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The Detection of Radar Echoes from the Sun

by
R. C. Barthle

Scientific Report No. 9
August 24, 1960

PREPARED UNDER
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RADIOSCIENCE LABORATORY
STANFORD ELECTRONICS LABORATORIES
STANFORD UNIVERSITY · STANFORD, CALIFORNIA

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SUMMARY

During several periods in 1958 and 1959 experiments were conducted at Stanford University for the purpose of obtaining a radar echo from the sun. Using a 40-kilowatt transmitter, coded sequences of transmitted pulses, a broadside array of rhombic antennas, conventional receiving equipment, and a detection process which included the use of a digital computer, solar echoes were obtained in April 1959. They were obtained again in a somewhat different experiment conducted in September, 1959.

As previously reported, the April 1959 experiments utilized a transmission code sequence of rectangular pulses of thirty-second duration with a period of one minute. Detailed analysis of echo indication curve shapes and echo-indication positions yielded conclusive evidence of the receipt of solar echoes. Numerical values of the probability that the received solar echo indications in these trials could be caused by chance were computed to be on the order of one chance in a million.

For the September 1959 trials a random length sequence of transmitted pulses was used with a higher-gain antenna arrangement. Increased solar activity during the period, along with equipment difficulties, prevented realization of the anticipated increase in the output signal-to-noise ratios.

A study of the correlation characteristics of adjacent frequency bands in the solar-echo background-noise spectrum revealed correlation coefficients of consistently high values. The results of this study were used in a data reduction technique which resulted in a net improvement of the output signal-to-noise ratios. The improvement made possible by this technique was sufficient to enable detection of radar echoes from the sun in the September, 1959, trials with computed error probabilities which are numerically smaller than those obtained in the April 1959 trials.

Information obtained from the results of the initial solar echo trials will permit the detailed planning of future experiments from which additional knowledge may be gained of the solar corona, the solar magnetic fields, and the characteristics of the solar radio noise spectrum.

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I. INTRODUCTION

On a number of mornings in September, 1958, April, 1959, and September, 1959, attempts were made at Stanford University to obtain radar echoes from the sun. The data have been intensively analyzed with the aid of an electronic digital computer. Results indicate that solar echoes were obtained on three mornings in April of 1959[1] and on two mornings in September of 1959.

The purpose of the experiment was to determine the feasibility of contacting the sun by radar. The experiment, if successful, would then open the way to controlled radar experiments for the purpose of gaining new scientific information about the sun and its effects on the earth. In 1952, Kerr[2] discussed the fundamental requirements for a solar radar experiment and suggested several interesting areas of investigation. Specifically, knowledge of the corona, which is extremely difficult to examine optically, could be increased by radar studies since the echoes are returned from high in the solar atmosphere. Also, streams of ionized particles emitted by the sun, which cause auroras, communication blackouts, and other terrestrial ionospheric effects, might be detected either in the vicinity of the sun or enroute toward the earth.

This dissertation is a detailed discussion of the data analysis methods used in the detection of hf (3 to 30 megacycle) radar echoes from the sun and is intended to complement and amplify the earlier publication[1] which summarizes the solar echo project and its initial success. The data analysis material for the publication, "Radar Echoes from the Sun", was assembled by the present author who accomplished the recording, reduction, and analysis operations with advice and guidance from the co-authors. Discussed for the first time in this dissertation are the results of the author's investigations of the correlation characteristics of adjacent frequency bands in the solar radio noise spectrum. Application of these correlation characteristics aided materially in confirmation of the receipt of radar echoes from the sun in September, 1959, notwithstanding the high solar noise activity during this period and the apparent loss of system sensitivity due to antenna difficulties.

II. GENERAL CONSIDERATIONS OF SOLAR ECHO DETECTION

A. INTRODUCTION

Radar originated with radio-echo studies of the ionosphere some 35 years ago. About 14 years ago the first radar detection of the moon was accomplished. Radar's use in studies of the solar system beyond the moon, however, is very recent since contact with the next nearest planet beyond the moon required an increase in system sensitivity of more than 70 decibels over moon echo requirements. Achievement of radar detection of Venus and the sun completes a large step in the use of radar in solar system studies since, in a logarithmic sense, it marks the half-way point to the limit of the solar system. A similar increase in system sensitivity (another 70 decibels) will make possible the detection of all of the planets, with the possible exception of Pluto.

Compared with the methods of optical and radio astronomy, radar astronomy has the primary advantage of controlled transmissions and the resultant possibilities of obtaining new knowledge through the observation of effects of both the reflecting surface and the intervening medium on transmitted electromagnetic energy of known characteristics.

The equipment and detailed techniques required for radar detection of the sun[1,2,4] differ in several ways from those required for detection of the nearer planets.[4-8] Specifically, the following fundamental differences exist:

(1) A relatively low radio frequency is needed for sun echoes, to avoid excessive absorption in the solar corona above the reflection points. On the other hand, the frequency must be high enough to pass through the earth's ionosphere. Both Kerr[2] and Bass and Braude[4], from considerations of electron temperature and density in the solar corona, estimate the optimum frequency to be within the range 25 to 30 megacycles. Much higher frequencies can be used for "hard" targets such as the planets, so that higher antenna gains can be realized for a given antenna size.

(2) The new low-noise receiving devices (masers, parametric amplifiers) which are extremely important for radar sensitivity at the higher frequencies, are of no value at the lower frequencies since galactic and

solar noise, rather than receiver noise, limit the detectability of weak signals.

(3) Solar noise is characterized by non-stationary statistics with occasional amplitude increases as great as tens of decibels. Solar noise bursts, with their large amplitude and long duration, can easily cause spurious echo indications which must be identified if solar echoes are to be detected. The task of recognizing a weak echo in a solar noise background which has a time-varying frequency spectrum is thus more difficult than if the echo were located in a background of random receiver or galactic noise, which can be easily handled using well-known statistical techniques.

One costly feature of the use of radar in solar system studies, that of reflected energy being inversely proportional to the fourth power of the target range, is well illustrated in the expression for power at the input terminals of the receiver of a radar system. The system is activated by a transmitter operating for t_1 seconds at a power level of P_t watts into an antenna of effective area A. Let the power-reflecting cross-section of a target such as the sun at range R be $f(\tau)$, a function of the time delay τ; $f(\tau)$ has zero value prior to $\tau = 0$ and also for values of τ up to the value of the minimum time delay (pulse travel time). The width of the main lobe of the antenna is assumed to be greater than the effective diameter of the target. The receiving antenna is assumed to have the same effective area as the transmitting antenna, A, and is at the same range from the target, R. The power in the received radio frequency signal as a function of time is then:

$$P_R(t) = \frac{P_t A^2 L}{4\pi\lambda^2 R^4} \int_{\tau}^{\tau+t_1} f(t - \tau)dt$$

The factor L, which is less than unity, is introduced to allow for system and possible polarization losses. The wavelength of the transmitted energy is λ. The integral expression is used to cover the situation where returns from differently delayed portions of the target are uncorrelated and must be added on a power basis.

B. THE SUN AS A RADAR TARGET

A study of the above relation indicates the effect of $f(\tau)$, the reflection cross-section, and t_1, the transmitted pulse length, on the reflected energy. With the other factors fixed by equipment capabilities and physical limitations, a careful study of the target and its effect on the received signal-to-noise ratio is warranted. As discussed by Kerr[2] and Green[3], the sun is a "soft" target on which major changes in the pattern of ionization levels in the corona, from which hf radar echoes are returned, presumably occur with a period on the order of days. Variations of a local nature, however, are assumed to occur with periods on the order of minutes. This results in a target which is not spherically symmetrical, but on which the contours of equal ionization density consist of irregular surfaces (see Fig. 1). These contours, from which radar reflections can occur, would then be rough surfaces with the upper limit on the scale of irregularities much less than a solar diameter, but probably larger than a wavelength. This uneven surface admits the possibility of echoes in depth, i.e., contributions to the reflected echo from varying distances. The reflection points might vary in distance from the center of the sun out to several solar radii.

Following the previously cited references, reflection at 26 megacycles back toward the receiver was estimated to be a factor of four greater than isotropic; the mean absorption loss estimated at 3.5 db for all rays contributing to the sun echo; and the mean reflecting surface (refractive index $\mu = 0$) estimated at 1.6 solar radii. These factors, plus the consideration of required bandwidth at the receiver (to be discussed later) combine to give an effective solar radar cross-section of 2.6 times the projected area of the photosphere (radius $R_0 = 6.96 \times 10^8$) meters.)

C. BACKGROUND NOISE

Essential to any discussion of the detection of an echo in the presence of noise is a discussion of the properties of the noise itself. The noise background for a solar radar echo attempt will be a combination of both solar and galactic or cosmic noise.[10,11] During periods when an antenna of 30-db gain is pointing toward a "quiet" sun, the noise energy at 30 megacycles is predominantly galactic noise—by a ratio of

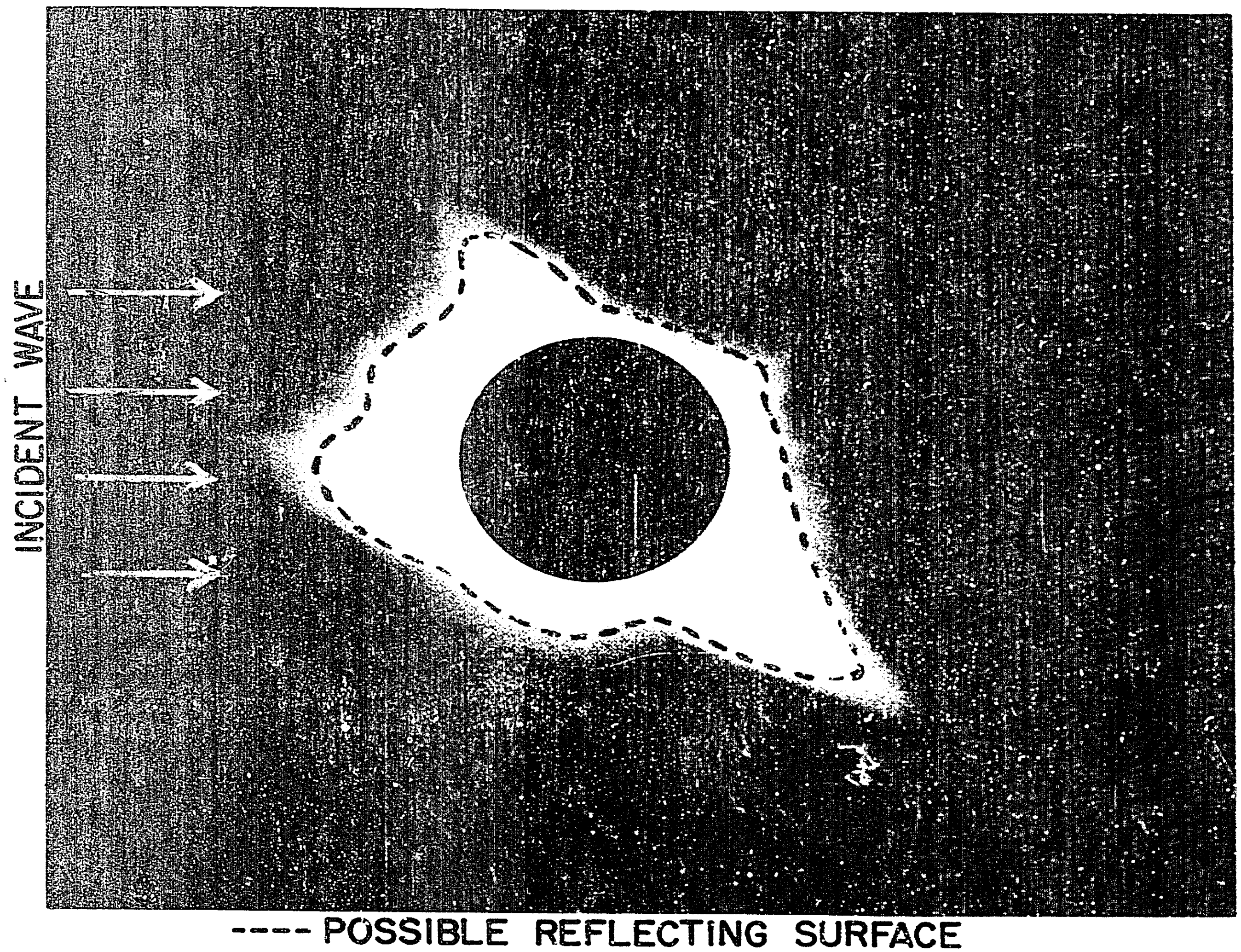


FIG. 1.--SOLAR CORONA--POSSIBLE REFLECTION SURFACE. (COURTESY OF NATIONAL GEOGRAPHIC SOCIETY AND G. VAN BIESBROCK OF YERKES OBSERVATORY.)

approximately 4:1. Following Smerd[10] and Shain[11], we note that the total background noise level, with quiet sun conditions, is approximately 3×10^{-19} watts per cycle per second in a system using a frequency of 30 megacycles and an antenna with a gain of 30 db. During periods of flares or other solar disturbances, however, the noise background may change by tens of decibels.

As a first approach to a probability distribution and frequency spectrum of the background noise, it is assumed that the galactic and quiet sun noise is gaussian in amplitude distribution and white across the relatively narrow (2000 cps) band of frequencies of interest. Superimposed on this distribution is a time-variable component which consists of random length, randomly spaced, solar noise bursts which vary in amplitude over extremely wide ranges. The variable component of the noise causes special difficulties in the detection of weak radar echoes from the sun.

