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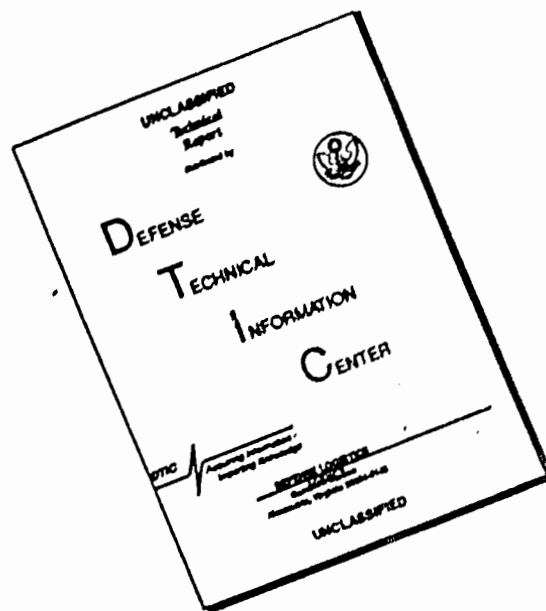
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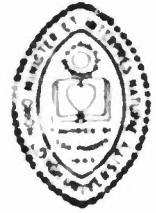
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Welded Continuous Frames and Their Components  
INTERIM REPORT NO. 43

# LITERATURE SURVEY

ON LATERAL INSTABILITY AND  
LATERAL BRACING REQUIREMENTS

By

George C. Lee



Fritz Engineering Laboratory Report No. 205H.2

October, 1959

Welded Continuous Frames and Their Components

Interim Report No. 43  
(not for publication)

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LATERAL INSTABILITY AND LATERAL BRACING REQUIREMENTS

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George C. Lee

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## 1. INTRODUCTION

In a plastically designed structure, a member must undergo large inelastic rotations within the region of a "plastic hinge" so that moments may be redistributed to develop the full strength of the structure. To achieve these large rotations, provisions must be made to prevent the member from failing prematurely due to various types of instability. One such important type of instability is lateral-torsional buckling.

Lateral-torsional buckling may be prevented by suitable lateral bracing. The ultimate object of the study of lateral-buckling is to determine the required stiffness of this bracing and to find the most economical method of design for the braces.

The various phenomena related to beams which fail by lateral-torsional buckling are illustrated in Fig. 1. A simply supported beam\* of narrow rectangular cross section is loaded about its strong axis by a concentrated load  $P$ . If it is assumed that the member contains no initial imperfection, the load vs. lateral deflection relationship will follow the

.....  
\*A simply supported end is defined as the following: The bending moments in the two principal directions are zero; twisting about the longitudinal axis is prevented. This definition is used throughout.

vertical axis from point "O" to point "A". At this stage the member has neither lateral deflection nor twisting; however it may be yielded in certain portions due to transverse bending. Point "A" is known as the point of bifurcation of equilibrium. Just as in an axially loaded straight column, such a bifurcation point is considered to be a criterion of instability. The member may buckle at point A or at point B depending on whether fiber unloading is prohibited (tangent modulus concept) or whether it is permitted (reduced modulus concept). These two loads represent the lower and the upper limits of the carrying capacity of the member.<sup>(69)</sup> The true maximum load\* that the member can support (point C) lies somewhere between these two limits.

The tangent modulus and reduced modulus solutions are so-called characteristic value type solutions. ("Eigen value" type solution). Since by the very nature of these two solutions the deflections at the bifurcation point are indeterminate, they are not suitable for the determination of the bracing forces. Point C in Fig. 1 is the ultimate strength of the beam; with regard to lateral buckling in this case lateral forces and twisting moments develop due to lateral and

~~with regard to lateral buckling, the ultimate strength is a post-buckling problem.~~

torsional deformations of the member. The deflection at the ends of a member or at the lateral supports are dependent on the stiffness of the bracing. It is believed that an analytical solution to the bracing stiffness may be achieved by considering the lateral and the torsional deformations at the supports.

> This literature survey is limited to a general survey of the existing solutions to the problem of lateral buckling of metal structures, with the emphasis on the following three points.

- (1) An extensive list of references including those that may be helpful for the lateral bracing problem.
  - (2) Description of several important papers particularly concerned with inelastic solutions.
  - (3) A general historical sketch of the problem of lateral instability. Suggestions are made for possible ways of extending knowledge in the field of plastic design.
- 

