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Methods Of Calculating The Effects Of AM Interference On AM Systems

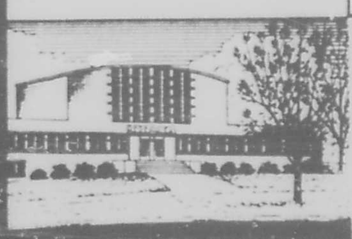
Interim Technical Note No. 3
Project No. A-345

By
Wm. B. Jones, Jr.

Prepared for
Rome Air Development Center
Air Research and Development Command
United States Air Force Base, New York

Contract No. AF 30 (602)-1789

20 March 1959



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METHODS OF CALCULATING THE EFFECTS OF AM INTERFERENCE ON AM SYSTEMS

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ABSTRACT

Methods for calculating the audio frequency output of amplitude demodulators when the input signal is the linear sum of two amplitude modulated carriers are described for the envelope detector, square-law demodulator and product demodulator. Defining the larger carrier as the desired signal, various audio frequency terms which result from the presence of the interference can be calculated. It is concluded that the carrier, which itself conveys no information, contributes greatly to the interference. Other questions concerning the significance and usefulness of these calculations and some related topics are summarized.

I. INTRODUCTION

The problem considered here is the calculation of audio output from various demodulators when the received signal consists of an amplitude modulated carrier and an interfering signal. The interfering signal is assumed to be amplitude modulated; several other kinds of interference could be represented as special cases of the amplitude modulated interference considered.

The received signal at the input to the demodulator has the form:

$$v = A(t) \cos \omega_1 t + B(t) \cos \omega_2 t \ .$$

The first term will be defined as the desired signal and the second as interference. In general ω_1 and ω_2 are different frequencies.

The significant output parameter is audio-frequency power. The power output of the demodulator corresponding to the desired signal and to other audio-frequency signals is of interest. The ratio of power in the desired signal to total interfering power is a rough measure of the quality of the demodulated signal.

II. CONCLUSIONS

The amount of interference produced between two amplitude modulated carriers is primarily dependent upon the ratio of the interfering carrier to the desired carrier. The degree of modulation on the interfering carrier is of secondary importance. In all three types of demodulators considered here, the total interfering audio power becomes equal to the desired audio power when the carrier ratio K is approximately equal to the degree of modulation of the desired signal. Most of this interfering power is in the carrier beat tone and relatively little is in the demodulated interfering signal.

In an envelope detector when K is less than one, the demodulated interference is smaller than it would be if demodulated alone by a factor of approximately $K/2$.

In the product demodulator and the square-law demodulator with enhanced carrier, the total interfering audio power is proportional to the total power in the interfering signal. The total desired audio power is proportional to the power in the sidebands of the desired signal; the transmitted carrier contributes nothing (directly) to the audio output.

These results lead to the conclusion that in these cases, the power transmitted in the carrier is essentially wasted and that, in the case of the interfering signal, the interference would be significantly reduced if the carrier were not transmitted. This conclusion is based solely on the consideration of mutual interference between two signals. It is assumed that the sideband power remains the same with or without the carrier. The practical problems of regenerating a carrier at the receiver are not considered to be a factor in this discussion.

III. RECOMMENDATIONS

The results described in this Note leave several questions unanswered. The use of signal and interfering audio power as a measure of performance leaves much to be desired. The principal merit of this approach is that these quantities can be calculated in several cases of interest. The ultimate measure of performance for a voice communication system is the intelligibility of the received signal; this is normally measured with articulation tests. The effects of interfering signals could be predicted in a more meaningful way if interfering power, such as might be calculated using the results of this Note, could be related to Articulation Scores or intelligibility in a reasonably direct manner.

The analysis of the envelope detector is valid for only a limited range of relative amplitudes of the two signals. When the effective modulation index of the sum of the desired and interfering signals exceeds unity, distortion of the envelope results. This probably would cause discontinuities in the slopes of the curves of Figures 1, 2, and 3. It would be desirable to study this effect experimentally to indicate if some kind of linear or nonlinear extrapolation of the calculated results is reasonable.

