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OPERATIONS RESEARCH OFFICE
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The Johns Hopkins University

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Optimum Duplex Spread (U)

by

K. L. Yudowitch

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TACTICS DIVISION
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Optimum Duplex Spread (U)

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K. L. Yudowitch

OPERATIONS RESEARCH OFFICE
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Optimum Duplex Spread

I. Controlled Dispersion

BACKGROUND

The SALVO program is concerned with enhancing the effectiveness of the infantry rifleman by increasing the rifleman's hit probability. The primary technique proposed for boosting this chance of hitting is the use of multiple projectiles.[1] Under the aegis of the OCO SALVO Steering Committee, following the specific suggestion of ORO,[2] a contract for the development of multiple bullet ammunition was let by the Ordnance Corps to Olin-Mathieson Winchester Div. This contract produced several varieties of multiple bullet rounds, notably tandem duplex and triplex rounds. In the summer of 1956, a field experiment was conducted to determine the hit probability of these prototype multiple rounds compared with ordinary single ball rounds in a combat-simulating context. The duplex used in that test has emerged quite successful.[3]

The contractor at the time of the test was able to supply either of two types of duplex ammunition. The so-called random dispersion yields separate groups for the front and rear bullets, the rear bullet group being at about 11o'clock relative to the front bullet group. The spread apparent in each group is similar to that obtained in an ordinary single ball group. Ammunition supplied for the test however is the so-called controlled dispersion. For this ammunition the front round maintains a dispersion comparable with ordinary single ball ammunition. The rear rounds fall

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[1] ORO-T-160
[2] ORO-T-245
[3] ORO-SP-2

within a narrow ring which is approximately concentric about this small front round group. For either dispersion, the angular separation between bullets is 3 mils. This controlled dispersion duplex thus provides in its front bullet alone all the hit probability of single ball, with all rear bullet hits clear bonus.

CONTROLLED DISPERSION MODEL

To deal analytically with the controlled dispersion duplex ammunition tested, a simplified model of the dispersion pattern was assumed. The simplifications basic to the model are: 1) the dispersion of front bullets is normal and symmetrical about the line of fire; 2) the ring of second bullet impacts is narrowed to a circle of negligible width and 3 mils radius; 3) the circle of second bullet impact is concentric about the corresponding front bullet impact; 4) the angular location of second bullet impacts on the circle is random. A fifth simplifying condition is used in our computations: 5) the target is circular.

From the geometry of Fig. 1, the fraction of the rear bullet circle which lies on the target is given by:

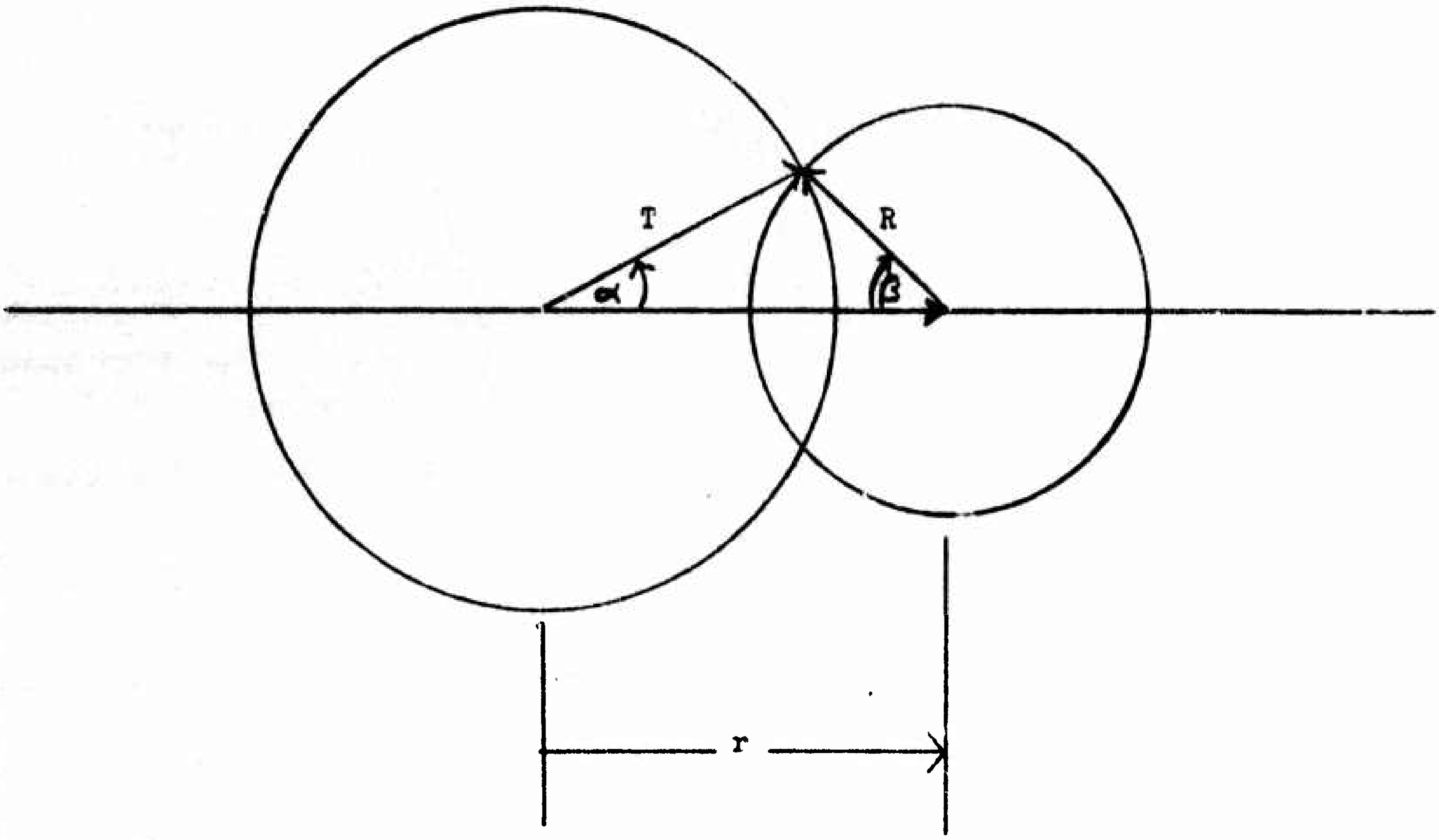
$$F = (1/\pi) \arccos \left[(R^2 - T^2 + r^2) / 2Rr \right] \qquad (1)$$

For a radially normal distribution of front bullet impacts, the probability of a front bullet impact at a distance r to $r + \delta r$ from the target center is given by:

$$\delta G = r \delta r / \sigma^2 \ \exp(-r^2/2\sigma^2) \qquad (2)$$

Fig. 1

Geometry of Controlled Dispersion Duplex Hits



T = target radius
R = rear bullet circle radius
r = radius vector from target center to front bullet impact

Using the fraction (F) and the probability element (δG) with the geometry of Fig. 1, duplex hit probabilities are readily deduced.

The single ball hit probability is:

$$N_1 = \int_{r=o}^{T} \delta G = 1 - \exp\left(-T^2/2\sigma^2\right) \qquad (3)$$

The duplex hit probabilities of interest are:

$$P_s = N_1 - N_2 + \int_T^{T+R} - \int_{|T-R|}^{T} \qquad (4)$$

$$P_d = N_2 + \int_{|T-R|}^{T} \qquad (5)$$

$$P_t = P_s + 2\,P_d = N_1 + N_2 + \int_{|T-R|}^{T+R} \qquad (6)$$

-- where:

$$\begin{cases} \int = \int F\,\delta G \\ P_s = \text{Probability of a single hit} \\ P_d = \text{Probability of a double hit} \\ P_t = \text{Total hit probability (All hits / shots)} \\ N_2 = \int_{r=o}^{T-R} \delta G = 1 - \exp\left(-[T-R]^2/2\sigma^2\right) \end{cases}$$

$$\begin{cases} \text{For } \underline{T<R},\ \underline{N_2} \text{ vanishes} \\ \text{For } \underline{T<R/2},\ \int_{|T-R|}^{T} \text{ vanishes} \end{cases}$$

Numerical integration has been substituted for expressions which are not amenable to integration. The hit probabilities are functions of three variables: the duplex spread (R), angular target size (T), and the angular aiming error (σ). It is quite possible then to compute the hits of each type which may be expected with a duplex round of known spread on a target of a given angular size under conditions of known aiming error.

DISCUSSION

The question has been raised whether the 3 mil separation of the test ammunition is optimum for controlled dispersion duplex. In an attempt to answer this question, we herein answer precisely a more restricted question: What is the optimum angular spread for the controlled dispersion duplex round satisfying the five assumptions of the model and applied to the particular target system and context of the experiment? The reasonableness of the simplified model has already been verified by the agreement of predicted duplex hits with experimental data.