In the detection of solar radar echoes, the presence of small amounts of echo energy must be made evident, and a single solar noise burst could completely blank the radar echo. If the solar noise burst happened to have a duration near t_1 seconds, a spurious echo indication would result. For this reason, repetitive signals (or coded sequences) should be used so that the statistical nature of the bursts themselves can be taken into account in looking for weak echoes.

D. PULSE LENGTHS

The choice of pulse length is influenced by the reflection characteristics of the target and by data reduction considerations. If we assume that the transmission code sequence of pulse lengths t_1 and period $2t_1$ is specularly reflected from a single point, the detection process would ideally consist of the use of a pre-detection matched filter, i.e., a filter whose bandwidth is $1/t_1$. See the appendix. In radar echoes from the sun, however, we can expect a certain amount of time-spreading, so that the ideal situation cannot be achieved. In the case in which the characteristics of the reflecting surgace are known for the duration of the pulse length t_1, the post-detection signal-to-noise ratio can still be maximized as indicated in the Appendix.

However, the solar echo frequency will also be spread over a band determined by the rotation of the sun, which precludes the use of narrow pre-detection bandwidths,[2] and the lack of specific knowledge of the reflecting surfaces prevents the use of the ideal matched filters.

In the design of the solar echo experiment an attempt was made to approach the ideal case for post-detection filtering by choosing transmission pulse lengths sufficiently long so that the expected pulse time spreading could be neglected, relative to the length of the pulse. With this condition fulfilled and with sufficiently wide pre-detection bandwidths to accept the greater portion of the echo energy, an idealized post-detection filtering with an equivalent bandwidth of $1/t_1$ could be used.

The minimum pulse length is dictated by the possible spread-in-depth of the echoes. If we consider the possibility of echoes from a spread of three solar radii, then the minimum pulse length and separation should be 14 seconds. See Table I. Similarly, the minimum pulse length based on the assumption of echoes in depth from 0 to 1.5 solar radii would be seven seconds.

TABLE I.--CONVERSION: DIFFERENTIAL DELAY TIME TO SOLAR RADII OF REFLECTION

ARRIVAL TIME OF ECHO AHEAD OF COMPUTED DELAY TIME TO CENTER OF SUN (SECONDS)	REFLECTION POINT (SOLAR RADII OF REFLECTION)
0	0
4.6	1
9.3	2
13.9	3
⋮	⋮
60.3	13

SOLAR RADIUS (OPTICAL)	6.96×10^8 METERS
SPEED OF LIGHT	2.9986×10^8 METERS/SECOND
TIME OF LIGHT TRAVEL PER A.U.	498.46 (BASED ON VENUS ECHO RESULTS) SECONDS

The maximum pulse length is determined from considerations of the total length of transmission time available (approximately 16 minutes), the desire for a large enough number of pulses to allow probability discussions and to permit observation of possible pulse to pulse variations, and the requirements of system frequency and gain stability with extremely long pulses. This would indicate that the upper limit should be near one minute.

An additional consideration in the case in which the pulse (or coded sequence) is repeated, is the range ambiguity which results. For example, if the pulses (or coded sequences) are repeated on a one-minute basis, the possible range ambiguity resulting from the inability to distinguish between pulses (or code sequences) would be 60/4.6 or 12.9 solar radii. Thus an echo from one solar radius would be indistinguishable from an echo from 13.9 solar radii or 26.8 solar radii, etc., except for possible identification either by relative intensity or, in the case in which echo strength is sufficient to permit the identification of each echo, by identifying each transmitted pulse with an echo pulse.

E. DATA REDUCTION

With the length of pulses involved in sun echo trials the construction of electronic filters with bandwidths on the order of $1/t_1$ cycles per second becomes extremely difficult. Also, extreme accuracy is required in the consideration of the change in echo frequency due to the relative motion between the antenna and the target (Doppler frequency). However, spreading of the frequency spectrum of the echo pulse due to rotation and massive changes in the reflecting surface (Doppler spreading) indicates that much larger receiver bandwidths must be used. It is not possible, therefore, to accomplish the data reduction in a single step and still achieve a maximized output signal-to-noise ratio. In the case for which the frequency spectrum of the background noise can be assumed to be fairly uniform over the region in the immediate vicinity of the echo enter frequency, the receiver bandwidths should be sufficiently wide to accept essentially all of the echo energy. Then, by subsequent processing, either in analog or in digital form, equivalent filtering can be performed by integration and correlation techniques. In cases in which interference-type signals are accepted by the wider bandwidths, and this interference is sufficiently strong to cause overloading of the receiving system, narrower bandwidths are essential.

Considering that the Doppler spreading of the echo is limited by the rotation of the corona at the estimated mean echo radius in the sun's central strip, the Doppler spreading would amount to approximately $\pm$ 500 cycles per second at a frequency of 25.6 megacycles. Similarly, if the possibility of echoes from out to three solar radii is considered, the Doppler spreading would be two kilocycles, i.e. essentially 100 percent of the echo energy would be returned in a 2000-cycle bandwidth.

The relative motion of the antenna array pattern and the target is made up of two components—one due to the rotation of the earth and the second due to the radial motion of the earth in its orbit around the sun. In Table II are tabulated the expected Doppler shifts and echo delay times for the echo trial periods. It should be noted that during the spring equinox the contributions differ in sign, making the resultant Doppler shift in frequency very small. During the autumnal equinox the frequencies are both positive, resulting in a Doppler shift of approximately 140 cycles per second.

F. DETECTION

With the transmitter power, antenna gain, and pulse lengths presently possible, the successful detection of a sun echo still depends on the ability to detect the target with a signal-to-noise ratio at the receiver IF output which is much less than unity—insufficient to give an audible or visible solar echo indication. Integration of a number of returns is thus necessary in order to establish an echo indication. Since the phases of the individual returns are unknown, the addition must be accomplished on a mean rectified amplitude or energy basis after envelope detection of the video signals has been accomplished.

The envelope detection process, independent of the power law of the detector, results in a "small-signal suppression effect".[8] When the signal-to-noise ratio $(S/N)_1$ is less than unity, the envelope detection process results in an output signal-to-noise ratio which is equal to the square of the original ratio, $(S/N)_1^2$. Addition of n such

TABLE II.--EXPECTED DOPPLER FREQUENCIES AND SOLAR PULSE DELAY TIMES

DATE	DOPPLER FREQUENCIES (CPS) EARTH ROTATION**	ORBIT CHANGE	NET	PULSE DELAY TIMES* EARTH-SUN CENTER
10 SEPT. '58	+63.0	+76.1	+139.1	1003.72
11 SEPT. '58	+63.0	+77.1	+140.1	1003.46
12 SEPT. '58	+63.0	+78.1	+141.1	1003.20
7 APR. '59	+63.0	-85.1	-22.1	997.89
10 APR. '59	+63.0	-83.6	-20.6	998.92
12 APR. '59	+63.0	-82.6	-19.6	999.48
7 SEPT. '59	+63.0	+77.0	+140.0	1004.56
8 SEPT. '59	+63.0	+77.9	+140.9	1004.30
15 SEPT. '59	+63.0	+78.6	+141.6	1002.42

*BASED ON SOLAR PARALLAX AS DETERMINED BY LINCOLN LABORATORY, MIT, FROM VENUS RADAR ECHOES.

**COMPUTED FOR ANTENNA CONFIGURATION POINTING EAST SHORTLY AFTER SUNRISE AT ZERO ELEVATION ANGLE.

returns yields a resultant signal-to-noise ratio of

$$(S/N)_0 = n(S/N)_1{}^2$$

The small-signal suppression effect thus produces a situation in which a given number of decibels improvement in transmitter power or any operation <u>prior</u> to detection is equivalent to twice the improvement in integration time <u>after</u> detection, expressed in decibels.

In summary, then, the detection process should consist of: (1) sampling the data at an adequate rate to reproduce the input wave faithfully (sampling rate $\geq$ 2W where W is the bandwidth of the original signal), (2) processing the samples to reduce the effects of short solar noise bursts (Chapter IV), (3) low pass filtering of the resultant output to a one cycle per second bandwidth (accomplished by integration over one-second intervals), (4) cross-correlating the sample sums with the transmitted code sequence (which amounts to additional filtering to an equivalent bandwidth indicated by the width of the central peak in the autocorrelation function of the transmitted code sequence), and finally (5) integrating over a number of pulses (or code sequences) to improve the signal-to-noise ratio by a factor equal to the number of pulses or code sequences.

III. SYSTEM DESCRIPTION - SOLAR ECHO EXPERIMENT

All of the sun echo tests were conducted with a transmitter (Collins FRT-22) having an average output power of 40 kilowatts and which was operated in the 25 to 30 megacycle range. The transmitter was pulsed on and off in predetermined code patterns for identification purposes. The antenna system consisted of a broadside array of either four or eight rhombic antennas with estimated gains as listed in Table III. These antennas cover a rectangular area of 1600 by 725 feet.

TABLE III.--SOLAR-ECHO SYSTEM CHARACTERISTICS

ITEM	SEPT. '58	APR. '59	SEPT. '59
WAVELENGTH (METERS)	11.7	11.7	11.5-11.7
RADIATED POWER (KILOWATTS)	40	40	40
PULSE LENGTH (SECONDS)	180	30	9-43
RECEIVER BANDWIDTH (CPS)	100	2000	2000
ANTENNA ARRAY GAIN (DB)	26	26	29
ESTIMATED ECHO S/N* RATIO (PRE-DETECTION)(DB)	-23	-22	-16

*INCLUDES THE THEORETICAL EFFECT OF RECEIVER BANDWIDTH, DIRECTIVITY OF REFLECTION SURFACE, AND ABSORPTION IN THE SOLAR CORONA.

The main lobe of the array pattern points generally in an east-west direction, and was used only for transmissions toward (and reception from) the east. The antenna orientation is such that the sun rises through the main lobe of the array only twice yearly—during periods early in April and September.

For the early trials only four of the eight rhombics were used since the eight-rhombic array produced a main lobe which was too narrow to allow both transmission and reception of the coded sequence in the same fixed position of the main lobe. During the period of approximately 16 minutes and 30 seconds required for the electromagnetic energy to travel to the sun and return, the earth rotates through a sufficient angle that the antenna is no longer oriented for reception. Use of only four of the rhombics in the broadside array provided a sufficiently broad lobe pattern to include the sun for approximately 30 minutes after sunrise each day during the equinoctial periods.

In later trials (September, 1959) the entire array of eight rhombics was used, with the main lobe position switched to follow the sun's apparent motion during the transmission and reception periods.

The lobe switching mechanism of the antenna array had been designed for short pulse operation only, with the result that serious overloading of the delay line contacts caused numerous antenna failures. Due to the complexity and physical size of the switching system it was not possible to check the connections after each switching operation, so that no positive indications could be obtained as to the proper functioning of the antenna array during the tests.

Identical Collins R-390 receivers were used with conventional pre-amplifiers in all three trial periods. In each case the transmitted and received signals were translated down to audio frequencies (without rectification) in the receivers and recorded on magnetic tape for subsequent data reduction. The recording was done on Ampex FR-100 and Precision Instruments Company recorders at 15-inch-per-second speeds. A time reference was recorded from station WWV for each trial run.

The data were subsequently converted from analog to digital form, and the detection process and post-detection integration were accomplished on an IBM 797 digital computer.

IV. DATA REDUCTION AND EXPERIMENTAL RESULTS

A. INTRODUCTION

Data reduction methods for the three sun-echo trial periods, while being similar in theory, differed sufficiently in practice to preclude general discussion of all phases of the detection process. Certain phases, such as clipping procedures and cross-correlation methods, were common to all data reduction and will be discussed in general. To aid in the segregation of the considerations which led to the positive results obtained in the April 1959 and September 1959 sun-echo trials, the remaining portions of the data reduction process for these periods will be discussed separately.

The September 1958 solar-echo trials were of a preliminary nature. They provided valuable experience in the conduct of the experiment although no positively identified echoes were obtained. Subsequent study of the data indicated that the pulsewidth and receiver bandwidth used in these first trials were probably not optimum. However, from these first tests new information was obtained about the character of the radio noise that would be present in future sun-echo experiments using radio frequencies in the hf (3 to 30 megacycle) band. In addition to the previously discussed noise of solar and galactic origin, local noise sources (power lines, car ignition, appliances, and industrial equipment) sometimes also caused noise level changes in the period during which the sun passed through the main lobe of the rhombic array.

Initial radar echo detection in the presence of the background noise was attempted only on records which contained no long-duration, large-amplitude level changes. Amplitude-time paper recordings were made of the received signals for each echo trial, and only those records with uniform mean amplitude and noise bursts of less than one or two seconds duration were chosen for further data reduction.

B. CLIPPING PROCEDURES

The effect of the short solar noise bursts was reduced by amplitude clipping in the data reduction process. Some clipping took place in the analog-to-digital sampling process due to the fact that the sampling unit was limited to sample sizes of three digits or less. This resulted in any sample which converted to a number greater than $\pm$ 999 on the arbitrary amplitude scale being replaced by the value $\pm$ 999. Voltage levels of the wave to be sampled were adjusted so that the input signal was clipped at approximately five times the mean magnitude of the rectified samples. On the data runs sampled, this peak magnitude clipping occurred on only 0.02 percent of the samples.

The samples were next summed over a time interval which could be considered short compared to the length of the pulses used in that specific solar echo trial. Four-second sums were used for the September 1958 data when 180-second pulses were transmitted, while one-second sums were used for the April and September 1959 data when the mean transmitted pulse lengths were 30 seconds.

Additional clipping of these sample sums was accomplished in digital form in the few cases in which the individual sums exceeded a value which was 10 per cent greater than a running mean of four samples. This clipping was restricted to only those cases in which individual sample sums were extremely high compared with the adjacent-interval sums and was performed on less than 3 per cent of the sample sums.

