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AN ANALYTICAL INVESTIGATION
OF AIRPORT CAPACITY

Report No. TA-1358-G-1
Internal Research

JUNE 1960

B U F F A L O , N E W Y O R K



CORNELL AERONAUTICAL LABORATORY, INC.
Buffalo 21, New York

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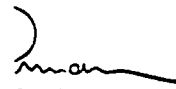
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ABSTRACT

The problems of air traffic control, particularly those of capacity in the terminal area, are examined indicating the need for increased airport capacity. Analytical models are formulated expressing the capacity of an airport runway as a function of parameters characterizing the airport, the air traffic control system, and the arriving aircraft.

Landing capacity is considered to be restricted by the minimum space separation required at the common-path gate and by the minimum time separation required at the runway. A model is formulated for estimating the landing capacity of a runway used for landings only which expresses the following system characteristics: 1) minimum space separation required at the beginning of the common landing path, 2) minimum time separation required at the runway, 3) length of the common landing path, and 4) the parameters of the landing-velocity distribution.

Take-offs are considered to be interposed between landings wherever possible and performed as a separate run where necessary. A model is formulated for estimating the operations capacity of a runway used for landings and take-offs which expresses the parameters of the

landing model as well as the following: 1) minimum time separation required between departures, 2) the runway-occupancy time of a landing aircraft, and 3) the minimum distance separation required to permit a departure to be interposed before an arrival. Extensions of these basic models are indicated, including extension to the investigation of a multiple-runway airport.

The effect of the various system parameters on capacity is investigated by means of the models developed. It is shown that landing capacity is negligibly improved by a reduction in runway time separation by such techniques as construction of high-speed turnoffs. The greatest improvement would result from reduction of the present 3-mile separation required at the beginning of the common landing path. A particular sequencing rule is examined as a potential means of increasing landing capacity, and is indicated to be of little value, although it could appreciably increase operations capacity. Other techniques for improving landing capacity are investigated. Operations rate is shown to represent a complex interaction of system parameters, and can often be increased by actions that tend to decrease landing capacity, but which increase the rate at which take-offs can be interposed between landings. The high-speed turnoff is shown to contribute to an increase in operations capacity by removing landings from the runway earlier, thereby providing increased opportunity for interposing a take-off.

PREFACE

This report was submitted as a thesis to the Graduate School of Cornell University in partial fulfillment of the requirements of the degree of Doctor of Philosophy in operations research. The author appreciates the advice and suggestions offered by the members of his special committee, Professors A. S. Schultz, Jr. (Chairman), A. E. Bechhofer, R. W. Conway, and P. E. Ney.

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TABLE OF CONTENTS

		<u>Page No.</u>
1	INTRODUCTION	1
1.1	Outline of the Report	2
1.2	Contributions of the Study	4
2	AIR TRAFFIC CONTROL	5
2.1	The Present System of ATC	7
2.1.1	Development of ATC	8
2.1.2	Instrument Flight Rules	9
2.1.2.1	Enroute Control	9
2.1.2.2	Terminal-Area Air Traffic Control	11
2.1.2.2.1	Separation Standards	14
2.2	Some Previous Studies of ATC Problems	17
2.3	Some Inherent Problems in ATC	25
2.3.1	Conflicting Needs of Airspace Users	25
2.3.2	Need for Setting Conservative Separation Standards	28
2.3.3	Obsolescence of See-and Be-Seen Operation	30

		<u>Page No.</u>
2.3.4	Introduction of Jet Aircraft	32
2.4	The Problem of ATC System Capacity	33
2.5	The Terminal Area Problem	38
2.6	Efforts to Improve Terminal Operation	40
2.6.1	Radar	40
2.6.2	Data Processing Central Equipment	40
2.6.3	Landing Aids	43
2.6.4	Collision-Avoidance and Proximity-Warning Equipment	45
2.6.5	High-Speed Turnoffs	47
2.6.6	Additional Construction	48
3	ANALYSIS OF THE LANDING CAPACITY OF A RUNWAY	50
3.1	Previous Investigations of the Landing Problem	50
3.1.1	Analytical Studies	51
3.1.2	Simulation Investigations	55
3.2	An Analytical Model of Runway Landing Capacity	63
3.2.1	Assumptions	63

		<u>Page No.</u>
3.2.2	Notation	64
3.2.3	General Formulation	67
3.2.3.1	Discrete Velocity Distribution	58
3.2.3.2	Uniform Velocity Distribution	68
3.2.4	Model When Gate Separation Maintained	
	Along Glide Path	72
3.3	Results of the Landing Capacity Analysis	73
3.3.1	Landing Capacity with the Discrete	
	Velocity Distribution	74
3.3.2	Landing Capacity with the Uniform	
	Distribution	78
3.4	Effect of Velocity Distribution on Landing	
	Capacity	86
3.5	Estimation of Landing Capacity of an	
	Individual Airport	86
3.6	Increasing Landing Capacity	89
3.6.1	Reducing Gate Separation (s_0)	90
3.6.2	Sequencing of Landing Aircraft	93
3.7	Queuing Considerations	108
3.7.1	Queue Relationships	109

		<u>Page No.</u>
3.7.2	Landing-Time Distribution	114
3.7.3	Parametric Investigation of Queue Characteristics	118
4	ANALYSIS OF THE OPERATIONS CAPACITY OF A RUNWAY USED FOR LANDINGS AND TAKE-OFFS	123
4.1	Previous Investigations of Mixed Operations	123
4.2	The Basic Analytical Model of Runway Operations Capacity	126
4.2.1	Assumptions	126
4.2.2	Notation	129
4.2.3	Time for a Sequence of N Operations	131
4.2.4	Interposition of Take-Offs	
4.2.4.1	Operations Rate with Interposition	133
4.2.4.2	Interposition Rate	135
4.2.4.2.1	Interposition Probabilities	137
4.2.4.2.2	Interposition Probabilities When $a < r_0 / (t_0 - \tau_L) < b$	143

		<u>Page No.</u>
4.3	Extensions of the Basic Operations	
	Capacity Model	144
4.3.1	Treatment of T_{DD} and T_{AD} as Discrete Chance Variables	144
4.3.2	Operations Rate for $\omega > 0$	146
4.3.3	Interposition of Landings between Take-Offs	149
4.3.4	Conservative Estimate of q_p	158
4.3.5	Operation of Multiple Runways	163
4.4	Results of the Operations Capacity Analysis	
4.4.1	Parametric Analysis	168
4.4.1.1	Effects on Interposition Rate (ν_D)	169
4.4.1.2	Effects on Operations Rate (μ)	171
4.4.2	Operations Capacity at Idlewild and LaGuardia	175
5	CONCLUSIONS	183
5.1	Improving Landing Capacity	184
5.2	Increasing Operations Capacity	186
5.3	Limitations of the Models	189
5.3.1	Use of the Rectangular Velocity Distribution	190
5.3.2	Runway Occupancy-Time Assumption	193

	<u>Page No.</u>
5.3.3 Representation of the Controller	194
5.3.4 Computation of Operations Capacity	196
5.4 Extensions of the Research	198
REFERENCES	201
APPENDIX A - Determination of Mean Landing Interval τ_{AA}	211
APPENDIX B - Derivation of Landing-Time Distribution $S_{AA}(t)$	215
APPENDIX C - Derivation of τ_I	221
APPENDIX D - Derivation of q_P and τ_I when $a < r_o / (t_o - \tau_L) < b$	225
APPENDIX E - Investigation of the Relationship between Discrete V Distributions and their Uniform Equivalents	232
APPENDIX F - Runway Performance Measures as a Function of $\bar{V}$, R , r_o , m , t_o , k_R , t_{oD} , s_o	251
APPENDIX G - Runway Performance Measures at LaGuardia and Idlewild	255

CHAPTER 1

INTRODUCTION

The air traffic volume in the United States has increased markedly since World War II, and is expected to increase at an even faster rate in coming years. This increased demand has not been accompanied by a commensurate increase in the capacity of the nation's airports and air traffic control facilities, and so has led to an increase in the number and length of delays experienced by aircraft, particularly during instrument weather. The problem is most critical in the terminal areas, since most of the delay is experienced while aircraft wait their turns to land or to take off.

This investigation is directed at the question of airport capacity. The problem has been examined in the past primarily by the techniques of real-time simulation with human subjects in a simulated environment, and fast-time digital simulations are currently being developed. Analytical formulation of this operational problem in terms of a system of equations has a number of advantages over simulation. Once developed, the equations can be solved for any number of conditions and parameter combinations far more cheaply and quickly than a simulation can be operated. Their simplicity often aids in achieving an understanding of the system operation more readily than with a complex

simulation. Analytical models can provide complete distributions or true expected values rather than sampled data. These advantages are generally associated with a system description that is less realistic than that possible in a simulation, but the lack of realism need not necessarily be a handicap in the context of the questions being investigated.

The objective of this study is the development of such analytical models expressing the landing capacity and the operations capacity (i. e., the rate of landings and take-offs under saturation conditions) of a runway as a function of the airport and air traffic parameters. With these models, the effect of the various parameters on airport capacity are examined with a view to understanding the operation of the system and the effect of the parameters and their interactions on its operation, ultimately with the goal of learning the most effective means of raising the capacity of airports in processing arriving and departing air traffic.

1.1 Outline of the Report

Following this introduction, Chapter 2 contains a brief description of the present air traffic control (ATC) system as it operates in the United States, particularly in conditions of poor visibility, when the strain on the system is most severe. This information is presented as an introduction to the reader unfamiliar with ATC and as a background for some of the assumptions made in the formulations of the analytical models. The

conclusions of some of the major national post-war studies of ATC are summarized, and some of the major remaining problems are indicated. The increasing problems of capacity are presented, and the problems of the terminal area in particular are discussed, indicating the crucial role of the terminals in the present problem.

