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FLOW FIELDS PRODUCED BY EXPLODING WIRES

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BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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ABSTRACT

Experimental evidence already reported suggests that the strong shock waves produced by exploding fine cylindrical wires, after an initial transition phase, closely follow trajectories of the parabolic type characteristic of similarity flows, studied by S. C. Lin. Shock data obtained in air at a sequence of pressures below atmospheric show large deviations from the $\rho^{-1/4}$ dependence on ambient density, given by Lin; yet in a limited time interval indicate approximately the expected parabolic shock trajectories as before. In view of the work on similarity flows, of Guderley, v. Weizsacker and others, the questions are discussed whether the data may be represented by other similarity flows, than the one prescribed by Lin, and whether there is in the cylindrical case a convergence to a particular flow such as is found by numerical means for the case of strong plane shocks.

I. INTRODUCTION

When a fine cylindrical wire is exploded into an ambient atmosphere by passage of a heavy current pulse, a complex flow with closely cylindrical symmetry is induced in the surrounding medium. For wire diameters of the order 0.1 mm or less and with available energies larger than 5 joules/cm of wire length, one of the most prominent features of such a flow is the primary shock of a cylindrical shock wave system which is responsible for the loud, sharp noise of the explosion. While the presence of a head shock wave has been known for sometime, and a number of authors[1] have studied various aspects of shock phenomena produced by exploding wires, it was not until the schlieren, Kerr-cell camera, studies of Müller[2] that the presence of both a first and second shock wave was established and quantitative data made available on the expanding fronts of the head shock, metal vapor boundary and discharge canal.

More recently the author has been able to study the early stages of shock and flow initiation[3] through the use of a modification of the rotating-mirror, streak camera. This method of investigation shows that, for a typical case, the shock and luminous boundary coincide up to about 1 μ sec after which the non-luminous shock traces out a closely parabolic trajectory for the next 2-3 μ sec. The luminous boundary itself approximates a smaller parabola up to 3 μ sec. At 6 μ sec. from the flash tip, a sharp luminous wedge appears which delineates the outward trajectory of the second shock wave after reflection at the axis from its earlier inward course.

Comparison of the experimental head shock trajectory with the parabolic law given by S. C. Lin[4] and A. Sakurai* for the constant energy, cylindrical blast wave, allows a determination to be made of the apparent

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* At a slightly earlier date A. Sakurai (see our reference 13) treated the constant energy, similarity flow problem for spherical, cylindrical and planar symmetry from a series approximation approach. His lowest order approximations for spherical and cylindrical symmetry agree with those of Taylor (reference 12) and Lin respectively. In addition he provides higher order approximations. We became aware of Sakurai's work after using Lin's solution. Our preliminary checks of his second approximation will be discussed below in § 3.5.

energy release per unit axial length at the virtual time of initiation of the shock found by extrapolating the parabola-test plot back to the time axis.

This application of the constant energy similarity solution may be criticized on the grounds that the experimental energy release is neither instantaneous nor along a line, but takes place over a finite interval into a finite massive metal cylinder. Furthermore the ambient gas is not perfect but real and exhibits both relaxation and radiation phenomena. Finally, the agreement between the experimental points and a theoretical parabola extends considerably beyond the time where a conservative estimate would credit the shock with a large enough Mach number ($M > 8$) so that the strong shock conditions could hold, providing the ambient fluid were a perfect gas.

Despite these objections to the use of strong shock, similarity theory, the method has a certain success. Energy values derived from parabola-test plots show that for the wires producing the stronger shock waves about half the stored electrical energy reappears as energy of the flow behind the shock. Another 25% can be accounted for as heat lost in the residual circuit resistance. Up to 10% may be used in heating the metal wire through its transition points to a temperature of several thousand degrees. This leaves 15 - 20% to be divided among heat conduction and radiation, and flow phenomena connected with departures from similarity. Thus there is no crude violation of conservation of energy principle and the indicated energy within the shock is in approximate agreement with the expectations derived for strong blast waves energized by chemical explosives or other means.