The analysis of the envelope detector is limited to simple tone modulation of each carrier. Experimental evaluation of the usefulness of this analysis for calculating the output of a system using complex speech waveforms and other kinds of interfering signals seems desirable.

If suppressed carrier transmission is considered (for the purpose of reducing mutual interference), then the problem of regenerating a carrier at the receiver cannot be ignored. This may require a receiver local oscillator with Automatic Frequency Control (AFC) which is itself subject to some degradation by the effects of interference. Then the behavior of the receiver AFC in the presence of interference must be evaluated.

IV. DISCUSSION

A. Envelope Detector

The received signal can be represented as:

$$v = A \cos \omega_1 t + B \cos (\omega_1 + \gamma)t$$

$$= V \cos(\omega_1 t + \varphi)$$

where

$$V = \sqrt{A^2 + B^2 + 2AB \cos \gamma t} \quad .$$

The angle φ is also a function of A, B, and γ, but is not needed in the following analyses of AM demodulators.

An ideal envelope detector produces an output signal which is a faithful reproduction of the envelope of the input signal. Thus, with an input voltage

$$v_i = V(t) \cos (\omega_1 t + \varphi)$$

the output should be

$$v_o = \alpha V(t) \ ,$$

where α is a constant, assumed hereafter to be unity. The problem is then to try to find a spectral analysis of V(t) in order to identify the various audio terms in the output.

No general solution for the spectrum of V(t) is known. However, several methods are available for use over limited ranges of the signal amplitudes. Perhaps the most useful for the present purposes is described by Aiken.*

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* C. H. Aiken, "Theory of the Detection of Two Modulated Waves by a Linear Rectifier," Proc IRE 21, 601 (1933).

Aiken's equation (62), which is summarized in the Appendix of this report, makes possible the calculation of the d-c term and the amplitudes of the two modulating frequencies, the carrier beat frequency, and several harmonics and intermodulation products of these three frequencies. Using this equation and the equation for total envelope power developed below, the audio terms of interest can be found for a limited range of input signal amplitudes.

The amplitudes of the two compenents of the input signal are

$$A(t) = E(1 + M \cos Pt)$$

and

$$B(t) = e(1 + m \cos pt) .$$

The voltage output of the envelope detector is

$$V(t) = \sqrt{A^2(t) + B^2(t) + 2A(t) \cdot B(t) \cos \gamma t} .$$

The power output of the detector is $V^2(t)$. The "instantaneous power" varies as the square of the instantaneous amplitude. The average power is the constant term in the expansion of $V^2(t)$. This can be calculated from the above equations:

$$V^2(t) = A^2(t) + B^2(t) + 2A(t) \cdot B(t) \cos \gamma t .$$

The third term gives no constant terms except in the trivial case where two of these frequencies involved are equal. Squaring A(t) and B(t) and collecting constant terms, the result is

$$\overline{V^2} = E^2(1 + \tfrac{1}{2} M^2) + e^2(1 + \tfrac{1}{2} m^2) .$$

Power is proportional to $\overline{V^2}$; the constant of proportionality will be taken as unity. The carrier ratio is

$$K = e/E .$$

With these definitions,

$$P_{total} = E^2[(1 + \tfrac{1}{2} M^2) + K^2(1 + \tfrac{1}{2} m^2)] .$$

This total power includes the power due to the average or d-c component of V(t) as well as the various audio power terms of interest. By using Aiken's equation to calculate the d-c power and audio power at the desired frequency and some interfering audio frequencies, the total audio power, total interfering audio power, and a partial breakdown into specific interfering terms can be found. This procedure is illustrated by the following example:

<u>Example</u>

When M = m = 0.3 and K = 0.2 the total envelope power is

$$\frac{P_t}{E^2} = 1 + \tfrac{1}{2}\,0.3^2 + 0.2^2\left(1 + \frac{0.3^2}{2}\right)$$
$$= 1.0868$$

