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SAFETY AND SAFETY FACTORS FOR AIRFRAMES

by

A. M. FREUDENTHAL

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SAFETY AND SAFETY FACTORS FOR AIRFRAMES

by

A.M. Freudenthal

This Report was presented at the Sixth Meeting of the Structures and Materials Panel, held from 4th to 8th November, 1957, in Paris

SUMMARY

The concept of structural safety of airframes is analyzed on the basis of its relation to the probability of structural failure, with a view of establishing procedures of quantitative evaluation of safety factors for a predetermined 'acceptable' risk of failure.

The difference in the approach to the concept of safety for ultimate strength and for fatigue is discussed, considering recent developments in fatigue research, particularly the results of fatigue tests under random loading, and methods of safety analysis for both conditions are proposed.

In this analysis the 'limit load' or 'limit load factor' is a basic concept. It should, however, be noted that this concept is not identical with the conventional structural design criterion of the same name. The difference is fundamental: while in conventional design the 'limit load' is a derivative concept, obtained simply by dividing the 'ultimate load' by an arbitrary 'safety' factor, usually 1.5, the concept as used here is the primary load criterion defining, independently of the ultimate load, a limiting condition of service by the aid of which fatigue design and design for limit load can be correlated.

Since it is a purpose of the present analysis to discuss and develop procedures for the rational evaluation of safety factors, expediently defined in terms of ratios of the ultimate to the limit load factor, it is obvious that both load factors have to be independently derived from operational criteria. Thus, while the meaning of the concept of 'ultimate load' used in this report does not differ significantly from its meaning in conventional design, the 'limit load' concept is significantly different.

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SOMMAIRE

La notion de la sécurité des cellules d'avion du point de vue structure est analysée sur la base de son rapport à la probabilité de la rupture de la structure, en vue d'établir des procédures d'évaluation quantitative des facteurs de sécurité pour un risque de rupture 'admissible' prédéterminé.

Les différentes façons d'aborder la question de la notion de la sécurité, selon que l'on étudie la rupture ou la fatigue, sont évoquées, compte tenu des développements récents concernant la recherche de la fatigue, en particulier, des résultats d'essais de fatigue obtenus sous des charges appliquées au hasard, et des méthodes d'analyse de la sécurité pour les deux cas sont proposées.

Dans cette analyse la 'charge limite' ou le 'facteur de charge limite' représente une notion de base. Il est à noter toutefois que cette notion n'est pas identique au classique critère de calcul des structures, de la même désignation. La différence essentielle, c'est que, alors que le calcul classique entend par la 'charge limite' une notion de dérivée obtenue en divisant la 'charge de rupture' par un coefficient de sécurité arbitraire, en général égal à 1.5, la notion telle qu'employée dans ce rapport est le critère de charge primaire, définissant, indépendamment de la charge de rupture, une limite d'utilisation qui permet de mettre en corrélation le calcul de fatigue et le calcul de charge limite.

Puisque la présente analyse a pour but d'étudier et d'élaborer des procédures pour l'évaluation rationnelle de coefficients de sécurité définis, de façon convenable, en fonction des rapports du coefficient de charge de rupture au coefficient de charge limite, il va sans dire que les deux coefficients de charge doivent être dérivés chacun des critères d'utilisation. C'est ainsi que, bien que la signification de la notion de la charge ultime, telle que l'on représente dans cette communication, ne diffère pas beaucoup du sens dans lequel on l'entend dans le calcul classique, il existe une différence importante quant à la notion de la charge limite.

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NOTATION

A	construction cost of airplane
C_F, C_L	cost of failure, unserviceability
d	differential sign
$l, l(x)$	probability of survival, survivalship function ($x = L, N, S, R, t$, etc.)
l_S, L_R	probability of exceeding S, of not attaining R
L	operational life
m	number of ultimate load repetitions
n	load factor, number of planes in fleet
n_U	ultimate load factor
N	number of (constant) load cycles
N_R	number of (random) load cycles
p, p_F	probability, failure rate
P, P_F	cumulative probability, probability of failure
$R, \tilde{R}, \bar{R}$	strength characteristic (variable, mode, mean)
$S, \tilde{S}, \bar{S}$	load intensity (variable, mode, mean)
t	time
$T, T_F, T_L, T_U, T(x)$	return period, return period of failure, limit load, ultimate load, return period of variable $\geq x$
y, z	variables
α_S, α_R	scale factors of extremal distribution ($\alpha = \pi/\sigma\sqrt{6}$)
$\nu, \bar{\nu}$	safety factors, with respect to mode, with respect to designated probability
σ_S, σ_R	standard deviation of variables, S, R

SAFETY AND SAFETY FACTORS FOR AIRFRAMES

A.M. Freudenthal*

1. INTRODUCTION

A critical re-evaluation of the concept of safety and of the safety factor is a task of considerable urgency, if the elaborate and refined methods of analysis made possible by computer development are to be effectively utilized in airframe design. The most careful and rigorous structural analysis is largely deprived of its merits if the accuracy of its results is diluted by the use of empirical multipliers, so-called safety factors, selected rather arbitrarily on the basis of considerations that are not always relevant or even rational. While an adequate degree of structural safety, has nevertheless been achieved by experience alone, such experience has not been acquired without serious setbacks[1].

The increasing use of so-called high-strength structural materials, the rapidly changing operating conditions, the rising cost and the resulting increase in operational life and utilization of the modern transport planes, as well as the increasing complexity and severity of the operational requirements with respect to military aircraft, have, however, largely invalidated the safety experience gained directly from the performance of airframes of the past. Moreover, these effects, singly and jointly, tend to reduce the structural safety of the airframe by intensifying the effects of fatigue[2] which, therefore, has recently emerged as a major cause of structural failure. This fact, in conjunction with the increasingly rapid rates of development of new types of transport planes and of obsolescence of military aircraft, makes it impossible to rely on experience acquired through past performance in terms of accident rates of airframes designed with empirically specified safety factors, apart from the wastefulness of such a procedure. Thus, methods must be devised with the aid of which the safety of a structure can be effectively analyzed, and design requirements specified on a rational basis that would ensure the correlation of the selected safety - or load factors with specified failure rates (for a fleet of airplanes) or probabilities of failure (for the individual airplane).

In the U.S. Civil Air Regulations[3], Part 4b paragraph 1.e, the following statements appear:-

(a) A limit load is the maximum load anticipated in *normal* conditions of operation

(b) An ultimate load is a limit load multiplied by the appropriate factor of safety

(c) The factor of safety is a design factor used to provide for the possibility of loads greater than those anticipated in *normal* conditions of operation and for uncertainties in design.

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It is implied that the load specified, in particular the gust load envelope, is a limit load envelope.

