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MEMORANDUM

DETAILS OF EXACT LOW PRANDTL NUMBER BOUNDARY-LAYER
SOLUTIONS FOR FORCED AND FOR FREE CONVECTION

By E. M. Sparrow and J. L. Gregg

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MEMORANDUM 2-27-59E

DETAILS OF EXACT LOW PRANDTL NUMBER BOUNDARY-LAYER
SOLUTIONS FOR FORCED AND FOR FREE CONVECTION

By E. M. Sparrow and J. L. Gregg

SUMMARY

↓ A detailed report is given of exact (numerical) solutions of the laminar-boundary-layer equations for the Prandtl number range appropriate to liquid metals (0.003 to 0.03). Consideration is given to the following situations: (1) forced convection over a flat plate for the conditions of uniform wall temperature and uniform wall heat flux, and (2) free convection over an isothermal vertical plate. Tabulations of the new solutions are given in detail. Results are presented for the heat-transfer and shear-stress characteristics; temperature and velocity distributions are also shown. The heat-transfer results are correlated in terms of dimensionless parameters that vary only slightly over the entire liquid-metal range. Previous analytical and experimental work on low Prandtl number boundary layers is surveyed and compared with the new exact solutions. ↗

INTRODUCTION

Interest in the use of liquid metals as heat-transfer media has been stimulated by nuclear-reactor applications. Because of their high thermal conductivity, liquid metals are characterized by Prandtl numbers that lie far below those of conventional media such as gases and ordinary liquids. As a consequence, the large body of heat-transfer information available for conventional fluids cannot be used directly for liquid metals. This circumstance has provided the motivation for heat-transfer research in the low Prandtl number range.

It is the purpose of this report to present exact (numerical) solutions of the laminar-boundary-layer equations for the Prandtl number range appropriate to liquid metals (0.003 to 0.03). Consideration is given to both forced-convection and free-convection boundary layers. For forced convection, solutions are obtained for flow over a flat plate for both the uniform-wall-temperature and uniform-heat-flux cases. The free-convection solutions are for the isothermal vertical plate.

Previous investigations of the low Prandtl number heat-transfer characteristics of the forced-convection boundary layer have been carried out with approximate analytical techniques. The method of reference 1 is based on the fact that the velocity boundary layer is much thinner than the thermal boundary layer when the Prandtl number is small. Thus, in the outer part of the thermal boundary layer, the velocity was taken from the potential-flow solution; and only in the inner part was an approximate correction made for the nonuniformity of the velocity distribution. Another approximate solution is given in reference 2, where the well-known Kármán-Pohlhausen procedure is used. The results of references 1 and 2 will be compared with those from the exact solutions obtained herein.

For the free-convection boundary layer on an isothermal vertical plate, isolated numerical solutions for the low Prandtl number range have been reported in reference 3 ($Pr = 0.01$) and reference 4 ($Pr = 0.03$). The Kármán-Pohlhausen approximation method has been applied to the problem in references 5 and 6. A somewhat different approximation procedure is used in reference 7, where polynomials are also used but the coefficients of the polynomial are found by satisfying the boundary-layer equations at selected points. Heat-transfer results are given for $Pr = 0.03$. An experiment utilizing liquid mercury as working fluid ($Pr = 0.025$) is also reported in reference 7. Again, comparisons will be made between the previous work and the new exact solutions.

An abbreviated presentation of some of the heat-transfer results corresponding to the new exact solutions* has been made in reference 8 (forced convection) and reference 9 (free convection). Other aspects of these solutions could not be given there because of space limitations. In the present report, the complete details are presented, including among other information the temperature and velocity distributions and full tabulation of the solutions. These tabulations and curves should prove useful to future investigators of low Prandtl number boundary layers; for example, as a source of information from which special characteristics of the boundary layer may be computed (e.g., thickness parameters), or as input data for the solution of related problems, or as a standard against which to compare experimentally determined temperature and velocity profiles. In addition, the present report brings together the existing work on low Prandtl number boundary layers and attempts to provide an integrated picture of what is currently known. Finally, the forced-convection solutions for $Pr = 0.003$, not available at the time reference 8 was published, are also given here. The forced-convection and free-convection boundary layers are treated separately.

*After a preliminary printing of this MEMORANDUM, the authors found that heat-transfer results corresponding to exact solutions for the forced-convection, uniform-wall-temperature case had also been published in reference 13.

SYMBOLS

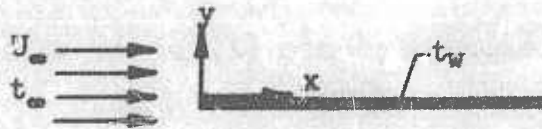
c_p	specific heat at constant pressure
c_f	friction factor, $2\tau/\rho U_\infty^2$
c_f^*	modified friction factor for free convection, $\tau/\rho\left(\frac{v}{x}\right)^2$
F	Blasius velocity function for forced convection, eq. (6b)
f	velocity function for free convection, eq. (26b)
Gr	Grashof number
Gr_L	Grashof number, $g\beta t_w - t_\infty L^3/\nu^2$
Gr_x	Grashof number, $g\beta t_w - t_\infty x^3/\nu^2$
g	acceleration due to gravity
h	local heat-transfer coefficient, $q/(t_w - t_\infty)$
\bar{h}	average heat-transfer coefficient, $Q/L(t_w - t_\infty)$
k	thermal conductivity
L	plate length
Nu	Nusselt number
Nu_L	average Nusselt number, $\bar{h}L/k$
Nu_x	local Nusselt number, hx/k
Pr	Prandtl number, $c_p\mu/k$
p	static pressure
Q	over-all heat-transfer rate, $\int_0^L q dx$
q	local heat-transfer rate per unit area
Re	Reynolds number

Re_L	Reynolds number, $U_\infty L/\nu$
Re_x	Reynolds number, $U_\infty x/\nu$
St	Stanton number, $Nu/RePr$
t	static temperature
t_w	wall temperature
t_∞	ambient temperature
U_∞	free-stream velocity
u	velocity component in x-direction
v	velocity component in y-direction
x	coordinate measuring distance along plate from leading edge
y	coordinate measuring distance normal to plate
α	thermal diffusivity, $k/\rho c_p$
β	coefficient of thermal expansion, $-\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_p$
ζ	free-convection similarity variable, eq. (26a)
η	forced-convection similarity variable, eq. (6a)
θ	dimensionless temperature, $(t - t_\infty)/(t_w - t_\infty)$
μ	absolute viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress at plate surface
ψ	stream function

FORCED-CONVECTION BOUNDARY-LAYER SOLUTIONS

Brief Review of Theory

First, attention is focused on the flow and heat transfer about a flat plate aligned parallel to a uniform free stream, as pictured in the following sketch:



(a)

The problem is governed by the basic conservation laws: mass, momentum, and energy. The boundary-layer form of these equations for laminar, constant-property, nondissipative flow over a flat plate is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2} \quad (3)$$

The viscous dissipation term has been deleted from the energy equation (3) because of its negligible effect on the heat transfer to low Prandtl number fluids (ref. 10).

As a consequence of the constant-property assumption, the velocity problem for the forced-convection flow can be solved without recourse to the temperature. Turning first to the velocity, the statement of the problem is completed by giving the boundary conditions

$$\left. \begin{array}{l} u = 0 \\ v = 0 \end{array} \right\} y = 0 \quad \left. \begin{array}{l} u \rightarrow U_\infty \\ \end{array} \right\} y \rightarrow \infty \quad (4)$$

The conditions that $u = v = 0$ at the wall ($y = 0$) arise respectively from the requirements of no slip and impermeability of the wall to mass. The mathematical problem represented by equations (1) and (2) with the boundary conditions (4) was first solved by Blasius in 1908. Equation (1) is immediately satisfied by the usual stream function ψ :

$$u = \frac{\partial \psi}{\partial y}, \quad v = - \frac{\partial \psi}{\partial x} \quad (5)$$

For equation (2), Blasius introduced his famous similarity variable η :

$$\eta = \frac{y}{2x} \sqrt{\frac{U_{\infty} x}{\nu}} \quad (6a)$$

and a dimensionless stream function F , given by

$$F = \psi / \sqrt{\nu U_{\infty} x} \quad (6b)$$

This transformation reduces equation (2) to the well-known Blasius equation,

$$F''' + FF'' = 0, \quad F(0) = F'(0) = 0, \quad F' \rightarrow 2 \text{ as } \eta \rightarrow \infty \quad (7)$$

where the boundary conditions have been evaluated from (4), and the primes denote differentiation with respect to η . Although a solution of equation (7) was given by Blasius, it was necessary to re-solve to greater accuracy for the purposes of this investigation.

For the isothermal wall, the energy equation (3) was first solved by E. Pohlhausen. Using Blasius' similarity assumption (6a) together with a dimensionless temperature

$$\theta(\eta) = \frac{t - t_{\infty}}{t_w - t_{\infty}} \quad (8)$$

he reduced equation (3) to the form

$$\theta'' + (\text{Pr})F\theta' = 0 \quad (9a)$$

where Pr represents the Prandtl number. The physical boundary conditions that $t = t_w = \text{const}$ at $y = 0$ and $t \rightarrow t_{\infty}$ as $y \rightarrow \infty$ are transformed to

$$\theta(0) = 1, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (9b)$$

The solution of the transformed energy equation (9a) depends upon the prior specification of the Prandtl number. Previous investigators have restricted themselves to $\text{Pr} \geq 0.6$.

For the situation of uniform wall heat flux, the surface temperature will not be constant but, as will be shown later, will vary along the plate according to the law

$$t_w - t_{\infty} = Ax^{1/2} \quad (10)$$

where A is a constant to be determined from the analysis. Taking cognizance of this variation in t_w , the energy equation (3) can be reduced to an ordinary differential equation by using the same transformation variables η , F , and θ as before. The result of the transformation is

$$\theta'' + \text{Pr}(F\theta' - F'\theta) = 0 \quad (11a)$$

The physical boundary conditions that $t = t_w$ at $y = 0$ and $t \rightarrow t_\infty$ as $y \rightarrow \infty$ become

$$\theta(0) = 1, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (11b)$$

Previous solutions of equation (11a) have been confined to the range $\text{Pr} \geq 0.7$.

Solutions and Results

Solutions. - The governing equations (9) and (11) for the uniform-wall-temperature and uniform-heat-flux problems have been solved numerically for Prandtl numbers of 0.03, 0.01, 0.006, and 0.003 utilizing an IBM 650 electronic computer. The numerical technique, described in detail in reference 11, is a forward integration procedure that requires both the function and its derivative to be specified at the starting point of the calculation for a second-order equation. In terms of the present problem, it is necessary that the pair $(\theta(0), \theta'(0))$ be given. As is seen from the boundary conditions (9b) or (11b), the derivative $\theta'(0)$ is not known. Therefore, the computational problem reduces to a systematic search for the appropriate values of the derivative that lead to solutions of equations (9) and (11) satisfying the condition $\theta \rightarrow 0$ as $\eta \rightarrow \infty$. In this way, the two-point boundary-value problem is rephrased as an initial-value problem.

The values of $\theta'(0)$ that correspond to solutions of equations (9) and (11) are listed in table I:

TABLE I. - FORCED-CONVECTION

TEMPERATURE DERIVATIVES

Pr	$[-\theta'(0)]_{\text{UWT}}$	$[-\theta'(0)]_{\text{UHF}}$
0.03	0.16878	0.24838
.01	.10318	.15512
.006	.081449	.12345
.003	.058742	.089850

In addition to their computational importance, these magnitudes are directly related to the heat-transfer characteristics of the flow (as will be shown later) and are therefore of immediate practical interest. In the table, the subscripts UWT and UHF are used to denote uniform wall temperature and uniform heat flux.

The low Prandtl number solutions of equations (9) and (11) are presented in detail in tables II and III (see pp. 23 to 30). For each of the eight cases considered, the dimensionless temperature θ and its derivative θ' are tabulated¹ as a function of the independent variable η . These tabulations should prove useful to future investigators of low Prandtl number boundary layers.

Temperature and velocity profiles. - Some insight into the thermal and flow fields may be obtained by inspection of the temperature and velocity distributions across the boundary layer. These are plotted in figure 1(a) for the case of uniform wall temperature and in figure 1(b) for the uniform-heat-flux case. From both of these figures, two important characteristics are immediately evident: First, that the thermal boundary layer is significantly thicker than the velocity boundary layer;² and secondly, that this disparity in thickness increases with decreasing Prandtl number. This suggests that the velocity boundary layer will play an ever-diminishing role in the heat-transfer process as the Prandtl number becomes smaller. Thus, for fluids with very small Prandtl numbers, the heat transfer will be essentially the same as that convected by an inviscid fluid. The solution for the heat transfer to an inviscid flow (ref. 1) thus appears as a limiting case.

Comparison of figures 1(a) and (b) indicates that, for a fixed Prandtl number, the thermal boundary layer corresponding to uniform wall temperature is somewhat thicker than that for uniform heat flux.³ It would thus be expected that, at a given Prandtl number, the uniform-wall-temperature situation will be closer to its inviscid limit than the uniform-heat-flux case will be to its limit.

¹The tables represent a condensation of the actual machine computations, which were run at a step size $\Delta\eta$ of 0.025 and eight figures.

²This is in contrast to gases or ordinary liquids, where the thermal-boundary-layer thickness is either the same order as or less than the velocity-boundary-layer thickness.

³It may be interesting to note that, for the wall-temperature variation $t_w - t_\infty = Ax^n$, of which equation (10) is a special case, the thermal boundary layer is thicker as n decreases.

Heat-transfer results. - The local rate of heat transfer from the surface to the fluid may be calculated using Fourier's law:

$$q = -k \left(\frac{\partial t}{\partial y} \right)_{y=0} \quad (12)$$

In terms of the variables of the analysis as given by equations (6a) and (8), the expression for q becomes

$$q = -\frac{k}{2} (t_w - t_\infty) \sqrt{\frac{U_\infty}{\nu x}} \theta'(0) \quad (12a)$$

where $\theta'(0)$ is a function of Prandtl number found from solutions of equations (9) and (11) and listed in table I. Clearly, for q to be independent of x , the temperature difference $t_w - t_\infty$ must vary as $x^{1/2}$, as prescribed by equation (10).

It is customary to phrase the local heat-transfer results in terms of a heat-transfer coefficient and a Nusselt number defined as follows:

$$h = \frac{q}{t_w - t_\infty}, \quad Nu_x = \frac{hx}{k} \quad (13)$$

Utilizing these definitions, equation (12a) becomes

$$\frac{Nu_x}{Re_x^{1/2}} = \frac{-\theta'(0)}{2} \quad (14)$$

where Re_x represents the Reynolds number. From equation (14), it is seen that the values of $\theta'(0)$ appearing in table I are directly applicable to the Nusselt-Reynolds relation.

For low Prandtl numbers, it is fruitful to rephrase equation (14) as⁴

$$\frac{Nu_x}{(Re_x Pr)^{1/2}} = \frac{-\theta'(0)}{2 Pr^{1/2}} \quad (14a)$$

⁴This step is suggested by the fact that $Nu/(Re_x Pr)^{1/2}$ is a constant for the inviscid flow.

where the product $Re_x Pr$ is sometimes referred to as the Peclet number. The dimensionless heat-transfer results in the form given by equation (14a) are listed in table IV:

TABLE IV. - FORCED-CONVECTION
HEAT-TRANSFER RESULTS

Pr	$Nu_x / (Re_x Pr)^{1/2}$	
	UWT	UHF
0.03	0.4872	0.7170
.01	.5159	.7756
.006	.5257	.7969
.003	.5362	.8202
Pr \rightarrow 0 Inviscid	0.564	0.886

There are also included in the table entries corresponding to the limiting case of inviscid flow over a flat plate, for which $Nu_x / (Re_x Pr)^{1/2}$ is a constant (ref. 1).