Using Aiken's equations the voltage terms of interest are:

$$V_{dc} = 1.01102$$
$$V_P = 0.29658$$
$$V_p = 0.00634$$
$$V_\Delta = 0.19869$$

The d-c power is V_{dc}^2 and the a-c power is, in each case, $\frac{1}{2}V_{ac}^2$. Thus

$$P_{dc} = 1.02217$$
$$P_P = 0.04400$$
$$P_p = 0.00002$$
$$P_\Delta = 0.01974$$

The total audio power is

$$P_{audio} = 1.0868 - 1.0222$$
$$= 0.0646$$

The audio power terms of principal interest are the desired signal P_p and the carrier beat P_Δ. The total of these two is 0.06374. The total remaining audio power is then 0.00086. This includes the demodulated interference and the harmonics and many sum and difference frequency terms.

In this example, the desired signal contains 68 per cent of the total audio power, the carrier beat note 30.6 per cent, and all other terms a total of 1.4 per cent.

Curves showing the variation of the more significant audio power terms with carrier ratio K are plotted in Figures 1, 2 and 3. In each case, the calculations on which the curves are based are valid only for the range of K indicated on the Figures.

Several observations can be made from an inspection of these curves. Perhaps the most striking is that the total interfering audio power is only slightly greater than the power in the carrier beat tone. It is also apparent that the amplitude of the desired signal decreases as the interfering carrier is made larger, and that the interfering audio power becomes equal to the desired signal power when K is approximately equal to M. These observations can be verified by a close inspection of Aiken's equations in the Appendix.

Another effect which can be pointed out is the de-emphasis of the interfering modulation due to the presence of the larger signal. The amplitude of the demodulated interfering term is reduced by a factor of approximately K/2. In the numerical example above, V_p is 0.00634. If this same signal with carrier amplitude K and modulation index m were demodulated alone in a clear channel, the V_p would be Km = 0.06. In this case it is slightly greater than $0.06 \cdot \frac{K}{2}$. The fact that

$$V_p \doteq \frac{K^2 m}{2}$$

can be verified by manipulating Aiken's results.