Another agreement follows as a consequence of the simultaneous study of 1) energy imparted to the shock and 2) the damping conditions in the exploding wire circuit.[5] It is shown that, for a series of copper wires, the maximum axial energy release to the shock flow does not occur for the same wire that gives minimum damping time in the exploding wire circuit. Furthermore the difference between the wires is in the right direction and close to the right magnitude to be accounted for by the effect of resistance in the circuit. Thus relative energy changes obtained from the use of the flow similarity theory agree with those obtained from the electrical theory of the circuit. This correspondence is taken as strong support for the correctness of shock flow energy values measured for wires neighboring the one that most rapidly damps the circuit.

A deviation from the similarity law which has been noted in all streak data from the first, is a tendency for the parabola-test plots to deviate from linearity by concavity upwards. This tendency for the later points to lie progressively above the linear fit to earlier data, was first thought to be connected with an optical distortion inherent in the experimental technique.[3] Later results[6] obtained by a method free of this particular distortion show the same concavity; so it is presently concluded that this is a real deviation from the parabolic, similarity trajectory probably to be explained by a closer theoretical representation of the physical flow. (see § 3.2).

Because of the simplicity and easy interpretation of the similarity flow theory, and its apparent success in correlating prominent aspects of the flows produced by exploding wires, it is important to delineate in some detail the range of flows and shock waves to which this type of analysis is applicable.

In the following we discuss some exploratory experiments performed with the end in view of investigating the density dependence of the head shock wave trajectory.

II EXPERIMENTAL

2.1 Experimental Data

Figure 1 shows streak camera pictures of 5-mil copper wires exploded into ambient room temperature air at pressures ranging from 1 atm down to 1/8 atm. As in the previous experiment, the stored energy is 118 joules at 28 kv in a circuit with a ringing frequency of about [illegible] mc. The electrodes and slit are enclosed in an evacuable bakelite chamber fitted with an optical glass window through which the flash is photographed. Pressure is determined to within 1% on a precision Wallace and Tiernan gauge. Because the refractive effect of the shock wave is difficult to observe at low gas densities, a method more sensitive than the mirror backlighting had to be found in order to study the shock wave at pressures below 1/5 atm. A method of linear streak backlighting[6] has been used to obtain 1/5 atm and 1/8 atm pictures shown in two of the panels. While a useful gain in sensitivity results from the linear streak technique, the extension to lower pressures is hardly a factor of two.

General features of the rotating mirror photographs are to be noted as follows. The shock wave which is quite clear at 1 atm is considerably less definite as pressure decreases. The luminous area, in the meantime, changes from smooth continuous luminosity to pronounced streakiness and the duration diminishes. Below 1/2 atm no second shock wave is visible. Most of the luminosity is finally concentrated at 1/8 atm into two more or less symmetrical bands or "wings" of light. It is likely that at 1/8 atm the wire explosion is on the verge of producing a moving plasma of the type encountered by Bohn et.al[7] at pressures below 10 cm of mercury.

The parabola test plot of Fig. 2 summarizes measurements from the streak pictures of Fig. 1. When $(2R)^2$ vs t is plotted it is known[3] that a straight line should result if the trajectory is that of the constant energy similarity flow given by Lin.[4] The slope of the line is $m = 4 S^2 (E/\rho_o)^{1/2}$ where S is a function only of the γ of the ideal gas, E is axial energy release per cm. and ρ_o is the density of the ambient atmosphere, also assumed to be a perfect gas.

If the axial energy release is held constant, then one would expect a monotonic increase in slope m as density ρ_o decreases. This means that the latus rectum of the shock trajectory should increase with decreasing density. Clearly no such increase can be observed except for 1/2 atm in Fig. 1. The plot in Fig. 2 demonstrates the failure of slope to increase monotonically. It is a fact worth noting that the lower pressure data return practically to the locus of the 1 atm points.

The trajectories of Fig. 2 are all still close to parabolic although each shows the concavity upwards discussed earlier.[3,6] From the slopes m and measured pressures, the axial shock energies E_s have been calculated. From these one sees that, although the wire and the available electrical energy remain the same, the apparent energy communicated to the flow decreases sharply with decrease of ambient density ρ_o.

