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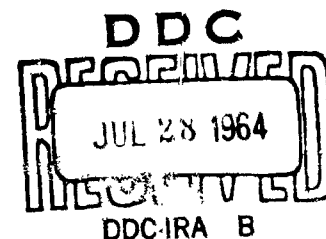
STRUCTURAL
MECHANICS

APPLIED
MATHEMATICS

ENVIRONMENTAL CONDITIONS OF SHIP MOTIONS AND
VIBRATIONS FOR DESIGN OF RADAR SYSTEMS ON
DESTROYERS AND AIRCRAFT CARRIERS

by

E. Buchmann, Ph.D., and J.D. McConnell



STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

July 1959

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NE 050-500**

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NOTATION

A	Single amplitude of vibration
d	Distance from center of gravity to base of radar mast
e	Base of Napierian logarithms
E	Mean value of x^2
E_m	Mean value of x^2 for most severe condition
f	Value for the risk taken
h	Height of radar mast above neutral axis of ship
I	Mass moment of inertia of radar unit about axes fixed in its center of gravity
M	Mass of radar unit
N	Number of variations in a sample
n	Whipping frequencies
$p(x)$	dP/dx , Relative probability of encountering a value of magnitude x
$P(x_i)$	Fraction of all members of a sample with values $x < x_i$
t	Time
x	Magnitude of a variable
x_i	A given value of x
x_m	Value of x which will be exceeded at least once in a fraction, f , of a great number of similar tests
y	Function of the risk
ψ	Roll angle
ϕ_p	Pitch angle
ω_r	Roll frequency
ω_p	Pitch frequency

ABSTRACT

From rigid-body motions and environmental vibrations measured on aircraft carriers and destroyers over a wide range of service conditions, environmental conditions of vibration and ship motions are defined for use in the design of radar installations. Statistical distribution patterns of motions applicable to ship operation over a long period and those for the most adverse operating conditions experienced in a severe sea are used to predict extreme values. Maximum linear accelerations in the longitudinal, athwartship, and vertical directions and maximum angular accelerations at the radar platform are estimated.

Applications of the data to design problems are discussed.

INTRODUCTION

The design and operation of search and height-finding radar have long been hampered by lack of information concerning the motions and accelerations to which this equipment is subjected. Accordingly, in January 1952 the Bureau of Ships¹ requested the David Taylor Model Basin to conduct tests on various types of ships to determine design conditions, with respect to ship's rigid-body motions, for stabilized radar installations. In October 1955² the problem was broadened to include vibratory motions associated with the elastic response of the structure as well, since successful aircraft intercept had become extremely sensitive to elevation errors in height-finding radar. Ship flexure between the radar antenna and the reference gyro equipment, which is usually remote from the antenna, and environmental vibrations at the actual antenna location can be sources of error.

Data have been collected to determine the rigid-body motions and environmental vibrations at the service platforms on radar masts, particularly on carriers and destroyers. This report evaluates the existing data on carriers and destroyers in a manner which, it is hoped, will be useful to the designer of radar installations.

The analysis of existing data is given in three parts: evaluation of ship motion data, evaluation of vibration data, and application to design.

RIGID-BODY MOTION OF SHIPS

Motion data necessary to specify structural design requirements are given in terms of maximum acceleration and maximum pitch and roll angles to be expected over the operational life of the ship. Operational requirements are given in terms of the probability of exceeding or not exceeding given angles of roll and pitch for the most severe operating condition

*References are listed on page 27.

expected. In this report ship motions are given in terms of their peak-to-peak variation; i.e., a roll angle of 30 deg will mean that the variation of roll angle is 30 deg from port to starboard.

A brief background of statistical analysis needed for the evaluation is outlined in the next section.

STATISTICAL BACKGROUND

The ship motions experienced under a given set of conditions can be described in terms of their distribution functions. It has been shown³ that the applicable distribution functions are approximated by the Rayleigh distribution for a given set of steady operating conditions such as sea state, ship speed, and heading (short-term), and by log-normal distributions if the operating conditions are allowed to vary over a wide range such as would occur over a typical year (long-term).

The Rayleigh distribution of a variable x is defined by the single parameter E , the mean square value of x ; i.e., $E = x^2$. The log-normal distribution of x is defined by two parameters—the mean value of $\log x$ and the variance of $\log x$. Both the Rayleigh (Figure 5) and log-normal distributions (Figures 2, 3, 4, 6, 7, and 8) can be represented by straight lines when plotted on special graph paper. The statistical methods utilized here are discussed in References 3 and 4.

For illustrative purposes, consider one of the variables—for example, pitch angle. All pitch angles, peak to peak, are considered to be members of a statistical population. The distribution function indicates the relative probability $p(x)$ of encountering a pitch angle of the magnitude x . Figure 1 illustrates this distribution function. The area under the curve of Figure 1 up to a value x_i is the fraction P of all members of the population which have values less than x_i . Therefore the probability of exceeding the value x_i is 100 $(1-P)$ percent. For the Rayleigh distribution $P(x) = 1 - e^{-x^2/E}$.

The distribution patterns give the probability of exceeding any given magnitude of motion and can be utilized in designing for endurance strength. For any set of operating conditions, characteristic and extreme values can be predicted from a knowledge of the corresponding value of E . Useful statistical estimates are made as follows:⁵

- a. The most frequent magnitude of variations is $0.707\sqrt{E}$.
- b. The average magnitude of variations is $0.866\sqrt{E}$.
- c. The most probable extreme value x_m experienced in a sample of N variations is $x_m \approx \text{constant}\sqrt{E}$. For large values of N , the constant is approximately equal to $\sqrt{\log_e N}$. A variation is defined as the magnitude of the change from a maximum value to the succeeding minimum value.

For design purposes, a statistical estimate of the extreme value of the various variables may be made. Let the value of E corresponding to the most severe condition expected be E_m . If the ship is expected to experience N variations during the time it is exposed to this

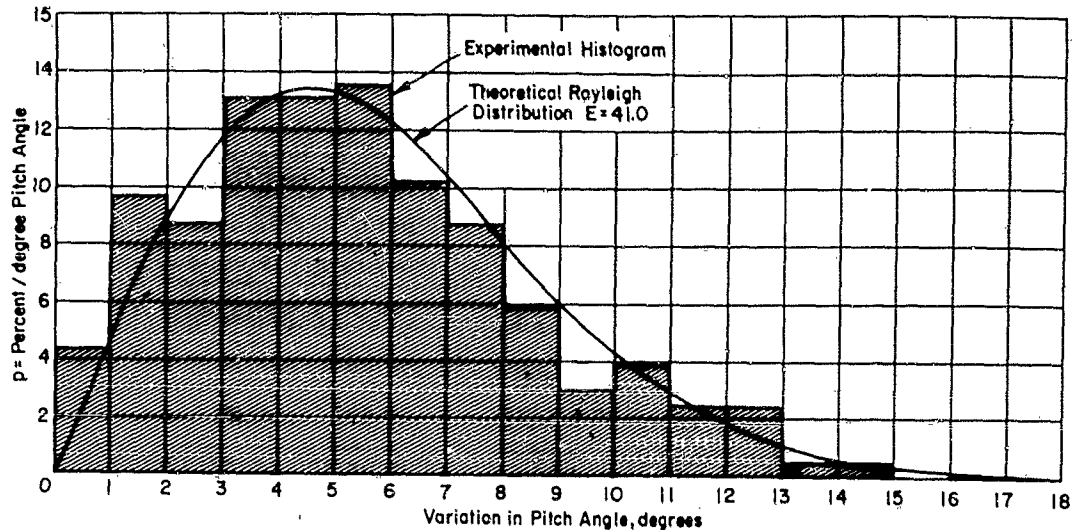


Figure 1 — Distribution of Variation in Pitch Angle for USS SPERRY (DD 697)

operating condition, then

$$x_{m1}^2 = E_m [y + \log_e N]$$

where N is assumed large.

The value of y is a function of the risk f (selected by the designer) which is the fraction of all samples of size N , belonging to a distribution specified by $p(x)$, which will have at least one value of $x > x_{m1}$. Table 1 of Reference 6 gives y as a function of f . For example, if one chance in a thousand is taken, $f = 0.001$ and $y = 7.0$. The value x_{m1} is then that magnitude of the variable which, on the average, is exceeded only by the fraction f of many similar ships operating under the most severe service conditions.