In Chapter 3, the problem of landings on a single runway is studied. This problem has practical importance of itself, since many runways often are used for landings only, but also serves as a necessary first step in the analysis of the larger problem of a runway used for mixed operations. A model is formulated permitting determination of the landing capacity of a runway at any airport as well as the distribution of the landing-time intervals. Possible extensions of the model are indicated. The model is used to examine the parametric effects on landing capacity, and to estimate the capacity of some typical airports. Various means of improving landing capacity are considered, and reduction of space separation along the landing path is shown to be most fruitful.

The problem of a runway used for both landings and take-offs is treated in Chapter 4. An analytical model is developed that relates operations capacity to the eight principal parameters affecting it. Extensions to this basic model are formulated, indicating how the basic model may be applied to situations excluded by the assumptions. Operations

capacity is shown to be affected by a complex interaction of the system parameters, and it can often be increased by steps that tend to decrease landing capacity.

In Chapter 5, the results of the analyses are discussed to indicate the relative effectiveness of various techniques for increasing airport capacity. The limitations of these results due to model assumption are indicated, and extensions of the research reported here are discussed.

1.2 Contributions of the Study

This study contributes to the solution of present and future problems of airport capacity as well as to the literature of operations research. The major contribution is to the body of models available to the operations analyst for studying terminal-area operations -- a classical problem area which has concerned operations research almost from its beginning -- by the formulation of new analytical models and extensions which indicate airport performance as a function of system characteristics, and which are widely applicable. In addition to developing these techniques for analyzing airport problems, they have been applied to current operations, resulting in specific recommendations for improving performance.

CHAPTER 2

AIR TRAFFIC CONTROL

The ATC system exists primarily to permit aircraft to fly in conditions of minimum visibility without the accompanying large risk of a mid-air collision. To be of value, this protection must be achieved with a minimum of inconvenience (primarily in the form of delay) or expense to the user. The protection is accomplished by following the position of each aircraft under control and assigning to it an exclusive volume of airspace. This cushion is large enough so that, despite the errors in position information, sufficient time is available for the control system to detect potentially dangerous proximity of two aircraft and to issue appropriate control orders to the aircraft to remedy the situation.

The ATC system must contain subsystems performing data-collection (to obtain knowledge of aircraft positions and intended flight paths), data-processing (to predict potential conflict situations), decision-making (to select the best means of resolving the conflicts), and communication (to inform the pilots of action required to resolve the conflicts). Today, the data collection is accomplished by radar observation and by radio reports sent by the pilots when they pass over specified radio transmitters; the data processing and decision-making is performed manually by controllers using hand-written records; and communication is performed by voice radio.

The controllers clearly have a predominant role in the system's operation. They are aided by a set of basic principles (e.g., "first-come-first-served") and by a codified set of "ANC procedures" [1]* which cover many situations, but much is still left to the controller's judgment. Equipment is now being developed which will aid the controllers in data-processing but which will continue to entrust all decision-making only to the versatile and flexible human controllers.

This chapter contains a brief introduction to ATC, primarily to provide some background reference for the analysis that follows and to indicate some of the practical motivation of the problems studied. Following a brief history of ATC development, the present methods of performing air traffic control are summarized, and only those detailed rules which are used in the later analyses are presented. Some previous major studies of ATC are also summarized. The discussion is concluded with some basic ATC problems which relate especially to those of system capacity and terminal operation, and particularly to terminal capacity, which forms the subject of the later analyses.

* The numbers in square brackets denote the references, which are listed on pages 201-210.

2.1 The Present System of ATC

The fundamental rules governing air traffic operation are contained in Part 60 of the Civil Air Regulations [2]. These define two systems of rules: Visual Flight Rules (VFR), which apply only when visibility is good, and Instrument Flight Rules (IFR), which apply under poorer (or "instrument") weather conditions or whenever a pilot requests control under these rules to obtain additional collision protection. All aircraft must fly according to IFR whenever they are:

less than 500 feet vertically under, 1,000 feet vertically over, and 2,000 feet horizontally from any cloud formation; or beneath the ceiling when it is less than 1,000 feet, . . . [or] when flight visibility is less than 1 mile . . . [or in the vicinity of an airport] when the visibility is less than 3 miles. [2, p. 3.]

The VFR rules are based on the "see-and-be-seen" principle, in which all responsibility for collision avoidance is placed with the pilots. The IFR rules compensate for the pilots' limited visibility by investing this responsibility in the control system, which surrounds each aircraft with a volume of airspace which may not be violated by any other IFR aircraft.

IFR control is exercised in accordance with a promulgated set of procedures jointly arrived at by military and civilian air authorities [1] and designated as "ANC Procedures." In this section, these procedures are briefly summarized, partly to indicate some of the operational concepts, but primarily to serve as a reference for some of the procedural assumptions that enter into the models of Chapters 3 and 4.

2.1.1 Development of ATC

Air Traffic control appeared relatively late in the development of aviation. The early pilots flew only in clear weather, and worried little about collision with the few other aircraft operating. As commercial aviation developed, with the concomitant demand for safe operation in almost all weather conditions, facilities were created to meet these demands. Radio equipment was first installed in a control tower in 1930 for airport control, but it was not until 1935 that any organized control of enroute IFR traffic was effected. As stated in a traffic control training manual:

At that time, air carriers operating between Newark, Cleveland, and Chicago established centers from which a limited amount of control information could be given to pilots to provide separation from other traffic. The following year, the Civil Aeronautics Administration (CAA) took over the operation. [3, p.1.]

Until 1944, all aircraft operating under IFR conditions were controlled by these Airways Centers from shortly after take-off until final approach to landing. The control towers at the airports were responsible for the control of traffic on the airport itself and for VFR traffic operating below the clouds. To remove some of the inefficiencies and delays in this system, an Approach Control function was then assigned to many of the towers to extend their control beyond their visual limits, and to permit them to coordinate activity on the airport with that in the surrounding airspace. Today, the enroute IFR separation is effected by Air Route Traffic Control Centers

(ARTCC), Approach Control in the airport towers directs the movement of IFR traffic to and from specific fixes in the vicinity of the terminal, and Airport Control directs the flow of traffic on the surface and in the visible portion of the airspace immediately adjacent to the airport.

2.1.2 Instrument Flight Rules

2.1.2.1 Enroute Control

Control during the enroute portion of a flight (i.e., the entire flight except that portion in the vicinity of the origination and destination terminals), is effected by means of a network of "airways," which also serve as a navigational route structure, similar to a highway system. The airways are defined between pairs of radio transmitters, or "fixes," located throughout the country. A pilot, before taking off, files a flight plan with the ATC system, indicating his intended airway route. From his flight plan, the Center (ARTCC) determines his estimated time of arrival (ETA) at each of his first few fixes. A controller is assigned to each fix, and he maintains a list of ETA's and scheduled altitudes of all aircraft intending to pass there.

When he is ready to take off, the pilot requests clearance to his first few fixes. The controller assigned to each of these fixes checks to see that no conflict with other traffic would be created by the aircraft arriving there. A conflict would occur if two aircraft have ETA's less than the minimum separation time (generally 10 minutes) and altitude separation less

than 1,000 feet. If no conflict exists, the pilot is cleared to the fix; if a conflict does exist, one of the aircraft is delayed, or is ordered to another altitude level. In accordance with the first-come-first-served principle, the pilot with the later ETA receives the control order.

As a pilot passes each of his enroute fixes, he reports his actual time of arrival, from which his ETA's at succeeding fixes are up-dated. He must then obtain clearance to each successive fix. Conflicts are generally resolved by holding (flying a specified pattern over the fix or reducing speed slightly) or altitude changes.

When radar facilities are available, the separation standards are far less severe than those imposed by the ANC requirements, since the position information is far more precise. Aircraft may approach as close as three miles if they are within 40 miles of the radar, five miles if beyond. This contrasts sharply with 50 miles required of 300-knot aircraft in the time-separation (ANC) system, or 100 miles required of 600-knot aircraft. The capacity improvements to be derived by the installation of radar in enroute control are obvious.

Radars, on the other hand, introduce additional problems: identification of the radar return from an aircraft which does not carry a beacon transponder can consume considerable controller and pilot effort; some targets are not seen on the radar, while spurious returns can clutter

the picture; vectoring aircraft observed on radars can be very time-consuming. Furthermore, since radar has not yet reached the reliability of radio, the system, even with radar, tends to operate at a low capacity level so that it can revert to ANC operation in the event of a radar failure without catastrophic effects. As these problems are resolved, the installation of enroute radar will accelerate and the enroute capacity will be appreciably increased beyond current levels.

2.1.2.2 Terminal-Area Air Traffic Control

Transition between enroute and terminal control is achieved at the "stacks," or holding fixes, located over some radio facility in the vicinity of every major terminal. The lower altitudes at the stacks are under the jurisdiction of Approach Control at the airport, while the upper altitudes are controlled by the Center. Approach Control keeps the Center informed of altitude availability at the stacks, and the Center clears an aircraft destined for the airport to the lowest available altitude in one of its stacks. Its further progress is then under the direction of Approach Control.

In addition to providing a convenient transition to terminal control, the stack serves as a buffer storage in which aircraft wait their turns to land. If no other aircraft are approaching or waiting to land when an aircraft arrives in the terminal area, it is cleared to land, and follows a standard procedural maneuver to final approach. If other aircraft are waiting to land, however, the arriving aircraft is ordered to the lowest vacant altitude in the

stack; when an aircraft leaves the bottom of the stack for an approach to the airport, the remaining aircraft all descend one altitude layer (1,000 feet) and continue to wait their landing turns.

Most major terminals maintain at least two stacks, partly to accommodate aircraft with different radio equipment, partly to accommodate aircraft arriving from different directions, and partly to provide an opportunity to interleave aircraft from the different stacks while maintaining vertical separation in the stack and alleviating the limitations of aircraft descent rate.