Thus if we assumed at the moment that because of our choice of both a particular wire and a constant electrical energy, a fixed amount of energy would be deposited in the wire and thence communicated to the flow, a consequence of the similarity theory is the prediction that slopes values should increase as $m \alpha \rho_o^{-1/2}$. The experimental results show that this relationship is not obeyed.

If, because of the nearly parabolic trajectories, we choose to assume that the shock paths at low density are those predicted by similarity theory, then a consequence of this assumption is the conclusion that the axial energy release E_s is a function of the ambient density ρ_o. If further investigation bears out this conclusion, then we have a new fact about the exploding wire phenomenon, viz., the wire acts as a transducer between electrical and fluid mechanical forms of energy and by inference its apparent resistance in the circuit depends upon the density of the surrounding medium.

Other alternatives arise if we vary the initial assumptions but these lead to more elaborate theoretical investigations which we shall not pursue at the present.

III DISCUSSION

3.1 Deviations From Similarity Flows

Exploding wires, as already noted, can hardly be expected to produce similarity flows because of fundamental differences between the physical situation and that represented by the assumptions of the theory. Nevertheless, the shock waves from exploding wires do over appreciable intervals of space and time behave like similarity flow shocks. Two principal deviations from the similarity flow predictions have been noted.

The first is a gross failure of the shock waves from a fixed size of wire supplied a fixed electrical energy to traverse parabolae whose breadth at the latus rectum increases as the inverse one-fourth power of ambient density. Comparison of the low pressure, streak pictures of Fig. 1 with the low shock energy flash for 8-mil copper previously published[5] shows similarities in shape, short duration and lack of a luminous second shock. This evidence is strong enough to render highly probable the assumption stated earlier in [illegible], viz., that E_s is dependent on ρ_o. Thus the deviation is caused by failure of our assumption about energy deposition in the wire, not necessarily [illegible] failure of the shocks to traverse the appropriate similarity [illegible]. Further substantiation of this interpretation can be supplied when current and voltage oscillograms are obtained for a sequence [illegible] given in Fig. [illegible]. Then experimental values of electrical energy [illegible] to the wire can be compared with energy values E_s indicated by the shock trajectory. The electrical techniques for such measurements have been discussed in several places.[7,8,9,10]

The second type of deviation from the similarity flow trajectories is the concavity of the parabola-test plots. The most strongly curved region, lying between [illegible] has been interpreted earlier[3] to be caused by energy [illegible] from the wire explosion. The much smaller tendency toward concavity in the region from [illegible] was initially thought to be a systematic error of the experimental method. As more recent data [illegible] the question has been examined more closely to see whether any interpretation is possible still within the framework of similarity flow theory. The shock data of Fig. 2 have been re-plotted using the variables $(2R)^n$ vs t in an attempt to see whether a general parabola with $n \neq 2$ may provide a better fit to the experimental data. Fig. 3 shows the curves so obtained together with the values of n employed. In each case the entire curve is considerably straightened, and in three cases the exponent is close to 5/3, i.e. a $t^{0.6}$ power law for the shock trajectory provides a better fit than the $t^{1/2}$ law previously used. Plots of the data on log paper do not permit of much refinement in the determination of n but do indicate the possibility that the data lying below 0.2 μ sec in time may in some cases fall on a different line with considerably lower n.

3.2 Variable Energy Similarity Flows

Theoretical justification for similarity flows with n values other than 2, may be found in a paper by Guderley.[11] While the main problem he considers is that of a strong spherical (or cylindrical) shock converging toward the origin, his development of the theory for spherical or cylindrical symmetry is perfectly general. With the assumption of a (1/n)th order parabola as the shock trajectory in the neighborhood of the origin and with the strong shock relations, he is led to introduce generalized similarity conditions for the entire flow behind the strong shock. These scaling relations reduce the conservation laws for mass, momentun and entropy from partial to ordinary differential equations. After a further reduction, based on dimensional analysis, which shows the solutions to be functions of only two, independent dimensionless variables, he is able to examine the entire solution field for the system by study of the solutions and singular points of a single non-linear, ordinary differential equation. One of the results of this study is the proof that in spherical or cylindrical coordinates there is only one non-trivial solution for which the particle at the origin can remain at rest. Comparison shows that this must be the constant energy solution found by Taylor for the spherical case[12] and Lin[4] and Sakurai[13] for the cylindrical case.

