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# TABLES FOR DETERMINATION OF FLOW VARIABLE GRADIENTS BEHIND CURVED SHOCK WAVES

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BALLISTIC RESEARCH LABORATORIES

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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## ABSTRACT

The shock wave relations and the equations of isoenergetic two-dimensional or axisymmetric flows can be combined to yield expressions for various useful quantities behind a shock wave if the curvature is known in addition to the position and slope. Tables are presented here of computations, for $\gamma = 1.4$, of seven functions of Mach number and shock wave slope, from which the following are easily computed: 1) streamline curvature, 2) velocity gradient along streamline, 3) angle between streamline and density contour, and 4) angle between streamline and Mach number contour. In addition, the Mach number and angle of inclination of streamline are listed. A Mach number range from 1.1 to 10 is covered by these computations.

A derivation of the shock gradient functions is presented; and several applications of the calculations are given, including a set of tables for determining the slope of the sonic line.

## SYMBOLS

| | |
|---|---|
| $(x, r)$ | - rectangular coordinates |
| $(n, s)$ | - natural coordinates normal and tangential respectively to streamline |
| $A$ | $= (\cos^2 \beta + g^2 \sin^2 \beta)^{1/2}$ |
| $a$ | - speed of sound $= (\partial p/\partial \rho)^{1/2}$ |
| $F_1, F_2 \ldots F_7$ | - shock gradient functions |
| $g$ | - inverse of density ratio across shock $= \rho_1/\rho$ |
| $h$ | - enthalpy |
| $K_s$ | - curvature of shock wave |
| $K_\psi$ | - curvature of streamline |
| $\ell$ | - inverse of pressure ratio across shock $= p_1/p$ |
| $M$ | - Mach number |
| $m$ | $= M_1 \sin \beta$ |
| $p$ | - pressure |
| $q$ | - velocity |
| $R$ | - ratio of stagnation densities across shock |
| $S$ | - entropy |
| $T$ | - temperature |
| $\beta$ | - angle between shock wave and x-axis |
| $\gamma$ | - ratio of specific heats (= 1.4 for air) |
| $\epsilon$ | - 0 for 2-dimensional flow<br>1 for axisymmetric flow |
| $\zeta$ | - angle between streamline and line of constant density |
| $\theta$ | - angle between streamline and x-axis |
| $\xi$ | - angle between streamline and line of constant Mach number |

$\rho$ - density

$\sigma$ - arc length along shock wave

<u>Subscripts</u>

1 - free stream

t - stagnation

## I. INTRODUCTION

The problem of determining the gradients of the flow variables behind a shock wave in isoenergetic two-dimensional flow has been considered by several investigators[1,2,3]. These gradients are very useful; for instance, the slope of the streamline at shock polar points in the hodograph plane can be obtained from them (the Busemann "hedgehog"). A particularly simple derivation was indicated by Sternberg[4], using natural coordinates. It is found that the flow variable gradients are proportional to the shock wave curvature. Tables of coefficients for streamline curvature have been computed by Thomas[2] for a limited range of Mach number.

The same gradients can be computed for axisymmetric flow. However, they are now linear combinations of the two shock curvatures, $K_s$ in the meridianal plane and $1/r$ in the azimuthal plane. (The two-dimensional result in obtained simply by neglecting the $1/r$ term.) Because of this the general usefulness of these gradient functions is restricted in that one can no longer, as in two-dimensional flow, obtain certain quantities (e.g., slope of sonic line) without having to specify the values of r and $K_s$. Expressions for the slope of these contours (density, pressure, Mach number) have been given by Wood and Gooderum[5].

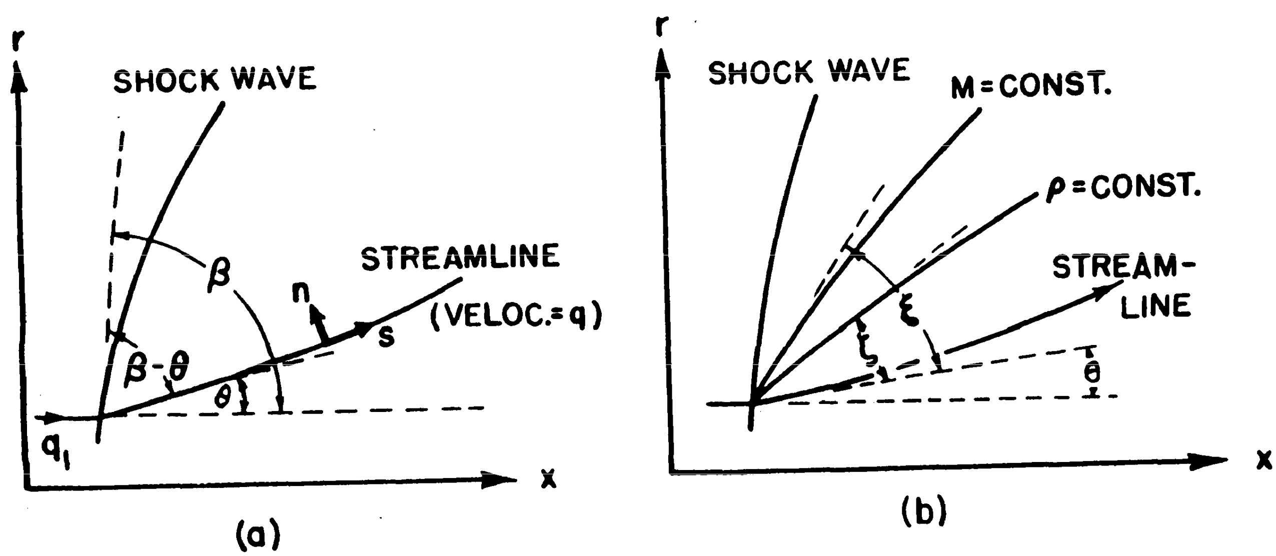


FIGURE 1.1

Aside from the results of Thomas cited above no calculations of the above-mentioned gradient quantities have to the authors' knowledge been published. This report supplies such a set of calculations ("shock gradient functions") which can be used to determine significant flow information from physical measurements. It is felt that these computations can also be helpful in obtaining a clearer general picture of flow behind a curved shock wave. These functions will find further application in the study of flow of a relaxing gas behind shock waves at large Mach numbers.

The particular quantities to be obtained here are (using the notation of Fig. 1.1): 1) $\partial q/\partial s$, the velocity gradient along the streamline; 2) $\partial \Theta/\partial s$ ($= K_\psi$), the curvature of the streamline; 3) $\Theta + \zeta$, the slope angle of the isopycnal, or density contour; and 4) $\Theta + \xi$, the slope angle of the Mach number contour. It is assumed that the gas is ideal, and that the flow is non-viscous and isentropic along streamlines.

The above flow quantities are obtainable from expressions having the following simplified forms:

$$\begin{aligned}
\partial\Theta/\partial s &= F_1(\gamma, M_1, \beta)\, K_s + \epsilon\, F_2(\gamma, M_1, \beta)\, (1/r) \\
(1/q_1)\partial q/\partial s &= F_3(\gamma, M_1, \beta)\, K_s + \epsilon\, F_4(\gamma, M_1, \beta)\, (1/r) \\
\tan\zeta &= \frac{F_3(\gamma, M_1, \beta)\, K_s + \epsilon\, F_4(\gamma, M_1, \beta)\, (1/r)}{F_5(\gamma, M_1, \beta)\, K_s + \epsilon\, F_6(\gamma, M_1, \beta)\, (1/r)} \\
\tan\xi &= \frac{F_3(\gamma, M_1, \beta)\, K_s + \epsilon\, F_4(\gamma, M_1, \beta)\, (1/r)}{F_7(\gamma, M_1, \beta)\, K_s + \epsilon\, F_6(\gamma, M_1, \beta)\, (1/r)}
\end{aligned} \tag{1.1}$$

where $\epsilon$ is equal to zero for two-dimensional flow and one for axisymmetric flow. $M_1$ and $q_1$ are the Mach number and velocity, respectively, in the free stream; $\tan\beta$ is the slope of the shock wave; $\gamma$ is the ratio of specific heats. The angle $\Theta = \Theta\,(M_1, \beta)$ is known from the shock wave relations (see, e.g., Ref. 6).

The gradient functions $F_1$, $F_2$, ...., $F_7$ are derived in detail in Section II. These functions for the most part are long and complicated expressions in $M_1$ and $\beta$ which do not lend themselves to analytical treatment or easy hand computation. It is, however, feasible to calculate them on a high speed computing machine. Thus the functions $F_1$, $F_2$, ...., $F_7$, $\Theta$, and M have been programmed for computation by the EDVAC in the Ballistic Research Laboratories, and they can be obtained quickly for any particular case of supersonic flow by specifying $M_1$ and $\gamma$ in the computing machine input.

The simple form of the expressions in Eqs. (1.1) suggests the practicability of a compilation of the coefficients of $K_s$ and $1/r$ which could be employed as a labor saving means of determining the desired quantities. To this end, the present paper provides a series of tables of shock wave gradient functions versus $\beta$ for $\gamma = 1.4$, with $M_1$ as parameter.

The angle $\zeta$ is of special interest to the authors, particularly in calculating a flow field (velocity, streamlines, etc.) from given interferometric density data (see Ref. 5, 7, and 8). The information resulting from the measured $\beta$ and $K_s$ reduces the uncertainty in density near a shock wave which is inherent in the interferometric method.

Sample hand computations have verified that the formulas used in this report yield the same values as formulas given in the references.

## II. DERIVATION OF GRADIENT FUNCTIONS

The present derivation follows the one indicated in Ref. 4. In natural coordinates the equations of isoenergetic flow of an ideal non-viscous gas are

$$
\begin{aligned}
&(1/\rho)\ \partial\rho/\partial s + (1/q)\ \partial q/\partial s + \partial\theta/\partial n + \epsilon(\sin\theta)/r = 0 && \text{(a)} \\
&\partial p/\partial s + \rho q\ \partial q/\partial s = 0 && \text{(b)} \\
&\partial p/\partial n + \rho\ q^2\ \partial\theta/\partial s = 0 && \text{(c)} \qquad (2.1) \\
&h + q^2/2 = \text{constant} && \text{(d)} \\
&TdS = dh - dp/\rho && \text{(e)}
\end{aligned}
$$

where $\epsilon = 0$ for two-dimensional, $= 1$ for axisymmetric flow. The coordinates s and n are arc lengths along the streamline and along the orthogonal trajectory of the streamline, respectively; q is the speed of flow, and $\theta$ is the angle between the streamline and the free stream direction (x axis); $\rho$ is the density, and p the pressure. T is the temperature; M is the Mach number ( $= q/a$, where $a = (\partial p/\partial\rho)^{1/2}$ = speed of sound).

The two essential equations to be used here are

$$
\begin{aligned}
&(1 - M^2)\ (1/q)\ \partial q/\partial s + \partial\theta/\partial n = -\ \epsilon(\sin\theta)/r && \text{(a)} \\
&\partial q/\partial n - q\ \partial\theta/\partial s = -\ (T/q)\ dS/dn, && \text{(b)}
\end{aligned}
\qquad (2.2)
$$

obtained from Eqs. (2.1) and the definition of the velocity of sound. (The foregoing equations are given in Ref. 9 and 10.) Entropy remains constant along streamlines.

If $\sigma$ is arc length along the shock wave, it is seen from the relation

$$\partial/\partial\sigma = (\partial/\partial s)\ (\partial s/\partial\sigma) + (\partial/\partial n)\ (\partial n/\partial\sigma)$$

that at the shock wave

$$\frac{\partial}{\partial\sigma} = \left(\frac{\partial}{\partial\beta}\right)\left(\frac{\partial\beta}{\partial\sigma}\right) = K_s\,\frac{\partial}{\partial\beta} = \left(\frac{\partial}{\partial s}\right)\cos(\beta - \theta) + \left(\frac{\partial}{\partial n}\right)\sin(\beta - \theta) \qquad (2.3)$$

where $\tan \beta$ and $K_s$ are the slope and curvature, respectively, of the shock wave (Fig. 1.1). As the coordinates are defined, curvatures are positive where curves are concave upward and negative where concave downward.

