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THEORETICAL ANALYSIS OF RADIOMETER PERFORMANCE

Research and Development Technical Report USNRDL-TR-

U.S. NAVAL RADIOLOGICAL DEFENSE LABORATORY

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THEORETICAL ANALYSIS OF RADIOMETER PERFORMANCE

Research and Development Technical Report USNRDL-TR-311
NS-081-001

13 January 1959

by

H.G. Ferris

Physics and Mathematics

Technical Objective
AW-7

Thermal Radiation Branch
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ABSTRACT

A theoretical analysis of the thermal behavior of water-cooled thin foil radiometers is carried out. The expression for the heat balance in the circular foil of the radiometer is:

$$\frac{1}{k}\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q}{kd} = \frac{\rho c}{k}\frac{T}{t}.$$

The steady state solution, $T/t = 0$, subject to certain boundary conditions, yields the thermal sensitivity: $\Delta/q = R^2/4k_0 d \left(\frac{^\circ C}{Cal/cm^2 \cdot sec}\right)$.

This relationship when applied to the three radiometers in common use at NRDL indicates that they operate linearly over their entire ranges (0 to 10, 0 to 50, and 0 to 100 Cal/cm^2) with a maximum error of 2 percent when maximum irradiance is incident on the foil. Considering k constant, the heat balance equation becomes:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q}{kd} = \frac{1}{\kappa}\frac{\partial T}{\partial t}$$

Solving, subject to the same boundary conditions, by means of the Laplace Transformation, an expression for reaction time results. $\tau = \frac{0.416 R^2}{\kappa}$. Applying this relationship to the three NRDL instruments yields reaction times of 0.016, 0.004, and 0.004 sec. respectively.

SUMMARY

The Problem

High intensity beams of thermal radiation as a function of time have, in recent years, been measured through the use of thin foil radiometers. One of the principles of operation of these instruments has been that a heat sink in the instrument remains at a constant temperature throughout the measurement of a thermal flux. The thin foil changes in temperature and hence measures the flux. However, it is believed that in actual field tests involving high intensity thermal radiation, the heat sink does not remain at a constant temperature.

Findings

The present study is a theoretical analysis of the thermal behavior of a water-cooled radiometer with particular consideration given to (1) the steady state sensitivity of the thin foil and (2) the reaction time of the foil (time to come to 90 percent of steady state temperature above its heat sink after onset of irradiance). The three radiometers in common use at NRDL (1) operate linearly over their entire ranges (0 to 10, 0 to 50, or 0 to 100 Cal/cm^2) with a maximum error of less than 2 percent when maximum irradiance is incident on the foil; and (2) have reaction times of 0.016, 0.004 and 0.004 sec., respectively.

ADMINISTRATIVE INFORMATION

I. BACKGROUND OF WORK

During FY 1951, the U. S. Naval Radiological Defense Laboratory was given the responsibility by the Armed Forces Special Weapons Project to develop instrumentation for measuring the irradiances and thermal exposures for nuclear detonations. The field radiometer that was developed under this program was found inadequate for laboratory measurements. The need for a reliable instrument for measuring the spot distribution of thermal irradiance in the focal plane was mandatory for the successful prosecution of laboratory-type programs.

The water-cooled radiometer was then conceived and developed by the Thermal Radiation Studies (now Thermal Radiation Branch), Nucleonics Division, Scientific Department, in approximately October 1951 for the primary purpose of continually monitoring the NRDL 36" thermal source. Subsequent developments of the water-cooled radiometer led to fabrication of improved water-cooled radiometers designed for determining the spot distribution of the NRDL 36" and modified Mitchell thermal sources capable of simulating thermal pulses produced by a nuclear detonation. The 36" and Mitchell thermal sources are capable of producing a range of energy levels from 3/8" spot diameter at approximately 100 cal/cm^2/sec and a range from [illegible] diameter spot at intensities from 28 cal/cm^2/sec to 15 cal/cm^2/sec. The water-cooled radiometer has been utilized as a secondary standard in the calibration of the NRDL thermal sources.

Water-cooled radiometers have been developed, fabricated and calibrated by this laboratory for use by virtually all Department of Defense organizations that are engaged in the calibration of thermal sources and solar furnaces.