AIRCRAFT CARRIERS

Rigid-body motions for ESSEX (CVA 9) Class aircraft carriers were obtained from tests conducted on USS VALLEY FORGE (CVS 45) during the period September 1955 to February 1957 in the North Atlantic, and on USS ESSEX (CVA 9) and USS ORISKANY (CVA 34) during trips around Cape Horn in the (local) winter seasons of 1957 and 1952, respectively. Data for ESSEX-Class carriers are given in References 7 and 8.

Long-Term Distributions

Figures 2 through 4 give the probability, for an ESSEX-Class aircraft carrier, of exceeding or not exceeding a given value of motion over a number of years. For example, only 1 percent of the variations in pitch angle would, on the average, exceed a value of 4 deg peak to peak; see Figure 2.

As pointed out by Jasper,³ the usefulness of the long-term distributions does not extend

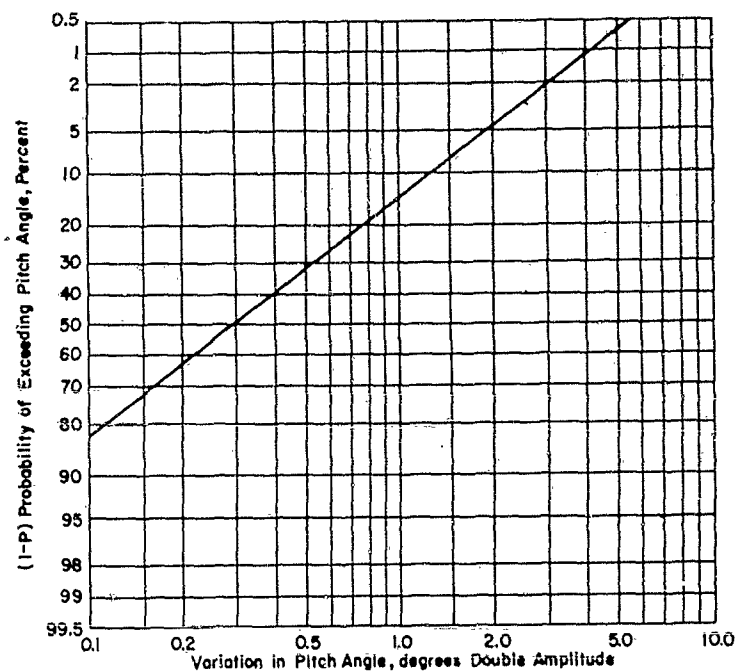


Figure 2 - Long-Term Cumulative Distribution of Pitch Angle for ESSEX-Class Aircraft Carriers

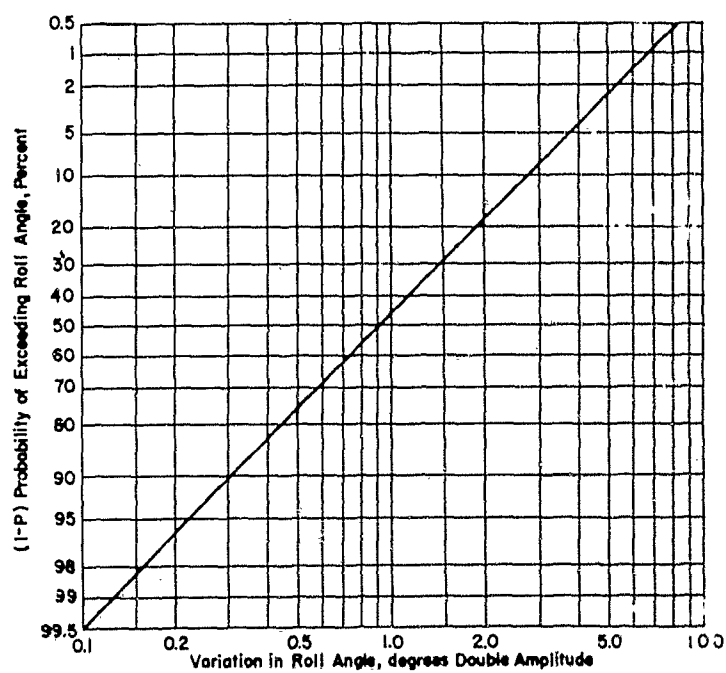


Figure 3 - Long-Term Cumulative Distribution of Roll Angle for ESSEX-Class Aircraft Carriers

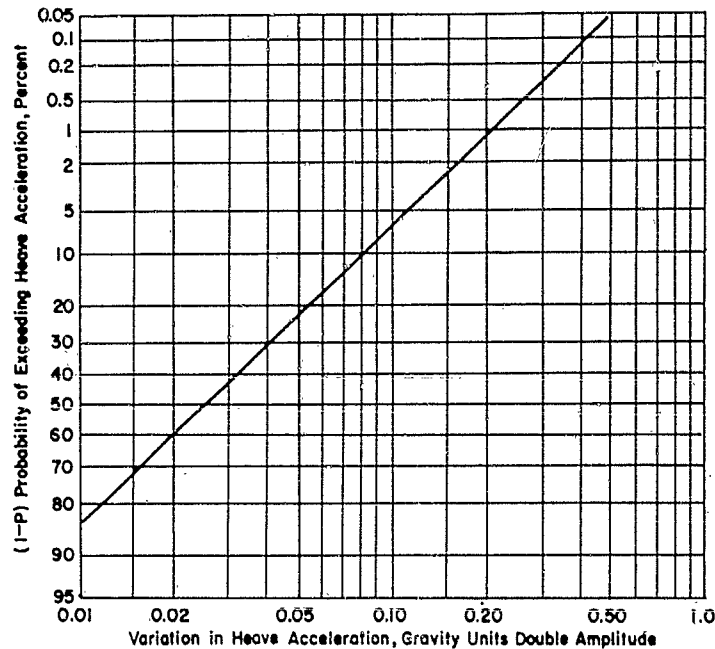


Figure 4 — Long-Term Cumulative Distribution of Heave Acceleration for ESSEX-Class Aircraft Carriers

to prediction of extreme values. The long-term distributions can probably be considered valid up to variations of 4 deg in pitch, 6.8 deg in roll, and 0.20 g in heave acceleration (corresponding to $1 - P = 0.01$).

Distribution for Most Severe Operating Conditions

The maximum values of E that have been computed from experimental data on ESSEX-Class carriers for the most severe conditions encountered during the tests are $12.8 (\text{deg})^2$ for pitch, $61 (\text{deg})^2$ for roll, and $0.014 (g)^2$ for heave. The actual operating conditions associated with these maximum values of E are listed in Table 1. From these values and Figure 5, the probability of exceeding or not exceeding a given value of ship motion under these most severe conditions can be obtained. As an example, consider the probability of exceeding a roll angle of 15 deg under the most severe operating conditions observed:

$$x^2/E = (15)^2 / 61.0 = 3.7$$

From Figure 5

$$(1 - P) = 2.5 \text{ percent.}$$

TABLE 1

Maximum Ship Motions for ESSEX-Class Aircraft Carriers

All values are peak-to-peak variations. Data were taken from Table 7 of Reference 7.

Quantity	Conditions for which Extreme Value is Predicted					Number of Variations in 4 Hours for Steady Conditions	Estimated Most Probable Largest Variation in 4 Hours	Maximum Variation for L.H. of Ship ($\gamma = 0.001$)	Largest Measured Variation	Maximum Variation for Design Purposes
	Sea State	Characteristic Wave Height ft	Direction of Sea Relative to Ship Course degree	Ship Speed knots	E_{max}					
Heave Acceleration	6	20	0	10	$0.014 (g)^2$	1350	0.32 g	0.49 g	0.30 g	0.50 g
Roll Angle	6	20	45	10	$61.0 (deg)^2$	980	19.6 deg	32.0 deg	19.0 deg	32 deg
Pitch Angle	6	24	45	8	$12.8 (deg)^2$	1180	9.5 deg	14.9 deg	9.5 deg	15 deg
Rolling Acceleration	6	20	45	10	--	980	0.067 rad/sec^2	0.10 rad/sec^2	--	0.10 rad/sec^2
Pitching Acceleration*	6	24	45	8	--	1180	0.07 rad/sec^2	0.10 rad/sec^2	--	0.10 rad/sec^2

*Angular acceleration for pitching is not sinusoidal. Peak values were computed from measured displacement data assuming the maximum value to be 1.5 times the maximum value of a single sinusoidal motion with the same amplitude and frequency. The rolling motion is considered to be sinusoidal.