By means of Eq. (2.3), Eqs. (2.2) are replaced by two simultaneous linear algebraic equations for $\partial q/\partial s$ and $\partial \Theta/\partial s$. The coefficients of these equations are now converted into the desired form; namely, that of Eqs. (1.1). For this purpose use is made of the following definitions and relations which can be found in many textbooks and reference books (the authors employed Ref. 6 mainly):

$$m = M_1 \sin \beta \tag{2.4}$$

The subscript 1 always refers to free stream conditions.

$$g = \rho_1/\rho = \left[(\gamma - 1)\, m^2 + 2\right] \Big/ \left[(\gamma + 1)\, m^2\right] \tag{2.5}$$

$$\ell = p_1/p = (\gamma + 1) \Big/ \left[2\gamma m^2 - (\gamma - 1)\right] \tag{2.6}$$

$$q = A\, q_1, \tag{2.7}$$

where

$$A = (\cos^2 \beta + g^2 \sin^2 \beta)^{1/2} \tag{2.8}$$

$$\sin(\beta - \Theta) = (g \sin \beta)/A, \quad \cos(\beta - \Theta) = (\cos \beta)/A \tag{2.9}$$

$$\sin \Theta = \left[(1 - g) \sin \beta \cos \beta\right] /A \tag{2.10}$$

$$T/T_1 = g/\ell \tag{2.11}$$

$$M/M_1 = A(\ell/g)^{1/2} \tag{2.12}$$

$$S = \left(\frac{q_1}{T_1}\right) \left[\frac{q_1}{\gamma(\gamma - 1)M_1^{\,2}}\right] \left[\log_e \left(\frac{1}{\ell}\right) - \gamma \log_e \left(\frac{1}{g}\right)\right] + S_1 \tag{2.13}$$

After differentiation for $\partial q/\partial \beta$, $\partial \Theta/\partial \beta$, and $\partial S/\partial \beta$ (noting that $\partial S/\partial s = 0$), one obtains expression for $F_1$, $F_2$, $F_3$, and $F_4$ given by the following sequence of formulas:

$$L(m) = g^2 - 1 - 4g/(\gamma m^2 + m^2) \tag{2.14}$$

$$K(m) = (1 - m^2 g \ell)^{-1} \tag{2.15}$$

$$J(m) = \frac{-\left[g/(\gamma + 1)\right] \left[4m^2 \ell - 2(\gamma + 1) + 2(\gamma - 1)/g\right]}{(\gamma - 1)\, \ell\, m^2\, L(m)} \tag{2.16}$$

$$f_1 = g \left[L(1 - J) - K(1 - g)\right] \tag{2.17}$$

$$f_2 = - K \left[(L - g + 1)/g + m^2\, \ell\, L\, (1 - J)\right] \tag{2.18}$$

$$f_3 = g\, K(1 - g) \tag{2.19}$$

$$f_4 = K \left[(L - g + 1) + L(1 - J)\right] \tag{2.20}$$

$$F_1 = \left[(\cos \beta)/A^3\right] (f_1 \sin^2 \beta + f_2 \cos^2 \beta) \tag{2.21}$$

$$F_2 = \left[(\cos \beta)/A^3\right] (f_3 \sin^2 \beta \cos \beta) \tag{2.22}$$

$$F_3 = \left[(\sin \beta)/A^2\right] (f_4 \cos^2 \beta + g\, f_3 \sin^2 \beta) \tag{2.23}$$

$$F_4 = - (g\, f_3 \sin^3 \beta \cos \beta)/A^2 \tag{2.24}$$

$$\partial\theta/\partial s = F_1\, K_s + \epsilon\, F_2\, (1/r)$$

$$(1/q_1)\, \partial q/\partial s = F_3\, K_s + \epsilon\, F_4\, (1/r)$$

The following relations, obtained from the flow equations, are used to find $\zeta$ and $\xi$:

$$p = \left(\frac{p_t}{\rho_t^{\gamma}}\right) \rho^{\gamma} = \left[\frac{(p_t/p_{t_1})}{(\rho_t/\rho_{t_1})^{\gamma}} \quad \frac{(\rho_1/\rho_{t_1})^{\gamma}}{(p_1/p_{t_1})} \quad \frac{p_1}{\rho_1^{\gamma}}\right] \rho^{\gamma} \tag{2.25}$$

where the subscript t denotes the stagnation value $(\partial p_t/\partial s = \partial \rho_t/\partial s = 0)$, and

$$p_t/p_{t_1} = \rho_t/\rho_{t_1} = R = \ell^{\,1/(\gamma - 1)}\; g^{-\gamma/(\gamma - 1)}$$ (See Ref. 6)

$$\rho_1/\rho_{t_1} = \left[1 + (\gamma - 1)\, M_1^{\,2}/2\right]^{-1/(\gamma - 1)}$$

$$p_1/p_{t_1} = \left[1 + (\gamma - 1)\, M_1^{\,2}/2\right]^{-\gamma/(\gamma - 1)}$$

By means of Eqs. (2.3) and (2.25), $\partial p/\partial s$ and $\partial p/\partial n$ can be expressed in terms of $\partial \rho/\partial s$, $\partial \rho/\partial n$, and other quantities already determined in this report. Substituting these expressions into Eqs.(2.1), one then derives expressions for $\partial \rho/\partial s$ and $\partial \rho/\partial n$; and putting these into the relation for the angle between the density contour and the streamline, namely,

$$\tan \zeta = - (\partial \rho/\partial s)/(\partial \rho/\partial n) \qquad (2.26)$$

one obtains

$$\tan \zeta = \frac{F_3\, K_s + \epsilon\, F_4\, (1/r)}{F_5\, K_s + \epsilon\, F_6\, (1/r)}\text{, where}$$

$$F_5 = \frac{4 \cos \beta}{\gamma + 1} \left[\frac{1}{g \ell m^4} - 1\right] - AF_1 \qquad (2.27)$$

$$F_6 = - AF_2 \qquad (2.28)$$

A positive value of $\zeta$ indicates a counterclockwise rotation from the streamline to the density contour.

The angle $\xi$ between the Mach number contour and the streamline is determined from

$$\tan \xi = - (\partial M/\partial s)/(\partial M/\partial n) = - (\partial M^2/\partial s)/(\partial M^2/\partial n), \qquad (2.29)$$

where $\xi$ has the same sign convention as $\zeta$. Since

$$M^2 = (\rho q^2)/(\gamma p),$$

$\partial M^2 \partial s$ and $\partial M^2/\partial n$ can be expressed in terms of the derivatives of $\rho$ and $q$, which in turn are expressible in terms of previously determined quantities.

Consequently one obtains

$$\tan \xi = \frac{F_3 K_s + \epsilon F_4 (1/r)}{F_7 K_s + \epsilon F_6 (1/r)} \quad \text{, where}$$

$$F_7 = - \left[ \frac{4 \cos \beta}{(\gamma + 1) g^2 m^2 \sin^2 \beta} \left\{ A^2 - \frac{2g (m^2 - 1)^2 \sin^2 \beta}{(\gamma + 1) m^2} \right\} + \left\{ \frac{(A^3 \gamma \ell m^2/g) + 2 A \sin^2 \beta}{\sin^2 \beta} \right\} F_1 + \left\{ \frac{A^2 m^2 \ell \cot \beta}{g^2 \sin^2 \beta} \right\} F_3 \right] \cdot \left[ 2 + (\gamma - 1) \frac{A^2 m^2 \ell}{g \sin^2 \beta} \right]^{-1} \tag{2.30}$$

## III. DISCUSSION OF COMPUTATIONS

Tabulations of the functions $F_1$, $F_2$, ...., $F_7$, $\Theta$, and M, calculated for $\gamma = 1.4$, are presented in Section 4 for values of $M_1$ ranging from 1.1 to 10.0. The functions are listed in order of decreasing $\beta$, from $\beta = 90^{\circ}$ to a value somewhat greater than $\beta = \sin^{-1}(1/M_1)$. The listed values of $M_1$ and $\beta$ were chosen so as to permit the determination of the functions to a reasonable degree of accuracy for any $M_1$ and $\beta$ by means of graphical interpolation.

Figure 3.1 shows an example of results obtainable from these calculations. The right hand curve shows the variation in slope of the constant density lines along the shock wave of a supersonic sphere in nitrogen (shown at the left) determined from shock wave coordinate measurements and the appropriate shock gradient functions. For reference purposes, the left hand figure shows the sonic point and "Crocco point" (defined below), and indicates the location of typical values of $rK_s$. It must be noted that the accuracy of results like those in Figure 3.1 is restricted by the limited accuracy of $\beta$ and $K_s$

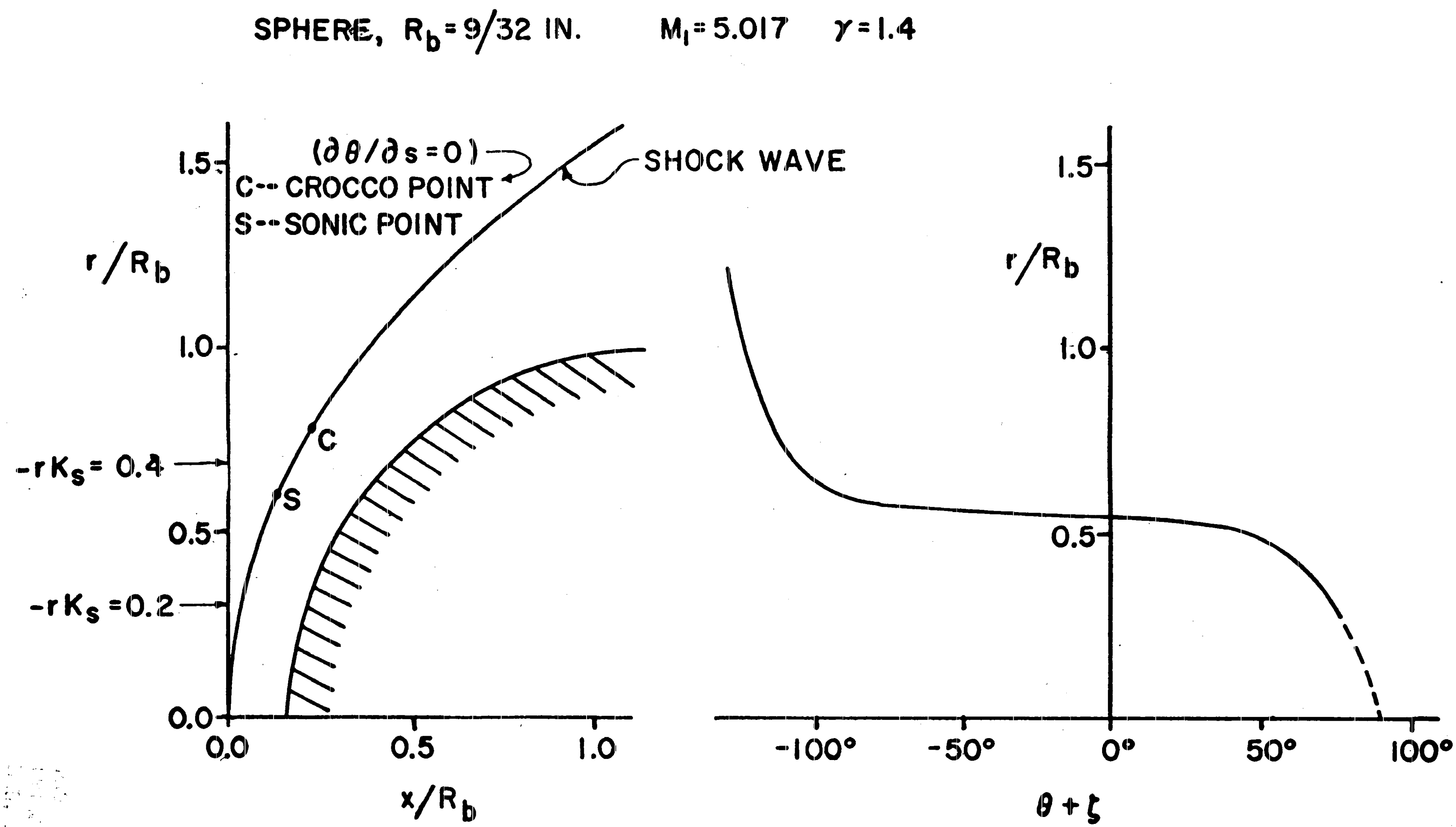


Figure 3.1. EXAMPLE OF VARIATON OF ANGLE OF CONSTANT DENSITY LINES ALONG A DETACHED SHOCK WAVE.

determined from experiment. (See Ref. 5 and 8 for discussion on determination of $\beta$ and $K_s$.)