From table IV it is immediately seen that the variation of the group $Nu_x / (Re_x Pr)^{1/2}$ is rather small over the entire liquid-metal range, being of the order of 10 percent. A more careful inspection of the table reveals that the variation is somewhat greater among the uniform-heat-flux results than it is among the uniform-wall-temperature results. This occurrence may be understood by recalling that the thermal boundary layer is thinner for the uniform-heat-flux problem and hence is more aware of the presence of the velocity boundary layer. The heat-transfer results based on the boundary-layer solutions smoothly approach that of the inviscid flow, the uniform-wall-temperature situation always being a little closer to its limiting value than the uniform-heat-flux case is to its limit. The heat-transfer results appearing in table IV are also plotted in figure 2.

The utility of the Reynolds-Prandtl product as a correlation parameter for low Prandtl number, laminar-boundary-layer heat transfer is noteworthy; especially since it has also served successfully in correlating turbulent-heat-transfer results for liquid-metal flow in tubes.

In the case of uniform wall temperature, it is often useful to know the over-all heat transfer Q from the entire surface. For a unit width of plate, Q is found from

$$Q = \int_0^L q \, dx \quad (15)$$

The integration may be carried out utilizing the local heat transfer q as given by equation (12a). The final result may be cast in a dimensionless form by defining an average heat-transfer coefficient and Nusselt number,

$$\bar{h} = \frac{Q}{L(t_w - t_\infty)}, \quad \overline{Nu}_L = \frac{\bar{h}L}{k} \quad (16)$$

from which it follows that

$$\frac{\overline{Nu}_L}{Re_L^{1/2}} = -\theta'(0) \quad (17a)$$

or

$$\frac{\overline{Nu}_L}{(Re_L Pr)^{1/2}} = \frac{-\theta'(0)}{Pr^{1/2}} \quad (17b)$$

The numerical values of the dimensionless heat-transfer group given by equation (17b) are simply twice those listed in table IV.

Comparison with previous investigations. - As has been noted in the INTRODUCTION, approximate solutions for the low Prandtl number, forced-convection boundary layer have been given in references 1 and 2. The heat-transfer results corresponding to these solutions may be expressed as follows:

(a) Morgan's velocity approximation (ref. 1):

$$\left[\frac{Nu_x}{(Re_x Pr)^{1/2}} \right]_{UWT} = 0.564 - 0.547 Pr^{1/2} \quad (18a)$$

$$\left[\frac{Nu_x}{(Re_x Pr)^{1/2}} \right]_{UHF} = 0.886 - 0.491 Pr^{1/2} \quad (18b)$$

(b) Kármán-Pohlhausen method (ref. 2):

$$\left[\frac{Nu_x}{(Re_x Pr)^{1/2}} \right]_{UWT} = \frac{0.529}{(1 + 0.82 Pr^{1/2})} \quad (19a)$$

$$\left[\frac{Nu_x}{(Re_x Pr)^{1/2}} \right]_{UHF} = \frac{0.816}{(1 + 1.064 Pr^{1/2})} \quad (19b)$$

To facilitate comparison with the exact boundary-layer solutions, these equations have been plotted in figure 2.

Turning first to the uniform-wall-temperature situation as shown in figure 2(a), it is seen that Morgan's results tend to approach the exact solution more and more closely as the Prandtl number decreases. This behavior is to be expected from the structure of Morgan's solution. The greatest deviation, at $Pr = 0.03$, is only 4 percent. The Kármán-Pohlhausen results fall about 5 percent below the exact solution over the entire range.

Now, passing to the uniform-heat-flux case (fig. 2(b)), it may be noted that, while Morgan's results still tend to approach the exact solution with decreasing Prandtl number, the deviations are larger and of different sign than those of figure 2(a). Because of the thinner thermal boundary associated with the uniform-heat-flux problem, this somewhat less successful performance of Morgan's method is not surprising. The results from the Kármán-Pohlhausen method continue to fall about 5 percent below those of the exact solution.

Modified Reynolds analogy. - It is of interest to determine the form of Reynolds analogy appropriate to low Prandtl number, forced-convection boundary-layer flows. It is to be recalled that Reynolds analogy compares the friction and heat-transfer characteristics of the flow.

As a prelude, it may be recalled that the friction factor c_f for flow over a flat plate is given by

$$c_f = \frac{\tau}{(\rho U_\infty^2 / 2)} = \frac{0.664}{Re_x^{1/2}} \quad (20)$$

Next, the Stanton number is introduced by its definition

$$St = \frac{Nu_x}{Re_x Pr} \quad (21)$$

Then, taking the ratio of (21) to (20), there is obtained

$$\frac{St}{c_f} = \frac{1}{0.664 Pr^{1/2}} \left[\frac{Nu_x}{(Re_x Pr)^{1/2}} \right] \quad (22)$$

As has been pointed out in connection with table IV, the bracketed factor varies only moderately over the liquid-metal range; therefore, to achieve a concise result here, an average value will be used. With this approximation, equation (22) becomes

$$\left(\frac{St}{c_f} \right)_{UWT} = \frac{0.770}{Pr^{1/2}} \quad (23a)$$

$$\left(\frac{St}{c_f} \right)_{UHF} = \frac{1.15}{Pr^{1/2}} \quad (23b)$$

It is important to observe that these relations deviate from the form of Reynolds analogy used for ordinary fluids, the difference being in the appearance of $Pr^{1/2}$ rather than in the more customary $Pr^{2/3}$.

FREE-CONVECTION BOUNDARY-LAYER SOLUTIONS

Brief Review of Theory

Now, attention is turned to the free-convection flow and heat transfer about an isothermal vertical plate. Two physical situations that come within the scope of the theory are shown in the following sketches:



Diagram (b) depicts the case in which the wall temperature exceeds ambient. For this situation, the buoyancy forces are upward, resulting in an upflow of fluid in the boundary layer. In diagram (c), the wall is cooler than ambient and the boundary-layer flow is downward. If the coordinates are taken as shown in the sketch, there will be no need to make any particular distinction between these two situations.

The free-convection flow and heat transfer are governed by the basic conservation principles. The boundary-layer form of these laws as given by equations (1) to (3) still applies, except that a buoyancy force

$$\pm g\beta(t - t_{\infty}) \quad (24)$$

is added to the right side of the momentum equation (2). The plus sign is associated with sketch (b), while the minus sign is used with sketch (c). The appearance of a temperature term in the velocity equation means that it is no longer possible to solve for the velocity independently of the temperature; instead, simultaneous solution is necessary. In this regard, the free-convection problem becomes more complex than the forced-convection problem.

In addition to the governing equations, it is necessary to give the boundary conditions in order to complete the statement of the problem. They are

$$\left. \begin{array}{l} u = 0 \\ v = 0 \\ t = t_w \end{array} \right\} y = 0 \quad \left. \begin{array}{l} u \rightarrow 0 \\ t \rightarrow t_{\infty} \end{array} \right\} y \rightarrow \infty \quad (25)$$

where t_w is a constant

The free-convection boundary layer on an isothermal vertical plate was first solved by Schmidt and Beckmann. The conservation of mass equation (1) was satisfied by the usual stream function ψ as given by equation (5). Then, turning to momentum and energy conservation, new independent and dependent variables were introduced as follows:

$$\zeta = \frac{y}{x} \left(\frac{g\beta |t_w - t_{\infty}| x^3}{4\nu^2} \right)^{1/4} \quad (26a)$$

$$f(\zeta) = \frac{\psi}{(64g\beta |t_w - t_{\infty}| x^3)^{1/4}}, \quad \theta(\zeta) = \frac{t - t_{\infty}}{t_w - t_{\infty}} \quad (26b)$$

where ζ is called a similarity variable, while θ is a dimensionless temperature and f is related to the velocities of the problem. The absolute magnitude signs have been introduced to make the results applicable to both $t_w > t_\infty$ and $t_w < t_\infty$. Under the transformation defined by equations (26a) and (26b), conservation of momentum and energy is reduced to the following pair of ordinary differential equations:

$$f''' + 3ff'' - 2(f')^2 + \theta = 0 \quad (27a)$$

$$\theta'' + 3(\text{Pr})f\theta' = 0 \quad (27b)$$

while the boundary conditions (25) become

$$\left. \begin{array}{l} f = 0 \\ f' = 0 \\ \theta = 1 \end{array} \right\} \zeta = 0 \quad \left. \begin{array}{l} f' \rightarrow 0 \\ \theta \rightarrow 0 \end{array} \right\} \zeta \rightarrow \infty \quad (27c)$$

The primes denote differentiation with respect to ζ , and Pr represents the Prandtl number. Since f and θ appear in both equations, simultaneous solution is required.

Previous investigators have concentrated mainly on the range $\text{Pr} \geq 0.7$; the existing solutions for low Prandtl number have already been mentioned in the INTRODUCTION.

Solutions and Results

Solutions. - Numerical solutions of equations (27a) and (27b) have been carried out for Prandtl numbers of 0.03, 0.02, 0.008, and 0.003 on an IBM 650 digital computer. The numerical scheme previously described for the forced-convection problem must now be modified to include simultaneous equations. Instead of looking for a single quantity $\theta'(0)$ as before, a pair of quantities $(\theta'(0), f''(0))$ must now be found that leads to solutions of equations (27a) and (27b) satisfying the conditions $\theta \rightarrow 0$ and $f' \rightarrow 0$ as $\zeta \rightarrow \infty$.

The values of $\theta'(0)$ and $f''(0)$ for which solutions were obtained are listed in table V:

TABLE V. - FREE-CONVECTION TEMPERATURE
AND VELOCITY DERIVATIVES

Pr	$-\theta'(0)$	$f''(0)$
0.03	0.13464	0.93841
.02	.11164	.95896
.008	.072464	.99550
.003	.045139	1.0223

These quantities are not only of importance in the execution of any forward integration procedure, but they are also related to the heat-transfer and shear-stress characteristics of the flow. Hence, they are of direct practical utility.

A detailed listing of the solutions of equations (27a) and (27b) is given in table VI (see pp. 31 to 37). For each of the four Prandtl numbers considered, the dependent variables θ , θ' , f , f' , and f'' are tabulated as functions of the independent variable ζ . These listings should provide useful information in future studies of low Prandtl number boundary layers.

Temperature and velocity profiles. - The distribution of temperature and velocity across the boundary layer is plotted in figures 3(a) to (d), each graph corresponding to a specific Prandtl number. The velocity profiles have their characteristic free-convection shape, rising rapidly to a maximum near the wall and then subsiding relatively slowly to zero with increasing values of ζ . All velocity profiles contain an inflection point just beyond the maximum. The temperature profiles have their usual simple shape, always concave upward.

Two features of this set of graphs are worth noting. The first is that, with decreasing Prandtl number, the region of high velocity gradients occupies a relatively smaller and smaller portion of the thermal boundary layer. This suggests that, as the Prandtl number approaches zero, the effects of viscosity on the heat transfer will steadily diminish and become negligible. In the limit, the heat transfer would be expected to approach that of an inviscid fluid. The second has to do with the apparent increase of the boundary-layer thickness with decreasing Prandtl number. That this trend may not be real is easily realized by observing that fluid properties appear in the abscissa variable ζ . With changing Prandtl number, these fluid properties will change, tending to affect the actual physical dimensions of the boundary layer.

Heat-transfer results. - For free convection, the heat transfer is the quantity of prime practical interest. The local heat-transfer

rate q is again computed from Fourier's law (eq. (12)). Introducing the dimensionless variables of equations (26a) and (26b), the expression for q becomes

$$q = -k(t_w - t_\infty) \left(\frac{g\beta|t_w - t_\infty|}{4\nu^2} \right)^{1/4} \theta'(0) \quad (28)$$

A rephrasing of equation (28) in terms of dimensionless variables leads to

$$\frac{Nu_x}{Gr_x^{1/4}} = \frac{-\theta'(0)}{\sqrt{2}} \quad (29)$$

where Nu_x is the Nusselt number as previously defined and Gr_x is the Grashof number. Since the values of $-\theta'(0)$ depend upon the Prandtl number, so will the Nusselt-Grashof relation.

The Prandtl number dependence of the dimensionless heat-transfer results may be considerably reduced by rewriting equation (29) as

$$\frac{Nu_x}{(Gr_x Pr^2)^{1/4}} = \frac{-\theta'(0)}{(2Pr)^{3/2}} \quad (30)$$

Such a step is suggested by the fact that $Nu_x/(Gr_x Pr^2)^{1/4}$ is a constant for the inviscid free-convection flow. Numerical values of this heat-transfer parameter for low Prandtl number boundary-layer flows are given in table VII, along with the limiting inviscid result (ref. 12):

TABLE VII. - FREE-CONVECTION

HEAT-TRANSFER RESULTS

Pr	$\frac{Nu_x}{(Gr_x Pr^2)^{1/4}}$
0.03	0.5497
.02	.5582
.008	.5729
.003	.5827
Pr $\rightarrow \infty$ } Inviscid }	0.6004

Inspection of this table shows that the group $Nu_x/(Gr_x Pr^2)^{1/4}$ possesses the desired characteristic of being almost independent of the Prandtl number, the variation over the entire liquid-metal range being about 6 percent. It is also seen that the boundary-layer heat-transfer results smoothly approach the inviscid-flow result as the Prandtl number decreases. The information appearing in table VII is also plotted in figure 4.

For engineering purposes, a simple and very adequate representation of these results is

$$Nu_x = 0.565(Gr_x Pr^2)^{1/4} \quad (31)$$

The maximum deviation of this expression from the entries of table VII is 3 percent.

Aside from the local values, the over-all heat transfer Q from the entire surface may be of interest. An expression for the over-all heat transfer may be found by integrating equation (28) in accordance with (15). The result of integration may be put into the following dimensionless forms:

$$\frac{Nu_L}{(Gr_L)^{1/4}} = \left(\frac{4}{3}\right) \frac{-\theta'(0)}{\sqrt{2}} \quad (32a)$$

or

$$\frac{Nu_x}{(Gr_L Pr^2)^{1/4}} = \left(\frac{4}{3}\right) \frac{-\theta'(0)}{(2Pr)^{1/2}} \quad (32b)$$

The right side of equation (32b) may be immediately evaluated by multiplying the entries of table VII by $4/3$.

Friction-factor results. - The friction force exerted on the wall by the free-convection flow may be computed by the Newtonian shear law,

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (33)$$

In terms of the variables of the analysis, equation (33) may be evaluated as

$$\tau = 4\mu\nu x^{1/4} \left(\frac{g\beta |t_w - t_{m1}|}{4\nu^2} \right)^{3/4} f''(0) \quad (33a)$$

In constructing a dimensionless form of equation (33a), the usual friction factor cannot be used because there is no characteristic velocity in a free-convection flow. An alternative friction factor that is sometimes used in free convection is as follows:

$$c_f^* \equiv \frac{\tau}{\rho \left(\frac{v}{x}\right)^2} \quad (34)$$

where (v/x) plays the role of a velocity. With this, equation (33a) can be rephrased as

$$\frac{c_f^*}{(4Gr_x^3)^{1/4}} = f''(0) \quad (35)$$

The numerical values of $f''(0)$, given in table V, vary only by 8 percent over the entire Prandtl number range of liquid metals. For engineering purposes a satisfactory representation of the shear-stress results would be

$$c_f^* = 0.98(4Gr_x^3)^{1/4} \quad (35a)$$

Comparison with previous investigations. - Studies of the low Prandtl number free-convection boundary layer that were performed before the present investigation are described in the INTRODUCTION. The heat-transfer results as reported by previous analytical workers are summarized in table VIII:

TABLE VIII. - SUMMARY OF PREVIOUS ANALYTICAL
HEAT-TRANSFER RESULTS FOR FREE CONVECTION

Pr	$Nu_x / (Gr_x Pr^2)^{1/4}$	Reference investigation
0.03	0.544	7
.03	.555	4
.01	.574	3
All	$0.508 / (Pr + 0.952)^{1/4}$	5 and 6

To facilitate comparison with the present results, the contents of this table are plotted in figure 4. The experimental data of reference 7 also appear in the figure.