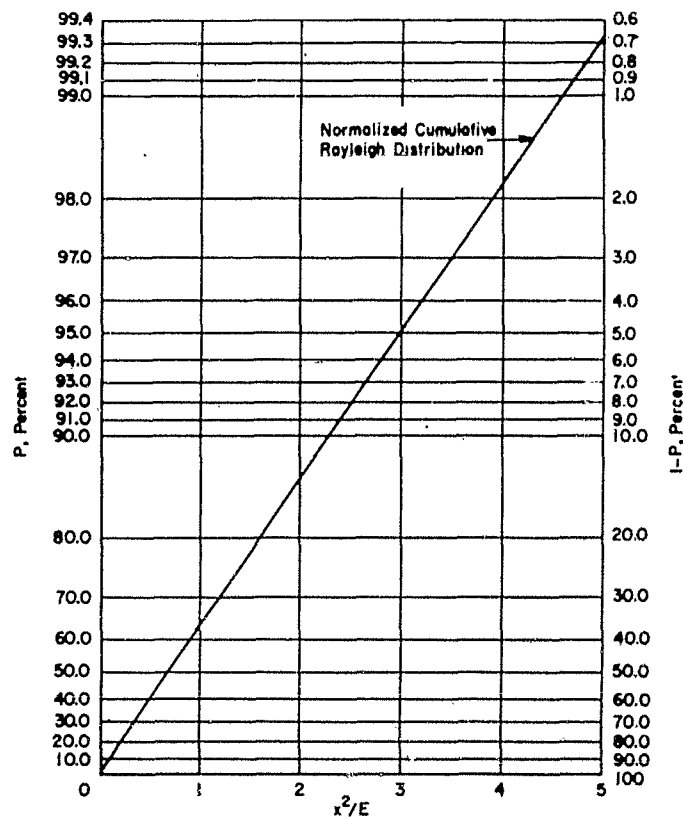


Figure 5 -- Short-Term Cumulative Distribution of Variation in Pitch Angle, Roll Angle, and Heave Acceleration

Extreme Values

The maximum predicted ship motions for an ESSEX-Class aircraft carrier computed by statistical methods are listed in Table 1. Table 1 gives the maximum motions that can be expected during any 4-hr period when operating conditions are the most severe expected for the motion under consideration, and also an estimate of the maximum motions that may be expected during the useful life of the ship. For the purpose of this report, the useful life of a ship was taken as 20 years, and a 4-hr period at the most severe operating condition was assumed to occur on an average of once every year.

The extreme values in Table 1 are based on the assumption that all variations are independent, random values. This is, in fact, not the case because consecutive motions may be shown to be partially correlated. Analysis by means of autocorrelation functions indicates that roughly every fourth cycle of motion may be considered independent. In practice this means that the predicted extreme values are conservative; i.e., they are likely to be somewhat too high. Actually these predictions of extreme values should be regarded as rough estimates.

DESTROYERS

The rigid-body motions for SUMNER (DD 692) Class destroyers were obtained from tests on USS CHARLES S. SPERRY (DD 697) during the period from September 1955 to February 1957 in the North Atlantic.⁹

Long-Term Distributions

Figures 6 through 8 give the probability of exceeding or not exceeding a given value of motion over a number of years. For example, only 1 percent of the variations in pitch angle would, on the average, exceed a value of 9.5 deg; see Figure 6.

The long-term distributions can probably be considered valid up to variations of 9.5 deg in pitch, 39 deg in roll, and 0.55 g in heave acceleration (corresponding to $1 - P = 0.01$).

Distribution For Most Severe Operating Conditions

The maximum values of E that have been computed from measurements made during sea tests of a SUMNER-Class destroyer are 41 (deg)² for pitch, 459.2 (deg)² for roll, and 0.096 (g)² for heave. Actual operating conditions associated with these maximum values of E are listed in Table 2. From these values and Figure 5, the probability of exceeding or not exceeding a given value of ship motion under these most severe conditions can be obtained. As an example, consider the probability of exceeding a roll angle of 40 deg under the most severe operating conditions observed:

$$\frac{x^2}{E} = \frac{(40)^2}{459.2} = 3.49, \text{ and}$$

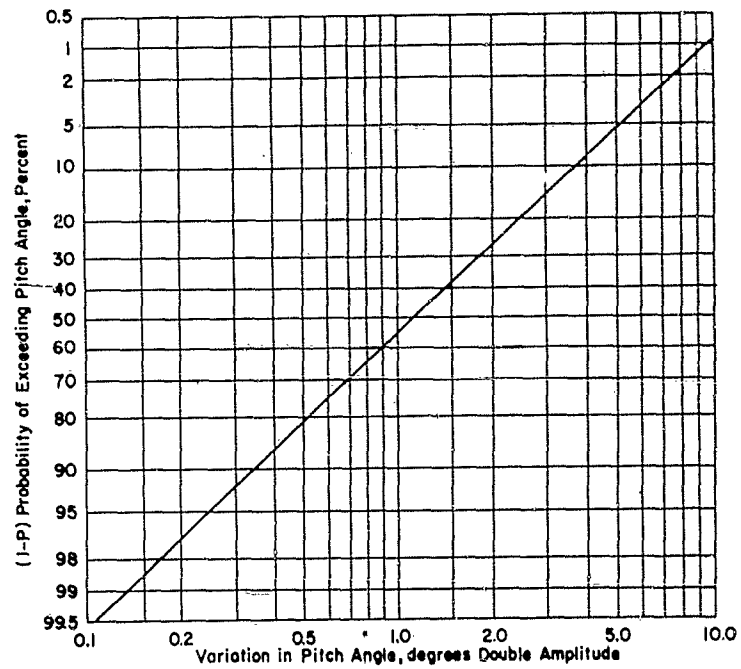


Figure 6 – Long-Term Cumulative Distribution of Pitch Angle for SUMNER-Class Destroyers

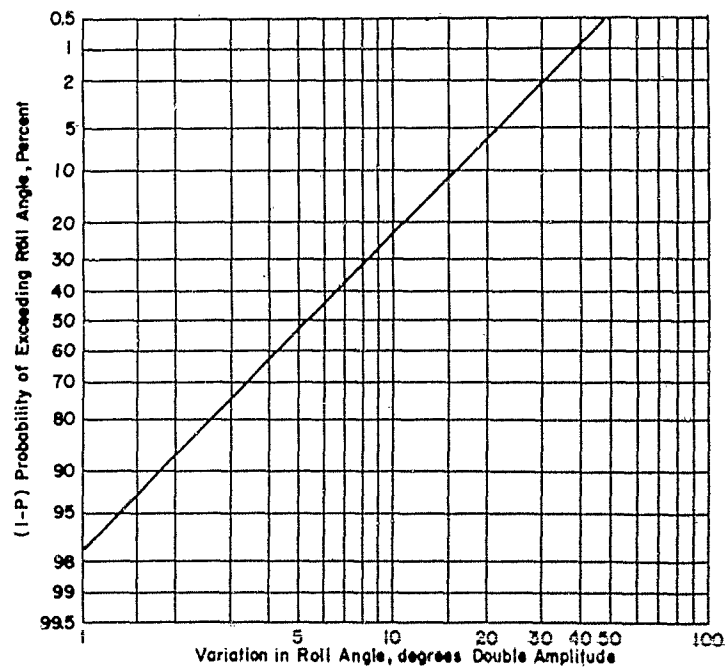


Figure 7 – Long-Term Cumulative Distribution of Roll Angle for SUMNER-Class Destroyers

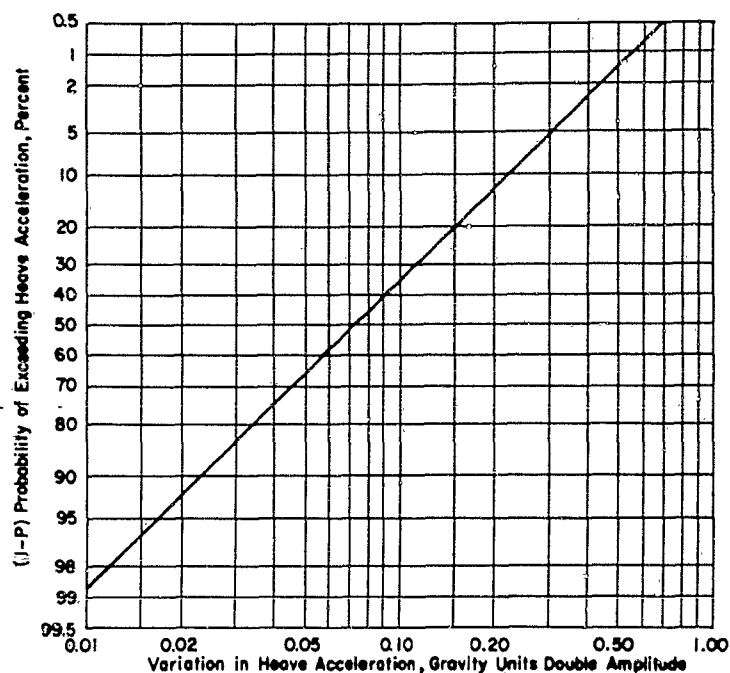


Figure 8 -- Long-Term Cumulative Distribution of Heave Acceleration for SUMNER-Class Destroyers

