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## SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO.

ON THE OSCILLATIONS OF FREE CONVECTION  
BOUNDARY LAYERS

THESIS

Presented to the Faculty of the School of Engineering of  
the Institute of Technology  
(Air University)  
in Partial Fulfillment of the  
Requirement for the Degree of  
Master of Science

By

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Preface

This investigation was conducted to provide detailed experimental data on the oscillations of free convection boundary layers. The object was to provide a basic step in furthering the understanding of the complex phenomenon of boundary layer transition.

The experimental study was conducted in conjunction with the Fluid Dynamics Branch of the Aeronautical Research Laboratory, Wright Air Development Center. I am very grateful to Mr. Erich Soehngen and Lt. Jack P. Holman for their invaluable aid and supervision of this project.

I also wish to express my appreciation to Major Harry R. Bulmer, my faculty advisor, and the members of the faculty of the Mechanical Engineering Department of the Air Force Institute of Technology, for the aid I received in conducting this study.

Harold E. Gartrell

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## \*Abbreviations:

- |     |                     |   |
|-----|---------------------|---|
| (a) | P. D.               | -Pulse Duration                             |
| (b) | O. E. B. L.         | -Pulse wire at outer edge of boundary layer |
| (c) | $\frac{1}{2}$ P. L. | -Pulse wire at $\delta/2$                   |
| (d) | Wall                | -Pulse wire near Wall                       |

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List of Symbols

Symbol		Units
$Gr_x$	Grashof Number based on x	-
$t$	Ambient air temperature	$^{\circ}F$
$t_w$	Wall (or plate) temperature	$^{\circ}F$
$T$	Absolute temperature	$^{\circ}R$
$f$	Defined "natural" frequency of Boundary Layer	cps
$u$	Natural flow velocity	in./sec
$x$	Distance from leading edge of plate	in.
$y$	Distance from surface of plate	in.
$\beta$	Thermal Expansion coefficient	$1/^{\circ}R$
$\delta$	Boundary Layer Thickness	in.
$\theta$	$t_w - t_o$	$^{\circ}F$
$\nu$	Kinematic Viscosity	ft. <sup>2</sup> /sec
$\lambda$	Wave Length	in.
$\omega$	Frequency	cps
$a$	Wave Amplitude	in.
$c$	Wave Speed	in./sec

Abstract

The purpose of this study was to conduct an extensive, definitive study of forced oscillations of the free convection boundary layer. *was conducted*

Various disturbances were introduced at different positions in the laminar boundary layer of a vertical flat plate, heated to approximately 40°F above ambient temperature. The resulting wave motion was studied from initial formation to the onset of turbulence.

Wave length, amplitude, speed and frequency were found to vary with distance from the plate surface, as well as with the distance from the plate's leading edge. No apparent relation existed among the various parameters which could be applied to the general flow conditions. No correlation between actual wave frequency and disturbance pulse frequency was found to exist.

Empirical relations developed in a previous study <sup>(AD-140652)</sup> (Ref. 1) for a defined "natural" boundary layer frequency and wave length with a Grashof-Prandtl Number product were found to be applicable only to the conditions under which they were developed. Therefore, it was concluded that at the present time there is no known relation or empirical equation which will predict the nature of oscillations of general boundary layer flow. ↗

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ON THE OSCILLATIONS OF FREE CONVECTION

BOUNDARY LAYERS

1. Introduction

Boundary layer transition from laminar to turbulent flow has posed many challenging problems. Although a considerable amount of contributing information has been obtained from numerous investigations, many questions concerning the detailed process of transition remain unanswered. There exists a wide gap in the correlation of theoretical and experimental analyses.

Turbulence in the boundary layer has a pronounced effect on the drag and heat transfer of a body in fluid motion. With the advent of supersonic flight, the mechanisms of flow which control the stability of the boundary layer have become increasingly important. Although free convective flow differs considerably from forced convection, there is a need for basic information concerning the general phenomena of boundary layer instability. A definitive study of free convection boundary layer oscillations will contribute to a more general understanding of boundary layer transition.

The purpose of this investigation was to conduct a definitive experimental study of forced oscillations of the free convection boundary layer on a vertical flat plate. In view of the complexity

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of theoretical analyses, which require many simplifying assumptions and often the use of high speed computers, it is essential to any hopes of successful correlation of these analytical results with experimental data that the basic flow parameters be completely defined and understood.

The objective of this study has been to thoroughly analyze and define, by observation and measurement, the wave propagation produced by artificial disturbances in the natural boundary layer. Basic parameters were chosen to yield information over a wide range of flow conditions. Disturbance characteristics were defined and analyzed with the intent that future studies might benefit from a standardized outlay of data as a possible means of closing the existing gap between theory and experiment. Each parameter was treated individually to completely determine its variations, in an attempt to learn the detailed mechanisms of the boundary layer oscillations. Results of the individual studies were compared and contrasted in attempts to correlate observations. Finally, comparisons were made with results reported in other studies (Refs. 1 & 3) and conclusions drawn which depict the action of the boundary layer in transition.

II. Background

Since Lorenz and Reynolds began the investigation of fluid flow in the 1880's, free convection studies have not received the emphasis that forced convection studies have received. A great deal of effort has been and is being spent on experimental investigations of forced flow. Almost every reference on boundary layer flow contains a history of the major developments marking the progress in forced flow studies. A resumé of these events is not included in this report as it would add very little to the understanding of this investigation.

Plapp (Ref.4) attempted a theoretical analysis by adapting two-dimensional, small-oscillation theory to the stability of a free convection boundary layer on a semi-infinite flat plate. Such an analysis proved extremely lengthy and complicated. His analytical predictions concerning the instability in natural flow were found to agree qualitatively but not quantitatively with experimental observations.

Experimental investigations of free convection flow have been made by Eckert and Soehngen (Ref. 3) and Birch (Ref. 1). Eckert and Soehngen conducted a study of the stability of laminar flow in free convection. Birch attempted to define a natural frequency for the free convection boundary layer by introducing artificial disturbances and studying the resulting wave motion.

In the study by Eckert and Soehngen it was determined that the instability region, e.g., the region where small oscillations did not

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damp out by viscous forces, occurred at a Grashof Number of  $4 \times 10^8$ . Using interference photographs, they observed that the wave lengths produced by natural disturbances were 3.1 times the boundary layer thickness. The wave velocity was determined to be 0.73 times the maximum flow velocity in the boundary layer at the point of observation. This compared favorably with the theoretical boundary layer velocity at the inflection point of the velocity profile, which was  $0.683 u_{\max}$ . From these observations they concluded:

This agreement permits the conclusion that in free-convection as in forced convection, the flow becomes unstable against small oscillations of certain wave length as soon as the boundary layer reaches a certain thickness and where the outside disturbance level is small enough. . . .

Eckert and Soehngen also concluded that the boundary layer obviously filtered out fluctuations of critical frequencies and amplified these until turbulence waves were produced. Birch attempted to define the critical frequency of the boundary layer by introducing controlled disturbances using an electrically pulsed thin wire positioned in the boundary layer. He concluded that the boundary layer responded to certain frequencies which he termed the "natural" frequency of the boundary layer, as specified by the relation

$$e^f = 1.65(Gr_x Pr)^{0.08}$$

He also defined the disturbance wave length at the natural frequency by the relation.

$$e^{\lambda} = 18.2(Gr_x Pr)^{.091}$$

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The present study was initiated to obtain a more definitive analysis than could be obtained with the limited data available in the previous investigations. It was desired to completely define the flow parameters by observations and measurements of many wave variations, in an attempt to discover general characteristics of the boundary layer in transition.

### III. Experimental Approach

This investigation of the free-convection boundary layer was accomplished through visual observation and recording by motion picture. To accomplish this, the boundary layer on a heated vertical flat plate was viewed through a Mach-Zehnder interferometer arrangement in conjunction with a 35mm motion picture camera. The interferometer offered definite advantages in that, for the measurements, no recording obstruction had to be introduced into the flow to cause additional turbulence or unusual flow variation during test runs. Wave formation appeared directly in the fringe lines viewed in the interferometer.

#### Test Parameters

Since general conclusions can result only from extensive and exhaustive studies of as many conditions as possible, the test parameters were carefully selected. However, it was not considered feasible or necessary to vary all parameters in this study. To narrow the scope of the investigation, it was decided to conduct the experiment using a constant, uniform plate temperature. The disturbance pulse wire was maintained at a constant  $x$  position approximately six inches from the leading edge of the plate. The voltage supplied to the wire was maintained constant at 24 volts.

Variable parameters during the investigation were designated to produce disturbance waves over as wide a range of influence as possible.

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The disturbance pulse frequency was varied from 0.5 cps to 3.0 cps. Pulse duration was set for fixed values of 0.1 and 0.5 seconds. This caused the time-average disturbance energy to vary with the frequency. The position of the pulse wire was varied through three positions in the boundary layer, starting next to the wall. The second position was at one-half the boundary layer thickness and the third position was at the outer fringe of the boundary layer.

The flow was photographed at four positions, starting at the pulse wire. Successive positions were used to record the flow at 6, 12, and 18 inches above the pulse wire. In this manner, the wave propagation was observed and recorded from the initial formation of the wave to the region where turbulence was observed.

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#### IV. Description of Apparatus

The equipment used to conduct this study was generally the same as used by Birch (Ref. 1) in a free convection study in 1957. Modifications were made to allow the pulse wire to be varied through the boundary layer thickness and to accommodate the use of a motion picture camera to record the flow phenomena.

##### Test Section

The test section for this experimental study was an enclosed air tunnel extending from just above the floor to within a few feet of the ceiling (Fig. 1). It was located in a closed room and all air circulating units were turned off during test runs to reduce random disturbances in the boundary layer. In addition, the openings between the viewing windows of the test section and the interferometer were enclosed with heavy cloth air shields. A wooden air wall was placed on the floor around the test section to prevent the circulation of air currents on the floor due to movement in the room during test runs.

##### Test Model

The test model was a vertical flat plate constructed of 47 electrically heated plates (Fig. 2). Each plate contained a heating element controlled by an individual rheostat. Each contained a thermocouple installed to sense the plate surface temperature. Figure 1 shows the wiring to the thermocouples and heater elements in the back of the plate.

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The individual plates were nickel-plated copper bars 10 inches long by  $\frac{3}{4}$  inch wide. The entire unit measured 10 inches by  $35\frac{1}{4}$  inches and could be moved up and down on a guide rail by an electric motor and pulley arrangement. Thus the plate could be moved so that any desired section could be viewed through the interferometer. The distance from the leading edge of the plate was indicated by needles placed 1 inch apart at the edge of the plate with letters designating every other one.

#### Thermocouples

Copper-constantan thermocouples were installed in each individual plate to sense the temperature at the surface. The thermocouples were threaded through hollow bolts which were screwed into holes in the back of the plate. The thermocouple junction was positioned just beneath the plate surface. Indicated temperatures were recorded by a Brown strip-chart electronic recorder. An automatic stepping switch was installed to record individual plate temperatures at the rate of one every 5 seconds. This allowed the Brown recorder sufficient time to stabilize during each reading.

#### Control Panel

The control panel (Fig. 3) housed all the controls and circuits required to control and record the temperature of the individual plates. The electrical power input was maintained constant and the individual rheostats were varied until an even plate temperature was recorded by the Brown recorder. Solenoid operated telephone step switches provided connections with the thermocouples and the Brown recorder. They also

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completed the circuits to the lights shown on the front of the control panel. These lights indicated which plate thermocouple was being sensed by the recorder.

The recorder was calibrated by using a hollow, oil-filled copper heat source. The heat source was electrically wired and the temperature controlled by varying the power input. A precision thermometer was inserted into the oil bath with a thermocouple of the same type and approximately the same length wire as those used in the test plate. The thermocouple wires were connected to the Brown recorder and readings taken at five precise temperatures of the oil bath. In this method, a straight line calibration curve was obtained for use in recording plate and ambient temperatures used during test runs.

#### Pulsing Unit

The disturbance wire unit, shown in Figure 2 as it appeared in front of the vertical plate, consisted of a hinged mounting which allowed the pulse wire to be moved in and out of the boundary layer by an electric motor. The horizontal mount on which the motor was attached was positioned in the test section by clamps installed on a single bar located on each side of the test plate. A needle was soldered on each side of the hinged vertical mount through a bolted spring arrangement designed to maintain tension in the wires during pulsing. Molybdenum wire, .001 inch in diameter, was stretched between the two needles.

A Dumont cathode-ray oscillograph was used to measure the time duration and the magnitude of each pulse. The oscilloscope was adjusted

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so that one vertical bar on the screen represented 0.1 volt. With the oscillograph connected across a 0.25 ohm resistance and the voltage maintained a constant 24 volts, the trace on the screen represented the current flow to the wire impressed by each square wave pulse. A Polaroid camera, type 2620, mounted on the oscilloscope was used to record the screen trace for each pulse variation. Motion picture recording was accomplished with an Arriflex 35mm camera using Kodak Photofluore green sensitive film.

Figure 4 shows the complete wiring diagram for the pulsing unit.

#### Interferometer

The basic interferometer was composed of eight-inch optical plates installed in a mirror-splitter arrangement in conjunction with a compensator unit. A high intensity light from a mercury vapor lamp was filtered through a green filter designed for 5461 angstrom wave length. The interferometer was suspended from the ceiling by an arrangement of springs and pulleys (Fig. 5). The springs were used to dampen the system and the pulleys were used to level the interferometer.

## V. Test Procedure

### Method

The power to the plate heating element was turned on approximately 24 hours prior to the initial test run. It had been previously established that a minimum of 12 hours was necessary for the plate to become uniformly heated.

The average plate temperature stabilized at 1190F, with no individual plate varying from this temperature by more than one degree. This accuracy was considered essential to avoid any irregularities which might result from a non-uniform temperature distribution.

Since the pulse wire was to be positioned 6 inches above the leading edge of the plate and at various positions in the boundary layer, the thickness of the boundary layer was computed at the 6-inch position to obtain the theoretical value as shown in Appendix C. The computed thickness was 0.60 inches. The undisturbed flow was then viewed in the interferometer and the pulse wire positioned just inside the outer fringe. Measurement showed that the wire was 0.6 inches from the plate. This position was used during the test runs. To study the effects of varying the position of the disturbances in the boundary layer thickness, the pulse wire was also positioned at 0.3 inches and 0.08 inches from the plate. The pulse frequency was varied from 0.5 cps to 3.0 cps at both 0.5 and 0.1 second pulse duration while the pulse wire was placed at each of the three positions.

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The plate was lifted until the pulse wire was centered in the test section viewing window. The initial formation of all disturbances was recorded at this position for all three positions of the pulse wire in the boundary layer. The plate was successively moved to view the sections at 6, 12, and 18 inches above the pulse wire. Approximately five-second recordings were made of each event with a 35mm motion picture camera operating at a shutter speed of 24 frames per second.

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### Reduction of Data

Approximately 900 feet of 35mm film were used to record the boundary layer motion during the test runs. Although no problem was encountered in obtaining a picture of a complete wave, as in the case of still camera photographs, the problem of determining which waves were representative and which were unique proved to be of major concern. It was decided that at least three different waves of each event recorded should be measured and an average obtained. This procedure proved to be extremely time consuming considering the large number of pictures which were recorded. Also, each wave was considered to be composed of individual waves (across the boundary layer thickness) defined by the fringe line contours. Therefore, each picture contained approximately six waves which were to be measured. It was decided to measure one complete wave and then check the measurements against one or two other waves to determine if the values were representative. If there was a wide variation, an average value was then used in place of the original measurement.

Consideration was given to taking measurements directly from photographs enlarged from the movie negatives. The quantity of pictures necessary made this procedure impractical. It was found that by viewing the film on the screen of a 35mm film recordak, the picture could be enlarged several times and measurements scaled directly from the screen image.

Accurate measurements of the wave parameters were very difficult to determine. A considerable amount of time was spent trying to define an accurate and correct procedure for obtaining measurements.

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Wave length was measured over a complete wave from minimum point to minimum point whenever possible. When such points existed the wave length was fairly easily defined. However, the leading side of a large amplitude wave generally sloped very gradually toward the wall (Fig. 6, C-2). Figure 6, D-2 shows that for small amplitude waves this was true for both sides of the wave. In this case, the wave length was measured from the point of zero amplitude where the wave fringe was observed to be parallel to the plate surface. Using this definition of the limits of the wave, the amplitude was then measured from the wave peak normal to a line connecting the two minimum points.

Wave speed was determined by measuring the distance a wave peak traveled up the plate during a specified time. The elapsed time was calculated using camera shutter speed and the number of frames required to record the wave displacement.

An oscilloscope trace of the input to the pulse wire was recorded by a Polaroid camera. One bar on the ordinate represented 0.1 voltage drop across a 0.25 ohm resistor. Using the computed current flow and the known voltage input of the battery, the energy input in BTU's was directly calculated (Table 1).