The major restriction then upon the interpretation of the results to be reported here is that they apply precisely only to the target system of our experiment. It is, however, a unique experiment, in that extraordinary pains were taken to make the test conditions as combat-like as possible. All the essential characteristics of the target system and environment were carefully designed to reproduce as exactly as feasible those conditions of combat affecting target size and aiming error. It is therefore not unreasonable to generalize with caution the results from this unique target system to the battlefield. It was arbitrarily decided in the first instance to limit consideration to daytime firing. A similar computation is possible for the nighttime data.

It is necessary, as indicated above, to determine the angular size and aiming error to be associated with each target of the system in order to predict the specific duplex hit probabilities. The angular target sizes are readily computed from the target dimensions and ranges used in the experiment. The aiming error associated with each target is computed from the single ball hits on each target as measured in the test. To ensure statistical significance (particularly on those targets which were hit the least), we lump together all 22 runs of single ball daytime firing:

Table 1

No. of Runs	Weapon	Firing Position
8	M1 cal. 30	Sitting
4	M1 cal. 30	Standing
4	T48 cal. 22	Sitting
2	T48 cal. 22	Standing
4	Carbine cal. 22	Sitting

Table 2 lists the angular size (T) in mils of each of the 22 targets of our daytime target system. Also listed in Table 2 are the number of shots (S) fired at each of these targets during the 22 semiautomatic daytime runs. The final column of the table gives the number of hits (H) scored on each of the targets. These 66 numbers comprise the basis for the computation to follow. First, the aiming error (σ) is computed on the assumption of a normal distribution from equation (3):

$$N_1 = H/S = 1 - \exp\,(-T^2/2\sigma^2) \qquad (7)$$

From Table 2 we have directly the values for T, and from equation (7) we compute the values for σ. It is necessary only to supply values for the duplex spread (R). Preliminary theoretical investigations have already suggested that the optimum value for (R) should approximate the width of a target at combat range. A reasonable average value for such angular target width is just about 3 mils. Several values of duplex spread (R) centering about a value of 3 mils should include the true optimum. We arbitrarily select the values R = 1, 2, 3 and 4 mils. We thus have 22 x 4, or 88 evaluations to perform by numerical integration in order to arrive at the desired hit probabilities.

Table 2

T	S	H
3.87	270	83
3.72	1238	436
4.61	421	192
3.22	1137	384
2.58	997	210
2.25	441	79
2.06	135	10
2.61	619	111
2.45	606	88
2.42	970	188
2.40	2099	521
2.34	199	23
2.25	217	18
1.32	188	3
1.31	251	27
1.62	381	53
1.53	756	100
1.48	92	3
1.06	1011	62
0.86	232	13
0.85	216	1
0.84	838	32
	13,314	2637

To determine the optimum spread (R), we must maximize a single expression which is a proper measure of effectiveness. As there is no significant velocity or weight difference between front and rear bullets of a duplex pair, we may conclude that either bullet is equally effective on a live target. The compound effect of a double hit on one target is a matter of some conjecture to be explored later. -- For a first model, however, we assume no compound effect. Thus in the case of a double hit, the second bullet is assumed to have precisely the same effectiveness as the first bullet, except that if the target is already killed by the first bullet, the second bullet's effectiveness is wasted. This overkilling effect is simply accounted for in the expression for kill probability (K):

$$K = (P_s + P_d) P_L + P_d (1-P_L) P_L \qquad (8)$$

where P_s is the probability of a single hit, P_L is the kill probability per hit or lethality and P_d is the probability of a double hit.*
To be unconcerned with possible lethality differences between single ball and duplex ammunitions, we use the probability of effective hits H' rather than kills as our measure:

$$H' = K/P_L = P_s + 2 P_d - P_d P_L \qquad (9)$$

The first two terms in Equation 9 are clearly just the total number of hits (P_t):

$$H' = P_t - P_d P_L \qquad (10)$$

It is clear now that effective hit probabilities may be computed for each of the 22 subject targets from Equations (1), (2), (5), (6) and (10).

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* A double hit refers to two hits on a target from a single trigger pull.

This has been done for each of the four values of R. In order to evaluate the integrated effectiveness on the entire target system, it is clearly necessary to consider the effectiveness on all targets. To achieve this integration of effect with appropriate weighting on each target, we merely multiply the computed value of H' by the corresponding value of S, the number of shots fired at that target. The product, of course, gives us the number of effective hits predicted for each target. The integrated effect is then simply the sum of these products for all 22 targets, which is just the number of effective hits on the entire target system:

$$\sum S\,(P_t + P_d P_L) = \sum(SP_t) + P_L \sum(SP_d) \qquad (11)$$

Inasmuch as we have designated no value for the lethality (P_L), our resultant expressions retain that term unevaluated.