TABLE 2
Maximum Ship Motions for SUMNER-Class Destroyers

Quantity	Conditions for which Extreme Value is Predicted					Number of Variations in 4 Hours for Steady Conditions	Estimated Most Probable Largest Variations in 4 Hours	Maximum Variation for Life of Ship ($\gamma = 0.501$)	Largest Measured Variation	Maximum Variation for Design Purposes
	Sea State	Characteristic Wave Height ft	Direction of Sea Relative to Ship Course degree	Ship Speed knots	E_{max}					
Heave Acceleration	4	8.0	45	25	0.896 (g) ²	2890	0.874 g	1.39 g	0.72 g	1.40 g
Roll Angle	5	10.0	confused Sea	8	450.2 (deg) ²	1616	58.2 deg	95 deg	93.0 deg	96 deg
Pitch Angle	5	10.0	90	7	41.0 (deg) ²	1485	17.3 deg	28 deg	15.6 deg	28 deg
Rolling Acceleration*	5	10.5	Confused Sea	8	--	1616	0.48 rad/sec ²	0.80 rad/sec ²	--	0.80 rad/sec ²
Pitching Acceleration*	5	10.5	90	7	--	1485	0.19 rad/sec ²	0.30 rad/sec ²	--	0.30 rad/sec ²

*Angular acceleration for pitching is not sinusoidal. Peak values were computed from measured displacement data, assuming the maximum value to be 1.5 times the maximum value of a simple sinusoidal motion with the same amplitude and frequency.

from Figure 5

$$(1 - P) = 3.0 \text{ percent}$$

Extreme Values

Table 2 gives the maximum predicted ship motions for a SUMNER-Class destroyer operating in the North Atlantic Ocean under wartime conditions. It lists the maximum value of motion expected to occur during a 4-hr period when operating conditions are the most severe expected for the motion under consideration and gives an estimate of the maximum motion expected to occur during the life of the ship as before, useful life was taken as 20 years, and a 4-hr period at the most severe operating condition was assumed to occur on the average of once every year.

As in the case of aircraft carriers, the predicted extreme values may be somewhat high because of the assumption that all variations are independent.

VIBRATIONS

This section presents environmental vibration data at the service platform of radar masts. The data may be used to establish design specifications of environmental vibration for shipboard radars. These vibrations may be specified in terms of the angular and linear motions together with the associated frequencies.

The vibration at the radar platform may be considered to have the following major components:

1. Vibration due to the hydrodynamic forces generated by the propeller. These oscillations are predominantly at propeller-blade frequency (shaft rpm times the number of propeller blades).
2. Vibration due to mass unbalance of the propeller and propulsion shafting. These oscillations are at shaft frequency (shaft rpm).
3. Transient vibrations of the radar-mast structure corresponding to its own natural frequencies, induced by ship maneuvers, such as turns and crash-ahead crash-astern operations.
4. Severe whipping vibrations of the ship and the mast structure induced by slamming of the ship in heavy seas.

The amplitudes of blade-frequency and transient vibrations, Items (1) and (3), were measured largely during smooth-water operations. The values so obtained must be increased to allow for rough-water operation, gunfire, and maneuvers. The limited data available indicate that the multiplication factors given in Table 8 are reasonable. The contributions of Items (1) through (4) are added linearly to arrive at maximum values for design purposes.

TABLE 3

Factors for Converting Vibration Amplitudes in Calm Seas to Extreme Conditions

The vibration amplitude for calm-sea operation is taken as A.

Type of Vibration	Multiplication Factor						Vibration Amplitude for Extreme Conditions	
	Rough Seas		Ship Maneuvers		Gunfire			
	CVA	DD	CVA	DD	CVA	DD	CVA	DD
Propeller-excited	2	2	2	2	1	1	4A	4A
Excited by unbalance of propeller-shaft system	1	1	1	1	1	1	A	A
Transient vibration during maneuvers	2	2	1	1	1	2	2A	4A
Whipping vibration due to slamming	1	1	1	1	1	1	A	A

Data for aircraft carriers were obtained from tests on USS MIDWAY (CVA 41),¹⁰ USS FRANKLIN D. ROOSEVELT (CVA 42),¹¹ and ESSEX.⁷ Data for destroyers were obtained from tests on USS GREENE (DD 711),¹² USS DECATUR (DD 938),¹³ and SPERRY,⁹ and from Reference 14.

All amplitudes of vibration are given in terms of single amplitudes, as is customary. This differs from the values for the rigid-body motion given previously which are all in terms of total variations; i.e., peak-to-peak values. The values for the environmental conditions at the radar platform derived from the rigid-body motions are given in single amplitudes. The reference coordinate system is assumed to be fixed in and move with the ship. The coordinate directions are always referred to as vertical, longitudinal, and athwartship directions, even though the axes may rotate.

AIRCRAFT CARRIERS**Maximum Vibration at Radar Locations**

The maximum amplitudes of vibration and the corresponding frequencies recorded on the radar masts of aircraft carriers are listed in Tables 4 and 5. Vibrations due to unbalance in propulsion system were negligibly small; hence they are omitted. From Tables 4 and 5 the conditions which give the largest accelerations in the vertical, longitudinal, and athwartship directions are selected. This gives the following probable maximum values, making due allowance for the factors listed in Table 3.

Table 4 — Maximum Vibrations Measured on Radar Masts of MIDWAY-Class Aircraft Carriers during Steady-Speed Operation in Calm Seas.

Type of Mast	Location of Mast	Direction of Vibration	MIDWAY (CVA 41)						FRANKLIN D. ROOSEVELT (CVA 42)		
			Four-Bladed Propellers			Three-Bladed Propellers			Three-Bladed Propellers		
			Shaft Speed rpm	Single Amplitudes mils	Frequency cps	Shaft Speed rpm	Single Amplitudes mils	Frequency cps	Shaft Speed rpm	Single Amplitudes mils	Frequency cps
Pole Mast (SR-3 Antenna)	Frame 98	Vertical	200	0.3	25.0	190	2.1	9.5**	—	—	—
		Longitudinal	180	75.0	2.8	200	134.0	2.5*	—	—	—
		Athwartship	200	45.5	3.4	200	133.5	3.3	155	14.1	7.7*
Tripod Mast (SX Antenna)	Frame 105	Vertical	120	5.9	8.0**	200	1.5	10.0**	165	3.8	7.7*
		Longitudinal	180	15.0	3.0	150	56.4	2.5	185	1.4	10.0**
		Athwartship	170	5.5	3.0	150	42.8	2.6*	95	5.8	5.3**
Pole Mast (SR-2 Antenna)	Frame 125	Vertical	140	1.8	5.7*	190	29.9	9.5**	—	—	—
		Longitudinal	190	24.5	4.5*	200	29.5	5.0*	—	—	—
		Athwartship	160	13.4	6.1*	200	31.9	6.0*	115	25.0	6.0**

*This frequency is believed to correspond to a natural frequency of the structure.
 **This vibration is believed to correspond to propeller-blade frequency.