## 2. A HISTORICAL SKETCH OF LATERAL BUCKLING THEORY

Lateral stability is an important criterion in the design of metal structures. Prandtl<sup>(1)</sup> and Michell<sup>(2)</sup> simultaneously published a rigorous theoretical analysis of lateral stability in 1899. Since then research in this field has been performed by many investigators. Notably, the basic general differential equations were derived by Timoshenko (Refs. 3, 7, 35), Goodier (Refs. 22, 25, 28, 64), Wagner (Refs. 11, 12), and Kappus (Ref. 15). Johnston (Refs. 9, 21, 27), Horne (Refs. 41, 47, 77, 92, 95), Winter (Refs. 29, 33, 38, 42, 83, 85, 101), Salvadori (Refs. 62, 66, 82, 87), Nylander (Refs. 31, 43, 94), Pettersson (Refs. 44, 67), Flint (Refs. 46, 53, 54, 65, 74, 89), Hill (Refs. 13, 20, 26, 32, 50, 56, 58, 76, 112) and Clark (Refs. 56, 58, 86, 99, 106, 112) have extensively developed the theory for many loading cases, both theoretically and experimentally. Thürlimann has clearly presented the physical picture of the relationship between load and deformation as well as mathematical derivations of the problem of lateral-torsional buckling in Ref. 114.<sup>5</sup> Bleich has summarized and presented, in detail, a general introduction to this problem in his book.<sup>(69)</sup>

The above mentioned authors, as well as other investigators have mostly restricted themselves to an elastic material.

In other words, the proportional limit of a stress-strain relationship is considered to be the upper limit for critical stress.\* This assumption permits elegant mathematical solutions, but they are limited to such structures where inelastic behavior of the material would not occur. When yielding of the material commences, the stress-strain relationship no longer obeys Hooke's Law, and an exact solution is usually impossible.

In recent years, plastic design of structures has been extensively developed. For certain types of structures, great saving in material and design time can be achieved by plastic design. Since plastic design requires the structure to undergo rather severe inelastic deformations before failing occurs, the problem of lateral stability must be investigated rather critically for these structures.

Inelastic lateral buckling solutions have been attempted by only a few investigators. Neal<sup>(45)</sup> was the first to present an inelastic solution for a beam of rectangular cross section subjected to uniform moment. This material was a structural-grade steel possessing an ideal elastic-plastic stress-strain curve. Neal discussed the effect of yielding on the lateral ~~\*This assumption is based on the hypothesis that there are no~~ residual stresses or initial deformations contained in the structure.

buckling rigidity and initial torsional rigidity of the yielded beam. His solution was obtained by means of a step-by-step calculation procedure. Horne<sup>(47,92)</sup> extended Neal's work to the case of I-beams, indicated an outline of a solution, and presented charts for critical buckling lengths of two particular loading cases. Wittrick<sup>(68)</sup> has solved the problem of a beam of rectangular cross section made out of material having a monotonically increasing stress-strain diagram. The difference between Wittrick's solution and Neal's solution is that the former has used the tangent modulus concept, while the latter has adopted the reduced modulus idea. Agnababian and Popov<sup>(63)</sup>, as well as Barrett<sup>(71)</sup>, have studied inelastic biaxial bending of beams with rectangular cross-section; Holland, Egger, Mayerjak and Munz<sup>(80)</sup> have also studied inelastic general bending of I-sections neglecting the effect of warping. The latter three reports, however, do not concern themselves with lateral buckling, but rather with studies of inelastic moment-curvature relationships. In 1956, White<sup>(91)</sup> obtained an inelastic solution for I-sections subjected to a non-uniform moment gradient using the finite difference method. He assumed that the material is either elastic or strain-hardened. The value of White's work is that it forms a basis for the determination

of the distance between lateral braces in the vicinity of a plastic hinge. White's solution was further modified on the basis of experimental results, and a simple design procedure for the lateral support spacing was presented by Kusuda, Sarubbi and Thürlimann<sup>(106)</sup>. This recommendation has been adopted as a design rule by the AISC.<sup>(108)</sup> In this study<sup>(106)</sup> the restraining influence of adjacent members on the critical span was analyzed, treating the system as a continuous beam, and the necessary correction factor for the restraining influence was proposed. Experimental results in this study indicated that White's solution was conservative. In a recent dissertation, Galambos<sup>(109)</sup> has obtained an inelastic lateral-torsional buckling solution of WF beam-columns. He considered the effect of residual stresses and used the tangent modulus concept. Critical moment versus length curves for important WF shapes were obtained by numerical calculations and the theory showed very close agreement with test results.

### 3. EVALUATION OF THE PRESENT STATE OF KNOWLEDGE AND FUTURE RESEARCH TRENDS

The preceeding section represents a brief history of the problem of lateral instability. It should be noted that all the inelastic solutions mentioned above were "Eigenvalue" solutions. In other words, solutions correspond to the first lateral and/or torsional movement of the member (point A or B in Fig. 1). Therefore the bracing length is only determined in terms of loading. It cannot yield any information about how to proportion the bracing. In almost every known solution the bracing points are assumed to be either simple supports or fixed-end supports. Studies by Zuk<sup>(88)</sup> indicate that in elastic analysis the magnitude of bracing force is negligibly small, and that the present day elastic design value of the bracing force as 2% of the applied load is sufficient. It is believed that according to the inelastic bifurcation analysis the bracing force can not be determined because of the nature of this method of approach.