In the excellent historical summary prepared by members of the Los Angeles office of C.A.A. and given in Appendix A of G.N. Mangurian's paper[4] of 1954, it is shown that the foregoing definitions are relatively recent and represent essentially an inversion of the previous definitions according to which the ultimate load was specified as 'design load', while a load, obtained by dividing this design load by a factor of safety, called the applied load, was introduced as the load that could be withstood without permanent set. It is that load which was considered as the 'actual' or 'most probable' maximum load 'expected' in service, and which was later converted into the 'limit load' representing a flight load limitation for airworthy operation. That the numerical value of the 'factor of safety' associated with the earlier design concept is derived from the ratio of ultimate strength to yield strength of structural members is implied by the wording of the strength requirements in the 1934 edition of the Aeronautical Bulletin 7-A: 'The minimum factor of safety for any aircraft structure or component shall be 1.50 unless otherwise specified. This requires that the ultimate strength of any member shall be at least 1.5 times as great as its critical applied load', - when coupled with the added requirement that the applied load should be withstood without permanent deformation. Considering moreover that, as Mangurian points out, 'the ultimate factor of safety of 1.5 was established in 1934 for commercial use at about the same time that 24S-T aluminum alloy, having approximately the same value of ultimate to yield strength ratio, came into general use in aircraft design', it is obvious that there is a discrepancy between the theoretical concept of the safety factor implied in its present definition and the method of selection of its numerical value.

The same numerical value of the safety factor is retained, although the design philosophy changed. It is only mathematically irrelevant whether the 'limit load' factor is obtained by dividing the ultimate load factor by 1.5 or whether the 'ultimate load factor' is obtained by multiplying the 'limit load factor' by 1.5. In design there is a great difference between the two procedures. If the latter procedure is preferred, it is only logical to expect that the method of evaluation of the safety factor be consistent with its theoretical definition.

The definition given in the U.S. Civil Air Regulation is rather vague, particularly with respect to the meaning of the terms 'normal' operating conditions and 'anticipated'. Comparing the specified gust envelope with evaluated V-g data for 50,000 hours of flight[3] the 'normal' limit load, as given by the gust envelope, would be exceeded twice in about 10^6 flight miles, or once every 2000 hours for piston-engine aircraft; this is about once a year for every aircraft. For a fleet of 20 aircraft, one plane in the fleet must therefore be expected to experience the limit load in an average of 100 hours of flight, which is not the remote condition which the occurrence of the limit load is generally 'anticipated' to be, at least in civilian airplanes. While in the design of fighters a not too infrequent occurrence of the 'limit load' is considered 'normal', although the estimate of the 'anticipated' number of such occurrences varies considerably, it is not only a question of semantics whether, in such cases, the design loads represents 'limit loads'. These may be 'normal' conditions and, considering the actual safety record of airframes designed on this basis, they most likely represent adequate design conditions for presently used structural materials, present aircraft types and operating conditions. Since, however, the numerical probability of failure or expected rate of failure associated with these design conditions is actually unknown, and since changes in structural materials, aircraft types and operating conditions might significantly affect this probability, and thus the actual safety record, it seems necessary to establish rational methods by which the numerical value of the safety factor could be related to the associated probability of failure in terms of the expected variability of the acting loads and of the strength of the structure. For new types of aircraft-structures under new operating conditions there is no 'experience' or 'engineering common sense' that could help in 'anticipating' maximum loads under 'normal conditions' of operation.

The specifications of the British Air Registration Board[6] as well as the International Specifications[7] do not differ significantly from the U.S. requirements. In the discussion of those requirements W. Tye[8] points out their ambiguity, as well as the fact that this ambiguity can only be removed by the specification of a 'tolerable' rate of occurrence of limit-load, or rather, proof-load-failures and of ultimate-load-failures, on the basis of which the corresponding values of ultimate-and proof-or limit-loads can be established with reference to existing flight-load records. Tye stresses the crux of the problem when he states that 'in practice the number of hours of flying for which records are available is usually considerably less than the number of hours which would be tolerated by airworthiness authorities as sufficient period between proof failures. A problem which (therefore) immediately arises is the extrapolation of the data based on a limited number of hours to cover a much more extended period'.

The realization that the safety factor has to provide for:-

(a) the possibility of the operational loads exceeding the design load value,

(b) the possibility of the airframe having a strength less than the assumed design strength,

immediately produces an operational definition of the safety factor as well as a rational procedure for its evaluation, provided the possibilities (a) and (b) can be expressed quantitatively, that is in the form of probability functions. These relate load-intensities S and strength values R, with their frequencies of occurrence expressed either by the respective 'return periods' T(S) and T(R), in terms of flying distances, flying times or number of observations between expected recurrences of values respectively equal to or exceeding S and equal to or not attaining R, or by probabilities of, respectively, being exceeded $P(S) = 1/T(S)$ and not being attained $P(R) = 1/T(R)$ (see Reference 9, p.1337).

The safety factor to be applied to the design load is necessarily a function of the specified 'expected' or average return period of the design-or 'limit'-load in relation to the anticipated return-periods of the failure load and of the 'unserviceability'-load, expediently identified with the 'proof-load', as well as a function of the expected or tolerated variability of the yield-and of the failure-strength of the structure. Thus, two safety factors are implied by this definition, one with respect to 'unserviceability' and one with respect to failure. It is only when the unserviceability load ('proof load') is identified with the limit load that the consideration of the first safety factor is usually omitted, although this procedure is unjustified considering the fact that the yield strength of the structure, as well as the specified limit load, are usually associated with 'average' return periods, and therefore subject to errors of estimation.

It is obvious that this approach to the safety factor precludes its identification with a simple strength ratio related to the structural material, or even to the structure as a whole. The specification of 'tolerable' rates of occurrence of structural failure and of unserviceability failure with respect to design for both ultimate strength and fatigue can not be avoided. However repulsive the specifications as 'tolerable' of a finite numerical failure rate, no matter how small, may appear to be to airworthiness authorities, as well as to airframe designers and manufacturers, it is an unavoidable prerequisite to the evaluation of rational values of the safety factor.

2. FAILURE CONDITIONS AND DESIGN CRITERIA

The principal structural aspects of the operating conditions of an airframe are the limiting condition of service and the condition of failure; both are usually defined in terms of the 'load factor' n, characteristic for the specified condition. In the first conditions, usually referred to as 'unserviceability', the structure has attained a state in which it can no longer function adequately, although it is structurally intact: the second condition is one of collapse of the structural resistance, either by structural instability or as a result of fracture of primary load-carrying members.