Table 5 — Maximum Vibrations Measured on Radar Masts of MIDWAY-Class Aircraft Carriers during Maneuvers in Calm Seas

Type of Mast	Location of Mast	Direction of Vibration	MIDWAY (CVA 41)						FRANKLIN D. ROOSEVELT (CVA 42)		
			Four-Bladed Propellers			Three-Bladed Propellers			Three-Bladed Propellers		
			Type of Maneuver	Single Amplitudes mils	Frequency cps	Type of Maneuver	Single Amplitudes mils	Frequency cps	Type of Maneuver	Single Amplitudes mils	Frequency cps
Pole Mast (SR-3 Antenna)	Frame 98	Vertical	Crash ahead from full speed astern	0.8	6.0	Hard turn to starboard 30 deg to 60 deg	4.4	9.5**	—	—	—
		Longitudinal	Crash ahead from full speed astern	335.0	3.3*	Hard turn to starboard 30 deg to 60 deg	169.8	2.7*	—	—	—
		Athwartship	Crash ahead from full speed astern	192.5	2.7*	—	—	—	15 deg right rudder at 30 knots	27.3	9.1**
Tripod Mast (SX Antenna)	Frame 105	Vertical	Crash astern from full speed ahead	3.0	5.8	Hard turn to starboard 60 deg to 90 deg	3.0	9.5**	Crash back to full power	11.6	4.5*
		Longitudinal	Crash ahead from full speed astern	143.0	2.8*	Hard turn to starboard 120 deg	101.6	2.4*	Crash back to full power	12.2	5.0
		Athwartship	Crash ahead from full speed astern	192.5	2.7*	Hard turn to starboard 120 deg	104.5	2.5*	Crash back to full power	39.3	5.0
Pole Mast (SR-2 Antenna)	Frame 125	Vertical	Hard turn 60 deg to port	19.5	10.0**	Hard turn to starboard 180 deg	4.7	9.0**	—	—	—
		Longitudinal	Crash ahead from full speed astern	103.5	4.4*	Hard turn to port 120 deg	18.3	8.7**	—	—	—
		Athwartship	Crash astern from full speed ahead	55.0	5.5	Hard turn to starboard 180 deg	17.5	8.9**	Crash back in shallow water	190.00	5.9*

*This frequency is believed to correspond to a natural frequency of the structure.
 **This vibration is believed to correspond to propeller-blade frequency.

Maximum propeller-blade-excited vibration:

Vertical direction;	$29.9 \times 4 = 120$ mils at 9.5 cps = 1.08 g
Longitudinal direction;	$14.1 \times 4 = 56$ mils at 7.7 cps = 0.33 g
Athwartship direction;	$8 \times 4 = 32$ mils at 12 cps = 0.46 g

Maximum transient vibration at the natural frequency of structure during maneuvers:

Vertical direction;	$11.6 \times 2 = 23$ mils at 4.5 cps = 0.47 g
Longitudinal direction;	$335 \times 2 = 670$ mils at 3.3 cps = 0.73 g
Athwartship direction;	$130 \times 2 = 260$ mils at 5.9 cps = 0.90 g

The maximum transient vibration due to slamming is more difficult to evaluate. When the ship slams in rough seas, the hull responds by vibrating in its flexural modes of vibration. The predominant response is in the first two modes of vertical and horizontal flexural vibration. Modes of deformation calculated for ESSEX by the method of Reference 15 are plotted in Figure 9. The difference in slope between various longitudinal locations along the ship (which may give rise to radar errors) will depend on the amplitude of vibration in the several modes. From the relative amplitudes and the maximum whipping motions expected in rough seas for ESSEX-Class carriers, the angular and linear vibratory motions at the base of the mast may be calculated.

Measurements made on ESSEX gave a maximum vertical deflection of 1.9 in. amidships corresponding to the two-noded vertical mode of vibration with a frequency of 0.8 cps. For the worst sea conditions it is estimated that a vertical deflection of $1.9 \times 1.5 = 2.85$ in. amidship, in this mode, may occur. The whipping motion for the second vertical mode has been estimated from the response in the first mode by assuming that the whipping is induced by a step load at the bow. Whipping in the vertical modes of vibration results in motions at the mast top in the vertical and longitudinal directions and rotation about an athwartship axis.

Whipping motions of the ship in the athwartship direction also occur. The limited measurements obtained during the vibration tests on ESSEX indicate that the maximum expected athwartship displacement at the stern of the ship is about 1 in. in the two-noded mode. This produces motions at the mast top in the athwartship direction, together with a rotation about the axis of the mast.

The motion of the hull due to whipping in the vertical and athwartship directions at selected locations and the corresponding angles of rotation are listed in Table 6. The

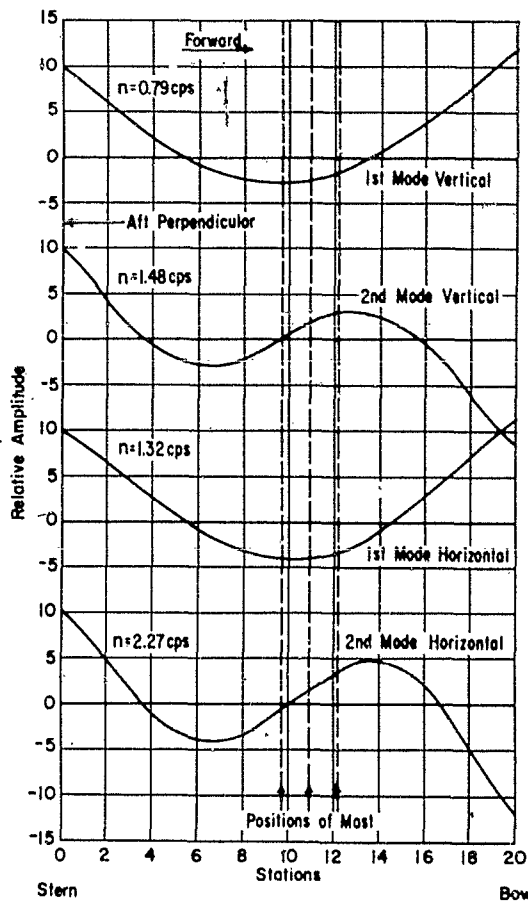


Figure 9 - Normal Modes of Vibration for ESSEX-Class Aircraft Carriers Calculated on UNIVAC
Displacement = 38,300 tons.

motions at the top of the mast positioned at the selected locations are listed in Table 7. For the calculations, the mast was assumed to be rigid and to extend 166.4 ft above the center of gravity of the ship; see Figure 10. Maximum accelerations resulting from propeller blade, transient, and whipping excitations are given in Table 8.

Elastic Distortion between Radar and Its Reference Gyro

There will, in general, be angular distortions between the location of the radar and the reference gyro. These differences in angle may produce radar errors. The differences arise from two sources: the elastic deformation of the hull, and the elastic deformation within the radar mast.

The maximum relative angular motion due to hull vibration will be computed on the assumption that both the mast and the stable element are located within one-half a ship length

TABLE 6

Vertical and Athwartship Whipping Motions of Carriers Due to Slamming

Longitudinal Location of Mast	Vertical Whipping				Athwartship Whipping			
	First Mode 0.79 cps		Second Mode 1.48 cps		First Mode 1.32 cps		Second Mode 2.27 cps	
	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$
Stern	9.5	3.85	3.7	2.2	1.0	2.5	0.32	1.32
Mast at Forward End of Island	1.43	2.70	1.13	0.35	0.3	0.16	0.10	0.12
Mast at Center of Island	2.38	1.32	0.75	1.3	0.6	0.07	0.04	0.12
Mast at Aft End of Island	2.85	0	0.19	1.3	0.4	0.12	0.03	0.08

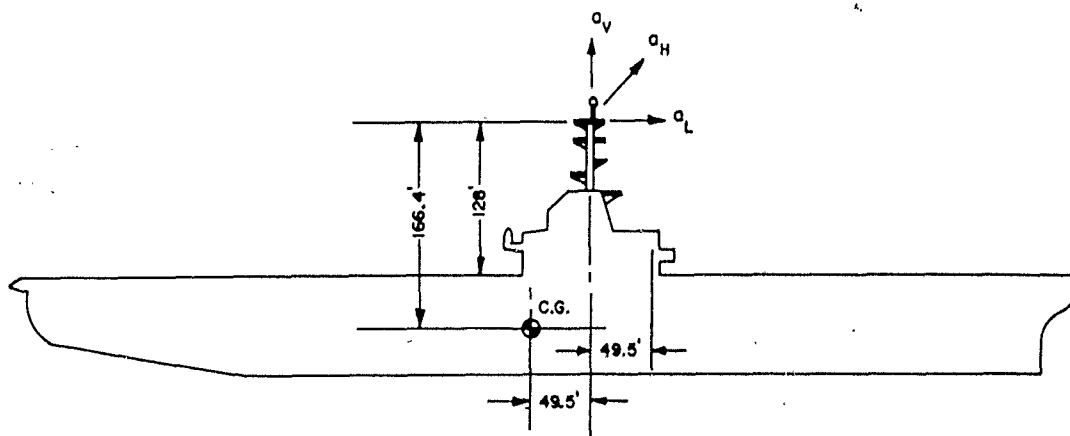


Figure 10 - Inboard Profile for ESSEX-Class Aircraft Carriers

Table 7 — Displacements and Accelerations at Radar Service Platform
on Carriers Due to Slamming (Whipping Response)