At the ultimate load large deflections develop and the magnitude of the bracing force and the required stiffness necessary to hold a member in its laterally undeformed position up to this stage is of practical importance in plastic design.

It is expected that the required bracing force would be no longer negligibly small. Research at Fritz Laboratory on plastically designed gable frames<sup>(114)</sup> and welded corner connections<sup>(113)</sup> show that adequate lateral supports are definitely necessary for a structure to undergo large rotations and to develop its ultimate strength.

Future research about inelastic lateral buckling should be carried out along the following path:

- (1) To obtain "Eigenvalue" solutions for as-rolled WF sections based on the reduced modulus approach in order to compare with the tangent modulus solution.<sup>(91)</sup> The former should yield an upper bound as compared to the lower bound solution obtained by White. This first step should give a proper guidance to the ultimate strength solution.
- (2) To obtain the maximum carrying capacity of a member with regard to lateral-torsional buckling (point C in Fig. 1).
- (3) Appropriate bracing systems should be analyzed in order to determine the required bracing stiffness and bracing spacing to prevent lateral buckling at the fully plastic ultimate load of the member.

Table 1 gives the present status of research and the necessary future research for the complete solution of the problem. Although in Appendix B known theoretical solutions are summarized and tabulated for reference, it should be noted that those solutions offer very little help toward the determination of the ultimate strength, neither in method of approach nor in their solution. Therefore to achieve the final purpose of obtaining the appropriate bracing requirement, the ultimate strength solution should be first accomplished.

#### 4. REFERENCES

A list of references on lateral-torsional buckling and lateral bracing requirements are given in Appendix C. They are listed according to the sequence of their date of publication, so that a general idea of the history of development of the particular phase of the science of mechanics may be seen. The primary interest of this study is limited to the lateral buckling of beams, (known solutions are tabulated in Appendix B). However, owing to the close similarity of the problem with a beam-column, it was decided to include all solutions including axial force in this list of references.

A page of author's index is given immediately after the list of reference.

## 5. OUTLINES OF THE MOST IMPORTANT REFERENCES

In order to avoid summarizing all the pertinent research papers, the tables in Appendix B are prepared. In these tables all known theoretical solutions are listed. Items included are:

- 1) Beam and loading
- 2) Cross sectional shape and axis of loading
- 3) Investigators and reference numbers
- 4) Remarks

A few selected references are described in the present section, since it is believed that they may furnish some suggestions toward the solutions of the problem of the bracing requirements

THE LATERAL INSTABILITY OF YIELDED MILD STEEL  
BEAMS OF RECTANGULAR CROSS SECTION

This work was the first investigation on lateral buckling in the inelastic range. The author's method of approach led other investigators later to solve more general problems in this field. (For example, Horne,<sup>(47)</sup> and Galambos<sup>(109)</sup>.)

The method of solution is a typical "Eigenvalue" approach, in which the critical moment (applied about the strong axis) is expressed in terms of the span length, the weak axis bending stiffness, and the torsional stiffness at the instant when lateral-torsional buckling begins. At this stage the beam is partially yielded and is deflected only in the strong direction under the applied load. The author showed that the bending stiffness in the weak direction decreases as yielding of the cross section increases. He also proved that the torsional rigidity remains constant before lateral buckling, even though some portions of the cross section are yielded. This theory was later adopted by other investigations for solving more extensive problems. The analysis was performed for rectangular and circular cross sections. (the warping torsional effect is negligible for rectangular cross sections) Solutions were

obtained by a step-by-step calculation procedure, and the theory was checked experimentally. Good agreement was observed between theory and experiment.

CRITICAL LOADING CONDITIONS IN ENGINEERING STRUCTURES(CHAPTER 8: THE LATERAL INSTABILITY OF I-BEAMS STRESSED BEYOND  
THE ELASTIC LIMIT

This study is an extension of Neal's paper to I-shaped beams, bent about the strong axis. The author was the first who attempted an inelastic analysis for WF section taking into account the effect of warping torsion.

The paper contains the following:

- (1) Horne determined the lateral flexural rigidity and the differential flange rigidity of a partially yielded I-section. The torsional rigidity is shown to be unaffected by yielding.
- (2) Two loading cases are solved: A simply supported beam under uniform moment, and a simply supported beam under a concentrated load acting at mid-span.
- (3) Horne discusses the effect of end restraints on the critical buckling length of a partially fixed beam. Charts are presented for various end fixities.

Horne's solution is modified by practical considerations, in Chapter 12 of Ref. 92, where the limiting critical lengths of standard British rolled I-beams are presented as a basis for design.

UNSYMMETRICAL BENDING OF RECTANGULAR BEAMS BEYOND  
THE ELASTIC LIMIT

This paper is not concerned with lateral instability. It deals with the general bending theory in which the inelastic moment-curvature relationship for a rectangular cross section is determined both theoretically and experimentally.