The sample sums which exceeded the mean by more than 10 per cent were replaced by the running mean value, rather than the extreme value. This was done to eliminate, insofar as possible, the effect of solar noise bursts whose duration was less than the time interval over which the integration was made.

C. DETECTION PROCESS

Final data reduction was accomplished by cross-correlating the received signal with the transmitted code. This process, in effect, constituted the final filtering of the signal in a filter whose bandwidth is $1/t_1$, since the process is selectively sensitive to variations in echo energy or amplitude (magnitude only) which occur at the same rate as variations in the transmitted wave.

The cross-correlation process consisted of comparing the received signal sums with a transmitted code made up of digital values of +1 corresponding to the "on" time of a transmitted pulse and -1 corresponding to the "off" period. Since all transmissions were 50-per cent duty cycle (equal "on" and "off" times) this simplified procedure could be used, making the computer operation merely one of adding and subtracting the received-signal sample sums in a specified manner.

Using the April 1959 data as an example, the cross-correlation of the received data with a single cycle of the transmitted code was accomplished as follows: Since the transmission code was a series of 30-second pulses, each followed by 30 seconds "off" time, the first point on the cross-correlation curve was obtained by adding sample sums 1 through 30 and subtracting sample sums 31 through 60; the second point was obtained by adding sample sums 2 through 31 and subtracting 32 through 61. This procedure was continued until the single cycle (30 values of +1 and 30 values of -1) was moved through the entire received-sample ensemble. The resulting single-cycle cross-correlation curve provided echo indication versus time information from which echo delay times could be obtained.

Multiple pulse integration was accomplished by merely summing the corresponding individual pulse values determined by the method just described. The first point was obtained by adding 1 through 30, subtracting 31 through 60, adding 61 through 90, etc., out to as many cycles as the integration period included. The second point was obtained by adding 2 through 31, subtracting 32 through 61, adding 62 through 91, etc., out to the same number of total samples with sample number 1 being included in the last subtraction period. This process yields a multiple-pulse integrated cross-correlation curve which provides an integrated echo indication on a base-length of a single period.

D. SUN ECHO TRIALS - APRIL 1959

1. EXPERIMENTAL PROCEDURES. During this period sun echo trials were conducted on seven days: 5, 6, 7, 9, 10, 11, and 12 April. Preliminary examination of the recorded data revealed that of the seven days only three had provided reliable usable data—7, 10 and 12 April. Data from the remaining four days were not analyzed in detail for reasons of large-amplitude level shifts in the recordings or equipment malfunctions.

The transmitted code consisted of a series of 30-second pulses with a 50-per cent duty cycle. Fifteen of these pulses were transmitted, after which the transmitter and pulse keying equipment were turned off. The antenna was then physically disconnected from the transmitter, connected to the receiving system, and the receiver output recorded on magnetic tape for a 16-minute period. Receiver bandwidths of 2000 cycles were used. The received signals were translated down in frequency through the use of a beat frequency oscillator so that the signal bandwidth extended from 0 to 2000 cycles.

On several days the antenna switching and the receiver adjustments were not completed prior to the expected return time of the first pulse. This, plus the fact that the main lobe of the antenna was broad enough to receive only about 13 of the sequence of 15 pulses, resulted in a maximum common echo period, for the three days, of only 12 minutes.

2. METHODS OF DATA REDUCTION. Initially the received signal was filtered to a 200 to 2000-cycle bandwidth to eliminate the effect of any 60 to 120-cycle pickup during the recording process. The data were then converted from analog to digital form, 4000 samples per second being taken at a 5-kilocycle rate in 0.8 seconds. The samples were then envelope-detected, clipped where required to eliminate the effect of solar noise bursts of one-second duration or less, and integrated into one-second sums.

The one-second sums were then cross-correlated with the transmitted code of rectangular pulses. The equivalent "matched filter" operation was completed by this cross-correlation of the received sample sums with a single cycle of the transmitted wave. This single cycle was moved through the entire group of sample sums in one-second intervals to provide an echo indication as a function of time. (See Fig. 2). Also plotted in Fig. 2 is the ideal echo-indication curve positioned to correspond to a reflection distance of 1.7 solar radii.

To improve the final signal-to-noise ratio by an additional factor equal to the number of pulses, the sample sums were cross-correlated with 12 minutes of the transmitted square wave. Figure 3 (a), (b), and (c) shows the 12-minute integrated cross-correlation plots for April 7, 10, and 12 respectively. The vertical coordinate on these

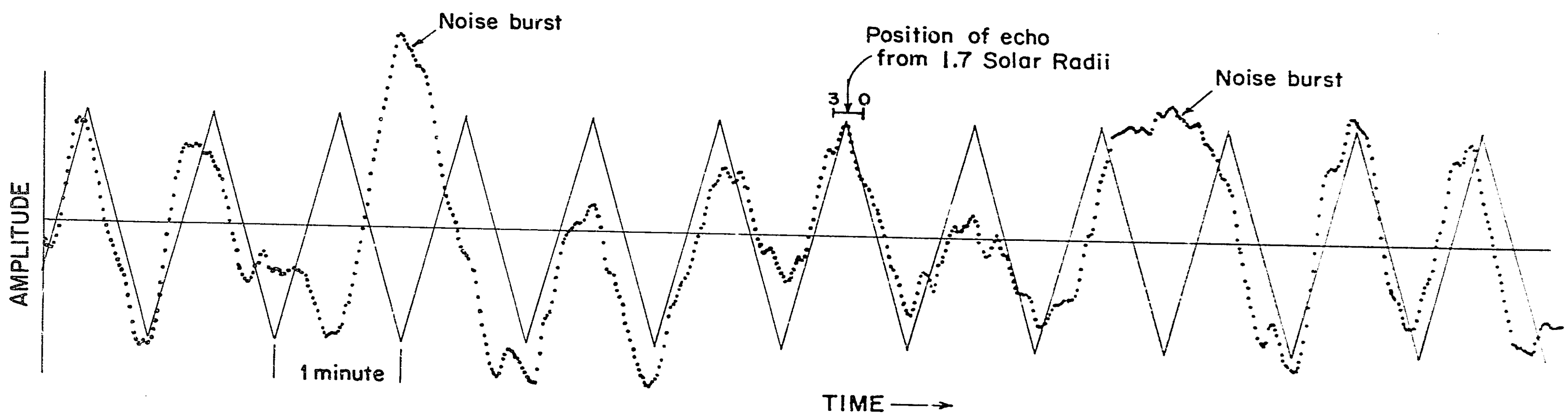


FIG. 2.--CROSS-CORRELATION OF ONE MINUTE OF TRANSMITTED CODE WITH 12 MINUTES OF RECEIVED SIGNAL SHOWING INDIVIDUAL ECHO RETURNS AND DISTURBING NOISE BURSTS COMPARED WITH IDEAL ECHO CURVE FOR 7 APRIL 1959.

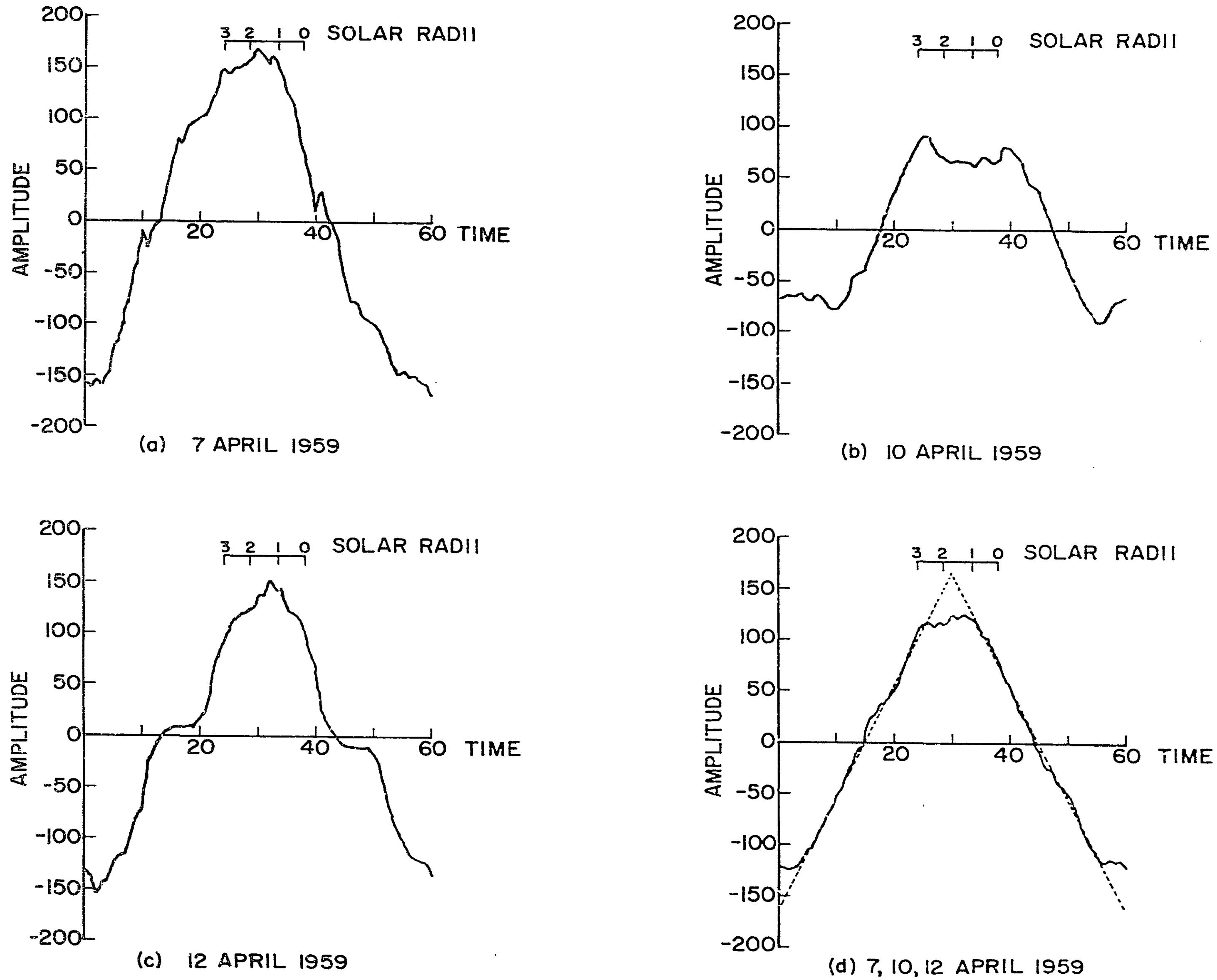


FIG. 3.--CROSS-CORRELATION OF 12 MINUTES OF TRANSMITTED CODE WITH 12 MINUTES OF RECEIVED SIGNAL SHOWING EVIDENCE OF RADAR ECHOES FROM THE SUN. THE IDEAL ECHO CURVE (DOTTED) IN FIG. 3(D) IS POSITIONED AT A REFLECTION POINT OF 1.7 SOLAR RADII. (A) 7 APRIL 1959; (B) 10 APRIL 1959; (C) 12 APRIL 1959; (D) COMPOSITE OF 7, 10, 12 APRIL 1959.

plots is relative amplitude, from which an average signal-to-noise figure can be obtained. The horizontal coordinate is time in seconds, adjusted so that an echo from a distance corresponding to that from the center of the sun would fall at second 38. An echo indication at 30 seconds corresponds to an echo from a mean distance of 1.7 solar radii.

It appears that echo indications were received on each of the three days. Additional integration to show the echo characteristics with greater clarity was performed by averaging the results of the three days. [See Fig. 3(d)].

3. EXPERIMENTAL RESULTS. Verification of the receipt of echoes from the sun during the April 1959 trials is derived from two different approaches: first, qualitative analysis of the 12-minute cross-correlation curves of Fig. 3, in which the significance of curve shapes and positions is considered, and second, a statistical consideration of the positions of the peaks in the one-cycle cross-correlation curves as a function of the delay times at which they occur. This second assembly of data, compared to similar data on "background noise only" periods of reception, permits the discussion of numerical probabilities that an echo was received, in terms of the probability that background noise alone could cause the resultant echo indication.

The qualitative discussion of curve shapes and peak positions is based upon a comparison of the actual cross-correlation curves with ideal echo curves. The ideal echo curve is the auto-correlation curve of the transmitted code, or, equivalently, the cross-correlation curve of the transmitted code with itself.

The ideal echo curve for the single-cycle cross-correlation process can be derived by considering an echo with no distortion and with no interfering noise. The resulting single-cycle cross-correlation curve would be a triangular wave with positive peaks located, in time, at positions corresponding to the mean delay time (τ) of each pulse. (See Fig. 2). If all of the echo energy were returned from a single distance from the transmitter, a sharp peak would result. If the echo pulse contained energy from reflection points at a spread of time delays small compared to the pulse length, the peak would be flattened slightly, but the sides would still remain straight, and the position of the triangular pulse would be centered at the mean delay time.

In data reduced by this method a solar noise burst of appreciable length will result in only a single peak in the cross-correlation curve Under the assumption that the occurrence of solar noise bursts is random in time, the positions of correlation-curve peaks caused by solar noise could fall at any point along the time axis. Correlation peaks caused by solar echoes, however, will occur at points along the time axis which correspond to reasonable reflection distances.

The ideal echo indication curve for the 12-minute cross-correlation process is merely the superposition of 12 of the individual triangles of the single-cycle curve and is the Λ (capital lambda) curve shown dotted in Fig. 3(d).

