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On a Criterion of the Applicability of the Magnetohydrodynamic Equations to a Plasma

S. I. BRAGINSKII

Vopr. Magnitnoi Gidrodin. i Dinamiki Plazmy, Akad. Nauk Latv. SSR
Riga, 1959, pp. 67 - 71

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ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE RESEARCH DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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The magnetohydrodynamic equations are often used in considering the phenomena in a rarefied plasma, in particular, in considering the phenomena of astrophysics. A sufficiently rigorous description of the rarefied plasma s yielded by the use of a system of kinetic equations, however, this method appears to be much too complex. A simple criterion can be obtained which will indicate under which conditions use of the magnetohydrodynamic equations and, in particular, their fundamental corollary on the ''adhesion'' of the magnetic lines of force to the substance, affords a possibility of obtaining at least qualitatively correct results in a very rough approximation. In order to obtain this criterion, it is convenient to use the model of a plasma in the form of a mixture of two charged gases, an electronic and an ionic gas [1-3]. From the equations of particle balance and momentum balance, we obtain by summing the appropriate equations for ions and electrons:

(1) $$\frac{\partial \rho}{\partial t} + \text{div}\, \rho\vec{v} = 0$$

(2) $$\rho \frac{d\vec{v}}{dt} = -\nabla P + \frac{1}{c}\left[\vec{j}\vec{H}\right]$$

where ρ is the density; $\vec{j} = en(\vec{v} - \vec{v}_e)$ the current density; n the number of ions per unit volume; $\vec{v}$ the velocity of the ionic gas; $\vec{v}_e$ the velocity of the electronic gas; $\vec{H}$ the magnetic field; ∇P the divergence of the pressure tensor which reduces, in the simplest cases, to the scalar pressure gradient $\nabla(nT_e = nT_i)$, where T_e, T_i are the electron and ion temperatures. Quasi-neutrality of the plasma is assumed

We neglect electron inertia in equation (2) since we are here interested only in low-frequency motions related to displacement of the.substance.

Furthermore, let us also assume that the frequencies of the motions are small in comparison with $\frac{c}{L}$ (where L is the characteristic dimension) so that displacement currents can be neglected.

The equation for the magnetic field which leads to ''adhesion'' of the magnetic lines of force

(3) $$\frac{\partial H}{\partial t} = \text{rot}\,\left[\vec{v}\,\vec{H}\right]$$

follows from the Maxwell equation

(4) $$\frac{\partial \vec{H}}{\partial t} = - c \cdot \text{rot}\, \vec{E}$$

and from the condition

(5) $$\vec{E} + \left[\frac{\vec{v}}{c}\vec{H}\right] = 0$$

This latter is obtained in magnetohydrodynamics from the conventional Ohm's law for a moving medium under the assumption of infinite conductivity.

Using the two-fluid model of a plasma and assuming infinite conductivity we obtain in place of (5)

(6) $$-\nabla P_e - en\left(\vec{E} + \left[\frac{\vec{v}}{c}\vec{H}\right]\right) + \frac{1}{c}\left[\vec{j}\,\vec{H}\right] = 0$$

Equation (6) is the equilibrium condition for an electron gas; here we neglect electron inertia, as we did in (2), since high frequencies are not considered.

In order that (5) could be used, the second term in (6) must be the principal one. Let us compare the terms $\vec{F}_H = \frac{1}{c}[\vec{j}\vec{H}]$ and $\frac{en}{c}[\vec{v}\vec{H}]$. Using $\vec{j} = \frac{c}{4\pi}\,\text{rot}\,\vec{H} \sim \frac{c}{4\pi}\frac{\tilde{H}}{L}$, where L is the characteristic dimension of the system, $\tilde{H}$ the part of the magnetic field varying along the extent of the system, and using also the relation $\rho v^2 \sim \frac{\tilde{H}^2}{4\pi}$, which is usual in magnetohydrodynamics, we obtain in order of magnitude

(7) $$\frac{F_H}{\frac{en}{c}vH} \sim \frac{j}{env} \sim \left(\frac{Mc^2}{4\pi e^2 nL^2}\right)^{\frac{1}{2}}$$

where M is the ion mass (or the order of magnitude of the ion mass during incomplete ionization).

Hence, the condition that the last term in (6) be small is $\Pi \gg 1$, where

(8) $$\Pi = \frac{4\pi e^2 nL^2}{Mc^2}$$

or, if the number of particles per unit length of the system under consideration is introduced $N \sim nL^2$, then

(8') $$\Pi = \frac{e^2 N}{Mc^2}$$

A similar estimate shows that the first term in (6) is also small for $\Pi \gg 1$ if $nT \leq \frac{H^2}{8\pi}$.

