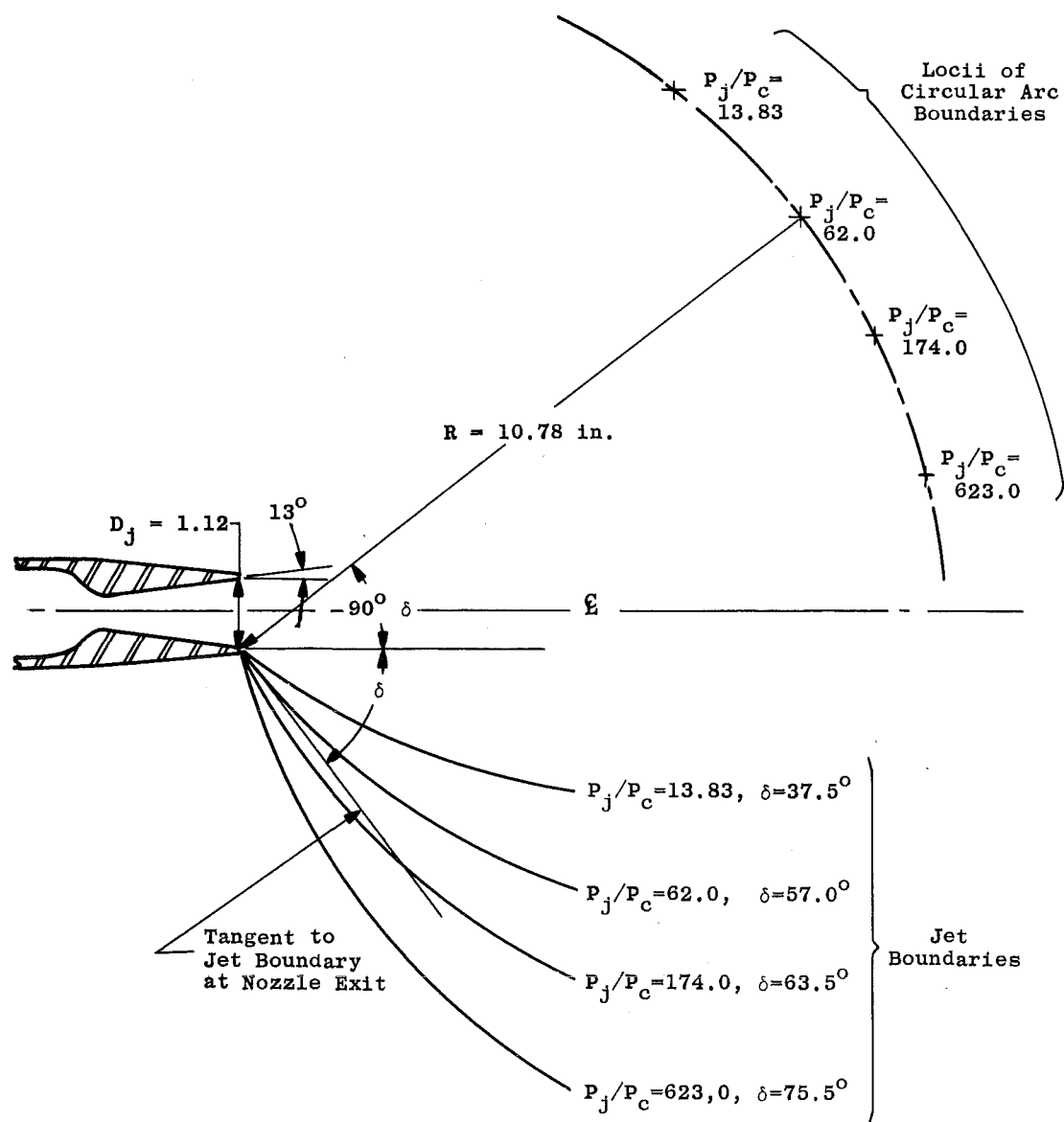


Fig. 4. Initial Portion of Jet Boundary