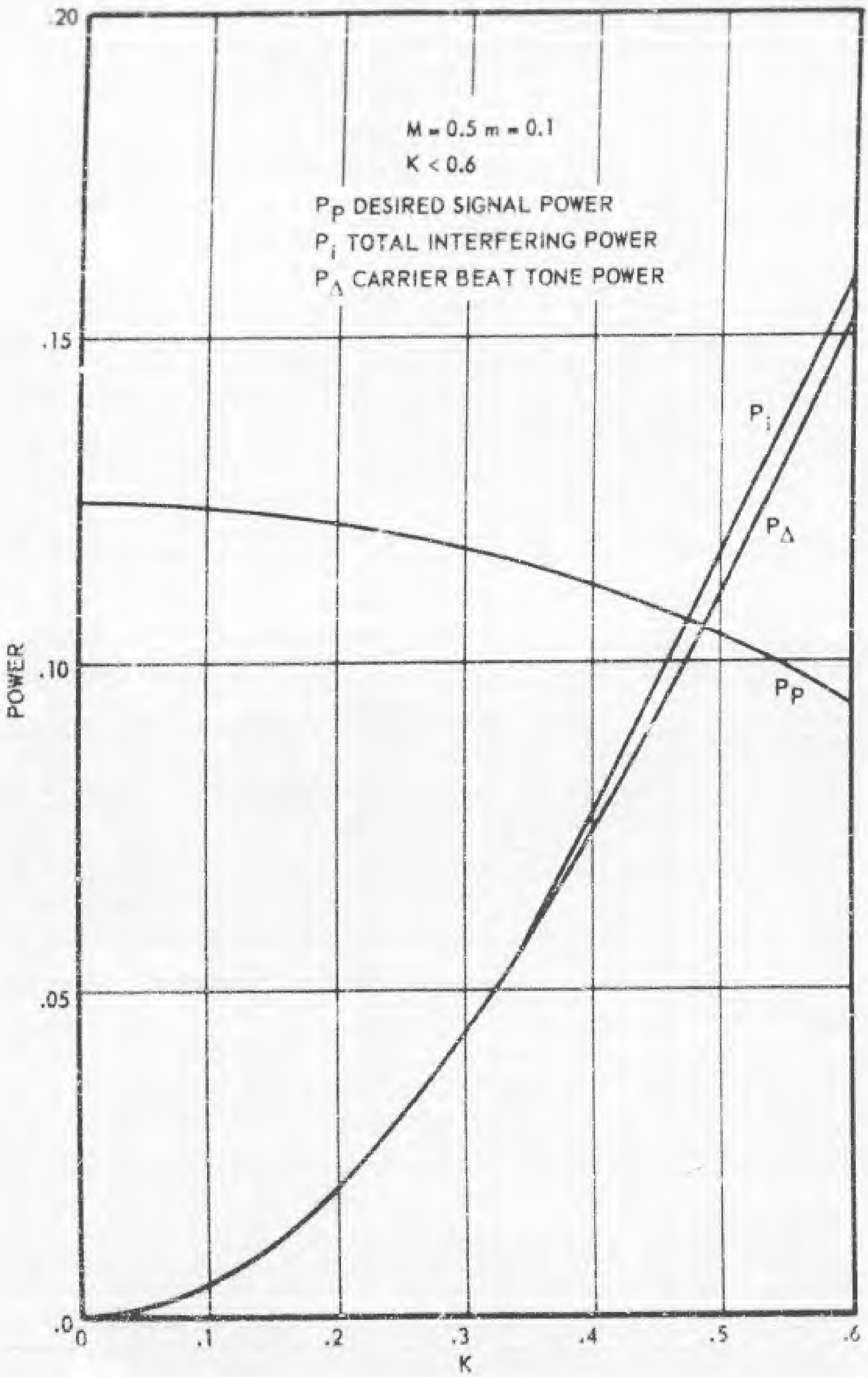

Figure 1. Audio Frequency Power Versus Carrier Ratio.