Under service conditions the structure should be able to sustain an infinite or, in the case of members subject to fatigue damage, a specified finite number of load applications without noticeable effect on its usefulness or 'serviceability'. Because such usefulness, prior to failure, can be affected structurally only by a change of shape, transient or permanent, which would impair the aerodynamic performance and thus the effective functioning of the airplane, the specification of the limiting service condition is necessarily based either on the requirement of perfect reversibility of the limiting transient deformation of the structure, or on an aerodynamically based limitation of the total permanent deformation whenever creep in the airframe proceeds during operation at elevated temperatures.

To establish the critical condition of catastrophic failure of the airframe, the different alternatives of collapse of the structural resistance must be considered. Collapse can be produced by

(a) instability of one or several primary structural elements under a single application of an excessive load;

(b) fracture of primary structural elements or structural connections under a single application of an excessive load;

(c) fracture of primary structural elements or connections by fatigue, resulting from the application of a random sequence of loads the intensity and frequency of occurrence of which is described by a characteristic 'load-spectrum';

(d) fracture of primary structural elements or connections as a result of creep under the sustained or intermittent application of a random sequence of loads and temperatures, the intensity, duration, and frequency of occurrence of which is described by a three-dimensional 'load-time-temperature' spectrum.

The critical condition of failure of a particular structure is that condition which, compared to the alternative conditions, has the highest probability P_F of occurrence during the specified operational life L. This life is therefore one of the main factors determining the relevant failure criterion; a change in the specified operational life may significantly affect the design criterion and the safety of the designed structure, either by changing the intensity of the maximum load (ultimate load factor n_U) to be expected during this period, or by changing the total number N of load cycles or the time t under sustained load at elevated temperature.

In the case of collapse under the single application of one 'ultimate' load cycle associated with the load factor n_U, the safety of the idealized structure, as expressed by the probability of its survival $l = (1 - P_F)$ during its operational life, is affected by the specified life, even if this life is appreciably shorter than the 'return period' $T_U = 1/P_F$ of the ultimate load, and failure occurs as soon as the specified ultimate load factor n_U is attained or exceeded, independently of any statistical variation of the ultimate 'strength'. The probability of survival can, in this case, be estimated simply on the basis of the Poisson equation

$$p(0) = \exp(-L/T_U) = l(L) \tag{1}$$

being the probability that during the life L the aircraft will not experience this load even once, which is the probability of survival $l(L)$ for an operational life L. To keep this probability sufficiently high, or the probability of failure $P_F = (1 - l)$ sufficiently low, the ratio (L/T_U) must be rather small: with $(L/T_U) = 0.01$, the probability of survival at L is $l(L) = 0.99$, with $(L/T_U) = 0.1$ the value $l(L) = 0.90$.

These figures show that, for an expected service life of the order of 10^5 hours, the return period for the ultimate load factor must not be less than 10^6 hours in order to keep the probability of failure sufficiently low.

The conditions are quite different when failure occurs as the result of repeated load cycles (fatigue) or sustained load (creep-fracture). In this case the probability of failure increases with every load cycle or with every unit-period of sustained load. The survivorship functions $l(N)$ or $l(t)$ in terms of load cycle, N or times, t, of sustained load application, assuming a certain load spectrum, express the gradual decrease of the probability or survival with increasing N or t. Up to the end of the expected operational life this probability must not drop below a value associated with an acceptable risk of failure, and must therefore be appropriately higher at the start. The 'acceptable risk' will obviously be much higher with respect to non-catastrophic failure of components that can be economically replaced, than with respect to vital structural parts the failure of which would result in structural collapse. For the latter, the probability of survival at the end of the operational life as a result of fatigue should just equal the probability of survival for the ultimate load factor. This is schematically shown in Figure 1, where the probability of survival for ultimate load and different operational lives according to a Poisson function is compared with an empirical fatigue survivorship function under a specified load spectrum, using extreme-value probability paper.

It can be assumed that, for the same structural dimensions, the probability of survival for ultimate load of an aircraft is increased by using a high-strength material such as AA-7075 instead of AA-2024 aluminum, while the fatigue performance remains unaffected. The basis for this assumption, which closely reproduces the real conditions, is the fact that by increasing the ultimate strength of the structure the return period T_U of a load equal to or exceeding this strength is increased together with the probability of survival $l = 1 - 1/T_U$.

Under the assumed conditions, design ranges for ultimate load and for fatigue are delimited by the points of intersection of the curves $l(L)$ and $l(N)$, with abscissae L_1 and L_2 for the two materials. Since $L_1 < L_2$, it is clearly demonstrated that an increased strength with respect to ultimate load, expressed in terms of an increased probability of survival, without appropriate improvement of the fatigue performance, will significantly increase the safety of only those aircraft the operational lives of which are much shorter than the operational life initially specified. For operational lives $L_1 < L$ for AA-7075 and $L_2 < L$ for AA-2024, the probability of survival under the fatigue load spectrum is lower than that for ultimate load and, for a specified operational life L, must therefore be increased by increasing the dimensions of the critical structural parts or sections beyond those required for resistance to ultimate load, so as to reduce the rate of fatigue damage to a figure which would increase the mean fatigue life by $\Delta\bar{N}$ and raise the probability of survival at life L by $\Delta l(L)$. If, for a specified operational life L, the probability difference $\Delta l(L)$ cannot be completely eliminated, possibly because of performance limitations imposed by the resulting increase of structural weight, the safety of the airplane and its expected failure rate will be determined by its fatigue performance rather than by its 'ultimate' resistance; the design must be concerned primarily with fatigue. In this case the safety with respect to the alternative failure criteria will be unbalanced.

It is interesting to note that the probability of survival for ultimate or for limit load can be very appreciably increased if the structure is designed to sustain, without collapse or without unserviceability, one or a very few applications of the ultimate or the limit load, instead of failing under its first application. If the structure is designed so as to fail only at m repetitions or to sustain at least (m-1) repetitions of the ultimate load without collapse, the probability of survival of the airplane will be equal to the probability of the airplane to encounter not more than (m-1) application of the ultimate load

$$l(L) = \exp(-L/T_U) \sum_{k=0}^{k=m-1} \frac{1}{k!} (L/T_U)^k = 1 - P(m) \qquad (2)$$

Thus, for instance, for $(L/T_U) = 0.1$ even the assumption of $m = 2$ increases the probability of survival from $l = 0.9$ according to Equation (1) to $l = 0.99$ according to Equation (2). For $(L/T_U) = 1.0$, the same assumption increases l from $l = 0.37$ to $l = 0.74$; design for $m = 4$ would further increase the probability of survival to $l = 0.98$. The functions $l(L)$ for $m = 2$ and $m = 4$ are shown in Figure 1. It is obvious that the increased and extended safety (high probability of survival) resulting from a design for collapse or unserviceability under a *few* rather than under the *first* application of the ultimate load or the limit load is significant only if fatigue failure (or creep-rupture) is not the dominant design criterion within the considered range of operational life.