It is instructive to reduce the number of effective hits to only the number of extra effective hits over the number obtained with single ball ammunition. This amounts to merely subtracting the term $\sum SN_1$ from equation (11). This term is of course identical with the sum of the column headed (H) in Table 2, merely the total of single ball hits on all targets. Our measure of relative increase in effectiveness (I) is then given by:

$$I = \frac{\sum SP_t - \sum SN_1}{\sum SN_1} - \frac{\sum SP_d}{\sum SN_1} P_L$$

-- or -- (12)

$$I = a - P_L$$

For simplicity, coefficients a and b have been substituted in equation 12. The boundary cases for zero and infinite values for duplex spread R are also included in Table 3:

Table 3

R	a	b
0	100	100
1	96.5	76.6
2	87.3	53.4
3	76.3	34.1
4	59.5	18.5
∞	0	0

Now, let us assume values of P_L over the entire range from zero to unity. Relative effectiveness increases I are tabulated for each of the several selected values of P_L:

Table 4

R	I_0	$I_{0.2}$	$I_{0.4}$	$I_{0.6}$	$I_{0.8}$	$I_{1.0}$
0	100	80	60	40	20	0
1	96.5	81.2	65.1	50.5	35.2	19.9
2	87.3	76.6	65.9	55.3	44.6	33.9
3	76.3	69.5	62.7	55.9	49.1	42.3
4	59.5	55.8	52.1	48.4	44.7	41.0
∞	0	0	0	0	0	0

The relative effectiveness increases (I) of Table 4 are shown graphically in Fig. 2.