A point often referred to in two-dimensional flow studies (e.g., Ref. 3) is the "Crocco point", where the streamline curvature is zero. Figure 3.2 shows the geometrical conditions which must exist at the Crocco point in plane and axisymmetric flows for $\gamma = 1.4$. It can be seen that in two-dimensional flow the Crocco point always lies below the sonic point on a shock wave of continuosuly decreasing slope. The two points approach coincidence with increasing Mach number. On the other hand, in axisymmetric flow the sonic point lies below the Crocco point for all values of $M_1$ and $rK_s$ shown. By continuity there would be values of these two parameters for which the sonic point lies above the Crocco point.

The slope of the sonic line is a quantity of considerable interest. It can be otained from the tables by evaluating the $\beta$ and shock gradient functions corresponding to M = 1 (e.g., by graphical interpolation); it is found to be given by

$$\tan\,(\theta + \xi)_{son.} = \frac{B_1\,(M_1)\; rK_s + 1}{B_2\,(M_1)\; rK_s + B_3(M_1)},$$

where $B_1$, $B_2$, and $B_3$ are given in Table 3.1 for $\gamma = 1.4$. It is seen that, formally, $rK_s = -\infty$ corresponds to two-dimensional flow. This particular case is plotted, in slightly different form, in Ref. 11.

Figure 3.3 shows plots of the variation of $\xi_{son.}$, the angle between the streamline and Mach number contour at the sonic point. For two-dimensional flow $\xi$ is negative, indicating that the sonic line has a smaller slope than the streamline. For axisymmetric flow the figure indicates that the sonic line can have a greater slope than the streamline. (The streamline curvature must be positive in this case. This can be seen from Eqs. (2.1b) and (2.1c) and the fact that $dq/d\sigma < 0$ and $dM/d\sigma > 0$ along a shock wave with continuously decreasing slope.) If $rK_s = -.2$ at the sonic point, the sonic line and streamline are nearly tangent to each other over a very wide range of Mach number.

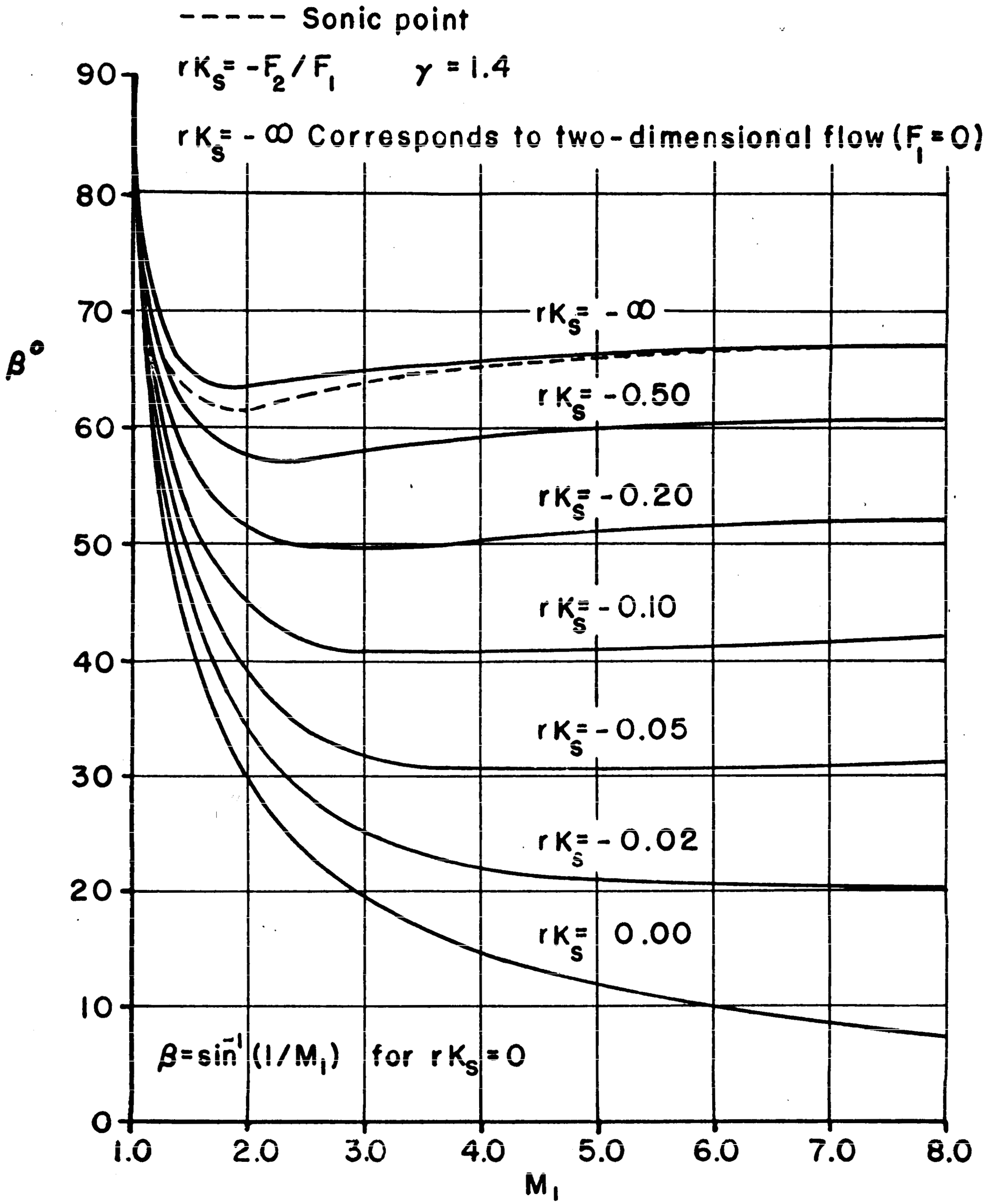


Figure 3.2. Conditions for zero curvature of streamline ($\partial\theta/\partial s = 0$) at shock wave (Crocco Point).

(NOTE: FOR THE $rK_s = 0$ CURVE, $\theta + \xi = \beta$)

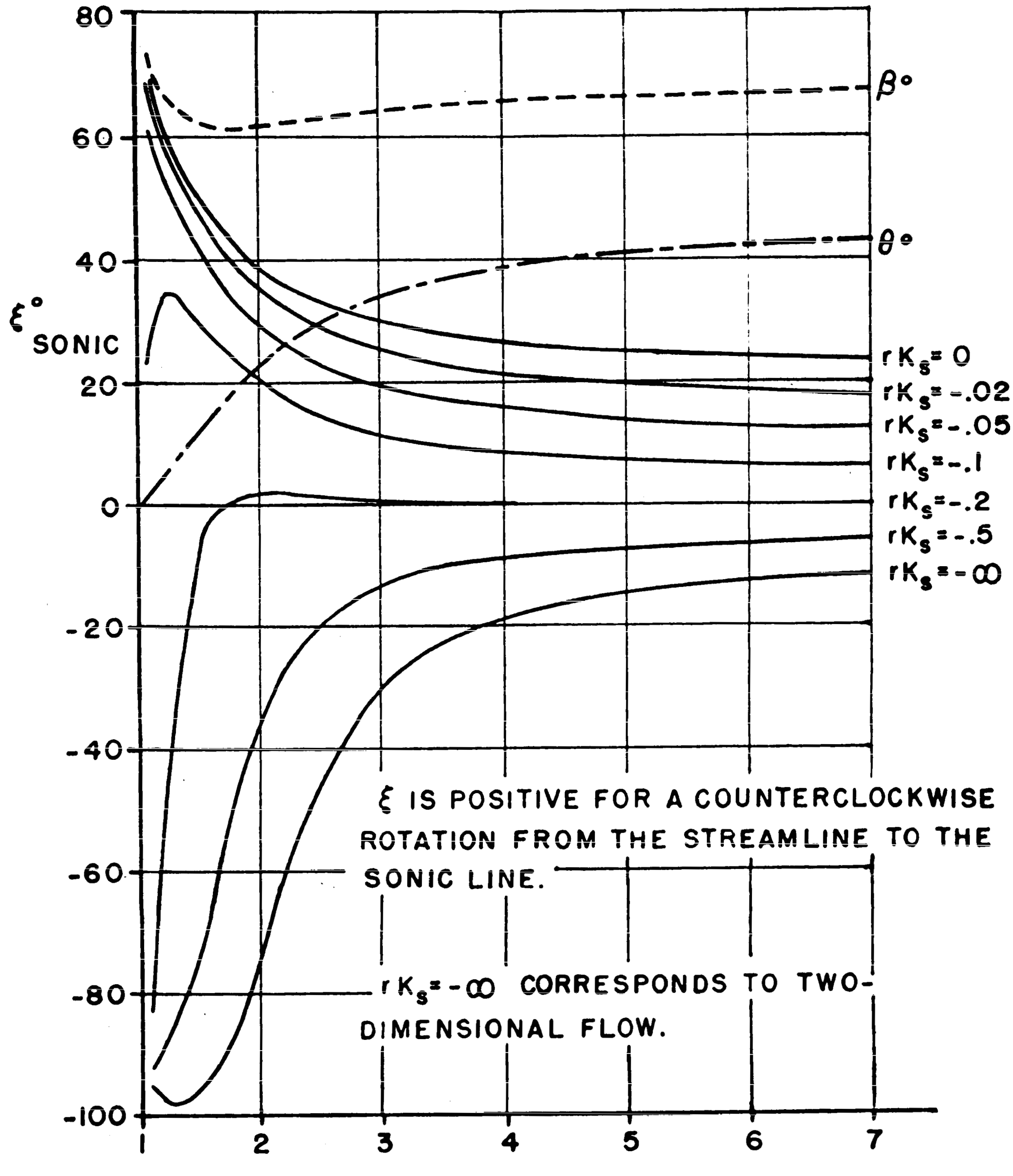


FIGURE 3.3. VARIATION OF ANGLE BETWEEN STREAMLINE AND SONIC LINE AT SHOCK WAVE.

The values of M are also useful in finding the slopes of the characteristics at the shock wave in the supersonic portion of the flow field.