Consideration of the curves of Fig. 3 yields the following five justifications for believing that they represent the successful detection of radar echoes from the sun:

a. Position of the Peaks. Solar noise, as previously discussed, is equally likely to produce a peak in the cross-correlation curve at any point along the time base. The ideal Λ echo curve of Fig. 3(d) is drawn at a position corresponding to an assumed reflection at 1.7 solar radii. A position scale corresponding to reflection points from 0 to 3 solar radii is shown over the positive crest of each curve. It can be seen that the position of each of the daily cross-correlation peaks corresponds to a reflection point near that which would be expected from theory (1 to 2 solar radii).

b. Symmetry. The use of the symmetrical form of the transmitted wave in the cross-correlation process results in an inverted repetition of values at 30-second intervals. However, solar noise in general would not produce a mirror symmetry about any specific time coordinate. Twelve-minute periods of "background noise only" were not available for experimental verification of this statement, although shorter (7-minute) periods of solar noise recorded in September 1959 produced smaller amplitudes and inverted repetition without obvious mirror symmetry about any time coordinate. If echoes are present, there would be such mirror symmetry about the peak time. Examination of the curves of Fig. 3 indicates that this mirror symmetry about the peak time is evident.

c. Linearity of the Sides. As previously discussed, echo indications from varying levels in the solar corona would result in flattened tops of the Λ echo indication, while still retaining the straight sides of the curve. This linearity of the sides of the curves of Fig. 3 is evident.

d. Improvement in the Composite Curve. Assuming that variations from the ideal Λ curve in curves (a), (b), and (c) are due to random background noise variations, the composite curve for the three days should be a closer approximation to the ideal echo shape. This is strikingly evident in Fig. 3(d), and is considered a reliable indication of the presence of solar echoes since it indicates that the effects of random noise variations tend to nullify each other, while the indication of solar echoes is greatly improved.

e. Signal-to-Noise Ratio. Estimated echo strengths following the computations of Kerr[2], indicate that the system used should obtain sun echoes at a signal-to-noise power ratio of about -22 decibels in the intermediate-frequency amplifier, or -44 decibels immediately after the detector. Computations of the mean signal-to-noise voltage after detection, but before post-detection integration, of the 7 April data indicate a voltage ratio of 1/155. This voltage ratio, with mean power being approximately the square of the mean voltage, yields a signal-to-noise power ratio of -43.8 decibels or -21.9 decibels before detection. The signal-to-noise power ratios before detection and integration for 10 and 12 April are -24.4 and -22.4 decibels, respectively. Significance can be derived from the fact that the echo indications are at least of the expected order of magnitude.

As a check on the ability of the receiving system and the data reduction process to detect signal-to-noise ratios of this magnitude, two test recordings were made and processed in exactly the same manner (and at the same amplitudes) as the solar echo recordings. The first of these was a system noise recording made with the pre-amplifier input terminated in a resistor. The second recording was made under identical conditions but with a -20-decibel signal added at the input. The curves of Fig. 4 indicate the results of the test, demonstrating the ability of the receiving system and data reduction process to detect signals when the pre-detection signal-to-noise ratio is on the order of -20 decibels.

A clearer understanding of the effects of impulse and burst noise can perhaps be obtained from attempts to detect each individual solar echo, rather than the integrated effect of many echoes. As can be seen from Fig. 2 there are serious departures from the ideal curve near the third and ninth cycles, but the cross-correlation curve always returns to approximately the ideal position and shape. Such departures could be caused by outbursts of noise. Examination of the photographic records of the Radio Astronomy Station of Harvard College Observatory indicates that one of the indicated noise bursts (that near the ninth cycle) corresponds in time to a small group of fast drift bursts of intensity 1 on their amplitude scale (fairly low level). This is not considered experimental verification that this particular group of fast drift bursts did, in fact, cause the departure from the cross-correlation curve, but is considered as a possible source of interference.

4. PROBABILITY DISTRIBUTION OF CROSS-CORRELATION PEAK POSITIONS. The data for the numerical probability discussion of solar echoes was obtained from cross-correlation curves of a single cycle of the transmitted square wave with 12 minutes of the one-second sums of the three days with usable records—7, 10, and 12 April 1959. Figure 2 shows one such curve.

As previously discussed, bursts of solar noise which cause random peaks in the cross-correlation curve normally exceed the energy content of the expected echo pulses by many decibels. For this reason a method of analysis was used in which the relative position of the pulse indications were tabulated in terms of solar radii of reflection, without regard to the relative magnitude of the peaks. The positions of all peaks representing a change in amplitude level of greater than -25 decibels were tabulated in terms of their delay times. With all transmitted pulses being identical, inter-pulse recognition was not possible, so all delay times were converted to the delay time corresponding to the first pulse. This yielded a scatter plot of cross-correlation peaks (or possible solar echo indications) on a time base of 60 seconds or 12.9 solar radii of reflection. Figure 5 is a bar graph of these peak locations combined on a 9.3 second (two solar radii of reflection)

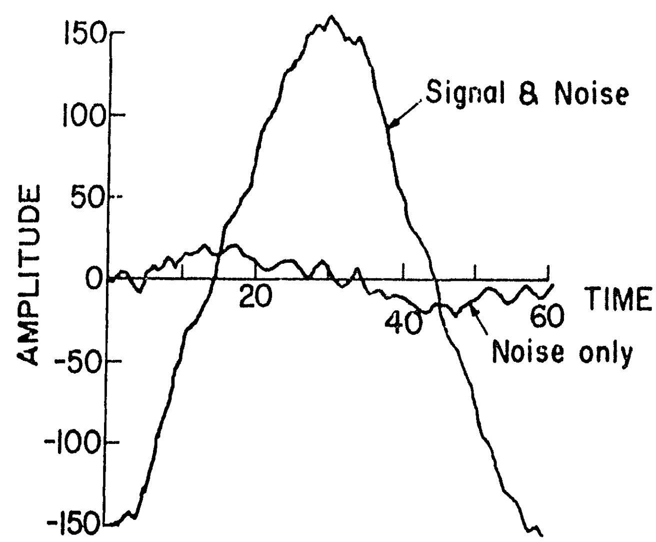


FIG. 4.--SYSTEM TEST RESULTS. THESE CURVES DEMONSTRATE THE ABILITY OF THE RECEIVING SYSTEM AND THE DATA REDUCTION PROCESS TO DETECT SIGNALS WHEN THE PRE-DETECTION SIGNAL-TO-NOISE RATIO IS NEAR −20 DECIBELS.

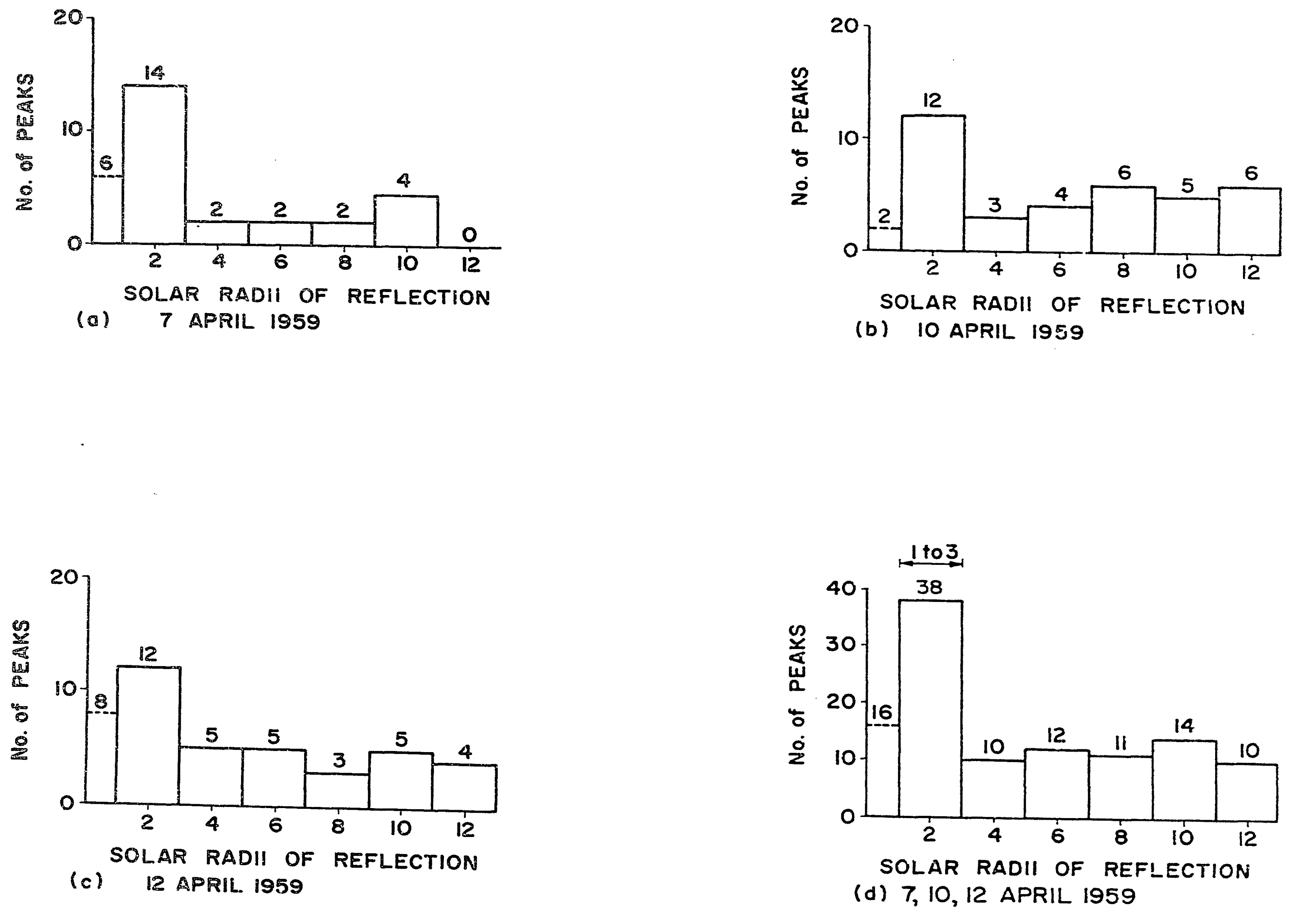


FIG. 5.--SOLAR-ECHO-CURVE. CROSS-CORRELATION-PEAK LOCATIONS PLOTTED AGAINST DELAY TIMES (EXPRESSED IN SOLAR RADII OF REFLECTION). THE BASIC GROUPING IS A TIME INTERVAL CORRESPONDING TO TWO SOLAR RADII OF REFLECTION. THE GRAPHS INDICATE THE PEAK POSITION BUNCHING IN THE APRIL, 1959, SOLAR ECHO INDICATION CURVES. (A) 7 APRIL 1959; (B) 10 APRIL 1959; (C) 12 APRIL 1959; (D) COMPOSITE OF 7, 10, 12 APRIL 1959.

basis. Figures 5(a), (b), and (c) demonstrate the peak location bunching on 7, 10 and 12 April 1959 respectively. Figure 5(d) is the combined data over the three day period.

One would expect the occurrence of solar noise bursts to be random in time and hence Poisson distributed. To investigate this probability, background noise samples were cross-correlated with the April 1959 transmitted code and a "noise only" plot was made of a number of peaks roughly corresponding to the number of peaks in the combined 7, 10 and 12 April 1959 data. For ease in plotting, a basic period of one minute was used. Peak indications in the second and succeeding minutes were translated back into the first minute. Also, to facilitate comparison with the echo data of 7, 10, 12 April 1959, a scale of solar radii of reflection was used. Note that the time scale for the plot of these graphs in Fig. 6 has no significance in terms of absolute time. The zero point is purely arbitrary.

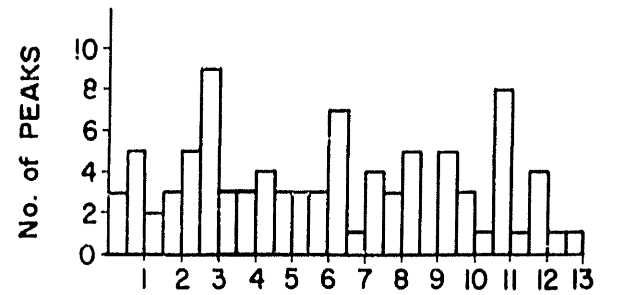
Figure 6(a) is a bar graph indication of the peak-time distribution of noise-correlation peaks combined in 2.3 second groupings (corresponding to 1/2 solar radius of reflection). Figure 6(b) indicates the same data combined in a 9.3 second grouping (2 solar radii of reflection).

These data were then compared with ideal Poisson distribution curves for the same number of samples and mean value. Figure 7 shows this comparison. Examination of various combinations of data indicates that with greater numbers of peak indications, the histograms approach the ideal Poisson curves more closely. The use of Poisson distribution probability relations in the determination of numerical probability of error in echo indications was thus considered justified.

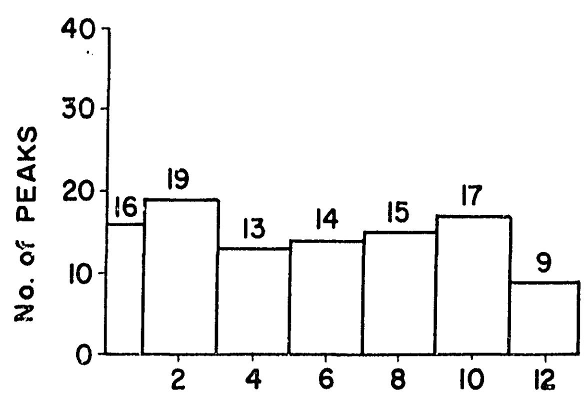
Using Poisson probability statistics for the peak locations, the probability that k or more events occurred in a time interval of length b is

$$P(k, b) = \frac{(ab)^k e^{-ab}}{k!}$$

where a is the average number of occurrences per time interval. The individual and combined probabilities are tabulated in Table IV.[13] In the table, P(b) is the probability that the given number of peak indications would occur in <u>any</u> two-solar-radii period on the indicated date.