## VI. Results and Discussion

### Generated Waves

The limits which defined or comprised the disturbance waves in the boundary layer appeared to be largely a matter of conjecture or opinion of the investigator. Figure 6 shows that waves generated in the boundary layer did not ordinarily conform to a sine wave configuration. Although some small amplitude waves appeared smooth and generally symmetric about the wave peak, the larger amplitude waves varied greatly from this pattern. The majority had a pronounced rolling tendency which produced a long and a short side of the wave. This phenomenon was mentioned in Reference 3, but no explanation was given as to what the limits of the wave were defined to be.

The limits of the wave as defined in this study were outlined on page 15. The high energy (or large amplitude) waves were of particular concern since the long side of the wave often sloped very gradually in toward the wall and it was extremely difficult to detect exactly where the wave should end. Some consideration was given to defining the wave as twice the trailing half-wave, which usually could be easily measured from a minimum point to the wave peak. However, this was not considered representative since it would lead to very short waves compared to the small amplitude waves which were relatively symmetric about the peak and generally did not have a definite minimum point on either side of the wave peak.

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It was impossible to consider a wave from maximum to maximum point since this required at least two pulsed waves to be combined to form a continuous wave pattern. This occurred only at specific frequencies, which were defined by Birch (Ref. 1) as "natural" boundary layer frequencies. This will be discussed in some detail later. However, it is sufficient to point out here that the original wave generated by the disturbance pulse, such as shown in Figure 6, D-1, was the one of interest in this study and the induced, or trailing, waves were not considered of primary concern.

Difficulties in defining wave limits were also encountered as the wave patterns approached the transition region. Small harmonics appeared throughout the wave and made it difficult to determine the exact extent of the original wave formation. The harmonics in the wave increased in number and magnitude until turbulence (as defined by erratic flow patterns) occurred. In the transition and turbulence regions, it was impossible to define wave limits with any accuracy (Fig. 6, A-4). Neither was it possible to obtain any correlation between measurements of waves chosen in the same event.

#### Wave Length

As mentioned previously, each boundary layer wave was considered to be composed of individual waves as defined by the fringe lines. The wave parameters at each position along the plate varied through the boundary layer. Each fringe line represented the various wave parameters

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at some particular distance from the surface of the plate. By observing the behavior of each of these fringe lines, a more definitive study could be made of the exact nature of the wave than by studying the behavior of the outer fringe, which represented only the wave phenomena at the outer edge of the boundary layer. In this way, a detailed observation was made of the wave propagation mechanisms throughout the boundary layer. The variations of wave length with distance from the plate are shown in Figures 7 thru 13, which indicate that in most cases the wave length did vary through the boundary layer and an average would have to be used to define the overall wave movement.

General observation of wave motion revealed that the large energy waves appeared somewhat shorter than small energy waves. By large and small energy waves, it is inferred that large amplitude implies large energy. This will be explained further in another section. However, no definite relation between wave length and disturbance energy input seemed to exist.

Figure 7 shows that during the initial stages of wave growth the wave length profile through the boundary layer was similar to the velocity profile shown in Figure 14. The average maximum point was at  $y/\delta = 0.536$  compared to the maximum boundary layer velocity point at  $y/\delta = 1/3$ . Comparison of Figures 7 and 8 indicate that variation of the pulse position in the boundary layer effected the profile of the wave length, but did not effect the range of magnitudes of wave lengths.

Maximum wave length occurred at approximately  $x = 11-13$  ins. (Fig. 15). At this position the wave length was 6.676 for a disturbance frequency of 1.5 cps. Wave length through the boundary layer in this region appeared to be approximately constant.

Increasing the energy input of the disturbance pulse generally produced a shortening of the waves (Fig. 9). Maximum wave length with frequency variation occurred at 3.0 cps at pulse origin, but at 1.5 cps for all other positions.

#### Wave Amplitude

The variations of wave amplitude through the boundary layer at the pulse origin had a profile (Figs. 16 and 17) similar to the wave length and velocity profiles previously mentioned. The point of maximum amplitude occurred at  $V/\xi = .396$ . As in the case of the wave length, the profile changed as the wave propagated up the plate. However, the amplitude did not tend to become constant through the boundary layer, but increased rapidly from the inner edge to the outer edge of the boundary layer wave. The amplitude of the outer edge waves varied from 2 to 4 times the values of the inner waves near the plate.

Wave amplitude did not vary in any constant pattern with variation of disturbance frequency. However, a marked increase in amplitude occurred with the increase in energy input of the pulse (Fig. 18). It was concluded from this that the amplitude of the wave was an indication of the relative energy of the wave. Therefore, the wave amplitude should have increased in all cases until the wave was completely developed and

then began a gradual decline as the wave energy was spent in propagation up the plate. Although this phenomenon did occur as expected, the exact opposite also occurred when only the position of the pulse in the boundary layer was changed (Fig. 15). It appeared that the boundary layer initially damped the wave. However, as the wave moved further up the plate, the boundary layer began amplifying or developing the wave energy. This indicated that the position of the pulse origin can have a definite effect on the wave development and consequent formation of turbulence regions.

#### Wave Speed

As previously mentioned in this paper, one of the disadvantages of a study made with an interferometer is the inability to follow particle motion. However, this was not of major concern in this study since the wave motion was of primary importance. Consequently, wave speed was defined by the time displacement of the maximum point of amplitude of the wave being considered and does not represent particle velocities in the boundary layer.

Profiles of the wave velocity through the boundary layer showed a marked resemblance with the boundary layer velocity profile during initial development of the waves (Figs. 23 & 24). The average maximum wave speed occurred at  $y/\delta = .354$ , which correlated with the theoretical point of maximum boundary layer velocity at  $y/\delta = 1/3$ .

The variation of the wave speed with distance from the leading edge of the plate is shown in Figure 15 at a constant relative position:  $y/\delta = .354$ .

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Pulse position in the boundary layer appeared to have very little effect on the wave velocity profiles. However, a definite variation in maximum velocity of the wave was noted in Figure 15. When the pulse was positioned at  $\delta/2$ , the wave speed peaked at approximately 10 in/sec at  $x = 13.5$  inches from the leading edge of the plate. When the pulse wire was moved near the wall where the viscosity effects were greatest, the resulting wave velocities were considerably reduced and showed only a slight increase with  $x$  position up the plate. Another explanation for this occurrence was that a loss of disturbance energy to the flow may have occurred when the wire was pulsed near the wall. Since the wire was already near the plate temperature due to its proximity, the energy input of the pulse would be expected to raise the temperature of the wire above that of the plate and force a certain amount of heat to be absorbed by the plate.

The data obtained for the pulse at the outer edge of the boundary layer was too incomplete for conclusive evaluation. However, as shown in Figure 8, a general wave velocity increase was noted from the outer edge of the boundary layer in toward the wall.

An interesting result was obtained when the disturbance energy input was increased. It was anticipated that the velocity profile would remain the same, but would be shifted to yield greater velocities through the boundary layer. However, a definite change in profile occurred when the energy input was increased at a pulse frequency of 1.5 cps (Fig. 25). This was a vivid example of the rolling tendency

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of the waves which was mentioned earlier. The inner waves of a high energy disturbance moved ahead of the center while the outer waves fell behind and produced a backward rolling motion.

An inverse velocity profile occurred at 1.5 cps disturbance frequency, with the pulse wire next to the wall (Fig. 28). This phenomenon points out the unique nature of the boundary layer in transition, which seems to defy general rules and conclusions.

#### Boundary Layer Frequency

The wave frequency of the boundary layer was defined as the ratio of wave speed to wave length. Since both the wave speed and wave length were measured, the frequency was calculated directly from the data obtained.

It was observed that the disturbance wave frequency near the pulse origin was approximately 4 to 6 times the pulse frequency (Fig. 29). The frequency decreased rapidly with distance from the wall. As the waves propagated up the plate, the frequency stabilized at approximately 1.4 times the disturbance frequency and remained generally constant through the boundary layer.

#### Summary of Observations and Comparison with Work of Others

The results of this investigation proved that boundary layer parameters vary with both x and y distance on a vertical flat plate. This precluded the use of wave data based entirely on x position unless average values were obtained. Failure to define basic parameters and

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insure standard notation and common language in studies of the boundary layer has undoubtedly played an important role in further complicating boundary layer phenomena.

Error introduced by measurement techniques or uncertainties as to what accurately comprised a wave in the boundary layer was found to be from 15 to 25%. Furthermore, the problem of detecting which waves were representative of the flow was very difficult as the turbulence region was approached. In the region of beginning transition, wave harmonics became more frequent and further complicated wave definition. As transition to apparently complete turbulence occurred, the harmonics appeared to completely reduce the wave propagation to a continuously erratic movement of small waves of varied configurations.

It was concluded in Ref. 3 that the natural instability region began at  $Gr_x = 4 \times 10^8$ . This figure correlated well with the general instability region observed in the present study using a disturbance frequency of 1.5 cps. Figure 6; D-3,4 and A-3,4 show that complete laminar flow existed at  $Gr_x = 2.28 \times 10^8$  for the lower energy disturbance waves and that generally erratic motion was observed at  $Gr_x = 5.55 \times 10^8$ . However, transition of the higher energy disturbances did begin in the outer waves of the boundary layer at  $Gr_x = 2.28 \times 10^8$  (Fig. 6, C-3) and this was also observed when the disturbance frequency was varied from 1.5 cps. When the position of the pulse wire was changed to the outer edge of the boundary layer, turbulence of the outer waves occurred at very low Grashof Numbers, although the inner

waves were generally undisturbed. This indicated that natural disturbances originating in the outer edge of the boundary layer could cause the passage of turbulence waves observed on occasion in the otherwise laminar boundary layer.

The boundary layer wave frequency as defined by Birch (Ref. 1), was the frequency of the disturbance pulse. It has been shown in the present study that the actual, or calculated, boundary layer frequency varied as much as 600% from the frequency of the disturbance pulse. Motion picture photographs of the pulsed disturbances developing in the boundary layers showed that at a specific frequency and energy input, the waves formed in the proper time sequence to augment the development of succeeding waves. Thus a continuous wave pattern was formed as the waves propagated up the plate. This pattern was obtained at a specific position in the boundary layer and a specific energy input to the pulsed disturbance. A more correct terminology for this particular phenomenon would have been natural "disturbance" frequency for specified flow and pulse input conditions. At lower disturbance frequencies the pulsed wave moved out of the range of influence before another wave was pulsed. At higher frequencies, the pulsed wave had not progressed sufficiently in its development before another wave was formed and interference occurred. Thus the process is basically one of timing and cannot be correlated with actual boundary layer frequency.

The relations developed by Birch ( $e^f = 1.65 (Gr_x \times Pr)^{0.08}$  and  $e^{\lambda} = 18.2 (Gr_x \times Pr)^{.091}$ ) did not correlate with data obtained in this investigation. Both wave frequency and wave length have been shown

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to vary through the boundary layer during this investigation. The equations developed by Birch define only one value at any  $x$  position. Substitution of actual boundary layer frequency in the first equation in place of the frequency ( $f$ ) used by Birch showed that these terms are not interchangeable.

Considerable time and effort was spent in an attempt to find a correlation for actual boundary layer frequency with wave speed and amplitude which would apply to the general boundary layer flow. Variations of frequency with a non-dimensional amplitude ratio were also investigated. However, no correlation was found which could be considered a general relation.

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## VII. Conclusions and Recommendations

### Conclusions

This investigation shows that the basic parameters of boundary layer oscillation vary with distance through the boundary layer as well as with position along the flow. Accurate wave movement cannot be defined, in general, by the analysis of a single wave contour at the outer edge of the boundary layer.

Boundary layer wave frequency must be defined as a function of wave speed and wave length and cannot, in general, be correlated with disturbance pulse frequency. Empirical relations developed by Birch (Ref. 1) apply only to the specified test conditions of that study.

No general relation exists at the present time which will accurately predict the influence of variations of flow conditions and disturbances in the boundary layer. The extension of any correlation of oscillations in free convection boundary layers to general boundary layer flows will be difficult indeed. However, the collection of systematic and definitive experimental data, such as presented in this study, should materially aid in the general understanding of boundary layer oscillation.

### Recommendations

Before additional experimentation is attempted using the techniques and procedures of this investigation, a detailed mathematical analysis should be made of the instability problem in free convection flow.