Aircraft are dispatched from the stack in accordance with a first-come-first-served priority rule, specified by the ANC procedures:

2.1001. Priority: The first aircraft estimated to arrive over the point from which approaches are commenced will normally be the first aircraft to approach. Other aircraft will normally have priority in order of their estimated arrival time over such point. [1, p. 11.]

Approach Control directs the aircraft from the stacks to final approach and the landing is completed with the use of a landing aid, normally the Instrument Landing System (ILS). The path flown onto final approach is normally a standard procedural maneuver. When radar control is available, radar vectoring over short-cut routes is often performed, consistent with the need to keep workload on the radar operators low in order to recover easily from a radar failure and to revert to standard ANC procedures.

Airport Traffic Control (the Tower), operating in the glass-enclosed portion of the airport tower, exercises visual control over all operations on the airport surface and the immediately adjacent airspace, and assigns all landing and take-off clearances. In controlling the runway use, the Tower follows the basic principle that the runway shall never be occupied by more than one aircraft. After Approach Control has cleared an aircraft for an approach, the pilot must contact the Tower for clearance to land. The Tower is in a position to hold departures to give the priority to the landing aircraft, or to order the landing aircraft to go around for another landing attempt if the runway is occupied and does not permit a landing.

Since landings are ordinarily given priority over take-offs, a long take-off queue often develops because landings have been operated at a high rate, and the landing intervals have been too short to interpose the waiting take-offs. From observations of controllers in operation, it has been noted that the Tower, under such circumstances, notifies Approach Control of the situation, and Approach Control then discontinues the landing process while the queue of departures is dismissed, or dispatches the arrivals from the stack at longer intervals so that more take-offs can be executed in each landing interval.

2.1.2.2.1 Separation Standards

An aircraft is not dispatched from the stack unless it is certain that its predecessor is well ahead of it. Without radar, timed approaches are used in dispatching aircraft. The ANC procedures indicate some of the considerations involved.

4.413. The time interval to be used between successive approaches shall be determined by the approach controller and is dependent upon the speed of the aircraft, existing weather conditions, the distance from the holding fix to the airport and type of approach. Under optimum conditions, a two-minute interval is the absolute minimum; this interval being increased as necessary in poorer weather conditions, or because of high-speed aircraft following slower speed aircraft. [1, p. 63.]

This two-minute requirement is equivalent to 5 miles at 150 knots, 4 miles at 120 knots, and 3 miles at 90 knots.

Since most important airports are equipped with radar, the radar separation is more often used. This separation is specified in a radar procedures manual as:

1.4. Separation Standards. A minimum of 3 miles separation shall be maintained between aircraft. . . . When radar separation is utilized more than 40 miles from a radar site, a minimum of 5 miles is maintained.

1.40. Additional separation shall be provided as required by the speed variations of aircraft, wind conditions, runway acceptance rate, or other reasons in order to insure that the required separation is maintained or increased as aircraft progress through the system. [4, p. 4.]

Separation is also required at the runway, as specified in the ANC procedures.

3.141. Sufficient separation shall be effected between arriving aircraft so that the succeeding landing aircraft on the same runway will not cross the airport boundary* in its final glide until the preceding aircraft has cleared the runway-in-use. [1, p.26.]

It has been determined, in discussions with air traffic controllers, that no corrective action is ordinarily taken to restore the 3-mile radar separation when it has deteriorated on final approach if it is clear that the runway separation will be maintained.

Standards defining the necessary interval between departures under ANC procedures are given as:

2.0801. Three-minute separation at the time altitude levels are crossed if a departure will be flown through the altitude level of a preceding departure and both departures propose to follow the same course. . .

2.08010. Two-minute separation at the time courses diverge if aircraft propose to fly the same course immediately after take-off and then follow different courses, providing aircraft will follow diverging courses within 5 minutes after take-off. . .

2.08011. One minute separation if aircraft propose to fly different courses and lateral separation is provided immediately after take-off. This minimum may be reduced when aircraft are using parallel runways. . .
[1, pp. 9, 10.]

* "Airport boundary" was changed to read, "approach end of the runway" in August, 1959. [5]

The restrictions on the runway-in-use are expressed by.

3.142. Controllers shall not clear a departing aircraft for take-off until the preceding departing aircraft on the same runway has crossed the end of that runway, or has started a turn away from the runway. [1, p. 26.]

Approach control, with its radar capability, also serves as Departure Control, and normal radar procedures apply, which are often less inhibiting than ANC procedures. For mixed arrivals and departures, the ANC procedures specify:

4.03. Take-off Limitations: An approach controller may, at his discretion authorize take-off. . . under the following conditions:

- a) When the arrival is sighted by the controller; or
- b) Until the arrival, making a straight-in approach to the airport, reports leaving a holding-fix located not less than 4 miles from the airport; or
- c) When the arrival, making a contact [visual] approach, reports over a visual reporting point not less than 4 miles from the airport; or
- d) When the arrival, in radar contact and positively identified, is observed to be not less than 2 miles from the airport. [1, p. 58.]

The above limitations apply to a departure when it precedes a landing. They presumably are designed so that the departure has sufficient time to clear the runway before the landing arrives. When the departure follows a landing, the ANC procedures require:

3.143 (2). A departing aircraft will not be cleared for take-off until the preceding landing aircraft on the same runway has cleared the runway. [1, p. 26.]

2.2 Some Previous Studies of ATC Problems

Air traffic control has been the subject of a number of major national studies since World War II. These have been directed primarily at indicating the best configuration of future ATC systems, but have been necessary largely because of the fragmentation of authority for ATC within the Federal government.

The first major study was initiated in June, 1947, by the Radio Technical Commission for Aeronautics (a government-industry association concerned with aeronautical use of electronics) at the request of the Air Coordinating Committee. The goal of the study, undertaken by RTCA's Special Committee 31, was to:

undertake a study of air traffic control for the purpose of developing recommendations for the safe control of expanding air traffic. [6, p. 2.]

Many of the questions SC-31 was asked to investigate remain the most heatedly-argued questions of ATC today, such as:

- 1) A determination as to whether aircraft operations shall be based upon utilization of airways only or of the entire navigable airspace.
- 2) Investigation of the advisability of establishing a maximum aircraft acceptance rate for landing and take-off at each airport to govern the flow of traffic into any given airport.

- 3) A determination as to whether traffic control should be made applicable to all flight movements and, if so, whether separate procedures should be established for unrestricted and restricted visibility weather conditions. [6, p.2.]

In addition, the still-current questions of how best to perform scheduling, flight planning, collision avoidance, and the appropriate division of responsibility between the air and the ground were to be studied.

The study resulted in a comprehensive 15-year plan [6], [7] to be enacted in two phases - a transition phase to be completed by 1953 and an ultimate phase to be completed by 1963 - to provide a Common System of ATC to be used by both military and civil aviation. The present ATC system, in most respects, resembles the recommended transition system, except that the transition is still not complete.

The proposed ultimate system was characterized by considerable automation of communication, decision-making, and even actual control of the aircraft. The recommendations were far ahead of their time. The memory of the phenomenal rate at which electronic equipment was developed under wartime motivation was still fresh, so the committee recommended equipment which, even today, has not yet been effectively developed. Most of the committee had had their experience in military electronics, where the ability to impose external discipline over the pilots was far greater than could ever be attained with civilian pilots. Implementation of the committee's

recommendations was made even more difficult by the lack of a central aviation authority in the government, and no single one of the many agencies was sufficiently powerful to withstand the pressures from the interested segments of aviation, each of which had its own ideas about the best system to develop.

The RTCA study was followed by a much more modest one [8] -- primarily a review of action effected as a result of the RTCA study and a delineation of the most essential action that could be taken to implement the Common System -- conducted in 1950 by the Operational Policy Group (Special Working Group 5) of the Air Coordinating Committee. The task assigned the group was the development of "operational policies and procedures which would permit a safe and orderly transition for the Common System" [8, p. ii] which, in the two years since the RTCA study, had progressed only slightly toward its goal. The group's report noted that:

The group's studies have disclosed that traffic congestion in a few major terminal areas is the prime cause of delays, confusion, and disarrangement of traffic throughout the entire system. Terminal area congestion is particularly serious during the high percentage of borderline weather where some traffic is flying on visual flight rules while other traffic is flying on instrument flight rules. [8, p. 2.]

The report further pointed out, with respect to the terminal area:

The most critical bottleneck in handling airborne traffic is found in the terminal area. . . . On close scrutiny, it becomes apparent that the traffic handling capacity of the terminal area determines the over-all capacity of the entire air traffic control system. [8, p. 23.]

In the terminal area, the group proposed increased use of radar, and radar approach and departure procedures as well as additional air/ground communication facilities, and radar beacons for identification and echo enhancement. The group also recommended the installation of surface detection radars for expediting traffic on the airport's surface. With some exceptions (e. g. , beacons), the group's program which was far more limited than that of SC-31, has been largely implemented by this time, at least at the major airports.

A somewhat different investigation, oriented more toward the general public than the aviation community, was undertaken in February, 1952 by the ad hoc President's Airport Commission under James H. Doolittle [9]. The study grew out of the several landing accidents that occurred in the winter of 1951-52, and particularly the two incidents at Elizabeth, N. J. , where aircraft attempting to land at Newark Airport crashed into residential areas with disastrous loss of life. The study was primarily oriented towards the reduction of both the hazard and nuisance imposed on the communities adjacent to airports. The study served effectively to absorb the public indignation, and generated a number of recommendations to reduce both the hazard and nuisance.

By May, 1955, development of the air traffic control system had slowed to a dangerously low rate, while the number and performance of the aircraft constituting the air traffic was increasing rapidly. The Bureau of

the Budget organized a committee under William Harding to determine whether "a study of long-range needs for aviation facilities and aids should be undertaken" [10, p. v] and, if one should be initiated, to indicate the scope of such a study.