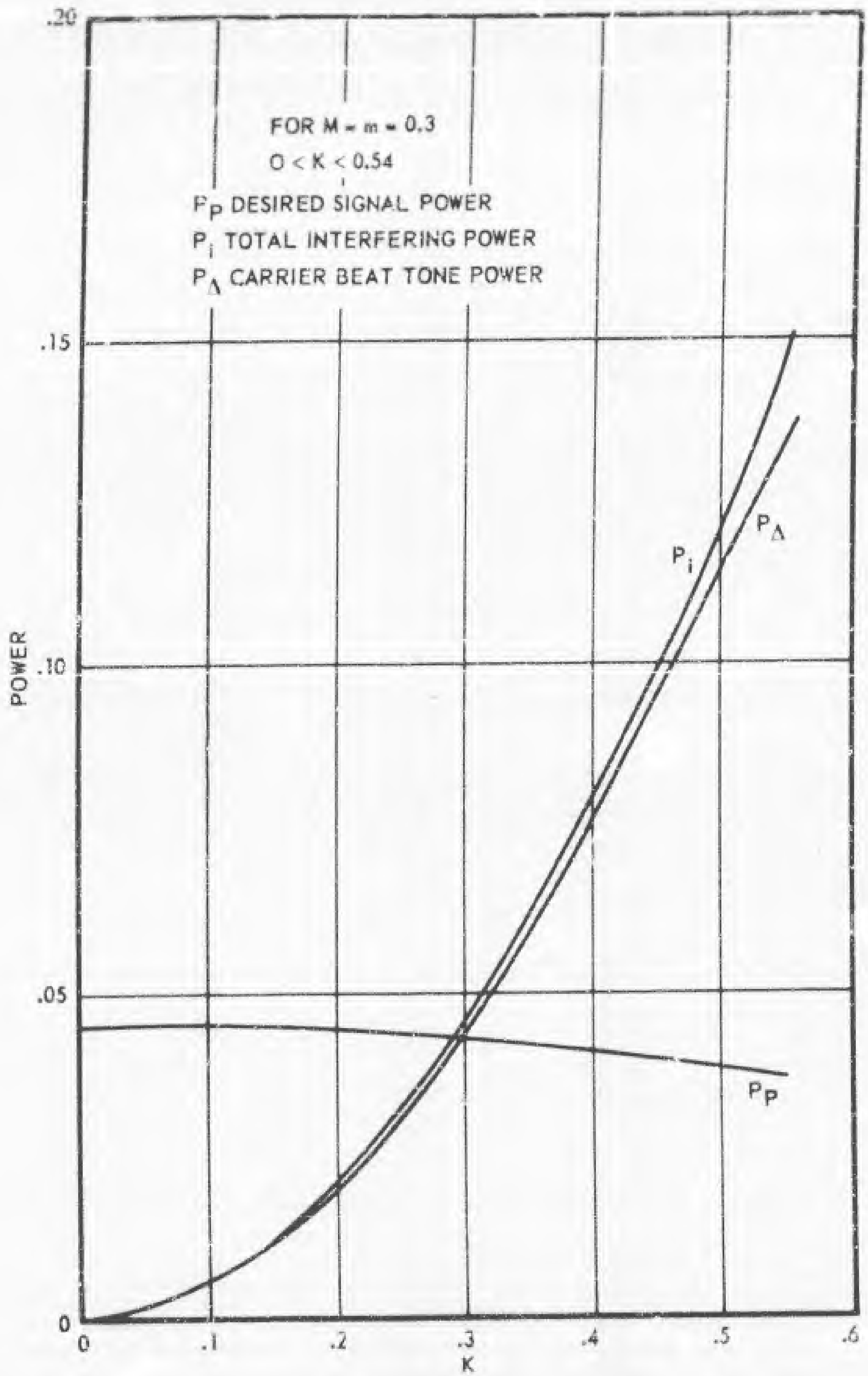

Figure 2. Audio Frequency Power Versus Carrier Ratio.