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Appendix A  
Figures

GAE-59-11

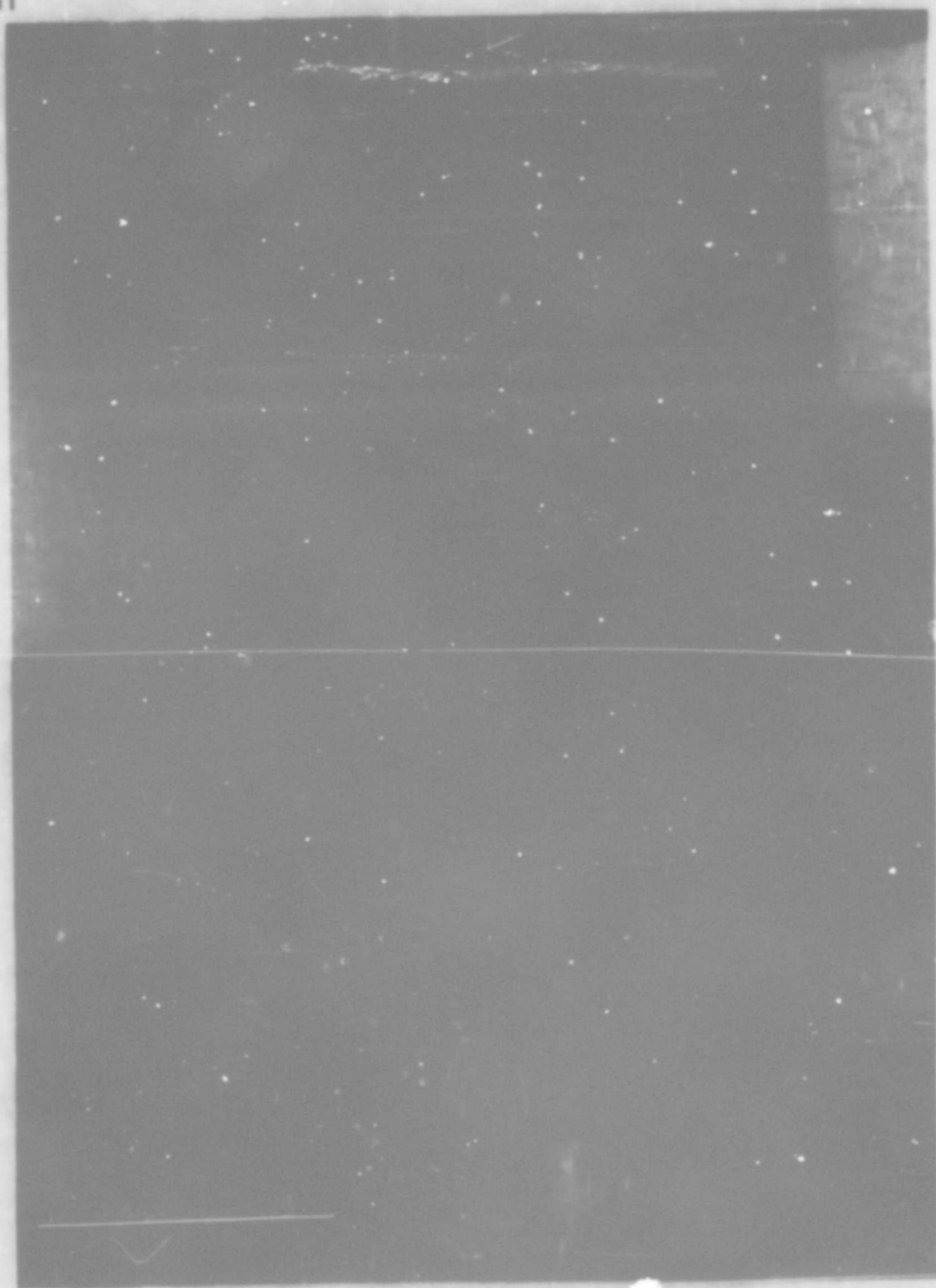


Fig. 1 Back of Test Plate shown in Test Tunnel

GAJ-59-11

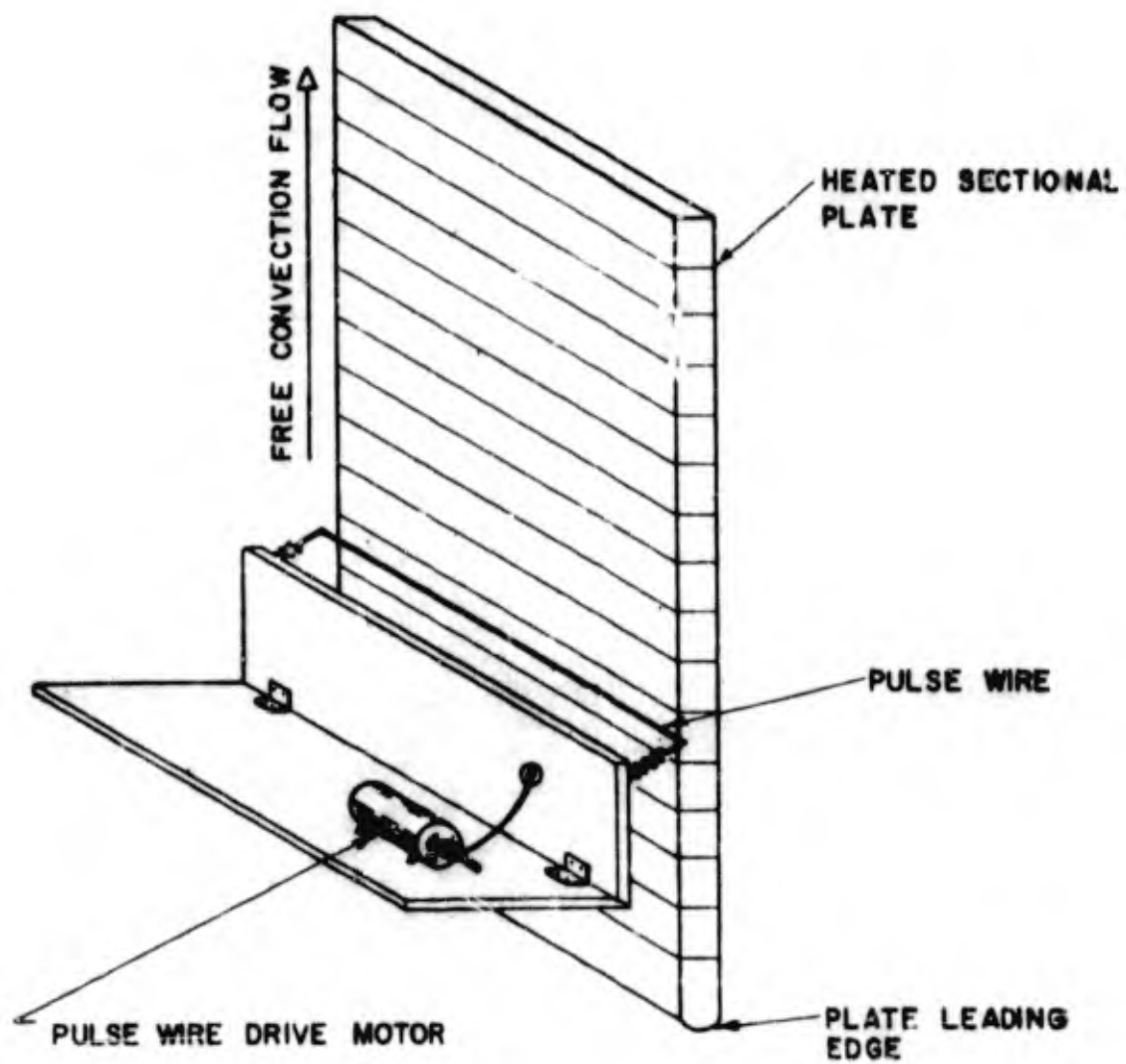


Fig. 2 Test Plate and Pulse Wire Unit

GM-59-11



Fig. 3 General Layout of Control Section



QAB-59-11



Fig. 5 Interferometer Suspension Unit



(4)  
 $Gr_x = 5.5 \times 10^8$

(3)  
 $Gr_x = 2.28 \times 10^8$

(2)  
 $Gr_x = 6.5 \times 10^7$

(1)  
 $Gr_x = 1.34 \times 10^7$

(A) Pulse at wall 1.5 cps 0.1 P.D.    (B) Pulse at wall 1.5 cps 0.5 P.D.    (C) Pulse at  $\xi/2$  1.5 cps 0.5 P.D.    (D) Pulse at  $\xi/2$  1.5 cps 0.5 P.D.  
 Fig. 6 Boundary Layer Wave Propagation