The Harding Committee quickly discovered the pressing need for some rapid action to shore up the nation's air traffic capability. They noted that:

much of our airspace is already overcrowded and that, in many important areas, the development of airports, navigation aids, and especially our air traffic control system is lagging far behind both aeronautical development and the needs of our mobile population and of our industry. [10, p. 1.]

They pointed out that the plight was due, not to lack of scientific capability for developing a solution to the problem, but in part because of the unexpected rapid increase in aviation capability and demand, and:

it is also due to the lack of general appreciation of the need for a "systems" approach to our aviation facilities development. [10, p. 1.]

The committee decried the dilution of administrative responsibility and authority among the Federal government, pointing out that there were then 75 governmental organizations concerned with aviation facilities matters.

The committee strongly urged a study to be performed by a special assistant to the President who would have authority over all of the many segments of the government, and who could supersede the parochial interests of each of the smaller groups.

Following the recommendations of the Harding Committee, with added impetus provided by the disastrous mid-air collision of two airliners over the Grand Canyon on June 30, 1956, President Eisenhower, in a letter dated February 10, 1957, appointed Edward P. Curtis as Special Assistant to the President for aviation facilities planning to make a comprehensive study of future requirements for aviation facilities and the most effective means of meeting these requirements. [11, p. vii.]

In his final report, Curtis summarized the nature of the present problem:

Between the combined demands of civil and military aviation, the existing control apparatus is already taxed to saturation whenever instrument conditions prevail around and between the principal metropolitan centers. Under such conditions, the normal flow of air traffic in and out of the New York area, for example, may fall as much as 60 percent. At the present time, our aviation facilities are hard-pressed to manage, within the bounds of prudence, even the fair-weather flow of traffic now contending for room on the principal routes and at the metropolitan approaches. They are wholly incapable, in their present form of meeting more than a fraction of the far heavier demands upon our airspace which the national strategy and the market are certain to impose by 1975 -- the period covered by this report. [11, p. 1.]

He noted the lack of progress that has characterized recent years:

. . . Except for the addition of radar control around major airports, the air traffic control system now in use is, in its principal technical particulars, much the same system that was devised two decades ago to accommodate the growth in traffic stimulated by the DC-3 -- an airplane that cruised at about 160 miles an hour. [11, p. 2.]

While the system has changed negligibly, the demand for assistance from the system has increased drastically. The Curtis Committee, in a four-volume final report [12, 13, 14, 15] forecast the traffic requirements for aviation facilities. In a Systems Engineering Team report [16], they presented some of the precepts of a future system design, and emphasized the need for the systems analysis approach to the solution of the problems facing air traffic control.

Probably the most significant practical result of the work of the Curtis Committee was the acceptance of their recommendations for the centralization of the ATC system development and operation in the government. The Airways Modernization Board was first created by Congress in August, 1957, with the exclusive authority to:

develop, modify, test, and evaluate systems, procedures, facilities, and devices, as well as define the performance characteristics thereof, to meet the needs for safe and efficient navigation and traffic control of all civil and military aviation except for those needs of military agencies which are peculiar to air warfare and primarily of military concern, and select such systems, procedures, facilities, and devices as will best serve such needs and will promote maximum coordination of air traffic control and air defense systems. [17, p. 1.]

This was followed one year later by the creation of the Federal Aviation Agency, which absorbed the AMB and had exclusive authority to:

provide for the safe and efficient use of the airspace by both civil and military aircraft. [18, p. 1.]

Both the AMB and the FAA were led by E. R. Quesada, a retired Air Force general.

In these two organizations, for the first time since the development of commercial aviation, all responsibility for national aviation facilities and operating procedures (including all military aviation other than tactical operations) was lodged in a single Federal organization which had the authority to impose its decisions on all segments of aviation.

Upon the creation of the FAA, the AMB was incorporated into the FAA as the Bureau of Research and Development (BRD) now under the direction of James Anast. The BRD is concerned with the development of all new navigational and control equipment and with all systems analysis and engineering for air traffic control. The core of the BRD's program has been the Data Processing Central [19, p. 1] equipment to automate many of the house-keeping tasks of air traffic controllers. Additional research and development is being conducted in many other areas, most notably in the development of an automatic data-link for ground-air communications and in the evaluation of potential automatic landing systems that will permit landing in essentially zero-zero weather conditions.

2.3 Some Inherent Problems in ATC

Many of the problems hindering the future development of improved ATC capacity are inherent in the nature of the air traffic situation in the United States. We discuss some of these here as a preliminary to a discussion of terminal and capacity problems in the next sections.

2.3.1 Conflicting Needs of Airspace Users

In the Federal Aviation Act as well as in the previous Civil Aviation Act, Congress declared the national policy concerning air traffic:

Sec. 104. There is hereby recognized and declared to exist in behalf of any citizen of the United States a public right of freedom of transit through the navigable airspace of the United States. [18]

With this expressed national policy, it is essential that no airspace user be discriminated against, and since the individual needs of the different airspace users are so diverse and often conflicting, an action that aids one group often works to the detriment--and the resulting vocal response -- of some other group.

The airlines constitute one end of the spectrum. They desire the utmost in protection of their aircraft, which carry many more passengers and represent considerably greater investments than in the past. The larger airlines are willing and anxious to equip their aircraft with the most sophisticated and expensive equipment to provide increased protection. Since direct

operating costs are weighed heavily in their decisions, they desire to operate as closely as possible to straight-line routes between their terminals, but are ready to adhere to a precisely-determined route which is filed before take-off. Even this degree of cooperation is qualified, since they need freedom to detour bad weather.

Military aviation is characterized by a wide variety of mission types. A sizeable portion of their activity is not of the point-to-point transportation type typical of airline operations. Training flights and test flights particularly require complete freedom of movement, independence of route structures, and freedom to change the route at any time during the flight. Safety, while an important consideration as in all flying, is of considerably less significance than with the air carriers.

General aviation (i. e., all aviation other than military or air carrier) represents the greatest number of flying hours and by far the largest number of aircraft in the nation. It also represents the most diverse interests. Pleasure-flying aircraft are characterized primarily by their light weight and minimum investment, with the consequent sensitivity to increases in weight and cost that might result from requiring additional electronic equipment. The owners generally are the least skillful pilots, and most of them do not operate in IFR weather conditions. General aviation also includes business flying, which itself represents a large range of capability from four-engine

craft flown by professional pilots for the larger corporations to the single-engine craft whose operation differs little from pleasure flying, and consequently represents a comparable range of needs. The third component is commercial flying, which includes such diverse activity as crop dusting, aerial surveys, and transportation for hire. The fourth major component is the instructional flying, which is generally performed in VFR weather in light aircraft, largely by unskilled pilots. Of all the components of aviation, general aviation is most limited in its skill and its financial resources; it consequently desires a minimum of restriction to its free operation, either in terms of route of flight or equipment requirements.

When the control of aviation was dispersed among the many Federal agencies, none of the agencies appeared powerful enough to resist the pressures of whatever aviation group felt itself wronged by a new regulation, and in order to offend no one, little was done to improve the situation. With the creation of strong central authority in the Federal Aviation Agency, the government is in a better position to resist this pressure, which is already being heavily applied.

2.3.2 Need for Setting Conservative Separation Standards

The ANC separation standards currently in effect were set at least 10 years ago on the basis of competent experiential judgment. Controllers were asked their opinions regarding appropriate separations, elementary trials were performed, and the standards were promulgated to be highly conservative so as to insure safety. That they were well designed for safety is attested to by the fact that no collision has ever occurred between two aircraft both operating within the IFP system.

Safety is extremely difficult to treat rationally. Only a portion of the economic cost of an aviation accident is represented by the physical damage and by insurance claims. Aside from the personal loss sustained by the survivors of the victims, which is truly immeasurable, significant secondary economic loss is sustained by aviation. The enraged public reaction that invariably follows a serious accident (such as the accidents at Elizabeth, N. J., the mid-air collision that killed several children in a school-yard in Pacoima, California, and the Grand Canyon collision) often forces the imposition of severely restrictive regulation of flight, which inevitably results in severe economic cost. To avoid accidents, then, separation standards must be set extremely conservatively.

Additionally, the air traffic controllers, on whom blame would be placed if two aircraft under IFR control collided, must be protected, so that standards must be set conservatively for their sake. To further protect himself, the controller often imposes an additional safety factor, further increasing the effective separation.

It is conceivable that highly conservative standards are today undesirable even from the standpoint of safety. When the demand on the system is heavy, large separations increase the delays experienced by aircraft. To avoid this delay, pilots may violate the rules, or try to operate outside the separation rules, and possibly create hazardous situations. As a result of the additional delay, most of which is absorbed in the terminal areas, the airspace around the terminals, which is already densely filled with aircraft, becomes even more congested, and may possibly provide greater opportunity for collision.

Separation standards have been established so that a controller need remember only a few simple rules of thumb which encompass all likely situations, and he need not perform any calculations. With the advent of computing equipment as an aid to ATC, it is becoming possible to develop more sophisticated rules that could handle each situation individually.

Since capacity is directly related to separation standards, it would appear desirable for a thorough investigation to be conducted into the setting of separation standards most appropriate to today's traffic situation.

2.3.3 Obsolescence of See-and-Be-Seen Operation

Probably the most critical problem ATC faces today is the rapid obsolescence of see-and-be-seen collision avoidance now used by aircraft flying under VFR. In the early days of aviation, the airspace was very sparsely populated, and aircraft were characterized by speeds well under 200 knots enroute and under 100 knots in the terminal area. In today's operation, some aircraft operate beyond the speed of sound and at about 200 knots in the terminal area. In the Thirties, a pilot flying under VFR was continually searching out the window, primarily for navigational checkpoints, but also on the lookout for other aircraft. Today, a pilot flying in VFR weather, particularly if he is flying on an IFR plan, spends much of his time poring over navigational charts, switching communication channels, switching channels on his navigation receiver, and watching the aircraft and navigation instruments, and so has little time to spend "seeing" other aircraft.