This report is concerned with a refinement of the theoretical analysis of the radiometer for both the steady and transient states for the measurement of thermal irradiance.

II. AUTHORIZATION AND FUNDING

The work discussed in this report was initially sponsored by the Bureau of Ships, under Project Number NS-081-001, entitled "The Effects of Atomic Warheads and Radiological Shielding," Subtask 3.4, "Department of Laboratory Sources of Thermal Radiation," Technical Objective AW-7. Financial support

support of this type of program has been continuously funded by the Armed Forces Special Weapons Project.

III. DESCRIPTION OF WORK

The work reported herein is concerned with the application of the LaPlace Transformation Theory to the solution of the transient Heat Flow Problem. The solutions to the problem give a measure of the sensitivity of the water-cooled radiometer in terms of its physical dimension as well as the response time required to reach 90 percent of the steady state condition.

INTRODUCTION

Thin foil radiometers for the measurement of high intensity beams of thermal radiation as a function of time have come into increasing use in the determination of the flux from solar furnaces and nuclear detonations, as well as from laboratory sources of high intensity thermal radiation. One such type of instrument is shown in Fig. 1. It consists basically of a thin foil mounted over a hole in a heat sink which is maintained at a constant temperature by a circulating stream of water. The temperature differential between the center and the edge of the foil is a measure of the incident flux. This temperature differential is measured by means of a silver-constantan thermocouple which generates an electrical voltage corresponding to the temperature differential developed. Another type of foil radiometer, developed particularly for use in field tests, is provided with a large copper block for a heat sink having no water cooling system. The relatively large thermal capacity of the copper block tends to maintain the temperature of the sink constant with respect to that of the foil when the incident irradiance is not at too high a level. However, it is believed that in actual field tests involving high intensity thermal radiation, e.g., nuclear weapons tests, the temperature of the copper block does not, in fact, remain constant. This provides a source of error which is difficult to estimate.

In the present study a theoretical analysis of the thermal behavior of an instrument of the water-cooled type is undertaken in which particular consideration is given to the steady state sensitivity and to the reaction time, which for present purposes is defined as the time for the foil to come to 90 percent of its full steady state temperature above that of the sink after the onset of irradiance. Any effects on the performance of the instrument due to the reaction of the thermocouple are entirely neglected. However, it is noted that these effects may not be negligible, and hence the actual performance of any instrument of the type discussed may differ somewhat from its performance herein predicted.

Consider a circular foil of thickness, d, and radius, R, which is in contact at its circumference with a large heat sink supposedly maintained at a constant temperature, T_O. This assumption represents quite accurately the physical situation existing in radiometers of the