3. LOAD POPULATIONS AND LOAD SPECTRA

The schematic presentation in Figure 1 of the effect of the specified operational life on the failure criteria disregards, in first approximation, the fact that the ultimate and the limit strength of the structure, as well as the load spectrum, are subject to statistical variation. Only the statistical variations of the occurrence of the ultimate load (or the limit load) and of the fatigue life under a specified spectrum are considered in terms of the return periods T_U or T_L of the ultimate and limit loads and of the (empirical) survivorship function in fatigue $l(N_R)$ associated with a specified, randomly applied load spectrum.

The return periods of the ultimate load and, in general, also of the limit load are, obviously, not parameters that can be directly obtained from records of observations; their values must be deduced from extrapolation of actual records towards the region of extremely low frequencies. They are, therefore, subject to the uncertainty of such extrapolation. Moreover, the probability associated with the failure or unserviceability criterion cannot be defined in terms of the occurrence of the respective load factor alone, but only in terms of the probability of coincidence of this load factor with a value of the ultimate or the limit strength of the structure that would not be sufficient to resist it (Ref.9, p.1337); it can only be derived by suitably combining the 'load-population' with the population of 'ultimate strength' or of 'limit strength' of the aircraft so as to produce the 'critical design condition' (combination of applied load and structural resistance) associated with a specified level of safety.

The ultimate load factor n_U is that ordinate of the significant load spectrum the return period of which is long enough to make the probability of catastrophic failure by collapse within a fleet of aircraft so low as to be 'acceptable'; the limit load factor n_L is the ordinate the return period of which, while much shorter than that of the ultimate load, is long enough to make it very unlikely that the limit of serviceability will be attained more than once during the anticipated operational life of an individual airplane. Since the load spectrum for a particular type of plane varies with the operational use of the individual plane, the design should be based on a certain range of load spectra representing the anticipated variation of operational conditions and of operational requirements, rather than on a single observed load spectrum.

It is generally accepted that gusts are the dominating cause of fatigue with respect to the wings of transport airplanes, while the dominant fatigue damage of wings of fighter or trainer planes is due to maneuver loads. This will usually also apply to the ultimate and the limit load factors, although it is theoretically possible that the critical fatigue damage of long-range fighters might be determined by gusts, while the ultimate and the limit loads form part of the maneuver load spectrum.

The purpose of the application of statistical methods to problems of maximum loads and of load spectra is to obtain by extrapolation estimates of the probability of encountering the extremely high design load factors of very low expected frequency of occurrence, as well as to obtain estimates of the rate of accumulation, under random loading associated with certain load spectra, of fatigue or creep-damage. It should be obvious that, because of the necessity of such extrapolation, the selection of a probability function involves more than simple curve fitting. Unless the selected probability function is germane to the problem, and adequately represents the inherent statistical variability of the phenomenon which results from certain basic assumptions concerning its origin, extrapolation towards the extremes (tails of the function) will result in erroneous predictions within this range of variation, which is just the relevant design range.

It appears that the probability distribution of the maximum values obtained from successive series (samples) of records can be utilized for an effective estimate of the required extremes for design. The statistical theory of the distribution of extreme values indicates that for initial distributions of the most common type which approach zero exponentially, a limiting form exists for the distribution of the maximum values of large samples[10]. The use of this asymptotic form should therefore increase the reliability of the estimate in comparison to direct extrapolation from the observed initial distribution.

While the character of the probability function of extreme gust-intensities, with the aid of which the ultimate load factor can be estimated, is thus completely determined by the extremal nature of the phenomenon, independently of the nature of the underlying distribution of gusts (gust spectrum), this distribution can either be obtained by statistical interpretation of actual gust records or by a theoretical study of the effects on the airplane of atmospheric turbulence and of buffeting, and of the response of the airplane to these random disturbances, by means of generalized harmonic analysis and power spectra technique.

The design gust spectrum should extend from the limit load downward. However, the small probability of actual occurrence, during the operational life, of loads in the vicinity of the limit load makes this load region rather insignificant with respect to damage accumulation over this life. The specific shape of the spectrum is therefore less significant in the limit load region than in the region of higher frequencies; thus, discrepancies between actual load distribution and the assumed form of the spectrum can be tolerated in the vicinity of the limit load. The conclusion appears, therefore, to be justified that the initial frequency function of gusts can be fairly well represented by a simple exponential probability function over the entire range. The same can also be done for maneuver loads for fighters[11].

There are other loads that may be significant in design for failure and for unserviceability, or represent important sources of fatigue damage for various parts of the airplane, such as take-off and landing loads, taxi-ing loads and pressure cabin loads. The total frequency of these loads, which occur only once or a few times per flight, is low in relation to the frequency of gust- and maneuver-loads; with the exception of the landing load, their range of variation is relatively narrow and cannot be dealt with statistically, because of the essentially non-random character of this variation. Only the landing loads are of sufficiently random nature to be represented by a spectrum.

The maximum intensity and number of repetitions of the non-random loads can, in general, be estimated from limiting operational conditions, and suitable design limit and ultimate loads specified on this basis. With respect to fatigue design, the damaging effect of the non-random loads may alone be critical, as in the pressure cabin, or it should be considered together with the damaging effects of the maneuver- and gust-load spectra. Where non-random loads represent the critical design conditions, the statistical design aspect associated with the load spectrum vanishes; the variability of the strength and of the fatigue life of nominally identical structures designed for a single, closely specified loading condition remains as the only significant statistical aspect.

4. THE VARIABILITY OF STRUCTURAL RESISTANCE

The variation of the structural resistance at the limits of serviceability and of failure is the result of the variation of the pertinent mechanical properties of the structural materials, of the dimensional tolerances and of the inaccuracies and variations in the manufacturing process. The variation in the fatigue life is the result of the same effects, magnified by the inherent highly statistical character of fatigue damage and its sensitivity to residual stresses of all kinds, local surface conditions and atmosphere.

The approach to the analysis of the scatter of the characteristic structural strength values, such as the yield limit or the ultimate fracture strength, is similar to that used in the analysis of load spectra, with the difference that the predictions that have to be made on the basis of the observed data refer to values that are smaller than those that have actually been observed. As for loads, such extrapolation can be made with some confidence only if the fitted probability function represents the characteristic pattern of variation that can be expected as a result of the nature of the phenomena.