For quick reference, Eqs. (1.1) are restated here.

$$\partial\theta/\partial s = F_1 K_s + F_2 (1/r)$$

$$(1/q_1)(\partial q/\partial s) = F_3 K_s + F_4 (1/r)$$

$$\tan \zeta = \frac{F_3(rK_s) + F_4}{F_5(rK_s) + F_6}$$

$$\tan \xi = \frac{F_3(rK_s) + F_4}{F_7(rK_s) + F_6}$$

TABLE 3.1

Sonic Point Table ($\gamma = 1.4$)

| $M_1$ | $B_1$ | $B_2$ | $B_3$ | $\beta^o$ | $\theta^o$ |
|---|---|---|---|---|---|
| 1.10 | 10.03 | .793 | .301 | 73.26 | 1.40 |
| 1.30 | 6.15 | .175 | .464 | 65.11 | 6.31 |
| 1.50 | 4.78 | - 0.548 | .524 | 62.26 | 11.71 |
| 1.75 | 3.71 | - 1.40 | .546 | 61.31 | 17.77 |
| 2.00 | 2.76 | - 2.29 | .541 | 61.50 | 22.70 |
| 2.50 | 1.16 | - 3.91 | .517 | 62.66 | 29.65 |
| 3.00 | - 0.31 | - 5.23 | .492 | 63.77 | 33.99 |
| 3.50 | - 1.53 | - 6.27 | .474 | 64.62 | 36.82 |
| 4.00 | - 2.56 | - 7.09 | .458 | 65.26 | 38.74 |
| 4.50 | - 3.38 | - 7.70 | .451 | 65.73 | 40.06 |
| 5.00 | - 4.04 | - 8.20 | .444 | 66.08 | 41.03 |
| 6.00 | - 5.04 | - 8.92 | .435 | 66.58 | 42.32 |
| 7.00 | - 5.70 | - 9.38 | .428 | 66.88 | 43.16 |
| 8.00 | - 6.19 | - 9.72 | .424 | 67.09 | 43.64 |
| 9.00 | - 6.50 | - 9.92 | .420 | 67.22 | 44.03 |
| 10.00 | - 6.76 | -10.09 | .419 | 67.35 | 44.27 |

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## IV - TABLES OF SHOCK GRADIENT FUNCTIONS

$M_1 = 1.10$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $10F_2$ | $F_3$ | $F_4$ | $F_5$ | $10F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 0.000 | 0.0000 | .00000 | 0.8574 | .00000 | 0.0000 | .00000 | 0.0000 | 0.912 |
| 89 | 0.169 | -0.0543 | .00357 | 0.8502 | -.01495 | 0.0463 | -.00305 | 0.0469 | 0.912 |
| 88 | 0.336 | -0.1074 | .01421 | 0.8283 | -.02985 | 0.0916 | -.01217 | 0.0928 | 0.913 |
| 87 | 0.498 | -0.1581 | .03181 | 0.7917 | -.04463 | 0.1351 | -.02728 | 0.1369 | 0.915 |
| 86 | 0.655 | -0.2054 | .05612 | 0.7399 | -.05923 | 0.1759 | -.04823 | 0.1781 | 0.917 |
| 85 | 0.804 | -0.2479 | .08683 | 0.6724 | -.07361 | 0.2128 | -.07482 | 0.2156 | 0.920 |
| 84 | 0.944 | -0.2844 | .12354 | 0.5885 | -.08770 | 0.2450 | -.10681 | 0.2482 | 0.923 |
| 83 | 1.072 | -0.3136 | .16580 | 0.4871 | -.10147 | 0.2712 | -.14390 | 0.2747 | 0.927 |
| 82 | 1.187 | -0.3341 | .21309 | 0.3668 | -.11485 | 0.2902 | -.18575 | 0.2940 | 0.932 |
| 81 | 1.287 | -0.3444 | .26484 | 0.2258 | -.12782 | 0.3005 | -.23200 | 0.3046 | 0.938 |
| 80 | 1.371 | -0.3427 | .32045 | 0.0616 | -.14034 | 0.3006 | -.28224 | 0.3048 | 0.944 |
| 78 | 1.483 | -0.2941 | .44071 | -0.3513 | -.16390 | 0.2611 | -.39298 | 0.2653 | 0.957 |
| 76 | 1.514 | -0.1645 | .56876 | -0.9167 | -.18532 | 0.1476 | -.51442 | 0.1515 | 0.974 |
| 74 | 1.454 | 0.0901 | .69956 | -1.7246 | -.20447 | -0.0838 | -.64288 | -0.0805 | 0.992 |
| 72 | 1.295 | 0.5642 | .82849 | -2.9781 | -.22131 | -0.5283 | -.77479 | -0.5259 | 1.013 |
| 70 | 1.032 | 1.5152 | .95145 | -5.2381 | -.23582 | -1.4444 | -.90674 | -1.4430 | 1.037 |

$M_1 = 1.30$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 0.000 | 0.0000 | .00000 | 0.8900 | .00000 | 0.0000 | .00000 | 0.0000 | 0.786 |
| 89 | 0.515 | -0.0796 | .00062 | 0.8856 | -.01552 | 0.0513 | -.00041 | 0.0556 | 0.786 |
| 88 | 1.027 | -0.1578 | .00248 | 0.8725 | -.03095 | 0.1019 | -.00164 | 0.1104 | 0.787 |
| 87 | 1.532 | -0.2335 | .00553 | 0.8506 | -.04621 | 0.1510 | -.00366 | 0.1637 | 0.789 |
| 86 | 2.028 | -0.3054 | .00973 | 0.8202 | -.06122 | 0.1981 | -.00647 | 0.2149 | 0.792 |
| 85 | 2.511 | -0.3724 | .01501 | 0.7814 | -.07590 | 0.2423 | -.01001 | 0.2632 | 0.795 |
| 84 | 2.979 | -0.4334 | .02127 | 0.7344 | -.09018 | 0.2832 | -.01425 | 0.3079 | 0.799 |
| 83 | 3.427 | -0.4873 | .02842 | 0.6794 | -.10399 | 0.3201 | -.01914 | 0.3484 | 0.804 |
| 82 | 3.855 | -0.5335 | .03634 | 0.6168 | -.11727 | 0.3523 | -.02462 | 0.3841 | 0.810 |
| 81 | 4.260 | -0.5711 | .04491 | 0.5466 | -.12998 | 0.3795 | -.03063 | 0.4146 | 0.816 |
| 80 | 4.638 | -0.5996 | .05400 | 0.4693 | -.14208 | 0.4011 | -.03711 | 0.4391 | 0.823 |
| 78 | 5.309 | -0.6269 | .07327 | 0.2938 | -.16429 | 0.4257 | -.05120 | 0.4689 | 0.839 |
| 76 | 5.856 | -0.6125 | .09317 | 0.0916 | -.18376 | 0.4228 | -.06636 | 0.4699 | 0.857 |
| 74 | 6.267 | -0.5544 | .11282 | -0.1370 | -.20043 | 0.3891 | -.08207 | 0.4386 | 0.878 |
| 72 | 6.535 | -0.4507 | .13148 | -0.3935 | -.21436 | 0.3210 | -.09785 | 0.3715 | 0.902 |
| 70 | 6.655 | -0.2991 | .14860 | -0.6813 | -.22569 | 0.2138 | -.11329 | 0.2637 | 0.928 |
| 65 | 6.299 | 0.3298 | .18228 | -1.5998 | -.24390 | -0.2803 | -.14829 | -0.2384 | 1.002 |
| 60 | 5.013 | 1.5662 | .20143 | -3.1378 | -.25056 | -1.3724 | -.17553 | -1.3461 | 1.089 |
| 55 | 2.836 | 5.0595 | .20685 | -7.3220 | -.24903 | -4.7334 | -.19342 | -4.7248 | 1.191 |

$M_1 = 1.50$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9105 | .00000 | 0.0000 | .00000 | 0.0000 | 0.701 |
| 89 | 00.86 | -0.1089 | .00096 | 0.9062 | -.01587 | 0.0557 | -.00052 | 0.0655 | 0.702 |
| 88 | 01.72 | -0.2159 | .00382 | 0.8933 | -.03162 | 0.1106 | -.00205 | 0.1301 | 0.703 |
| 87 | 02.56 | -0.3189 | .00850 | 0.8720 | -.04713 | 0.1638 | -.00459 | 0.1929 | 0.705 |
| 86 | 03.39 | -0.4162 | .01491 | 0.8425 | -.06229 | 0.2146 | -.00808 | 0.2531 | 0.708 |
| 85 | 04.20 | -0.5061 | .02290 | 0.8052 | -.07699 | 0.2621 | -.01248 | 0.3099 | 0.712 |
| 84 | 04.99 | -0.5872 | .03229 | 0.7604 | -.09114 | 0.3056 | -.01770 | 0.3626 | 0.717 |
| 83 | 05.75 | -0.6583 | .04289 | 0.7088 | -.10466 | 0.3447 | -.02368 | 0.4105 | 0.723 |
| 82 | 06.48 | -0.7184 | .05448 | 0.6507 | -.11748 | 0.3787 | -.03033 | 0.4530 | 0.729 |
| 81 | 07.17 | -0.7669 | .06685 | 0.5868 | -.12955 | 0.4073 | -.03756 | 0.4898 | 0.737 |
| 80 | 07.82 | -0.8032 | .07977 | 0.5176 | -.14082 | 0.4300 | -.04527 | 0.5203 | 0.745 |
| 78 | 09.00 | -0.8389 | .10644 | 0.3654 | -.16089 | 0.4569 | -.06176 | 0.5615 | 0.764 |
| 76 | 10.00 | -0.8262 | .13293 | 0.1982 | -.17762 | 0.4580 | -.07907 | 0.5751 | 0.785 |
| 74 | 10.81 | -0.7678 | .15799 | 0.0196 | -.19111 | 0.4329 | -.09656 | 0.5601 | 0.810 |
| 72 | 11.42 | -0.6680 | .18070 | -0.1682 | -.20156 | 0.3816 | -.11367 | 0.5166 | 0.837 |
| 70 | 11.84 | -0.5309 | .20047 | -0.3636 | -.20929 | 0.3042 | -.12997 | 0.4442 | 0.866 |
| 65 | 12.06 | -0.0472 | .23560 | -0.8890 | -.21883 | -0.0069 | -.16523 | 0.1332 | 0.949 |
| 60 | 11.16 | 0.6374 | .25133 | -1.5074 | -.21844 | -0.5192 | -.19094 | -0.3970 | 1.045 |
| 55 | 09.26 | 1.6434 | .25151 | -2.3673 | -.21215 | -1.3759 | -.20672 | -1.2875 | 1.152 |
| 50 | 06.44 | 3.5288 | .24037 | -3.9945 | -.20270 | -3.1431 | -.21319 | -3.0970 | 1.271 |

$M_1 = 1.75$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9269 | .00000 | 0.0000 | .00000 | 0.0000 | 0.628 |
| 89 | 01.28 | -0.1497 | .00146 | 0.9221 | -.01615 | 0.0607 | -.00064 | 0.0783 | 0.629 |
| 88 | 02.55 | -0.2961 | .00580 | 0.9078 | -.03212 | 0.1204 | -.00255 | 0.1554 | 0.630 |
| 87 | 03.80 | -0.4359 | .01288 | 0.8843 | -.04776 | 0.1777 | -.00569 | 0.2301 | 0.633 |
| 86 | 05.03 | -0.5660 | .02248 | 0.8520 | -.06289 | 0.2318 | -.00999 | 0.3013 | 0.637 |
| 85 | 06.22 | -0.6841 | .03430 | 0.8115 | -.07740 | 0.2816 | -.01536 | 0.3681 | 0.642 |
| 84 | 07.38 | -0.7880 | .04801 | 0.7636 | -.09114 | 0.3264 | -.02168 | 0.4296 | 0.648 |
| 83 | 08.50 | -0.8761 | .06323 | 0.7089 | -.10403 | 0.3655 | -.02884 | 0.4850 | 0.655 |
| 82 | 09.56 | -0.9475 | .07957 | 0.6483 | -.11600 | 0.3984 | -.03671 | 0.5339 | 0.663 |
| 81 | 10.57 | -1.0016 | .09663 | 0.5827 | -.12699 | 0.4247 | -.04514 | 0.5757 | 0.672 |
| 80 | 11.52 | -1.0385 | .11406 | 0.5130 | -.13697 | 0.4442 | -.05400 | 0.6103 | 0.682 |
| 78 | 13.23 | -1.0623 | .14869 | 0.3642 | -.15390 | 0.4626 | -.07252 | 0.6570 | 0.705 |
| 76 | 14.68 | -1.0255 | .18122 | 0.2077 | -.16693 | 0.4539 | -.09134 | 0.6741 | 0.731 |
| 74 | 15.87 | -0.9375 | .21012 | 0.0482 | -.17639 | 0.4200 | -.10968 | 0.6629 | 0.760 |
| 72 | 16.80 | -0.8087 | .23455 | -0.1110 | -.18273 | 0.3629 | -.12700 | 0.6253 | 0.792 |
| 70 | 17.47 | -0.6484 | .25418 | -0.2681 | -.18642 | 0.2851 | -.14290 | 0.5632 | 0.826 |
| 65 | 18.12 | -0.1583 | .28343 | -0.6476 | -.18713 | 0.0123 | -.17524 | 0.3118 | 0.922 |
| 60 | 17.48 | 0.4126 | .28988 | -1.0210 | -.18036 | -0.3636 | -.19666 | -0.0709 | 1.029 |
| 55 | 15.75 | 1.0718 | .28116 | -1.4300 | -.17018 | -0.8673 | -.20826 | -0.6105 | 1.147 |
| 50 | 13.08 | 1.9253 | .26322 | -1.9676 | -.15898 | -1.6036 | -.21162 | -1.4085 | 1.276 |
| 45 | 09.59 | 3.3278 | .23991 | -2.8902 | -.14797 | -2.9203 | -.20814 | -2.8039 | 1.415 |
| 40 | 05.30 | 7.0134 | .21332 | -5.4093 | -.13764 | -6.5463 | -.19877 | -6.5066 | 1.569 |