Longitudinal Location of Mast	Mode of Vibration	Linear Motion at Radar Service Platform						Rotational Motion at Radar Service Platform					
		Vertical			Athwartship			Longitudinal			About Vertical Axis		
		Amplitude in.	Acceleration g	Frequency cps	Amplitude in.	Acceleration g	Frequency cps	Amplitude in.	Acceleration g	Frequency cps	Angle radians $\times 10^{-3}$	Acceleration radians per sec ² $\times 10^{-2}$	Angle radians $\times 10^{-3}$
Forward End of Island	First	1.43	0.09	0.8	0.3	0.05	1.32	5.4	0.35	0.8	0.16	1.10	1.32
	Second	1.13	0.25	1.48	0.10	0.05	2.27	0.7	0.15	1.48	0.12	2.45	2.27
Center of Island	First	2.38	0.15	0.8	0.6	0.10	1.32	2.85	0.17	0.8	0.07	0.50	1.32
	Second	0.75	0.17	1.48	0.04	0.02	2.27	2.80	0.57	1.48	0.12	2.4	2.27
Aft End of Island	First	2.85	0.18	0.8	0.40	0.07	1.32	0	0	0.8	0.12	0.80	1.32
	Second	6.19	0.04	1.48	0.03	0.02	2.27	2.60	0.57	1.48	0.08	1.2	2.27

Table 8 — Maximum Vibrational Accelerations for Carriers at Radar Service Platform

Cause of Vibration	Maximum Linear Acceleration in g												Maximum Angular Acceleration radian per sec ²
	Mast at Forward End of Island						Mast at Aft End of Island						
	Vertical			Longitudinal			Vertical			Longitudinal			
	Athwartship	Longitudinal	Vertical	Athwartship	Longitudinal	Vertical	Athwartship	Longitudinal	Vertical	Athwartship	Longitudinal	Vertical	
Propeller	1.08	0.46	0.33	0.46	0.33	1.08	0.46	-0.33	0	0.083	0.118	0	Mast at Center of Island, About Indicated Axis
Maneuver	0.05	0.90	0.73	0.90	0.73	0.05	0.90	0.73	0	0.185	0.025	0	
Shaft	0	0	0	0	0	0	0	0	0	0	0	0	
Whipping, First Mode	0.05	0.05	0.35	0.15	0.17	0.18	0.07	0	0.095	0.034	0	0	
Whipping, Second Mode	0.25	0.05	0.15	0.17	0.57	0.04	0.02	0.57	0.024	0.111	0	0	Mast at Center of Island, About Indicated Axis
Total Acceleration	1.47	1.46	1.56	1.48	1.80	1.58	1.45	1.63	0.029	0.413	0.143	0	

aft of the assumed location of the mast. The computed maximum angular distortions within this length, about an athwartship axis and about the axis of the mast located at the center of the island, are listed in Table 9. The maximum angular rotations of the radar platform relative to the base of the mast were computed on the assumption that these angles are equal to the maximum amplitude of mast vibration divided by the height of the radar platform above the flight deck (128 ft). The sum of these angular components is assumed to represent the most severe, expected, angular error between the radar and its reference gyro. Table 9 gives the resulting values.

TABLE 9

Elastic Angular Distortion on Carriers between Radar Platform and Reference Gyro for Mast Position at Center of Island

Relative Angular Distortion about Indicated Axis	Contribution from Hull Distortion, radians $\times 10^{-3}$			Contribution from Mast Distortion, radians $\times 10^{-3}$		Total Angular Distortion radians $\times 10^{-3}$
	Vertical Vibration	Athwartship Vibration	Torsional Vibration	Athwartship Vibration	Longitudinal Vibration	
Vertical	0	2.50	0	0	0	2.50
Athwartship	8.7	0	0	0	0.47	9.17
Longitudinal	0	0	0	0.19	0	0.19

DESTROYERS

Maximum Vibration at Radar Locations

The maximum amplitudes of vibration and the corresponding frequencies recorded on the radar tripod mast on destroyers of SUMNER¹² and FORREST SHERMAN (DD 931)¹³ Classes are listed in Table 10. From Table 10 and Reference 14 the conditions which give the largest accelerations in the vertical, longitudinal, and athwartship directions are selected. This gives the following maximum values, making due allowance for the factors given in Table 8.

Maximum propeller-blade-excited vibration:

The propeller-blade-excited vibrations at the mast top will be the same order of magnitude as those normally experienced by the hull at Frame 72. These magnitudes for the longitudinal and athwartship directions were obtained from Table 1 of Reference 14.

Vertical direction; $6.0 \times 4 = 24.0$ mils at 13 cps
= 0.41 g

Longitudinal direction; $24 \times 4 = 96$ mils at 7.5 cps
= 0.54 g

Athwartship direction; $15 \times 4 = 60$ mils at 10 cps
= 0.6 g

TABLE 10

Maximum Vibrations Measured on Platform of Tripod Radar Masts on Destroyers
during Steady-Speed Operation and Maneuvers

Destroyer Class	Shaft Speed rpm	Direction of Vibration	Single Amplitude mils	Frequency cps	Type of Operation	Remarks
SUMNER	260	Vertical	6.0	13.0	Steady runs	-
	330	Longitudinal	27.0	6.0	Steady runs	-
	150		24.0	7.5	Steady runs	-
	200	Athwartship	15.0	10.0	Steady runs	-
FORREST SHERMAN	310	Vertical	4.0	6.5	Steady runs	-
	210		8.0	2.4	Steady runs	-
	310		12.5	6.0	Maneuvers	-
	310	Longitudinal	30.0	5.2	Steady runs	Shaft frequency
	160		70.0	1.4	Steady runs	Shaft frequency
	310		65.0	5.8	Maneuver	Natural frequency of mast
	310	Athwartship	28.0	5.2	Steady runs	Shaft frequency
	200		32.0	3.3	Steady runs	Shaft frequency
	310		37.0	6.3	Maneuver	Natural frequency of mast

Maximum transient vibration at the natural frequencies of the mast during maneuvers:

Vertical direction; $12.5 \times 4 = 50.0$ mils at 6.0 cps
= 0.18 g

Longitudinal direction; $65 \times 4 = 260$ mils at 5.8 cps
= 0.89 g

Athwartship direction; $37 \times 4 = 148$ mils at 6.3 cps
= 0.59 g

Maximum vibration excited by shaft unbalance:

Vertical direction; very small.

Longitudinal direction; $30 \times 1 = 30$ mils at 5.2 cps
= 0.081 g

Athwartship direction; $28 \times 1 = 28$ mils at 5.2 cps
= 0.076 g

Only few data are available for the first-mode vertical and athwartship flexural vibrations of a destroyer due to slamming. These values were obtained on SPERRY in April 1956 during operation in a State 6 sea superimposed on a swell. The direction of the seas was 45 deg relative to ship course. These values are used for estimation of the response in the corresponding second modes in the same way as used for carriers. The calculated mode shapes

are shown in Figure 11, and the inboard profile for SUMNER-Class destroyers with the necessary dimensions is shown in Figure 12. The vibrations of the hull and of the radar service platform resulting from slamming calculated for the position of the radar mast shown in Figure 12 are listed in Table 11.

The maximum linear accelerations at the location of the radar in three orthogonal directions and the three maximum angular vibrations are listed in Table 12.

Elastic Distortion Between Radar and its Reference Gyro

The maximum angular distortion of the radar platform relative to its gyro is computed on the assumption that the gyro position is located one-half a ship length aft of the mast. The total angular distortion, the sum of hull and mast distortion, is listed in Table 13.