(a) 2.3 SECOND GROUPING-1/2 SOLAR RADII



(b) 9.3 SECOND GROUPING-2 SOLAR RADII

FIG. 6.--BACKGROUND-NOISE, CROSS-CORRELATION-PEAK LOCATIONS PLOTTED AGAINST DELAY TIMES (EXPRESSED IN SOLAR RADII OF REFLECTION). THE GRAPHS INDICATE PEAK POSITION DISTRIBUTION OBTAINED FROM BACKGROUND NOISE DATA COMBINED IN BOTH TWO SOLAR RADII AND ONE-HALF SOLAR RADIUS INTERVALS. (A) 2.3-SECOND GROUPING---$\frac{1}{2}$ SOLAR RADIUS; (B) 9.3-SECOND GROUPING---2 SOLAR RADII.

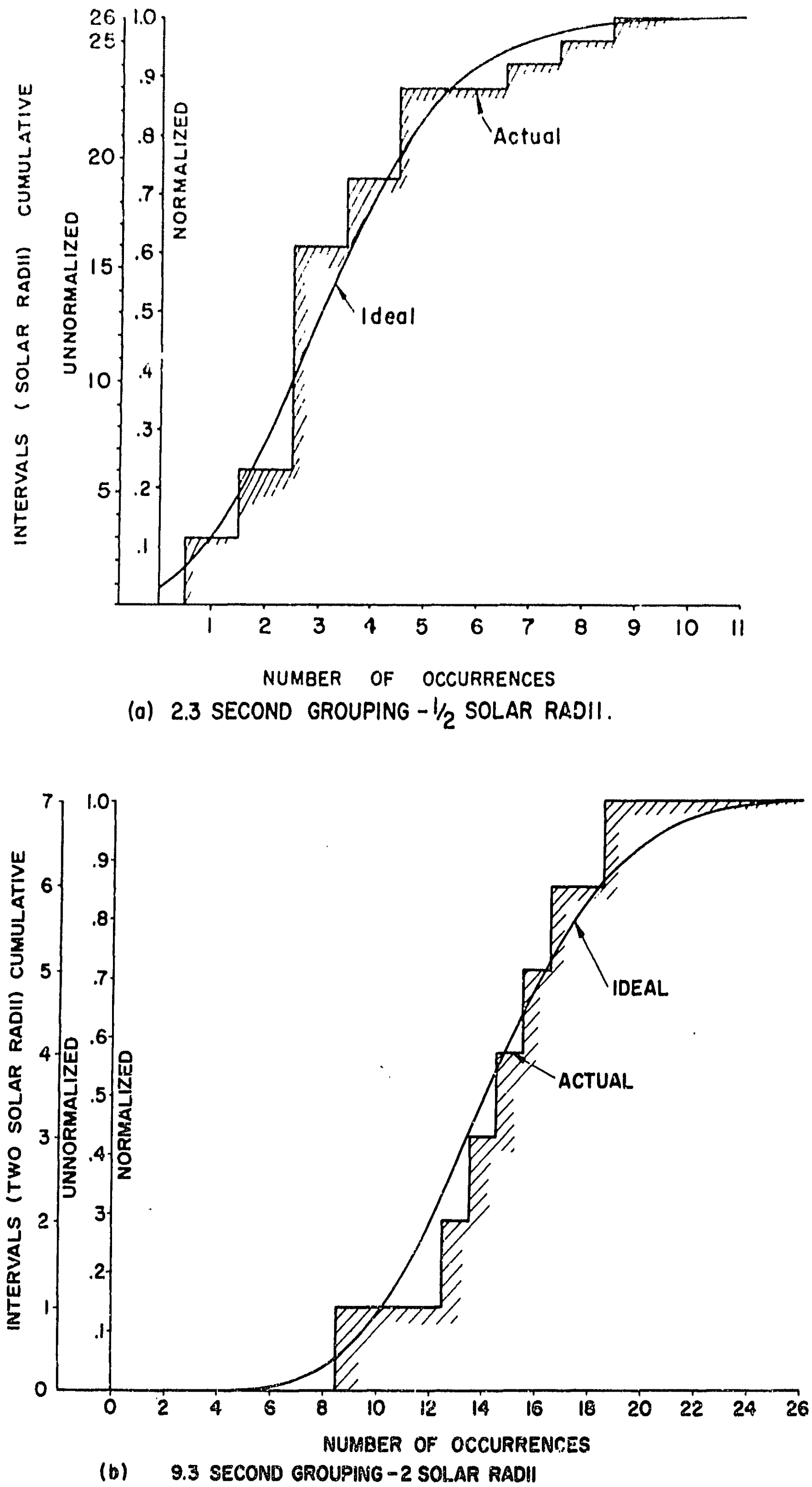


FIG. 7.--COMPARISON OF BACKGROUND-NOISE, CROSS-CORRELATION-PEAK POSITION DISTRIBUTION WITH IDEAL POISSON PROBABILITY DISTRIBUTION. (A) 2.3-SECOND GROUPING---$\frac{1}{2}$ SOLAR RADIUS; (B) 9.3-SECOND GROUPING---2 SOLAR RADII.

TABLE IV.--SOLAR ECHO ERROR PROBABILITIES - APRIL 1959

DATE	P(B)	P(1 TO 3)	P(ECHO)
7 APRIL	1.35×10^{-4}	2.1×10^{-5}	$1 - (2.1 \times 10^{-5})$
10 APRIL	9.23×10^{-3}	1.43×10^{-3}	$1 - (1.43 \times 10^{-3})$
12 APRIL	1.13×10^{-2}	1.75×10^{-3}	$1 - (1.75 \times 10^{-3})$
COMBINED	1.0×10^{-6}	1.6×10^{-7}	$1 - (1.6 \times 10^{-7})$

P(1 to 3) is the probability that the given number of echo indications would fall specifically in the 1 to 3 solar radii of reflection grouping. P(echo) is the resultant probability of echo for that date. The combined probability figures in Table IV are not, in a statistical sense, correct. The actual probability, considering that the trials on the three days were completely independent, should be the product of the probabilities of the individual days. Thus the over-all probability that background noise would cause the given echo indications is computed to be 5.3×10^{-11}. If the method used here for obtaining numerical probabilities is accepted, there can be no further doubt that radar echoes from the sun were obtained.

E. SUN ECHO TRIALS—SEPTEMBER 1959

1. EXPERIMENTAL PROCEDURES. Sun echo trials were conducted on a total of 16 days in the period 1—17 September, 1959. In addition to previously discussed antenna troubles during the transmissions, records of solar noise bursts in the hf frequency range, as tabulated by Dr. Alan Maxwell of the Radio Astronomy Station of Harvard College Observatory, indicate high levels of solar activity during this period. Using only solar noise background considerations, conditions for echo detection were relatively good on 5, 7, 8, 9, 15 and 16 September. Equipment failures and extremely large-amplitude level shifts attributed to local noise sources reduced the available echo periods to the first

code period on 5 September and to both code periods on 7 September. The noise recording after the echo periods on 8 September also met the short noise burst and uniform mean amplitude requirements.

The transmission code used for the September 1959 sun-echo trials was a random code sequence of rectangular pulses. This sequence was designed to simulate random-length pulses randomly spaced, and was implemented by keying the transmitter from the output of a recorder utilizing a carefully prepared keying tape. The pulse lengths and times between pulses are listed sequentially in Table V.

TABLE V.--TRANSMITTED CODE SEQUENCE - SEPTEMBER 1959

ON (SECONDS)	OFF (SECONDS)
9	
	11
16	
	23
25	
	43
41	
	37
35	
	29
21	
	17
13	
160 (TOTAL)	160 (TOTAL)

The coded sequence (160 seconds on, 160 seconds off) was transmitted twice during the transmission period, with 120 seconds of separation between the coded sequences. This procedure permitted reception of two minutes of background noise prior to the expected return of the first coded sequence, two minutes of noise between the sequences, and two minutes of noise after the second sequence. The total length and separation of the coded sequence was designed from considerations of the antenna lobe pattern and the movement of the sun. The antenna main lobe position was manually switched to five positions during a typical run—one position for the transmission of the first code, one for the transmission of the second code, one to receive the first code, one to receive the second code, and a final position to receive and record only background noise on the signal channel and on a channel 10 kilocycles below the echo center frequency. Two receivers were used for data reception for reasons which are explained below.

2. ADJACENT-CHANNEL NOISE-AMPLITUDE CORRELATION. In the September 1959 sun echo trials an additional feature was attempted for the purpose of investigating a method of eliminating or reducing the effect of variable solar noise bursts. Two receivers were used for reception—one tuned to the expected echo center frequency and the other tuned to a frequency 10 kilocycles below the echo frequency. The second receiver was assumed to be sufficiently detuned from the echo frequency so that no echo energy would be accepted by its pass band. The receiver settings, except for tuning, were identical.

Based on the assumption that the amplitude of galactic and "quiet sun" solar noise in this frequency range is essentially equal in two 2-kilocycle bandwidths separated by only 10 kilocycles,[7] and further that sharp solar noise bursts are sufficiently broad in frequency spectrum so that the amplitude of their contributions to the received energy in the two separated bands is equal, the amplitude of the noise in the "noise channel" should serve as an amplitude comparison reference for the echo receiver.

Since the two received noise envelopes (both band-limited to 2 kilocycles) are obtained from different frequency bands, coherent correlation between the wave shapes will be essentially zero since the phases are completely random. However, if the above mentioned assumptions are justified, there should be a high degree of correlation between the mean magnitudes of relatively long-period sample sums.

The required length of the sample sum periods can be developed as a function of the mean frequency of the noise envelope, the bandwidth of the noise, and the sampling rate. Since the expected correlation is between the mean magnitudes of the envelope samples, the basic requirement to be fulfilled is that the integration period be sufficiently long to provide an accurate mean of the rectified noise envelope. The required sampling rate is a function of the maximum frequency present in the wave to be sampled; e.g., complete reproduction of the original envelope is possible if the sampling rate is equal to or greater than two samples per cycle of the highest frequency present. Conversely, the length of the period over which those individual samples must be integrated in order to provide a reliable mean magnitude is determined by the lowest frequency present in the band-limited spectrum. Considering the determination of the mean to be a process of analyzing the frequency spectrum of the envelope, computing the mean amplitude of each frequency component present and then combining these into the combined mean, it is evident that the minimum required period would be that of the period of the lowest frequency component present. As an example, on a noise envelope with a mean frequency of 1100 cycles, band-limited to 200 to 2000 cycles per second, and sampled at a 4000 sample per second rate (minimum), the minimum integration period for the determination of mean amplitude would be 0.005 seconds.

Proceeding on the understanding that the samples from both channels have been prepared to represent long-period mean amplitudes from which phase information has been removed, and that they are from identical time periods, a method of establishing the degree of correlation between the mean magnitudes of the sample ensembles is seen to be required.

As a method of identification, samples from the "on frequency" echo channel will be designated s_i, while samples from the adjacent noise channel will be represented by n_i.

Sample group from echo channel: $s_i \quad 0 < i \leq M$

Sample group from noise channel: $n_i \quad 0 < i \leq M$

Individual samples on each channel (from which the mean amplitudes were derived) are not independent samples since they were obtained at a rate designed to reproduce the information content of the original envelope. The question to be resolved is the degree of independence or correlation between the two sample sets.

As a measure of the correlation between the mean noise samples on the two channels, the covariance σ_{12} will be used.[12]

$$\sigma_{12} = \frac{1}{M - 1} \sum_{i=1}^{M} (s_i - \bar{s})(n_i - \bar{n})$$

where $\bar{s}$ indicates the mean over the sample ensemble and M is the number of samples.

This product indicates a measure of the relationship between the two variables, but in this form is a function of the units of the variables. A more acceptable measure is the coefficient of correlation—defined as the ratio of the covariance of the sample ensembles to the product of the standard deviations of the individual sample ensembles.

$$r_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$

where

$$\sigma_1 = \left[\frac{1}{M - 1} \sum_{i=1}^{M} (s_i - \bar{s})^2\right]^{\frac{1}{2}}$$

$$\sigma_2 = \left[\frac{1}{M - 1} \sum_{i=1}^{M} (n_i - \bar{n})^2\right]^{\frac{1}{2}}$$

This is equivalent to considering that the coefficient of correlation is equal to the covariance of the two transformed variables $(s_i - \bar{s})/\sigma_1$ and $(n_i - \bar{n})/\sigma_2$.

The coefficient of correlation varies between unity limits:

$$-1 \leq r_{12} \leq 1$$

with +1 representing perfect correlation between the sample ensembles, zero representing total independence, and -1 indicating perfect negative correlation.

In the application of the process of using adjacent noise channel information to improve the signal-to-noise ratio on the echo channel, a unity coefficient of correlation would permit an infinite improvement:

$$\frac{S}{N - (1.0)N} \to \infty$$

while a zero coefficient would yield no improvement. Negative coefficients, properly interpreted, could again result in improvement of the signal-to-noise ratio although the occurrence in nature of a physical situation in which negative correlation would result is difficult to visualize.