In common with the current study, references 3 and 4 carried out numerical solutions of the boundary-layer equations. As seen from figure 4, their heat-transfer results fall slightly high relative to those of this investigation, the deviation being no more than 1 percent. Since the older work was performed on desk calculators and slower computers, such deviations are not at all unreasonable.

The approximation procedure of reference 7 also gives very good agreement with the present solution, falling only about 1 percent below at the point of comparison, $Pr = 0.03$. The result reported as corresponding to reference 7 is the average of three levels of approximation, the maximum spread among the three approximations being 16 percent.

The results based on the Kármán-Pohlhausen method lie from 7 to 12 percent below the exact solution. This agreement must be regarded as remarkably good when one considers the relatively broad assumptions used in carrying through the Kármán-Pohlhausen procedure for this problem.

The experimental data of reference 7 for mercury fall within the crosshatched band as shown in figure 4, the deviations from theory being confined to ± 6 percent. This very good agreement may be interpreted as a strong support of the analytical predictions.

CONCLUDING REMARKS

Although laminar-boundary-layer theory can supply information on heat-transfer and skin-friction characteristics, it cannot predict the region of applicability of these results. For sufficiently high Reynolds or Grashof numbers, the flow will become turbulent. On the other hand, for sufficiently low Reynolds or Grashof numbers, the boundary layer is relatively thick, and certain assumptions of the theory are no longer valid. It remains for experiment to define the limits of applicability of the theory.

For low Prandtl number forced-convection flows, transition to turbulence should occur in the same Reynolds number range (5×10^4 to 10^6) as for high Prandtl number fluids. On the other hand, in the absence of experiments involving liquid metals, it cannot be stated which Reynolds numbers are sufficiently low to invalidate the boundary-layer assumptions as a consequence of a too thick thermal boundary layer. All that can be stated is that the (thermal) boundary-layer assumptions will not remain valid to as low Reynolds numbers for low Prandtl number flows as they do for high Prandtl number fluids.

For free-convection flows, it is rather uncertain that information on laminar-turbulent transition for high Prandtl number fluids can be applied to low Prandtl number fluids. Therefore, at present, the extent

of the laminar regime for low Prandtl number fluids must be regarded as unknown. Further, a state of uncertainty exists as to when the boundary-layer assumptions become invalid because of a too thick boundary layer. For gases, it has been established that boundary-layer heat-transfer predictions are correct when $Gr_x > 5 \times 10^4$. For liquid metals, it may be conjectured that this limit will be at a higher Grashof number, but it is not known how much higher.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, December 9, 1958

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TABLE II. - FORCED-CONVECTION SOLUTIONS FOR UNIFORM-WALL-TEMPERATURE CASE

[Governing differential equation:

$$\theta'' + (\text{Pr})F\theta' = 0, \theta(0) = 1, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

where F is the Blasius function.](a) $\text{Pr} = 0.03$

η	θ'	θ	η	θ'	θ
0.00	-0.1688	1.0000	4.60	-0.1098	0.3073
.10	-.1688	.9831	4.80	-.1048	.2859
.20	-.1688	.9662	5.00	-.0999	.2654
.30	-.1688	.9494	5.20	-.0949	.2459
.40	-.1687	.9325	5.40	-.0900	.2274
.50	-.1686	.9156	5.60	-.0851	.2099
.60	-.1685	.8988	5.80	-.0803	.1934
.70	-.1684	.8819	6.00	-.0756	.1778
.80	-.1682	.8651	6.20	-.0710	.1631
.90	-.1680	.8483	6.40	-.0665	.1494
1.00	-.1677	.8315	6.60	-.0622	.1365
1.10	-.1673	.8147	6.80	-.0580	.1245
1.20	-.1669	.7980	7.00	-.0539	.1133
1.30	-.1664	.7814	7.20	-.0500	.1029
1.40	-.1658	.7648	7.40	-.0463	.0933
1.50	-.1652	.7482	7.60	-.0428	.0844
1.60	-.1644	.7317	7.80	-.0394	.0762
1.70	-.1636	.7153	8.00	-.0362	.0686
1.80	-.1627	.6990	8.20	-.0332	.0617
1.90	-.1617	.6828	8.40	-.0303	.0554
2.00	-.1607	.6667	8.60	-.0277	.0496
2.10	-.1595	.6507	8.80	-.0252	.0443
2.20	-.1583	.6348	9.00	-.0229	.0395
2.30	-.1570	.6190	9.20	-.0207	.0351
2.40	-.1556	.6034	9.40	-.0187	.0312
2.50	-.1541	.5879	9.60	-.0169	.0276
2.60	-.1525	.5726	9.80	-.0152	.0244
2.70	-.1509	.5574	10.00	-.0136	.0215
2.80	-.1492	.5424	10.50	-.0103	.0156
2.90	-.1474	.5276	11.00	-.0076	.0111
3.00	-.1456	.5129	11.50	-.0056	.0078
3.10	-.1437	.4984	12.00	-.0040	.0054
3.20	-.1417	.4842	12.50	-.0029	.0037
3.30	-.1397	.4701	13.00	-.0020	.0025
3.40	-.1376	.4562	13.50	-.0014	.0017
3.50	-.1355	.4426	14.00	-.0009	.0011
3.60	-.1333	.4291	14.50	-.0006	.0007
3.70	-.1311	.4159	15.00	-.0004	.0005
3.80	-.1289	.4029	15.50	-.0003	.0003
3.90	-.1266	.3901	16.00	-.0002	.0002
4.00	-.1243	.3776	16.50	-.0001	.0001
4.20	-.1195	.3532	17.00	-.0001	.0001
4.40	-.1147	.3298	17.50	.0000	.0000

TABLE II. - Continued.

(b) $Pr = 0.01$

η	θ'	θ	η	θ'	θ
0.00	-0.1032	1.0000	6.20	-0.0773	0.4102
.10	-.1032	.9897	6.40	-.0756	.3949
.20	-.1032	.9794	6.60	-.0740	.3799
.30	-.1032	.9690	6.80	-.0723	.3653
.40	-.1032	.9587	7.00	-.0705	.3510
.50	-.1031	.9484	7.20	-.0688	.3371
.60	-.1031	.9381	7.40	-.0670	.3235
.70	-.1031	.9278	7.60	-.0653	.3103
.80	-.1031	.9175	7.80	-.0635	.2974
.90	-.1030	.9072	8.00	-.0618	.2849
1.00	-.1030	.8969	8.20	-.0600	.2727
1.10	-.1029	.8866	8.40	-.0582	.2609
1.20	-.1028	.8763	8.60	-.0565	.2494
1.30	-.1027	.8660	8.80	-.0547	.2383
1.40	-.1026	.8558	9.00	-.0530	.2275
1.50	-.1024	.8455	9.20	-.0513	.2171
1.60	-.1023	.8353	9.40	-.0496	.2070
1.70	-.1021	.8251	9.60	-.0479	.1972
1.80	-.1019	.8149	9.80	-.0462	.1878
1.90	-.1017	.8047	10.00	-.0446	.1787
2.00	-.1015	.7945	10.50	-.0406	.1575
2.10	-.1013	.7844	11.00	-.0368	.1381
2.20	-.1010	.7743	11.50	-.0331	.1206
2.30	-.1007	.7642	12.00	-.0297	.1049
2.40	-.1004	.7541	12.50	-.0265	.0909
2.50	-.1001	.7441	13.00	-.0236	.0784
2.60	-.0998	.7341	13.50	-.0208	.0673
2.70	-.0994	.7241	14.00	-.0183	.0575
2.80	-.0990	.7142	14.50	-.0160	.0490
2.90	-.0986	.7043	15.00	-.0139	.0415
3.00	-.0982	.6945	15.50	-.0121	.0350
3.10	-.0978	.6847	16.00	-.0104	.0294
3.20	-.0973	.6749	16.50	-.0089	.0246
3.30	-.0969	.6652	17.00	-.0076	.0205
3.40	-.0964	.6556	17.50	-.0065	.0170
3.50	-.0959	.6460	18.00	-.0054	.0140
3.60	-.0954	.6364	18.50	-.0046	.0115
3.70	-.0949	.6269	19.00	-.0038	.0094
3.80	-.0943	.6174	19.50	-.0032	.0076
3.90	-.0937	.6080	20.00	-.0026	.0062
4.00	-.0932	.5987	21.00	-.0018	.0040
4.20	-.0920	.5802	22.00	-.0012	.0025
4.40	-.0907	.5619	23.00	-.0008	.0016
4.60	-.0894	.5439	24.00	-.0005	.0010
4.80	-.0880	.5261	25.00	-.0003	.0006
5.00	-.0866	.5087	26.00	-.0002	.0003
5.20	-.0852	.4915	27.00	-.0001	.0002
5.40	-.0837	.4746	28.00	-.0001	.0001
5.60	-.0821	.4580	29.00	.0000	.0001
5.80	-.0806	.4418	30.00	.0000	.0000
6.00	-.0789	.4258			

TABLE II. - Continued.

(c) Pr = 0.006

η	θ'	θ	η	θ'	θ
0.00	=0.0814	1.0000	7.00	-0.0648	0.4661
.10	--.0814	.9919	7.20	--.0639	.4532
.20	--.0814	.9837	7.40	--.0629	.4405
.30	--.0814	.9756	7.60	--.0619	.4281
.40	--.0814	.9674	7.80	--.0609	.4158
.50	--.0814	.9593	8.00	--.0599	.4037
.60	--.0814	.9511	8.20	--.0588	.3918
.70	--.0814	.9430	8.40	--.0578	.3802
.80	--.0814	.9349	8.60	--.0567	.3687
.90	--.0814	.9267	8.80	--.0557	.3575
1.00	--.0813	.9186	9.00	--.0546	.3465
1.10	--.0813	.9104	9.20	--.0535	.3356
1.20	--.0813	.9023	9.40	--.0525	.3250
1.30	--.0812	.8942	9.60	--.0514	.3147
1.40	--.0812	.8861	9.80	--.0503	.3045
1.50	--.0811	.8780	10.00	--.0492	.2945
1.60	--.0810	.8699	10.50	--.0465	.2706
1.70	--.0809	.8618	11.00	--.0439	.2480
1.80	--.0809	.8537	11.50	--.0412	.2267
1.90	--.0808	.8456	12.00	--.0386	.2068
2.00	--.0806	.8375	12.50	--.0361	.1881
2.10	--.0805	.8295	13.00	--.0336	.1707
2.20	--.0804	.8214	13.50	--.0312	.1545
2.30	--.0803	.8134	14.00	--.0288	.1395
2.40	--.0801	.8054	14.50	--.0266	.1257
2.50	--.0800	.7973	15.00	--.0245	.1129
2.60	--.0798	.7894	15.50	--.0225	.1012
2.70	--.0796	.7814	16.00	--.0205	.0904
2.80	--.0795	.7734	16.50	--.0187	.0806
2.90	--.0793	.7655	17.00	--.0170	.0717
3.00	--.0791	.7576	17.50	--.0154	.0635
3.10	--.0789	.7497	18.00	--.0139	.0562
3.20	--.0787	.7418	18.50	--.0126	.0496
3.30	--.0784	.7339	19.00	--.0113	.0436
3.40	--.0782	.7261	19.50	--.0101	.0383
3.50	--.0779	.7183	20.00	--.0090	.0335
3.60	--.0777	.7105	21.00	--.0071	.0254
3.70	--.0774	.7028	22.00	--.0056	.0191
3.80	--.0772	.6950	23.00	--.0043	.0142
3.90	--.0769	.6873	24.00	--.0033	.0105
4.00	--.0766	.6797	25.00	--.0025	.0076
4.20	--.0760	.6644	26.00	--.0018	.0055
4.40	--.0754	.6493	27.00	--.0013	.0039
4.60	--.0747	.6342	28.00	--.0010	.0027
4.80	--.0741	.6194	29.00	--.0007	.0019
5.00	--.0733	.6046	30.00	--.0005	.0013
5.20	--.0726	.5900	31.00	--.0003	.0009
5.40	--.0718	.5756	32.00	--.0002	.0006
5.60	--.0710	.5613	33.00	--.0002	.0004
5.80	--.0702	.5472	34.00	--.0001	.0003
6.00	--.0694	.5332	35.00	--.0001	.0002
6.20	--.0685	.5194	36.00	.0000	.0001
6.40	--.0676	.5058	37.00	.0000	.0001
6.60	--.0667	.4924	38.00	.0000	.0000
6.80	--.0658	.4791			

TABLE II. - Concluded.

(d) Pr = 0.003

η	θ'	θ	η	θ'	θ
0.00	-.0587	1.0000	8.60	-0.0490	0.5211
.10	-.0587	.9941	8.80	-.0486	.5113
.20	-.0587	.9883	9.00	-.0481	.5017
.30	-.0587	.9824	9.20	-.0476	.4921
.40	-.0587	.9765	9.40	-.0471	.4826
.50	-.0587	.9706	9.60	-.0467	.4732
.60	-.0587	.9648	9.80	-.0462	.4639
.70	-.0587	.9589	10.00	-.0457	.4548
.80	-.0587	.9530	10.50	-.0444	.4322
.90	-.0587	.9471	11.00	-.0431	.4104
1.00	-.0587	.9413	11.50	-.0418	.3891
1.10	-.0587	.9354	12.00	-.0404	.3686
1.20	-.0587	.9295	12.50	-.0391	.3487
1.30	-.0587	.9237	13.00	-.0377	.3295
1.40	-.0586	.9178	13.50	-.0363	.3110
1.50	-.0586	.9119	14.00	-.0350	.2932
1.60	-.0586	.9061	14.50	-.0336	.2760
1.70	-.0586	.9002	15.00	-.0322	.2596
1.80	-.0585	.8944	15.50	-.0309	.2438
1.90	-.0585	.8885	16.00	-.0295	.2287
2.00	-.0585	.8827	16.50	-.0282	.2143
2.10	-.0584	.8768	17.00	-.0269	.2006
2.20	-.0584	.8710	17.50	-.0256	.1875
2.30	-.0583	.8651	18.00	-.0243	.1750
2.40	-.0583	.8593	18.50	-.0231	.1631
2.50	-.0582	.8535	19.00	-.0219	.1519
2.60	-.0581	.8477	19.50	-.0207	.1413
2.70	-.0581	.8419	20.00	-.0196	.1312
2.80	-.0580	.8361	21.00	-.0174	.1128
2.90	-.0580	.8303	22.00	-.0154	.0964
3.00	-.0579	.8245	23.00	-.0135	.0820
3.10	-.0578	.8187	24.00	-.0118	.0694
3.20	-.0577	.8129	25.00	-.0102	.0584
3.30	-.0576	.8071	26.00	-.0088	.0489
3.40	-.0576	.8014	27.00	-.0076	.0407
3.50	-.0575	.7956	28.00	-.0064	.0337
3.60	-.0574	.7899	29.00	-.0055	.0278
3.70	-.0573	.7842	30.00	-.0046	.0228
3.80	-.0572	.7784	31.00	-.0038	.0186
3.90	-.0571	.7727	32.00	-.0032	.0151
4.00	-.0570	.7670	33.00	-.0026	.0121
4.20	-.0567	.7556	34.00	-.0022	.0097
4.40	-.0565	.7443	35.00	-.0018	.0078
4.60	-.0563	.7330	36.00	-.0014	.0062
4.80	-.0560	.7218	37.00	-.0012	.0049
5.00	-.0557	.7106	38.00	-.0009	.0038
5.20	-.0555	.6995	39.00	-.0007	.0030
5.40	-.0552	.6885	40.00	-.0006	.0023
5.60	-.0549	.6775	41.00	-.0005	.0018
5.80	-.0545	.6665	42.00	-.0004	.0014
6.00	-.0542	.6556	43.00	-.0003	.0010
6.20	-.0539	.6448	44.00	-.0002	.0007
6.40	-.0535	.6341	45.00	-.0002	.0006
6.60	-.0532	.6234	46.00	-.0001	.0004
6.80	-.0528	.6126	47.00	-.0001	.0003
7.00	-.0524	.6023	48.00	-.0001	.0002
7.20	-.0520	.5919	49.00	-.0001	.0002
7.40	-.0516	.5815	50.00	.0000	.0001
7.60	-.0512	.5712	51.00	.0000	.0001
7.80	-.0508	.5610	52.00	.0000	.0001
8.00	-.0504	.5509	53.00	.0000	.0001
8.20	-.0499	.5409	54.00	.0000	.0000
8.40	-.0495	.5309			