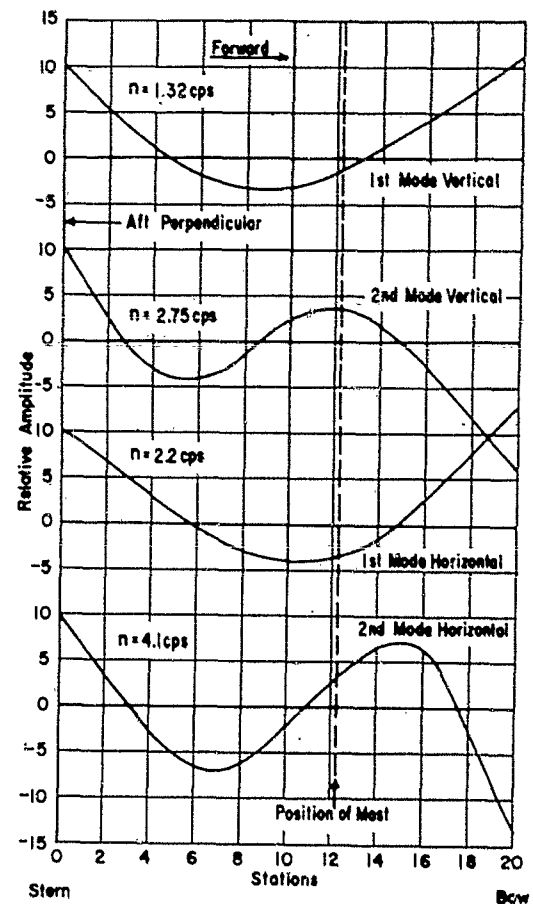


Figure 11 - Normal Modes of Vibration for SUMNER-Class Destroyers from Vibration Generator Tests

Displacement - 3255 - 3220 tons

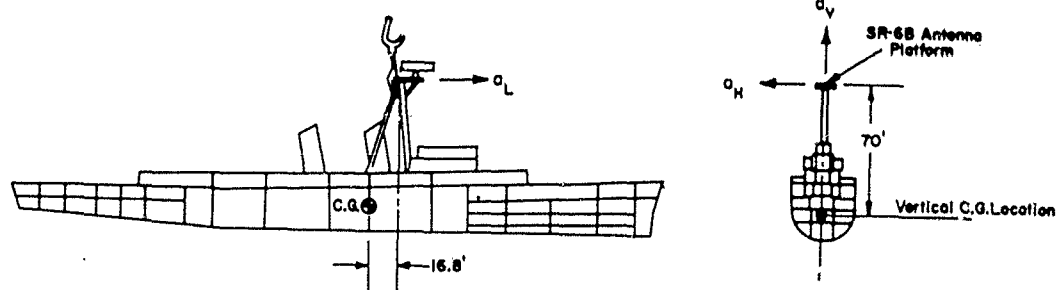


Figure 12 - Inboard Profile for SUMNER-Class Destroyers

Table 11
Hull and Radar Mast Whipping Vibration for Destroyers Due to Slamming

Longitudinal Location on Ship	Hull Whipping Motion						Motion at Radar Service Platform											
	Vertical						Linear						Rotation About Indicated Axis					
	First Mode 1.32 cps		Second Mode 2.75 cps		Athwartship First Mode 2.2 cps		Second Mode 4.1 cps		Vertical		Athwartship		Longitudinal		Vertical		Athwartship	
	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Angle radians $\times 10^{-3}$	Amplitude in.	Acceleration g	Frequency cps	Amplitude in.	Acceleration g	Frequency cps	Amplitude in.	Acceleration rad/sec ² $\times 10^{-2}$	Angle radians $\times 10^{-3}$	Frequency cps
Stern	3.3	3.32	0.75	1.23	0.5	0.376	0.15	0.226										
Point 16.8 ft fwd of C.G.	0.33	2.04	0.26	0.085	0.17	0.141	0.045	0.168										
Mast top 70 ft from C.G.									0.33	0.06	1.32	0.17	0.084	2.2	1.71	0.3	2.04	1.32
16.8 ft fwd of C.G.									0.26	0.20	2.75	0.045	0.077	4.1	0.07	0.06	0.005	2.75

TABLE 12

Maximum Vibrational Acceleration for Destroyers at Radar Service Platform

Cause of Vibration	Maximum Linear Acceleration in g		
	Vertical	Athwartship	Longitudinal
Propeller	0.41	0.6	0.54
Maneuver	0.18	0.59	0.89
Shaft	—	0.08	0.08
Whipping, First Mode	0.06	0.08	0.30
Whipping, Second Mode	0.20	0.08	0.06
Total	0.85	1.43	1.87
	Maximum Angular Acceleration about Axis in rad/sec ²		
Propeller	—	0.31	0.33
Maneuver	—	0.48	0.32
Shaft	—	0.05	0.06
Whipping, First Mode	0.03	0.14	—
Whipping, Second Mode	0.12	0.03	—
Total	0.15	1.01	0.69

TABLE 13

Elastic Distortion between Radar and Reference Gyro for Destroyers

The values given in this table are derived from the summation of the contributions from the first and second modes of flexural hull vibration.

Relative Angular Distortion about Axis	Contribution from Hull Distortion,			Contribution from Mast Distortion,		Total Angular Distortion
	radians $\times 10^{-3}$			radians $\times 10^{-3}$		radians $\times 10^{-3}$
	Vertical	Athwartship	Torsional	Athwartship	Longitudinal	
Vertical	0	0.91	0	0	0	0.91
Athwartship	6.92	0	0	0	0.54	7.46
Longitudinal	0	0	Negl.	0.33	0	0.33

APPLICATION TO DESIGN

DETERMINATION OF MAXIMUM ACCELERATIONS AT RADAR SERVICE PLATFORM

The extreme motions at the radar service platform on carriers and destroyers during operation in heavy seas may be used as a guide for structural design of radar antennas or for specifications of stabilization requirements. The following outline is intended only as an indication of possible procedures.

The maximum accelerations at the radar service platform in three linear and three rotational directions depend on the rigid-body motion and the elastic vibration of the ship. The maximum values due to the elastic vibrations for carriers and destroyers have been given in Tables 8 and 12, respectively. It is, therefore, necessary to add to these values those due to the rigid-body motions in rough seas. The amplitude of the rigid-body motions will be taken as one-half the magnitude of the "variation." Although maximum values may not occur simultaneously, they may conservatively be assumed to do so for purpose of design. The calculations have been made for the estimated, most probable, largest variation during a 4-hr operation and for the maximum variation for design purposes. The respective values of pitch, roll, and heave are listed in Tables 1 and 2 for ESSEX-Class carriers and SUMNER-Class destroyers, respectively. The ship motions are referred to orthogonal axes through the center of gravity which are fixed in the ship, and the coordinate directions are denoted vertical, longitudinal and athwartship directions.

The accelerations at the radar service platform due to the rigid-body motions depend on the location of the platform relative to the ship's center of gravity. Typical dimensions are shown in Figures 10 and 12. The calculations for the radar service platform for carriers will be made on the assumption that the radar mast is located at the center of the island.

The maximum heave acceleration is taken in the vertical direction. The rolling acceleration produces linear components in the vertical and athwartship directions, and the pitching acceleration produces linear components in the vertical and longitudinal directions. If h is the height of the mast above the center of gravity, ω_r the rolling frequency in radians per second, and ϕ_r the maximum angle of roll in radians, the accompanying vertical acceleration is $h \omega_r^2 \phi_r^2$ and the athwartship acceleration is $-h \omega_r^2 \phi_r$. These maxima do not occur at the same time; the maximum vertical acceleration occurs at zero angle and the maximum athwartship acceleration at maximum angle.

The accelerations due to pitching result in two terms for each direction. The vertical acceleration is

$$h \omega_p^2 \phi_p^2 \cos^2 \omega_p t - d \omega_p^2 \phi_p \sin \omega_p t,$$

where ω_p is the pitching frequency in radians per second,

ϕ_p is the maximum angle of pitch in radians ,

d is the distance from the center of gravity of the ship to the base of the mast, and

t is the time.

The acceleration in the longitudinal direction is

$$-h \omega_p^2 \phi_p \sin \omega_p t - d \omega_p^2 \phi_p \cos^2 \omega_p t$$

The sine and cosine factors are included to indicate that the maximum value in each case depends on the weight of each term.

The calculations of the maximum accelerations due to the rigid-body motions are summarized in Table 14 for carriers and in Table 15 for destroyers. The values of the linear accelerations for the most probable largest variation during a 4-hr operation in heavy seas and for the largest probable variation during the lifetime of a ship are both of the order of 2g. The vertical acceleration is superimposed on gravity. Components in the longitudinal and athwartship directions associated with gravity are automatically included. The estimated maximum angular accelerations for destroyers are considerably larger than those for carriers.