$M_1 = 2.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9375 | .00000 | 0.0000 | .00000 | 0.0000 | 0.577 |
| 89 | 01.66 | -0.1930 | .00202 | 0.9320 | -.01632 | 0.0652 | -.00076 | 0.0906 | 0.578 |
| 88 | 03.32 | -0.3807 | .00801 | 0.9158 | -.03241 | 0.1289 | -.00302 | 0.1797 | 0.580 |
| 87 | 04.94 | -0.5579 | .01771 | 0.8891 | -.04806 | 0.1896 | -.00671 | 0.2656 | 0.583 |
| 86 | 06.53 | -0.7204 | .03073 | 0.8528 | -.06304 | 0.2459 | -.01172 | 0.3471 | 0.588 |
| 85 | 08.08 | -0.8643 | .04657 | 0.8076 | -.07721 | 0.2969 | -.01794 | 0.4228 | 0.594 |
| 84 | 09.57 | -0.9869 | .06465 | 0.7546 | -.09040 | 0.3415 | -.02518 | 0.4919 | 0.601 |
| 83 | 10.99 | -1.0866 | .08436 | 0.6950 | -.10253 | 0.3789 | -.03329 | 0.5534 | 0.610 |
| 82 | 12.34 | -1.1626 | .10509 | 0.6300 | -.11351 | 0.4089 | -.04208 | 0.6070 | 0.619 |
| 81 | 13.61 | -1.2149 | .12627 | 0.5606 | -.12332 | 0.4310 | -.05137 | 0.6523 | 0.630 |
| 80 | 14.81 | -1.2443 | .14736 | 0.4880 | -.13195 | 0.4453 | -.06099 | 0.6891 | 0.642 |
| 78 | 16.94 | -1.2406 | .18759 | 0.3373 | -.14577 | 0.4511 | -.08060 | 0.7380 | 0.669 |
| 76 | 18.73 | -1.1664 | .22317 | 0.1847 | -.15538 | 0.4285 | -.09986 | 0.7555 | 0.699 |
| 74 | 20.18 | -1.0387 | .25268 | 0.0350 | -.16136 | 0.3811 | -.11799 | 0.7448 | 0.733 |
| 72 | 21.30 | -0.8735 | .27570 | -0.1085 | -.16435 | 0.3130 | -.13451 | 0.7092 | 0.769 |
| 70 | 22.12 | -0.6842 | .29246 | -0.2445 | -.16498 | 0.2277 | -.14915 | 0.6521 | 0.808 |
| 65 | 22.97 | -0.1643 | .31134 | -0.5506 | -.15966 | -0.0417 | -.17714 | 0.4314 | 0.917 |
| 60 | 22.41 | 0.3686 | .30713 | -0.8205 | -.14919 | -0.3714 | -.19380 | 0.1147 | 1.037 |
| 55 | 20.73 | 0.9094 | .28996 | -1.0815 | -.13714 | -0.7626 | -.20125 | -0.3025 | 1.167 |
| 50 | 18.13 | 1.5083 | .26635 | -1.3758 | -.12534 | -1.2544 | -.20160 | -0.8597 | 1.307 |
| 45 | 14.74 | 2.2937 | .23999 | -1.7828 | -.11460 | -1.9619 | -.19646 | -1.6672 | 1.456 |
| 40 | 10.62 | 3.6349 | .21267 | -2.5176 | -.10524 | -3.2449 | -.18695 | -3.0717 | 1.617 |
| 35 | 05.75 | 7.3335 | .18491 | -4.6213 | -.09724 | -6.9018 | -.17361 | -6.8444 | 1.795 |

$M_1 = 2.50$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9500 | .00000 | 0.0000 | .00000 | 0.0000 | 0.513 |
| 89 | 02.33 | -0.2793 | .00320 | 0.9430 | -.01652 | 0.0725 | -.00096 | 0.1124 | 0.514 |
| 88 | 04.64 | -0.5479 | .01259 | 0.9224 | -.03269 | 0.1427 | -.00380 | 0.2224 | 0.517 |
| 87 | 06.90 | -0.7958 | .02761 | 0.8889 | -.04818 | 0.2082 | -.00841 | 0.3276 | 0.521 |
| 86 | 09.09 | -1.0149 | .04736 | 0.8439 | -.06271 | 0.2671 | -.01458 | 0.4260 | 0.527 |
| 85 | 11.21 | -1.1994 | .07077 | 0.7889 | -.07604 | 0.3180 | -.02210 | 0.5160 | 0.535 |
| 84 | 13.22 | -1.3456 | .09666 | 0.7258 | -.08803 | 0.3596 | -.03069 | 0.5964 | 0.545 |
| 83 | 15.12 | -1.4524 | .12385 | 0.6564 | -.09858 | 0.3914 | -.04008 | 0.6666 | 0.556 |
| 82 | 16.91 | -1.5209 | .15129 | 0.5827 | -.10765 | 0.4131 | -.04999 | 0.7262 | 0.568 |
| 81 | 18.57 | -1.5534 | .17807 | 0.5062 | -.11527 | 0.4249 | -.06019 | 0.7753 | 0.582 |
| 80 | 20.10 | -1.5536 | .20345 | 0.4286 | -.12150 | 0.4273 | -.07044 | 0.8143 | 0.597 |
| 78 | 22.78 | -1.4741 | .24811 | 0.2746 | -.13019 | 0.4064 | -.09042 | 0.8644 | 0.631 |
| 76 | 24.96 | -1.3170 | .28300 | 0.1280 | -.13462 | 0.3565 | -.10887 | 0.8821 | 0.669 |
| 74 | 26.68 | -1.1134 | .30792 | -0.0073 | -.13578 | 0.2844 | -.12520 | 0.8735 | 0.711 |
| 72 | 27.97 | -0.8867 | .32382 | -0.1295 | -.13453 | 0.1959 | -.13917 | 0.8440 | 0.756 |
| 70 | 28.89 | -0.6533 | .33216 | -0.2387 | -.13161 | 0.0961 | -.15079 | 0.7979 | 0.804 |
| 65 | 29.80 | -0.0961 | .32971 | -0.4619 | -.12031 | -0.1816 | -.17053 | 0.6281 | 0.934 |
| 60 | 29.18 | 0.3904 | .30845 | -0.6328 | -.10714 | -0.4768 | -.17958 | 0.3966 | 1.077 |
| 55 | 27.44 | 0.8175 | .27971 | -0.7748 | -.09443 | -0.7818 | -.18097 | 0.1034 | 1.231 |
| 50 | 24.85 | 1.2221 | .24923 | -0.9102 | -.08307 | -1.1076 | -.17697 | -0.2682 | 1.395 |
| 45 | 21.57 | 1.6594 | .21961 | -1.0648 | -.07334 | -1.4884 | -.16924 | -0.7548 | 1.569 |
| 40 | 17.68 | 2.2266 | .19189 | -1.2814 | -.06525 | -2.0071 | -.15891 | -1.4357 | 1.754 |
| 35 | 13.17 | 3.1621 | .16625 | -1.6642 | -.05875 | -2.8952 | -.14670 | -2.5278 | 1.952 |
| 30 | 07.99 | 5.4086 | .14226 | -2.6251 | -.05370 | -5.0966 | -.13288 | -4.9407 | 2.169 |

$M_1 = 3.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9568 | .00000 | 0.0000 | .00000 | 0.0000 | 0.475 |
| 89 | 02.85 | -0.3576 | .00431 | 0.9484 | -.01662 | 0.0782 | -.00112 | 0.1298 | 0.476 |
| 88 | 05.67 | -0.6978 | .01687 | 0.9237 | -.03277 | 0.1530 | -.00441 | 0.2563 | 0.480 |
| 87 | 08.42 | -1.0051 | .03669 | 0.8840 | -.04804 | 0.2215 | -.00970 | 0.3763 | 0.485 |
| 86 | 11.07 | -1.2674 | .06229 | 0.8312 | -.06207 | 0.2812 | -.01671 | 0.4872 | 0.493 |
| 85 | 13.60 | -1.4771 | .09189 | 0.7678 | -.07461 | 0.3302 | -.02511 | 0.5872 | 0.502 |
| 84 | 15.99 | -1.6312 | .12368 | 0.6963 | -.08550 | 0.3675 | -.03453 | 0.6751 | 0.514 |
| 83 | 18.22 | -1.7304 | .15598 | 0.6193 | -.09469 | 0.3926 | -.04461 | 0.7504 | 0.527 |
| 82 | 20.29 | -1.7787 | .18737 | 0.5393 | -.10221 | 0.4058 | -.05502 | 0.8133 | 0.542 |
| 81 | 22.19 | -1.7819 | .21677 | 0.4583 | -.10813 | 0.4078 | -.06547 | 0.8643 | 0.558 |
| 80 | 23.93 | -1.7472 | .24343 | 0.3779 | -.11259 | 0.3993 | -.07574 | 0.9043 | 0.576 |
| 78 | 26.90 | -1.5923 | .28699 | 0.2240 | -.11777 | 0.3560 | -.09502 | 0.9554 | 0.616 |
| 76 | 29.25 | -1.3659 | .31716 | 0.0840 | -.11902 | 0.2850 | -.11197 | 0.9754 | 0.660 |
| 74 | 31.06 | -1.1072 | .33532 | -0.0395 | -.11751 | 0.1949 | -.12626 | 0.9720 | 0.708 |
| 72 | 32.38 | -0.8421 | .34371 | -0.1467 | -.11418 | 0.0924 | -.13789 | 0.9516 | 0.760 |
| 70 | 33.28 | -0.5856 | .34465 | -0.2388 | -.10971 | -0.0175 | -.14707 | 0.9183 | 0.814 |
| 65 | 34.07 | -0.0209 | .32680 | -0.4158 | -.09647 | -0.3039 | -.16101 | 0.7958 | 0.961 |
| 60 | 33.32 | 0.4288 | .29544 | -0.5393 | -.08308 | -0.5874 | -.16532 | 0.6285 | 1.123 |
| 55 | 31.49 | 0.7935 | .26084 | -0.6319 | -.07099 | -0.8604 | -.16316 | 0.4140 | 1.296 |
| 50 | 28.86 | 1.1114 | .22734 | -0.7104 | -.06058 | -1.1276 | -.15668 | 0.1382 | 1.482 |
| 45 | 25.62 | 1.4234 | .19659 | -0.7901 | -.05185 | -1.4046 | -.14737 | -0.2227 | 1.681 |
| 40 | 21.85 | 1.7821 | .16905 | -0.8897 | -.04469 | -1.7277 | -.13629 | -0.7090 | 1.894 |
| 35 | 17.58 | 2.2839 | .14465 | -1.0436 | -.03897 | -2.1835 | -.12419 | -1.4041 | 2.122 |
| 30 | 12.77 | 3.1850 | .12300 | -1.3429 | -.03458 | -3.0261 | -.11152 | -2.5422 | 2.367 |
| 25 | 07.28 | 5.6371 | .10337 | -2.1959 | -.03142 | -5.4158 | -.09835 | -5.2321 | 2.638 |