TABLE III. - FORCED-CONVECTION SOLUTIONS FOR UNIFORM-HEAT-FLUX CASE

[Governing differential equation:

$$\theta'' + \text{Pr}(F\theta' - F'\theta), \theta(0) = 1, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

where F is the Blasius function.](a) $\text{Pr} = 0.03$

η	θ'	θ	η	θ'	θ
0.00	-0.2484	1.0000	4.40	-0.0941	0.2122
.10	-.2482	.9752	4.60	-.0877	.1940
.20	-.2476	.9504	4.80	-.0815	.1771
.30	-.2466	.9257	5.00	-.0757	.1614
.40	-.2453	.9011	5.20	-.0701	.1468
.50	-.2436	.8766	5.40	-.0649	.1333
.60	-.2416	.8524	5.60	-.0599	.1209
.70	-.2392	.8283	5.80	-.0551	.1094
.80	-.2366	.8045	6.00	-.0507	.0988
.90	-.2337	.7810	6.20	-.0465	.0891
1.00	-.2305	.7578	6.40	-.0426	.0802
1.10	-.2271	.7349	6.60	-.0389	.0720
1.20	-.2234	.7124	6.80	-.0355	.0646
1.30	-.2196	.6902	7.00	-.0323	.0578
1.40	-.2156	.6685	7.20	-.0294	.0516
1.50	-.2115	.6471	7.40	-.0266	.0460
1.60	-.2073	.6262	7.60	-.0241	.0410
1.70	-.2030	.6057	7.80	-.0217	.0364
1.80	-.1986	.5856	8.00	-.0196	.0323
1.90	-.1942	.5659	8.20	-.0176	.0286
2.00	-.1898	.5467	8.40	-.0158	.0252
2.10	-.1853	.5280	8.60	-.0141	.0222
2.20	-.1809	.5097	8.80	-.0126	.0196
2.30	-.1764	.4918	9.00	-.0113	.0172
2.40	-.1720	.4744	9.20	-.0100	.0151
2.50	-.1676	.4574	9.40	-.0089	.0132
2.60	-.1633	.4409	9.60	-.0079	.0115
2.70	-.1589	.4248	9.80	-.0070	.0100
2.80	-.1547	.4091	10.00	-.0061	.0087
2.90	-.1504	.3938	10.50	-.0044	.0051
3.00	-.1463	.3790	11.00	-.0032	.0042
3.10	-.1421	.3646	11.50	-.0022	.0029
3.20	-.1381	.3506	12.00	-.0015	.0019
3.30	-.1341	.3370	12.50	-.0011	.0013
3.40	-.1301	.3237	13.00	-.0007	.0008
3.50	-.1262	.3109	13.50	-.0005	.0005
3.60	-.1224	.2985	14.00	-.0003	.0003
3.70	-.1186	.2865	14.50	-.0002	.0002
3.80	-.1149	.2748	15.00	-.0001	.0001
3.90	-.1113	.2635	15.50	-.0001	.0001
4.00	-.1077	.2525	16.00	-.0001	.0000
4.20	-.1007	.2317	16.50	.0000	.0000

TABLE III. - Continued.

(b) $Pr = 0.01$

η	θ'	θ	η	θ'	θ
0.00	-0.1551	1.0000	6.00	-0.0720	0.2975
.10	-.1551	.9845	6.20	-.0694	.2833
.20	-.1549	.9690	6.40	-.0668	.2697
.30	-.1545	.9535	6.60	-.0643	.2566
.40	-.1541	.9381	6.80	-.0618	.2440
.50	-.1535	.9227	7.00	-.0594	.2319
.60	-.1528	.9074	7.20	-.0570	.2203
.70	-.1520	.8922	7.40	-.0547	.2091
.80	-.1511	.8770	7.60	-.0525	.1984
.90	-.1501	.8619	7.80	-.0503	.1881
1.00	-.1489	.8470	8.00	-.0482	.1782
1.10	-.1477	.8322	8.20	-.0461	.1688
1.20	-.1465	.8174	8.40	-.0441	.1598
1.30	-.1451	.8029	8.60	-.0422	.1512
1.40	-.1437	.7884	8.80	-.0403	.1429
1.50	-.1422	.7741	9.00	-.0385	.1350
1.60	-.1406	.7600	9.20	-.0367	.1275
1.70	-.1391	.7460	9.40	-.0350	.1203
1.80	-.1375	.7322	9.60	-.0334	.1135
1.90	-.1358	.7185	9.80	-.0318	.1070
2.00	-.1342	.7050	10.00	-.0302	.1008
2.10	-.1325	.6917	10.50	-.0266	.0866
2.20	-.1309	.6785	11.00	-.0234	.0741
2.30	-.1292	.6655	11.50	-.0204	.0632
2.40	-.1275	.6527	12.00	-.0177	.0537
2.50	-.1258	.6400	12.50	-.0154	.0454
2.60	-.1241	.6275	13.00	-.0133	.0383
2.70	-.1225	.6152	13.50	-.0114	.0321
2.80	-.1208	.6030	14.00	-.0097	.0268
2.90	-.1191	.5910	14.50	-.0083	.0223
3.00	-.1175	.5792	15.00	-.0070	.0185
3.10	-.1158	.5675	15.50	-.0059	.0153
3.20	-.1141	.5560	16.00	-.0050	.0126
3.30	-.1125	.5447	16.50	-.0042	.0103
3.40	-.1109	.5335	17.00	-.0035	.0084
3.50	-.1092	.5225	17.50	-.0029	.0068
3.60	-.1076	.5117	18.00	-.0024	.0055
3.70	-.1060	.5010	18.50	-.0019	.0044
3.80	-.1044	.4905	19.00	-.0016	.0036
3.90	-.1028	.4801	19.50	-.0013	.0028
4.00	-.1012	.4699	20.00	-.0010	.0023
4.20	-.0981	.4500	21.00	-.0007	.0014
4.40	-.0950	.4307	22.00	-.0004	.0009
4.60	-.0920	.4120	23.00	-.0003	.0005
4.80	-.0890	.3939	24.00	-.0002	.0003
5.00	-.0860	.3764	25.00	-.0001	.0002
5.20	-.0831	.3595	26.00	-.0001	.0001
5.40	-.0803	.3431	27.00	.0000	.0001
5.60	-.0775	.3273	28.00	.0000	.0000
5.80	-.0747	.3121			

TABLE III. - Continued.

(c) Pr = 0.006

η	θ'	θ	η	θ'	θ
0.00	-0.1235	1.0000	7.20	-0.0599	0.3258
0.10	-.1234	.9877	7.40	-.0583	.3120
0.20	-.1233	.9753	7.60	-.0566	.3005
0.30	-.1231	.9630	7.80	-.0550	.2893
0.40	-.1228	.9507	8.00	-.0534	.2785
0.50	-.1225	.9384	8.20	-.0518	.2680
0.60	-.1221	.9262	8.40	-.0503	.2577
0.70	-.1216	.9140	8.60	-.0488	.2478
0.80	-.1210	.9019	8.80	-.0473	.2382
0.90	-.1204	.8898	9.00	-.0458	.2289
1.00	-.1197	.8778	9.20	-.0444	.2199
1.10	-.1190	.8659	9.40	-.0430	.2112
1.20	-.1182	.8540	9.60	-.0416	.2027
1.30	-.1173	.8423	9.80	-.0403	.1945
1.40	-.1165	.8306	10.00	-.0389	.1866
1.50	-.1156	.8190	10.50	-.0358	.1679
1.60	-.1146	.8075	11.00	-.0328	.1508
1.70	-.1136	.7960	11.50	-.0300	.1351
1.80	-.1127	.7847	12.00	-.0273	.1208
1.90	-.1116	.7735	12.50	-.0249	.1078
2.00	-.1106	.7624	13.00	-.0226	.0959
2.10	-.1096	.7514	13.50	-.0204	.0852
2.20	-.1086	.7405	14.00	-.0184	.0754
2.30	-.1075	.7297	14.50	-.0166	.0667
2.40	-.1065	.7190	15.00	-.0149	.0588
2.50	-.1054	.7084	15.50	-.0134	.0517
2.60	-.1044	.6979	16.00	-.0120	.0454
2.70	-.1033	.6875	16.50	-.0107	.0398
2.80	-.1023	.6772	17.00	-.0095	.0347
2.90	-.1012	.6671	17.50	-.0084	.0303
3.00	-.1002	.6570	18.00	-.0074	.0263
3.10	-.0991	.6470	18.50	-.0066	.0228
3.20	-.0981	.6372	19.00	-.0058	.0198
3.30	-.0970	.6274	19.50	-.0051	.0170
3.40	-.0960	.6178	20.00	-.0044	.0147
3.50	-.0950	.6082	21.00	-.0034	.0108
3.60	-.0939	.5988	22.00	-.0025	.0079
3.70	-.0929	.5894	23.00	-.0019	.0057
3.80	-.0919	.5802	24.00	-.0014	.0041
3.90	-.0909	.5710	25.00	-.0010	.0029
4.00	-.0899	.5620	26.00	-.0007	.0020
4.20	-.0879	.5442	27.00	-.0005	.0014
4.40	-.0858	.5269	28.00	-.0004	.0010
4.60	-.0839	.5099	29.00	-.0003	.0007
4.80	-.0819	.4933	30.00	-.0002	.0005
5.00	-.0799	.4771	31.00	-.0001	.0003
5.20	-.0780	.4613	32.00	-.0001	.0002
5.40	-.0761	.4459	33.00	-.0001	.0002
5.60	-.0742	.4309	34.00	.0000	.0001
5.80	-.0724	.4162	35.00	.0000	.0001
6.00	-.0705	.4019	36.00	.0000	.0001
6.20	-.0687	.3880	37.00	.0000	.0001
6.40	-.0669	.3745	38.00	.0000	.0001
6.60	-.0651	.3613	39.00	.0000	.0001
6.80	-.0634	.3484	40.00	.0000	.0000
7.00	-.0616	.3359			

TABLE III. - Concluded.

(d) Pr = 0.003

η	θ'	θ	η	θ'	θ
0.00	-0.0898	1.0000	8.20	-0.0511	0.4109
.10	-.0898	.9910	8.40	-.0501	.4008
.20	-.0898	.9820	8.60	-.0492	.3908
.30	-.0897	.9731	8.80	-.0483	.3811
.40	-.0895	.9641	9.00	-.0474	.3715
.50	-.0894	.9552	9.20	-.0465	.3621
.60	-.0891	.9462	9.40	-.0456	.3529
.70	-.0889	.9373	9.60	-.0447	.3439
.80	-.0886	.9285	9.80	-.0438	.3350
.90	-.0883	.9196	10.00	-.0429	.3264
1.00	-.0880	.9108	10.50	-.0408	.3054
1.10	-.0876	.9020	11.00	-.0388	.2855
1.20	-.0872	.8933	11.50	-.0368	.2667
1.30	-.0866	.8846	12.00	-.0348	.2488
1.40	-.0863	.8759	12.50	-.0329	.2318
1.50	-.0858	.8673	13.00	-.0311	.2158
1.60	-.0854	.8588	13.50	-.0294	.2007
1.70	-.0849	.8502	14.00	-.0277	.1864
1.80	-.0843	.8418	14.50	-.0261	.1730
1.90	-.0838	.8334	15.00	-.0245	.1604
2.00	-.0833	.8250	15.50	-.0230	.1485
2.10	-.0828	.8167	16.00	-.0216	.1373
2.20	-.0822	.8085	16.50	-.0202	.1269
2.30	-.0817	.8003	17.00	-.0189	.1171
2.40	-.0811	.7921	17.50	-.0177	.1079
2.50	-.0806	.7840	18.00	-.0165	.0994
2.60	-.0800	.7760	18.50	-.0154	.0914
2.70	-.0795	.7680	19.00	-.0143	.0839
2.80	-.0790	.7601	19.50	-.0133	.0770
2.90	-.0784	.7522	20.00	-.0124	.0706
3.00	-.0779	.7444	21.00	-.0106	.0591
3.10	-.0773	.7367	22.00	-.0091	.0492
3.20	-.0768	.7290	23.00	-.0077	.0408
3.30	-.0762	.7213	24.00	-.0066	.0337
3.40	-.0757	.7137	25.00	-.0055	.0277
3.50	-.0751	.7062	26.00	-.0046	.0226
3.60	-.0746	.6987	27.00	-.0038	.0184
3.70	-.0741	.6913	28.00	-.0032	.0149
3.80	-.0735	.6839	29.00	-.0026	.0120
3.90	-.0730	.6766	30.00	-.0022	.0096
4.00	-.0724	.6693	31.00	-.0018	.0077
4.20	-.0714	.6549	32.00	-.0014	.0061
4.40	-.0703	.6407	33.00	-.0011	.0048
4.60	-.0692	.6268	34.00	-.0009	.0038
4.80	-.0682	.6131	35.00	-.0007	.0029
5.00	-.0671	.5995	36.00	-.0006	.0023
5.20	-.0661	.5862	37.00	-.0005	.0018
5.40	-.0650	.5731	38.00	-.0004	.0014
5.60	-.0640	.5602	39.00	-.0003	.0010
5.80	-.0629	.5475	40.00	-.0002	.0008
6.00	-.0619	.5350	41.00	-.0002	.0006
6.20	-.0609	.5227	42.00	-.0001	.0005
6.40	-.0599	.5107	43.00	-.0001	.0003
6.60	-.0589	.4988	44.00	-.0001	.0003
6.80	-.0579	.4871	45.00	-.0001	.0002
7.00	-.0569	.4756	46.00	.0000	.0001
7.20	-.0559	.4644	47.00	.0000	.0001
7.40	-.0549	.4533	48.00	.0000	.0001
7.60	-.0539	.4424	49.00	.0000	.0001
7.80	-.0530	.4317	49.50	.0000	.0000
8.00	-.0520	.4212			

TABLE VI. - FREE-CONVECTION SOLUTIONS

[Governing differential equation:

$$f''' + 3ff'' - 2(f')^2 + \theta = 0, f(0) = f'(0) = 0, f' \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

$$\theta'' + 3(\text{Pr})f\theta' = 0, \theta(0) = 1, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty]$$