General expressions are obtained for the inter-relationship between the bending moment about the x-axis, the bending moment about the y-axis, and the inclination of the neutral axis for a rectangular beam subjected to pure bending as yielding progresses from zero to full plasticity. Equations are given for the special cases of the elastic limit and for full plasticity.

It is shown that the neutral axis undergoes considerable rotation between its position at the start of yielding to its position when the beam is fully plastic. In some cases this rotation of the neutral axis is more than six degrees. The authors assumed that the direction of the applied bending moment remains the same.

A number of experiments have been conducted and the results show reasonable correlation with the theory.

It is also shown that the shape factor for oblique loading varies from 1.5 to 2.0, thus indicating that for rectangular sections a greater economy is obtained by the use of the plastic method of design.

LATERAL INSTABILITY OF RECTANGULAR BEAMS OF STRAIN-  
HARDENING MATERIAL UNDER UNIFORM BENDING

This paper is an extension of Neal's work<sup>(45)</sup> on the lateral stability of beams of rectangular cross section in which part of the section yields before lateral buckling occurs.

Two main differences between this and Neal's work are as follows:

(1) Neal ignores strain-hardening of the material, while Wittrick extends Neal's study to materials in which strain-hardening occurs. He adopts a general shape of a monotonically increasing stress-strain curve and generalizes the problem to cover materials other than mild steel.

(2) Neal's solution is based on the reduced modulus concept and is checked experimentally with annealed specimens. Wittrick used the tangent modulus concept in his solution. He did not conduct any experiments.

The main contributions of this paper are:

(1) It is shown that it makes very little difference if the upper yield point is neglected and if only the lower yield point of the steel is used in the calculations.

(2) Neal has shown that for a yielded mild steel beam under the action of a uniform bending moment in the strong direction the torsional rigidity remains at its elastic value irrespective of the extent of yielding. This is also verified experimentally. Wittrick extended this further and showed that Neal's theory applies equally well to members of strain-hardening material.

(3) The following statement is quoted:

"The case of beams of rectangular section is not of great practical interest except insofar as it is a guide to the expected behavior of other sections. The problem of determining the lateral buckling loads for open sections, such as channels and I-beams, is complicated by the warping of the cross sections under torsion and the consequent increase in torsional rigidity when the torsion is non-uniform (see, for example, Timoshenko<sup>(7)</sup>)."

STATIC AND DYNAMIC LOAD-DEFLECTION TESTS  
OF STEEL STRUCTURES

This paper contains four phases of a study, namely:

- (1) Static Tests to Failure of Steel Beam-Columns
- (2) Model Studies of Frames Subjected to Static Lateral Loads
- (3) Static Oblique Loading Tests of Steel Beam-Columns
- (4) Dynamic Response of Beams

The third phase deals with the general bending theory of partially yielded members and is of interest for the problem of lateral instability. It presents a method of analysis for the determination of the load-deflection and moment-curvature relationships of rolled I or WF sections in the inelastic range under oblique load. The very important effect of cross bending (warping torsion) is neglected and therefore this method, as compared by the authors to experiments, is not able to correctly predict moment-curvature relationships under oblique loading.

LATERAL BUCKLING OF ROLLED STEEL BEAMS

This is a report of an experimental investigation of the lateral buckling of rolled steel beams. Thirty-three simply supported beams (depth from ten to eighteen inches) and twenty-one ten inch beams with two types of bolted semi-rigid end connections were tested. The purpose of this work is to check experimentally the theories and the design rules used in steel industry. Recommendations by de Vries<sup>(37)</sup> and Winter<sup>(38)</sup> are verified, and comparisons have been made between the test results and the AISC design formula as well as the AASHO and AREA design formulas. Factors of safety are discussed, and reasonably modified design formulas are presented.

The end conditions of these tests are not clearly defined in this report. It is known that the support conditions and loading conditions have a marked influence on the buckling load, and it is very difficult to design a fixture which simulates the conditions that are used in the theoretical analysis, particularly in prototype experiments.

THE LATERAL-TORSIONAL BUCKLING OF YIELDED STRUCTURAL  
MEMBERS

This paper is the only one that presents a general inelastic solution for WF beams. The author assumes that the material is either elastic or strain-hardened and adopts the tangent modulus concept in his solution. Under a non-uniform moment gradient, strain-hardening of material will commence from one end of the member; hence the resistance of the member in the strain-hardening zone is different from that in the elastic region, the stiffness of each portion of these two regions are governed by the elastic and strain-hardening moduli, respectively. At the juncture of these two portions of the beam, conditions of continuity are used. Solutions are obtained by finite difference procedure with the calculations performed by a digital computer. For the basic case of a simply supported beam of strain-hardening material subjected to a uniform moment with the twisting resisted by warping torsion only, it was found that the slenderness ratio of the basic critical buckling length is 18. This basic case was then modified for the effect of moment gradient, the extent of strain-hardening, St. Venant's torsional resistance and conditions of end fixity. White also presents a trial-and-

error procedure for design of lateral bracing.