$M_1 = 3.50$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9609 | .00000 | 0.0000 | .00000 | 0.0000 | 0.451 |
| 89 | 03.25 | -0.4242 | .00527 | 0.9513 | -.01667 | 0.0824 | -.00124 | 0.1434 | 0.452 |
| 88 | 06.46 | -0.8240 | .02056 | 0.9231 | -.03279 | 0.1606 | -.00488 | 0.2825 | 0.456 |
| 87 | 09.58 | -1.1782 | .04441 | 0.8782 | -.04784 | 0.2310 | -.01067 | 0.4136 | 0.462 |
| 86 | 12.57 | -1.4715 | .07471 | 0.8190 | -.06142 | 0.2905 | -.01827 | 0.5336 | 0.471 |
| 85 | 15.40 | -1.6952 | .10902 | 0.7489 | -.07328 | 0.3371 | -.02726 | 0.6405 | 0.482 |
| 84 | 18.06 | -1.8477 | .14498 | 0.6711 | -.08328 | 0.3700 | -.03718 | 0.7333 | 0.495 |
| 83 | 20.51 | -1.9326 | .18049 | 0.5888 | -.09141 | 0.3890 | -.04761 | 0.8118 | 0.510 |
| 82 | 22.77 | -1.9572 | .21393 | 0.5047 | -.09774 | 0.3948 | -.05820 | 0.8766 | 0.527 |
| 81 | 24.82 | -1.9313 | .24420 | 0.4210 | -.10244 | 0.3884 | -.06863 | 0.9286 | 0.545 |
| 80 | 26.67 | -1.8647 | .27061 | 0.3395 | -.10567 | 0.3713 | -.07868 | 0.9693 | 0.565 |
| 78 | 29.80 | -1.6480 | .31108 | 0.1875 | -.10855 | 0.3109 | -.09704 | 1.0221 | 0.609 |
| 76 | 32.22 | -1.3709 | .33603 | 0.0538 | -.10789 | 0.2247 | -.11259 | 1.0459 | 0.658 |
| 74 | 34.04 | -1.0764 | .34818 | -0.0606 | -.10493 | 0.1223 | -.12521 | 1.0492 | 0.711 |
| 72 | 35.34 | -0.7892 | .35068 | -0.1571 | -.10057 | 0.0103 | -.13509 | 1.0383 | 0.767 |
| 70 | 36.19 | -0.5220 | .34632 | -0.2378 | -.09545 | -0.1062 | -.14255 | 1.0176 | 0.827 |
| 65 | 36.85 | 0.0376 | .31858 | -0.3865 | -.08170 | -0.3997 | -.15270 | 0.9361 | 0.987 |
| 60 | 35.97 | 0.4587 | .28154 | -0.4834 | -.06870 | -0.6797 | -.15412 | 0.8203 | 1.163 |
| 55 | 34.05 | 0.7836 | .24401 | -0.5502 | -.05737 | -0.9399 | -.14987 | 0.6651 | 1.353 |
| 50 | 31.39 | 1.0524 | .20925 | -0.6012 | -.04780 | -1.1821 | -.14192 | 0.4569 | 1.558 |
| 45 | 28.16 | 1.3001 | .17821 | -0.6475 | -.03986 | -1.4148 | -.13166 | 0.1751 | 1.779 |
| 40 | 24.46 | 1.5642 | .15099 | -0.7007 | -.03338 | -1.6572 | -.12005 | -0.2098 | 2.018 |
| 35 | 20.35 | 1.9006 | .12736 | -0.7778 | -.02820 | -1.9537 | -.10784 | -0.7481 | 2.277 |
| 30 | 15.78 | 2.4312 | .10694 | -0.9159 | -.02420 | -2.4207 | -.09554 | -1.5503 | 2.558 |
| 25 | 10.68 | 3.5499 | .08920 | -1.2335 | -.02130 | -3.4561 | -.08343 | -2.9817 | 2.864 |
| 20 | 04.77 | 7.9361 | .07323 | -2.5313 | -.01942 | -7.7610 | -.07132 | -7.6495 | 3.212 |

$M_1 = 4.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9635 | .00000 | 0.0000 | .00000 | 0.0000 | 0.435 |
| 89 | 03.56 | -0.4790 | .00607 | 0.9529 | -.01671 | 0.0856 | -.00133 | 0.1538 | 0.436 |
| 88 | 07.07 | -0.9270 | .02362 | 0.9219 | -.03277 | 0.1664 | -.00523 | 0.3026 | 0.441 |
| 87 | 10.46 | -1.3176 | .05073 | 0.8727 | -.04763 | 0.2378 | -.01140 | 0.4420 | 0.447 |
| 86 | 13.71 | -1.6327 | .08471 | 0.8086 | -.06084 | 0.2967 | -.01943 | 0.5687 | 0.457 |
| 85 | 16.76 | -1.8634 | .12254 | 0.7334 | -.07215 | 0.3410 | -.02880 | 0.6805 | 0.469 |
| 84 | 19.60 | -2.0098 | .16137 | 0.6509 | -.08145 | 0.3699 | -.03903 | 0.7766 | 0.483 |
| 83 | 22.22 | -2.0787 | .19885 | 0.5648 | -.08877 | 0.3837 | -.04965 | 0.8571 | 0.499 |
| 82 | 24.60 | -2.0811 | .23326 | 0.4780 | -.09424 | 0.3834 | -.06026 | 0.9231 | 0.518 |
| 81 | 26.74 | -2.0295 | .26353 | 0.3929 | -.09805 | 0.3705 | -.07057 | 0.9759 | 0.538 |
| 80 | 28.66 | -1.9368 | .28912 | 0.3111 | -.10043 | 0.3467 | -.08036 | 1.0173 | 0.559 |
| 78 | 31.87 | -1.6726 | .32620 | 0.1615 | -.10180 | 0.2736 | -.09786 | 1.0723 | 0.607 |
| 76 | 34.32 | -1.3598 | .34660 | 0.0332 | -.09996 | 0.1766 | -.11227 | 1.1001 | 0.659 |
| 74 | 36.12 | -1.0421 | .35404 | -0.0743 | -.09616 | 0.0655 | -.12363 | 1.1099 | 0.716 |
| 72 | 37.39 | -0.7423 | .35224 | -0.1631 | -.09128 | -0.0530 | -.13226 | 1.1080 | 0.776 |
| 70 | 38.20 | -0.4704 | .34422 | -0.2360 | -.08588 | -0.1742 | -.13852 | 1.0981 | 0.839 |
| 65 | 38.74 | 0.0806 | .31014 | -0.3663 | -.07211 | -0.4736 | -.14616 | 1.0509 | 1.009 |
| 60 | 37.76 | 0.4801 | .26979 | -0.4469 | -.05960 | -0.7536 | -.14574 | 0.9764 | 1.196 |
| 55 | 35.77 | 0.7782 | .23076 | -0.4984 | -.04891 | -1.0091 | -.14018 | 0.8682 | 1.398 |
| 50 | 33.07 | 1.0159 | .19547 | -0.5339 | -.03998 | -1.2407 | -.13134 | 0.7126 | 1.619 |
| 45 | 29.85 | 1.2254 | .16444 | -0.5623 | -.03263 | -1.4530 | -.12047 | 0.4904 | 1.860 |
| 40 | 26.20 | 1.4366 | .13755 | -0.5919 | -.02665 | -1.6568 | -.10850 | 0.1759 | 2.123 |
| 35 | 22.18 | 1.6883 | .11445 | -0.6334 | -.02188 | -1.8778 | -.09615 | -0.2698 | 2.412 |
| 30 | 17.78 | 2.0544 | .09475 | -0.7066 | -.01818 | -2.1812 | -.08396 | -0.9183 | 2.728 |
| 25 | 12.94 | 2.7353 | .07801 | -0.8645 | -.01545 | -2.7671 | -.07230 | -1.9546 | 3.077 |
| 20 | 07.44 | 4.6432 | .06361 | -1.3444 | -.01364 | -4.5626 | -.06124 | -4.2388 | 3.466 |

$M_1 = 4.50$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9654 | .00000 | 0.0000 | .00000 | 0.0000 | 0.424 |
| 89 | 03.80 | -0.5236 | .00674 | 0.9539 | -.01673 | 0.0881 | -.00140 | 0.1619 | 0.425 |
| 88 | 07.54 | -1.0103 | .02611 | 0.9206 | -.03274 | 0.1707 | -.00550 | 0.3182 | 0.430 |
| 87 | 11.15 | -1.4290 | .05583 | 0.8680 | -.04744 | 0.2428 | -.01196 | 0.4639 | 0.437 |
| 86 | 14.58 | -1.7593 | .09267 | 0.8000 | -.06035 | 0.3010 | -.02029 | 0.5955 | 0.447 |
| 85 | 17.80 | -1.9929 | .13311 | 0.7208 | -.07123 | 0.3432 | -.02994 | 0.7108 | 0.460 |
| 84 | 20.78 | -2.1317 | .17396 | 0.6349 | -.07998 | 0.3688 | -.04036 | 0.8092 | 0.475 |
| 83 | 23.50 | -2.1856 | .21265 | 0.5461 | -.08668 | 0.3782 | -.05106 | 0.8911 | 0.492 |
| 82 | 25.97 | -2.1684 | .24744 | 0.4576 | -.09150 | 0.3731 | -.06164 | 0.9579 | 0.512 |
| 81 | 28.17 | -2.0957 | .27734 | 0.3716 | -.09468 | 0.3550 | -.07180 | 1.0114 | 0.533 |
| 80 | 30.14 | -1.9822 | .30197 | 0.2900 | -.09646 | 0.3262 | -.08134 | 1.0534 | 0.556 |
| 78 | 33.39 | -1.6821 | .33598 | 0.1428 | -.09679 | 0.2435 | -.09812 | 1.1106 | 0.606 |
| 76 | 35.84 | -1.3439 | .35269 | 0.0187 | -.09420 | 0.1386 | -.11164 | 1.1424 | 0.661 |
| 74 | 37.63 | -1.0110 | .35657 | -0.0834 | -.08991 | 0.0213 | -.12206 | 1.1581 | 0.720 |
| 72 | 38.86 | -0.7039 | .35166 | -0.1667 | -.08474 | -0.1018 | -.12978 | 1.1638 | 0.783 |
| 70 | 39.63 | -0.4301 | .34108 | -0.2341 | -.07923 | -0.2265 | -.13521 | 1.1631 | 0.849 |
| 65 | 40.07 | 0.1122 | .30281 | -0.3517 | -.06560 | -0.5307 | -.14112 | 1.1437 | 1.027 |
| 60 | 39.01 | 0.4953 | .26045 | -0.4215 | -.05352 | -0.8124 | -.13948 | 1.1025 | 1.222 |
| 55 | 36.96 | 0.7746 | .22059 | -0.4634 | -.04333 | -1.0670 | -.13307 | 1.0321 | 1.435 |
| 50 | 34.24 | 0.9914 | .18510 | -0.4892 | -.03489 | -1.2946 | -.12363 | 0.9190 | 1.669 |
| 45 | 31.02 | 1.1760 | .15419 | -0.5068 | -.02797 | -1.4977 | -.11236 | 0.7450 | 1.926 |
| 40 | 27.41 | 1.3541 | .12757 | -0.5227 | -.02237 | -1.6824 | -.10013 | 0.4860 | 2.210 |
| 35 | 23.45 | 1.5555 | .10485 | -0.5446 | -.01791 | -1.8639 | -.08766 | 0.1078 | 2.526 |
| 30 | 19.17 | 1.8313 | .08561 | -0.5849 | -.01444 | -2.0823 | -.07549 | -0.4456 | 2.877 |
| 25 | 14.51 | 2.3049 | .06947 | -0.6731 | -.01186 | -2.4564 | -.06403 | -1.2948 | 3.270 |
| 20 | 09.31 | 3.4366 | .05595 | -0.9157 | -.01010 | -3.4544 | -.05350 | -2.8660 | 3.709 |