(a) Pr = 0.03

ζ	f''	f'	f	θ'	θ
0.00	0.9384	0.0000	0.0000	-0.1346	1.0000
0.10	.8392	.0889	.0045	-.1346	.9865
0.20	.7423	.1679	.0174	-.1346	.9751
0.30	.6484	.2374	.0378	-.1346	.9596
0.40	.5585	.2977	.0646	-.1345	.9432
0.50	.4732	.3493	.0970	-.1344	.9327
0.60	.3934	.3926	.1342	-.1343	.9193
0.70	.3195	.4282	.1753	-.1341	.9058
0.80	.2521	.4567	.2196	-.1339	.8924
0.90	.1913	.4788	.2664	-.1336	.8791
1.00	.1374	.4952	.3152	-.1332	.8657
1.10	.0902	.5065	.3653	-.1328	.8524
1.20	.0496	.5134	.4163	-.1324	.8392
1.30	.0153	.5166	.4678	-.1318	.8260
1.40	-.0133	.5167	.5195	-.1313	.8128
1.50	-.0365	.5141	.5711	-.1306	.7997
1.60	-.0551	.5095	.6223	-.1299	.7867
1.70	-.0696	.5032	.6729	-.1292	.7737
1.80	-.0807	.4957	.7229	-.1283	.7609
1.90	-.0887	.4872	.7721	-.1275	.7481
2.00	-.0944	.4780	.8203	-.1266	.7354
2.10	-.0981	.4684	.8676	-.1256	.7228
2.20	-.1002	.4585	.9140	-.1246	.7102
2.30	-.1012	.4484	.9593	-.1236	.6978
2.40	-.1012	.4383	1.0037	-.1225	.6855
2.50	-.1006	.4282	1.0470	-.1214	.6733
2.60	-.0994	.4182	1.0893	-.1202	.6613
2.70	-.0979	.4083	1.1306	-.1190	.6493
2.80	-.0962	.3986	1.1710	-.1178	.6375
2.90	-.0943	.3891	1.2104	-.1165	.6257
3.00	-.0923	.3797	1.2488	-.1152	.6142
3.10	-.0903	.3706	1.2863	-.1139	.6027
3.20	-.0882	.3617	1.3229	-.1126	.5914
3.30	-.0862	.3530	1.3586	-.1112	.5802
3.40	-.0842	.3444	1.3935	-.1099	.5691
3.50	-.0822	.3361	1.4275	-.1085	.5582
3.60	-.0802	.3280	1.4607	-.1071	.5474
3.70	-.0783	.3201	1.4931	-.1057	.5368
3.80	-.0765	.3123	1.5248	-.1043	.5263
3.90	-.0746	.3048	1.5556	-.1028	.5159
4.00	-.0728	.2974	1.5857	-.1014	.5057
4.20	-.0694	.2832	1.6438	-.0985	.4857
4.40	-.0661	.2696	1.6990	-.0955	.4663
4.60	-.0630	.2567	1.7517	-.0926	.4475
4.80	-.0600	.2444	1.8018	-.0897	.4293
5.00	-.0572	.2327	1.8495	-.0868	.4116
5.20	-.0545	.2216	1.8949	-.0839	.3946
5.40	-.0519	.2109	1.9381	-.0811	.3781
5.60	-.0494	.2008	1.9793	-.0783	.3621
5.80	-.0471	.1911	2.0185	-.0755	.3467
6.00	-.0448	.1820	2.0558	-.0728	.3319

TABLE VI. - Continued.

(a) Concluded. Pr = 0.03

ξ	r''	r'	r	θ'	θ
6.20	-0.0427	0.1732	2.0913	-0.0701	0.3176
6.40	-.0407	.1649	2.1251	-.0675	.3039
6.60	-.0388	.1569	2.1573	-.0650	.2906
6.80	-.0369	.1494	2.1879	-.0625	.2779
7.00	-.0352	.1421	2.2170	-.0600	.2656
7.20	-.0335	.1353	2.2448	-.0577	.2539
7.40	-.0319	.1288	2.2712	-.0554	.2426
7.60	-.0304	.1225	2.2963	-.0532	.2317
7.80	-.0289	.1164	2.3202	-.0510	.2213
8.00	-.0275	.1110	2.3430	-.0489	.2113
8.20	-.0262	.1056	2.3646	-.0469	.2017
8.40	-.0249	.1005	2.3852	-.0449	.1926
8.60	-.0237	.0956	2.4048	-.0430	.1836
8.80	-.0226	.0910	2.4235	-.0412	.1753
9.00	.0215	.0866	2.4412	-.0394	.1673
9.20	-.0205	.0824	2.4581	-.0377	.1596
9.40	-.0195	.0784	2.4742	-.0361	.1522
9.60	-.0186	.0746	2.4895	-.0345	.1451
9.80	-.0177	.0709	2.5040	-.0330	.1384
10.00	-.0168	.0675	2.5179	-.0315	.1319
10.50	-.0149	.0596	2.5496	-.0281	.1170
11.00	-.0131	.0526	2.5776	-.0251	.1038
11.50	-.0116	.0464	2.6023	-.0223	.0919
12.00	-.0102	.0410	2.6242	-.0198	.0814
12.50	-.0090	.0362	2.6434	-.0176	.0721
13.00	-.0080	.0319	2.6604	-.0156	.0638
13.50	-.0070	.0282	2.6754	-.0139	.0564
14.00	-.0062	.0249	2.6887	-.0123	.0499
14.50	-.0055	.0219	2.7004	-.0109	.0441
15.00	-.0048	.0194	2.7107	-.0096	.0389
15.50	-.0043	.0171	2.7198	-.0085	.0344
16.00	-.0038	.0151	2.7278	-.0075	.0304
16.50	-.0033	.0133	2.7349	-.0067	.0268
17.00	-.0029	.0117	2.7411	-.0059	.0237
17.50	-.0026	.0103	2.7466	-.0052	.0209
18.00	-.0023	.0091	2.7515	-.0046	.0185
18.50	-.0020	.0081	2.7558	-.0041	.0163
20.00	-.0014	.0055	2.7659	-.0028	.0112
21.00	-.0011	.0043	2.7708	-.0022	.0087
22.00	-.0008	.0033	2.7746	-.0017	.0068
23.00	-.0007	.0026	2.7775	-.0013	.0053
24.00	-.0005	.0020	2.7798	-.0010	.0041
25.00	-.0004	.0016	2.7816	-.0008	.0032
26.00	-.0003	.0012	2.7830	-.0006	.0025
27.00	-.0002	.0010	2.7841	-.0005	.0019
28.00	-.0002	.0007	2.7849	-.0004	.0015
29.00	-.0001	.0006	2.7856	-.0003	.0012
30.00	-.0001	.0004	2.7861	-.0002	.0009
31.00	-.0001	.0004	2.7865	-.0002	.0007
32.00	-.0001	.0003	2.7868	-.0001	.0006
33.00	-.0001	.0002	2.7871	-.0001	.0004
34.00	.0000	.0002	2.7872	-.0001	.0003
35.00	.0000	.0001	2.7874	-.0001	.0003
36.00	.0000	.0001	2.7875	-.0001	.0002
37.00	.0000	.0001	2.7876	-.0000	.0002
38.00	.0000	.0001	2.7877	-.0000	.0001
39.00	.0000	.0000	2.7877	-.0000	.0001
40.00	.0000	.0000	2.7878	-.0000	.0001
41.00	.0000	.0000	2.7878	-.0000	.0001
42.00	.0000	.0000	2.7878	-.0000	.0000

TABLE VI. - Continued.

(b) $Pr = 0.02$

ξ	r^0	r^1	r	θ^1	θ
0.00	0.9590	0.0000	0.0000	-.1116	1.0000
.10	.8597	.0909	.0646	-.1116	.9888
.20	.7624	.1720	.0179	-.1116	.9777
.30	.6681	.2435	.0387	-.1116	.9665
.40	.5776	.3058	.0662	-.1116	.9554
.50	.4917	.3592	.0996	-.1115	.9442
.60	.4112	.4043	.1378	-.1114	.9330
.70	.3367	.4416	.1802	-.1113	.9219
.80	.2686	.4716	.2259	-.1112	.9108
.90	.2072	.4956	.2743	-.1110	.8997
1.00	.1528	.5135	.3248	-.1108	.8886
1.10	.1052	.5263	.3768	-.1106	.8775
1.20	.0642	.5348	.4299	-.1103	.8665
1.30	.0296	.5394	.4837	-.1100	.8554
1.40	.0008	.5409	.5377	-.1097	.8445
1.50	-.0227	.5397	.5916	-.1093	.8335
1.60	-.0415	.5365	.6456	-.1089	.8226
1.70	-.0562	.5316	.6990	-.1085	.8117
1.80	-.0675	.5253	.7518	-.1080	.8009
1.90	-.0758	.5182	.8040	-.1075	.7901
2.00	-.0817	.5103	.8555	-.1070	.7794
2.10	-.0857	.5019	.9061	-.1064	.7687
2.20	-.0882	.4932	.9558	-.1058	.7581
2.30	-.0896	.4843	1.0047	-.1052	.7476
2.40	-.0900	.4753	1.0527	-.1045	.7371
2.50	-.0898	.4663	1.0998	-.1039	.7267
2.60	-.0892	.4573	1.1459	-.1032	.7163
2.70	-.0882	.4485	1.1912	-.1025	.7060
2.80	-.0869	.4397	1.2356	-.1017	.6958
2.90	-.0856	.4311	1.2792	-.1009	.6857
3.00	-.0841	.4226	1.3219	-.1002	.6756
3.10	-.0826	.4143	1.3637	-.0994	.6656
3.20	-.0810	.4061	1.4047	-.0985	.6557
3.30	-.0795	.3981	1.4449	-.0977	.6459
3.40	-.0779	.3902	1.4843	-.0968	.6362
3.50	-.0764	.3825	1.5230	-.0960	.6266
3.60	-.0749	.3749	1.5608	-.0951	.6170
3.70	-.0734	.3675	1.5980	-.0942	.6076
3.80	-.0720	.3602	1.6343	-.0933	.5982
4.00	-.0692	.3461	1.7050	-.0914	.5797
4.20	-.0665	.3326	1.7728	-.0895	.5616
4.40	-.0639	.3195	1.8380	-.0876	.5439
4.60	-.0614	.3070	1.9007	-.0857	.5266
4.80	-.0590	.2949	1.9609	-.0837	.5096
5.00	-.0568	.2834	2.0187	-.0817	.4931
5.20	-.0546	.2722	2.0742	-.0798	.4769
5.40	-.0524	.2615	2.1276	-.0778	.4612
5.60	-.0504	.2513	2.1789	-.0758	.4458
5.80	-.0485	.2414	2.2281	-.0738	.4306
6.00	-.0466	.2319	2.2755	-.0718	.4153
6.20	-.0448	.2227	2.3209	-.0699	.4021
6.40	-.0430	.2140	2.3646	-.0680	.3883
6.60	-.0414	.2055	2.4065	-.0660	.3749
6.80	-.0398	.1974	2.4468	-.0641	.3619

TABLE VI. - Continued.

(b) Concluded. Pr = 0.02

ξ	f''	f'	f	θ'	θ
7.00	-0.0382	0.1896	2.4855	-0.0623	0.3493
7.20	-.0367	.1821	2.5227	-.0604	.3370
7.40	-.0353	.1749	2.5584	-.0586	.3251
7.60	-.0339	.1680	2.5927	-.0568	.3136
7.80	-.0326	.1613	2.6256	-.0551	.3024
8.00	-.0313	.1549	2.6572	-.0534	.2915
8.20	-.0301	.1488	2.6876	-.0517	.2810
8.40	-.0289	.1429	2.7167	-.0500	.2708
8.60	-.0278	.1372	2.7448	-.0484	.2610
8.80	-.0267	.1318	2.7717	-.0468	.2515
9.00	-.0257	.1265	2.7975	-.0453	.2423
9.20	-.0247	.1215	2.8223	-.0438	.2334
9.40	-.0237	.1167	2.8461	-.0423	.2247
9.60	-.0228	.1120	2.8690	-.0409	.2164
9.80	-.0219	.1076	2.8909	-.0395	.2084
10.00	-.0210	.1033	2.9120	-.0382	.2006
10.50	-.0190	.0933	2.9611	-.0349	.1824
11.00	-.0172	.0843	3.0055	-.0320	.1656
11.50	-.0155	.0761	3.0455	-.0292	.1504
12.00	-.0140	.0687	3.0817	-.0266	.1364
12.50	-.0127	.0620	3.1143	-.0243	.1237
13.00	-.0115	.0560	3.1438	-.0221	.1121
13.50	-.0104	.0505	3.1704	-.0201	.1016
14.00	-.0094	.0456	3.1944	-.0183	.0920
14.50	-.0085	.0411	3.2161	-.0166	.0833
15.00	-.0076	.0371	3.2357	-.0151	.0754
15.50	-.0069	.0335	3.2533	-.0137	.0682
16.00	-.0062	.0302	3.2692	-.0124	.0617
16.50	-.0056	.0272	3.2836	-.0112	.0558
17.00	-.0051	.0246	3.2965	-.0102	.0505
17.50	-.0046	.0221	3.3082	-.0092	.0457
18.00	-.0042	.0200	3.3187	-.0083	.0413
18.50	-.0037	.0180	3.3282	-.0075	.0373
19.00	-.0034	.0162	3.3367	-.0068	.0337
19.50	-.0031	.0146	3.3444	-.0062	.0305
20.00	-.0028	.0131	3.3513	-.0056	.0275
21.00	-.0022	.0106	3.3632	-.0046	.0225
22.00	-.0018	.0086	3.3728	-.0037	.0183
23.00	-.0015	.0070	3.3805	-.0030	.0150
24.00	-.0012	.0056	3.3868	-.0025	.0122
25.00	-.0010	.0045	3.3918	-.0020	.0099
26.00	-.0008	.0036	3.3958	-.0017	.0081
27.00	-.0007	.0029	3.3991	-.0014	.0066
28.00	-.0005	.0023	3.4016	-.0011	.0054
29.00	-.0004	.0018	3.4036	-.0009	.0044
30.00	-.0004	.0014	3.4052	-.0007	.0036
31.00	-.0003	.0011	3.4065	-.0006	.0029
32.00	-.0002	.0008	3.4074	-.0005	.0024
33.00	-.0002	.0006	3.4081	-.0004	.0019
34.00	-.0002	.0004	3.4086	-.0003	.0016
35.00	-.0001	.0003	3.4089	-.0003	.0013
36.00	-.0001	.0002	3.4091	-.0002	.0010
37.00	-.0001	.0001	3.4092	-.0002	.0009
38.00	-.0001	.0000	3.4093	-.0001	.0007

TABLE VI. - Continued.