for Various  $P_j/P_c$  Values

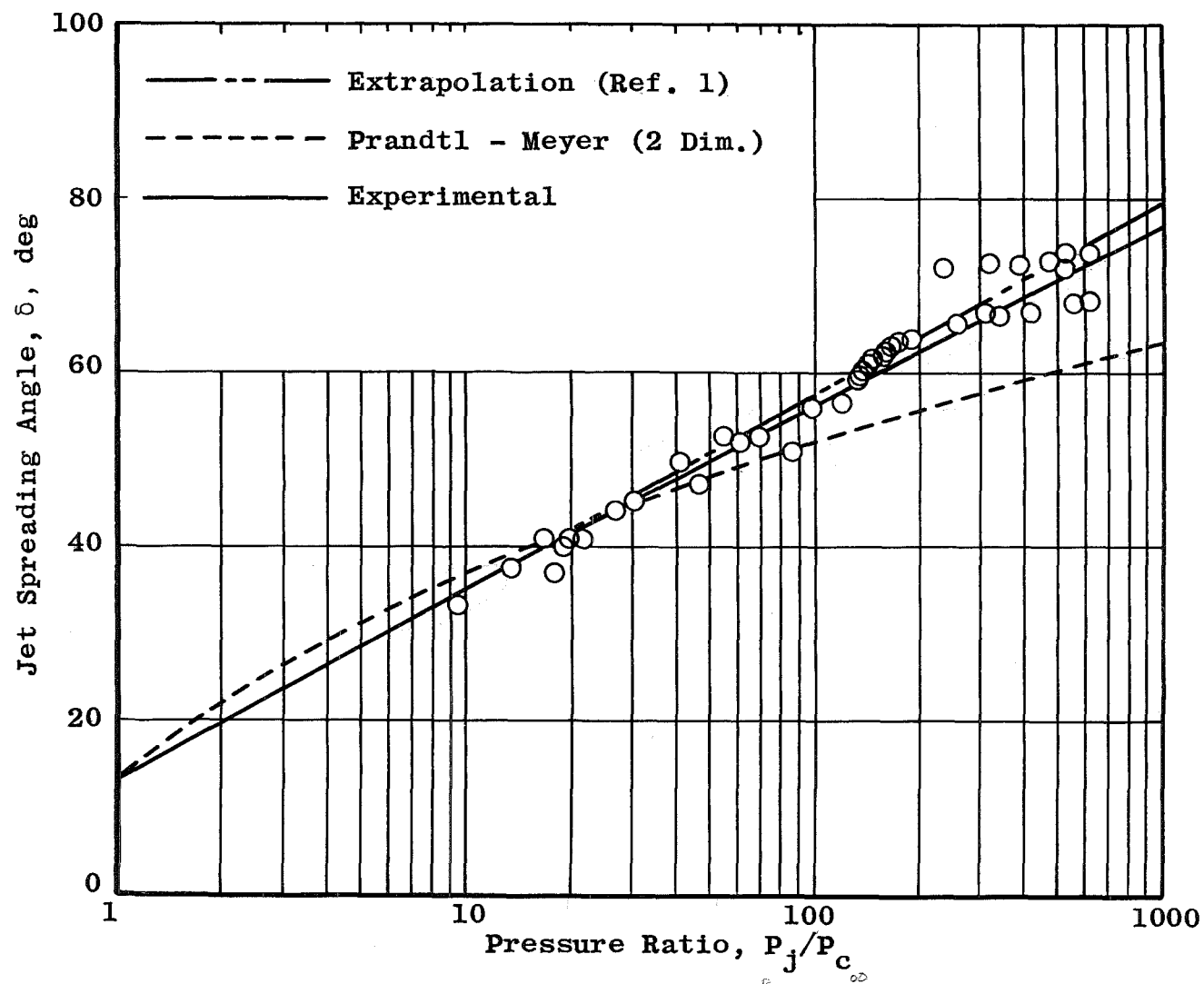