If the assumption of constant energy is given up, then an infinity of similarity solutions with a power law dependence of energy addition on time may be found. For the cylindrical case these take the form $E \propto t^{(4/n)-2}$. The constant energy case can be approximated as closely as we please by time dependent solutions with n values sufficiently close to 2. The particular values, $n = 5/3$, 1.43, used in Fig. 3 imply parabolic energy addition with t raised to the 0.4 and 0.8 powers respectively. Neither of these is close to constant energy release. Furthermore it is noteworthy that the generalization of our theoretical considerations to include similarity flows with time variable, energy addition has led us to the point where an expression is available against which an experimental check may be made.

Through measurements of current and voltage in the exploding wire circuit, energy delivered to the wire can be obtained as a function of time and compared with the power laws characterizing the variable energy addition given by similarity theory. Agreement of these values would be strong evidence that the flows involved are of the similarity type.

One puzzling feature of the theory is the fact that for the variable energy flows the particle at the origin is not permanently at rest. Symmetry considerations indicate that the axial particles in the real, exploding wire flows should remain at rest, and the streak data indicate that this is approximately true. Some light is shed on this question by the study of similarity flows over blunt-nosed, slender bodies given by Lees and Kubota.[14]

These authors treat the case of a slender body of revolution for which the flow in the axial direction can be neglected in comparison to the radial flow, i.e., flow values close to the nose are <u>not</u> required. Thus their equations are fundamentally the same as those used by Guderley although because of differences in notation and mathematical approach direct comparison is very difficult. Lees and Kubota show that 1) for similarity flows to exist the body shape must be similar to that of the shock, i.e., both vary as x^m where x is axial distance and m specifies the parabola $(m = 1/n)$;

2) the constant energy solution given by $m = 1/2$ is a singular solution passing through a saddle point, which agrees in a general way with Guderley's analysis; and 3) similarity flows for bodies of positive slope ($dE/dt > 0$) exist for $1/2 < m \leq 1$.

The point of interest for our discussion is that the fluid particle originally on the axis traverses the stream line of the body contour and thus does not remain at rest.

The existence of a variable energy similarity flow about the exploding wire would thus imply that the real flow caused by the expanding wire vapor lies within a particle path (surface of revolution) which has the same parabolic law of growth as the shock wave, and that the flow of energy across this surface is the same as would be caused by an identical blunt-nosed, slender body producing a flow of equal radial Mach number. This interesting fact suggests that closer analysis of luminous boundary paths like those measured in our earlier studies[3,5] may show them to be parabolas with the same power law as the shock wave, both differing from the $m = 1/2$ constant energy case by which they can be approximately fitted. Further experimental work in this direction is needed.

3.3 Higher Order Approximations

By virtue of an analytical treatment based on series expansions with the square of reciprocal shock Mach number as the variable, Sakurai[13] is able to provide higher order approximations to the flow than that given by the similarity solutions. For the constant energy case, which is the only one he treats, his first two terms yield a function for the shock trajectory which is an hyperbola rather than the parabola given by the first approximation. In connection with a preliminary study of shock data extending to 10 μ sec, obtained from a 5-mil copper wire[6], we have found that the difference between Sakurai's second approximation and the parabolic similarity solution is everywhere less than 2%. Thus it appears that for the ranges of data so far obtained not much is to be gained by preferring the second approximation to the first.