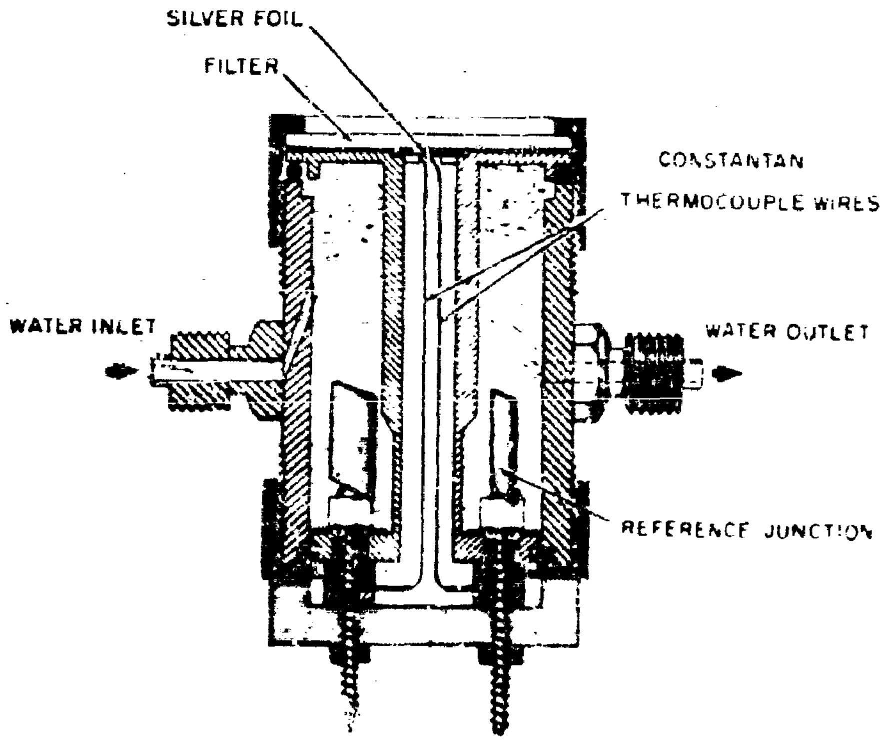

Fig. 1. Thin foil radiometer for measurement of high intens[illegible] of thermal radiation as a function of time.

water-cooled type. The foil is assumed to absorb thermal radiation of intensity, q, on one face, beginning at a particular instant of time. To obtain the differential equation for the temperature at any point in the foil, it is necessary to consider the heat balance for an elemental ring of the foil material of radius, r, and width, δr, as shown in Fig. 2. Neglecting any temperature gradient between faces of the foil, as well as heat losses from either face, the heat balance for the elemental ring can be expressed by the following equation:

$$\frac{\partial T}{\partial t}\cdot 2\pi r\cdot\delta r\cdot d\rho c = 2\pi r\cdot\delta r\cdot q - \frac{\partial T}{\partial r}\cdot k\cdot 2\pi r d$$
$$+ \left\{k\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)\delta r\right\} 2\pi (r+\delta r) d. \qquad (1)$$

Here T is foil temperature at radius, r, thickness, d, and time, t, measured from the instant at which irradiation begins, k is the thermal conductivity of the foil material, ρ is its density, and c the specific heat. Equation (1) ultimately simplifies to

$$\frac{1}{k}\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q}{kd} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \qquad (2)$$

and the solution, T, must be subject to the boundary conditions

$$T = T_o, \text{ at } t = 0, \text{ all } r, \qquad (3)$$
$$T = T_o, \text{ at } r = R, \text{ for } t>0. \qquad (4)$$

SENSITIVITY

To discuss the sensitivity of a circular foil radiometer, we need consider only the steady state solution to Eq (2), i.e., the solution for which $\partial T/\partial t = 0$ subject to the boundary condition (4). If we allow for variation of thermal conductivity according to the relation,

$$k = k_o(1 + aT), \qquad (5)$$

then with $\partial T/\partial t = 0$, Eq (2) becomes

$$\frac{d}{dr}\left\{(1 + aT)\frac{dT}{dr}\right\} + \frac{1 + aT}{r}\frac{dT}{dr} + \frac{q}{k_o d} = 0, \qquad (6)$$

or

$$\frac{1}{r}\frac{d}{dr}\left\{r(1 + aT)\frac{dT}{dr}\right\} + \frac{q}{k_o d} = 0. \qquad (7)$$

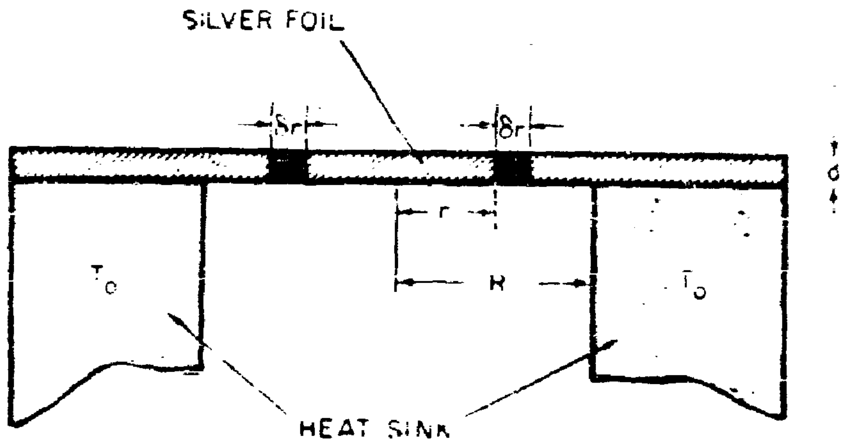

Fig. 2. Schematic diagram of thin foil mounted over a hole in a heat sink. For analysis of heat balance, an elemental ring of foil material of radius r and width δr is considered.