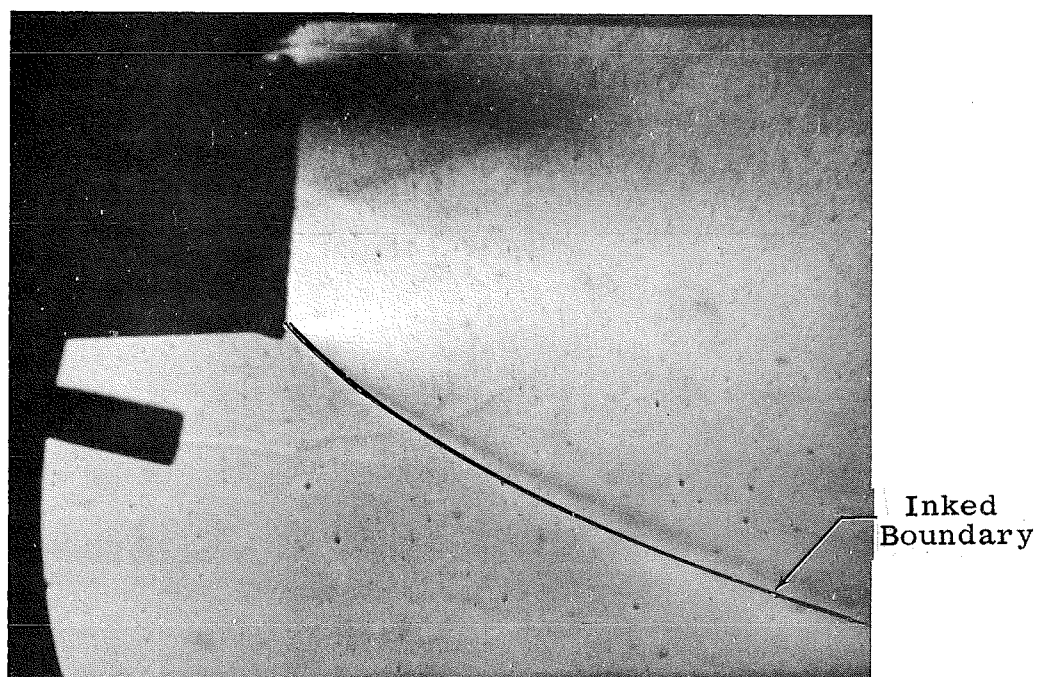
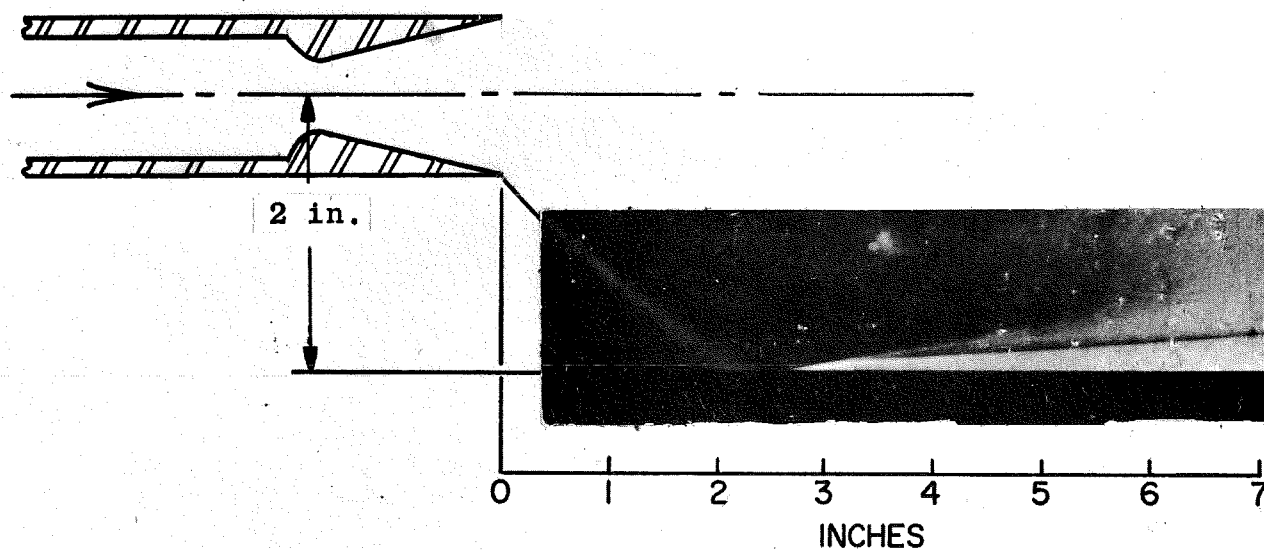