$M_1 = 5.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9667 | .00000 | 0.0000 | .00000 | 0.0000 | 0.415 |
| 89 | 03.99 | -0.5598 | .00728 | 0.9546 | -.01674 | 0.0900 | -.00146 | 0.1682 | 0.417 |
| 88 | 07.90 | -1.0774 | .02814 | 0.9194 | -.03272 | 0.1740 | -.00571 | 0.3303 | 0.422 |
| 87 | 11.68 | -1.5179 | .05995 | 0.8641 | -.04728 | 0.2466 | -.01238 | 0.4809 | 0.429 |
| 86 | 15.26 | -1.8593 | .09901 | 0.7929 | -.05995 | 0.3041 | -.02094 | 0.6162 | 0.440 |
| 85 | 18.60 | -2.0935 | .14143 | 0.7107 | -.07047 | 0.3444 | -.03079 | 0.7341 | 0.453 |
| 84 | 21.68 | -2.2246 | .18369 | 0.6222 | -.07880 | 0.3672 | -.04133 | 0.8341 | 0.469 |
| 83 | 24.48 | -2.2650 | .22314 | 0.5315 | -.08503 | 0.3732 | -.05207 | 0.9171 | 0.488 |
| 82 | 27.01 | -2.2316 | .25801 | 0.4418 | -.08937 | 0.3641 | -.06259 | 0.9845 | 0.508 |
| 81 | 29.26 | -2.1417 | .28742 | 0.3555 | -.09207 | 0.3421 | -.07261 | 1.0385 | 0.530 |
| 80 | 31.25 | -2.0118 | .31113 | 0.2741 | -.09342 | 0.3092 | -.08193 | 1.0812 | 0.554 |
| 78 | 34.53 | -1.6844 | .34252 | 0.1290 | -.09303 | 0.2194 | -.09813 | 1.1404 | 0.606 |
| 76 | 36.98 | -1.3280 | .35631 | 0.0084 | -.08993 | 0.1086 | -.11096 | 1.1758 | 0.663 |
| 74 | 38.74 | -0.9847 | .35749 | -0.0898 | -.08533 | -0.0133 | -.12068 | 1.1966 | 0.724 |
| 72 | 39.94 | -0.6731 | .35029 | -0.1689 | -.08001 | -0.1398 | -.12773 | 1.2087 | 0.789 |
| 70 | 40.68 | -0.3989 | .33789 | -0.2323 | -.07444 | -0.2671 | -.13255 | 1.2155 | 0.858 |
| 65 | 41.04 | 0.1358 | .29675 | -0.3410 | -.06100 | -0.5754 | -.13724 | 1.2187 | 1.041 |
| 60 | 39.91 | 0.5064 | .25310 | -0.4033 | -.04927 | -0.8592 | -.13475 | 1.2044 | 1.243 |
| 55 | 37.83 | 0.7720 | .21278 | -0.4386 | -.03947 | -1.1147 | -.12774 | 1.1647 | 1.465 |
| 50 | 35.09 | 0.9741 | .17723 | -0.4579 | -.03140 | -1.3417 | -.11789 | 1.0865 | 1.709 |
| 45 | 31.87 | 1.1415 | .14645 | -0.4684 | -.02481 | -1.5413 | -.10634 | 0.9524 | 1.979 |
| 40 | 28.28 | 1.2973 | .12007 | -0.4756 | -.01950 | -1.7167 | -.09394 | 0.7396 | 2.282 |
| 35 | 24.37 | 1.4659 | .09762 | -0.4855 | -.01527 | -1.8770 | -.08136 | 0.4161 | 2.622 |
| 30 | 20.17 | 1.6857 | .07870 | -0.5066 | -.01198 | -2.0474 | -.06917 | -0.0671 | 3.006 |
| 25 | 15.64 | 2.0418 | .06294 | -0.5578 | -.00952 | -2.3046 | -.05782 | -0.8026 | 3.441 |
| 20 | 10.67 | 2.8144 | .04995 | -0.6980 | -.00782 | -2.9304 | -.04757 | -2.0544 | 3.933 |

$M_1 = 6.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9684 | .00000 | 0.0000 | .00000 | 0.0000 | 0.404 |
| 89 | 04.25 | -0.6135 | .00808 | 0.9553 | -.01675 | 0.0928 | -.00154 | 0.1773 | 0.406 |
| 88 | 08.42 | -1.1762 | .03115 | 0.9174 | -.03267 | 0.1786 | -.00601 | 0.3476 | 0.411 |
| 87 | 12.43 | -1.6474 | .06598 | 0.8581 | -.04703 | 0.2517 | -.01298 | 0.5049 | 0.419 |
| 86 | 16.21 | -2.0027 | .10821 | 0.7825 | -.05934 | 0.3079 | -.02185 | 0.6453 | 0.431 |
| 85 | 19.73 | -2.2352 | .15329 | 0.6960 | -.06937 | 0.3454 | -.03194 | 0.7667 | 0.445 |
| 84 | 22.94 | -2.3527 | .19735 | 0.6040 | -.07709 | 0.3639 | -.04263 | 0.8689 | 0.462 |
| 83 | 25.85 | -2.3718 | .23756 | 0.5107 | -.08266 | 0.3648 | -.05337 | 0.9532 | 0.482 |
| 82 | 28.45 | -2.3136 | .27225 | 0.4197 | -.08634 | 0.3501 | -.06377 | 1.0215 | 0.503 |
| 81 | 30.76 | -2.1986 | .30069 | 0.3331 | -.08842 | 0.3224 | -.07355 | 1.0763 | 0.527 |
| 80 | 32.78 | -2.0453 | .32285 | 0.2523 | -.08919 | 0.2840 | -.08254 | 1.1201 | 0.552 |
| 78 | 36.08 | -1.6807 | .35029 | 0.1106 | -.08789 | 0.1844 | -.09788 | 1.1828 | 0.607 |
| 76 | 38.51 | -1.3007 | .35994 | -0.0051 | -.08418 | 0.0656 | -.10975 | 1.2239 | 0.667 |
| 74 | 40.23 | -0.9449 | .35743 | -0.0977 | -.07924 | -0.0624 | -.11851 | 1.2526 | 0.731 |
| 72 | 41.39 | -0.6288 | .34723 | -0.1712 | -.07377 | -0.1937 | -.12468 | 1.2744 | 0.799 |
| 70 | 42.08 | -0.3550 | .33249 | -0.2293 | -.06820 | -0.3246 | -.12870 | 1.2926 | 0.870 |
| 65 | 42.32 | 0.1676 | .28782 | -0.3264 | -.05509 | -0.6392 | -.13183 | 1.3293 | 1.063 |
| 60 | 41.11 | 0.5211 | .24270 | -0.3793 | -.04388 | -0.9274 | -.12826 | 1.3547 | 1.274 |
| 55 | 38.97 | 0.7685 | .20194 | -0.4064 | -.03462 | -1.1864 | -.12051 | 1.3609 | 1.508 |
| 50 | 36.20 | 0.9515 | .16642 | -0.4179 | -.02706 | -1.4160 | -.11015 | 1.3357 | 1.768 |
| 45 | 32.98 | 1.0973 | .13589 | -0.4199 | -.02092 | -1.6160 | -.09825 | 1.2634 | 2.059 |
| 40 | 29.42 | 1.2257 | .10985 | -0.4171 | -.01600 | -1.7865 | -.08561 | 1.1233 | 2.390 |
| 35 | 25.58 | 1.3552 | .08778 | -0.4134 | -.01210 | -1.9300 | -.07289 | 0.8865 | 2.770 |
| 30 | 21.49 | 1.5110 | .06927 | -0.4141 | -.00908 | -2.0563 | -.06066 | 0.5097 | 3.210 |
| 25 | 17.13 | 1.7423 | .05395 | -0.4289 | -.00682 | -2.2001 | -.04936 | -0.0791 | 3.724 |
| 20 | 12.44 | 2.1883 | .04152 | -0.4830 | -.00522 | -2.4907 | -.03936 | -1.0335 | 4.324 |

$M_1 = 7.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9694 | .00000 | 0.0000 | .00000 | 0.0000 | 0.397 |
| 89 | 04.43 | -0.6499 | .00863 | 0.9557 | -.01676 | 0.0946 | -.00159 | 0.1832 | 0.399 |
| 88 | 08.76 | -1.2429 | .03319 | 0.9159 | -.03263 | 0.1816 | -.00620 | 0.3589 | 0.404 |
| 87 | 12.92 | -1.7339 | .07005 | 0.8539 | -.04685 | 0.2549 | -.01336 | 0.5206 | 0.413 |
| 86 | 16.83 | -2.0971 | .11432 | 0.7754 | -.05892 | 0.3102 | -.02242 | 0.6642 | 0.425 |
| 85 | 20.45 | -2.3269 | .16106 | 0.6862 | -.06862 | 0.3455 | -.03266 | 0.7878 | 0.440 |
| 84 | 23.75 | -2.4338 | .20613 | 0.5920 | -.07595 | 0.3611 | -.04342 | 0.8913 | 0.458 |
| 83 | 26.73 | -2.4377 | .24667 | 0.4973 | -.08111 | 0.3586 | -.05414 | 0.9763 | 0.478 |
| 82 | 29.37 | -2.3624 | .28105 | 0.4055 | -.08437 | 0.3402 | -.06444 | 1.0453 | 0.500 |
| 81 | 31.71 | -2.2306 | .30869 | 0.3189 | -.08607 | 0.3088 | -.07404 | 1.1007 | 0.525 |
| 80 | 33.75 | -2.0623 | .32973 | 0.2387 | -.08650 | 0.2669 | -.08280 | 1.1453 | 0.551 |
| 78 | 37.05 | -1.6744 | .35446 | 0.0993 | -.08467 | 0.1610 | -.09757 | 1.2107 | 0.608 |
| 76 | 39.46 | -1.2804 | .36144 | -0.0131 | -.08063 | 0.0373 | -.10882 | 1.2559 | 0.669 |
| 74 | 41.16 | -0.9178 | .35663 | -0.1022 | -.07551 | -0.0945 | -.11700 | 1.2902 | 0.736 |
| 72 | 42.28 | -0.5997 | .34462 | -0.1722 | -.06999 | -0.2288 | -.12262 | 1.3188 | 0.806 |
| 70 | 42.94 | -0.3269 | .32851 | -0.2271 | -.06445 | -0.3621 | -.12617 | 1.3446 | 0.879 |
| 65 | 43.11 | 0.1873 | .28185 | -0.3173 | -.05158 | -0.6811 | -.12837 | 1.4041 | 1.077 |
| 60 | 41.84 | 0.5299 | .23598 | -0.3647 | -.04072 | -0.9730 | -.12417 | 1.4565 | 1.295 |
| 55 | 39.67 | 0.7663 | .19503 | -0.3871 | -.03181 | -1.2358 | -.11600 | 1.4944 | 1.537 |
| 50 | 36.88 | 0.9380 | .15960 | -0.3941 | -.02456 | -1.4693 | -.10534 | 1.5065 | 1.808 |
| 45 | 33.65 | 1.0711 | .12927 | -0.3915 | -.01871 | -1.6730 | -.09323 | 1.4788 | 2.114 |
| 40 | 30.11 | 1.1837 | .10346 | -0.3832 | -.01403 | -1.8457 | -.08045 | 1.3926 | 2.465 |
| 35 | 26.31 | 1.2914 | .08163 | -0.3724 | -.01034 | -1.9869 | -.06764 | 1.2218 | 2.875 |
| 30 | 22.28 | 1.4128 | .06335 | -0.3629 | -.00750 | -2.0992 | -.05536 | 0.9265 | 3.360 |
| 25 | 18.04 | 1.5811 | .04827 | -0.3608 | -.00538 | -2.1977 | -.04408 | 0.4419 | 3.941 |
| 20 | 13.52 | 1.8815 | .03612 | -0.3800 | -.00388 | -2.3494 | -.03416 | -0.3542 | 4.641 |