Upon two integrations, the general solution is found to be

$$T + \frac{\alpha T^2}{2} + \frac{qr^2}{4k_o d} = C_1 \log_e r + C_2 , \tag{8}$$

where C_1 and C_2 are arbitrary constants. Since T cannot become infinite at $r = 0$, we must specify that $C_1 = 0$.

The constant C_2 may now be evaluated by imposing the condition that $T = T_o$ at $r = R$, i.e., Eq (4). Thus,

$$C_2 = T_o + \frac{\alpha T_o^2}{2} + \frac{qR^2}{4k_o d} , \tag{9}$$

and the required steady state solution is

$$T - T_o + \frac{\alpha}{2}(T - T_o)(T + T_o) = \frac{q}{4k_o d}(R^2 - r^2) . \tag{10}$$

Defining Δ to be the difference between the steady-state temperature of the center of the foil and that of its circumference, we then have

$$\Delta = T - T_o , \text{ at } r = 0 . \tag{11}$$

and from (10),

$$\Delta\left\{1 + \frac{\alpha}{2}(T + T_o)\right\} = \frac{qR^2}{4k_o d} . \tag{12}$$

This can also be written as

$$\Delta\left\{1 + \alpha T_o + \frac{\alpha\Delta}{2}\right\} = \frac{qR^2}{4k_o d} . \tag{13}$$

Also, if we put

$$T_m = \frac{T_o + T}{2} , \tag{14}$$

then T_m is the arithmetic mean of T_o and T, the temperature at the center, and Eq (12) can be written as

$$\Delta\left\{1 + \alpha T_m\right\} = \frac{qR^2}{4k_o d} . \tag{15}$$

Equation (12), or the alternative forms (13) or (15), defines the sensitivity of a circular foil radiometer in terms of the temperature difference Δ produced by radiation of intensity q. Thus from (13)

$$\text{Sensitivity} = \frac{\Delta}{q} = \frac{R^2}{4k_o d} \quad \frac{1}{1 + \alpha T_o + \frac{\alpha\Delta}{2}} \tag{16}$$

Clearly Δ is not in general a linear function of q unless $\alpha = 0$, in which case there is no variation of thermal conductivity with temperature. Also it is evident that the sensitivity is dependent upon T_0, the temperature of the sink, as well as upon q unless $\alpha = 0$. This seems to have been overlooked in Gardon's Analysis[1].

To investigate more precisely the dependence of Δ on both q and T_0, we must examine the appropriate solution to Eq (13). Since Eq (13) is a quadratic in Δ, there will be two roots of which only one is of physical interest in the present discussion. Writing Eq (13) in the form:

$$\frac{\alpha \Delta^2}{2} + (1 + \alpha T_0)\Delta - q' = 0, \tag{17}$$

where q' stands for $qR^2/4k_0 d$, we find for the two roots of (17)

$$\Delta_1, \Delta_2 = \frac{-(1 + \alpha T_0) \pm \sqrt{(1 + \alpha T_0)^2 + 2\alpha q'}}{\alpha} \tag{18}$$

Physical considerations lead us to retain only that root, Δ_1, in which the positive sign is to be taken before the radical, this being the only root which reduces to zero for $q = 0$.

Introducing the series expansion of the radical and ultimately retaining only terms of the first power in the small quantities αT_0 and $\alpha q'$, we obtain the result:

$$\Delta_1 = q'(1 - \alpha T_0 - \frac{1}{2}\alpha q'), \tag{19}$$

from which the effects on Δ of variations in T_0 and q can be easily ascertained.

At this point it will be advantageous to discuss a specific numerical example. A common instrument in use by the Thermal Radiation Branch at NRDL is the RW-100 radiometer. This is a water-cooled, silver foil instrument designed to cover the range $0 \leq q \leq 100$ Cal/cm²·sec. For this instrument $R = 0.127$ cm and $d = 0.00254$ cm. The values of k_0 and α as listed in the International Critical Tables are $k_0 = 1.001$ Cal/cm·sec·°C, and $\alpha = -1.7 \times 10^{-4}$/°C, which are valid in the temperature range 0 - 100°C.