Fig. 5. Predicted and Experimental Jet Spreading Angle

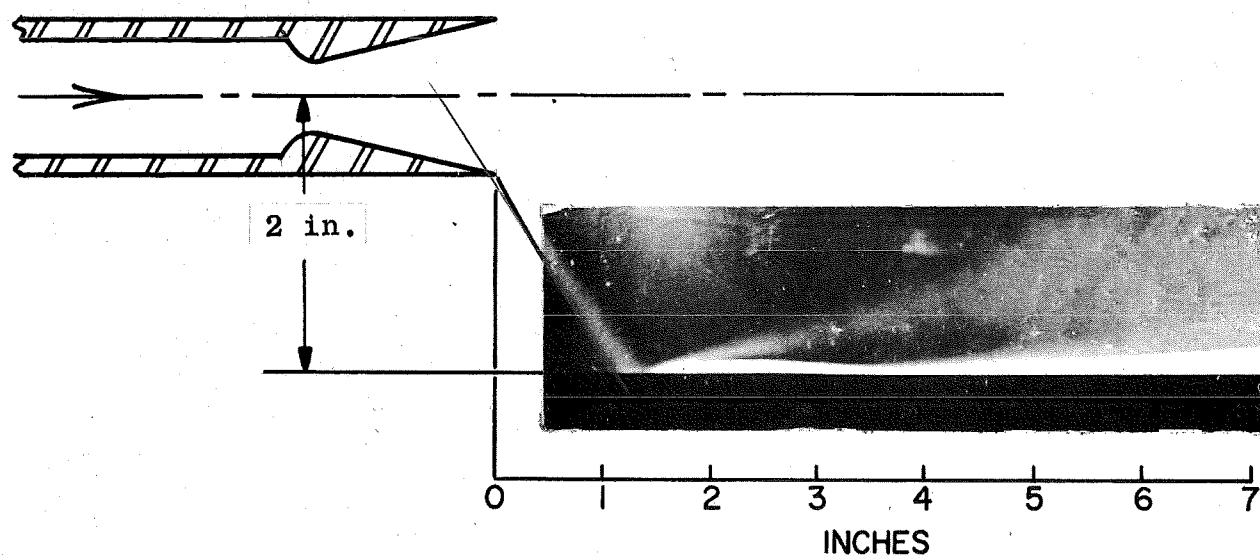


Experimental  $P_j/P_c = 38.0$   $\gamma = 1.4$   $\theta_n = 13^\circ$   
Theoretical (Ref. 3)  $P_j/P_c = 38.6$   $\gamma = 1.35$   $\theta_n = 15^\circ$