(c) $Pr = 0.008$

ζ	r''	r'	r	θ'	θ
0.00	0.9955	0.0000	0.0000	-0.0725	1.0000
.10	.8960	.0946	.0048	-.0725	.9928
.20	.7983	.1793	.0186	-.0725	.9855
.30	.7032	.2543	.0403	-.0725	.9783
.40	.6118	.3200	.0691	-.0724	.9710
.50	.5249	.3768	.1040	-.0724	.9638
.60	.4433	.4252	.1442	-.0724	.9565
.70	.3677	.4657	.1888	-.0724	.9493
.80	.2987	.4989	.2371	-.0723	.9421
.90	.2365	.5256	.2884	-.0723	.9348
1.00	.1813	.5463	.3420	-.0722	.9276
1.10	.1332	.5621	.3975	-.0722	.9204
1.20	.0918	.5733	.4543	-.0721	.9132
1.30	.0568	.5817	.5120	-.0720	.9060
1.40	.0278	.5849	.5704	-.0719	.8988
1.50	.0042	.5864	.6289	-.0718	.8916
1.60	-.0147	.5859	.6876	-.0717	.8844
1.70	-.0295	.5836	.7461	-.0716	.8772
1.80	-.0409	.5801	.8043	-.0715	.8701
1.90	-.0494	.5756	.8620	-.0713	.8629
2.00	-.0556	.5703	.9193	-.0712	.8558
2.10	-.0599	.5645	.9761	-.0710	.8487
2.20	-.0629	.5584	1.0322	-.0708	.8416
2.30	-.0647	.5520	1.0877	-.0706	.8345
2.40	-.0658	.5454	1.1426	-.0705	.8275
2.50	-.0662	.5388	1.1968	-.0703	.8204
2.60	-.0662	.5322	1.2504	-.0701	.8134
2.70	-.0659	.5256	1.3033	-.0698	.8064
2.80	-.0653	.5190	1.3555	-.0696	.7995
2.90	-.0649	.5125	1.4071	-.0694	.7925
3.00	-.0642	.5061	1.4580	-.0692	.7856
3.10	-.0634	.4997	1.5083	-.0689	.7787
3.20	-.0626	.4934	1.5579	-.0687	.7718
3.30	-.0619	.4871	1.6070	-.0684	.7649
3.40	-.0611	.4810	1.6554	-.0681	.7581
3.50	-.0603	.4749	1.7032	-.0679	.7513
3.60	-.0595	.4689	1.7504	-.0676	.7446
3.70	-.0587	.4630	1.7970	-.0673	.7378
3.80	-.0580	.4572	1.8430	-.0670	.7311
3.90	-.0572	.4514	1.8884	-.0667	.7244
4.00	-.0565	.4458	1.9333	-.0664	.7178
4.20	-.0551	.4346	2.0212	-.0658	.7045
4.40	-.0537	.4237	2.1071	-.0651	.6915
4.60	-.0523	.4131	2.1908	-.0644	.6785
4.80	-.0510	.4028	2.2724	-.0638	.6657
5.00	-.0497	.3927	2.3519	-.0631	.6530
5.20	-.0485	.3829	2.4295	-.0623	.6405
5.40	-.0473	.3733	2.5051	-.0616	.6281
5.60	-.0461	.3640	2.5789	-.0609	.6158
5.80	-.0450	.3549	2.6507	-.0601	.6037
6.00	-.0438	.3460	2.7208	-.0593	.5918
6.20	-.0428	.3374	2.7892	-.0585	.5800
6.40	-.0417	.3289	2.8558	-.0578	.5684
6.60	-.0407	.3207	2.9207	-.0570	.5569
6.80	-.0397	.3126	2.9841	-.0562	.5456
7.00	-.0387	.3048	3.0458	-.0554	.5344
7.20	-.0377	.2972	3.1060	-.0545	.5234
7.40	-.0368	.2897	3.1647	-.0537	.5126
7.60	-.0359	.2824	3.2219	-.0529	.5020
7.80	-.0350	.2754	3.2777	-.0521	.4915
8.00	-.0341	.2684	3.3321	-.0513	.4811
8.20	-.0333	.2617	3.3851	-.0504	.4709
8.40	-.0325	.2551	3.4368	-.0496	.4609
8.60	-.0317	.2487	3.4871	-.0488	.4511
8.80	-.0309	.2425	3.5363	-.0480	.4414
9.00	-.0301	.2364	3.5841	-.0472	.4319
9.20	-.0294	.2304	3.6308	-.0464	.4225

TABLE VI. - Continued.

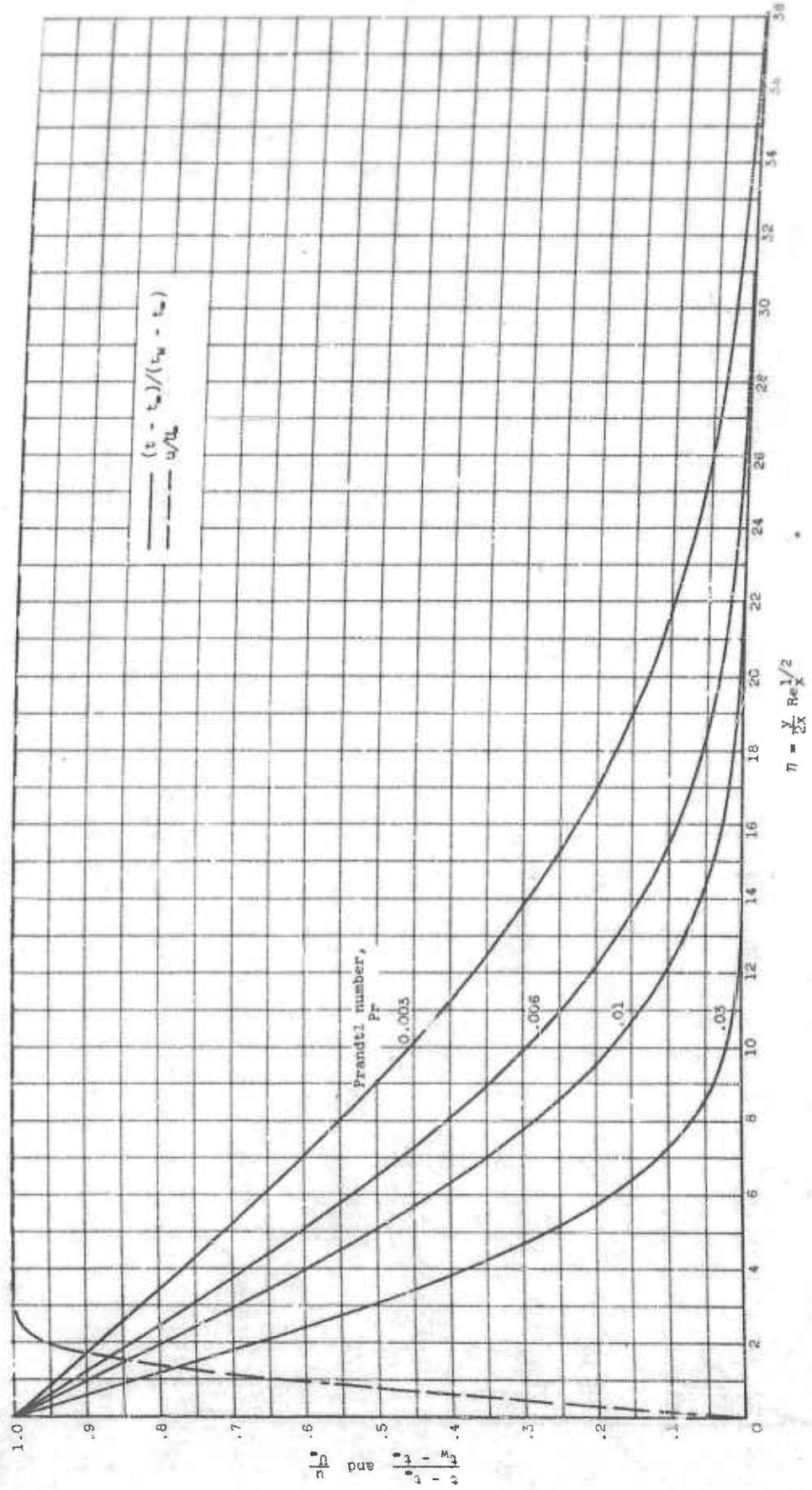
(c) Concluded. Pr = 0.008

ξ	f''	f'	f	θ'	θ
9.40	-.0287	0.2246	3.6763	-0.0456	6.4133
9.60	-.0279	.2189	3.7207	-.0448	4043
9.80	-.0273	.2134	3.7639	-.0440	.3954
10.00	-.0266	.2080	3.8060	-.0432	.3867
10.50	-.0250	.1952	3.9366	-.0412	.3656
11.00	-.0234	.1831	4.0013	-.0393	.3455
11.50	-.0220	.1717	4.0900	-.0375	.3263
12.00	-.0207	.1610	4.1731	-.0356	.3080
12.50	-.0194	.1510	4.2511	-.0339	.2907
13.00	-.0182	.1416	4.3242	-.0322	.2741
13.50	-.0171	.1328	4.3928	-.0305	.2585
14.00	-.0161	.1245	4.4571	-.0290	.2436
14.50	-.0151	.1167	4.5174	-.0274	.2295
15.00	-.0141	.1094	4.5739	-.0260	.2161
15.50	-.0133	.1026	4.6269	-.0246	.2035
16.00	-.0124	.0962	4.6766	-.0233	.1915
16.50	-.0117	.0901	4.7231	-.0220	.1802
17.00	-.0110	.0845	4.7668	-.0208	.1695
17.50	-.0103	.0792	4.8076	-.0196	.1595
18.00	-.0096	.0742	4.8460	-.0185	.1499
18.50	-.0089	.0695	4.8819	-.0175	.1409
19.00	-.0085	.0651	4.9155	-.0165	.1325
19.50	-.0080	.0610	4.9471	-.0155	.1245
20.00	-.0075	.0572	4.9766	-.0146	.1169
21.00	-.0066	.0502	5.0302	-.0130	.1032
22.00	-.0058	.0440	5.0772	-.0115	.0910
23.00	-.0051	.0386	5.1163	-.0102	.0802
24.00	-.0045	.0339	5.1547	-.0090	.0706
25.00	-.0039	.0297	5.1864	-.0079	.0622
26.00	-.0034	.0260	5.2142	-.0070	.0547
27.00	-.0030	.0228	5.2385	-.0062	.0481
28.00	-.0027	.0199	5.2598	-.0054	.0423
29.00	-.0023	.0175	5.2785	-.0048	.0372
30.00	-.0020	.0153	5.2948	-.0042	.0327
31.00	-.0018	.0133	5.3091	-.0037	.0287
32.00	-.0016	.0117	5.3216	-.0033	.0252
33.00	-.0014	.0102	5.3329	-.0029	.0222
34.00	-.0012	.0089	5.3421	-.0025	.0195
35.00	-.0011	.0078	5.3504	-.0022	.0171
36.00	-.0009	.0068	5.3576	-.0020	.0150
37.00	-.0008	.0059	5.3639	-.0017	.0131
38.00	-.0007	.0051	5.3694	-.0015	.0115
39.00	-.0006	.0044	5.3742	-.0013	.0101
40.00	-.0006	.0038	5.3783	-.0012	.0089
41.00	-.0005	.0033	5.3819	-.0010	.0078
42.00	-.0004	.0029	5.3850	-.0009	.0068
43.00	-.0004	.0025	5.3876	-.0008	.0059
44.00	-.0003	.0021	5.3899	-.0007	.0052
45.00	-.0003	.0018	5.3919	-.0006	.0045
46.00	-.0002	.0016	5.3936	-.0005	.0040
47.00	-.0002	.0013	5.3951	-.0005	.0035
48.00	-.0002	.0011	5.3963	-.0004	.0030
49.00	-.0002	.0010	5.3974	-.0004	.0026
50.00	-.0001	.0008	5.3982	-.0003	.0023
51.00	-.0001	.0007	5.3990	-.0003	.0020
53.00	-.0001	.0005	5.4001	-.0002	.0015
54.00	-.0001	.0004	5.4005	-.0002	.0013
55.00	-.0001	.0003	5.4009	-.0002	.0011
56.00	-.0001	.0002	5.4011	-.0001	.0009
57.00	-.0001	.0002	5.4013	-.0001	.0008
58.00	.0000	.0001	5.4015	-.0001	.0007
59.00	.0000	.0001	5.4016	-.0001	.0006
60.00	.0000	.0001	5.4017	-.0001	.0005
61.00	.0000	.0000	5.4017	-.0001	.0004
62.00	.0000	.0000	5.4018	-.0001	.0003

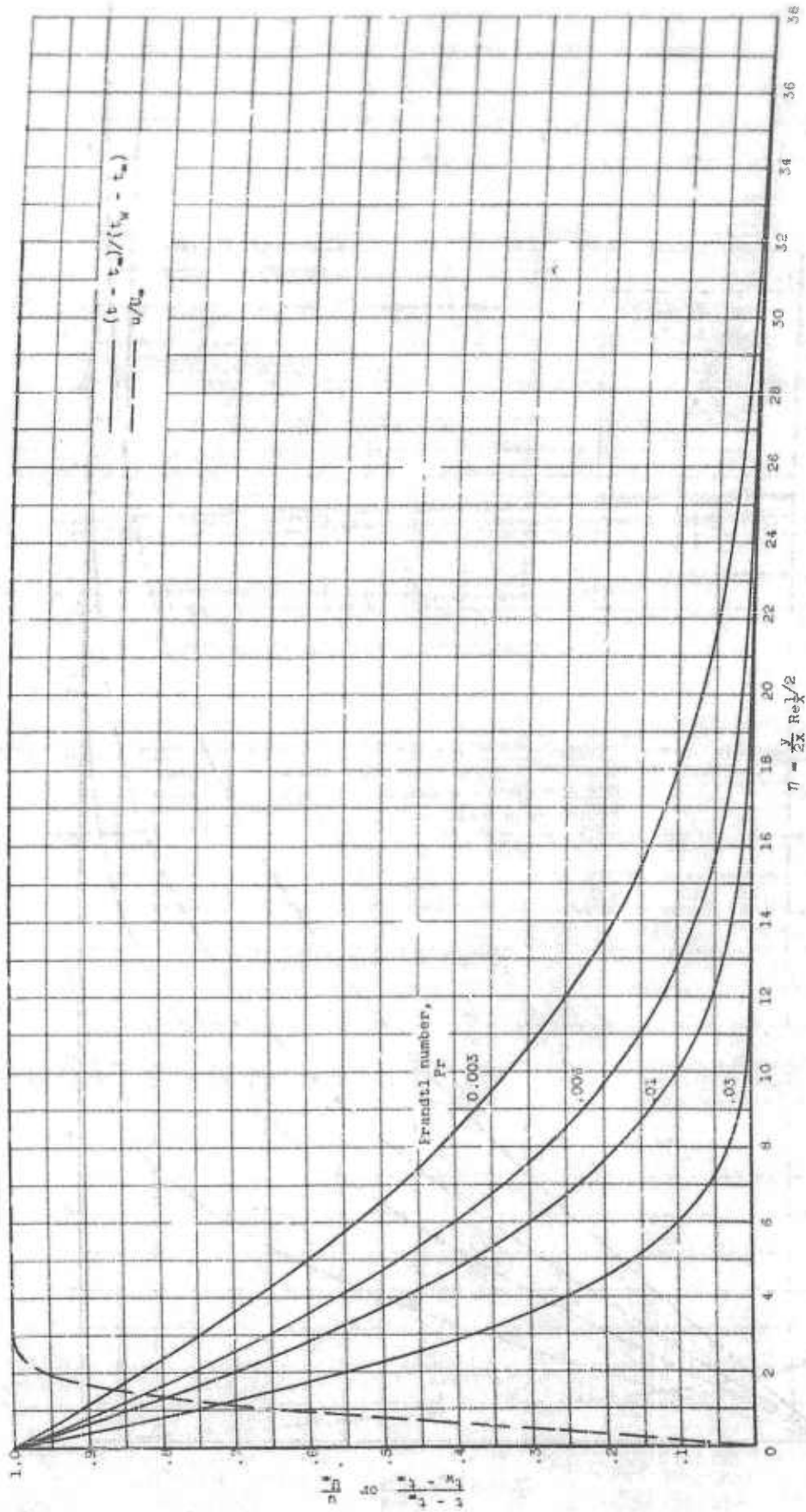
TABLE VI. - Concluded.