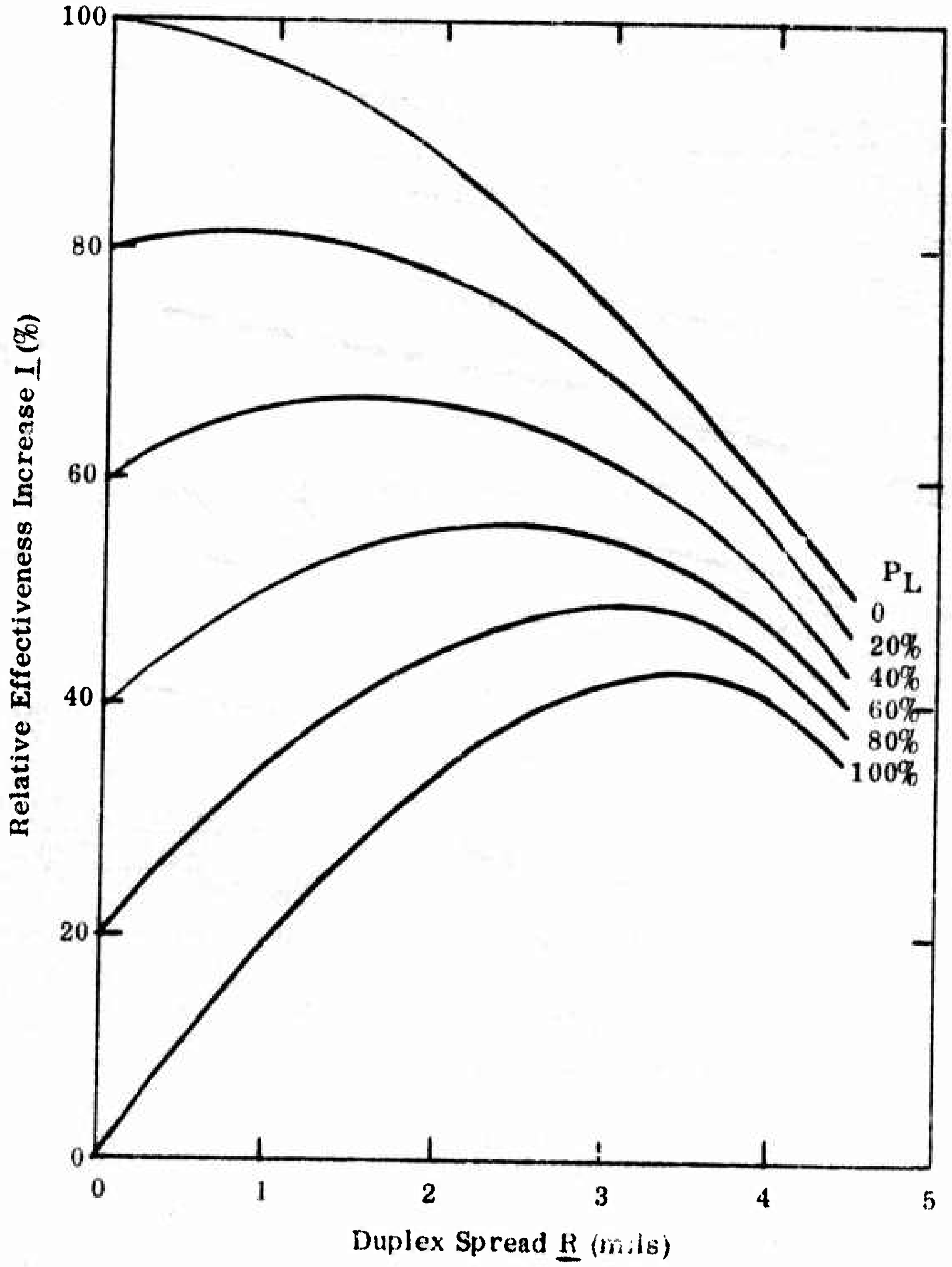

Fig. 2—Relative Effectiveness Increase (I) vs. Duplex Spread (R) [-for a range of values of lethality (P_L)]

INTERPRETATION OF RESULTS

It is clear that as the single bullet lethality increases from 0 to 100%, optimum duplex spread increases from to 3.4 mils. . Currently available figures on the lethality of either single ball or duplex ammunition are both about 45% at ranges up to 140 yards. This figure is based on a criterion of thirty second incapacitation against defending troops. From our curves, this corresponds to a optimum duplex spread of 1.9 mils. In interpreting these results we are now obliged to consider the validity of the lethality criterion and of the experimental context. It is necessary to resort to less quantitative considerations in contemplating the perturbations imposed by reevaluation of these conditions.

The lethality criterion is a source of possible modification of our conclusions. Experienced judgment suggests that the 45% value of 30 second incapacitation would be applicable more to the extraordinarily heroic soldier than to the average. The criterion is apparently based on actual physical incapacitation without regard for associated demoralizing effects. It is suggested that the operationally effective lethality criterion might be closer to double the 45% required for actual physical incapacitation. Such a change in the lethality criterion would increase the optimum duplex spread on our experimental target system from 1.9 to 3.4 mils.

It is quite apparent that the experimental environment knowingly fell short of true combat in its simulation of aspects of stress and camouflage in particular, as well as other characteristics affecting the aiming error. The targets were readily identified by the white paper faces which were more easily discerned than would be the darker uniforms of enemy soldiers.

Also, the riflemen were permitted to fire from fairly comfortable positions, likewise enhancing their marksmanship. Perhaps more important, the riflemen were not in mortal fear of their lives, and took full opportunity to aim at any targets which appeared. --That is, where in battle a rifleman might raise from his cover for only the minimum time required to get off a shot, in the experiment the men exposed themselves freely to take full advantage of the target's appearance as long as it remained visible. All of these differences from combat tended to reduce the aiming error in the test. Such reduction was essential in the experiment in order to obtain adequate hit data. Whereas in actual combat, the hit probability of the aimed fire is probably under 1%, the totals from Table 2 reveal that the experiment produced a hit probability of 20%.

As the duplex advantages accrue from inaccuracy of aim, it might be intuitively predicted that the existence of a larger aiming error in combat would dictate a larger optimum duplex spread for combat. This intuition is borne out by sample calculations made on several pairs of individual targets from Table 2. Comparing targets of approximately equal size (T) but different hit probabilities (S/H), for any value of lethality, the optimum duplex spread is larger for the target with the smaller hit probability. We may thus conclude, without attempting careful quantification, that in generalizing our results from experiment to combat, we must increase the optimum duplex spread.

CONCLUSION

Despite the qualitative nature of the perturbations imposed, particularly by the increased aiming error of true combat, it is required that our best judgment be utilized to recommend an optimum duplex spread for combat. Our conclusion is that a spread of 3 to 4 mils is approximately optimum, and that (from Fig. 2) a variation in either direction of 1 mil in dispersion induces less than 3% variation in the number of effective duplex hits. Thus, we recommend that future controlled dispersion duplex rounds be manufactured with this same dispersion of 3 mils, and note that up to a full mil variation in either direction from this value is of no substantial consequence.