Fig. 6. Theoretical Boundary Superimposed on Schlieren Photograph



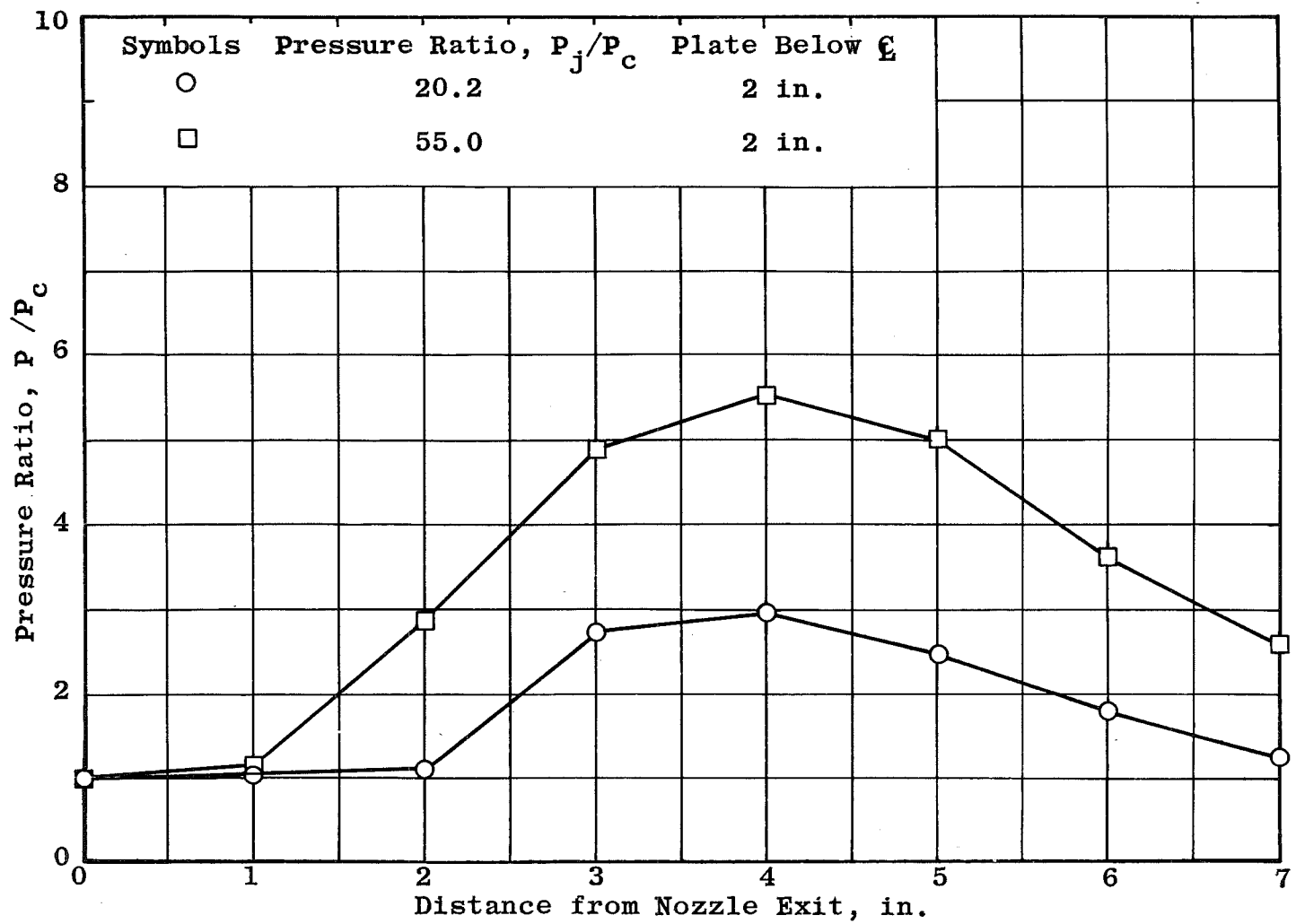
a.  $P_1/P_c = 20.2$



b.  $P_1/P_c = 55.0$

Fig. 7. Schlieren Photographs and Static Pressure Distributions, Plate 2 in. below Nozzle Centerline