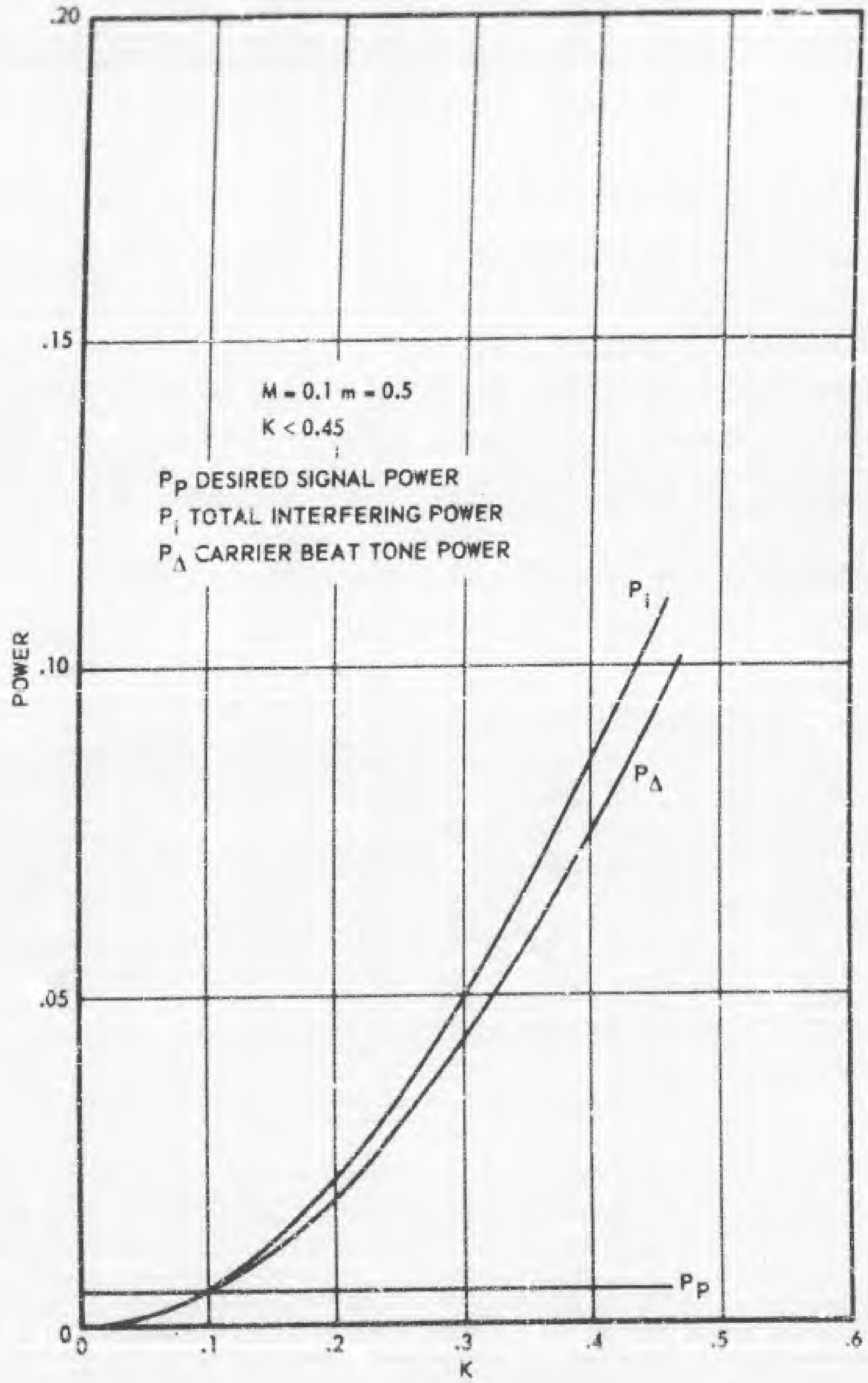

Figure 3. Audio Frequency Power Versus Carrier Ratio.

B. Square Law Demodulator

1. Theory of Operation

An amplitude modulated signal of the form

$$v = A(t) \cos \omega_1 t$$

is applied to a square-law device which generates v^2. This v^2 includes the desired envelope function and also some d-c, r-f and audio-distortion terms.

Let A(t) be represented by a cosine series. Then

$$v = E(1 + \sum_j m_j \cos P_j t) \cos \omega_1 t ,$$

and

$$v^2 = E^2(1 + \sum_j m_j \cos P_j t)^2 \cos^2 \omega_1 t$$

$$= E^2\{1 + 2 \sum_j m_j \cos P_j t$$

$$+ \sum_{\substack{j \\ j \neq k}} \sum_k m_j m_k [\cos(P_j + P_k)t + \cos(P_j - P_k)t]$$

$$+ \frac{1}{2} \sum_j m_j^2 (1 + \cos 2P_j t)\}(\frac{1}{2} + \frac{1}{2} \cos 2\omega_1 t) .$$

After the d-c and r-f terms have been filtered out, there will remain

$$2v^2/E^2 = 2 \sum_j m_j \cos P_j t$$

$$+ \sum_{\substack{j \\ j \neq k}} \sum_k m_j^2 \cos 2P_j t$$

$$+ \frac{1}{2} \sum_j m_j^2 \cos 2P_j t .$$

The first term represents the desired A(t); the others repr sent distortion generated by the demodulation.

The amplitude of the desired signal components are proportional to the m_j . The distortion terms are proportional to m_j^2 or $m_j m_k$; they are of the order of m^2 . Then if the m_j's are all very small, the distortion terms will be negligible in comparison with the signal.

The m_j's can be made very small by artificially boosting the carrier amplitude in the receiver before demodulation. The effective m's at the demodulator can thus be made small enough to avoid significant distortion. This is a more or less well-known technique, known as an "enhanced carrier" demodulator. It is shown in the following paragraph that enhanced carrier demodulation is also useful for reducing some effects of interference.