Simplifications of White's design procedure on an experimental basis were made in Ref. 106. In this reference it was suggested that the critical buckling length is

$$\begin{aligned} \frac{L_{cr}}{r_y} &\leq 30 && \text{for } 1.0 \geq \rho \geq 0.6 \\ &\leq 48-30\rho && \text{for } 0.6 \geq \rho \geq -1.0 \end{aligned}$$

where  $\rho = M/M_p$  is the end moment ratio. A further modification incorporating practical considerations was made in Refs. 108 and 110. It was shown that the critical buckling length is

$$\begin{aligned} \frac{L_{cr}}{r_y} &\leq 35 && \text{for } \rho > 0.625 \\ &\leq 60-40\rho && \text{for } \rho < 0.625 \end{aligned}$$

This last equation has been recommended for design in structural steel

INELASTIC LATERAL-TORSIONAL BUCKLING OF ECCENTRICALLY  
LOADED WIDE-FLANGE COLUMNS

This investigation is the first that is concerned with inelastic lateral-torsional buckling, taking into account the effect of residual stresses in as-rolled members. Curves relating the coefficients of the lateral-torsional buckling equation and any combination of axial force and bending moment are presented for partially yielded WF sections. Interaction curves between the axial force, the end bending moment, and the slenderness ratio are also calculated for four column shapes. (the 8WF31, the 14WF142, the 14WF246, and the 27WF94 shape)

It is found that the presence of residual stresses may reduce considerably the lateral buckling strength of an eccentrically loaded column. For usual column lengths, lateral-torsional buckling takes place in the elastic range. It is shown also that the lateral-torsional buckling strength of columns varies considerably with member size. The prediction curves showed very good agreement with existing experimental results. The author also checked White's solution for the case of a beam of uniform moment with the axial force of the column equal to zero.

## 6. CONCLUSIONS

As mentioned in Section 3 of this report, almost all existing solutions offer very little information toward the determination of ultimate strength. Nevertheless, this survey presents a brief history of the problem, the present situation, and possible future trends of research about lateral instability, as well as a list of references and a tabulation of all theoretical solutions. It is hoped that this report may offer a reasonable, complete picture of the problem of lateral buckling of metal structural members.

It is felt that, in attempting an ultimate strength solution, a method of approach, different from those already used, would be necessary. Research at Lehigh University is currently continued along this line, both theoretically and experimentally.

## 7. ACKNOWLEDGEMENT

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The author wishes to express his sincere appreciation to Dr. R. L. Ketter and Dr. T. V. Galambos, former and current supervisors of the research project, for their valuable suggestions and criticisms.

The manuscript was typed by Miss Grace Mann; her assistance is greatly acknowledged.