c. Static Pressure Distributions

Fig. 7. Concluded

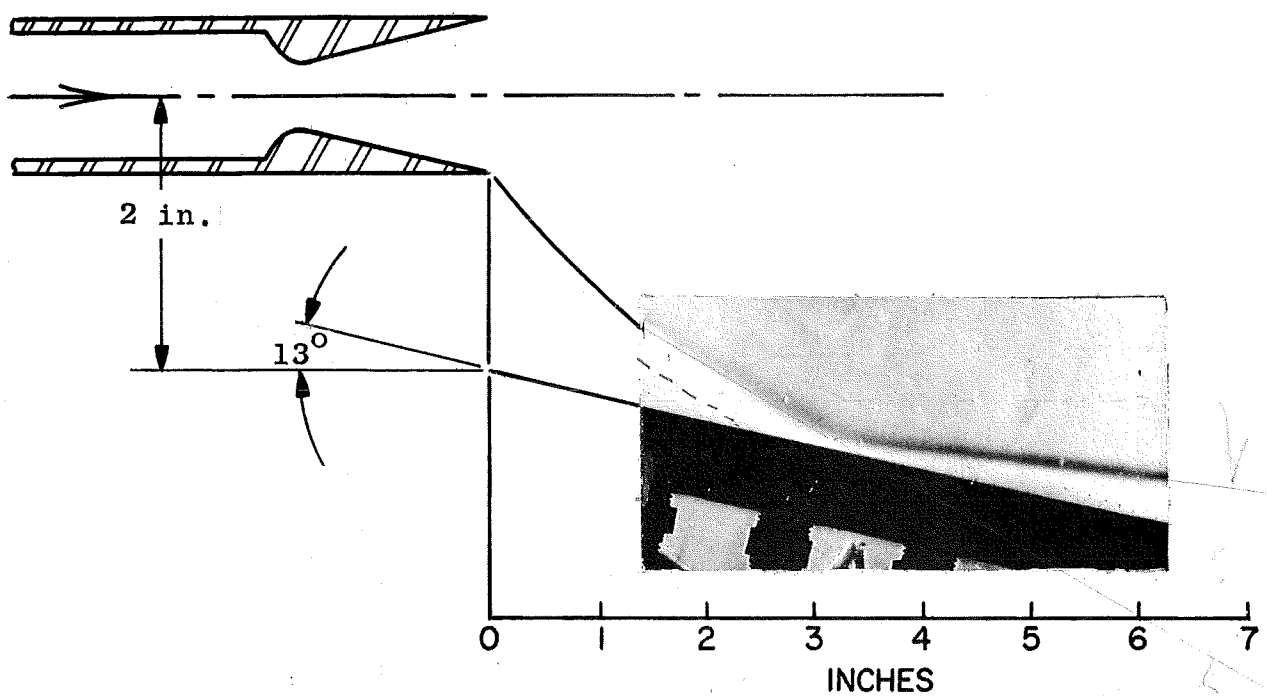