2. Effects of Interference

Now let us calculate the output of a square-law demodulator whose input v is the sum of a true signal and an amplitude modulated interfering carrier:

$$v = A(t) \cos \omega_1 t + B(t) \cos \omega_2 t .$$

In order to keep the equations reasonably compact, the envelope functions will be kept simple. After developing expressions for this simple case, the results for more general envelope functions will be stated without reproducing the complete development. For this purpose let

$$A(t) = E(1 + M \cos Pt) ,$$

$$B(t) = e(1 + m \cos pt) .$$

The square-law demodulator will generate the following audio terms:

$$V^2/E^2 = \tfrac{1}{2}(2M \cos Pt + \tfrac{1}{2} M^2 \cos 2Pt)$$

$$+ \tfrac{1}{2}(2K^2 m \cos pt + \tfrac{1}{2} K^2 m^2 \cos 2pt)$$

$$+ K \cos \Delta t$$

$$+ \tfrac{1}{2} Km[\cos(\Delta + p)t + \cos(\Delta - p)t]$$

$$+ \tfrac{1}{2} KM[\cos(\Delta + P)t + \cos(\Delta - P)t]$$

$$+ \tfrac{1}{4} KMm[\cos(\Delta + P + p)t + \cos(\Delta - P - p)t$$

$$+ \cos(\Delta + P - p)t + \cos(\Delta - P + p)t] ,$$

where $\Delta = \omega_1 - \omega_2$

and $K = e/E$

This includes all of the audio terms; depending on the values of Δ, P and p, some of these terms may fall outside the audio band of interest.

To use an enhanced carrier, we make E very large. Then M and K become small; m is not affected. Every term in the expression above has M or K in the coefficient. Those terms with M^2, K^2, or MK are negligible in comparison with terms with first order coefficients in M and K. The significant audio terms are then

$$V(t) = V \cos Pt + K \cos \Delta t$$

$$+ \tfrac{1}{2} Km[\cos(\Delta + p)t + \cos(\Delta - p)t] .$$

This expression includes the desired signal and an interference term for each frequency component of the interfering signal. In each case, the term due to interference has amplitude proportional to the appropriate frequency component, and audio frequency equal to the difference frequency between the interference and the enhanced carrier.

The signal power P_P and the interfering power P_I in the audio output are easily calculated:

$$P_P = \frac{1}{2} M^2$$

$$P_I = \frac{1}{2} K^2(1 + \frac{1}{2} m^2) \ .$$

When A(t) and B(t) are represented by the more general expressions

$$A(t) = E(1 + \sum_j M_j \cos P_j t)$$

$$B(t) = e(1 + \sum_k m_k \cos p_k t) \ ,$$

the equivalent results are as follows. The audio output of the enhanced carrier demodulator is

$$V(t) = \sum_j M_j \cos P_j t + K \cos \Delta t$$

$$+ \frac{1}{2} K \sum_k m_k [\cos(\Delta + p_k)t + \cos(\Delta - p_k)t].$$

The P_P and P_I are

$$P_P = \frac{1}{2} \sum_j M_j^2$$

$$P_I = \frac{1}{2} K^2(1 + \sum_k m_k^2) \ .$$

It is interesting to note that the total signal power is proportional to the total sideband power in the modulated carrier; the manner in which this power is divided among the various sideband-pairs makes no difference.

Similarly, the interfering power due to sidebands of the interfering signal may be in one sideband-pair or distributed over many with the same result.

C. Product Demodulator

In the product demodulator, or synchronous detector, the received signal is multiplied by a locally generated carrier, and the audio terms which result are removed by filtering. Using the same notation as before, let

$$v = A(t) \cos \omega_1 t + B(t) \cos \omega_2 t ,$$

$$A(t) = E(1 + \sum_j M_j \cos P_j t)$$

$$B(t) = e(1 + \sum_k m_k \cos p_k t) .$$

The local carrier is $E_c \cos \omega_1 t$. The audio terms in the product $v \times E_c \cos \omega_1 t$ are

$$\frac{1}{2} E E_c \sum_j M_j \cos P_j t + \frac{1}{2} e E_c \cos \Delta t$$

$$+ \frac{1}{4} e E_c \sum_k m_k [\cos(\Delta + p_k)t + \cos(\Delta - p_k)t] .$$

Normalizing, this can be written

$$V(t) = \sum_j M_j \cos P_j t + K \cos \Delta t$$

$$+ \frac{1}{2} K \sum_k m_k [\cos(\Delta + p_k)t + \cos(\Delta - p_k)t] ,$$

which is exactly the expression derived for the enhanced carrier square-law demodulator.