GAE-59-11

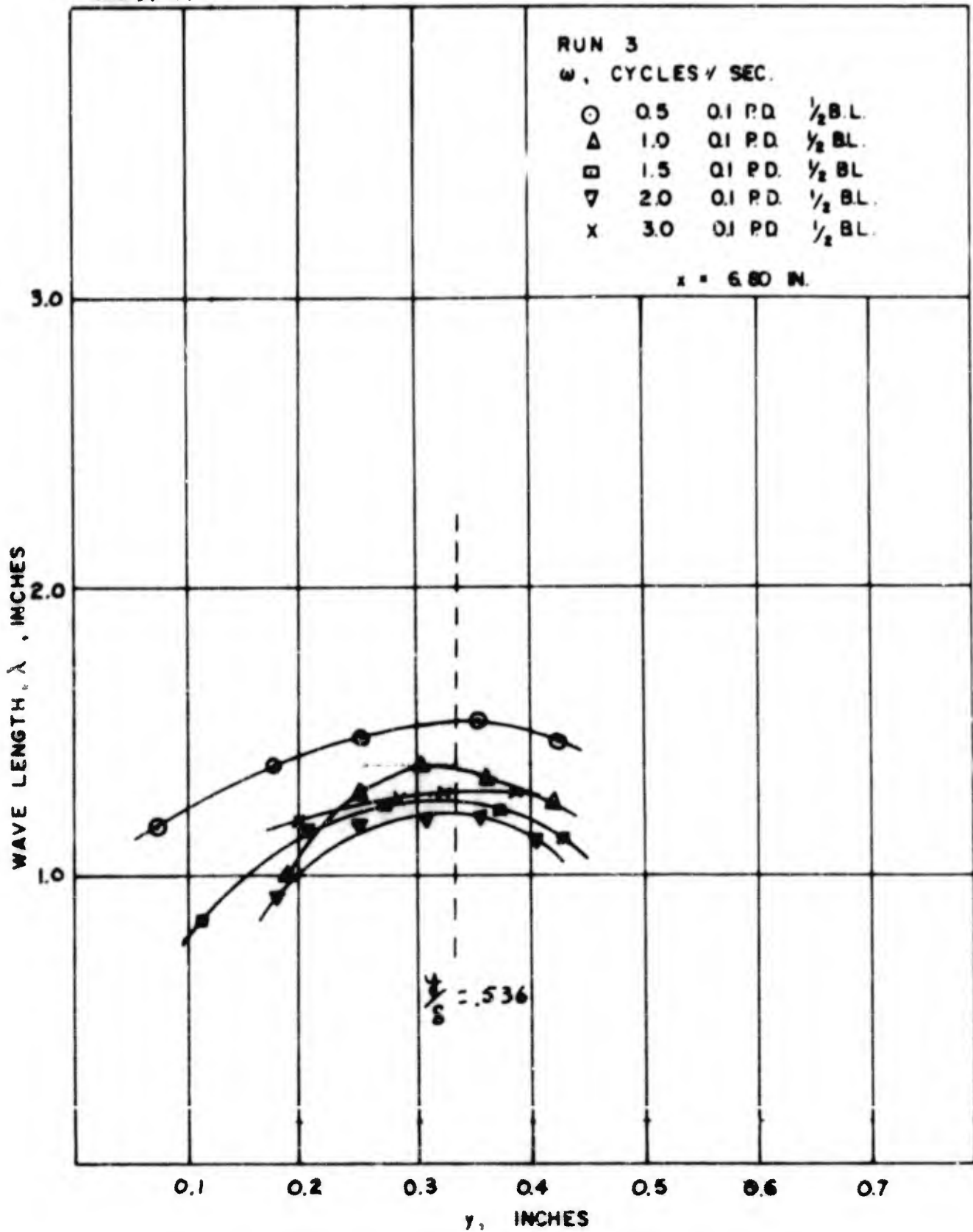


Fig. 7 Wave Length vs. Distance from Wall

GAE-59-11

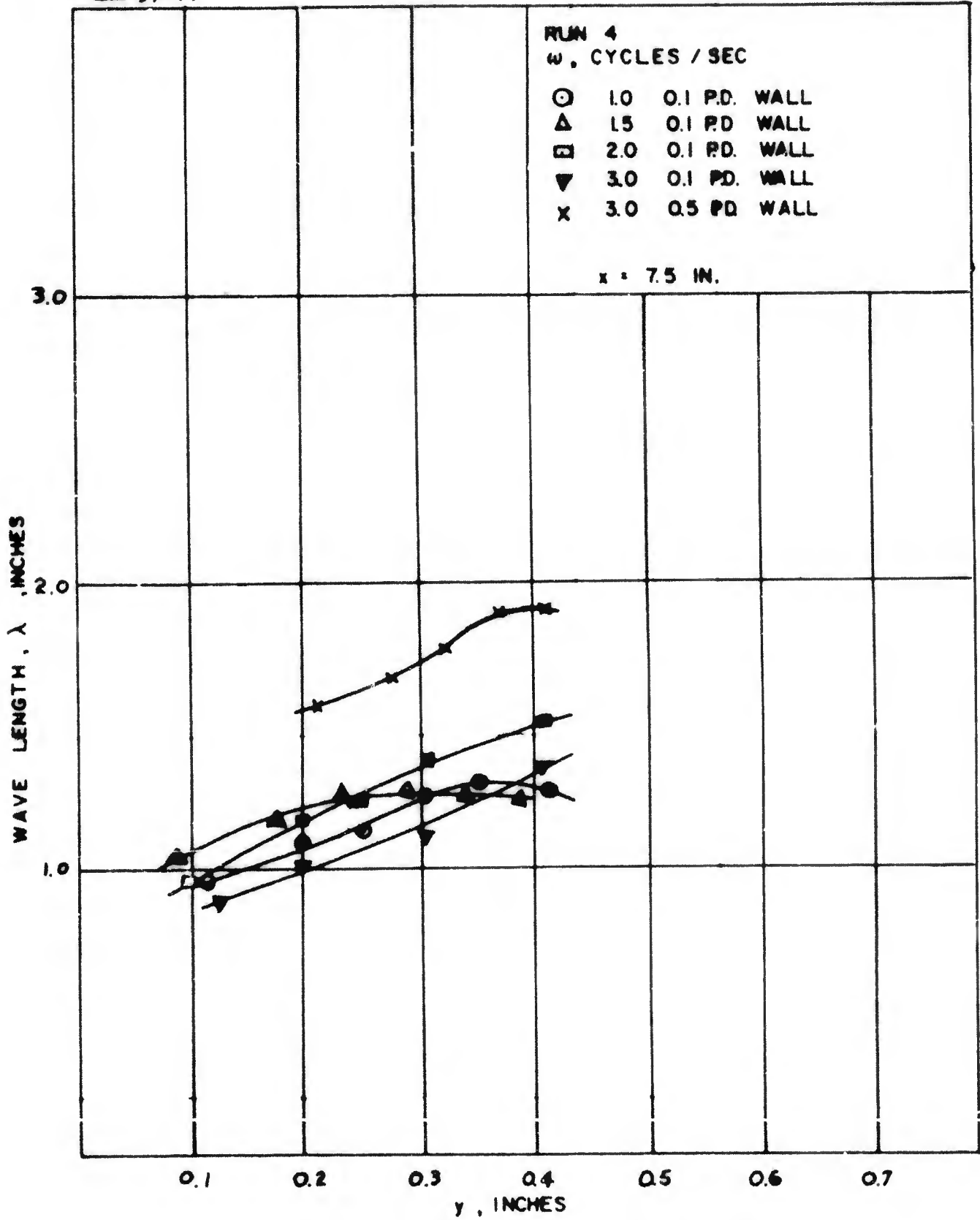


Fig. 8 Wave Length vs. Distance from Wall

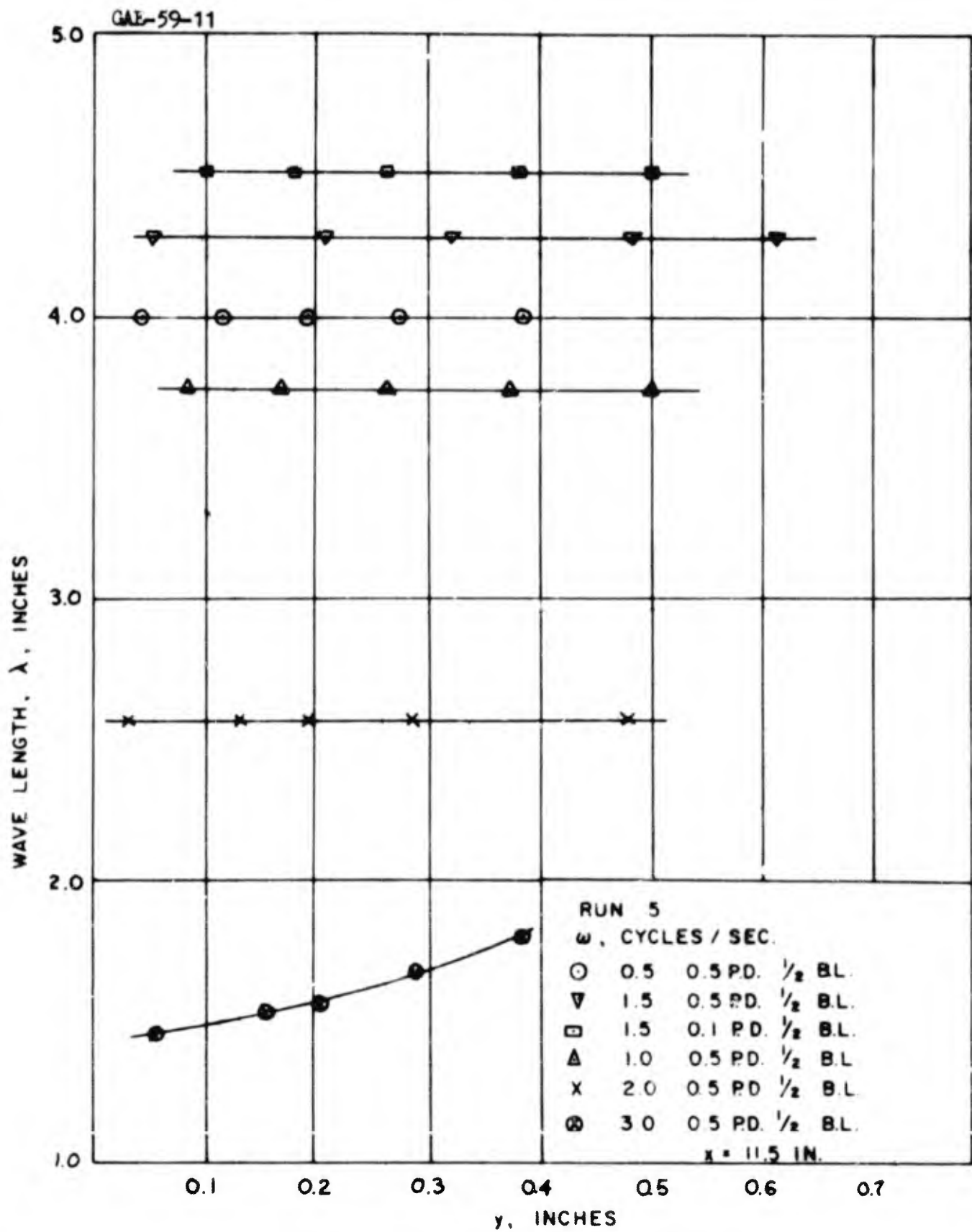


Fig. 9 Wave Length vs. Distance from Wall

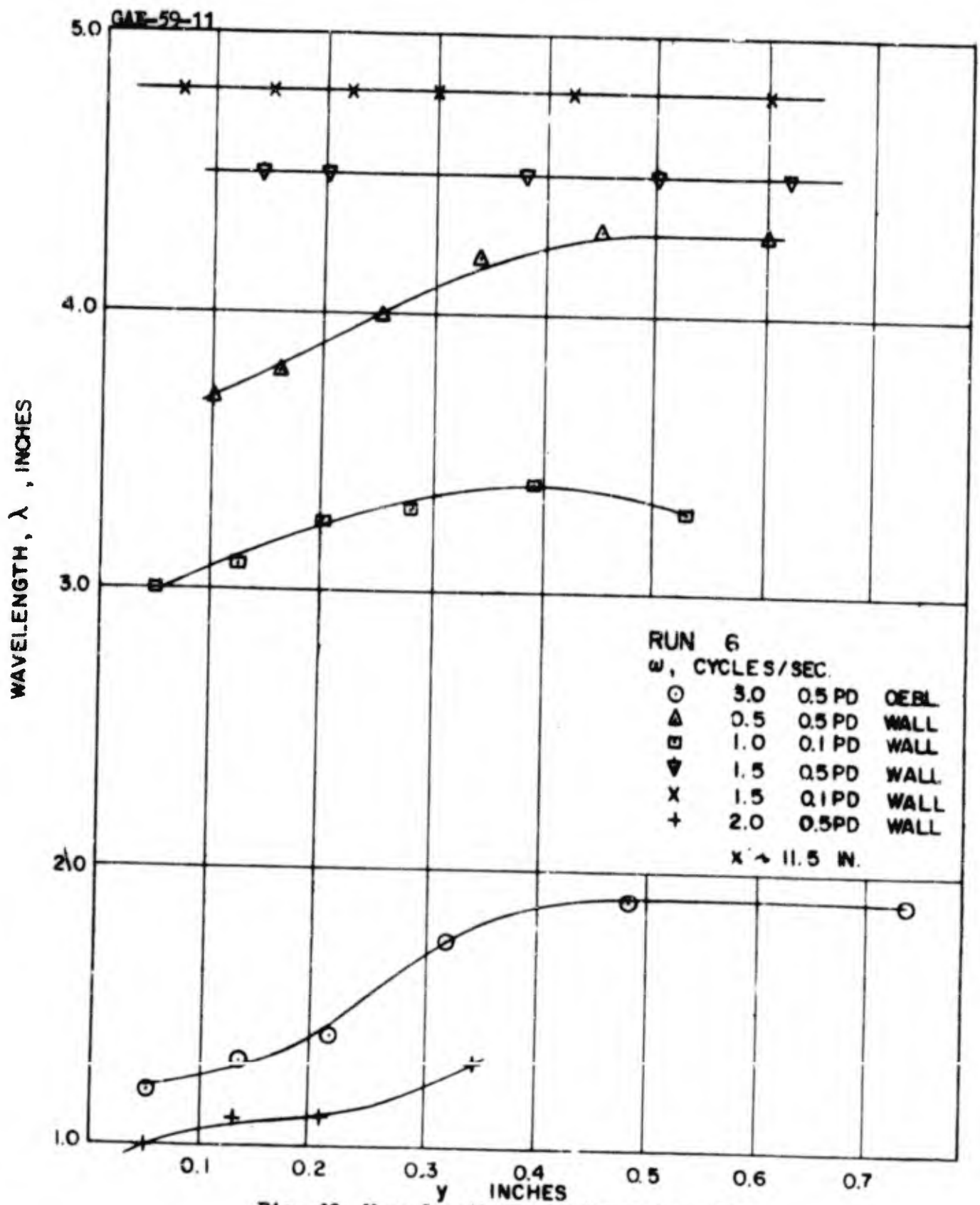


Fig. 10 Wave Length vs. Distance from Wall

GAP-59-11

RUN 7

$\omega$ , CYCLES / SEC

○	3.0	0.1 PD	OE	BL
△	3.0	0.1 PD	$\frac{1}{2}$	BL
□	2.0	0.5 PD	$\frac{1}{2}$	BL
▽	1.5	0.1 PD	$\frac{1}{2}$	BL