3.4 Convergence of the Flows

We discuss here, in a speculative way, two topics which may conceivably be related to convergence properties of the flow solution. The first of these has to do with the experimental fact, revealed in Fig. 2, that the actual shock trajectories for different ambient densities are not greatly different from one another. This is shown even more strikingly in Fig. 3 where we see that all the points lie close to a single line. The separate sets of experimental points actually are fitted by lines of slightly differing slope and differing behaviour at the origin; nevertheless the plot gives the distinct impression that some invariant feature of the flows is being exhibited. If the shock trajectories are of the variable energy type represented by $R = \xi_s t^{1/n}$ where ξ_s is the similarity constant for the shock depending on parameters defining the flow then the concordance of the data in Fig. 3 suggests that $(2R)^n/t = (2\xi_s)^n$ is a constant independent of energy and density. In our case its numerical value would be about 1.2. Thus a functional relationship of the type $n \ln \xi_s$ = const. for different time dependent flows seems to be implied. Further investigation of this question will probably require extensive numerical work; for the variable energy solutions have not been obtained in analytical form.

The second, and probably related, item of this discussion takes note of the theoretical model of a strong, plane shock proposed by von Weizsacker[15] in connection with problems encountered in astrophysics. v. Weiszacker defines a prototype, infinite plane shock wave to have a similarity (or homology) type of flow solution determined at the shock by the strong shock conditions. The other boundary is heuristically determined by the condition that the wave lose none of the mass over which it passes. This we see requires that pressure, and pressure gradient, vanish sufficiently far behind the shock wave and indeed the statement is made that the wave terminates in a vacuum. The numerical calculations of v. Weiszacker show the existence both of unsuitable singular solutions and a possibly satisfactory solution. The need for a deeper analysis is indicated.

Such an analysis is provided by the work of Häfele[16] who discusses in detail the plane shock case by recourse to a treatment modeled after the one given by Guderley for the spherical and cylindrical cases. The direction field and solutions in the projective plane for the differential equation representing the plane case are quite different from those given by Guderley. In discussing the various possible solutions, Häfele criticizes the solution corresponding to constant energy release on a plane surface at $t = t_0$, which solution is contained in Sakurai's first paper, and rejects it as unsuitable because while density vanishes at the origin, temperature becomes infinite in such a way as to maintain pressure constant. For a shock traveling only to the right (excluding the symmetrical case with identical shocks traveling in both directions), this boundary condition is interpreted as implying a displacement of the energy release plane. We infer that it would mean a leakage of mass in the negative direction, and thus to account for all the energy would require integration to negative infinity after $t = 0$.

Rejection of the constant energy solution on grounds of unsuitability leaves only variable energy solutions to be considered. Of these, Häfele determines that there is only one which he calls the "regular homology solution", which can be adjusted to pass through the point representing the strong shock conditions and in addition behave, sufficiently far behind the shock, so that both p and ρ vanish as T becomes infinite. For the case of air with $\gamma = 7/5$ the shock parabola is $x = \xi\, t^{0.6}$ and functions for the flow variables can be obtained explicitly. It is worth noting that in this case particle velocity decreases linearly with distance behind the shock front at a given time and for other values of γ the decrease is practically linear.

In a separate study Hain and Hoerner[17] examine numerically the effects of perturbing the regular homology solution by arbitrarily assigned initial values of the flow variables. They investigate only the case of astrophysical interest, viz., $\gamma = 5/3$. Their results show that for a variety of perturbations, the ensuing flows converge with time on the regular homology solution. Thus empirical evidence is provided for the stability of the regular solution. Further analytical studies[18,19] support this conclusion subject to the condition that the perturbations do not result in the formation of internal shock waves.

With these preliminaries we return now to the question of a possible connection between the experimental flows and these theoretical results. A strong plane shock may in principle be produced in three different ways; either directly or in the limit approached by strong spherical or cylindrical shocks. It is not known theoretically under what conditions, if any, the three flows so produced would be the same, but it may be conjectured not only that suitable conditions exist, but also that the flows so produced would converge to the regular homology solution of v. Weiszäcker and Häfele.

If this conjecture were true then we would expect the experimental shock trajectory with variable energy to approach a 0.6 power law with time, rather than the 2/3 power law required by the special condition of constant energy.