(d) Pr = 0.003

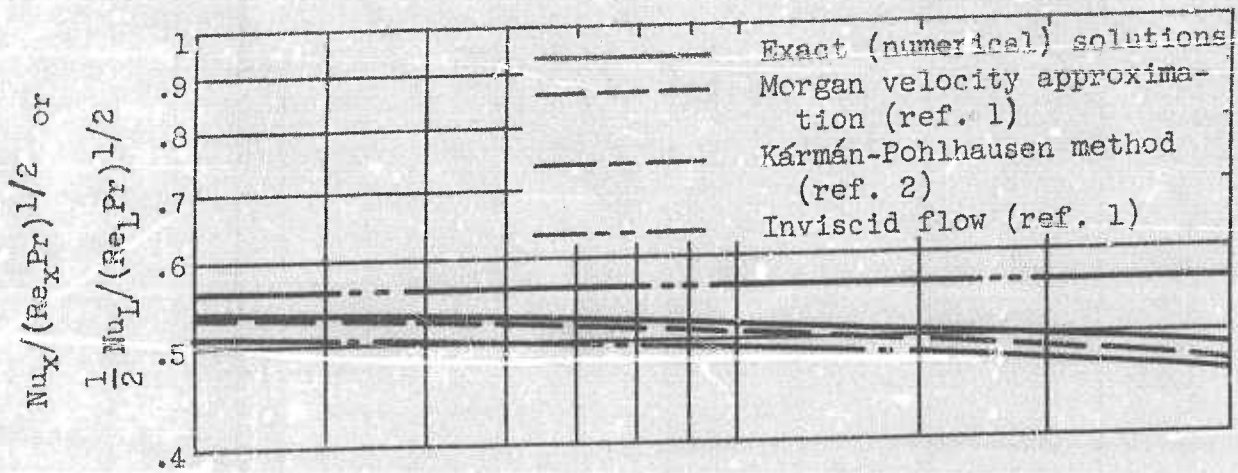
z	r ⁿ	r ¹	r ²	r ³	r ⁴	z	r ⁿ	r ¹	r ²	r ³	r ⁴
0.00	1.0223	0.0000	0.0000	-0.0451	1.0000	23.00	-0.0096	0.1185	7.0354	-0.0174	0.2161
0.10	.9227	.0072	.0049	.0451	.9959	23.50	.0092	.1198	7.0412	.0168	.2296
0.20	.8246	.0146	.0191	.0451	.9910	24.00	.0088	.1093	7.0479	.0163	.2213
0.30	.7290	.0222	.0413	.0451	.9865	24.50	.0085	.1049	7.0521	.0158	.2133
0.40	.6369	.0305	.0713	.0451	.9819	25.00	.0082	.1008	7.0572	.0152	.2055
0.50	.5494	.0398	.1073	.0451	.9774	25.50	.0079	.0967	7.0622	.0147	.1980
0.60	.4671	.0466	.1489	.0451	.9729	26.00	.0076	.0929	7.0676	.0142	.1908
0.70	.3909	.0534	.1952	.0451	.9684	26.50	.0073	.0892	7.0711	.0137	.1838
0.80	.3213	.0590	.2454	.0451	.9639	27.00	.0070	.0856	7.0758	.0133	.1771
0.90	.2586	.0649	.2988	.0451	.9594	27.50	.0067	.0822	7.0807	.0128	.1706
1.00	.2031	.0709	.3547	.0451	.9549	28.00	.0065	.0789	7.0841	.0124	.1643
1.10	.1548	.0772	.4128	.0451	.9504	28.50	.0062	.0757	7.0879	.0119	.1582
1.20	.1133	.0821	.4723	.0451	.9459	29.00	.0060	.0726	7.0917	.0115	.1523
1.30	.0783	.0868	.5331	.0450	.9414	29.50	.0058	.0697	7.0952	.0111	.1467
1.40	.0494	.0910	.5946	.0450	.9369	30.00	.0055	.0669	7.0984	.0107	.1412
1.50	.0259	.0947	.6566	.0450	.9324	30.50	.0053	.0642	8.0192	.0103	.1359
1.60	.0071	.0983	.7188	.0450	.9279	31.00	.0051	.0615	8.0506	.0100	.1309
1.70	-.0076	.1017	.7812	.0449	.9234	31.50	.0049	.0590	8.0807	.0096	.1260
1.80	-.0188	.1049	.8434	.0449	.9189	32.00	.0047	.0566	8.1094	.0093	.1212
1.90	-.0273	.1079	.9055	.0448	.9144	32.50	.0046	.0543	8.1374	.0089	.1167
2.00	-.0335	.1107	.9673	.0448	.9099	33.00	.0044	.0521	8.1640	.0086	.1123
2.10	-.0379	.1133	1.0288	.0448	.9054	33.50	.0042	.0499	8.1893	.0083	.1081
2.20	-.0410	.1158	1.0899	.0447	.9009	34.00	.0040	.0479	8.2139	.0080	.1040
2.30	-.0431	.1180	1.1506	.0447	.8965	34.50	.0039	.0459	8.2373	.0077	.1000
2.40	-.0444	.1200	1.2108	.0446	.8920	35.00	.0037	.0440	8.2598	.0074	.0963
2.50	-.0451	.1219	1.2707	.0446	.8875	35.50	.0036	.0421	8.2813	.0072	.0926
2.60	-.0455	.1235	1.3300	.0445	.8831	36.00	.0035	.0404	8.3019	.0069	.0891
2.70	-.0456	.1250	1.3889	.0445	.8786	36.50	.0033	.0387	8.3217	.0066	.0857
2.80	-.0455	.1264	1.4474	.0444	.8742	37.00	.0032	.0370	8.3406	.0064	.0824
2.90	-.0453	.1277	1.5054	.0444	.8698	37.50	.0031	.0355	8.3587	.0062	.0793
3.00	-.0450	.1289	1.5629	.0443	.8653	38.00	.0030	.0340	8.3761	.0060	.0763
3.10	-.0447	.1300	1.6200	.0442	.8609	38.50	.0028	.0325	8.3927	.0058	.0734
3.20	-.0444	.1311	1.6767	.0442	.8565	39.00	.0027	.0311	8.4086	.0056	.0705
3.30	-.0440	.1321	1.7329	.0441	.8521	39.50	.0026	.0298	8.4239	.0053	.0678
3.40	-.0437	.1330	1.7886	.0440	.8476	40.00	.0025	.0285	8.4385	.0051	.0652
3.50	-.0433	.1339	1.8440	.0440	.8432	40.50	.0024	.0273	8.4524	.0049	.0627
3.60	-.0429	.1347	1.8989	.0439	.8388	41.00	.0023	.0261	8.4657	.0047	.0603
3.70	-.0426	.1354	1.9533	.0438	.8345	41.50	.0022	.0249	8.4785	.0046	.0580
3.80	-.0422	.1361	2.0074	.0437	.8301	42.00	.0022	.0238	8.4907	.0044	.0558
3.90	-.0419	.1367	2.0610	.0437	.8257	42.50	.0021	.0228	8.5024	.0042	.0536
4.00	-.0415	.1373	2.1142	.0436	.8214	43.00	.0020	.0218	8.5135	.0041	.0515
4.10	-.0411	.1379	2.1673	.0435	.8171	43.50	.0019	.0208	8.5241	.0039	.0496
4.20	-.0408	.1384	2.2202	.0434	.8128	44.00	.0018	.0199	8.5343	.0038	.0476
4.30	-.0404	.1389	2.2729	.0433	.8084	44.50	.0018	.0190	8.5440	.0036	.0458
4.40	-.0399	.1394	2.3254	.0431	.8041	45.00	.0017	.0181	8.5533	.0035	.0440
4.50	-.0395	.1399	2.3778	.0429	.8000	45.50	.0016	.0173	8.5621	.0034	.0423
4.60	-.0391	.1403	2.4299	.0427	.7958	46.00	.0016	.0165	8.5706	.0032	.0407
4.70	-.0387	.1407	2.4819	.0425	.7917	46.50	.0015	.0157	8.5786	.0031	.0391
4.80	-.0383	.1411	2.5337	.0423	.7875	47.00	.0014	.0150	8.5863	.0030	.0376
4.90	-.0379	.1415	2.5854	.0421	.7834	47.50	.0014	.0143	8.5936	.0029	.0361
5.00	-.0375	.1419	2.6369	.0419	.7793	48.00	.0013	.0136	8.6005	.0028	.0347
5.10	-.0371	.1423	2.6882	.0417	.7752	48.50	.0013	.0129	8.6072	.0027	.0333
5.20	-.0367	.1427	2.7394	.0415	.7711	49.00	.0012	.0123	8.6135	.0026	.0320
5.30	-.0363	.1431	2.7904	.0413	.7670	49.50	.0012	.0117	8.6195	.0025	.0308
5.40	-.0359	.1435	2.8413	.0411	.7629	50.00	.0011	.0111	8.6252	.0024	.0296
5.50	-.0355	.1439	2.8920	.0409	.7588	50.50	.0011	.0106	8.6306	.0023	.0284
5.60	-.0351	.1443	2.9427	.0407	.7547	51.00	.0010	.0100	8.6358	.0022	.0273
5.70	-.0347	.1447	2.9932	.0405	.7506	51.50	.0010	.0095	8.6406	.0021	.0262
5.80	-.0343	.1451	3.0437	.0403	.7465	52.00	.0010	.0090	8.6453	.0020	.0252
5.90	-.0339	.1455	3.0940	.0401	.7424	52.50	.0009	.0085	8.6497	.0019	.0242
6.00	-.0335	.1459	3.1443	.0399	.7383	53.00	.0009	.0081	8.6538	.0019	.0232
6.10	-.0331	.1463	3.1944	.0397	.7342	53.50	.0008	.0077	8.6577	.0018	.0223
6.20	-.0327	.1467	3.2445	.0395	.7301	54.00	.0008	.0072	8.6615	.0017	.0214
6.30	-.0323	.1471	3.2944	.0393	.7260	54.50	.0008	.0068	8.6650	.0017	.0206
6.40	-.0319	.1475	3.3443	.0391	.7219	55.00	.0007	.0064	8.6683	.0016	.0198
6.50	-.0315	.1479	3.3940	.0389	.7178	55.50	.0007	.0061	8.6714	.0015	.0190
6.60	-.0311	.1483	3.4437	.0387	.7137	56.00	.0007	.0057	8.6744	.0015	.0182
6.70	-.0307	.1487	3.4932	.0385	.7096	56.50	.0007	.0054	8.6771	.0014	.0175
6.80	-.0303	.1491	3.5427	.0383	.7055	57.00	.0006	.0050	8.6797	.0014	.0168
6.90	-.0300	.1495	3.5920	.0381	.7014	57.50	.0006	.0047	8.6822	.0013	.0161
7.00	-.0296	.1499	3.6413	.0379	.6973	58.00	.0006	.0044	8.6845	.0013	.0155
7.10	-.0292	.1503	3.6904	.0377	.6932	58.50	.0006	.0041	8.6866	.0012	.0149
7.20	-.0288	.1507	3.7394	.0375	.6891	59.00	.0005	.0038	8.6886	.0012	.0143
7.30	-.0284	.1511	3.7882	.0373	.6850	59.50	.0005	.0036	8.6904	.0011	.0137
7.40	-.0280	.1515	3.8369	.0371	.6809	60.00	.0005	.0033	8.6922	.0011	.0131
7.50	-.0276	.1519	3.8854	.0369	.6768	60.50	.0005	.0031	8.6938	.0010	.0126
7.60	-.0272	.1523	3.9337	.0367	.6727	61.00	.0004	.0028	8.6952	.0010	.0121
7.70	-.0268	.1527	3.9819	.0365	.6686	61.50	.0004	.0026	8.6966	.0010	.0116
7.80	-.0264	.1531	4.0299	.0363	.6645	62.00	.0004	.0024	8.6979	.0009	.0111
7.90	-.0260	.1535	4.0778	.0361	.6604	62.50	.0004	.0022	8.6990	.0009	.0107
8.00	-.0256	.1539	4.1254	.0359	.6563	63.00	.0004	.0020	8.7000	.0009	.0102
8.10	-.0252	.1543	4.1729	.0357	.6522	63.50	.0004	.0018	8.7010	.0008	.0098
8.20	-.0248	.1547	4.2202	.0355	.6481	64.00	.0004	.0016	8.7018	.0008	.0094
8.30	-.0244	.1551	4.2674	.0353	.6440	64.50	.0003	.0014	8.7026	.0008	.0090
8.40	-.0240	.1555	4.3144	.0351	.6400	65.00	.0003	.0013	8.7033	.0007	.0086
8.50	-.0236	.1559	4.3613	.0349	.6359	65.50	.0003	.0011	8.7039	.0007	.0083
8.60	-.0232	.1563	4.4080	.0347	.6318	66.00	.0003	.0009	8.7044	.0007	.0079
8.70	-.0228	.1567	4.4547	.0345	.6277	66.50	.0003	.0008	8.7048	.0007	.0076
8.80	-.0224	.1571	4.5012	.0343	.6236	67.00	.0003	.0006	8.7052	.0006	.0073
8.90	-.0220	.1575	4.5477	.0341	.6195	67.50	.0003	.0005	8.7054	.0006	.0070
9.00	-.0216	.1579	4.5940	.0339	.6154	68.00	.0003	.0004	8.7057	.0006	.0067
9.10	-.0212	.1583	4.6403	.0337	.6113	68.50	.0002	.0003	8.7058	.0006	.0064
9.20	-.0208	.1587	4.6864	.0335	.6072	69.00	.0002	.0001	8.7059	.0005	.0061
9.30	-.0204	.1591	4.7324	.0333	.6031	69.50	.0002	.0000	8.7060	.0005	.0059



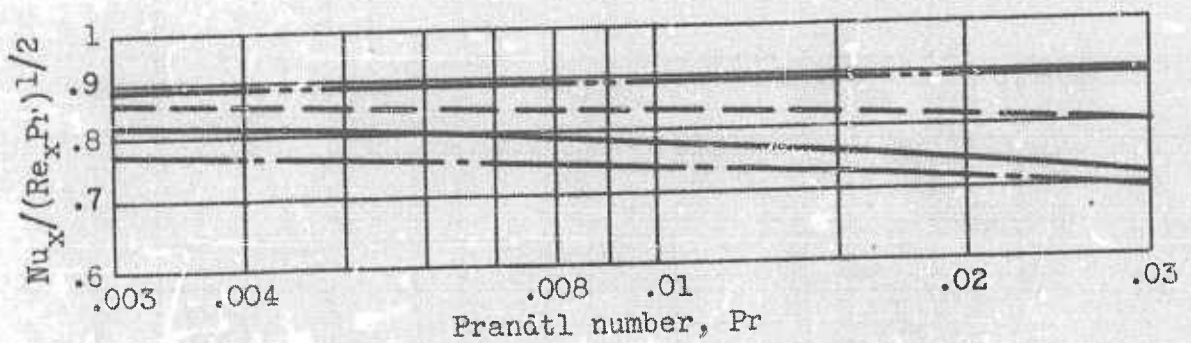
(a) Uniform-wall-temperature situation.
 Figure 1. - Forced-convection temperature and velocity profiles.



(b) Uniform-heat-flux situation.
 Figure 1. - Concluded. Forced-convection temperature and velocity profiles.