8 APPENDIX A

FIGURES AND TABLES

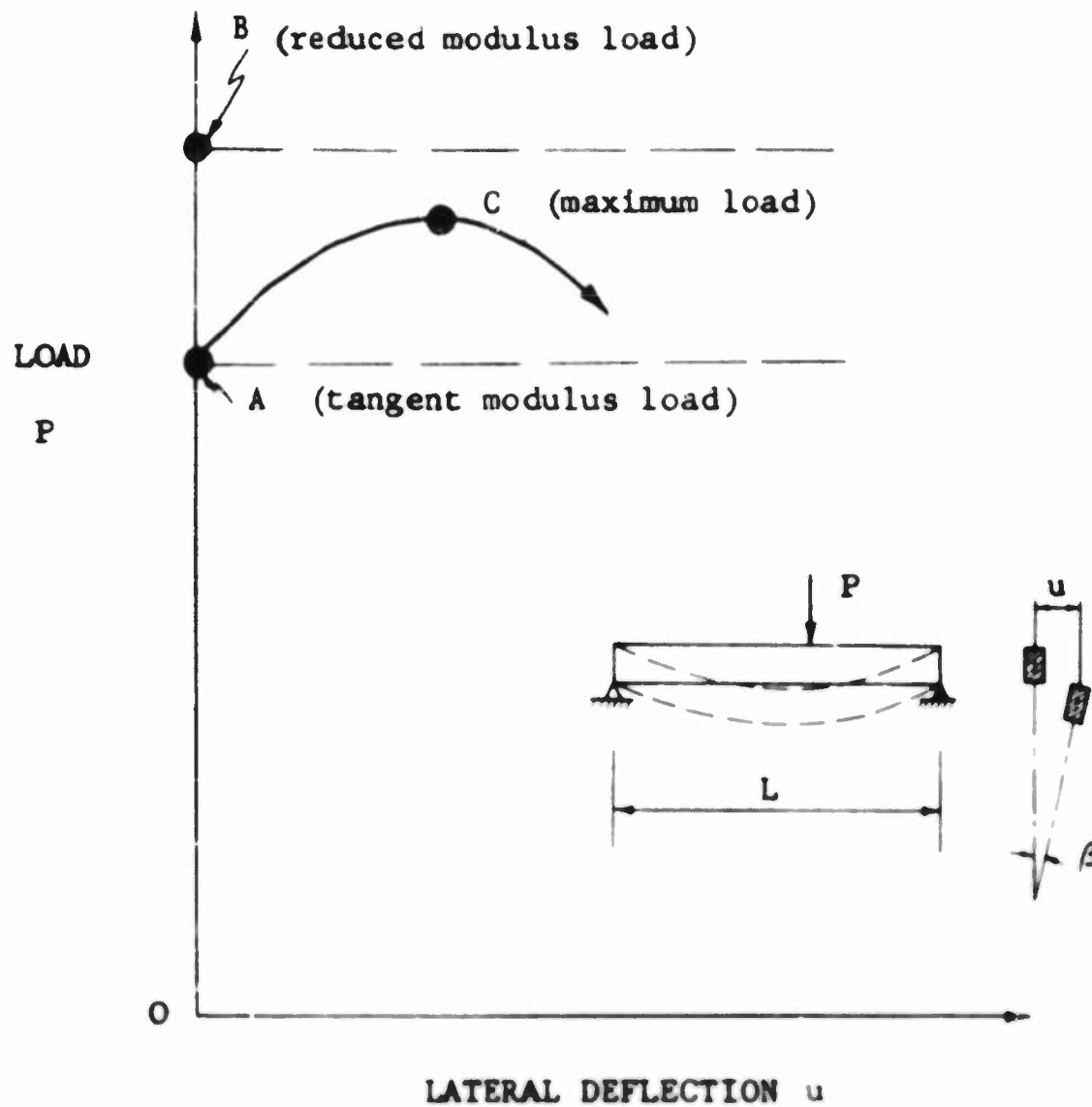


Fig. 1 Typical Load vs Lateral Deflection  
Relationship for a Beam Loaded in the Strong Direction.

| Structure   | Bifurcation Solution                                                     |                                              | Ultimate Strength Solution (Point C in Fig. 1) | Lateral Bracing Requirement to Ultimate Strength |
|-------------|--------------------------------------------------------------------------|----------------------------------------------|------------------------------------------------|--------------------------------------------------|
|             | Tangent Modulus Solution (Point A in Fig. 1)                             | Reduced Modulus Solution (Point B in Fig. 1) |                                                |                                                  |
| Beam        | White (91)<br>(General solution for elastic-strain-hardened mat'l.)      | Horne (47)<br>(Two special loading cases.)   | ?                                              | ?                                                |
| Beam-Column | Galambos (109)<br>(Solution not general, but covers important WF shapes) | ?                                            | ?                                              | ?                                                |

Table 1      Present Situation of the Inelastic  
Solution of WF Beams and Beam-Columns Available in  
Plastic Design.

## 9. APPENDIX B

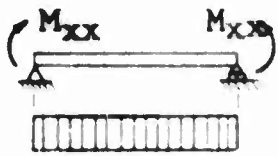
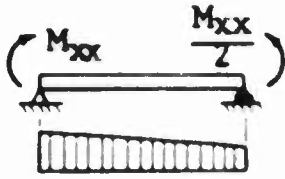
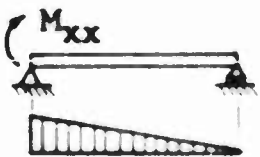
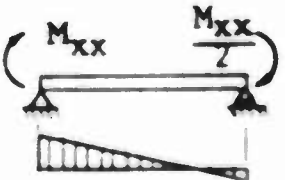
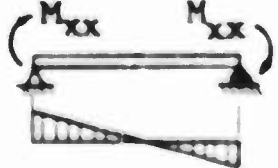
Table of Known Theoretical Solutions

## Nomenclature

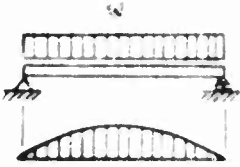
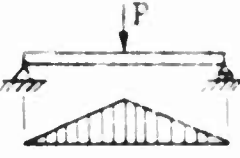
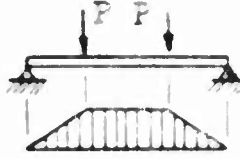
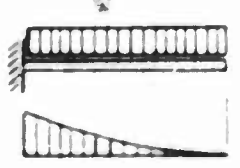
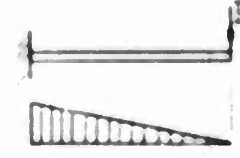
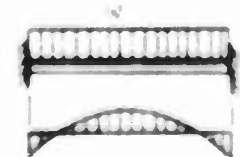
|          |   |                                                                   |
|----------|---|-------------------------------------------------------------------|
| $a, e$   | = | Eccentricity of loading position with respect to "S"              |
| $F$      | = | Lateral bracing force at support                                  |
| $f$      | = | Lateral bracing force per unit length                             |
| $M_T$    | = | Twisting moment about the longitudinal axis of a beam             |
| $M_{xx}$ | = | Bending moment in the weak direction of a section                 |
| $M_{yy}$ | = | Bending moment in the strong direction of a section               |
| $P$      | = | Concentrated load                                                 |
| $S$      | = | Shear center of a section                                         |
| $u$      | = | Displacement in the weak (x-) direction                           |
| $v$      | = | Displacement in the strong (y-) direction                         |
| $w$      | = | Uniformly distributed load                                        |
| $x, y$   | = | Coordinate axes in the weak and the strong direction respectively |
| $\rho$   | = | Ratio of end moments                                              |