The experimental data of Figs. 2 and 3 suggest that a thorough investigation of this conjecture may be worthwhile; for the several cases presented seem to favor the 0.6 power law rather than 2/3. In the only case where data is available out to 10 μ sec, viz. the 1 atm No. 130 firing, the points from 3 to 10 μ sec fit the 0.6 parabola very closely.

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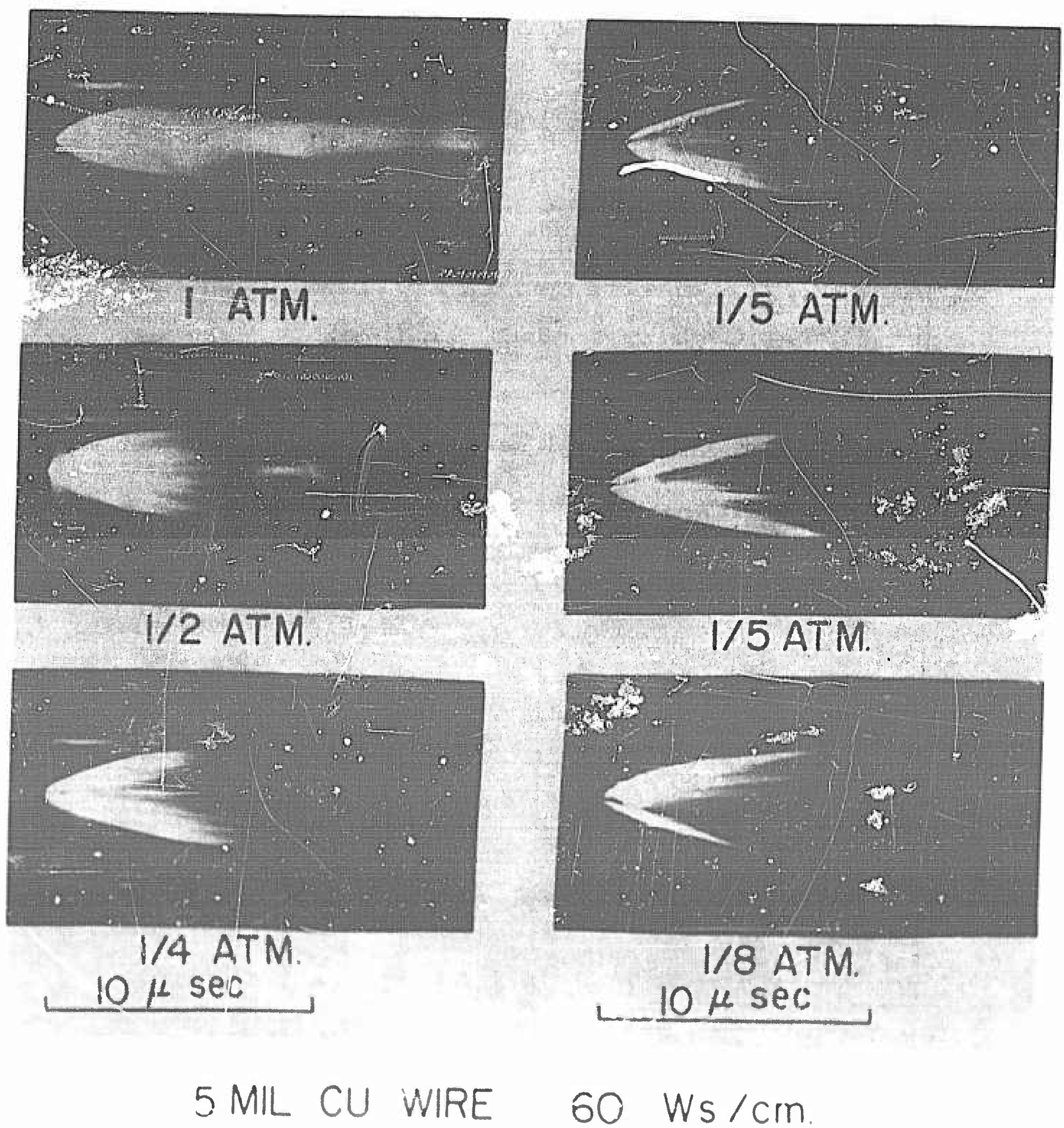

Fig. 1. Streak pictures of 5-mil exploding wire at different ambient pressures.