Coefficients of correlation between $0 < |r_{12}| < 1$ would permit varying amounts of improvement in the output signal-to-noise ratio. As an example, a coefficient of correlation of 0.9 would, after the noise samples were subtracted from the "signal plus noise" samples, yield a 20 db improvement in the signal-to-noise ratio, considering that S and N represent voltages.

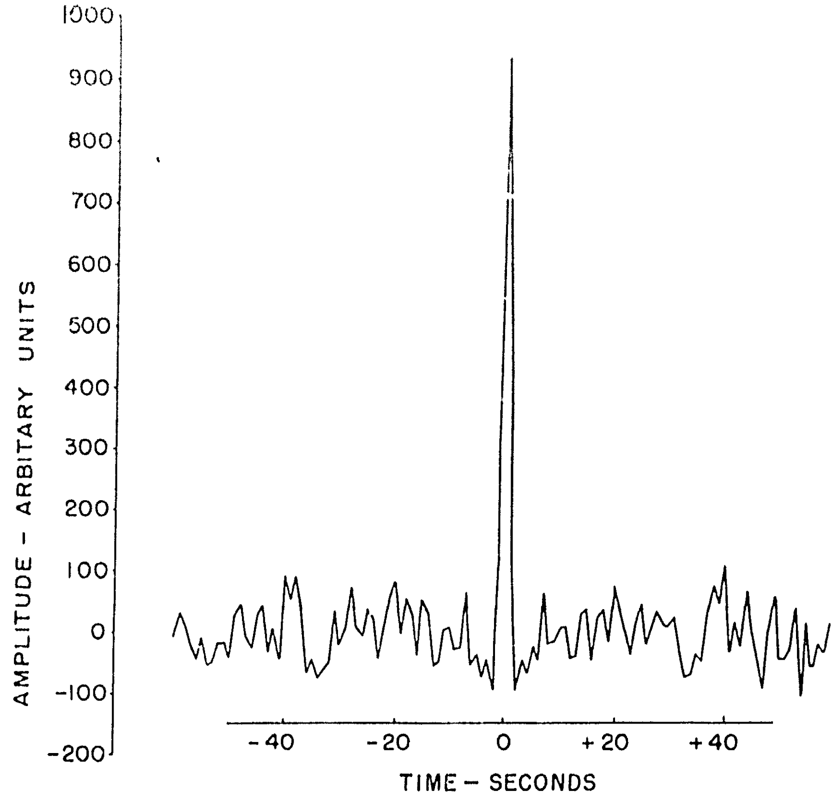
$$\frac{S}{N - 0.9N} = 10\,\frac{S}{N}$$

As an experimental check on the basic assumptions involved, samples of background noise were simultaneously recorded on two separate channels. The band-limited noise envelopes were processed in the same manner as the sun echo data—converted from analog to digital form, clipped, detected, and integrated into one-second sample sums. These sums were then cross-correlated to obtain a measure of the correlation between the amplitude of the background noise in closely spaced, but separated, channels.

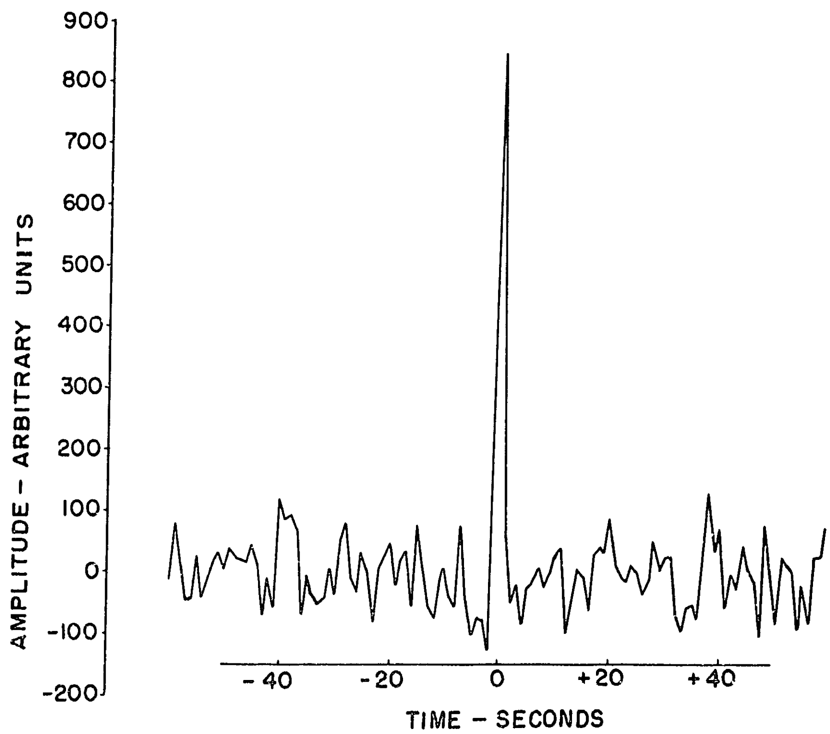
To provide an absolute measure of perfect correlation for comparison purposes ($r_{12} = +1$), the auto-correlation curve of the noise from the signal channel (no signal present) is plotted in Fig. 8(a).

$$r_{12} = \frac{\frac{1}{M-1}\sum_{i=1}^{M}(s_i - \bar{s})^2}{(\sigma_1)\,(\sigma_1)} = \frac{(\sigma_1)^2}{(\sigma_1)^2} = 1$$

The cross-correlation curve of background noise on the signal channel versus background noise on the noise channel (separated by 10 kilocycles) is plotted in Fig. 8(b) to the same scale. The curves indicate the high degree of correlation which was present on a one-cycle-per-second basis between the two channels on 8 September 1959. The coefficient of correlation for the data in the curve of Fig. 8(b) is $r_{12} = 0.86$. Comparison of the curve of the cross-correlation process with the auto-correlation curve of the noise on the signal channel indicates that individual variations on the separated channels are so nearly identical that all major features of the auto-correlation curve are reproduced in the cross-correlation curve. The values for the coefficient of correlation



(a) Auto Correlation – Noise on Signal channel



(b) Cross Correlation - Noise on signal channel vs Noise on Noise channel.

FIG. 8.--CORRELATION BETWEEN MEAN MAGNITUDES OF SOLAR ECHO BACKGROUND NOISE ENVELOPES FROM TWO BAND-LIMITED CHANNELS SEPARATED BY TEN KILOCYCLES. CURVES SHOW HIGH DEGREE OF CORRELATION PRESENT AT FREQUENCIES NEAR 25.6 MEGACYCLES ON 8 APRIL 1959. (A) AUTO-CORRELATION---NOISE ON SIGNAL CHANNEL; (B) CROSS-CORRELATION---NOISE ON SIGNAL CHANNEL VERSUS NOISE ON NOISE CHANNEL.

on 5 and 7 September 1959 were measured at 0.92 and 0.85 respectively. These values were obtained near a frequency of 25.6 megacycles, using one-second integrated sample sums and two-kilocycle bandwidths separated by ten kilocycles.

This experimental verification of the close correlation between background noise samples provides the foundation for a method of improving the signal-to-noise ratio on a signal channel through the use of noise from a parallel channel. The high degree of correlation will permit the effect of noise bursts to be removed as long as the amplitude level changes are not sufficient to overload either receiver. In general, the effect of any amplitude variation common to both receivers can be removed from the received signal using this method. This would also include such effects as gain variations of identical receivers caused by changes in applied voltage.

The over-all effect will be that of a receiving system in which the reference for comparison, rather than being from a different part of the sky or from a noise source (as in several commonly used systems in radio astronomy), is from the same part of the sky but observed at a slightly different frequency. This allows the echo detection process to be one of detecting a small change in energy level, rather than one of detecting a small amount of energy in the presence of large amounts of background noise.

The use of an adjacent channel also permits continuous recording on both channels and thus increases the effective duty cycle over that possible with a system in which the antenna is switched away from the signal channel for comparison purposes.

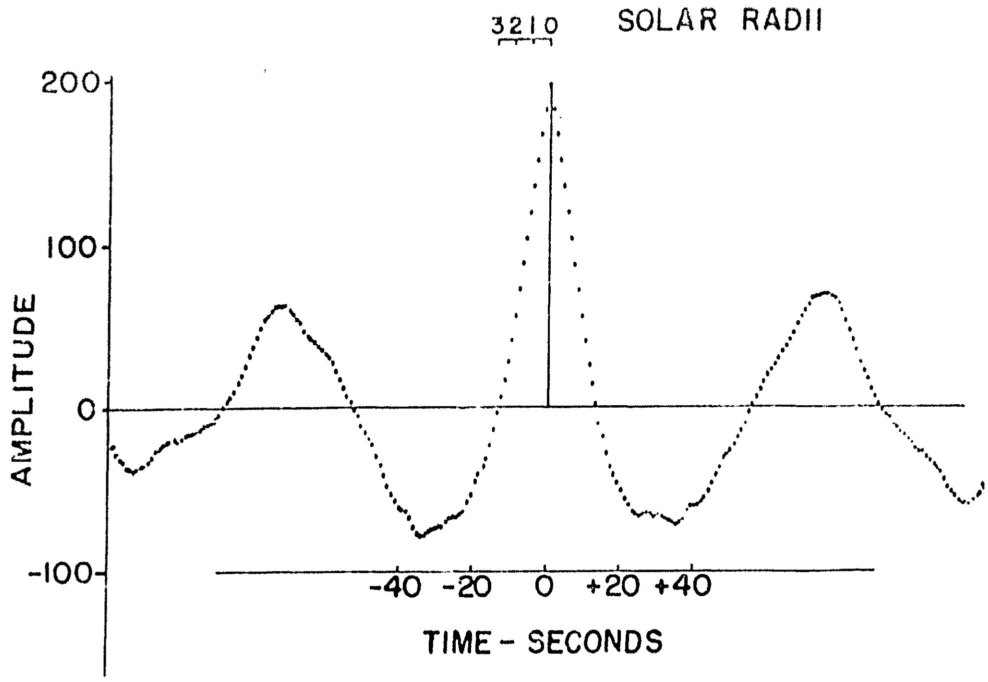
The measurement of the noise-amplitude-correlation between adjacent bands of solar noise and the use of these results to aid in the detection of weak signals in solar noise are perhaps the most important contributions of this investigation.

3. METHODS OF DATA REDUCTION. Data reduction methods were identical to those used for the April 1959 data up to the point of cross-correlation. The one-second sums (560 total---320 in each code sequence and 120 in each of two periods of background noise) were cross-correlated with the transmitted code sequence. The ideal echo-indication curve (auto-correlation curve of the transmitted code sequence) is shown in

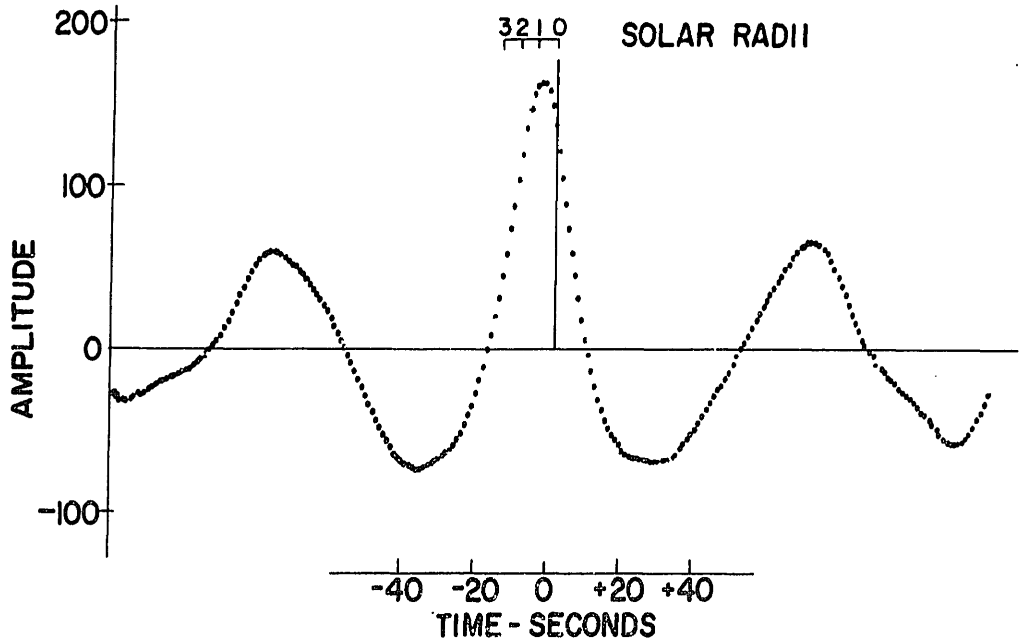
Fig. 9. As discussed in the Appendix, a measure of the effectiveness of the equivalent "matched filter" of the data reduction process can be determined from the width of the central peak of the auto-correlation function of the transmitted code. Figure 9(a) indicates that the approximate bandwidth of the equivalent filter is 1/30 cycle per second—equal to that of the April 1959 data reduction system. Figure 9(b) shows the effect of possible pulse broadening due to echoes in a depth of two solar radii from the sun.

The one-second sample sums were prepared for both the "on frequency" echo channel and the separated noise channel for a total of three code periods on 5 and 7 September 1959. The sample sums from the echo channels were first cross-correlated with the transmitted code sequence in an attempt to locate a peak indication and thus establish the presence of an echo. The cross-correlation curves of all three of the codes, one on 5 September and two on 7 September, gave very weak indications of the presence of solar echoes—all in the vicinity of -25-decibel signal-to-noise ratios. The low signal strengths were attributed to the fact that the expected antenna gains were not being realized.