[1] Gardon, R. Rev. Sci. Inst. p 366 May 1953

For this case we then have:

$$-a T_o = 1.7 \times 10^{-4} \times T_o, \tag{20}$$

$$q' = \frac{R^2 q}{4 k_o d} = \frac{(0.127)^2 q}{(4)(1.001)(.00254)} = 1.59 q, \tag{21}$$

and $$-\frac{1}{2} a q = (0.85)(1.59) \times 10^{-4} q = 1.35 \times 10^{-4} q. \tag{22}$$

Thus, according to Eqs (19) and (20) a change in the sink temperature, T_o, from 0° C to 100° C would only produce a change in Δ_1 of 1.7 percent. However, assuming the value of a to remain about the same for higher temperatures, an increase of T_o to 1000° C could result in as much as 17 percent change in Δ_1 for the same value of q. This is an important fact to keep in mind when using foil radiometers at extremely high ambient temperatures.

Equation (19) indicates that Δ_1 is a quadratic equation in q, when first order terms in aq^2 are retained. However, according to Eq (22), the departure from linearity for the RW-100 radiometer is only 1.35 percent when q has its maximum value of 100 Cal/cm² · sec. For lower values of q the quadratic term is of even less importance.

Summarizing, we may say that the RW-100 radiometer operates linearly over its entire range of 0-100 Cal/cm² · sec with a maximum error of less than 2 percent when maximum irradiance is incident on the foil. With regard to the sink temperature, T_o, it has been shown that variations in T_o of as much as 100° C in the range of 0-100° will produce changes in Δ_1 of less than 2 percent.

The RW-10 and RW-50 are two other radiometers in use by the Thermal Radiation Branch at NRDL designed to cover the ranges $0 \leq q \leq 10$, and $0 \leq q \leq 50$ Cal/cm² · sec, respectively. Like the RW-100 they are both water-cooled, silver foil instruments, for which R = 0.254 cm and d = 0.00127 cm for the RW-10, and R = 0.127 cm and d = 0.00127 cm for the RW-50. As in the case of the RW-100 it can be shown that both the RW-10 and RW-50 radiometers operate linearly over their entire ranges of irradiance with a maximum error of less than 2 percent when the irradiance is a maximum. The effect of T_o is, of course, the same as for the RW-100 since this effect is independent of R, d, or q, and depends only on a.

In Table 1 the linear sensitivity for the three above-mentioned instruments is listed together with the maximum percent error at full scale operation due to the effect of non-linearity.

It should be noted that the present discussion has omitted any discussion of the effects of non-linearity in the thermocouples used for measuring the temperature difference Δ_1. Actually, the output of these instruments is a thermocouple voltage, which will reflect the non-linearity of the thermocouple as well as that of the radiometer itself. Also, the sink temperature, T_0, will affect the operation of the thermocouple to a certain extent. It should therefore not be expected that the measured performance of these instruments will conform exactly to the figures given above and in Table 1.

Table 1

Sensitivities and Reaction Times of Three Radiometers

	Instrument		
Factor	RW-10	RW-50	RW-100
R (cm)	0.254	0.127	0.127
d (cm)	0.00127	0.00127	0.00254
k_o (cgs)	1.001	1.001	1.001
$\Delta/q = R^2/4k_o d$ $\left(\frac{{}^oC}{cal/cm^2 \cdot sec}\right)$	1.59	3.17	12.68
ϵ (%)	1.26	1.35	1.35
τ (sec)	0.016	0.004	0.004

ϵ : maximum percent error at full scale due to non-linearity.

τ : time to reach 90 percent of full value.

SPEED OF RESPONSE

To determine the speed of response of a circular foil radiometer by analytic methods we require the solution to Eq (2) for the time-dependent case. If it be assumed that k is independent of T, i.e., that $a = 0$, it is not hard to find the time-dependent solution to Eq (2) which satisfies (3) and (4). The assumption of constant k should actually make little difference in the determination of the response time.