On the strength of the plausible assumption that only the low range of the observed strength values are of significance in analyzing the safety of a designed structure or its probability of failure or survival, extrapolation of the observed towards extremely low strength values of very low probability of not being attained can be based on the distribution function of extreme (lowest)values. Similar to the case of loads - where, however, extrapolation is required towards a maximum - asymptotic forms exist for the distribution of minimum values of large samples of observations of strength. The use of such distributions will necessarily increase the reliability of the estimate of design minima in comparison to direct extrapolation from the (observed) initial distribution; it is obvious that, within the range of high strength values, the initial distribution provides a better fit of the observations, but this range does not determine the safety or probability of failure.

Relatively few systematic observations exist concerning the scatter of yield stress and of fracture strength of aircraft materials, and still fewer concerning the strength of structural parts. Whenever structural parts were tested to failure it became evident that the scatter in the strength of the part was much wider than the scatter due to variations in the strength of the structural material itself[17]. The observed distributions of such strength were either normal or logarithmic normal, while the coefficient of variation of the material itself, including thickness tolerances, was usually not more than 7-10%.

No data are available concerning the scatter of the yield limit of structural parts. The yield stress of structural metals shows, in general, a narrower range of variation than the strength; it can therefore be assumed that the variation of the yield limit of structural parts is much narrower than that of their strength, particularly since the latter is strongly affected by the extensive plastic redistribution of stress that occurs before the ultimate strength is reached. The coefficient of variation of the yield stress observed in more than 1000 tests of a typical structural steel (Ref.9, p.1358) was found to be almost 10%; considering the more stringent controls in the selection of aircraft structural metals, it may be assumed that the coefficient of variation of the yield stress of such metals may be as low as 5-7%, with resulting coefficient of variation of the yield limit of structural parts of the order of at least 7-10%.

5. THE RISK OF FAILURE (DECISION RULES)

The probability of failure or of unserviceability should be specified in relation to the expected service life, so as to reduce the risk of failure or unserviceability, either to an arbitrarily specified 'acceptable' maximum, or to a value derived on the basis of an economic balance between the 'cost' of increasing the safety or of reducing the risk of failure and the 'cost' of such failure. The term 'cost' may refer to actual cost of construction alone, or it may also include operational costs or it may be defined in terms of (military) performance such as range, speed or climb, independently of any economic considerations.

In the first case, the cost of increasing the safety depends only on the rate at which the construction cost A of the whole structure increases with decreasing probability of failure P_F or unserviceability P_L, while the cost of failure or of unserviceability can be considered as a charge against the structure equal to the (capitalized) cost of failure C_F or of unserviceability C_L divided by the respective return period T_F or T_L, or multiplied by the respective probability of occurrence P_F or P_L. The optimal conditions are therefore defined by $(A + P_L C_L) \to \min$ and $(A + P_F C_F) \to \min$, or

$$\frac{dA}{dP_L} + P_L \frac{dC_L}{dP_L} + C_L = 0 \quad \text{and} \quad \frac{dA}{dP_F} + P_F \frac{dC_F}{dP_F} + C_F = 0 \qquad (3)$$

The cost of failure is made up of two parts: one part, C_{FO}, which is independent of P_F and includes all direct and indirect losses resulting from the failure, the other which represents the cost of reconstruction or of a new construction and depends on P_F in a similar way as the initial cost A. The latter can, in first approximation, be assumed to increase linearly with decreasing ($\log P_F$).

The economically optimal risk of failure or of unserviceability increases with decreasing ratio of cost of failure to initial construction cost, and with increasing range of scatter of the relevant strength properties. When the economic consideration are extended to include not only the initial cost but also the cost of operation, the cost of increasing the safety of a transport of specified gross weight will be made up of the (capitalized) total loss of income due to the increased ratio of structural weight to gross weight, less a possible capitalized gain in reduced maintenance costs, in addition to the increase in construction costs; the cost of failure will necessarily include a term representing the capitalized loss of income after failure.

An entirely different set of conditions has to be set up if the estimate of the optimal design risk of failure or unserviceability is to be based on (military) performance instead of on economic considerations. The cost of increasing the safety may be expressed, for instance, in terms of 'range' by relating the increase of the ratio of structural to gross weight to the resulting reduction of the ratio of fuel weight to gross weight, which determine the range, and by expressing the cost of reduction of the range in military terms, for instance in terms of the number of missions lost as a result of the reduced range. If the 'cost' of failure, which is the military effect of the loss of the aircraft as a result of structural failure, is formulated in the same terms, an equation equivalent to Equation (3) is obtained, which can be used for the estimate of the optimal design risk of failure. A similar approach can be based on 'speed', 'rate of climb' or any other relevant performance criterion.

The estimated optimal design probabilities of failure refer to the total number of aircraft in operation, and thus also express the acceptable risk of failure of the individual aircraft during its specified operational life L in hours or in miles. The associated optimal design *rate* of failure is therefore $p_F = P_F/L$ per hour or per miles of flight.

Using the optimal failure risks P_F, or failure rates p_F, the design probability of no failures in fleets of n airplanes or in a total of m flying hours or flying miles (survivorship function) can be specified on the basis of the Poisson distribution:

$$l(n) = \exp(-nP_F) \text{ and } l(m) = \exp(-mp_F) \qquad (4)$$

with similar expressions for unserviceability.

The correlation between design for limit load and ultimate load on the one hand, and for fatigue on the other, is established by the fact that the limit load should represent the maximum ordinate of the load spectrum forming the basis for the fatigue design, with an average return period equal to the operational life of the structure.

While in design for failure or unserviceability the probability of failure or survival is related primarily to the probability of encountering or not encountering certain critical but rare load intensities, the probability of fatigue failure, being the end result of a process of progressive damage during the operational life, essentially determined by normal operating conditions, depends on the statistical variation of the fatigue damage rate for individual airplanes of identical design under a set of nominally identical operating conditions, as well as on the statistical variation of these conditions. The difference in the basic approaches to risk and failure rate in design for ultimate load and for fatigue is therefore the same as that in the approaches to accident insurance and to life insurance: the accident insurance premium, which is a measure of the risk of occurrence involved, is practically independent of age, while the dependence of the life insurance premium on age reflects the fact that the mortality is a *function* of age, subject to random variation for individuals in a certain age group, depending on the physical constitution of the individual as well as on his occupation and mode of life. The operating conditions of the designed airplane are specified in terms of a single critical load spectrum or of a range of load spectra, in conjunction with the respective survivorship functions for the various spectra obtained from spectrum fatigue tests under different assumptions concerning the stress amplitude at the extreme or the most probable (characteristic) limit load in terms of the yield stress or the ultimate strength; they determine the design survivorship functions from which the expected probability of failure at the specified operational life, in terms of the total number of load cycles, can be deduced. Because the risk of fatigue failure at the end of the operational life must still be 'acceptably' low, the estimate of an optimal risk must be based on this life.