If we use (6) instead of (5), then additional terms appear in (3) and it becomes

(3') $$\frac{\partial \vec{H}}{\partial t} = \text{rot}\left[\vec{v}\,\vec{H}\right] - \text{rot}\left\{\frac{c}{en}\left(\frac{1}{c}\left[\vec{j}\,\vec{H}\right] - \nabla P_e\right)\right\}$$

If (2) is used, or if the equation of ion-momentum balance is taken at once instead of (6), then using the transformation $\mathrm{rot}\,\frac{d\vec{v}}{dt} = \frac{\partial\,\mathrm{rot}\,\vec{v}}{\partial t} - \mathrm{rot}\,[\vec{v}\,\mathrm{rot}\,\vec{v}]$, we obtain

$$(3'') \qquad \frac{\partial}{\partial t}\left(\frac{e\vec{H}}{Mc} + \mathrm{rot}\,\vec{v}\right) = \mathrm{rot}\left[\vec{v}, \frac{e\vec{H}}{Mc} + \mathrm{rot}\,\vec{v}\right] - \mathrm{rot}\left(\frac{\nabla P_i}{\rho}\right)$$

In those cases when we can put $P_i = nT_i$ and $[\nabla n, \nabla T_i] = 0$, the last term vanishes.

Hence, under the same conditions, when the theorem on the ''adhesion'' of vortex lines to a substance in valid in the conventional hydrodynamics of an ideal fluid, the lines $\frac{e\vec{H}}{Mc} + \mathrm{rot}\,\vec{v}$ are ''glued'' in plasma dynamics. If $\rho v^2 \sim \frac{\tilde{H}^2}{4\pi}$, then we obtain $|\mathrm{rot}\,\vec{v}|/(\frac{eH}{Mc}) \ll 1$ for $\Pi \gg 1$ [the last term in (3'') is small here] so that the representation of the ''adhesion'' of magnetic lines of force is approximately valid.

The ratio $\frac{j}{env} = \frac{|\vec{v} - \vec{v}_e|}{v} \sim \Pi^{-\frac{1}{2}}$ is small for $\Pi \gg 1$. The electron velocity is here close to the ion velocity even if collisions are absent. In this case the electrons are ''bound'' to the ions in this case as a result of collective interaction: An increase in the difference in the ion and electron velocities appears to be impossible because of the appearance of very large currents and magnetic fields. This circumstance is also the reason that even the condition $\Pi \gg 1$ permits the consideration that the magnetic lines of force are ''glued'' to the substance , i.e., to the ions, under infrequent collisions.

As is known, the quasi-neutrality condition of the plasma is that the Debye radius δ_D be small in comparison with the characteristic dimensions. It is easy to see that

$$(9) \qquad \frac{\delta_D^2}{L^2} \sim \frac{T}{Mc^2} \cdot \frac{1}{\Pi}$$

so that the condition $\Pi \gg 1$ is more rigorous and the quasi-neutrality is guaranteed with a large margin when it is fulfilled.

A number of other important plasma characteristics [4] are related to the parameter Π. Thus, for example, the Larmor radius of the ions r_i is small in comparison with the system dimensions of $\Pi \gg 1$ (if $nT \leq H^2/8\pi$):

$$(10) \qquad \frac{r_i}{L} \sim \frac{1}{\Pi^{\frac{1}{2}}}$$

Equalization of the temperatures between the electron and ion gases is also determined by the parameter Π . If the Joulean heat of the electrons $\sim \frac{j^2}{\frac{e^2 n\tau}{m}}$ and the heat transfer from the electrons to the ions $\sim \frac{m}{M}\frac{n}{\tau}(T_e - T_i)$ are equated in order of magnitude, where $\frac{1}{\tau}$ is the frequency of electron collision with ions, we obtain (for $nT \sim H^2/8\pi$):

$$\frac{T_e - T_i}{T} \sim \frac{1}{\Pi}$$

The condition $\Pi \gg 1$ is usually satisfied well for astrophysical objects. If M is the proton mass, then $\Pi \sim \frac{N}{10^{16}}$. Assuming $L \sim 10^9$ cm $n \sim 10^8$ particles/cm^3 for a solar protuberance, for example, we obtain $N \sim 10^{26}$ cm^{-1}, $\Pi \sim 10^{10}$ and assuming $L \sim 10^{19}$ cm , $n \sim 1$ for interstellar gas clouds, we obtain $N \sim 10^{38}$, $\Pi \sim 10^{22}$.

In those cases when the mean free path is small so that hydrodynamic considerations are applicable, the condition $\Pi \gg 1$ affords the possibility of transforming from the two gas model to conventional magnetohydrodynamics in a first approximation. If collisions are so infrequent that the hydrodynamic consideration is generally inapplicable, strictly speaking, then the magnetohydrodynamic equations with $\Pi \gg 1$ can still yield qualitatively true results, simulating more rigorous considerations, in a number of cases even under very rough simplifications while the results obtained by using magnetohydrodynamics are completely untrue for $\Pi < 1$. If, for example, small plasma oscillations are considered by using the two gas model [5], then the low-frequency branches of the oscillations agree with the Alfven waves obtained in magnetohydrodynamics and the two magnetosonic waves when $\Pi \gg 1$ (it is necessary to take $\frac{1}{k}$ for L , where k is the wave number). The dispersion relations and type of oscillations appear to be completely different for $\Pi \ll 1$. The kinetic consideration for a rarefied plasma shows, moreover, that it is impossible to take the adiabatic exponent equal to $\frac{5}{3}$ and also that strong wave attenuation can exist in a number of cases; these effects are not taken into account in magnetohydrodynamics.

Moscow July 1958

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