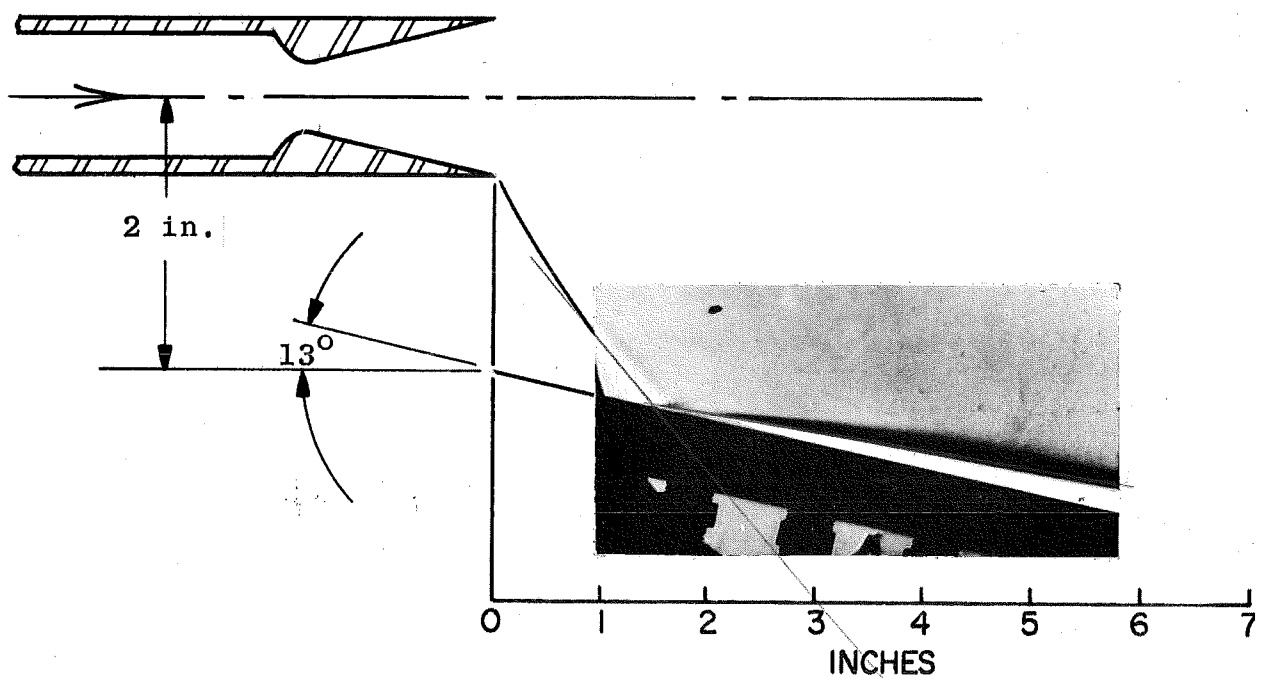
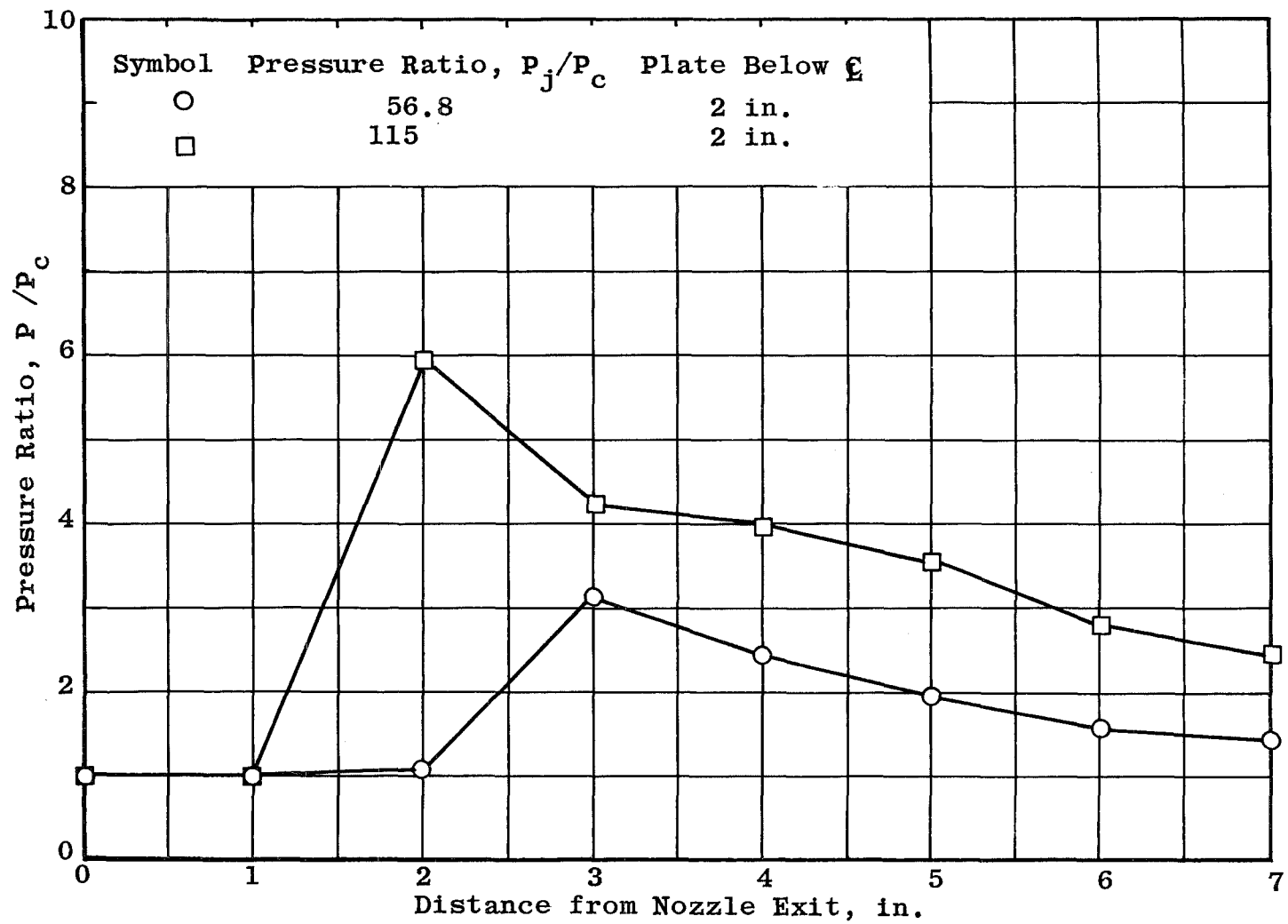
a.  $P_1/P_c = 56.8$ b.  $P_1/P_c = 115.0$ 