On the other hand, aircraft speeds have climbed to the point where little time is available for an aircraft to "be seen," even if the other pilot is looking. The Civil Aeronautics Board, in a report on a 1957 collision [20], noted that the two aircraft were closing at a rate such that the pilot of one would have had only 15 seconds from the time of earliest possible detection until collision, barely enough time to evade, even if he had been looking in the right place at the right time. Similar arguments have been presented in other accident investigation reports [21, 22, 23].

The serious hazard inherent in VFR operation has long been recognized by both the pilots and the Federal regulatory agencies. Unfortunately, eliminating VFR would drastically cripple the nation's air traffic activity. It has been estimated by the CAB [24, p. 150] that the present air traffic control system has a capacity to handle only about 20% of the aircraft that fly during VFR weather. Thus, the danger inherent in see-and-be-seen operation must be accepted as being sufficiently less than the economic penalty that would result from requiring completely controlled operation with today's facilities. The ultimate goal of most planning, however, is to place all air traffic under some sort of control wherever collision dangers exist.

The airlines, to whom safety is more significant than to any other component of aviation, now have a policy of conducting almost all their flights under IFR, even in VFR weather conditions. In this manner, they are positively separated from all other IFR aircraft, and thus effectively eliminate the possibility of another two-airliner collision similar to the Grand Canyon disaster of June, 1956 [23]. That this procedure does not insure protection is evidenced by the Las Vegas collision of April, 1957 [21], in which an airline flying IFR in VFR weather collided with an Air Force jet operating VFR. To prevent a recurrence of such a situation, and to provide additional protection in VFR weather, several positive control airways from which VFR are excluded were established in the 17,000-22,000 feet region. The jet

airliners are continually monitored by air defense radars, and are thus positively separated from all other air traffic; such an operation has so far been possible only because of the relatively small number of such aircraft operating. The ultimate goal is to provide, to all aircraft desiring it, positive separation from all other aircraft, regardless of weather conditions.

As a result of this desire for protection, the airlines, as well as many other segments of aviation, are flying IFR, and must be separated by IFR standards with the assumption that no pilot visibility is available. It is obvious that this places an additional burden on the ATC facilities and increases the problem of system capacity.

2.3.4 Introduction of Jet Aircraft

Within the last decade, the performance of aircraft flown by both the military and air carriers has increased markedly. The flying public's apparently insatiable demand for more speed as well as the airlines' desire for increased operating efficiency has resulted in the introduction of first the turbo-prop and more recently the jet aircraft into the airline fleet. Aircraft manufacturers [25] are already discussing the possibility of Mach 3.0 transports by 1965. The demand for improved performance from military combat aircraft has similarly forced the development of high-speed vehicles.

In addition to marking the obsolescence of VFR operation, the introduction of the large number of jet aircraft places additional responsibilities on the ATC system. As pointed out by Stevens [26], jet aircraft are extremely sensitive to off-design performance and to air traffic delays. Jets are particularly efficient when operated according to their planned flight profiles, which typically involve high-altitude flight at close to an optimum altitude and then rapid descent into the terminals. The economic penalties for deviation from these profiles are far more severe than that incurred by piston aircraft, and requires special consideration in air traffic control, particularly in current operation, where the airline jets constitute such a small portion of the operating aircraft. Policy, however, as stated by Deputy FAA Administrator, James Pyle, in October, 1958 [27, p. 206] dictates no preferential treatment for jet aircraft, although eventual recognition of this problem seems inevitable.

2.4 The Problem of ATC System Capacity

The demand on the nation's air traffic system is continually increasing because both the number and the proportion of airborne aircraft that can (by equipment and pilot skill) operate within the ATC system are increasing. In addition, the demand on the ATC system for collision prevention is increasing as a result of the growing awareness of the hazards inherent in VFR operation.

The recent growth in air operations was stated in October, 1958 by Pyle:

Our air traffic control system is handling an average of 22,400 IFR operations daily. I think that it is significant to note that this is a 10,000 per day increase in less than three years and an incredible jump of 5,000 a day since July 1 of this year. [27, p. 216.]

In contrast to this number of IFR operations, Pyle points out that "there are an average of 200,000 flights daily in the United States under VFR conditions and this number is increasing." Since the facilities are being taxed by operations at a rate of 22,000 per day, certainly a ten-fold increase would be needed to handle today's VFR load if all aircraft are to be controlled. Such an increase by 1975 has been called for by the Curtis Committee [16, p. 1].

In regard to the terminal areas, the Curtis Committee noted that:

While in 1936 there were 5 million take-offs and landings at the Nation's airports, there are now 65 million and 115 million are forecast in 1975. . . . We predict the need for a two-fold increase in the capacity of our nation's airports [by 1975]. [11, pp. 5, 6.]

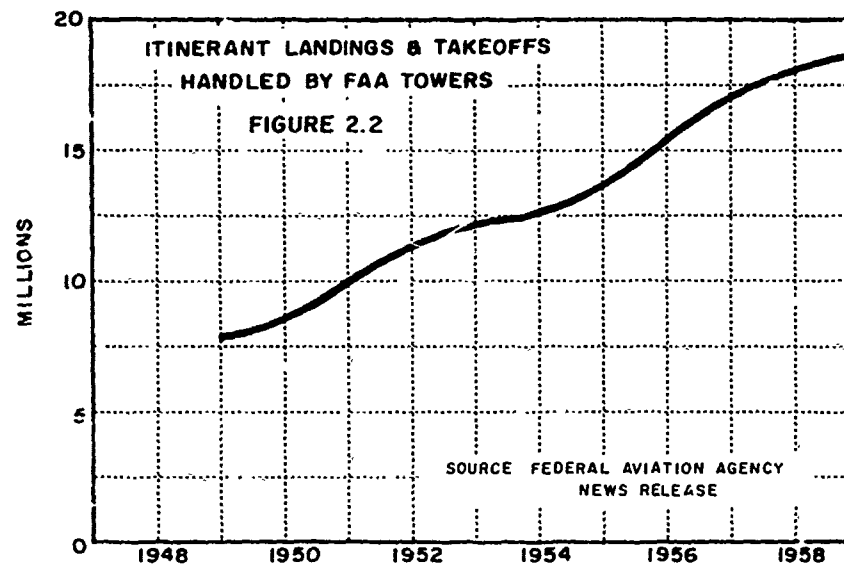
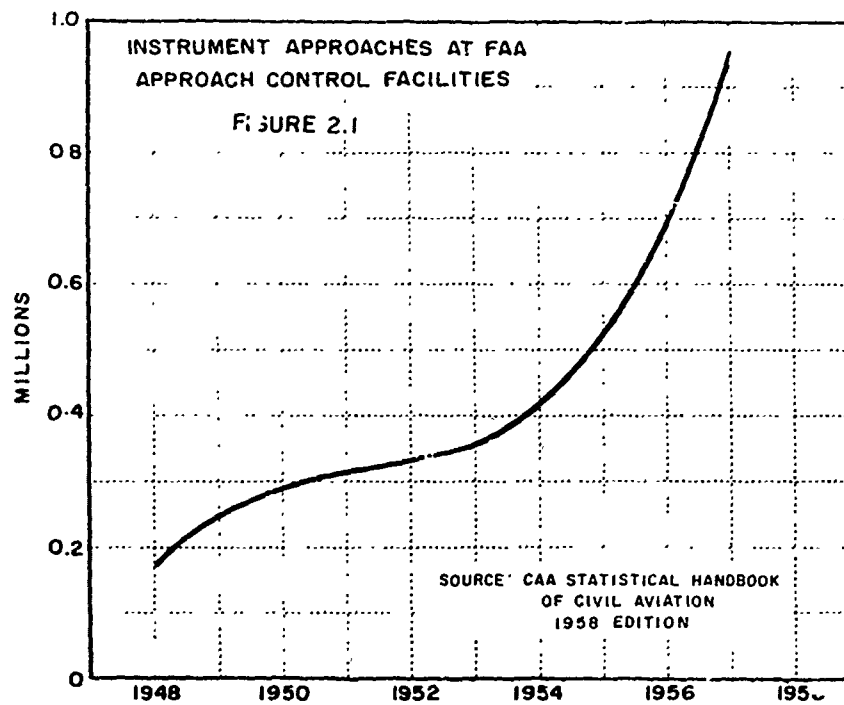
While these numbers indicate growth over the nation, situations at individual airports present the situation more specifically:

During peak hours on busy days in 1956, there were 175 aircraft simultaneously airborne in the New York area (which is the largest airline passenger generating area in the world). It is predicted that by 1975, this number will grow to 370. Similar statistics for Los Angeles (which generates the largest volume of general aviation in the world) show that in 1956 there were 270 aircraft simultaneously airborne under similar conditions. In 1975, this number will have grown to 730. [11, pp. 6, 7.]

The question of capacity has arisen in the proposed plan to expand the airport facilities in the New York City Area. A study by the Port of New York Authority [28] estimates the peak-hour movement requirements of the New York area as 169 movements in 1965 and 200 movements in 1975. The study estimates present capacity as 70 per hour for the dual instrument runways at Idlewild and 40 per hour for the single instrument runways at LaGuardia and Newark. On the basis of this study, the Authority is proposing to build a \$220,000,000 airport in Morris County, N. J., to meet the increasing demand

The rapid growth in instrument activity at the terminals is illustrated by the chart of Figure 2.1 (from [29]) which depicts the rise in number of instrument approaches at CAA approach control facilities during the 1948-58 period. The rapid growth in the curve in recent years represents the combined effect of increased traffic, more electronically-equipped aircraft, and increased inclination to operate under IFR control. The increase in air traffic activity alone is shown in Figure 2.2 (from data in [30]) which depicts itinerant (i. e., operations which do not both take-off and then land at the same airport) aircraft operations at FAA-controlled airports.