x	0.5	0.1 PD	$\frac{1}{2}$	BL

x - 17.5 IN

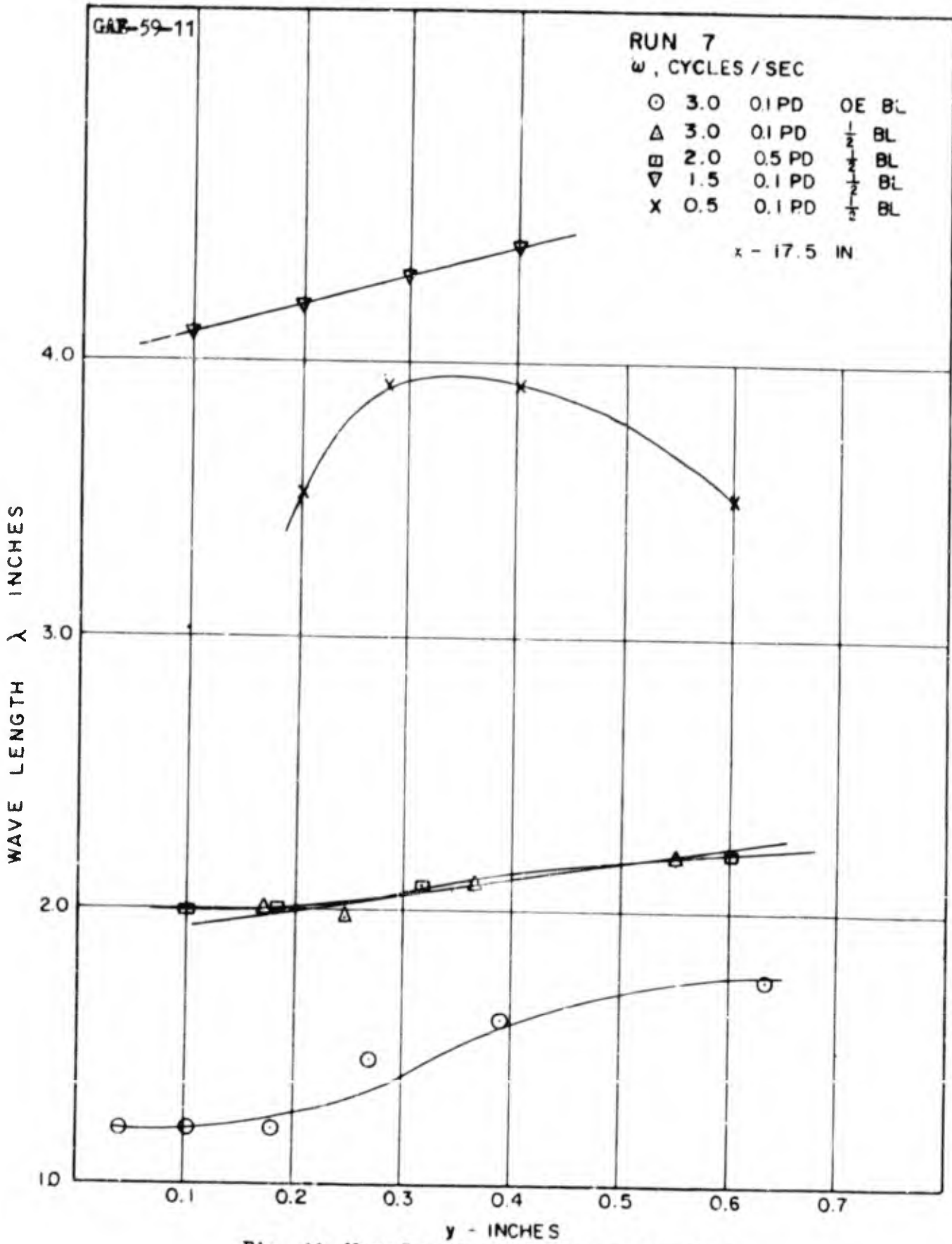


Fig. 11 Wave Length vs. Distance from Wall

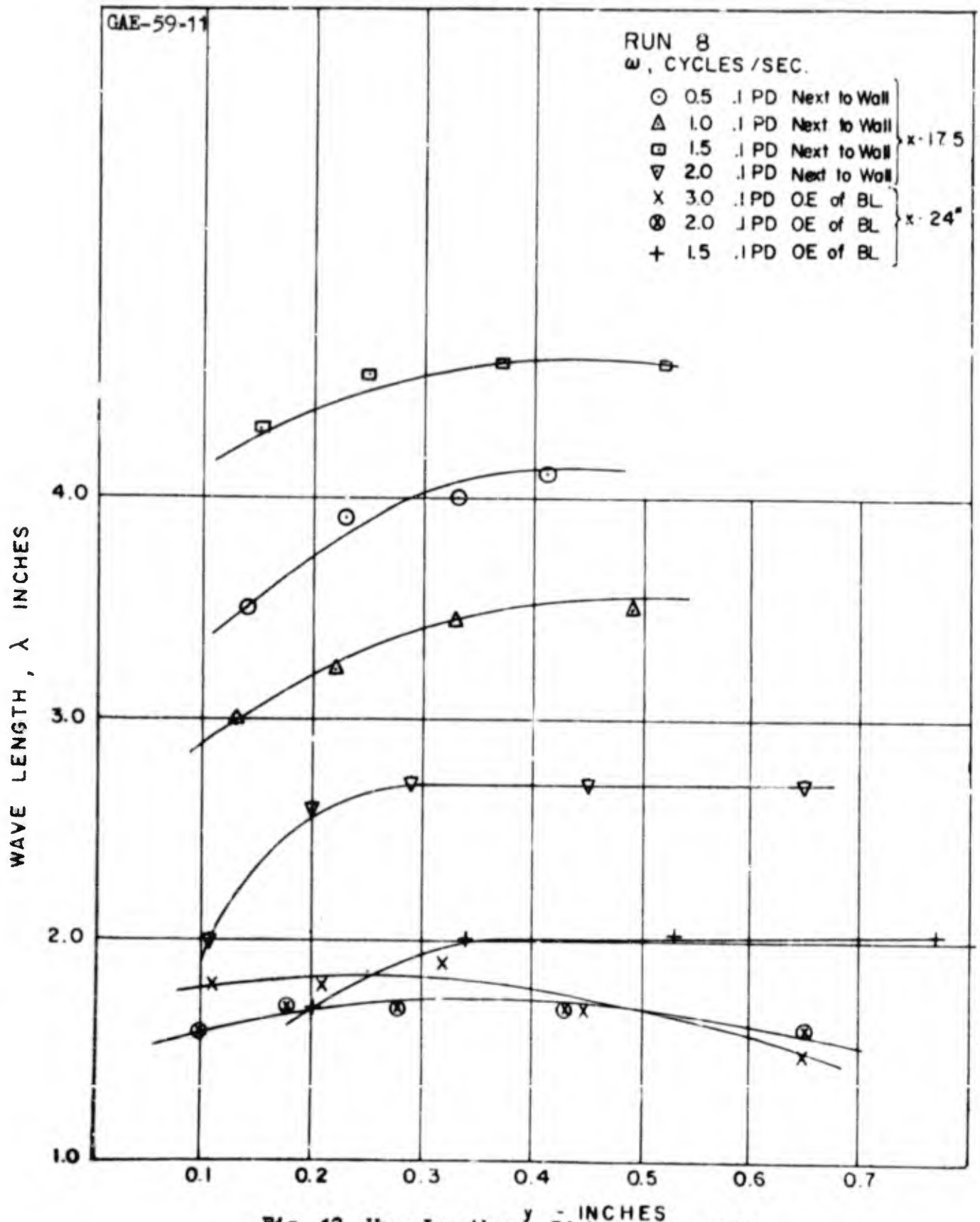


Fig. 12 Wave Length vs. Distance from Wall

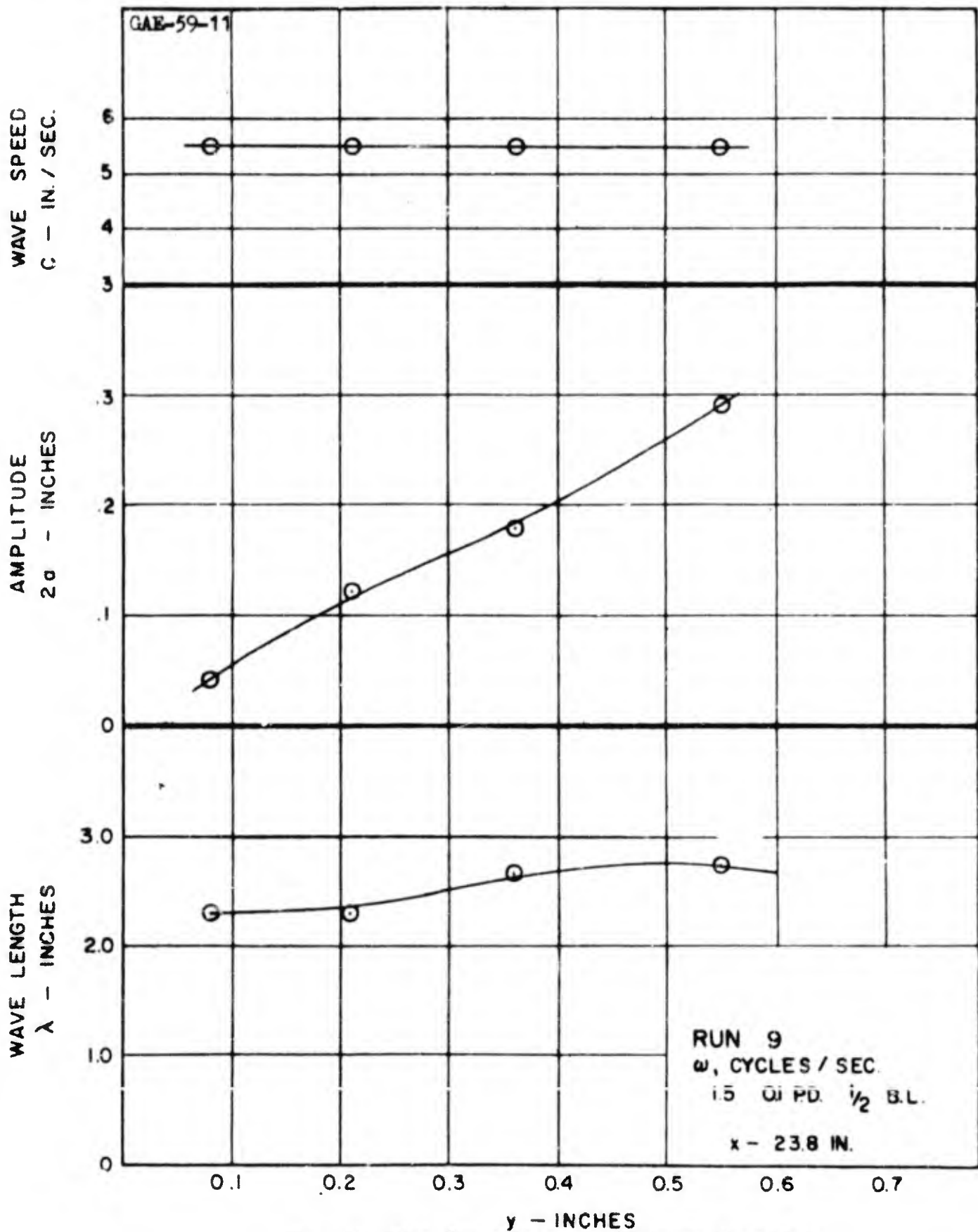


Fig. 13 Wave Parameters vs. Distance from Wall

GAE-59-11

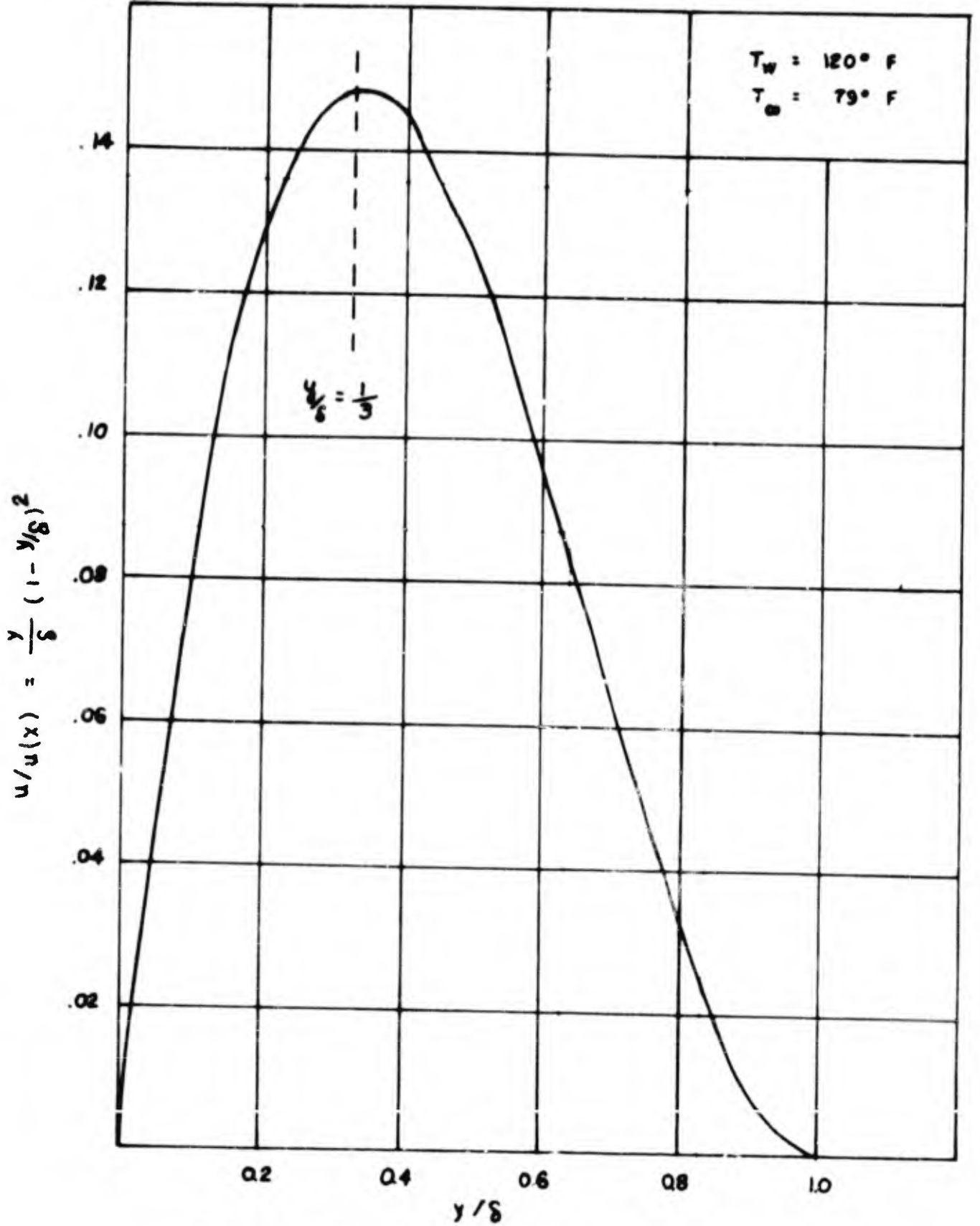


Fig. 14 Free Convection Boundary Layer Velocity Profile

GAE-59-11

$\omega$  CYCLES / SEC

○ 1.5 .1 PD.  $\frac{1}{2}$ BL

△ 1.5 .1 PD. WALL

(  $\frac{y}{\delta} = .354$  )