6. SAFETY AND SAFETY FACTOR

The only rational expression of the safety of an airplane structure is the specification of its probability of survival under the full range of possible operating conditions of various probabilities of occurrence, up to a critical condition under which the structure would be expected to fail, and the probability of occurrence of which should therefore be rather remote. This specification implies an expected or average operational life as a basic design parameter, with respect to which this probability for a fleet of airplanes can be expressed in terms of the expected failure rate per mile or per hour of flight or per mission of specified duration. On this basis there is no other difference between the approaches to the design for ultimate strength and for fatigue but the fact that the ultimate strength design is concerned with a single critical condition, while in fatigue design the whole range of operating conditions must be considered. In both design approaches, survivorship functions expressing the probability of survival as a function of the expected operational life can be constructed (Fig.1), and the difference is only in the basic character of the respective survivorship functions $l(L)$ and $l(N)$ reflected in the methods of their derivation: while the survivorship function $l(L)$ for ultimate strength design is determined by purely statistical considerations, the survivorship function $l(N)$ for fatigue design is the result of the combination of physical and statistical effects. Thus, a rational unified design procedure for a specified probability of survival under all conditions, including fatigue, can be devised without any reference to the concept of a 'safety factor'. It is, in fact, not immediately obvious how such a concept could be introduced without disturbing, by arbitrary assumptions, the sequence of this unified design for a specified probability of survival.

The necessity for the concept of a safety factor in the design for ultimate strength arises from the difficulty of performing an effective structural analysis of the airframe in the critical condition of imminent collapse, for which the usual idealizations of structural response (linear elasticity, small deformations) that make such analysis possible are no longer justified and for which, moreover the critical minimum values of the material strength characteristics are not easily obtained by materials tests, unless their number is increased far beyond the number usually considered as practical. It becomes therefore expedient to introduce design operating conditions for which the structural analysis can be performed, and to relate the structural response under these conditions, using the mean values of the respective strength parameters that are easily obtained from a small number of tests, rather than the extremes (minimum values), to the (minimum) ultimate strength developed under the critical conditions with the aid of a safety margin or 'safety factor'. The required safety factor will necessarily be higher, the larger the difference between the two conditions; its smallest value is obtained if the limiting operating conditions (limit load), which are the conditions at the limit of unserviceability, are introduced as design conditions.

7. SAFETY ANALYSIS FOR ULTIMATE STRENGTH

If S denotes the applied load or the stress in the critical section resulting from it, associated with the return period T(S) or with the probability $l_S = 1 - 1/T(S)$ of not being exceeded, and R denotes the carrying capacity of the structure under this type of load, or the strength of its critical section associated with a return period T(R), or with the probability $l_R = 1 - 1/T(R)$ of being exceeded, the probability of survival under the combination (S,R) is

$$l(S,R) = l_S \, l_R \tag{5}$$

and the probability of failure is therefore $(1 - l_S \, l_R)$, provided that the relation

$$R - S = 0 \tag{6}$$

delimits the range of survival, $R - S > 0$, and the range of failure $R - S < 0$.

Assuming extremal distributions of S (distribution of maxima) and of R (distribution of minima) the lines of equal probability of 'survival' are shown in Figure 2. In the same coordinate system, the condition $R = S$ is represented by the equation of a straight line. The ratio of modal resistance to modal load is defined as the 'safety factor' ν associated with the design condition based on the modes $\tilde{R}$ and $\tilde{S}$: to ensure that $R = S$, it is necessary that $\tilde{R} = \nu \tilde{S}$.

The optimal condition for a given safety factor is obviously that associated with the highest probability or survival along the respective straight line, which is defined by the coordinates of that point on this line at which is touches a contour line l = constant (see Figure 2).

Values of the optimal safety factors ν for the probabilities of survival $l = 1 - 10^{-2}$, $1 - 10^{-4}$ and $1 - 10^{-6}$ have been computed and are presented in Table I for different coefficients of variation of S and R.

Table I shows that a certain inter-relation exists between the specified coefficients of variation and the maximum probability of survival that can be attained, a conclusion that is borne out by a simple inspection of Figure 2. Thus for instance a probability of survival of $l = 1 - 10^{-4}$ cannot be attained unless the coefficient of variation of the strength charadteristic $v_R < 0.15$; for $l = 1 - 10^{6}$ the restriction is to $v_R < 0.10$. Failure to control the strength characteristics within the respective limits cannot be made up by increasing the safety factor, if the design is to a specified probability of survival, whil uncertainties in the load specification can be counteracted by improving the control of the variation of the strength parameter, which is most effectively done by attempting to reduce the considerable discrepancy between the variation in the material properties and the variation in the respective properties of the finished structural parts.

The values[13] of ν in Table I indicate that design on the basis of the modal values $\tilde{S}$ and $\tilde{R}$ would require relatively high safety factors, much higher than those used at present, even for the smallest values of the respective coefficients of variation.

If instead of the modal values $\tilde{S}$ and $\tilde{R}$ new values S_0 and R_0, with return period or return number of 100 (1% probability of values $> S_0$ or $< R_0$) are selected as design conditions at the limit of serviceability, on the basis of the consideration that the optimal probability of unserviceability is of the order of 10^{-2} (see Section 4), a new set of safety factors $\bar{\nu}$ referring to those conditions can be computed. These factors are presented in Table II. For limit load or 'proof load' design, with small variation of S and R the appropriate safety factors $1.0 \leq \bar{\nu} \leq 1.10$ are of the conventional order of magnitude.

8. METHODS OF SAFETY ANALYSIS IN FATIGUE AND CREEP DESIGN

A rational design for fatigue should ensure a probability of survival of the aircraft structure under conditions of fatigue at least equal to that on which the ultimate load design is based. However, whereas the latter probability is significantly determined by the expected risk of occurrence of a rather unlikely extreme value of the limit load or of the limit-load stress in a critical section, the former depends primarily on the load spectra representing conditions within the actual operational range, and is therefore more affected by the spectra associated with the likely average values of the limit load, than by the unlikely extremes.

The fatigue damage produced by a spectrum extending to the limit load or limit load stress can be associated with the return-period $T(S_L)$ of this load, which increases as its intensity is increased; the longer the return period of the limit load, the larger the amount of fatigue damage produced by the associated load spectrum. However, while the probability of survival under an unlikely, excessive value of the limit load, commonly identified as the 'ultimate load', is practically constant for an operational life selected on the basis of its return period, the probability of survival in fatigue decreases continually and, at any moment, depends essentially on the past load history.