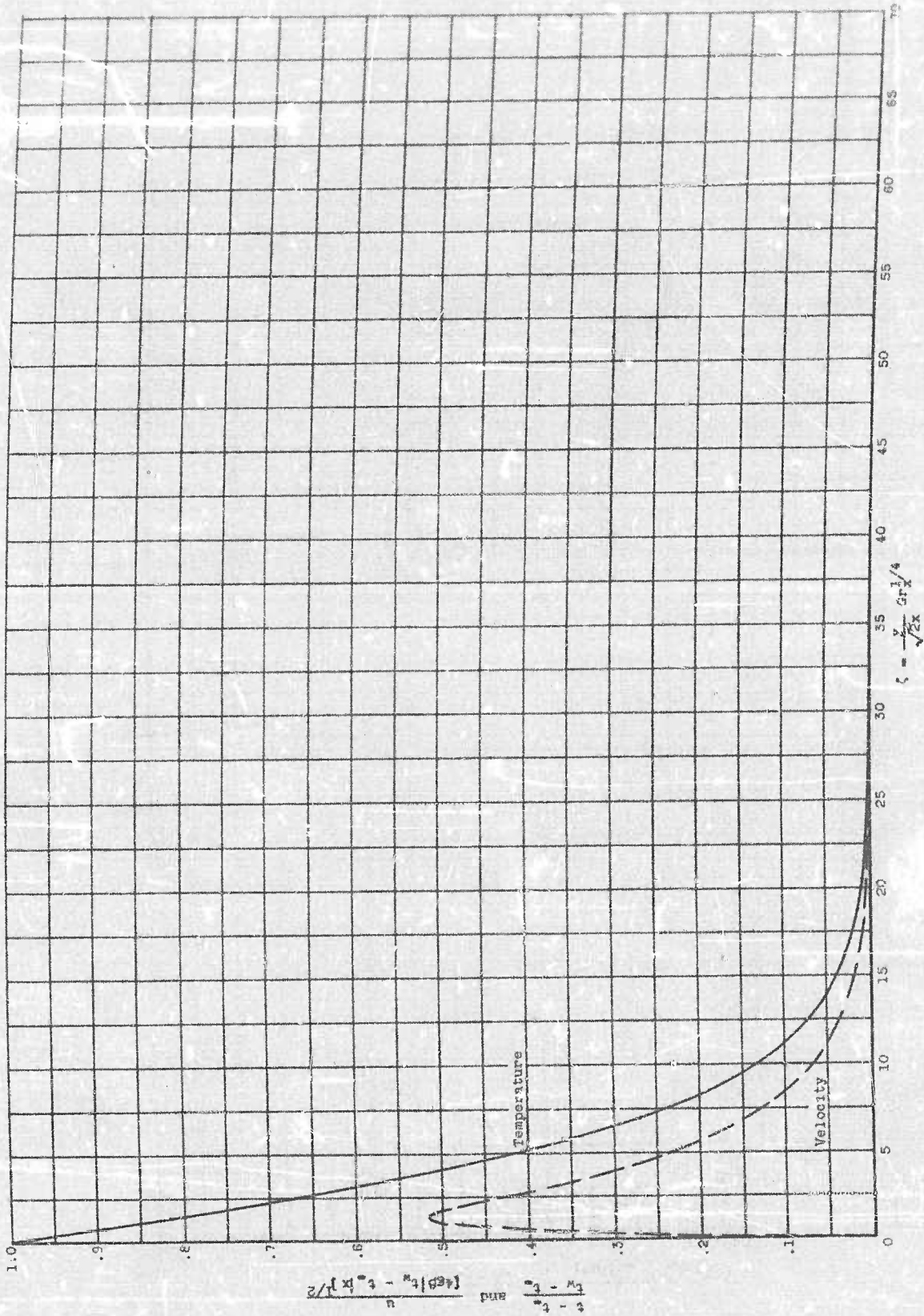


(a) Uniform-wall-temperature situation.



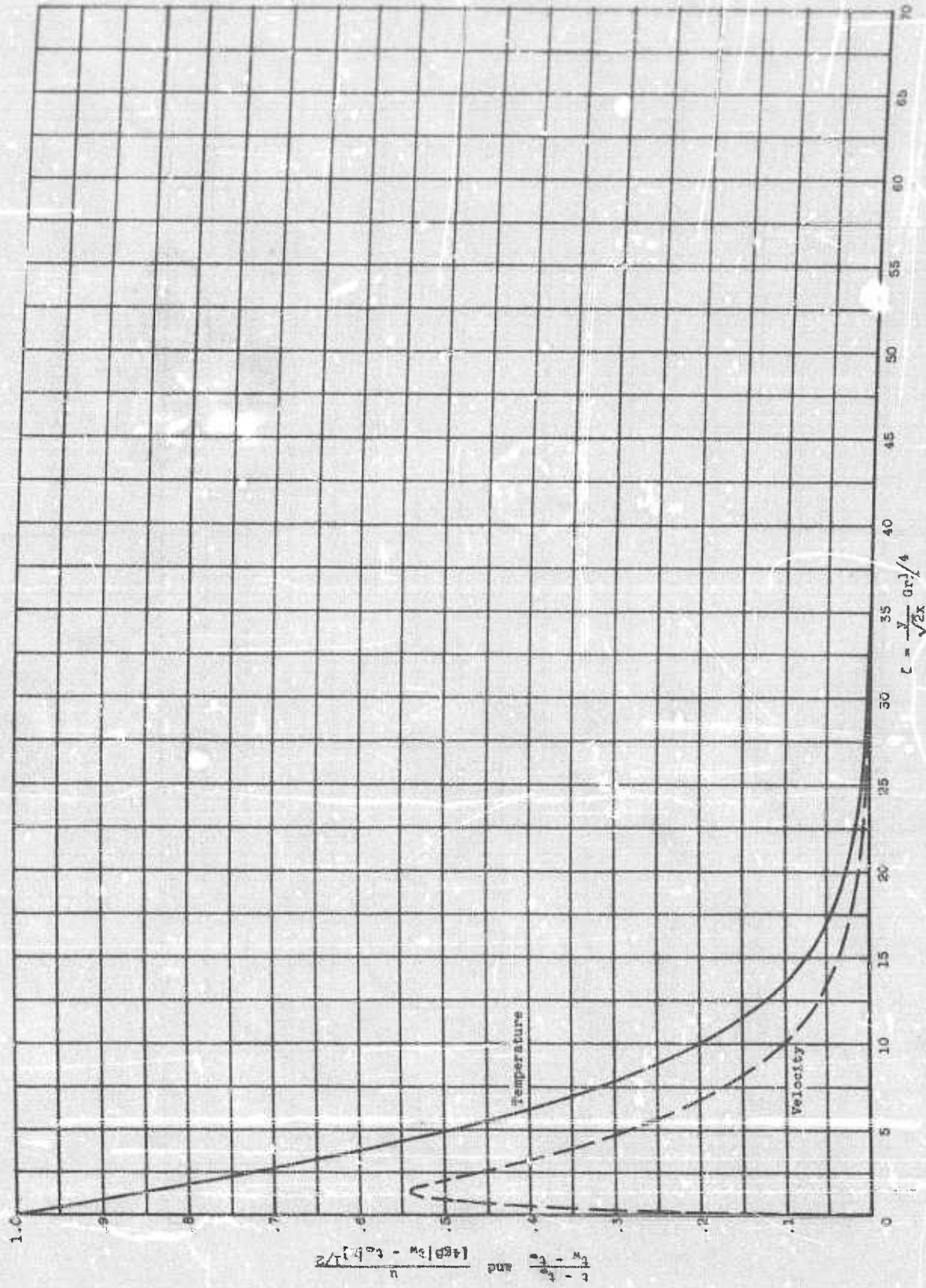
(b) Uniform-heat-flux situation.

Figure 2. - Forced-convection heat-transfer characteristics.

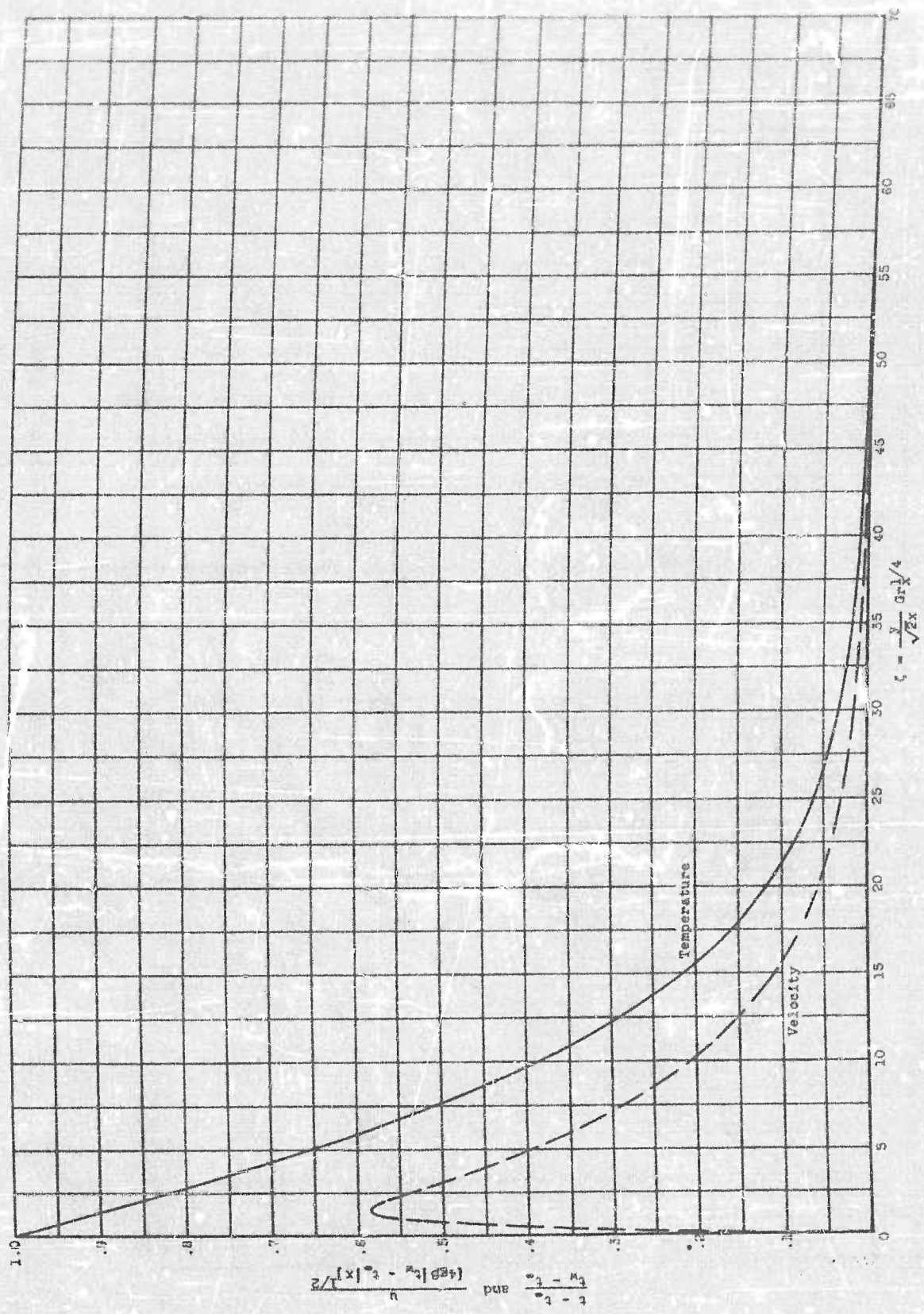


(a) Prandtl number, 0.03.

Figure 3. - Free-convection temperature and velocity profiles.



(b) Prandtl number, 0.02. Figure 3. - Continued. Free-convection temperature and velocity profiles.



(c) Prandtl number, 0.008.
Figure 3. - Continued. Free-convection temperature and velocity profiles.



(d) Prandtl number, 0.003.
 Figure 3. - Concluded, Free-convection temperature and velocity profiles.

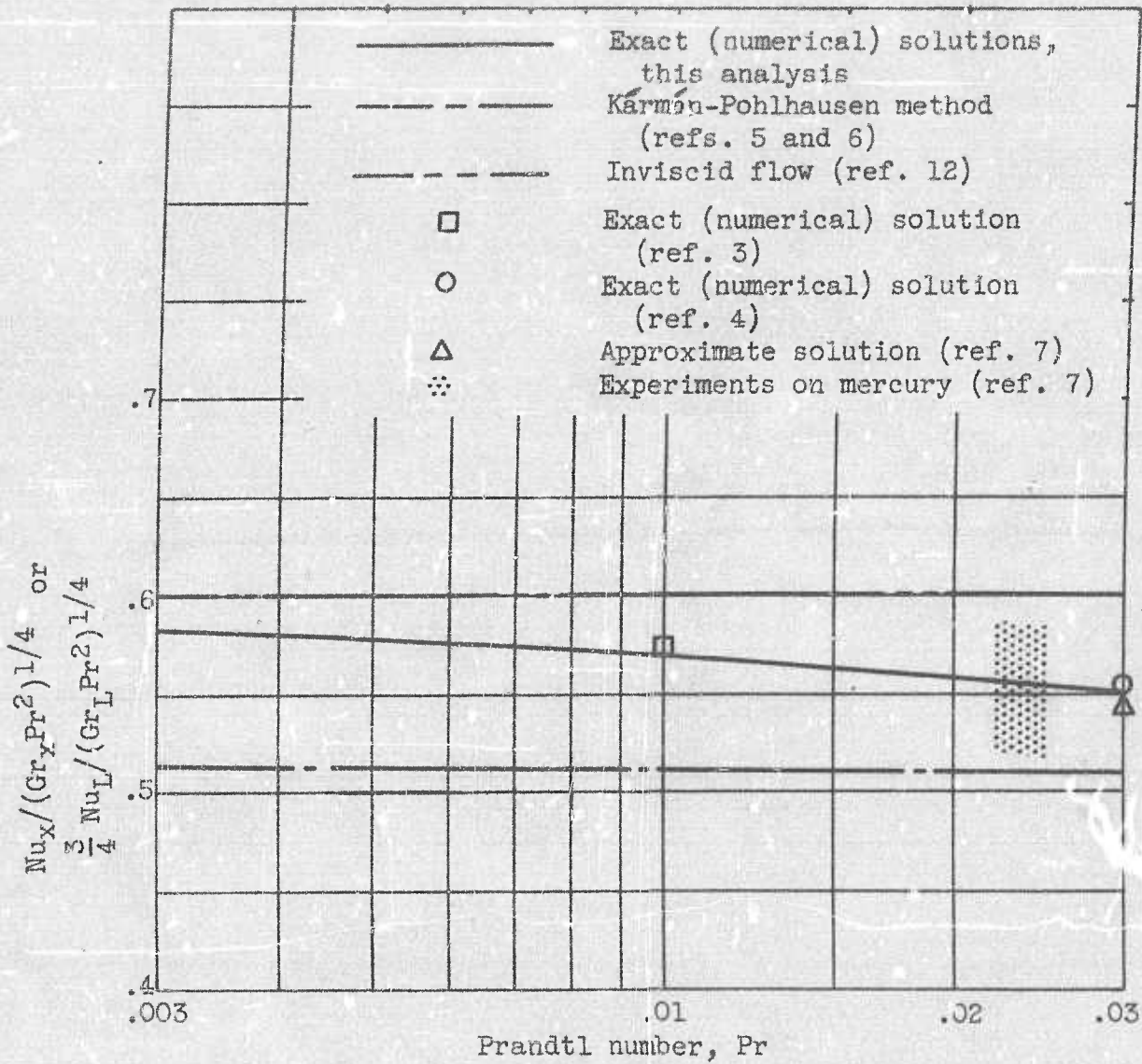


Figure 4. - Free-convection heat-transfer characteristics.