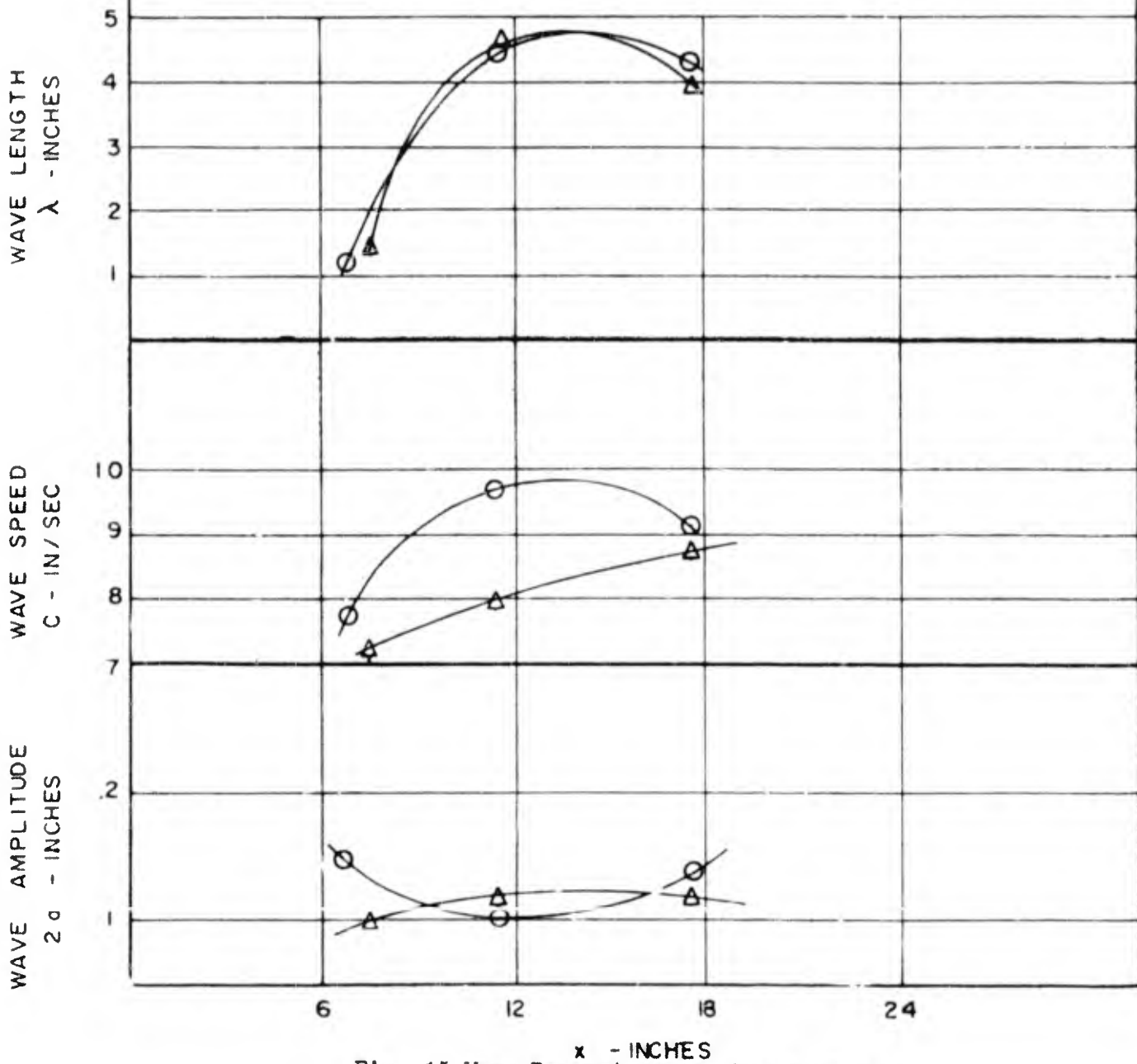


Fig. 15 Wave Parameters vs. Distance From Leading Edge of Plate

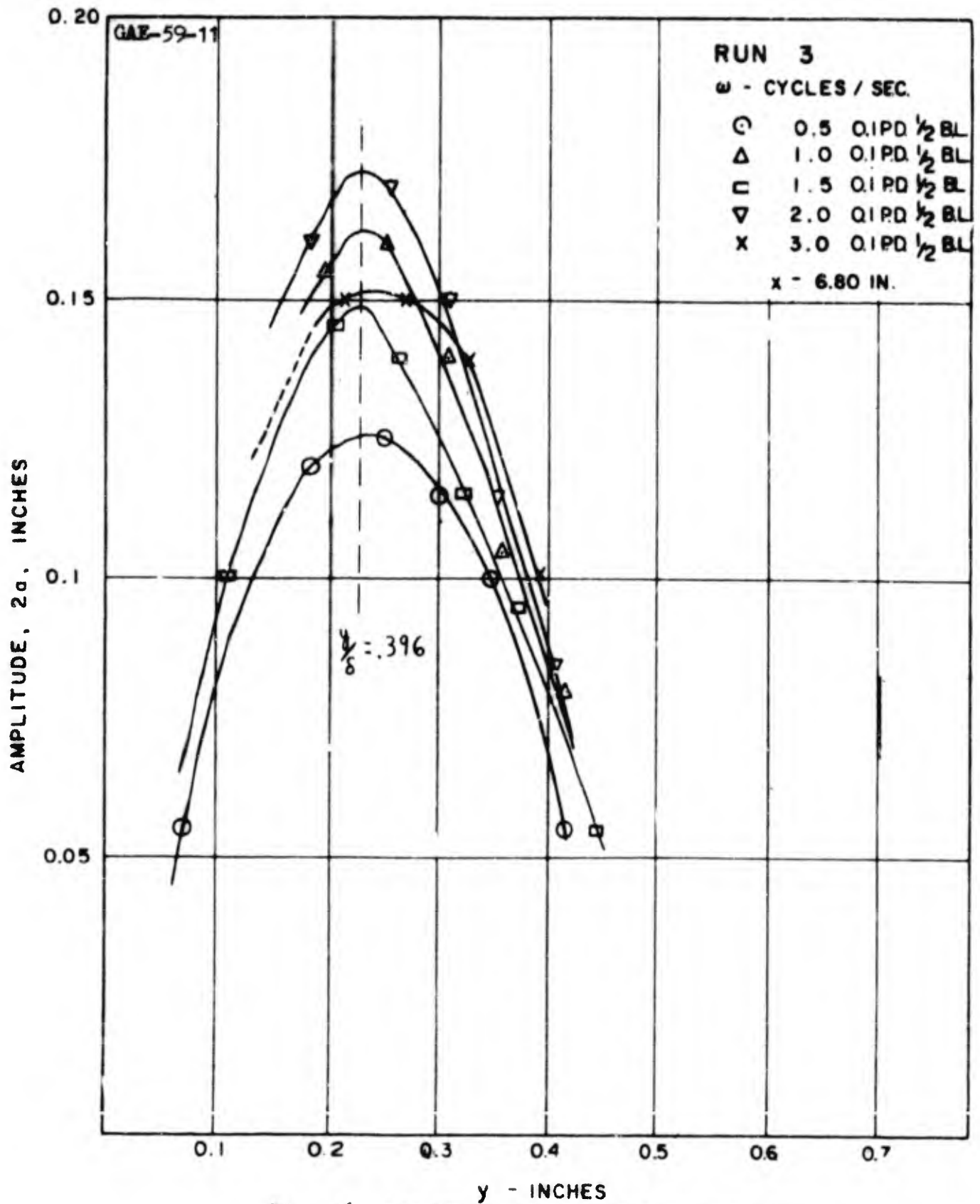


Fig. 16 Wave Amplitude vs. Distance from Wall

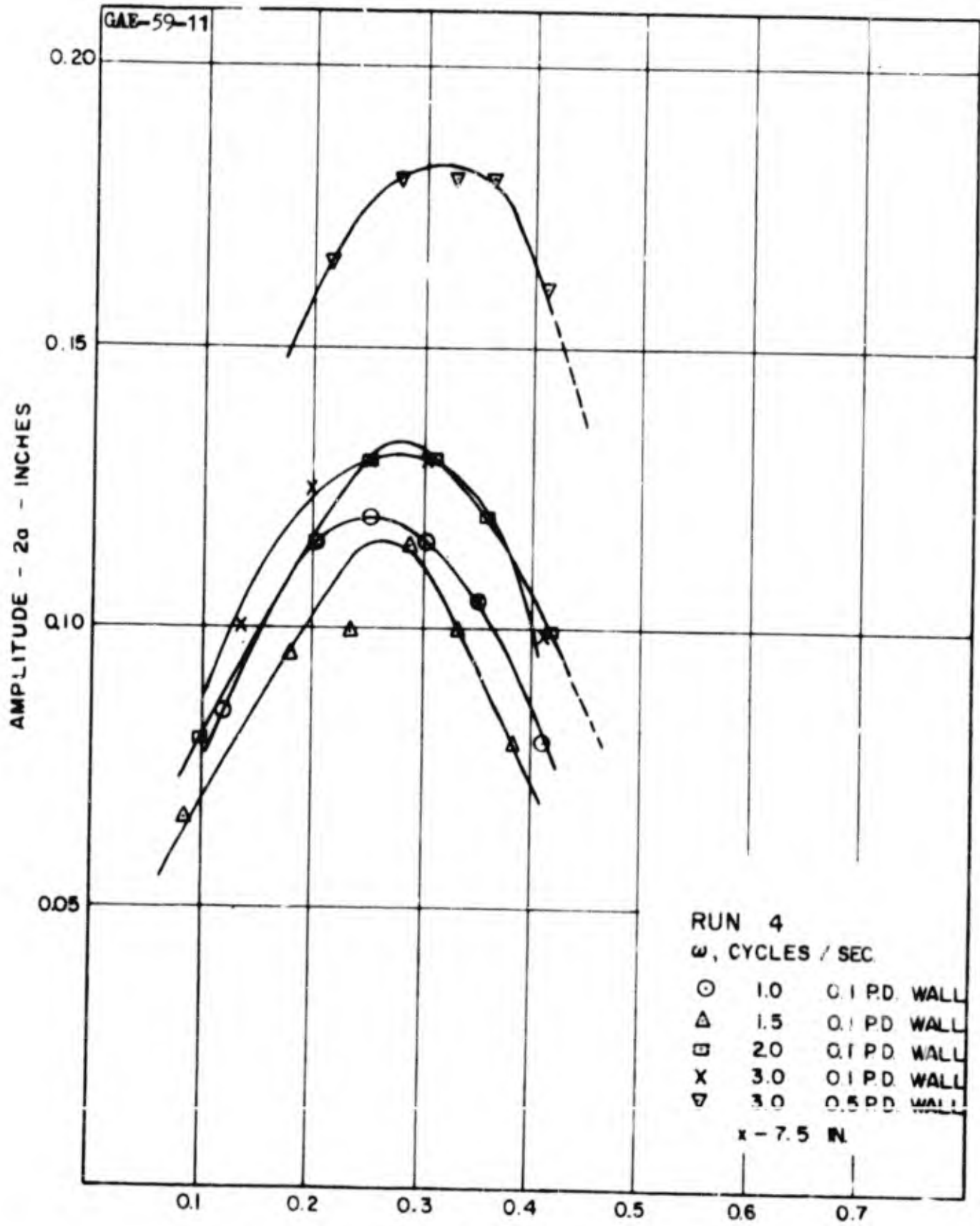


Fig. 17 Wave Amplitude vs. Distance from Plate

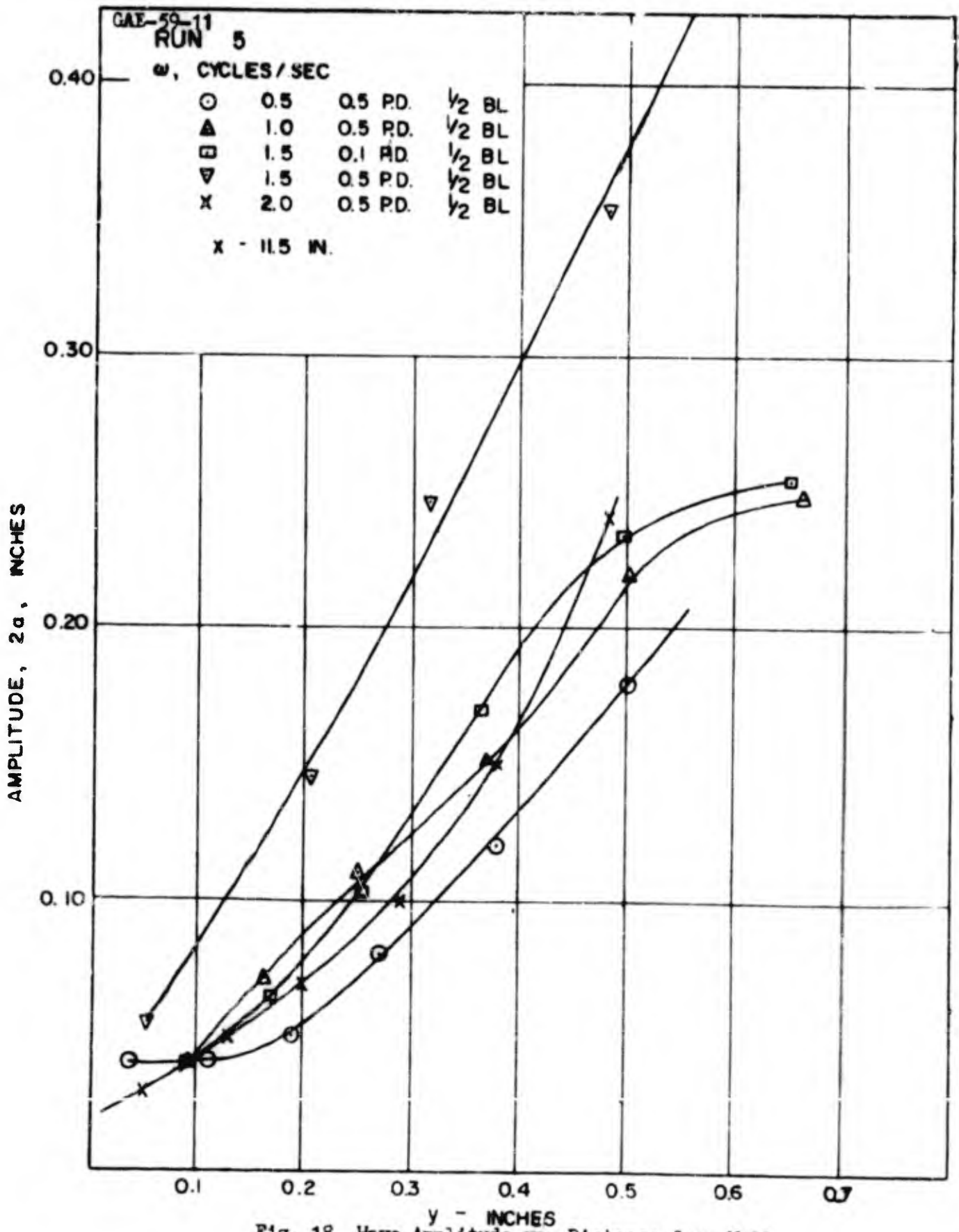


Fig. 18 Wave Amplitude vs. Distance from Wall

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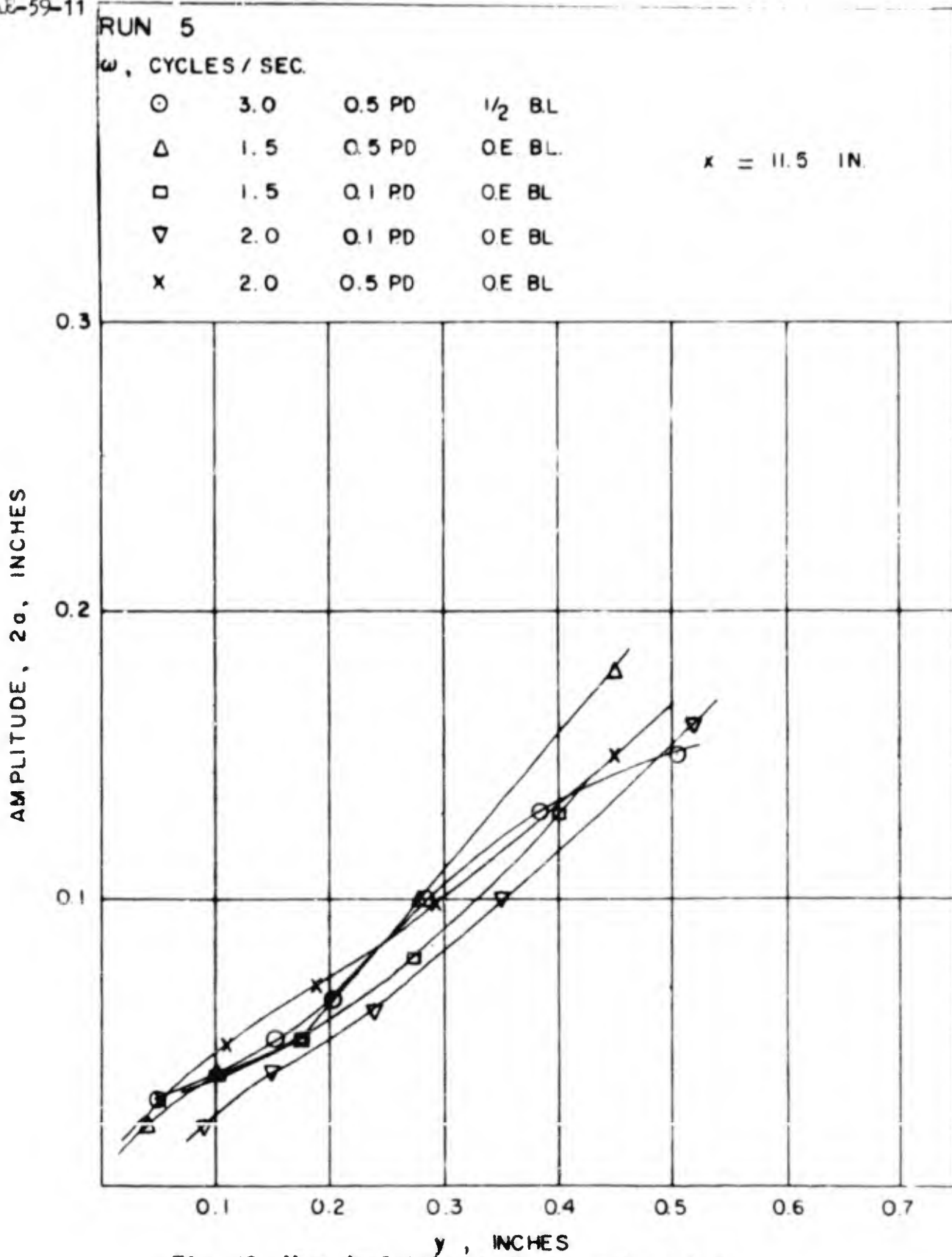


Fig. 19. Wave Amplitude vs. Distance from Wall

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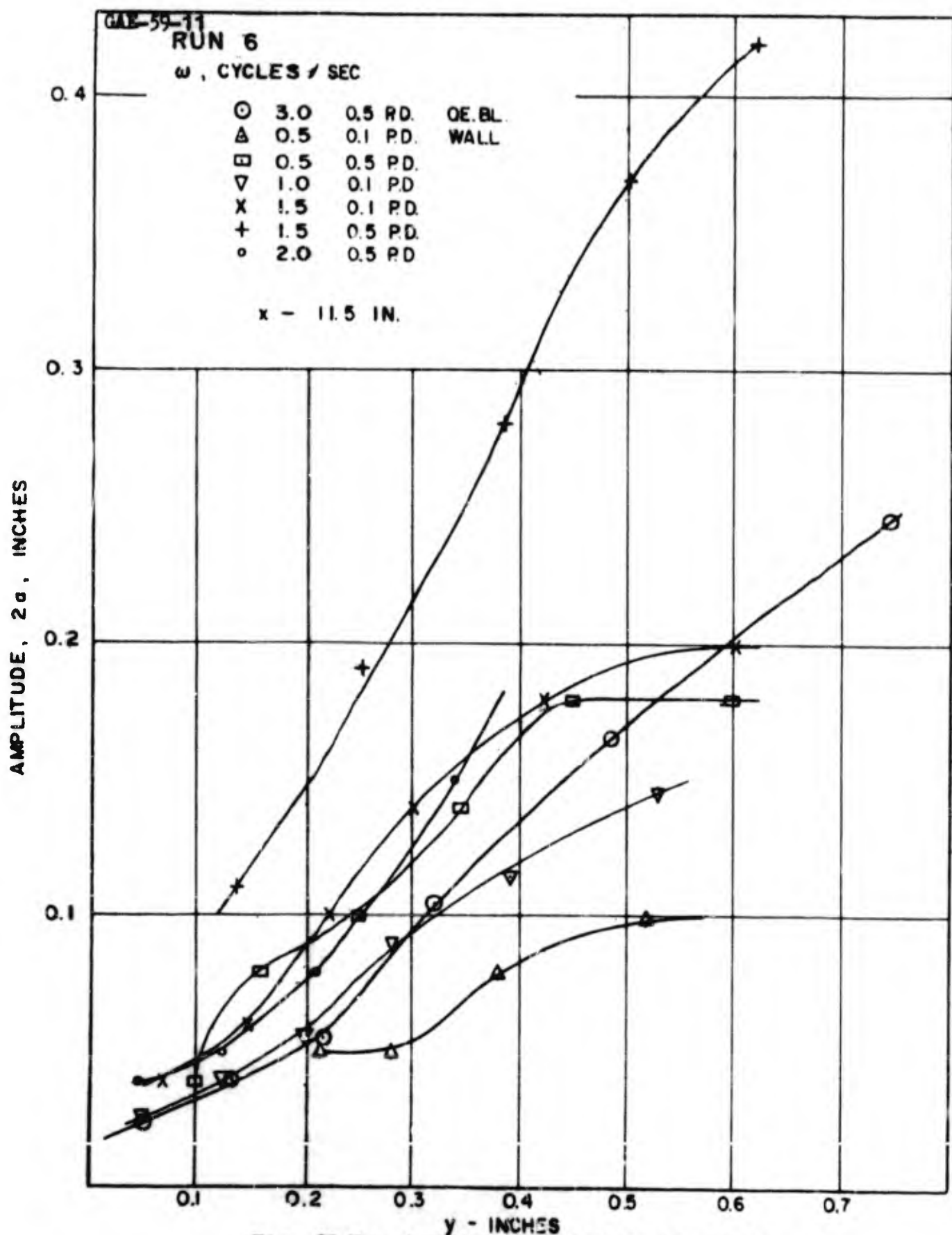


Fig. 20 Wave Amplitude vs. Distance from Wall

GAE-59-11

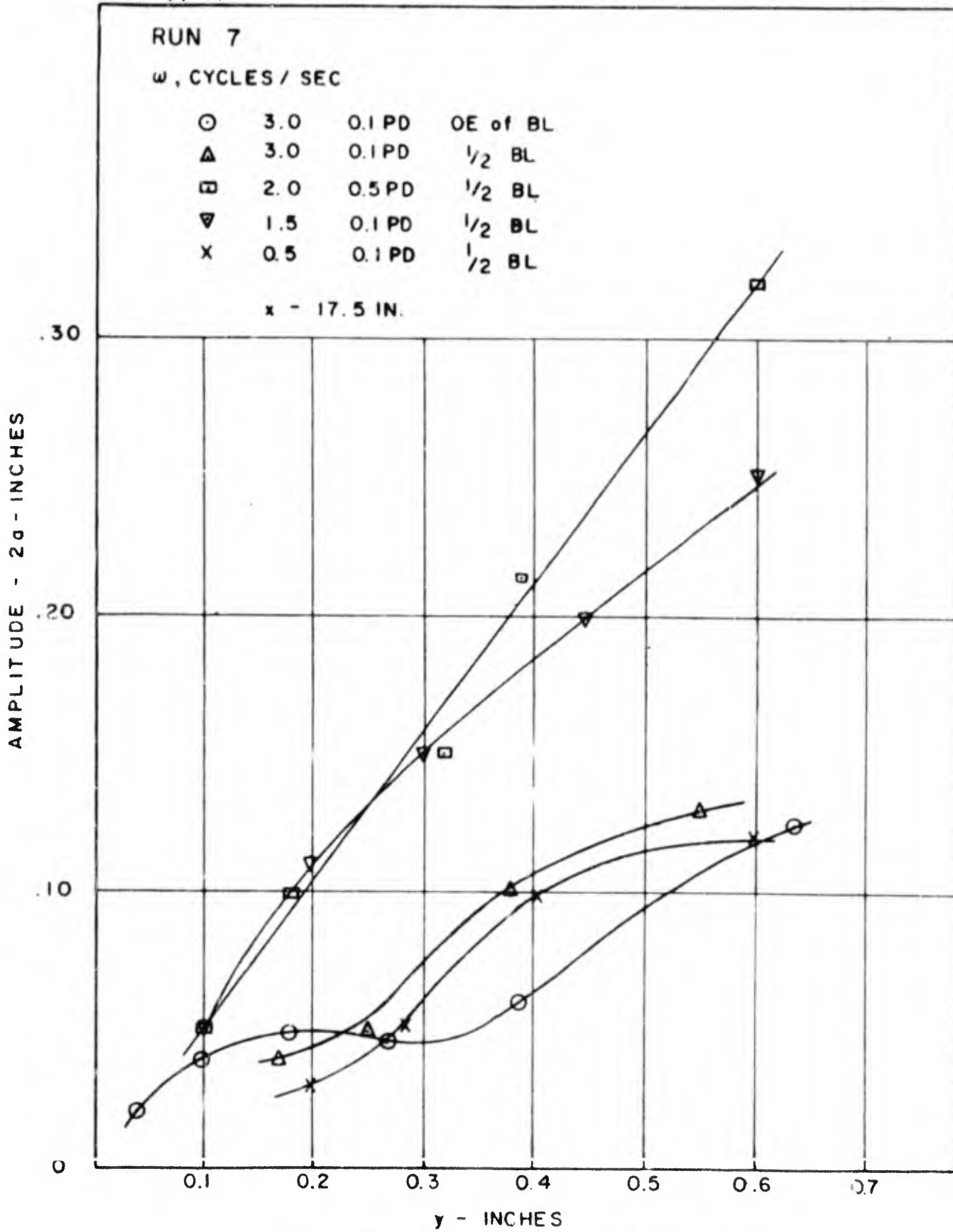


Fig. 21 Wave Amplitude vs. Distance from Wall

GAE-50-51

RUN 8

$\omega$ , CYCLES/SEC.

○	0.5	0.1 PD	NEXT TO WALL
△	1.0	0.1 PD	NEXT TO WALL
□	1.5	0.1 PD	NEXT TO WALL
▽	2.0	0.1 PD	NEXT TO WALL

x - 17.5 IN.