The similiarity in the safety analysis for fatigue and for creep is in the fact that complete load spectra representing conditions within the operational range, rather than highly improbable load maxima, must be expected to produce almost all the damage in terms of strength reduction or permanent deformation. The difference, however, is in the additional effect of time t and of temperature T, so that the creep-damage at a certain load level depends not only on this level itself, but also on its duration and the temperature during that time. The 'spectrum' for safety analysis under creep conditions should therefore be considered as four-dimensional, expressing the probabilities of encountering combinations more severe than the considered set of variables (S, t, T), unless definite functional relations between certain of these variables, for instance between S and t, or S and T, or t and T, can be established on an empirical basis, so that the basic four-dimensional spectrum can be reduced to three or even two dimensions. This could be done if, for instance, certain load intensities could always be correlated with the same times of application and/or the same temperatures. By introducing workable expressions for creep-damage accumulation and for cumulative creep-deformation under a specified load-time-temperature history and combibing these with the load-time-temperature spectrum, survivorship functions in creep $l(t)_{s(t),T}$ can be constructed from which the probability of surviving certain critical design load-time-temperature histories could be estimated. For a specified operational life this probability must not fall below the probability for ultimate load design. Because of the necessity to consider creep-rupture and maximum permanent deformation as alternative design criteria, two survivorship-functions [illegible] t),T are necessary, one with actual creep-rupture as a criterion for non-surviva[illegible]ne other with the maximum acceptable permanent deformation delimiting 'survival' and 'non-survival'.

The absence of adequate expression for creep and creep-damage accumulation makes it difficult at present to develop the outlined method in more detail.

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TABLE I

Safety Factor, ν, Based on Modal Design Values

$\frac{\sigma_S}{\bar{S}}$	$\alpha_S S'_0$ \ $\alpha_R R'_0$	$P_p = 10^{-2}$, $l = 1 - 10^{-2}$							$P_p = 10^{-4}$, $l = 1 - 10^{-4}$			$P_p = 10^{-6}$, $l = 1 - 10^{-6}$	
	$\frac{\sigma_R}{\bar{R}}$	0	0.05	0.10	0.15	0.20	0.25	0.30	0	0.05	0.10	0	0.05
	$\alpha_R R'_0$	∞	26.23	13.40	9.13	6.99	5.71	4.85	∞	26.23	13.40	∞	26.23
0	∞	1	1.2	1.5	2.0	2.9	5.1	19	1	1.5	3.2	1	2.1
0.05	26.23	1.2	1.5	1.9	2.6	3.8	6.9	27	1.4	2.2	4.7	1.5	3.4
0.10	13.40	1.3	1.7	2.2	3.0	4.6	8.4	33	1.7	2.8	6.1	2.0	4.6
0.15	9.13	1.5	1.9	2.5	3.5	5.2	9.8	40	2.0	3.4	7.4	2.5	5.8
0.20	6.99	1.7	2.2	2.8	3.9	5.9	11	46	2.3	3.9	8.6	3.0	6.8
0.25	5.71	1.8	2.3	3.1	4.3	6.5	12	51	2.6	4.4	9.8	3.4	7.9
0.30	4.85	1.9	2.5	3.4	5.6	7.0	14	57	2.9	4.9	11	3.6	8.9
0.35	4.24	2.1	2.7	3.7	5.0	7.6	15	62	3.2	5.4	12	4.3	9.8
0.40	3.78	2.2	2.9	3.9	5.4	8.1	16	67	3.4	5.8	13	4.7	11

TABLE II

Safety Factor, $\bar{\gamma}$, Based on Design Values at Return Period 10^7

		$P_F = 10^{-7}$, $l = 1 - 10^{-7}$							$P_F = 10^{-4}$, $l = 1 - 10^{-4}$			$P_F = 10^{-6}$, $l = 1 - 10^{-6}$	
	$\frac{\sigma_R}{\bar{R}}$	0	0.05	0.10	0.15	0.20	0.25	0.30	0	0.05	0.10	0	0.05
$\frac{\sigma_S}{\bar{S}}$	$\alpha_S S_0'$ \ $\alpha_R R_0'$	∞	26.23	13.40	9.13	6.99	5.71	4.85	∞	26.23	13.40	∞	26.23
0	∞	1	1.00	1.00	1.00	1.00	1.00	1.00	1	1.27	2.10	1	1.74
0.05	26.23	1.00	1.05	1.07	1.09	1.12	1.14	1.17	1.15	1.54	2.64	1.30	2.40
0.10	13.40	1.00	1.07	1.09	1.12	1.17	1.22	1.28	1.26	1.71	2.98	1.51	2.83
0.15	9.13	1.00	1.07	1.11	1.14	1.19	1.27	1.36	1.34	1.84	3.22	1.67	3.16
0.20	6.99	1.00	1.08	1.12	1.15	1.21	1.31	1.42	1.40	1.93	3.41	1.79	3.40
0.25	5.71	1.00	1.08	1.14	1.17	1.22	1.33	1.46	1.45	2.01	3.56	1.89	3.61
0.30	4.85	1.00	1.08	1.15	1.18	1.23	1.36	1.50	1.49	2.07	[illegible]	1.97	3.75
0.35	4.24	1.00	1.08	1.16	1.19	1.24	1.38	1.55	1.52	2.12	3.69	2.04	3.89
0.40	3.78	1.00	1.08	1.16	1.20	1.25	1.40	1.56	1.55	2.16	3.85	2.10	4.00