With the assumption of k constant, Eq (2) reduces to

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q}{kd} = \frac{1}{\kappa}\frac{\partial T}{\partial t} . \tag{23}$$

where $$\kappa = \frac{k}{\rho c} .$$

which is to be solved subject to Eqs (3) and (4).

The solution can conveniently be carried out by means of the Laplace Transformation.

Before proceeding with this, however, it will facilitate matters to introduce the new variable θ, defined as

$$\theta = T - T_0 . \tag{24}$$

Thus θ will satisfy Eq (23), as well as the boundary conditions

$$\theta = 0, \text{ at } r = R, \ t > 0, \tag{25}$$

and

$$\theta = 0, \text{ at } t = 0, \text{ for all } r. \tag{26}$$

Applying the Laplace transformation to the variable θ, and defining

$$\bar{\theta} = \int_0^\infty \theta e^{-pt} dt, \tag{27}$$

the subsidiary equations for $\bar{\theta}$ are

$$\frac{\partial^2 \bar{\theta}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\theta}}{\partial r} - s^2 \bar{\theta} = -\frac{q}{pkd} . \tag{28}$$

where $$s^2 = \frac{p}{\kappa} , \tag{29}$$

with the condition that

$$\bar{\theta} = 0, \text{ at } r = R. \tag{30}$$

The general solution to Eq (28) is

$$\bar{\theta} = A I_0(sr) + B K_0(sr) + \frac{q}{p^2 \rho c d} . \tag{31}$$

where A and B are arbitrary constants, and $I_0(sr)$ and $K_0(sr)$ are modified Dessel functions. Since $K_0(sr) \rightarrow \infty$ as $r \longrightarrow 0$, it is clear that we must put $B = 0$. The constant A can then be determined by imposing the condition expressed in Eq (30), i.e.,

$$A I_0(sR) + \frac{q}{p^2 \rho c d} = 0 . \tag{32}$$

Hence the desired solution is

$$\bar{\theta} = \frac{q}{p^2 \rho c d}\left\{1 - \frac{I_0(sr)}{I_0(sR)}\right\} \qquad (33)$$

$$= \frac{q}{\rho c d}\left\{\frac{1}{p^2} - \frac{1}{p^2}\,\frac{I_0(sr)}{I_0(sR)}\right\} .$$

To find θ, we now take the inverse Laplace Transform (L^{-1}) of $\bar{\theta}$. Thus

$$\theta = \frac{q}{\rho c d}\, L^{-1}\left\{\frac{1}{p^2}\right\} - \frac{q}{\rho c d}\, L^{-1}\left\{\frac{1}{p^2}\,\frac{I_0(sr)}{I_0(sR)}\right\} \cdot \qquad (34)$$

From any table of Laplace transforms

$$L^{-1}\left\{\frac{1}{p^2}\right\} = t , \qquad (35)$$

and by the complex inversion theorem

$$L^{-1}\left\{\frac{1}{p^2}\,\frac{I_0(sr)}{I_0(sR)}\right\} = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{\lambda t}}{\lambda^2}\,\frac{I_0(\mu r)}{I_0(\mu R)}\, d\lambda , \qquad (36)$$

where

$$\mu = \sqrt{\frac{\lambda}{\kappa}} \cdot \qquad (37)$$

The integrand in Eq (36) is a single valued function of λ over the entire complex λ plane. Therefore, it can be evaluated by finding the residues at the poles. Examination shows that there is a double pole at $\lambda = 0$, and simple poles at the zeros of $I_0(\mu R)$.

To find the residue at the pole $\lambda = 0$, use the power series for the Bessel functions, i.e.,

$$I_\nu(\mu r) = \sum_{m=0}^{\infty} \frac{(1/2\ \mu r)^{\nu + 2m}}{m!\ \Gamma(\nu + m + 1)}$$

and for $\nu = 0$ this reduces to

$$I_0(\mu r) = \sum_{m=0}^{\infty} \frac{(1/2\ \mu r)^{2m}}{m!\ m!}$$

$$= 1 + (1/2\ \mu r)^2 + \frac{(1/2\ \mu r)^4}{4} + \frac{(1/2\ \mu r)^6}{36} + \ldots$$

* Since the zeros of $I_0(sR)$ are all on the negative real axis and $p^2 = ks^4$, then $\frac{1}{p^2}\frac{I_0(sr)}{I_0(sR)}$ is analytic over any half plane to the right of but not including the origin.