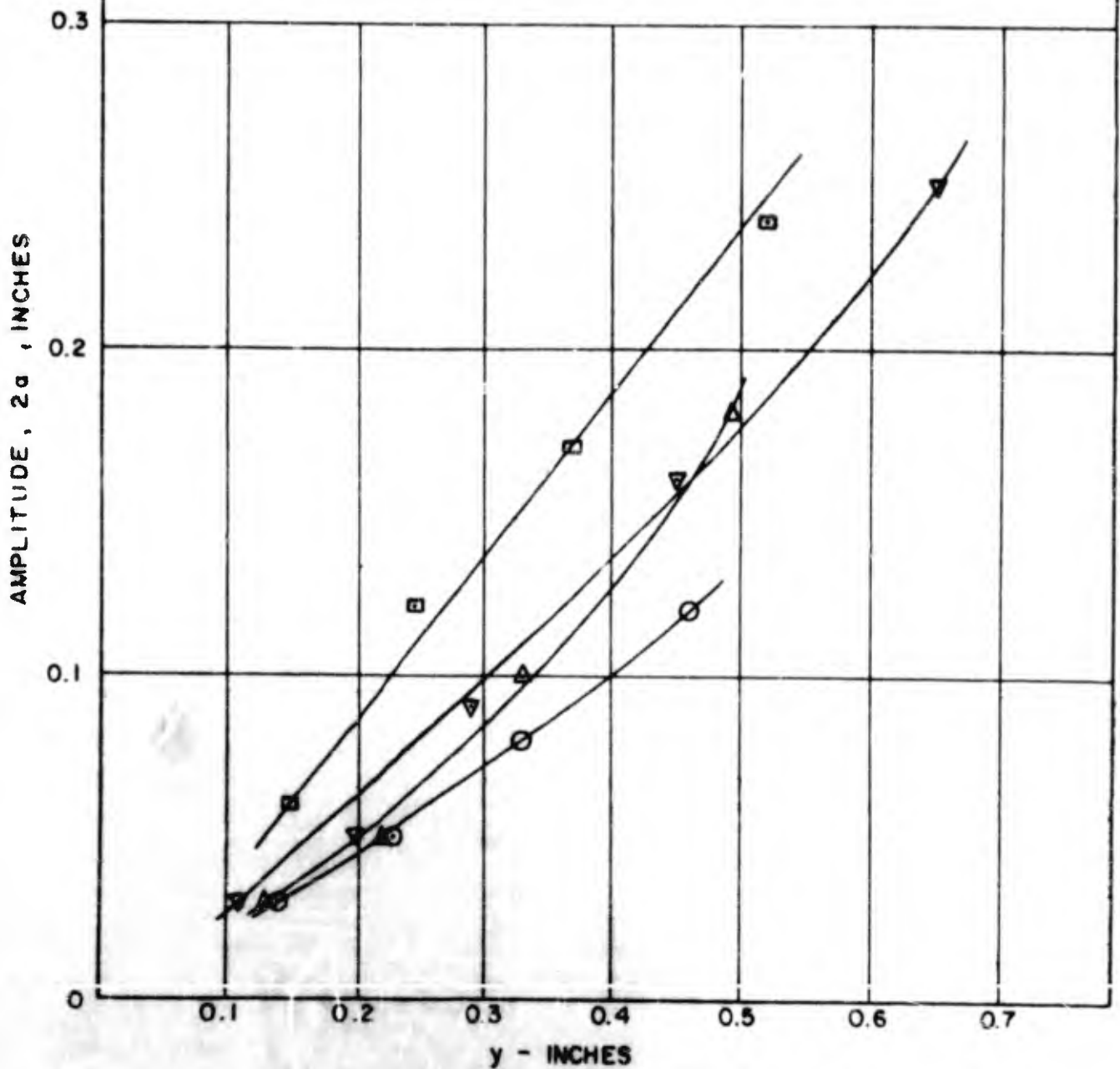


Fig. 22 Wave Amplitude vs. Distance from Wall

GA-59-11

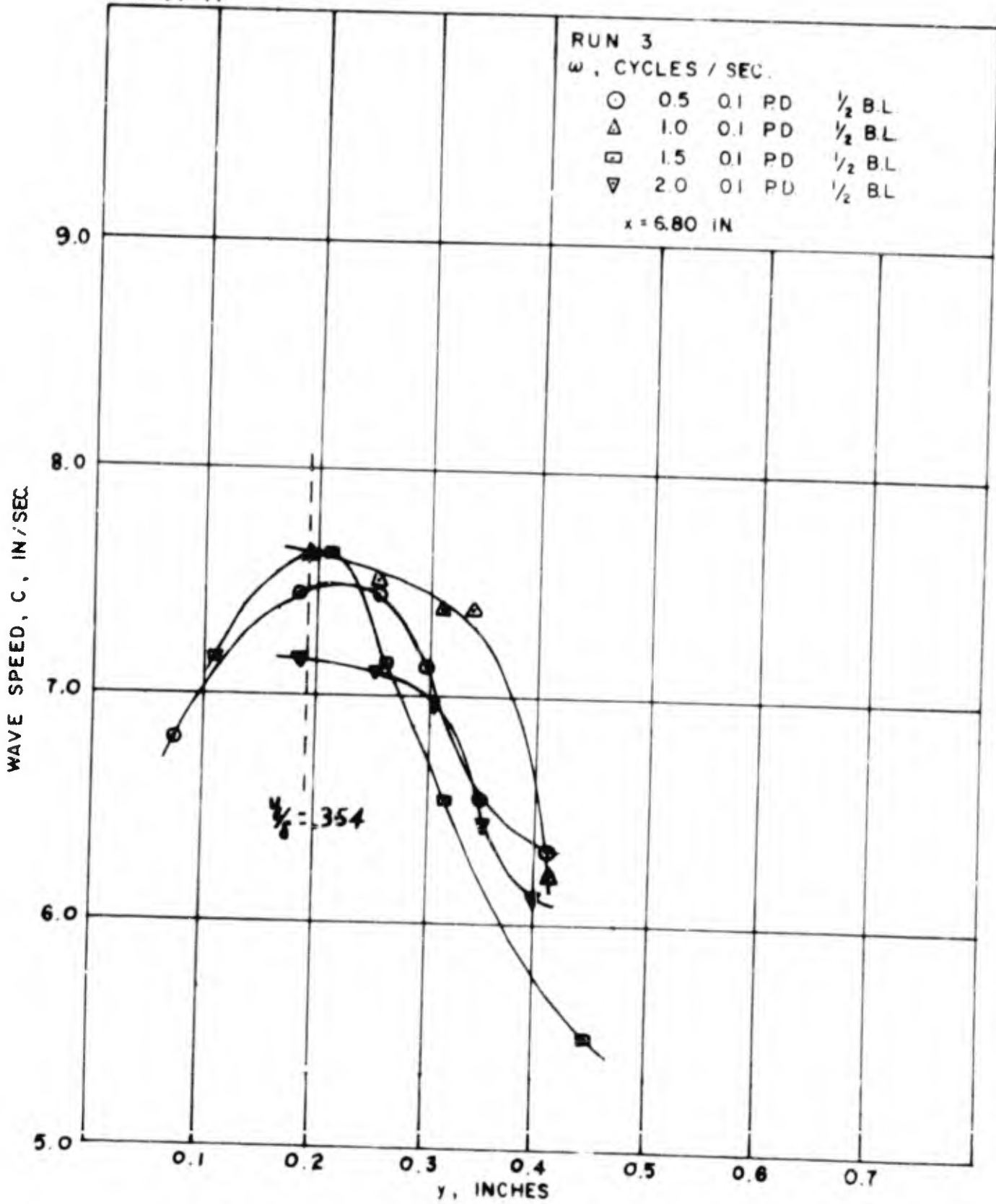


Fig. 23 Wave Speed vs. Distance from Wall

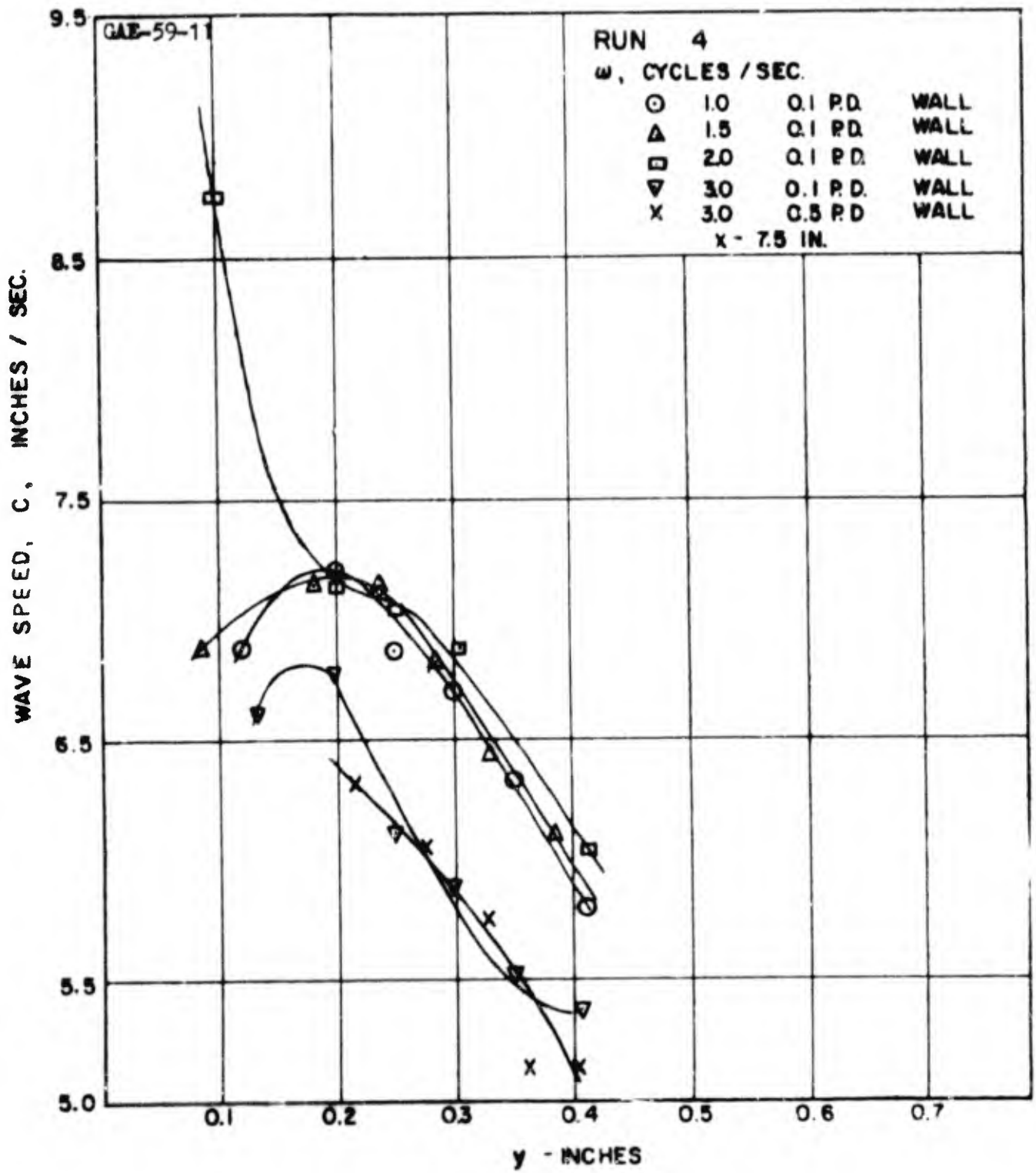


Fig. 24 Wave Speed vs. Distance from Wall

KA-59-11

RUN 5

$\omega$ , CYCLES/SEC.

○	0.5	0.5 PD.	$\frac{1}{2}$ BL
△	1.0	0.5 PD.	$\frac{1}{2}$ BL
□	1.5	0.1 PD.	$\frac{1}{2}$ BL
▽	1.5	0.5 PD.	$\frac{1}{2}$ BL
X	3.0	0.5 PD.	$\frac{1}{2}$ BL

$x \sim 11.5$  IN.