LATERAL BUCKLING OF BEAMS

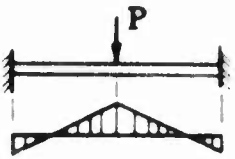
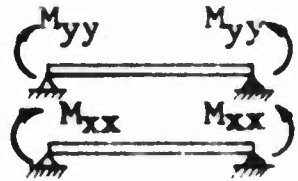
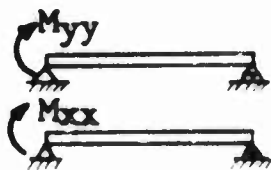
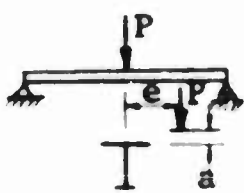
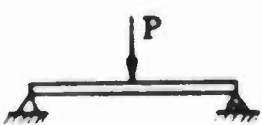
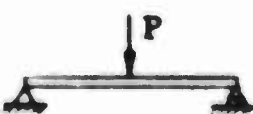
## I. KNOWN SOLUTIONS TO ELASTIC CASES

| BEAM AND LOADING                                                                    | SECTION                                                                                                                                                                | SOLUTION BY                                                                                                | REMARKS                                                |
|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|--------------------------------------------------------|
|    | <br> | Timoshenko <sup>(7)</sup><br>Goodier <sup>(25)</sup><br>Nylander <sup>(94)</sup><br>Winter <sup>(33)</sup> | Diff. equation solution.                               |
|   |                                                                                     | Salvadori <sup>(87)</sup>                                                                                  | Energy solution, experiment by Clark <sup>(99)</sup> . |
|  |                                                                                     | Salvadori <sup>(87)</sup>                                                                                  | Energy solution, experiment by Clark <sup>(99)</sup> . |
|  |                                                                                     | Salvadori <sup>(87)</sup>                                                                                  | Energy solution, experiment by Clark <sup>(99)</sup> . |
|  |                                                                                     | Salvadori <sup>(87)</sup>                                                                                  | Energy solution, experiment by Clark <sup>(99)</sup> . |

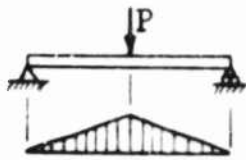
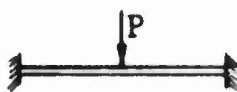
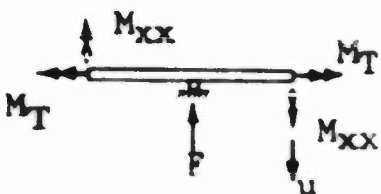
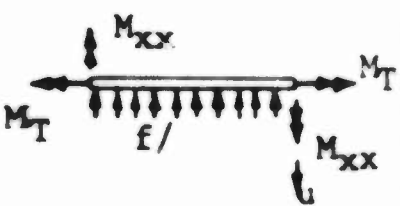
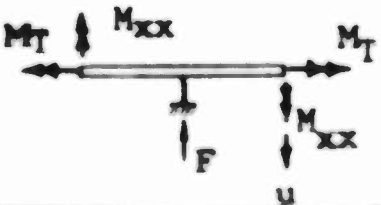
## KNOWN SOLUTIONS TO ELASTIC CASES

| BEAM AND LOADING                                                                    | SECTION                                                                                                                                                                                                                                                           | SOLUTION BY                                                | REMARKS                                                                                                           |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
|    |                                                                                                                                                                                  | Austin (98)<br>de Vries (37)<br>Schrader (30)              | Ref. 98 finite diff. method.<br>Ref. 37 energy solution.<br>Ref. 30 energy sol.<br>Experiments by Hill (26)       |
|    | <br><br>       | Flint (46)<br>Timoshenko (7)<br>Austin (98)<br>Winter (29) | Experiments by Flint (46) (65) on C and I sections. Ref. 7 diff. equation solution.                               |
|  | <br><br> | Winter (29) (42)<br>Schrader (30)                          | Energy solution.<br>Experiment by Winter (42), Hechtman (81) and Flint (65) and also by Hill on C and I sections. |
|  |                                                                                                                                                                                | Poley (90)                                                 | Finite difference method.                                                                                         |
|  | <br>                                                                                        | Timoshenko (7)<br>Kerensky (89)                            | In (89) local buckling of web, influence of restraints on stability are considered. Experiments by Flint (65).    |
|  |                                                                                                                                                                                | Austin (98)                                                | Solution by finite difference method.                                                                             |