Thus, near $\lambda = 0$, assuming $R > r$, the integrand looks like

$$\frac{1}{\lambda^2}\;\frac{\left(1+\lambda t+\frac{\lambda^2t^2}{2!}+\frac{\lambda^3t^3}{3!}+\cdots\right)\left(1+\frac{\mu^2r^2}{4}+\frac{\mu^4r^4}{64}+\cdots\right)}{1+\frac{\mu^2R^2}{4}+\frac{\mu^4R^4}{64}+\cdots}$$

$$=\frac{1}{\lambda^2}\left(1+\lambda t+\frac{\lambda^2t^2}{2!}+\frac{\lambda^3t^3}{3!}+\cdots\right)\left(1-\frac{\mu^2\left[R^2-r^2\right]}{4}+O(\mu^4)+\cdots\right),$$

$$=\frac{1}{\lambda^2}-\frac{1}{\lambda}\left(\frac{R^2-r^2}{4k}-t\right)\quad+\text{ higher terms in }\lambda\ .$$

Thus, the residue at $\lambda = 0$ is

$$t-\frac{R^2-r^2}{4k}\ .$$

To this must be added the contributions from the remaining poles at the zeros of $I_0(\mu R)$, i.e., at $\lambda = -k\beta_n^2$ where β_n are the roots (all real) of

$$J_0(\beta R) = 0\ .$$

Since these are all simple poles, the residues will be given by

$$\sum_{n=1}^{\infty}\left.\frac{e^{\lambda t}\;I_0(\mu r)}{(d/d\lambda)\left[\lambda^2 I_0(\mu R)\right]}\right|_{\lambda=-k\beta_n^2},$$

$$=\frac{2}{R}\sum_{n=1}^{\infty}\frac{I_0(i\beta_n r)\;ik\beta_n}{k^2\beta_n^4\;I_1(i\beta_n R)}\;e^{-k\beta_n^2 t}\ .$$

Using the fact that

$$I_0(i\beta_n r) = J_0(\beta_n r)\ ,$$

$$I_1(i\beta_n r) = iJ_1(\beta_n r)\ ,$$

this becomes

$$\frac{2}{Rk}\sum_{n=1}^{\infty}\frac{J_0(\beta_n r)}{\beta_n^3\,J_1(\beta_n R)}\;e^{-k\beta_n^2 t}\ .$$

where J_0 and J_1 are ordinary Bessel functions.

We then have the result that

$$L^{-1}\left\{\frac{1}{p^2} - \frac{1}{p^2}\,\frac{I_o(sr)}{I_o(sR)}\right\}$$

$$= t - \left\{t - \frac{R^2 - r^2}{4k} + \frac{2}{Rk}\sum_{n=1}^{\infty}\frac{J_o(\beta_n r)}{\beta_n^3 J_1(\beta_n R)}\, e^{-k\beta_n^2 t}\right\} \quad .$$

$$= \frac{R^2 - r^2}{4k} - \frac{2}{Rk}\sum_{n=1}^{\infty}\frac{J_o(\beta_n r)}{\beta_n^3 J_1(\beta_n R)}\, e^{-k\beta_n^2 t} \quad .$$

Thus, from Equation (34)

$$\theta = \frac{q}{kd}\left\{\frac{R^2 - r^2}{4} - \frac{2}{R}\sum_{n=1}^{\infty}\frac{J_o(\beta_n r)}{\beta_n^3 J_1(\beta_n R)}\, e^{-k\beta_n^2 t}\right\} . \tag{38}$$

For very large t the exponential terms may be disregarded, and θ reduces to the steady state value

$$\theta_s(r) = \frac{q}{kd}\,\frac{R^2 - r^2}{4} \quad , \text{ as } t \longrightarrow \infty \; . \tag{39}$$

This agrees with Eq (10) when $a = 0$ and θ is put equal to $T - T_o$.