The square-law and product demodulators considered are two ways of using a locally generated carrier to demodulate the received signal. Since they give equivalent results, the choice between them can be made on the basis of convenience, or other secondary considerations.

Respectfully submitted:

W. W. Wright
Project Director

Approved:

J. E. Boyd, Director
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V. APPENDIX

Notes from "Theory of the Detection of Two Modulated Waves by a Linear Rectifier," C. H. Aiken, Proc IRE 21 601 (1933).

The following is a summary of one of the results from Aiken's paper. No effort is made here to reproduce or summarize the logical steps of the development of this result. It is presented solely to provide a method of calculating the spectrum of $V(t)$. For the more complete equation and for the development of the equation, reference should be made to the original paper.

Using the notation introduced in Section IV, the two waves are

$$A = E(1 + M \cos Pt) \cos \omega_1 t$$

$$B = e(1 + m \cos pt) \cos(\omega_1 + \gamma)t .$$

The envelope function V will be

$$V = \sqrt{A^2 + B^2 - 2AB \cos \Delta t}$$

where Δ is introduced in place of γ so as to make the last term negative. This has no effect on the result but is a convenience in the development.

As before, K is the carrier ratio. The principal limitation of Aiken's result is

$$K = e/E < \frac{1 - M}{1 + m} .$$

This limits K to small values if either wave has a large modulation index.

Aiken's equation (62) may be described as follows:

$$\frac{V}{E} = V_{dc} + V_P \cos Pt + V_p \cos pt - V_\Delta \cos \Delta t + \ldots$$

$$V_{dc} = 1 + \frac{K^2 G_{20} a_{10}}{4 \times 2 \times 2} + \frac{K^4 G_{40} a_{30}}{64 \times 2 \times 2} + \frac{K^6 G_{60} a_{50}}{256 \times 2 \times 2} + \ldots$$

$$V_P = M + \frac{K^2 G_{20} a_{11}}{4 \times 2} + \frac{K^4 G_{40} a_{31}}{64 \times 2} + \frac{K^6 G_{60} a_{51}}{256 \times 2} + \ldots$$

$$V_p = \frac{K^2 G_{21} a_{10}}{4 \times 2} + \frac{K^4 G_{41} a_{30}}{64 \times 2} + \frac{K^6 G_{61} a_{50}}{256 \times 2} + \ldots$$

$$V_\Delta = K - \frac{K^3 G_{30} a_{20}}{8 \times 2 \times 2} - \frac{K^5 G_{50} a_{40}}{64 \times 2 \times 2} - \frac{5K^7 G_{70} a_{60}}{1024 \times 2 \times 2} + \ldots$$

Aiken's paper also gives amplitude equations for some of the harmonic and intermodulation frequencies. These terms are not needed here.

The a_{qn} and G_{qn} of these equations are tabulated below:

a_{qn}

q \ n	0	1
1	2α	$-2\alpha\mu$
2	$2\alpha^3$	$-2M\alpha^3$
3	$(2 + M^2)\alpha^5$	$-3M\alpha^5$
4	$(2 + 3M^2)\alpha^7$	$-M(4 + M^2)\alpha^7$
5	$2(1 + 3M^2 + \frac{3M^4}{8})\alpha^9$	$-5M(1 + \frac{3M^2}{4})\alpha^9$

$$\alpha = \frac{1}{\sqrt{1 - M^2}} \qquad \mu = \frac{M\alpha}{1 + \alpha}$$

G_{qn}

q \ n	0	1
1	2	m
2	$2 + m^2$	$2m$
3	$2 + 3m^2$	$3m(1 + \frac{m^2}{4})$
4	$2 + 6m^2 + \frac{3m^4}{4}$	$4m(1 + \frac{3m^2}{4})$
5	$2 + 10m^2 + \frac{15m^4}{4}$	$5m(1 + \frac{3m^2}{2} + \frac{m^4}{8})$
6	$2 + 15m^2 + \frac{45m^4}{4} + \frac{5m^6}{8}$	$6m(1 + \frac{5m^2}{2} + \frac{5m^4}{8})$

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