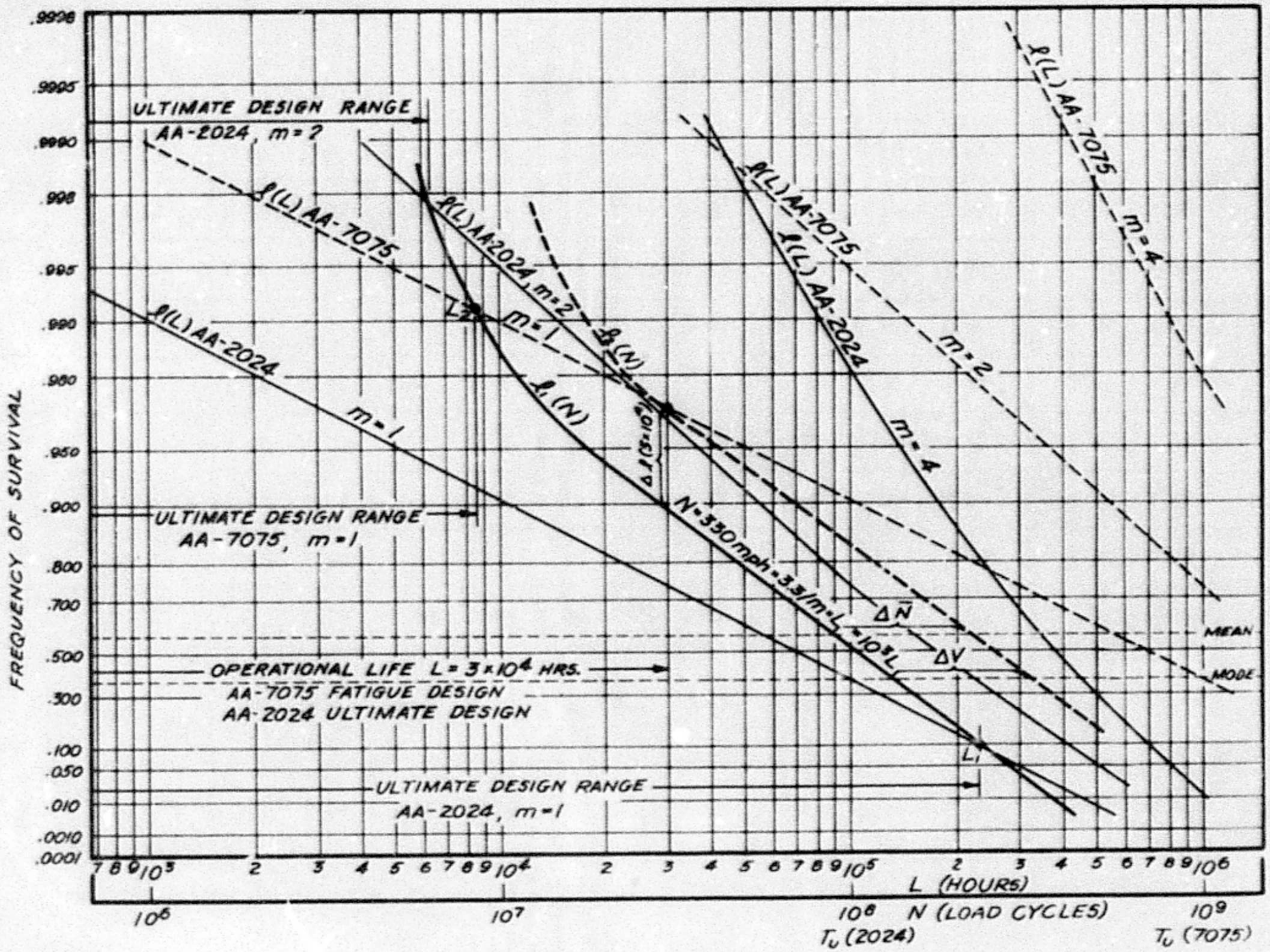

Fig.1 Comparison of relations $l(L)$ between expected frequency of survival and operational life for ultimate load design ($m = 1$) and repeated ultimate load design ($m = 2,4$), with schematic survivorship function for fatigue $l(N)$

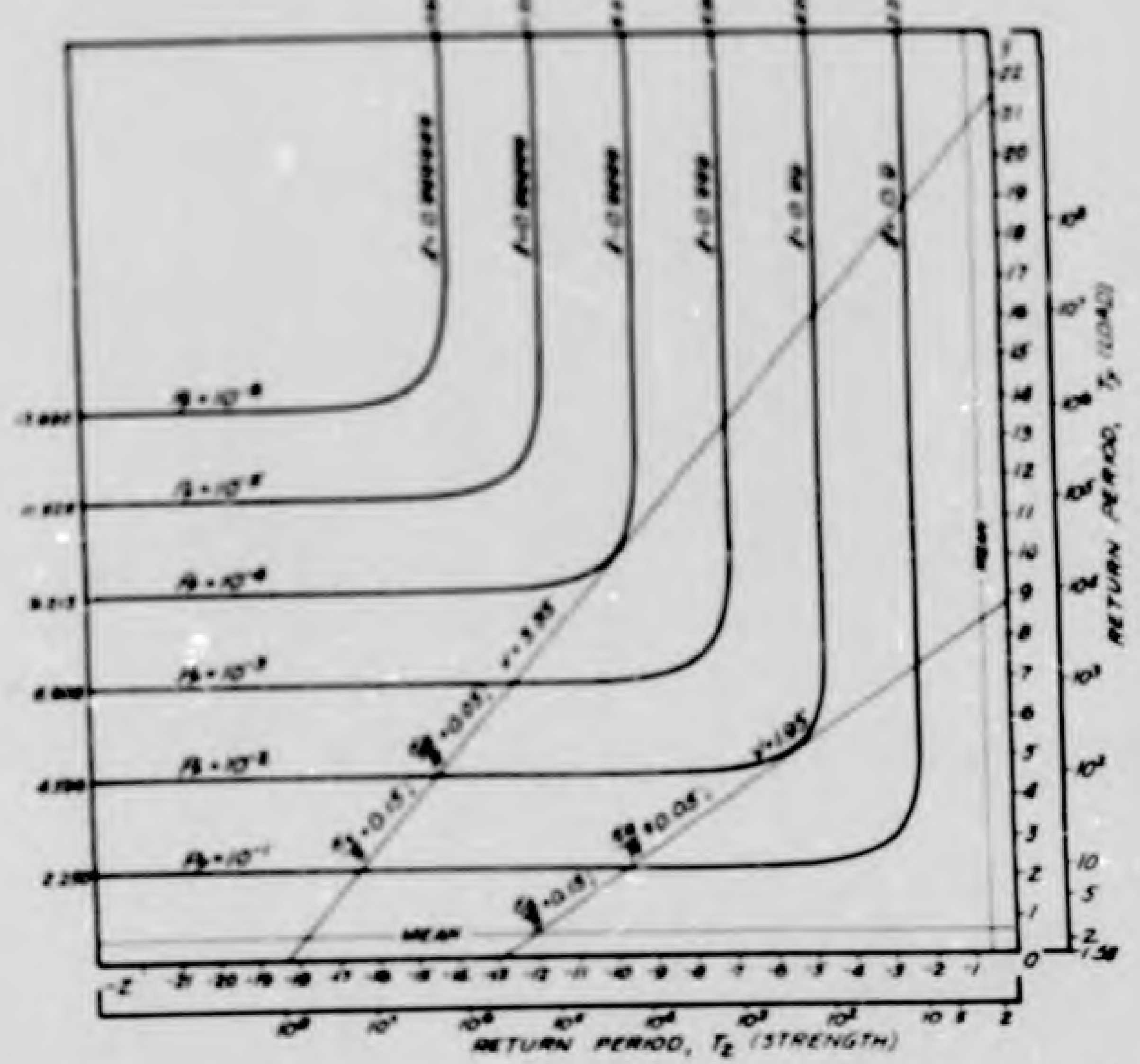

Fig.2 Lines of equal probabilities of failure P_F or of survival l associated with different return periods of maximum load T_y and minimum resistance T_z, based on extremal distributions of S and R $[y = \alpha_S(S - \tilde{S}),\ z = \alpha_R(R - \tilde{R})]$