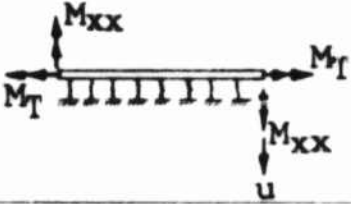
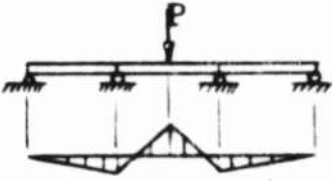
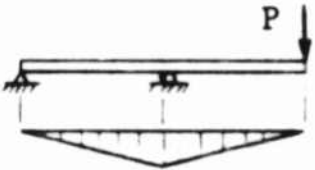
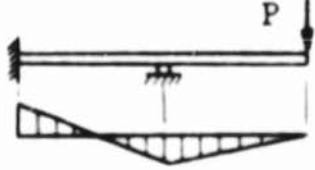
## KNOWN SOLUTIONS TO ELASTIC CASES

| BEAM AND LOADING                                                                    | SECTION                                                                             | SOLUTION BY     | REMARKS                                                                       |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-----------------|-------------------------------------------------------------------------------|
|    |    | Austin (98)     | Finite difference method.                                                     |
|    |    | Pettersson (67) | Diff. equation solution (implicit).                                           |
|  |  | Pettersson (67) | Diff. equation solution (implicit)                                            |
|  |  | Pettersson (67) | Diff. equation solution (implicit), load at any point. Experiments also made. |
|  |  | Pettersson (67) | Diff. equation solution (implicit), load at any point.                        |
|  |  | Pettersson (67) | Diff. equation solution (implicit), load at any point.                        |

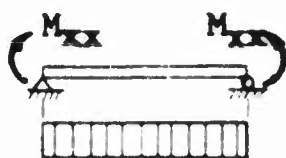
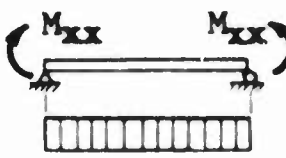
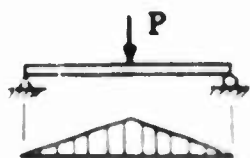
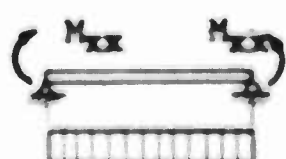
## KNOWN SOLUTIONS TO ELASTIC CASES

| BEAM AND LOADING                                                                                             | SECTION                                                                             | SOLUTION BY | REMARKS                                                                          |
|--------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------|----------------------------------------------------------------------------------|
|                             |    | Zetlin (83) | General bending theory. A method presented for analyzing unsymmetrical sections. |
| <br>$u=u''=0 \quad u=u''=0$ |    | Masur (97)  | Non-linear strain, energy solution. Experiments also made.                       |
|                           |  | Winter (29) | Energy solution.                                                                 |
|                           |  | Zuk (88)    | Diff. equation solution.                                                         |
|                           |  | Zuk (88)    | Diff. equation solution.                                                         |
|                           |  | Zuk (88)    | Energy solution.                                                                 |

## KNOWN SOLUTIONS TO ELASTIC CASES

| BEAM AND LOADING                                                                    | SECTION                                                                             | SOLUTION BY     | REMARKS                                       |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-----------------|-----------------------------------------------|
|    |    | Zuk (86)        | Energy solution                               |
|    |    | Pettersson (67) | Diff. equation solution (implicit)            |
|  |  | Kerensky (89)   | Determination of the influence of restraints. |
|  |  | Kerensky (89)   | Determination of the influence of restraints. |
|                                                                                     |                                                                                     |                 |                                               |
|                                                                                     |                                                                                     |                 |                                               |

## II KNOWN SOLUTIONS TO INELASTIC CASES

| BEAM AND LOADING                                                                    | SECTION                                                                                                                                                                | SOLUTION BY    | REMARKS                                                                                                                                                        |
|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    | <br> | Neal (45)      | Elastic-perfectly plastic material. Strain reversal considered, Eigenvalue solution. Experiments also done in (45).                                            |
|   |                                                                                       | Wittrick (68)  | Elastic-strain-hardened material. No strain reversal, Eigenvalue solution.                                                                                     |
|  |                                                                                     | Horne (47)     | Elastic-perfectly plastic material. Strain reversal considered. Eigenvalue solution. Experiments also made.                                                    |
|  |                                                                                     | White (91)     | Elastic-strain-hardened material. General loading, no strain reversal. Eigenvalue solution (finite diff. method). Experiments by White (91) and Sarubbi (106). |
|  |                                                                                     | Galambos (109) | Elastic-perfectly plastic mat'l. No strain reversal, Eigenvalue sol. Residual stress considered.                                                               |
|                                                                                     |                                                                                                                                                                        | Horne (47)     | Elastic-perfectly plastic material. Strain reversal considered. Eigenvalue solution.                                                                           |

10 APPENDIX C

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