It is thus clear that the demand on the ATC system has been increasing, and all estimates indicate that it will continue to increase at an even faster rate as a result of the increased aviation activity and of the increased tendency to operate within the control of the ATC system rather than under the see-and-be-seen VFR system.



GROWTH OF TERMINAL ACTIVITY

With increased load, increased delay inevitably follows if capacity remains fixed. This delay is expensive to the aircraft owners, particularly to the air carriers operating jets with their voracious fuel appetities at low altitudes. The expense to the airlines is compounded by the indignation of passengers missing connections or appointments.

As delay becomes intolerable, there is a tendency to violate or circumvent the traffic rules. The Grand Canyon collision [23] resulted in part from the fact that one of the two aircraft involved had requested clearance to the altitude occupied by the other, and failing to receive the clearance, chose a "VFR on top" clearance to the same altitude, which the ARTCC was powerless to refuse, and which, when accompanied by the limitations of see-and-be-seen, led to the fatal disaster.

Since the ATC system is now operating to capacity much of the time, as evidenced by the frequent delays encountered, the need for increased system capacity appears clear.

2.5 The Terminal Area Problem

The terminal is the critical bottleneck in ATC operation, from the viewpoint of both delay and safety -- the two most important operational performance measures of an ATC system.

A tabulation of mid-air collisions in civil flying from 1948 to 1957 [31] by the Civil Aeronautics Board (CAB) showed that, of 159 collisions that occurred during the period, only 22 (14%) occurred more than 5 miles from an airport; of the remainder, 41 (26%) occurred just over the airport, and an additional 68 (43%) occurred within 2 miles of an airport. Of those whose altitudes were known, all but 13 occurred below 3000 feet. Forty-three percent of the aircraft involved were in the landing approach or traffic pattern preparatory to landing, and were within a quarter-mile of the airport when the collision occurred. Similarly, Air Force statistics indicate that landings account for 43% of its accidents [32, p. 151]. Clearly, most collisions occur in the vicinity of the terminals.

No measured data on "delays" are available, although a study is currently under way at Cook Laboratories under FAA sponsorship to measure aircraft delays in the Chicago area. Some data are available from a simulation developed at IBM [33] of an IFR air traffic control system operating under ANC rules. The IBM study simulated a situation in which the landing rate at each of the New York terminals was set at 30 landings per hour and

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a "normal" traffic load of 200 aircraft in the New York ARTCC area in 2 hours (based on observations made by Franklin Institute Laboratories [34] and Airborne Instruments Laboratory [35]) was imposed.

In analyzing the results of this simulation, it was found by Cornell Aeronautical Laboratory [36] that the "landing delay" (delay in waiting to enter a stack prior to landing and delay waiting in the stack for a landing turn) constituted 76% of the total delay time measured in the simulation; the "terminal delay" (landing delay plus delay waiting for departure clearance but apart from the interaction effects between landing and departing aircraft) constituted 89% of the total delay. The landing delays represented 56% of the total number of delays experienced, while the terminal delays accounted for 79% of the delays, so that the large majority of aircraft encountered some terminal delay. Despite the limited validity of the simulation as an approximation to the real situation, the indication of these results that the major bottleneck is the terminal seems convincingly strong.

2.6 Efforts to Improve Terminal Operation

2.6.1 Radar

The single, most effective measure that has been taken in recent years to improve terminal capacity has been the installation of surveillance radars at all major terminals. This program is being extended to include the smaller airports. Radar beacon transceivers are being installed at a number of the radars to provide positive identification and echo enhancement of beacon equipped aircraft [37]; unfortunately, very few aircraft -- primarily airliners -- are equipped with beacons to provide a return signal.

2.6.2 Data Processing Central Equipment

The major development effort being sponsored by the Bureau of Research and Development of the FAA is the Data Processing Central (DPC) equipment being developed by the General Precision Laboratory. According to the FAA:

The keystone effort in the initial program to modernize air traffic control is the data processing central project. Its purpose is to provide for the development of a semi-automatic air traffic control data processing and display system to replace the existing manual system in all major high density areas of the nation's airways. . . . Delivery of these equipments will start in May of 1959. [19, p. 1.]

This program has already slipped a year so that testing of the first model is not expected to start until some time in the spring or summer of 1960. The DPC equipment is essentially a mechanization of the present ATC system, in which the flight strips will be automatically computed and up-dated as an

aircraft's ETA's at its fixes change. Thus, the controllers' workload will probably decrease as they are relieved of much of the routine connected with the control of aircraft, and it might be possible to control the present volume of traffic with fewer controllers. But it is important to note that this equipment will change only the data processing techniques, and that all the present control procedures and separation standards will be retained. Since system capacity is limited by these rather than by the speed of the data processing, the capacity of the system will remain unchanged even after the DPC is operational in about 1962-63, and the need for increased capacity will still remain unsatisfied.

The specifications for the DPC include specifications for Transition and Terminal Equipment [38]. This equipment will probably be the last to be developed, and includes display consoles which will aid the terminal controller in sequencing the landing aircraft and in maintaining appropriate separation between successive aircraft. The system envisions track-while-scan or rate-aided tracking (i. e. , progression of the tracked blip between radar scans) displays which will enable the controller to order the aircraft such that they arrive at the runway with a uniform spacing (equal to the reciprocal of the landing rate). The specifications state:

A standard time interval is desired between aircraft touching down (except when positions in the landing sequence are left open to permit take-offs). . . this will dictate some non-uniformity in aircraft spacing at the gate, inasmuch as the final approach speeds of all types of aircraft cannot be identical. [38, p.8.]

It is planned to assign to each aircraft a landing "slot" in one of the uniformly-spaced intervals. Only a portion of the available slots will be assigned in order to leave some slots open for departing aircraft. But once an aircraft is assigned a slot, it is expected to maintain that slot or to take a wave-off.

The specifications state:

Should the slot ahead or behind this aircraft's assignment be vacant, either because of being reserved for a take-off or because there was no traffic to fill it, this aircraft will be allowed to move up or fall back in the sequence if it encounters any difficulty in maintaining the progress required to stay with the slot originally assigned it. Furthermore, should an aircraft making a direct approach be unable to stay with its assigned slot, and the slot into which it intrudes has been assigned to an aircraft making an approach from the opposite direction, so that this second aircraft must follow a "trombone," then these two aircraft may possibly be interchanged, with the length of the "trombone" being adjusted to compensate for this change. Except for these cases, an aircraft that deviates from its scheduled progress to such a degree that its separation from the aircraft in the adjoining slot is below standard will be removed from its assigned positions and re-introduced into the landing sequence at a later point. [38, pp. 8, 9.]

The specifications call for equipment designed to "permit sustained arrival and departure rates as high as" 60 arrivals per hour and 60 departures per hour at Idlewild, assuming "separate non-interfering runways." Otherwise, "the number of movements (arrivals plus departures) shall be 60/hr." They do note that these arrival rates:

are not compatible with the present radar separation rules; any relaxation in these rules required to achieve the landing rates indicated will be the Government's responsibility. [38, p. 16.]

It appears probable that, when the slots become small enough to achieve the desired landing rates of 60 per hour, the wide variability of aircraft performance (particularly when a slow aircraft follows a fast one) will make this modern-day Procrustes' bed have an excessive number of unnecessary wave-offs, which will further complicate the traffic situation in addition to creating pilot resentment.

2.6.3 Landing Aids

Landing at an airport under zero-zero visibility and ceiling conditions is today impossible. Based on the particular conditions of terrain and landing aids at an airport, landing minimums are established, 200 feet ceiling and 1/4 mile visibility representing a lower bound to these. More widespread installation of ILS, the primary civil landing aid, will bring more airports closer to these values. The lack of an ILS glide-path indication is partly responsible for the crash of an Electra near LaGuardia in February, 1959.

Studies conducted by United Research, Inc., [39] have evaluated the economic potential of a landing system that would permit landings in zero-zero conditions. The studies, based on economic loss due to unreliability and insurance costs of property and personal damage, indicate that such a system would quickly repay the installation costs at most of the major airports, particularly those in the north where weather frequently closes the airport.

A number of development programs are currently under way to provide this zero-zero capability [32]. Systems have been developed for the military by Bell Aircraft and Autonetics to perform automatic landing, and these, along with a British landing system (BLEU), are currently being evaluated by the FAA to ascertain their adaptability to the present Common System. For the more distant future, systems under development by Gifillan and Airborne Instruments Laboratory are being investigated by the FAA.

Installation of zero-zero landing systems will significantly improve the ability of the airports to handle traffic under extreme weather conditions. Those systems which provide accurate range data on the landing aircraft may also contribute to increasing airport capacity by permitting reduced separations along the final approach path. But, when weather is above present minimums, only those systems that permit reduced separation will provide an increase in capacity.

Installation of additional ILS equipment will provide no increase of capacity on the runways currently provided with ILS -- those most frequently used under instrument conditions -- but will increase the capacity of the airport when the wind requires use of the newly-instrumented runways. A common technique for landing on an uninstrumented runway is to follow the available ILS until the ground is visible. The pilot then maneuvers over the field to line up visually with the runway he desires, and completes his landing visually. Thus, installation of the additional equipment avoids the need for this extra time-consuming maneuver and can increase capacity.

2.6.4 Collision Avoidance and Proximity Warning Equipment

The electronics industry has directed considerable effort at the development of collision-avoidance equipment that can be carried by the aircraft, thus making it free of any reliance on ground-based control. This effort has included proximity-warning equipment, cooperative collision-avoidance systems, and self-contained collision-avoidance systems.