$M_1 = 8.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9701 | .00000 | 0.0000 | .00000 | 0.0000 | 0.393 |
| 89 | 04.55 | -0.6755 | .00902 | 0.9558 | -.01677 | 0.0958 | -.00163 | 0.1873 | 0.395 |
| 88 | 09.00 | -1.2895 | .03463 | 0.9148 | -.03260 | 0.1836 | -.00633 | 0.3666 | 0.400 |
| 87 | 13.26 | -1.7939 | .07288 | 0.8510 | -.04672 | 0.2570 | -.01362 | 0.5313 | 0.409 |
| 86 | 17.26 | -2.1620 | .11854 | 0.7704 | -.05863 | 0.3115 | -.02281 | 0.6771 | 0.422 |
| 85 | 20.95 | -2.3891 | .16636 | 0.6794 | -.06810 | 0.3454 | -.03314 | 0.8020 | 0.437 |
| 84 | 24.30 | -2.4879 | .21206 | 0.5837 | -.07516 | 0.3590 | -.04393 | 0.9064 | 0.455 |
| 83 | 27.32 | -2.4808 | .25274 | 0.4881 | -.08004 | 0.3540 | -.05463 | 0.9919 | 0.476 |
| 82 | 29.99 | -2.3935 | .28682 | 0.3959 | -.08303 | 0.3331 | -.06485 | 1.0613 | 0.499 |
| 81 | 32.34 | -2.2503 | .31385 | 0.3094 | -.08448 | 0.2992 | -.07433 | 1.1172 | 0.524 |
| 80 | 34.39 | -2.0717 | .33407 | 0.2296 | -.08469 | 0.2549 | -.08292 | 1.1624 | 0.550 |
| 78 | 37.70 | -1.6685 | .35693 | 0.0919 | -.08253 | 0.1449 | -.09731 | 1.2298 | 0.608 |
| 76 | 40.10 | -1.2655 | .36210 | -0.0183 | -.07829 | 0.0179 | -.10814 | 1.2781 | 0.672 |
| 74 | 41.77 | -0.8989 | .35579 | -0.1050 | -.07308 | -0.1165 | -.11593 | 1.3163 | 0.739 |
| 72 | 42.87 | -0.5798 | .34259 | -0.1727 | -.06754 | -0.2527 | -.12120 | 1.3497 | 0.810 |
| 70 | 43.51 | -0.3080 | .32562 | -0.2255 | -.06202 | -0.3876 | -.12443 | 1.3810 | 0.885 |
| 65 | 43.62 | 0.2003 | .27774 | -0.3113 | -.04934 | -0.7099 | -.12605 | 1.4565 | 1.087 |
| 60 | 42.32 | 0.5357 | .23143 | -0.3552 | -.03871 | -1.0047 | -.12145 | 1.5279 | 1.310 |
| 55 | 40.12 | 0.7648 | .19041 | -0.3747 | -.03003 | -1.2708 | -.11300 | 1.5882 | 1.558 |
| 50 | 37.32 | 0.9292 | .15506 | -0.3789 | -.02299 | -1.5080 | -.10216 | 1.6272 | 1.836 |
| 45 | 34.09 | 1.0542 | .12487 | -0.3734 | -.01733 | -1.7157 | -.08992 | 1.6322 | 2.153 |
| 40 | 30.56 | 1.1569 | .09922 | -0.3617 | -.01281 | -1.8925 | -.07705 | 1.5867 | 2.519 |
| 35 | 26.78 | 1.2511 | .07755 | -0.3467 | -.00927 | -2.0364 | -.06418 | 1.4669 | 2.952 |
| 30 | 22.80 | 1.3516 | .05942 | -0.3313 | -.00655 | -2.1464 | -.05187 | 1.2366 | 3.471 |
| 25 | 18.63 | 1.4831 | .04449 | -0.3198 | -.00453 | -2.2275 | -.04057 | 0.8351 | 4.107 |
| 20 | 14.22 | 1.7040 | .03250 | -0.3213 | -.00311 | -2.3144 | -.03070 | 0.1529 | 4.896 |

$M_1 = 9.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9705 | .00000 | 0.0000 | .00000 | 0.0000 | 0.390 |
| 89 | 04.63 | -0.6939 | .00930 | 0.9560 | -.01677 | 0.0967 | -.00165 | 0.1902 | 0.392 |
| 88 | 09.16 | -1.3230 | .03566 | 0.9139 | -.03258 | 0.1851 | -.00643 | 0.3721 | 0.397 |
| 87 | 13.49 | -1.8369 | .07491 | 0.8489 | -.04663 | 0.2585 | -.01381 | 0.5389 | 0.407 |
| 86 | 17.56 | -2.2081 | .12155 | 0.7669 | -.05842 | 0.3124 | -.02308 | 0.6861 | 0.419 |
| 85 | 21.30 | -2.4330 | .17013 | 0.6746 | -.06772 | 0.3452 | -.03347 | 0.8120 | 0.435 |
| 84 | 24.69 | -2.5257 | .21624 | 0.5779 | -.07460 | 0.3573 | -.04428 | 0.9170 | 0.453 |
| 83 | 27.73 | -2.5105 | .25696 | 0.4816 | -.07928 | 0.3506 | -.05496 | 1.0029 | 0.475 |
| 82 | 30.42 | -2.4146 | .29080 | 0.3892 | -.08209 | 0.3279 | -.06512 | 1.0726 | 0.498 |
| 81 | 32.79 | -2.2631 | .31737 | 0.3027 | -.08336 | 0.2922 | -.07450 | 1.1289 | 0.523 |
| 80 | 34.85 | -2.0774 | .33698 | 0.2233 | -.08342 | 0.2463 | -.08298 | 1.1745 | 0.550 |
| 78 | 38.15 | -1.6636 | .35849 | 0.0868 | -.08104 | 0.1334 | -.09709 | 1.2435 | 0.609 |
| 76 | 40.54 | -1.2546 | .36240 | -0.0218 | -.07667 | 0.0041 | -.10764 | 1.2940 | 0.673 |
| 74 | 42.20 | -0.8853 | .35505 | -0.1068 | -.07140 | -0.1321 | -.11515 | 1.3351 | 0.742 |
| 72 | 43.28 | -0.5657 | .34106 | -0.1730 | -.06585 | -0.2697 | -.12018 | 1.3720 | 0.814 |
| 70 | 43.90 | -0.2947 | .32349 | -0.2243 | -.06036 | -0.4058 | -.12321 | 1.4072 | 0.889 |
| 65 | 43.98 | 0.2093 | .27481 | -0.3071 | -.04781 | -0.7304 | -.12442 | 1.4942 | 1.094 |
| 60 | 42.65 | 0.5396 | .22822 | -0.3487 | -.03735 | -1.0275 | -.11955 | 1.5794 | 1.320 |
| 55 | 40.43 | 0.7638 | .18717 | -0.3661 | -.02883 | -1.2962 | -.11092 | 1.6561 | 1.572 |
| 50 | 37.62 | 0.9231 | .15189 | -0.3685 | -.02194 | -1.5365 | -.09996 | 1.7150 | 1.856 |
| 45 | 34.39 | 1.0427 | .12181 | -0.3611 | -.01641 | -1.7479 | -.08763 | 1.7445 | 2.181 |
| 40 | 30.87 | 1.1388 | .09627 | -0.3473 | -.01201 | -1.9289 | -.07469 | 1.7300 | 2.559 |
| 35 | 27.11 | 1.2239 | .07471 | -0.3295 | -.00856 | -2.0770 | -.06178 | 1.6501 | 3.008 |
| 30 | 23.16 | 1.3107 | .05669 | -0.3104 | -.00593 | -2.1893 | -.04944 | 1.4719 | 3.555 |
| 25 | 19.03 | 1.4186 | .04186 | -0.2932 | -.00399 | -2.2654 | -.03814 | 1.1391 | 4.236 |
| 20 | 14.71 | 1.5908 | .02996 | -0.2843 | -.00262 | -2.3215 | -.02828 | 0.5492 | 5.101 |

$M_1 = 10.00$ $\gamma = 1.4$

| $\beta^o$ | $\theta^o$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | M |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 00.00 | 0.0000 | .00000 | 0.9708 | .00000 | 0.0000 | .00000 | 0.0000 | 0.388 |
| 89 | 04.70 | -0.7076 | .00951 | 0.9560 | -.01677 | 0.0973 | -.00167 | 0.1923 | 0.390 |
| 88 | 09.28 | -1.3479 | .03643 | 0.9133 | -.03256 | 0.1861 | -.00650 | 0.3761 | 0.395 |
| 87 | 13.67 | -1.8686 | .07642 | 0.8473 | -.04656 | 0.2595 | -.01394 | 0.5444 | 0.405 |
| 86 | 17.78 | -2.2419 | .12377 | 0.7642 | -.05826 | 0.3131 | -.02327 | 0.6928 | 0.417 |
| 85 | 21.55 | -2.4650 | .17288 | 0.6710 | -.06745 | 0.3451 | -.03371 | 0.8194 | 0.433 |
| 84 | 24.98 | -2.5531 | .21927 | 0.5736 | -.07419 | 0.3560 | -.04453 | 0.9248 | 0.452 |
| 83 | 28.03 | -2.5318 | .26002 | 0.4769 | -.07873 | 0.3481 | -.05520 | 1.0110 | 0.474 |
| 82 | 30.74 | -2.4295 | .29366 | 0.3843 | -.08140 | 0.3241 | -.06531 | 1.0808 | 0.497 |
| 81 | 33.11 | -2.2720 | .31986 | 0.2979 | -.08254 | 0.2871 | -.07462 | 1.1374 | 0.523 |
| 80 | 35.17 | -2.0810 | .33903 | 0.2188 | -.08250 | 0.2399 | -.08301 | 1.1834 | 0.550 |
| 78 | 38.47 | -1.6596 | .35935 | 0.0832 | -.07996 | 0.1249 | -.09691 | 1.2535 | 0.610 |
| 76 | 40.86 | -1.2464 | .36254 | -0.0243 | -.07551 | -0.0060 | -.10726 | 1.3057 | 0.674 |
| 74 | 42.50 | -0.8753 | .35444 | -0.1081 | -.07020 | -0.1435 | -.11457 | 1.3490 | 0.743 |
| 72 | 43.57 | -0.5555 | .33988 | -0.1731 | -.06465 | -0.2821 | -.11943 | 1.3885 | 0.816 |
| 70 | 44.18 | -0.2850 | .32190 | -0.2234 | -.05918 | -0.4190 | -.12231 | 1.4266 | 0.893 |
| 65 | 44.23 | 0.2158 | .27266 | -0.3041 | -.04673 | -0.7454 | -.12323 | 1.5222 | 1.099 |
| 60 | 42.89 | 0.5424 | .22589 | -0.3440 | -.03639 | -1.0443 | -.11818 | 1.6176 | 1.328 |
| 55 | 40.65 | 0.7630 | .18483 | -0.3600 | -.02799 | -1.3151 | -.10942 | 1.7066 | 1.583 |
| 50 | 37.84 | 0.9188 | .14960 | -0.3611 | -.02120 | -1.5580 | -.09837 | 1.7804 | 1.871 |
| 45 | 34.61 | 1.0345 | .11960 | -0.3524 | -.01576 | -1.7726 | -.08598 | 1.8287 | 2.201 |
| 40 | 31.09 | 1.1259 | .09414 | -0.3370 | -.01144 | -1.9574 | -.07300 | 1.8381 | 2.588 |
| 35 | 27.34 | 1.2047 | .07266 | -0.3174 | -.00807 | -2.1099 | -.06006 | 1.7898 | 3.051 |
| 30 | 23.41 | 1.2821 | .05471 | -0.2957 | -.00551 | -2.2262 | -.04770 | 1.6536 | 3.620 |
| 25 | 19.32 | 1.3738 | .03995 | -0.2747 | -.00362 | -2.3029 | -.03639 | 1.3780 | 4.337 |
| 20 | 15.05 | 1.5136 | .02812 | -0.2592 | -.00230 | -2.3453 | -.02653 | 0.8664 | 5.267 |

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