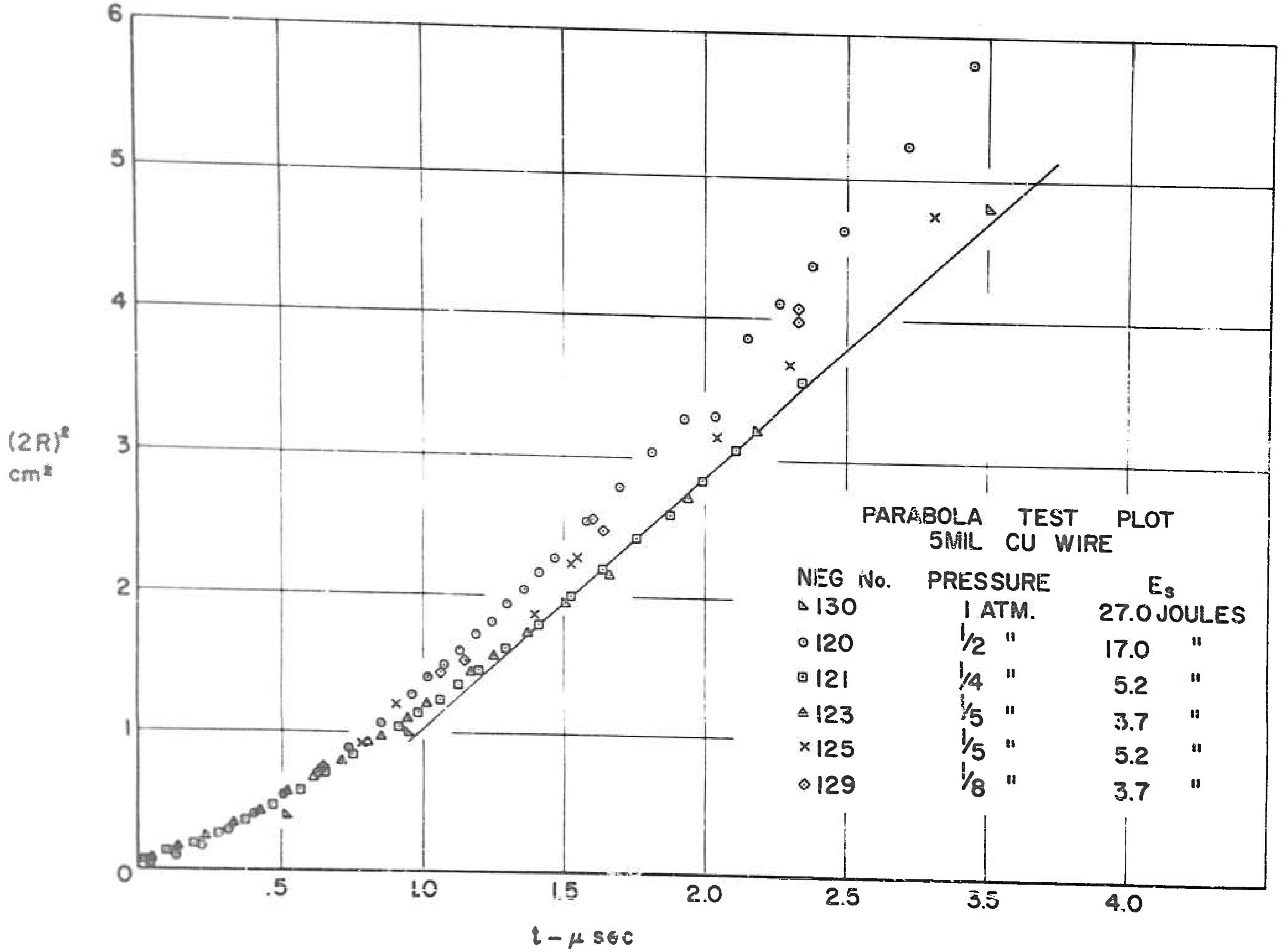

Fig. 2. Test of data for [illegible] energy similarity flow.