A proximity-warning device, such as infrared detectors or radar, indicates to the pilot the existence and possibly range and/or bearing of all other aircraft within the detection range of the equipment. Such a device is the easiest to build, but there is some question whether it provides sufficient information to avoid an impending collision, and it requires a set of well-defined "rules of the road" to be effective [40, p. 50]. In the terminal area particularly, the device has limited merit, since the alarm would be ringing constantly, and might thus be completely ignored.

A cooperative collision-avoidance system combines electromagnetic emanations from two aircraft to provide, to each of them, a signal of the other's presence and position. (An aircraft equipped with only a transmitter could inform other aircraft of its presence, but it would receive no information on the presence or position of the others.) Since such equipment is likely to be expensive and heavy, its installation would probably be limited to the larger aircraft -- primarily the airliners -- which often fly IFR to protect against each other, and need the protection primarily against the smaller and the faster aircraft.

A self-contained collision-avoidance system represents the ultimate in protection for the aircraft carrying it. It would indicate to the pilot (or directly to the autopilot) the optimum path for an aircraft to fly to avoid an impending collision. The equipment would thus require no assistance from other aircraft or from the ground control system, and would presumably indicate a maneuver only when one is necessary. Efforts at developing such a device, however, have met with little success. Some of the problems involve specification of whether a collision course exists when two aircraft are on non-linear courses, computation of an optimum avoidance maneuver that does not create a collision situation with another aircraft, and attaining sufficient precision to minimize the false-alarm rate. The last problem is particularly pertinent in the terminal area, where many aircraft are crowded into a small volume of airspace.

Development of effective airborne devices to permit collision-avoidance by the pilot without reliance on ground facilities appears unlikely in the near future, so that primary -- if not all -- reliance will continue to rest on the ground facilities in IFR weather. Many of the aircraft that fly by VFR -- especially the light private aircraft -- would be the last to install collision-avoidance devices, so that the development will probably not even affect them.

2.6.5 High-Speed Turnoffs

A major program aimed at increasing terminal capacity is the installation of high-speed runway turnoffs. With such a turnoff on a runway, the pilot need not slow to essentially zero speed before he leaves the runway, but rather, can leave it while moving as fast as 60 knots, and thus appreciably reduce his runway-occupancy time. Installation of these turnoffs has long been promoted, and in a paper given in 1954 by the Chairman of the Port of New York Authority, the prevailing concept of their merit was indicated:

CAA simulation studies show that use of high-speed turnoffs at [30 miles per hour] can reduce runway occupancy time to little more than half the present values. The corresponding increase in landing rate is obvious. [41, p. 3.]

Also, in a recent FAA training manual, it is stated that:

The installation of high-speed turnoffs will prove to be an important factor in the expeditious handling of arriving traffic. [42, p. 11.36.5.]

It appears quite clear that the high-speed turnoffs will appreciably reduce runway-occupancy times, and, since VFR landing rate is essentially a function of occupancy time, can increase the VFR landing rate. But the more serious problem is generally the landing rate during IFR conditions, and it is not at all clear that the turnoffs will improve operations under those conditions.

As will be shown in the next chapter, landing rate is far more sensitive to radar separation standards, and is little affected by even major reductions in runway occupancy time as long as present radar separations are maintained. The reduced runway-occupancy time resulting from the high-speed turnoffs will prove to be of advantage, however, in improving operations rate, as will be shown in Chapter 4, by permitting more frequent interpositions of take-offs between landings.

2.6.6 Additional Construction

Since it is recognized that the present capacity of the major terminals is inadequate to meet future demand, the obvious means of increasing capacity by providing additional runway is being actively pursued. Some airports (e.g., LaGuardia and Los Angeles) are expanding their present facilities. But, since land in the vicinity of most major airports is now prohibitively expensive because of the encroachment of industry around the airport, additional airports are being built. The Dulles Airport near Washington will soon begin operation, and the Port of New York Authority plans to build one in New Jersey at a cost of \$220,000,000. In view of the expense of such expansion, it would appear to be desirable to raise the capacity of existing facilities to the maximum attainable.

The remainder of this study is directed at the question of airport capacity. It is evident from the preceding discussion that capacity is a serious problem at all major airports, and will become more serious at the

smaller ones. The capacity is now generally sufficient during VFR operation because of the close spacing possible under such operation, but is insufficient during IFR operation, when the pilot-imposed separations are replaced by the more conservative ANC or radar separations. The situation is most critical during periods when the weather is changing from VFR to IFR, and the sky is occupied by the many more aircraft characteristic of VFR conditions. The investigations conducted here are concerned first with the questions of landing capacity of a single runway, and then with the operations capacity of a single runway when that runway is used for both landings and take-offs. The results of the study permit the potential capacity to be estimated for any airport for the conditions characteristic of that airport. This estimate will ordinarily tend to be high, since the analyses assume that controllers operate to separation standards, whereas, in practice, they generally err conservatively. A more important use of the results of the study is in an investigation of the effect of the various parameters on capacity measures in order to indicate how best to improve the capacity.

CHAPTER 3

ANALYSIS OF THE LANDING CAPACITY OF A RUNWAY

In this chapter, the landing capacity of a single runway used only for landing is analyzed. This problem has practical significance, since runways are often used only for landing, but serves also as a first step in the larger investigation of the operations capacity of a runway used for landings and take-offs, and of the still larger problem of multiple runways used simultaneously. Models are developed to study the effect of the various system parameters on landing capacity and some recommendations for improving capacity are derived from use of these models.

3.1 Previous Investigations of the Landing Problem

Most of the analytical research into the problem of landing aircraft has been directed at the estimation of delay measures and queue state probabilities using queuing models with assumed service-time distributions -- usually constant -- and Poisson arrival distributions. In contrast to these studies of delay for specified service rate, little analysis has been directed at estimating this service rate or to examine the effect of system parameters on it; landing capacity has been studied analytically only with a model based on discrete aircraft velocities.

In addition to the analytical studies, extensive simulation experiments -- primarily in real time using human operators -- have been conducted by Federal ATC agencies. Effort is currently being directed at development of fast-time simulation of the landing process on large digital computers.

3.1.1 Analytical Studies

One of the earliest analytical investigations of the landing problem was the queuing study reported by Bowen and Pearcey [43] in 1948, the mathematical details of which were presented by Pearcey [44]. They assumed that aircraft arrive at a single runway with a Poisson distribution (and justify this assumption with data from the Kingsford-Smith Airport in Sydney) and that successive aircraft must maintain a fixed time separation (t_0). They considered both continuous control (so that two aircraft arriving closer than t_0 land with a separation exactly equal to t_0) and quantized control (so that, if two aircraft require additional separation, then the second is held only an integral number of holding times). They developed delay distributions and showed that the fraction of aircraft delayed is equal to the runway utilization (ρ = arrival rate/service rate) and that the mean delay is $\rho^2/2(1-\rho)$, a result which had been obtained previously by queuing theorists.

Bell [45] reported on some observations of traffic operation in the vicinity of London and Northolt airports in England and on the results of some theoretical queuing studies similar to those of Bowen and Pearcey.

He summarized measurements of pre-take-off warm-up times, landing times, runway-occupancy times, delay times, taxi speeds, communication times, accuracy in estimation of ETA's, navigation accuracy, height-keeping accuracy, and geographical density of flying. These measurements pertain largely to the particular London situation and the aircraft and navigational equipment then available, and are of little value for extrapolation to current operation. In discussing his theoretical studies, he noted that arrivals are:

not necessarily Poisson, although observations at London and Northolt Airports and at Kingsford Smith Airport, Australia show that it is in fact nearly so. [45, p. 971.]

Regarding the service time of the runway, he stated that:

For the purpose of analysis, and considering runway congestion, it must be assumed that an arriving or departing aircraft occupies the runway for some definite and constant time, called for convenience the "holding time." Actually we know from Figures 1 - 3 that this assumption is not strictly valid, and it has been established that the holding times, in fact, have a distribution to which a Pearson type III curve can usually be fitted. [45, p. 971.]

Bell stated that the mathematics becomes "intractible" with a Pearson type III distribution^{*}, so that he developed his theory based on a constant holding time and an arbitrary arrival distribution specified by a set of probabilities, $\{a_i\}$, that i aircraft arrive in a unit holding time. He illustrated his model by computing delay and queue measures for the Poisson arrival distribution and the constant holding time.

^{*} The Pearson type III distribution is a version of the k -Erlang distribution, which is treated extensively in the recent queuing literature.

Bell also investigated the effect of orderly scheduling of arrivals (with variability in the arrival time about the scheduled time) and found that scheduling serves primarily to reduce the probability of long queues and long delays, but has little effect on mean delays until very precise scheduling is achieved.

Pollaczek has used integral equation techniques to develop the distribution of discrete delays to aircraft arriving at an s -runway airport with a Poisson arrival distribution and landing with constant and equal landing times [47] or with landing times having an arbitrary distribution [48]. He assumed, in these papers, that every s^{th} arrival lands on the same runway; in a later paper [49], he developed the equations for aircraft landing in the order of their arrival at the airport.

Galliher and Wheeler [50] demonstrated a means of simulating Poisson arrivals with a time-varying parameter, also using a constant holding time. They assumed that the time variation of the arrival rate is discrete, so that the process consists of a sequence of intervals each of which is equal to the service time and is characterized by a constant Poisson input. The intervals can be treated in succession, using the output state of one as the input state of the next.

In addition to the above queuing studies directed specifically at the airport problem, the large body of literature on queuing theory is available for determination of delay characteristics.

In his resume of queuing theory [51], Saaty points out that:

the time required to land (holding time) will be constant for a given type of aircraft, but if many different types are stacked, waiting to land, the landing time may be exponential. [51, p. 192.]