Fig. 8. Schlieren Photographs and Static Pressure Distributions, Plate 2 in. below Nozzle Centerline, and Tilted  $13^\circ$  from Horizontal



c. Static Pressure Distribution

Fig. 8. Concluded

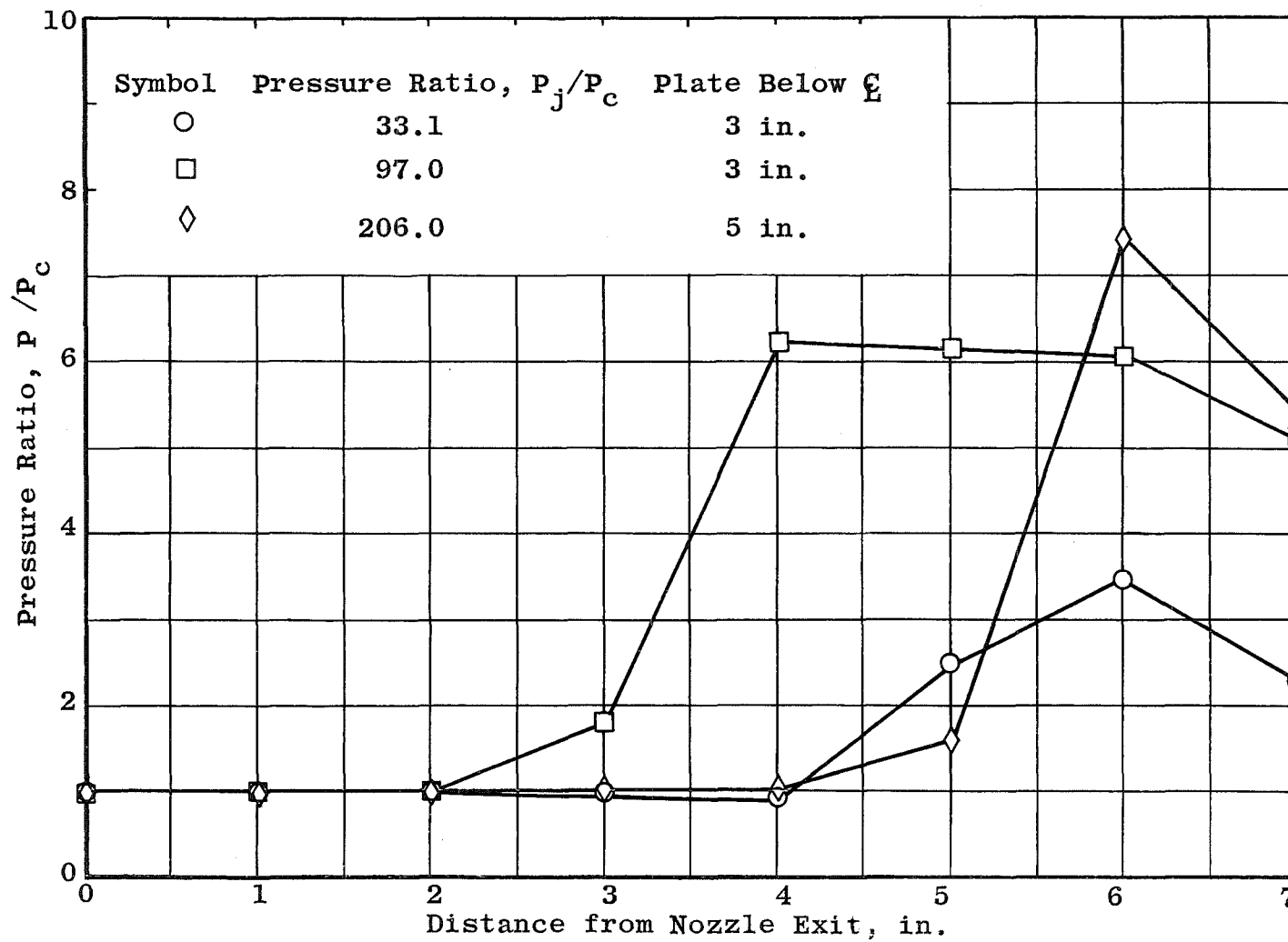


Fig. 9. Static Pressure Distributions on Plate Located 3 in. and 5 in. below Nozzle Centerline