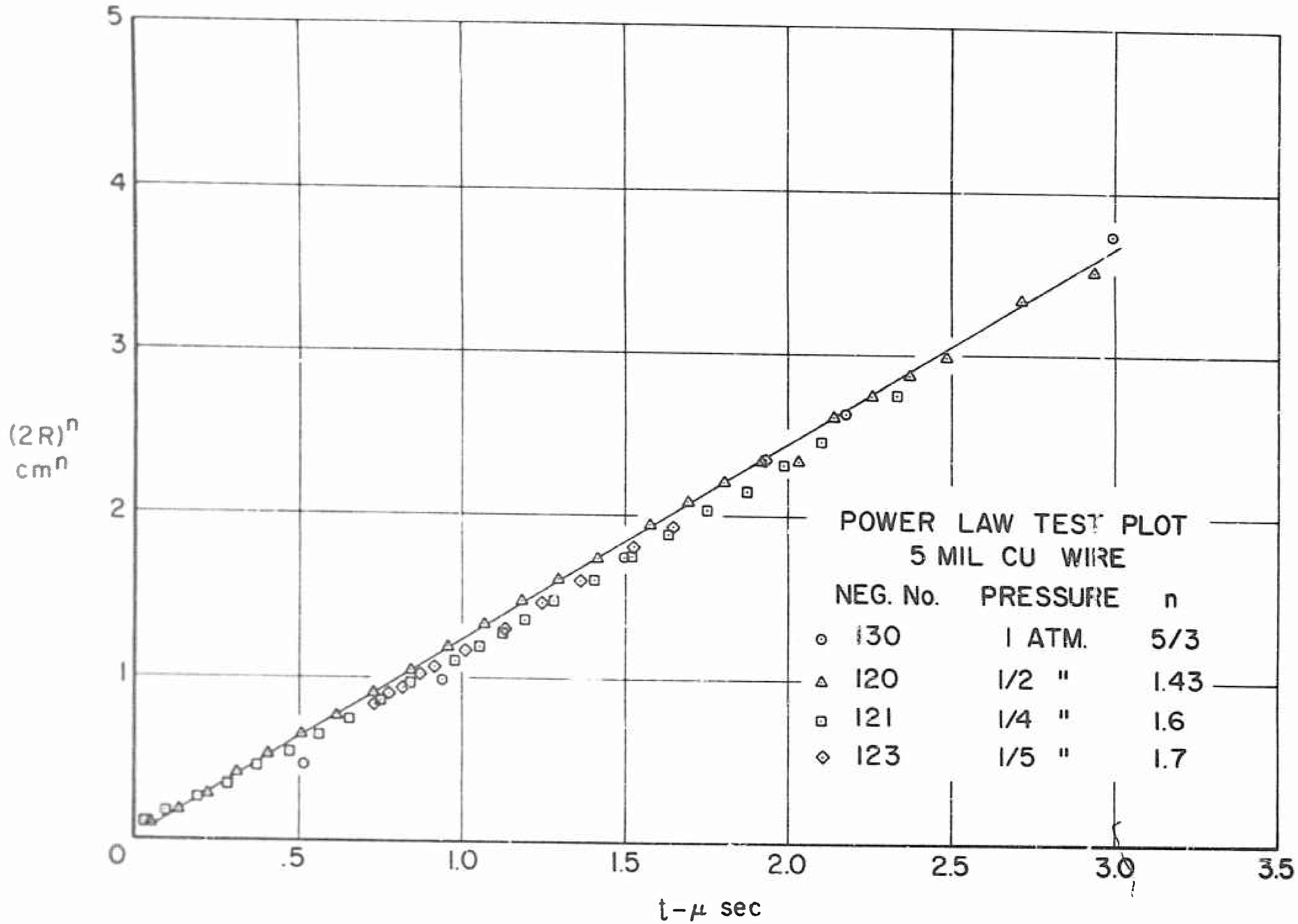

Fig. 5. Test of data for similarity flow of variable energy.