An attempt was then made to improve the signal-to-noise ratios by subtracting the noise samples recorded in the separated channel. This was accomplished after first adjusting the means of the noise samples to 0.99 of the means of the echo channel samples (allowing for a -20-db differential in mean due to the presence of the echo). The cross-correlation curves of each set of corrected data showed marked improvement in echo indication. Figure 10 shows the cross-correlation curves of the first code sequence of 7 September 1959. Figure 10(a) shows the cross-correlation curve prior to the correction of the data samples. In Fig. 10(b) the dotted curve shows the cross-correlation using data samples from which the noise has been subtracted. The peak indication in the original curve corresponds to a pre-detection signal level of -26.8 db. The peak indication in the corrected curve gives a signal-to-noise echo indication of -17.8 db based on the original noise level. If, however, the noise coefficient of correlation (0.85 for 7 September 1959) is taken into account, the final signal-to-noise ratio becomes -10.4 db. It should be noted that the peak position has been shifted

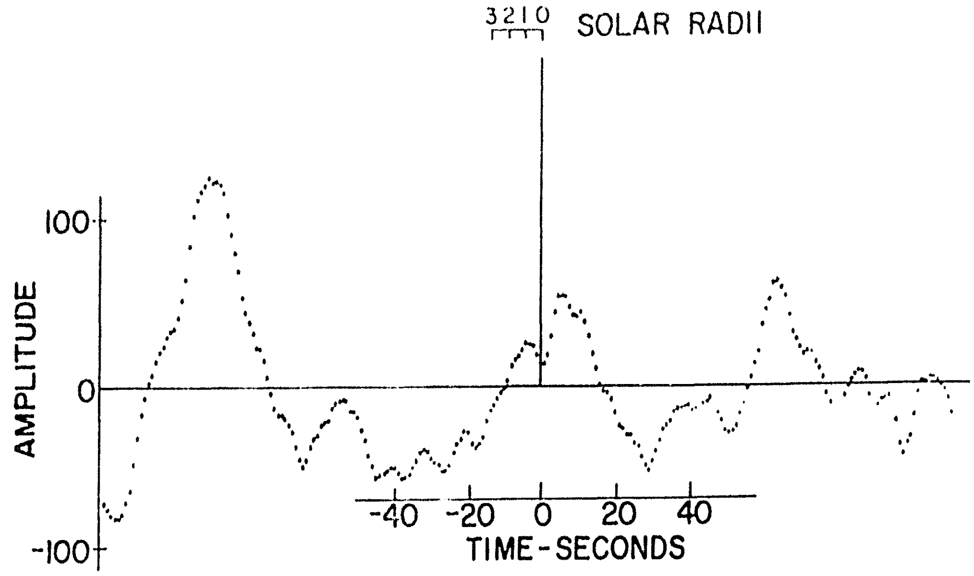


(a) AUTO CORRELATION OF SEPT. 1959. TRANSMITTED CODE SEQUENCE

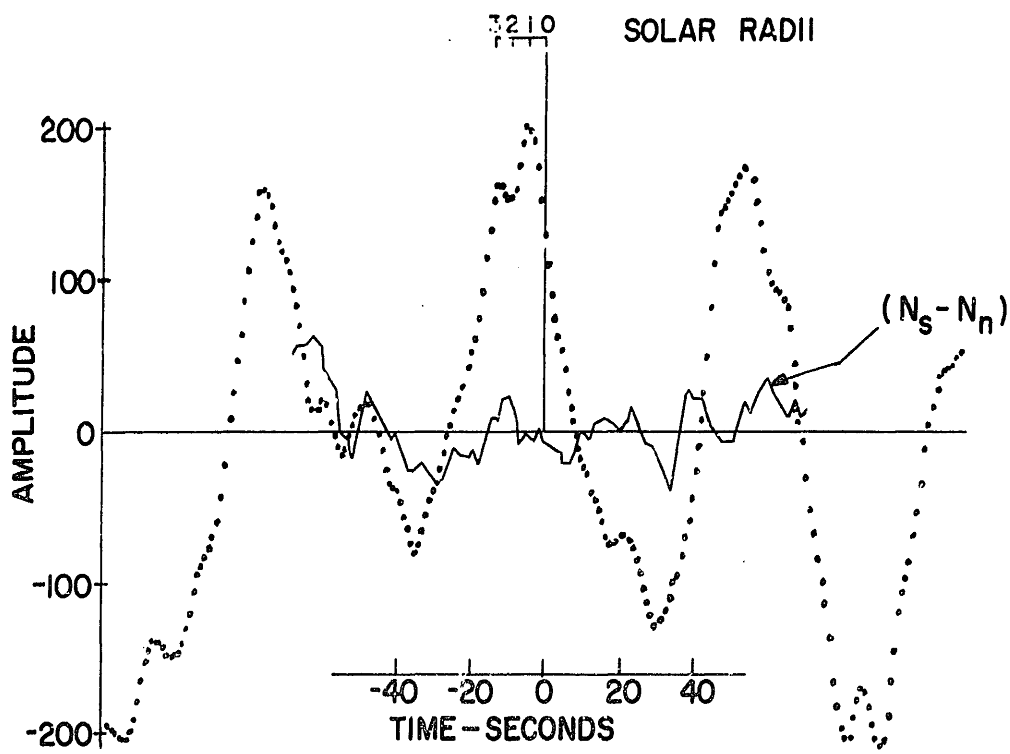


(b) CROSS CORRELATION OF SEPT. 1959. TRANSMITTED CODE WITH IDEAL ECHO-PULSES SPREAD TWO SOLAR RADII IN TIME

FIG. 9.--IDEAL ECHO INDICATION CURVES FOR SEPTEMBER 1959 DATA. THE CURVES INDICATE THE IDEAL ECHO INDICATION AND THE EFFECT OF PULSE SPREADING. (A) AUTO-CORRELATION OF SEPTEMBER 1959 TRANSMITTED CODE SEQUENCE; (B) CROSS-CORRELATION OF SEPTEMBER 1959 TRANSMITTED CODE WITH IDEAL ECHO-PULSES SPREAD TWO SOLAR RADII IN TIME.



(a) UN-CORRECTED CROSS-CORRELATION CURVE 7 SEPT. 1959
CODE 1



(b) CORRECTED CROSS CORRELATION CURVE 7 SEPT. 1959
CODE I

FIG. 10.--CROSS-CORRELATION CURVES OF THE SEPTEMBER 1959 TRANSMITTED CODE SEQUENCE WITH CODE 1, 7 SEPTEMBER 1959. THE CURVES INDICATE THE IMPROVEMENT IN SOLAR ECHO INDICATION THROUGH THE USE OF PARALLEL CHANNEL NOISE. THE (N_S - N_N) CURVE IN (B) (SOLID LINE) GIVES AN INDICATION OF THE VARIATION IN THE CROSS-CORRELATION CURVE VALUES OBTAINED DURING PERIODS OF NOISE ONLY. (A) UNCORRECTED CROSS-CORRELATION CURVE, 7 SEPTEMBER 1959, CODE 1; (B) CORRECTED CROSS-CORRELATION CURVE, 7 SEPTEMBER 1959, CODE 1 (DOTTED LINE) COMPARED WITH CROSS-CORRELATION CURVE OF NOISE ONLY (N_S - N_N) (SOLID LINE).

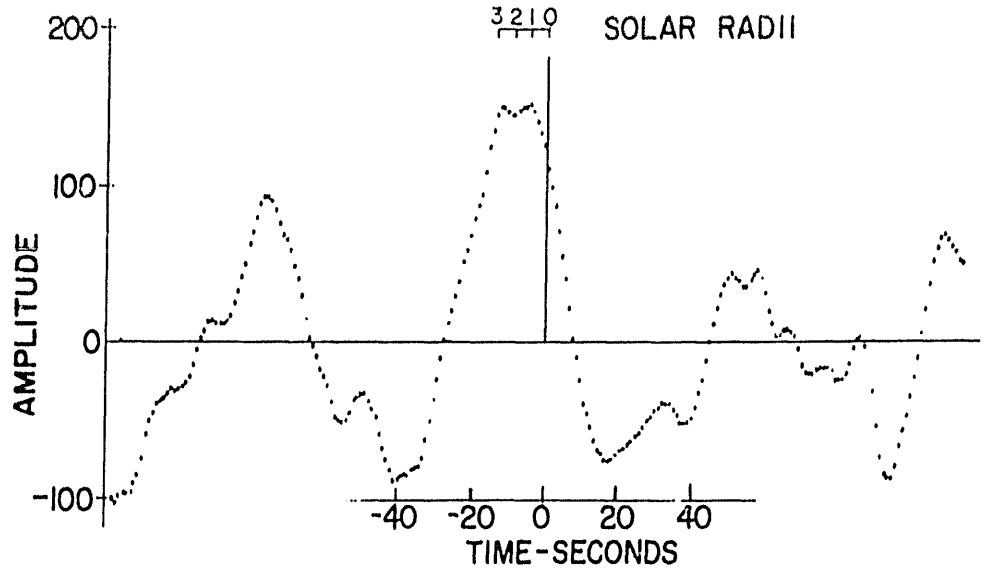
to a point where the mean reflection distance is 1.5 solar radii. The curve marked (N_s - N_n) in Fig. 10(b) (solid line) is a plot of the cross-correlation of the differences between the mean magnitudes of the noise on the two separated channels. This curve is discussed in detail in Section IV E 5.

As with the April 1959 data, assuming that variations in the daily curves from the ideal curve are caused by random noise bursts, a linear combination of the curves should result in an over-all improvement of the echo indication. Figure 11 indicates this combination in progressive steps. Figure 10(b) illustrates the corrected cross-correlation curve for 7 September, Code 1. Figure 11(a) shows the combined curve for Codes 1 and 2 on 7 September, and 11(b) the combined curve for 5 and 7 September (three code sequences).

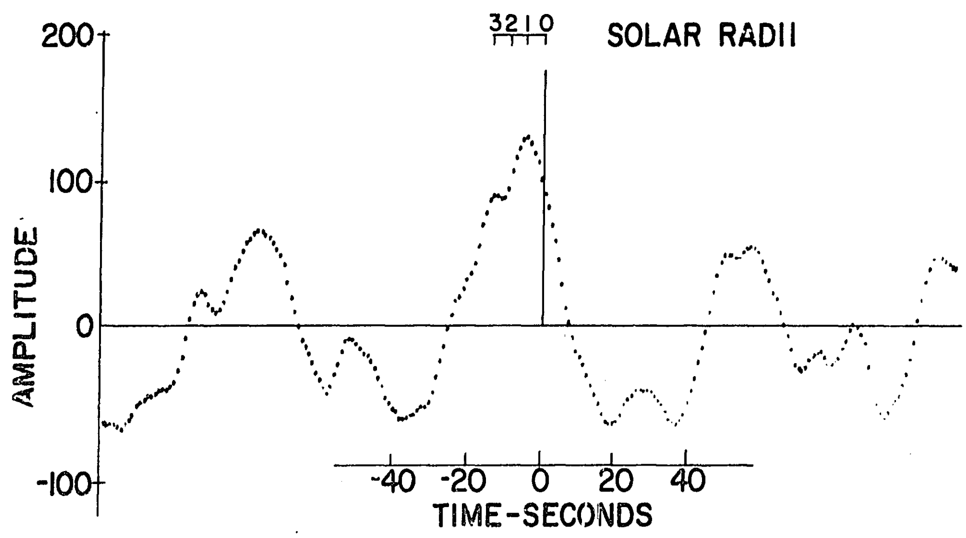
4. EXPERIMENTAL RESULTS. Analysis of the results of the data reduction process as applied to the September 1959 data was accomplished along the lines of the April 1959 analysis. Careful study of the cross-correlation curves of Figs. 10 and 11 indicate the following justifications for acceptance of these curves as indications of radar echoes from the sun:

a. Correlation Peak Position. The probability that a well-defined peak in the cross-correlation curve could be caused by random background noise is extremely small. Significance is therefore given to the occurrence of the central peak of the correlation function at a time position corresponding to an echo reflection distance of between zero and three solar radii. Less significance is assigned to the position of the smaller side peaks since the contribution to the major peak consists of 160 seconds of echo and 160 seconds of background noise, while the contributions to the side peaks are made up of about 230 seconds of background noise, and only 90 seconds of echo time.

b. Definition of Central Peak. A well-defined central peak is an indication that the contributions to the echo indication are not only of proper pulse durations, but are also separated by proper time intervals. The probability of this occuring with random noise bursts is extremely small. The central peak is well-defined in all curves of Fig. 11 and in the corrected curve of Fig. 10(b).



(a) COMBINED CROSS CORRELATION CURVE 7 SEPT. 1959
CODES 1 AND 2.



(b) COMPOSITE - 7 SEPT. 1959 (CODES 1 & 2) PLUS 5 SEPT. 1959
CODE 1

FIG. 11.--COMBINED SOLAR-ECHO-INDICATION, CROSS-CORRELATION CURVES OBTAINED BY AVERAGING VALUES OF TWO AND THREE SEPARATE SUN ECHO TRIALS. THE CURVES INDICATE THE CANCELLATION OF THE EFFECTS OF RANDOM NOISE AND THE RESULTANT IMPROVEMENT IN THE SOLAR ECHO INDICATION. (A) COMBINED CROSS-CORRELATION CURVE, 7 SEPTEMBER 1959, CODES 1 AND 2; (B) COMPOSITE, 7 SEPTEMBER 1959 (CODES 1 AND 2) PLUS 5 SEPTEMBER 1959 (CODE 1).

c. <u>Over-all Shape of Curve</u>. As with the April 1959 composite curves, assuming that the deviations from the ideal echo curve shape are due to random noise bursts, a composite curve in which daily contributions are averaged should yield a smoother curve with more precisely defined features. This is evident from Figs. 10(b), 11(a) and (b).

d. <u>Improvement of Echo Indication After Subtraction of Adjacent-Channel Mean Noise</u>. With the high degree of correlation between the amplitude of the noise on the echo channel and the noise on the adjacent channel, the subtraction of the "noise" samples from the "signal plus noise" samples should greatly improve the echo indication. This is strikingly evident in Figs. 10(a) and (b).