For large t, at $r = 0$, Equation (38) reduces to*

$$\theta = \frac{qR^2}{4kd} - \frac{2q}{kdR}\,\frac{e^{-k\beta_1^2 t}}{\beta_1^3 J_1(\beta_1 R)}$$

$$= \frac{qR^2}{4kd} - \left\{\frac{qR^2}{4kd} \cdot \frac{8}{(\beta_1 R)^3 J_1(\beta_1 R)}\right\} e^{-k\beta_1^2 t} \quad .$$

or

$$\theta = \theta_{s(o)} - \theta_{s(o)}\left\{\frac{8}{(\beta_1 R)^3 J_1(\beta_1 R)}\right\} e^{-k\beta_1^2 t} \quad . \tag{40}$$

Thus,

$$\frac{\theta_{s(o)} - \theta}{\theta_{s(o)}} = \frac{8}{(\beta_1 R)^3 J_1(\beta_1 R)}\, e^{-k\beta_1^2 t} \quad . \tag{41}$$

---

* since $J(0) = 1$ and since all the terms in the summation may be neglected compared to the first for very large t.

Using tables of Bessel functions, we find

$$\beta_1 R = 2.405 \quad ,$$

$$J_1(\beta_1 R) = 0.519 \quad ,$$

and, therefore,

$$\frac{\theta_s - \theta}{\theta_s} = 1.11 \; e^{-(2.405)^2 k t/R^2} \quad . \tag{42}$$

To find the time τ for the center of the circular foil radiometer to reach a temperature within 10 percent of its steady state temperature, put $(\theta_s - \theta)/\theta_s$ equal to 10^{-1} in Eq (42), i.e.,

$$10^{-1} = 1.11 \; e^{-(2.405)^2 k\tau/R^2}$$

and taking $\log_{10}$ of both sides,

$$-1 = \log_{10}(1.11) - \frac{(2.405)^2 k\tau}{R^2} \log_{10} e \ ,$$

or

$$\left.\begin{aligned} \frac{k\tau}{R^2} &= \frac{\log_{10}(1.11) + 1}{(2.405)^2 \log_{10} e} \quad , \\ &= \frac{1.0453}{(5.784)(0.4343)} = 0.416 \quad , \end{aligned}\right\} \tag{43}$$

Thus,

$$\tau = \frac{(0.416)R^2}{k} \quad . \tag{44}$$

In Gardon's analysis, previously referred to, an approximate exponential solution for the time-dependent problem is obtained which yields a value of

$$t = \frac{(0.25) R^2}{k}$$

corresponding to a value of

$$\frac{\theta_s - \theta}{\theta_s} = \frac{1}{e} = 0.368 \quad .$$

From this it follows that a value of

$$\tau = \frac{(0.575)R^2}{\kappa}$$

is required for θ to be within 10 percent of its steady state value, i.e., for

$$\frac{\theta_s - \theta}{\theta_s} = 0.10 .$$

However, the more rigorous solution derived above indicates a somewhat faster approach to the steady state since it has been shown that when Eq (44) is satisfied, θ is very nearly within 10 percent of its steady state value.

Actually, the rigorous solution to the time-dependent problem, given in Eq (38) shows that, in general, θ does not have a simple exponential time dependence, but only approaches an exponential behavior for large values of the time. It can be shown that the large time approximation is valid, with negligible error, for computing the reaction time τ as we have done.

Using Eq (44), the values of the reaction time τ required to reach 90 percent of full value have been calculated for each of the three instruments already discussed. These values are included in Table 1. The value of the diffusivity, κ, of silver was computed from the basic definition $\kappa = k/\rho c$, where $k = 1.001$ Cal/cm. sec. $^\circ$C, $\rho = 10.45$ gm/cc, and $c = 0.057$ Cal/gm $\cdot$ $^\circ$C.

Approved by:

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For the Scientific Director

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DATE ISSUED: 28 April 1959

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TRC 28 April 1997

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SUBJECT: Change of Distribution Statement on AD-216542

The Defense Special Weapons Agency Security Office (OPSSI) has approved the following report for **public release**:

AD-216542 AFSWP-1119 (USNRDL-TR-311)
Theoretical Analysis of Radiometer Performance,
13 January 1959 by H. G. Ferris.

Distribution statement "A" now applies.

Ardith Jarrett
ARDITH JARRETT
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