TABLE 14

Extreme Values of Accelerations Due to Rigid-Body Motions and Vibrations at Location of Radar for Carriers

All values are single amplitudes.

Source of Acceleration	Linear Accelerations, g			Angular Accelerations about Axes rad/sec ²		
	Vertical	Athwartship	Longitudinal	Vertical	Athwartship	Longitudinal
Case I - Based on most probable largest variation during 4 hr						
Gravity	1.0	0	0	0	0	0
Heave	0.16	0	0	0	0	0
Roll	0	0.17	0	0	0	0.034
Pitch	0	0	0.17	0	0.035	0
Total from Vibrations, Table 8	1.45	1.48	1.80	0.029	0.413	0.143
	2.61	1.65	1.97	0.029	0.448	0.177
Case II - Based on maximum variation for design purposes						
Gravity	1.0	0	0	0	0	0
Heave	0.25	0	0	0	0	0
Roll	0	0.28	0	0	0	0.05
Pitch	0	0	0.27	0	0.05	0
Total from Vibrations, Table 8	1.45	1.48	1.80	0.029	0.413	0.143
	2.70	1.76	2.07	0.029	0.463	0.193

TABLE 15

**Extreme Values of Accelerations Due to Rigid-Body Motions and
Vibrations at Location of Radar for Destroyers**

All values are single amplitudes.

Source of Acceleration	Linear Accelerations, g			Angular Accelerations, about Axes rad/sec ²		
	Vertical	Athwartship	Longitudinal	Vertical	Athwartship	Longitudinal
Case I - Based on most probable largest variation during 4 hr						
Gravity	1.0	0	0	0	0	0
Heave	0.16	0	0	0	0	0
Roll	0	0.55	0	0	0	0.24
Pitch	0	0	0.21	0	0.095	0
Total from Vibrations, Table 12	0.85	1.43	1.87	0.15	1.010	0.69
	<u>2.01</u>	<u>1.98</u>	<u>2.08</u>	<u>0.15</u>	<u>1.105</u>	<u>0.93</u>
Case II - Based on maximum variation for design purposes						
Gravity	1.0	0	0	0	0	0
Heave	0.25	0	0	0	0	0
Roll	0	0.90	0	0	0	0.40
Pitch	0	0	0.33	0	0.15	0
Total from Vibrations, Table 12	0.85	1.43	1.87	0.15	1.01	0.69
	<u>2.10</u>	<u>2.33</u>	<u>2.20</u>	<u>0.15</u>	<u>1.16</u>	<u>1.09</u>

It should be emphasized that the estimates of extreme values of accelerations that are not-expected to be exceeded in heavy-sea conditions were made on the assumption that maximum vibration and rigid-body motion occur at the same time and that the radar mast is a rigid mast flexibly attached to the ship. The direct superposition of maximum values will tend to give an overestimate; the assumption of a rigid mast, on the other hand, will tend toward an underestimate.

MAXIMUM ANGULAR ERRORS DUE TO HULL AND MAST DISTORTIONS

The maximum angular distortions that may occur during ship operation in rough seas are due to elastic deformation incident to: (a) slamming, (b) hull vibration, and (c) ordinary hogging and sagging. The angular distortions given in Tables 10 and 14 include the effects of (a) and (b) only, and must be augmented to allow for (c). Data obtained on ESSEX indicate that the distortion due to hog and sag is of the order of 40 percent of the first-mode whipping response induced by slamming. The total distortion resulting from the superposition of all three effects is given in Table 16. Proper location of the reference gyro with respect to the radar mast could reduce these values considerably.

TABLE 16

Maximum Angular Elastic Distortion between Radar and Reference
Gyro for Carriers and Destroyers

Relative Angular Distortion about Axis	Maximum Angular Distortion, deg	
	Carriers	Destroyers
Longitudinal	0.011	0.019
Athwartship	0.59	0.55
Vertical	0.144	0.052

DETERMINATION OF STABILIZATION REQUIREMENTS

The distribution functions for the ship motions in rolling and pitching, for both the long-term and the short-term distributions, may be used to determine stabilization criteria for the ship or for the radar antenna. A design which would provide stabilization against the most severe predicted, rolling and pitching motions would be uneconomical. It seems more expedient to limit the requirements so that a certain percentage of the motions will be stabilized. The designer will have to determine the conditions for which the stabilization appears to be necessary. This may require stabilization up to a certain sea state or stabilization for a certain percentage of all motions experienced during the lifetime of the ship. Possible application will be illustrated by examples which will consider roll stabilization only; the same procedure applies for pitch stabilization.

Example 1 - A carrier may be equipped with a stabilizer designed to reduce rolling motions of the ship from 8 deg to 2 deg. What stabilization of the radar system is required for 99-percent effectiveness against all motions encountered during the most severe operating conditions?

It will be assumed that the ship stabilization of 6 deg reduces all larger roll angles by the same amount. The motions of the ship during the most severe operating conditions are specified by the short-term distribution. Measurements on ESSEX-Class carriers gave $E_{\max} = 61$. From Figure 5, $\frac{x^2}{E} = 4.6$ for a probability of 99 percent. From this the roll angle is calculated as $\frac{x}{2} = 8.35$ deg, single amplitude. Stabilization of the radar antenna, therefore, would be required for 8.35 deg minus 6 deg, or 2.35 deg. Without stabilization of the ship, the radar antenna would require stabilization for 8.35 deg, single amplitude. The required torque for stabilization against roll depends on the angular accelerations and the mass moment of inertia of the equipment to be stabilized. The required accelerations are given in Table 14 for the unstabilized ship.

Example 2 - It is required to design a stabilizer on a destroyer that will effectively provide stabilization for a radar antenna for: (a) 95 percent of all rolling motions encountered during

the lifetime of the ship, and (b) the estimated most probable largest variation of roll in 4-hr operation in heavy seas.

The solution to (a) utilizes the long-term cumulative distribution of roll angle. The roll angle which is not expected to be exceeded by 95 percent of all rolling motions taken from Figure 7 is 22 deg; i.e., a roll angle of 11 deg single amplitude.

The most probable largest angle of roll in heavy seas (b) is listed in Table 2 as a variation of 58 deg; i.e., a single amplitude of 29 deg.

STRUCTURAL DESIGN OF MAST

Example 3 - It is required to design a mast structure for a carrier to carry a radar system. What are the loads, due to the radar, for which the mast should be designed? The radar unit has a mass M and mass moments of inertia I_z , I_y , I_x about vertical, athwartship, and longitudinal axes fixed at the center of gravity of the radar unit.

The required load may be estimated by taking the product of inertia and acceleration. The values of acceleration are taken from Table 14, Case II:

The axial load will be $2.70 M$.

The athwartship load will be $1.76 M$.

The longitudinal load will be $2.07 M$.

The moment about the athwartship axis through the center of gravity will be $0.463 I_y$; about the mast axis $0.029 I_z$; about the longitudinal axis $0.193 I_x$.

SUMMARY

1. Statistical methods have been used to predict probable extreme values of ship motion to be expected over the operating life of carriers and destroyers.
2. Graphs have been given from which the probability of exceeding any magnitude of motion may be read.
3. The environmental vibrations experienced at radar locations during operation in smooth seas have been given. Magnification factors are tabulated which, when applied to vibration amplitudes measured during smooth-sea operation, give an estimate of the amplitudes to be expected during operation in rough seas.
4. The extreme values of linear and angular accelerations expected at radar locations due to the combined rigid-body and vibratory motions have been tabulated.
5. Maximum angular distortions between the location of radars and the reference gyros have been computed. These distortions may give rise to inaccuracies in radar signals and thus limit radar capabilities.

6. The values given in this report refer to aircraft carriers and destroyers . They are based on reasonably well-established values for the rigid-body motions. Vibration measurements taken on ships at rough-sea conditions are scarce, and additional measurements over long operational periods and for severe sea states are needed to improve upon the numerical values given in this report. It may be reasonably expected that ships of similar design will be subjected to similar conditions during their operational lifetime. Little is known about extreme motions and vibrations of ships whose design departs considerably from the above-mentioned types.

ACKNOWLEDGMENT

The authors accumulated many of the needed data from existing reports. The calculations were checked by Mr. J.F. O'Donnell, Jr. Dr. N.H. Jasper contributed enlightening discussions and helpful suggestions during the preparation of this report.

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