5. NUMERICAL PROBABILITY CONSIDERATIONS. Discussion of the numerical probability of error in the detection of solar echoes in the September 1959 trials is complicated by the length of the code sequence of transmitted pulses used and by the shape of the ideal echo indication curve. (See Fig. 9). Peak position probabilities as utilized for the data from the April 1959 trials is a possibility; however, the number of code sequences transmitted and received in the September trials is insufficient to permit meaningful results to be derived therefrom.

Following the example of the Lincoln Laboratory group in their treatment of echoes from the planet Venus,[3] it would be desirable to discuss the amplitude of the cross-correlation peak in terms of the standard deviation of the cross-correlation curve magnitudes during periods of "noise only". Information for this discussion is readily available in an experiment such as the Venus radar echo trials, in which the ideal echo indication curve consists of a single narrow central peak separated by many pulse widths from small-amplitude side peaks with zero amplitude between the peaks. Information on the amplitude variation of the cross-correlation curve caused by noise alone can then be determined from curve values which lie between the indicated peaks.

With the length of pulses used in the sun echo trials, however, the central and side peaks in the ideal echo indication curve are not separated, and no sizable periods of zero amplitude occur. It then becomes a problem of curve-fitting, i.e., comparing the shape of the ideal echo curve to that obtained from the received data. This yields a confidence figure which can be expressed in terms of an echo probability, but does not have the familiar appearance of normal probability computations.

The probability consideration is complicated even more by the fact that the actual echo indication curves, one of which is shown in Fig. 10(b) (dotted curve), are obtained by a process which subtracts the mean amplitude of the "noise only" samples (N_n) from the "signal plus noise" samples (SN_s). This fact, however, leads to a possible approach which is essentially equivalent to the Venus echo probability discussion.

For the probability computations we need: (1) the magnitude of the cross-correlation-curve peak value, (2) the standard deviation of variations in the cross-correlation curve due to noise contributions only, and (3) a predicted time interval during which the actual sun echo should return.

Since the first item on the required list is obtained from a corrected echo indication curve (SN_s - N_n), to be consistent in our approach the variations caused only by noise (item 2) should also be obtained from differences between corresponding values of "noise only" on the separated channels. These data are not available from the identical time periods during which the echo indications were obtained since only two received channels were recorded. However, on most of the days in the last half of the trial period recordings were made of noise on the two adjacent channels during the period immediately after the last transmitted pulse had returned.

The values of the standard deviation of the variations in the cross-correlation curve due to noise alone were obtained using the following procedure. Recordings of "noise only" for periods immediately following the echo periods on 8 September 1959 were processed in exactly the same manner and at the same magnitude levels as the recordings of solar-echo periods. The one-second sample sums from the noise channel (N_n) were subtracted from the corresponding sample sums from the signal channel (N_s). The transmission code sequence was then cross-correlated with the difference sample sums (N_s - N_n) and a noise cross-correlation curve obtained [Fig. 10(b), curve labeled (N_s - N_n)].

This noise cross-correlation curve fluctuated randomly about the zero value with a standard deviation of 22.5 arbitrary amplitude units. The peak value of this noise correlation curve has a relative magnitude of 2.5 standard deviations.

The noise correlation curve was superimposed on the solar echo indication curve of Fig. 10(b) to provide a comparison of the magnitude of the echo indication as compared to variations due to noise values of identical amplitude and processed in the same manner as the echo indication data.

Error probability computations were made by first converting the peak value of the corrected solar echo cross-correlation curve (SN_s - N_n) to an equivalent number of standard deviations, using the standard deviation of the noise correlation curve values (N_s - N_n) as a normalizing factor. This was done only if the peak value of the echo indication curve was positioned at a time corresponding to a reasonable reflection height in the solar corona. The numerical value was then determined from standard error tables (gaussian distribution) of the probability that a value corresponding to the peak value of the echo indication (or greater) could be caused by noise alone. This probability was then considered the probable error in the determination of a solar echo for that particular solar echo trial.

The magnitudes of the central peaks in the solar echo cross-correlation curves for the various codes were computed to be:

5 September 1959	Code 1	4.1 Standard Deviations
7 September 1959	Code 1	9.0 Standard Deviations
7 September 1959	Code 2	6.5 Standard Deviations

As mentioned previously these peak locations were maximum values, and were located in the reasonable solar echo area.

Error probabilities corresponding to the above peak magnitudes are summarized in Table VI. These error probabilities can be reduced by considering that the figures below are actually the probabilities that the given magnitudes would occur at any point due only to background noise. The probabilities that the peak values would fall in a specific time interval corresponding to a solar echo from a point between

TABLE VI.--SOLAR ECHO ERROR PROBABILITIES - SEPTEMBER, 1959

DATE	ERROR PROBABILITY IN ECHO DETERMINATION
5 SEPTEMBER, CODE 1	2×10^{-5}
7 SEPTEMBER, CODE 1	1×10^{-17}
7 SEPTEMBER, CODE 2	8×10^{-9}

zero and three solar radii would be less by a factor of 13.9/560, where 13.9 seconds is the time interval equivalent to three solar radii of reflection, and 560 seconds is the time interval searched in the cross-correlation process.

It should be emphasized that the value of the standard deviation of the noise cross-correlation curve $(N_s - N_n)$ used in the probability computations is based on only a single seven-minute period of noise in the two separated channels. Additional periods of noise of reasonably uniform mean amplitude and noise bursts less than one or two seconds duration were not available.

V. CONCLUSIONS

The sun echo trials conducted at Stanford University during 1959 (both in April and September) yield conclusive evidence of the presence of radar echoes from the sun. As indicated previously these initial sun echo trials were intended only as a first step in the establishment of a potentially useful technique for the study of the solar corona and interplanetary space. Information beyond (a) the presence of a radar echo from the sun, and (b) the mean reflection radius of the 26-megacycle electromagnetic energy being 1.5 to 1.7 solar radii, is not claimed for this series of solar radar echo experiments.

Associated information, gathered in the process of establishing the above conclusions, provides additional knowledge of background (quiet sun plus galactic) noise and an experimentally verified method of improving the signal-to-noise ratio on a given channel through the use of noise recorded on an adjacent channel.

The method of improving received signal-to-noise ratios, applicable to band-limited systems where integration can be used in the detection process (echo-energy or magnitude-only detection), can be implemented through the use of the output of a second receiver channel separated in frequency from the signal channel. The method is based on the assumption that the interfering noise sources are broad enough in frequency spectrum to make correlated amplitude contributions to both channels. In cases in which sufficient integration can be accomplished to provide accurate mean values of the amplitude of the two received envelopes, the subtraction of the mean amplitude of the noise background (as obtained from the separated channel) will result in an improvement of the overall signal-to-noise ratio.

A measure of the signal-to-noise ratio improvement is obtained from the coefficient of correlation between the mean noise amplitudes from the two separated channels. Coefficients in the range $0 < |r_{12}| \leq 1$ provide improvement. Measured coefficients of correlation from the September 1959 sun echo trials ranged between 0.85 and 0.93.

VI. RECOMMENDATIONS

As a result of the reception of radar echoes from the sun and the associated background noise investigations, a number of areas for further study are indicated. Successful investigation in these areas will require larger signal-to-noise ratios than were obtained in the initial sun echo experiments to allow a more detailed study of the characteristics of each echo pulse.

1. ECHO FREQUENCY SPECTRUM STUDIES. A study of the frequency spectrum of an echo pulse as compared to the frequency spectrum of time-coincident background noise will yield information on the extent of Doppler spreading of the transmitted signal. This information would be related to rotation and other motion of the solar corona.

2. TIME VARIATION STUDIES. (a) The variation of mean echo reflection distance with time as determined from the mean reflection point of each echo pulse will provide information on the gross changes in the solar corona and may provide data for correlation studies with solar phenomena observed by optical or radio astronomy techniques. (b) Continued reception of radar echoes from the sun with polarization-sensitive antennas could provide interesting data on the solar corona and the associated solar magnetic field. (c) Constant-energy-level transmissions and reception at a fixed frequency continued for relatively long periods of time could provide data on echo intensity levels as a function of time. Removal of echo intensity variations due to reflection distance changes could then provide information on absorption characteristics in the solar corona.

3. FREQUENCY VARIATION STUDIES. Additional information could be gained in the above-mentioned areas as to variations as a function of transmitted frequency. These studies should probably be deferred until fairly reliable (or coincident) time variations are available, so that the effect of frequency variations could be isolated.

4. BACKGROUND (GALACTIC AND SOLAR) NOISE STUDIES. Studies of the characteristics of solar echo background noise over a long period of time could provide an experimentally verified probability distribution function for solar noise under both "quiet sun" and disturbed conditions.

Additional information on the method of improvement of signal-to-noise ratios through the use of adjacent-channel noise could be obtained from studies of the variation of the coefficient of correlation with:

a. Frequency Separation. Data obtained from adjacent noise channels (identical bandwidths, gain and detection) should provide a means to measure the useful range of the technique as a function of the amount of separation.

b. Integration Period. Studies of adjacent noise channel amplitude-correlation were restricted in the solar echo experiments to one-second integration periods. Useful information for future applications would be the effect of the length of the integration period on the coefficient of correlation, and the establishment of experimentally verified limits.

c. Solar Activity. Studies of the variation of the coefficient of correlation during periods of solar activity should provide interesting information.

5. SHORT-PULSE EXPERIMENTS. Sufficient increase in system sensitivity to permit the use of short pulses for solar echo work will make possible the detection of separate reflection centers in the solar corona. The combination of this sensitivity with a highly directive antenna system should provide data which can be analyzed for spectrum, range, and angle information and should lead to greater knowledge of the size, shape, and mass motions in the solar corona.

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APPENDIX. MAXIMIZING THE SIGNAL-TO-NOISE RATIO[8]

Consider the received signal during a period when the echo is present. The input to the receiving system is $y(t) = s(t) + n(t)$, where $s(t)$ is the echo signal and $n(t)$ the background noise. To observe the possibility of maximizing the output signal-to-noise ratio (defined as the ratio of the square of the peak signal voltage to the noise power), let L [ ] denote the linear operation performed by a filter. The output is

$$y_0(t) = L\,[s + n]\,(t) = L\,[s]\,(t) + L\,[n]\,(t)$$

and we define $L\,[s]\,(t)$ to be the output signal $s_0(t)$ and $L\,[n]\,(t)$ to be the output noise $n_0(t)$.

Specializing to a more specific case in which $s(t)$ is a known function of time (a priori information),

$$y(t) = s(t) + n(t)$$

where $n(t)$ is now a sample function from a wide-sense stationary random noise process.

The output signal-to-noise ratio will be a maximum at some arbitrary time t_2 (a function of the echo delay time)

$$(S/N)_0 = \frac{s_0^{\,2}(t)}{E[n_0^{\,2}(t)]}$$

where $E[n_0^{\,2}(t)]$ is the expected value of $n_0^{\,2}(t)$—the output noise power.

Assuming our filter acts on the input for a time T

$$s_0(t_2) = \int_0^T h(\tau)\; s(t_2 - \tau)\, d\tau$$

$$n_0(t_2) = \int_0^T h(\tau)\; n(t_2 - \tau)\, d\tau$$

where $h(\tau)$ is the impulse response of the filter.

For the case in which the noise can be assumed to be white, gaussian, and of spectral density N_0 watts per cps, over the bandwidth of interest, the maximum signal-to-noise ratio is obtained[9] when

$$h(\tau) = \frac{1}{N_0} s(t_2 - \tau) \qquad\qquad 0 \leq \tau \leq T$$

Thus the optimum filter has the form of the signal run backward, starting from the arbitrary time t_2. This is termed a "matched filter". This filtering operation can be accomplished by the cross-correlation of the received "signal plus noise" with the transmitted signal. The cross-correlation curve is then an indication of the signal strength under maximum signal-to-noise ratio conditions.

Now consider a transmitted wave $s(t)$ of a coded sequence of rectangular pulses. With the returned signal $ks(t)$ passed through a matched filter of impulse response,

$$h(t) = f(t_2 - t)$$

the receiver output is

$$y_0(t) = k \int_{-\infty}^{\infty} s(\tau)\, h(t - \tau)\, d\tau = k \int_{-\infty}^{\infty} s(\tau)\, f(\tau + t_2 - t)\, d\tau$$

$$= kR_{11}(t - t_2)$$

The quantity R_{11} is the auto-correlation function of the transmitted wave form $s(t)$.

With a transmitted code of rectangular pulses of length and separation t_1 seconds, the auto-correlation function R_{11} (which may be considered as the ideal echo indication curve) is a triangular wave with length of base t_1 seconds. The function $y_0(t)$ will then have a bandwidth of approximately $1/t_1$ cps.

With a random sequence of rectangular pulses in the transmitted code, the approximate bandwidth of the equivalent matched filter is determined by the width of the base of the central peak in the auto-correlation function of the transmitted code.

The assumption of white, gaussian noise over the narrow bandwidth of interest is an idealized one and was chosen to arrive at an idealized signal (echo) indication curve which would be used for comparison purposes. Actual noise conditions depart from the idealized case in varying amounts, depending upon the solar activity at any given time.

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