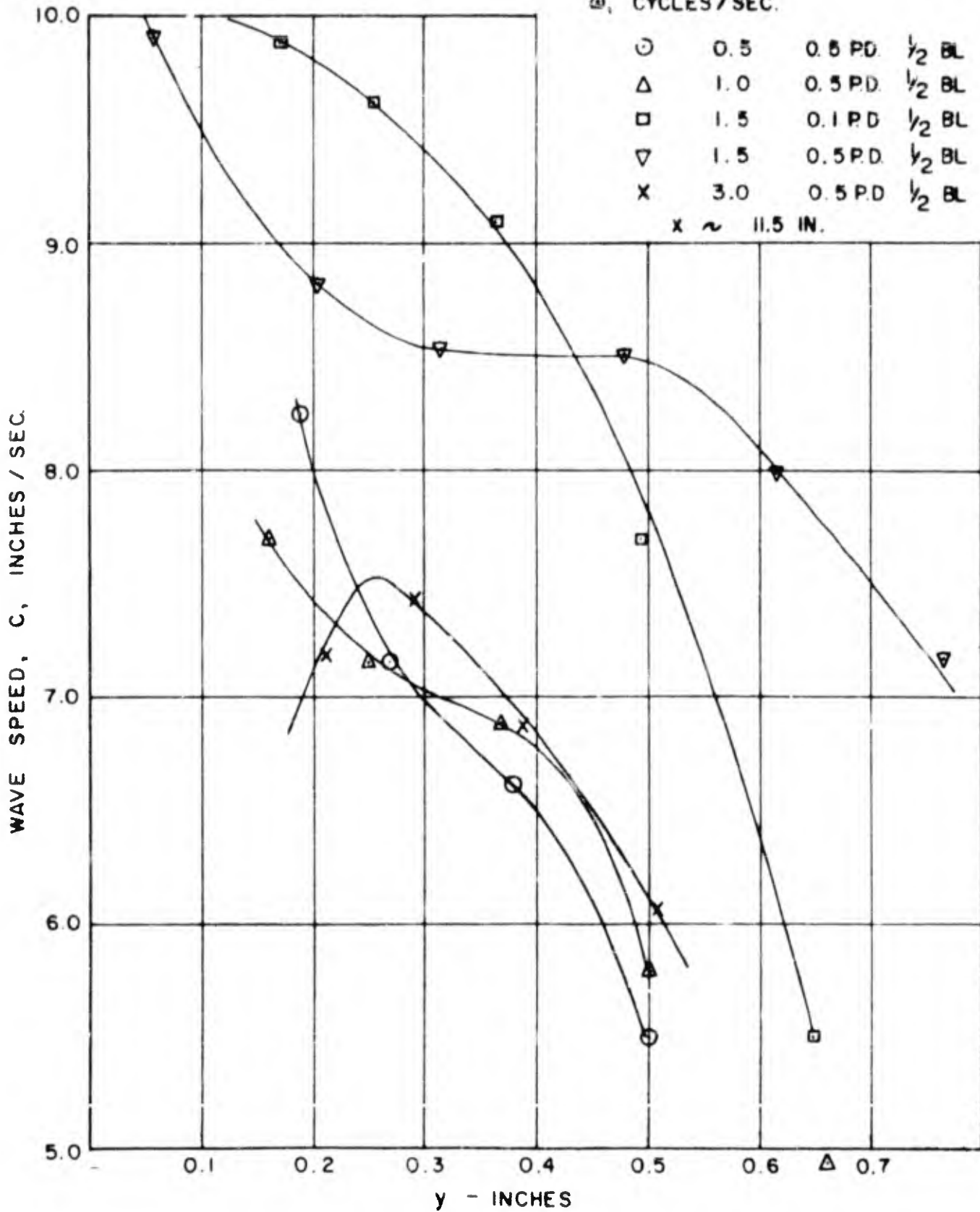


Fig. 25 Wave Speed vs. Distance from Wall

GAE-59-11

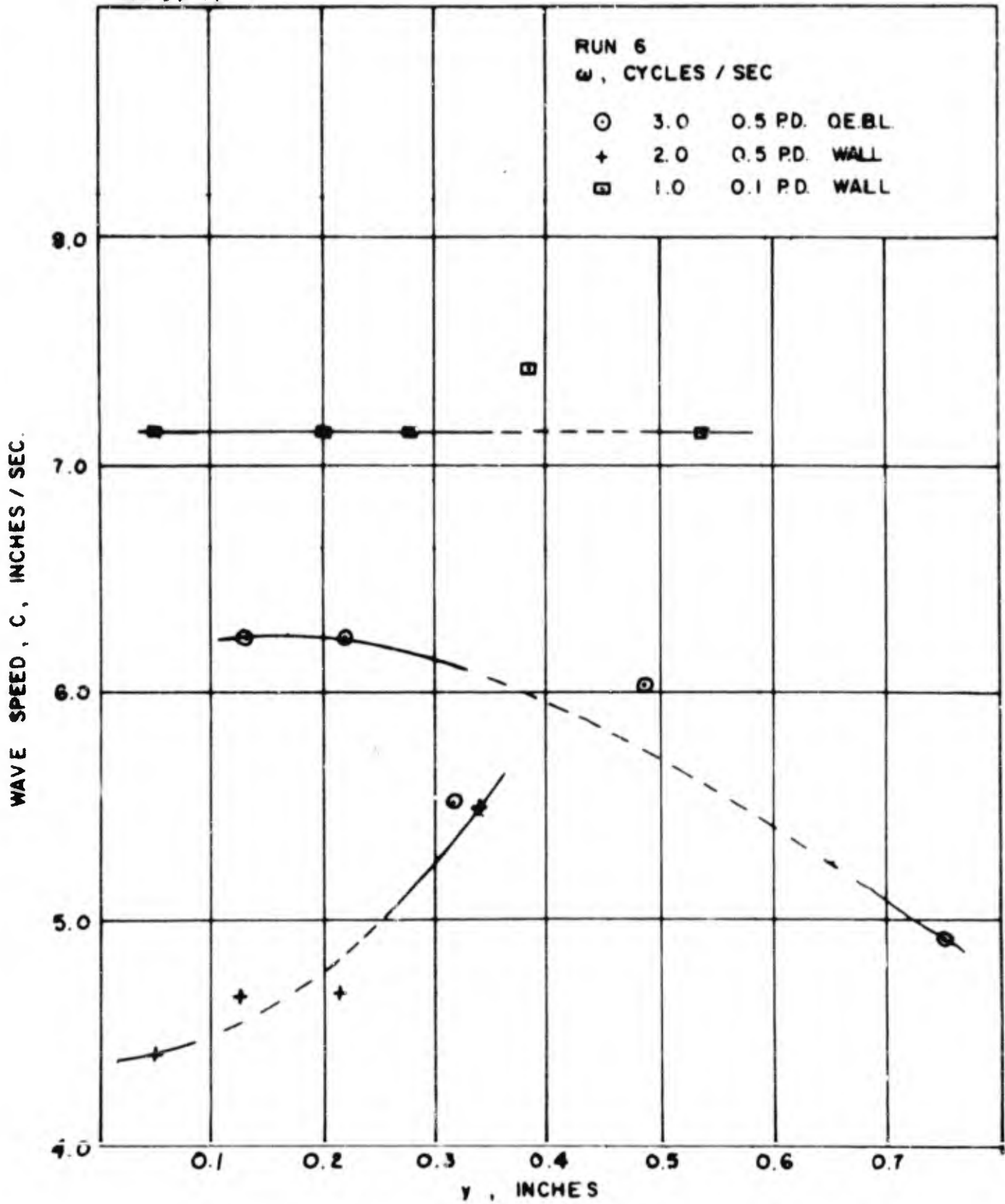


Fig. 26 Wave Speed vs. Distance from Wall

GAE-59-11

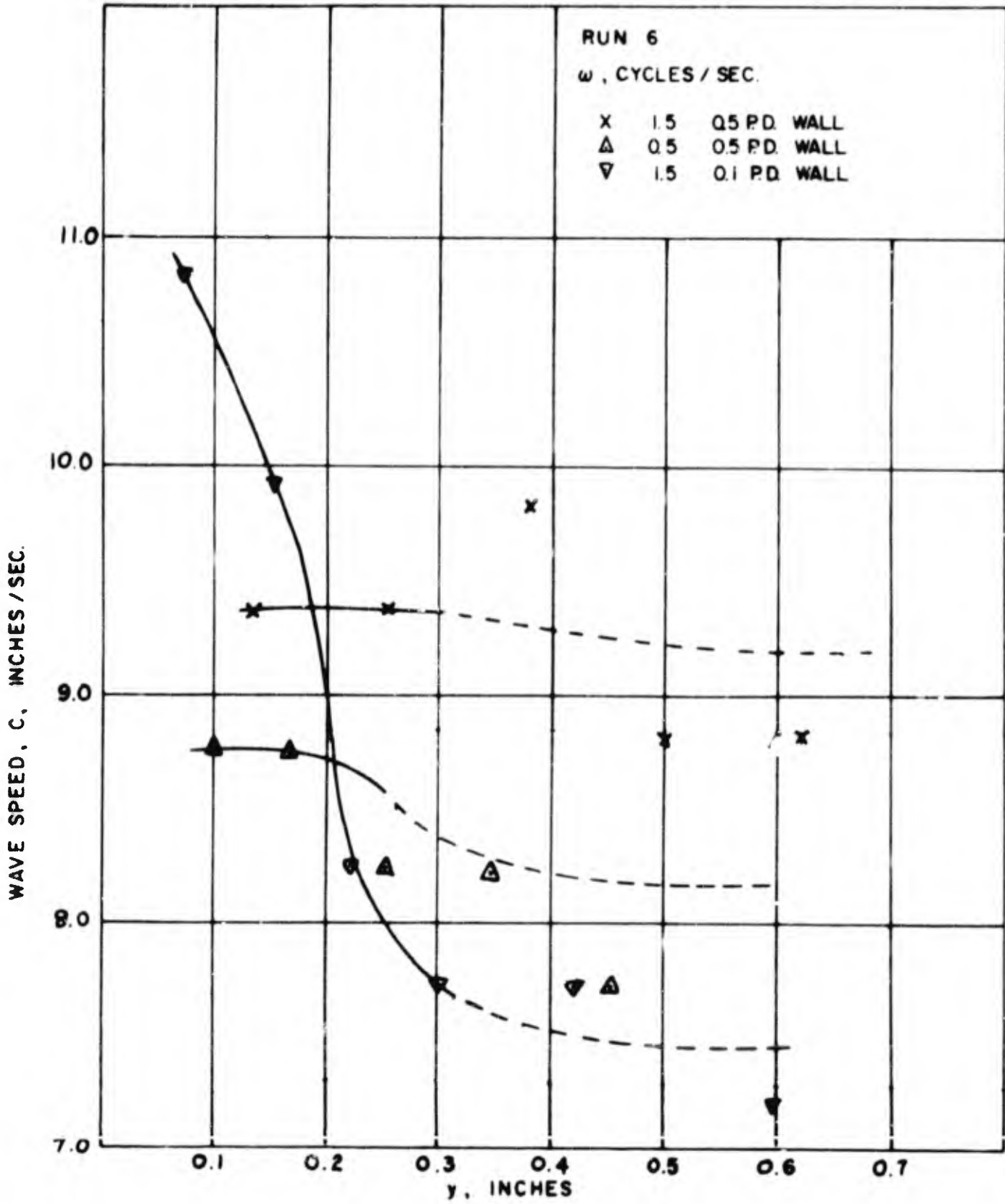


Fig. 27 Wave Speed vs. Distance from Wall

GAE-59-11

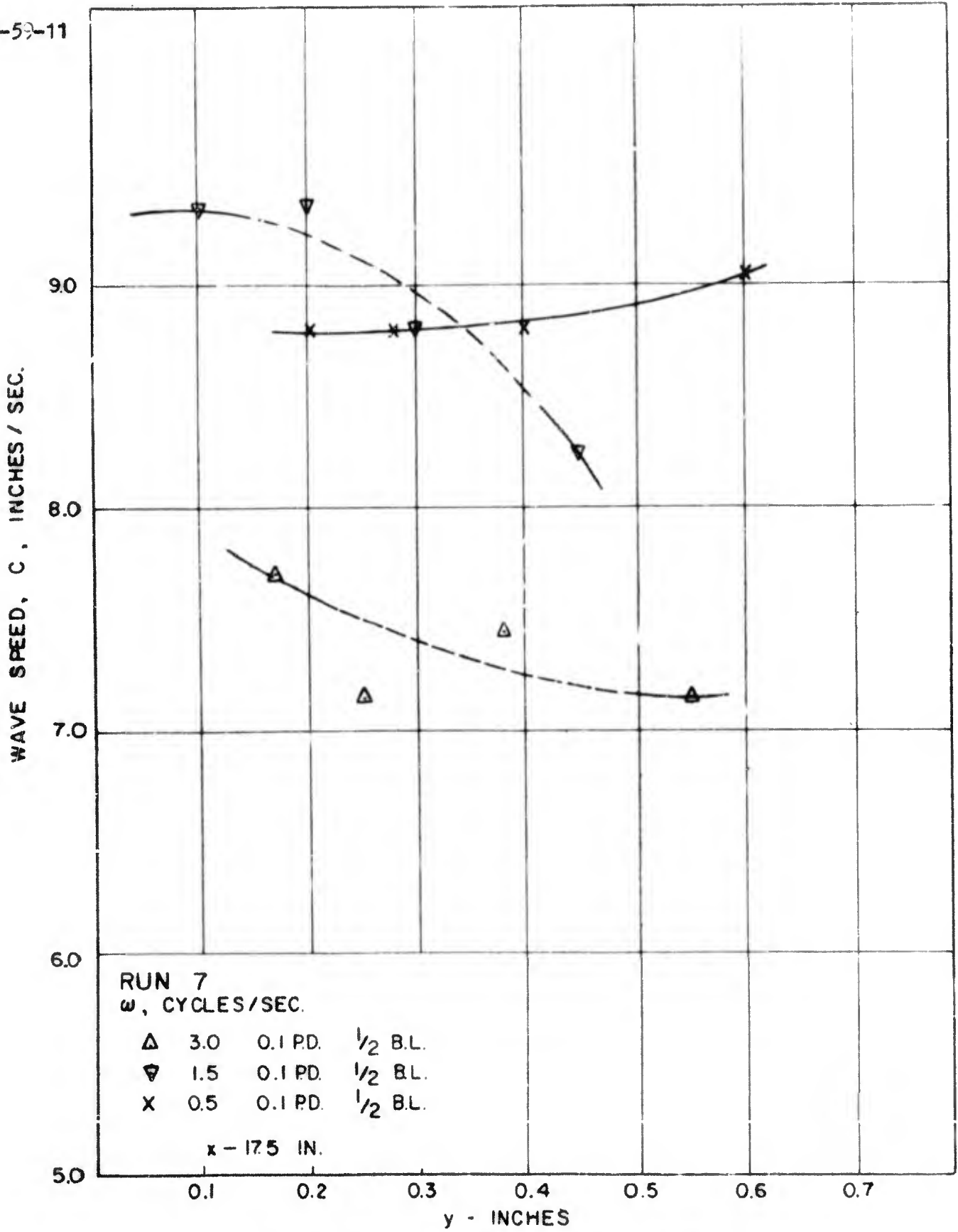


Fig. 28 Wave Speed vs. Distance from Wall

GAT-59-11

**RUN 8**  
 $\omega$ , CYCLES / SEC.

x - 17.5 IN.	{	○	0.5	0.1 PD	NEXT TO WALL
		△	1.0	0.1 PD	NEXT TO WALL
		□	1.5	0.1 PD.	NEXT TO WALL
x - 24 IN.	{	▽	3.0	0.1 PD	OE. of BL
		x	0.5	0.1 PD	OE. of BL

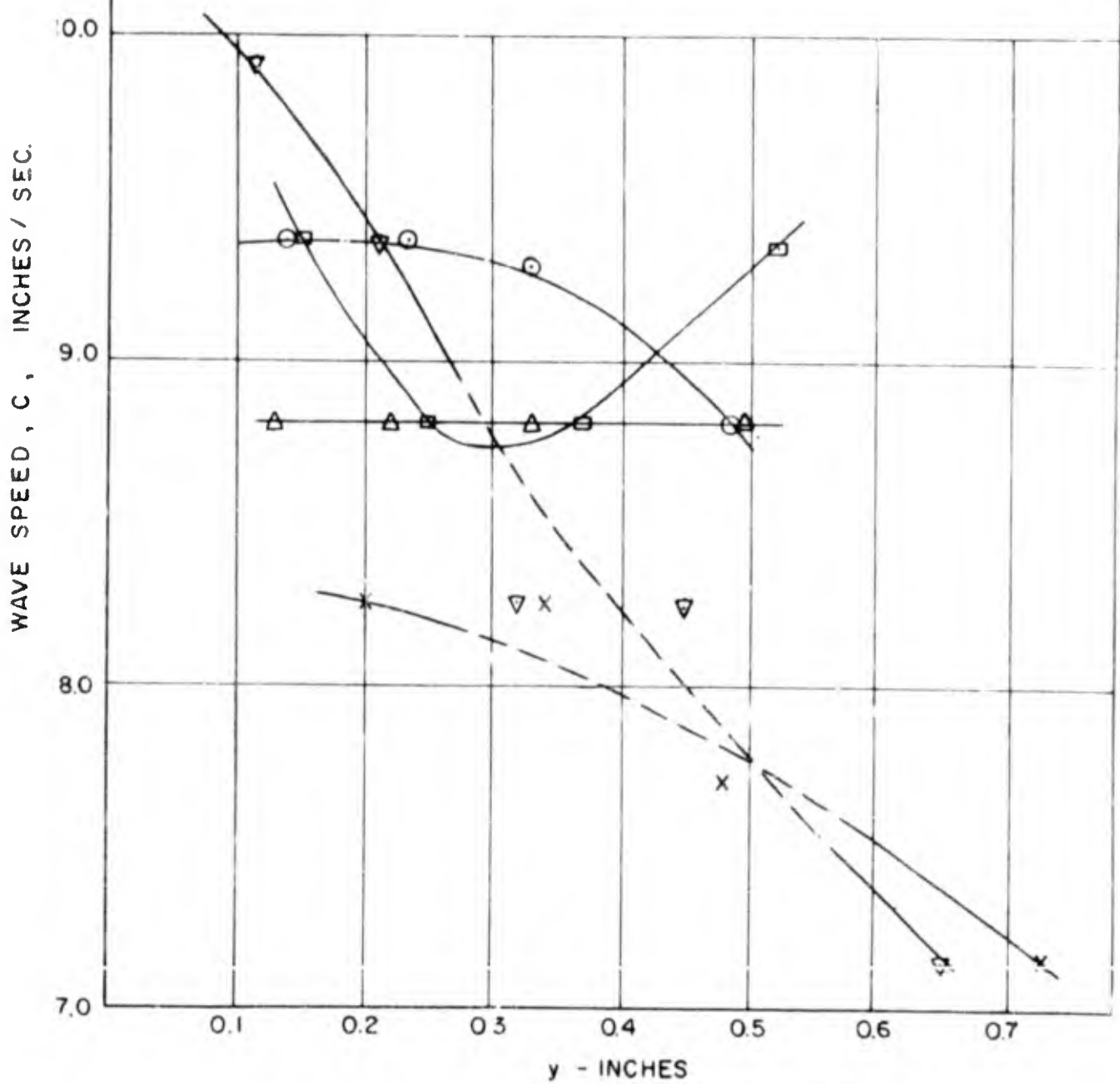


Fig. 29 Wave Speed vs. Distance from Wall

GAB-59-11

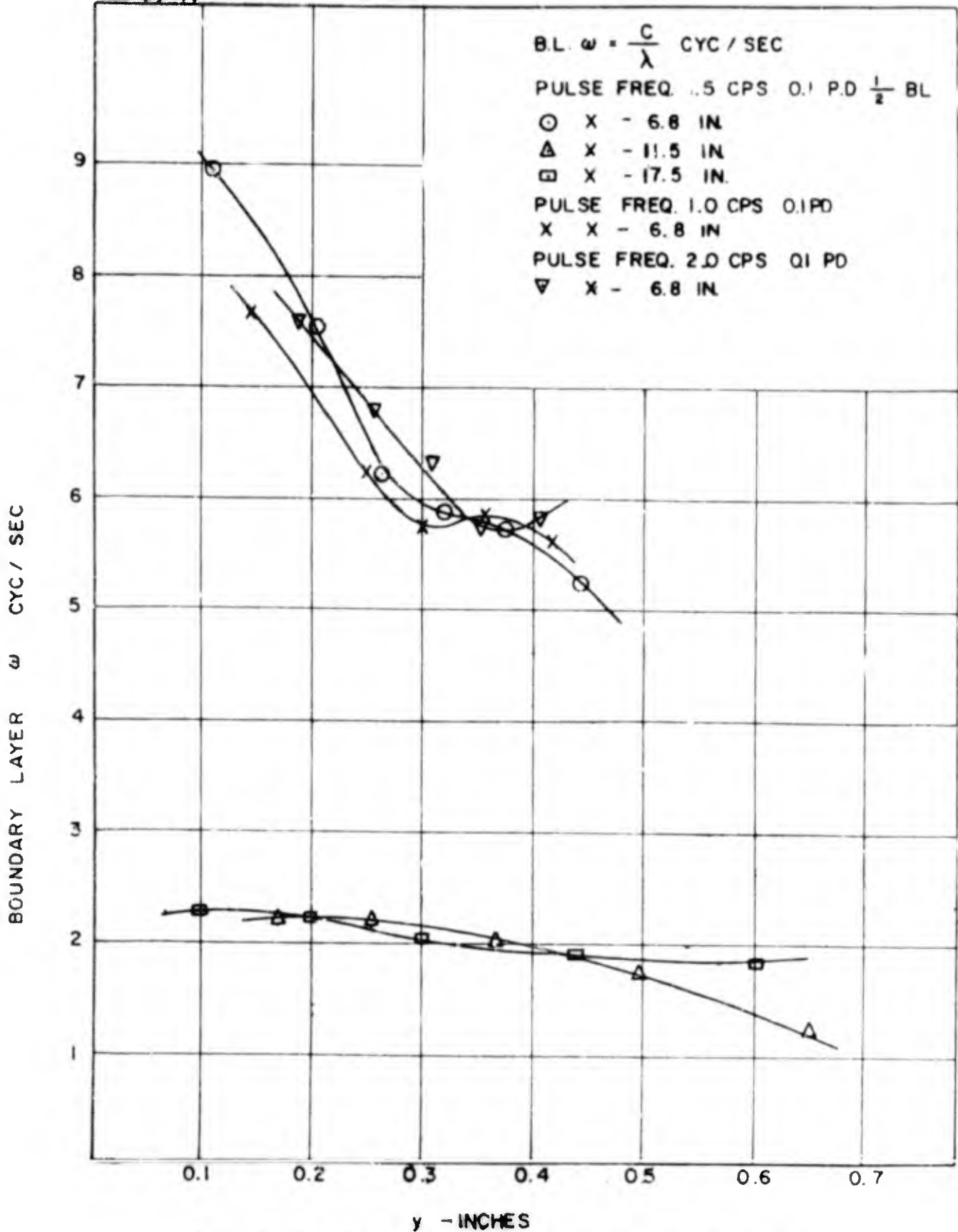


Fig. 30 Boundary Layer Wave Frequency vs. Distance from Wall

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Appendix B  
Tables

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Table 1  
Data Sheet

Pulse Frequency cps	Pulse Timer Setting	Pulse Length Sec.	Energy per Pulse BTU x 10 <sup>-4</sup>
0.5	0.1	0.0222	0.20
1.0	0.1	0.0834	0.76
1.0	0.5	0.2890	2.63
1.5	0.5	0.2780	2.53
1.5	0.1	0.0778	0.71
2.0	0.1	0.0682	0.62
2.0	0.5	0.2510	2.28
3.0	0.5	0.1781	1.62
3.0	0.1	0.0667	0.61

Run No.	Wave Position x in	Gr <sub>x</sub> x 10 <sup>7</sup>	t <sub>0</sub> of	t <sub>w</sub> of
3	6	1.34	76	119
4	6	1.34	79	121
5	6	1.34	75	118
6	6 12	1.34 6.5	75 75	118 118
7	12	6.5	75	118
8	12 18	6.5 22.8	79 79	120 120
9	18 24	22.8 55.5	79 79	120 120

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Appendix C  
Sample Calculations

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Sample Calculations

Grashof Number

$$Gr_x^d = \frac{g \beta \theta x^3}{\nu^2}$$

$$\beta = \frac{1}{T_0} = \frac{1}{539} = \frac{1}{R}$$

$$\theta = (t_w - t_0) = 40 \text{ } ^\circ\text{F}$$

$$x = 11.5 \text{ in} = .96 \text{ ft}$$

$$\nu = 18 \times 10^{-5} \text{ ft}^2/\text{sec} \text{ (Ref 2: 504)}$$

$$Gr_x^d = \frac{32.2 \cdot \frac{40}{539} \cdot (.96)^3}{(18)^2 \times 10^{-10}} = 6.5 \times 10^7$$

Pulse Energy Input

Trace on screen of oscilloscope gives actual pulse duration and voltage drop:

(a) at 1.0 cps, 0.1 P.D.; actual pulse duration is shown to be .0834 sec.; Voltage drop = 0.01 volt

$$(b) I = \frac{E}{R} = \frac{0.01}{.25} = .04 \text{ amps}$$

$$(c) \text{ Energy} = E I T \text{ (where } E \text{ is constant } 24 \text{ volts used during test)}$$
$$= 24 (.04) (.0834) = .081 \text{ watt-sec.}$$
$$= .081 \times .948 \times 10^{-3} = .76 \times 10^{-4} \text{ BTU}$$

Boundary Layer Thickness

(Ref 2: 314)

$$\frac{\delta}{x} = 3.93 Pr^{-\frac{1}{4}} (0.952 + Pr)^{\frac{1}{4}} (Gr_x)^{-\frac{1}{4}}$$

at  $x = 6 \text{ in}$

$$\delta = 6 (3.93) (.705)^{-\frac{1}{4}} (.952 + .705)^{\frac{1}{4}} (1.34 \times 10^7)^{-\frac{1}{4}} = .595 \text{ in}$$

GAE-59-11

Vita

Harold Eugene Gartrell [REDACTED]

[REDACTED] He entered the United States Military Academy in July, 1949. In June, 1953, he was graduated from the Academy and received a Bachelor of Science Degree in Military Science. He was also granted a permanent commission in the United States Air Force and assigned to pilot training at Goodfellow Air Force Base, Texas.

Captain Gartrell's military assignment prior to entering the Air Force Institute of Technology was flight instructing at Reese Air Force Base, Texas.

[REDACTED]

This thesis was typed by Miss Addie Sproles

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