He thus expresses a hope more than an actuality, since queuing theory analysis is far more difficult unless one of his service-time conditions is met. But, in view of the narrow range within which delay varies with the form of the service-time distribution, the actual form of the distributions is relatively unimportant for Poisson arrivals and a fixed mean service time, and the value of this mean service time is the important consideration.

Analytical models of runway landing capacity have recently been formulated at the University of California by Horonjeff et al.[52] for the purpose of optimizing the location of high-speed runway turnoffs to maximize landing rate. Their models consider aircraft arriving at the runway with a constant separation (one model considers the time separation constant while another considers the distance separation constant) dictated by a specified arrival rate. They treat runway-occupancy time as a chance variable, and indicate its effect on landing rate by causing an arrival to be waved off if the preceding arrival is still on the runway when it is ready to land, thereby decreasing the runway acceptance rate below the arrival rate. Turnoff locations that minimize the runway-occupancy time thus serve to maximize the landing rate by minimizing the wave-off rate. Since the models assume

constant separation at the runway, events along the common landing path are not considered. These events, however, create a separation that is not constant, since a slow aircraft following a fast one will generally land with a separation greater than in the reverse situation because of the minimum separation required at the beginning of the common path. The results of the California study indicate that the optimum location depends strongly on the runway-occupancy characteristics of the aircraft using the runway, and on the number of turnoffs provided.

Additional analytical investigations are currently being conducted by Jackson, Ottison, and Pardee of Thompson-Ramo-Wooldridge under Project Radir [53]. Their investigations involve an extension of the work reported in this chapter, primarily by the inclusion of controller error in gate separation (s_0). Results from this investigation have not yet been published.

3.1.2 Simulation Investigations

Adler and Fricker [54] investigated queuing delays by simulating several aircraft arrival distributions and assuming a constant landing time. They simulated three bounded distributions -- rectangular, triangular, and parabolic -- of enroute delay imposed upon scheduled uniformly-spaced arrival times, primarily to investigate various techniques for effecting enroute flow control of arrivals at the airport.

Extensive simulation studies of the landing problem have been conducted jointly by the former CAA Technical Development Center (TDC) and the Franklin Institute Laboratories (FIL). These studies included deterministic models, graphical Monte Carlo models, and real-time simulated experiments. In each case, a sample traffic sequence was generated with the use of random-number tables based on probabilities of arrival of three speed classes (slow, medium, fast), an assumed Poisson arrival distribution, and a distribution of direction of arrival. Three traffic samples were thus generated [55].

The paper studies were extensions of work reported by Philpott [56], which used now-standard Monte Carlo procedures for sampling from a distribution to select an aircraft's time in the various phases of the landing operation. Approach speed, on-runway deceleration rates, and taxi speeds are sampled from independent normal distributions defined for each of three aircraft types.

For each of the nine possible aircraft type pairs, a procedure is described for establishing separations prior to landing. Given an aircraft landing-time distribution, a "tolerance band" of $T\%$ (arbitrarily selected) about the mean landing time is determined such that $T\%$ of the distribution lies within this band. Separation between successive aircraft at the entrance to the common path is then selected so that no separation violations occur if both aircraft remain within the tolerance band. [Unfortunately, since he uses

a two-stage division of the landing process (gate to wave-off point, and wave-off point to runway exit) and uses the tolerance bands in both phases, T is not usable in a probability statement, and generally has little physical significance.] The separation required at the gate between any pair of aircraft types (speed class) is such that, for the assumed value of T , the first aircraft at the slow end of its tolerance band will not conflict with the second aircraft at the fast end of its tolerance band. Having established these criteria, a sequence of landings is then generated. The number of wave-offs that occur because an aircraft was outside the tolerance band is then determined by selecting a random number for each aircraft; $T\%$ arrive within the tolerance band, $[(100 - T) / 2]\%$ arrive early, and $[(100 - T) / 2]\%$ arrive late.

Philpott illustrated the method for the 0% tolerance band, so he indicates only the effect of sequences of successive late and early arrivals, where the wave-off requirement is obvious. For $T > 0$, a sequence in which one of the pair is on time would presumably not result in any wave-offs.

Philpott presents the results of the computation for the range of tolerance bands, and notes that, as T decreases, the aircraft are spaced at closer separation, but result in a larger percentage of wave-offs because of variation in aircraft performance; as T increases, the separation required is larger and fewer wave-offs result. The value of T that gives the maximum acceptance rate is then used.

The paper has a number of technical errors (e.g., a strange discrete approximation to the normal distribution, incorrect compounding of distributions) and suffers from incomplete description. The independent normal distributions for times in successive phases of the landing might better have been correlated. The requirement for choosing a T value, with no physical meaning to the quantity, leaves much to be desired in the method, as does the associated method for determining separation, which has little relation to the system operation. Despite these shortcomings, the paper was valuable, since it was the first step in the development of a theoretical means of estimating runway capacities, and provided the starting point for the TDC-FIL studies.

The team of investigators at FIL and TDC built on Philpott's methods, and investigated a number of important questions with their simulation facility ([55], [57], [58]). Realizing that wave-offs were an infrequent occurrence, they designed a method that would provide the initial separation they desired without invoking Philpott's tolerance bands. Based on assumed truncated (at $\pm 2.5 \sigma$) normal distributions of aircraft velocity in each type class, separations are provided so that a wave-off probability of less than 1% exists. It is not indicated how this is determined, but it appears to be by the empirical cut-and-try method. This separation is added to that required to maintain a minimum glide-slope separation along the entire common path. It is never stated explicitly, but it appears that mean landing interval is

determined from. $\sum_{i=1}^3 \sum_{j=1}^3 T_{ij} p_i p_j$ where T_{ij} is the time between landings when an aircraft type i is at the gate and an aircraft of type j is at the appropriately-selected separation, and both aircraft fly with the velocity characteristic of their class. The values p_i and p_j represent the frequency with which aircraft of that type (three types are considered) appear at the runway.

Other than the above analytical computation, the remainder of the studies are simulation efforts. Two simulation techniques are discussed -- "graphical" and "dynamic." A special case of the graphical is the "ideal," in which many details of terminal operation (e.g., altitude changes) are ignored to reduce the complexity of the problem. It was noted in most of the comparisons of the graphical and ideal methods that the results by the two methods were very similar, so that the additional complexity was generally superfluous. In the graphical method, aircraft motion is depicted on a distance-time coordinate system.* Using a sample of successive arriving aircraft generated on the basis of traffic statistics, the path of each aircraft is plotted on the distance-time plot. For each aircraft, any potential conflicts with other aircraft are noted, and are resolved by appropriate delay (i.e., a constant-distance line or a sinusoidal curve to represent a turn) to the aircraft with the later ETA. In this manner, appreciable detail of operating procedure can be expressed and many aspects of the ATC operation investigated.

* See Figure 4.2 for an illustration of such a figure.

The "dynamic" simulation was a real-time simulation in which the traffic sample served as a set of flight plans to be presented to trained controllers operating in their normal environmental situation, communicating orders and clearances in the normal manner. The aircraft were represented by human "pilots" operating controls on analogue computing equipment into which they insert airspeed, rates of turn, climb, or descent, turn orders, and other important aircraft characteristics. By presenting a pre-determined traffic script, a reasonably realistic representation of the actual operating system was available for observation.

Using their graphical and dynamic simulators, the FIL-TDC team investigated aircraft delay as a function of a number of combinations of system parameters. They also investigated landing capacity as a function of location of the entrance gate, and of two other factors which enter implicitly through the initial gate separation: glide-slope wave-off minimum and existence of speed control along the glide path. The existence of speed control is represented by a further truncation (at $\pm 1.25 \sigma$) of the normal distribution of velocities within aircraft classes; the velocity distribution and wave-off minimum combine to produce the initial separation for the class pair.

The simulation experiments produced many important conclusions regarding terminal operation. These were summarized by Berkowitz, Fritz, Grubmeyer, and Miller [59]. Among other things, they inferred a

need for attaining closer separation on the landing glide path and advocated the use of high-speed runway turnoffs to prevent runway wave-offs. These were supplemented by recommendations regarding stack and airway configuration, communication facilities, radar operation and displays, controller procedures and aids, and navigational facilities.

A fast-time digital simulation of the final approach to landing has recently been developed by Rosenshine and Rosenbaum [60] at Cornell Aeronautical Laboratory. Their simulation, an extension of the Monte Carlo model of the Ground Controlled Approach described by Blumstein [61], considers aircraft released from the common-path gate at either pre-assigned separation times or at times that just satisfy the separation requirements and provides landing rates and separation distributions along the common path. Only preliminary results have been generated with this model.

The simulation efforts, especially of Berkowitz et al., contributed significantly to the body of techniques for examining air traffic problems without the necessity of performing expensive aircraft operation. With simulation it became possible to examine, in extensive detail, many of the characteristics of an ATC situation with varying degrees of realism.

As has been shown in many fields, it is desirable to be able to investigate systems problems by analysis as well as by simulation. Simulations, while capable of far greater realism than any analytical approaches,

have the disadvantage that they are only samplings from an actual operation, so that there is a finite probability, albeit arbitrarily small, that they will produce erroneous results. In general, in a simulation, many extensive runs must be performed in order to get statistically reliable results. Performing simulation, while far cheaper than actual physical experimentation, still requires tedious computation by a number of people in the paper (or "graphical") simulation and expensive machine programming and running in the digital simulation. The "dynamic" or real-time simulation requires expensive installations and many human operators to perform the roles in the simulated operation as well as to monitor, record, and analyze the output data.

Analytical formulations, on the other hand, while requiring more extensive simplification than simulation, permit the investigation of a large number of combinations of system configuration inexpensively and with a minimum of effort. The structure of the analytical formulation often provides valuable insight into the operation of the system, and provides a means for efficiently searching for an optimum system configuration.

The desirability of an analytical model was pointed out in the preface to a report on a symposium on Monte Carlo methods [66]:

More than one user of [Monte Carlo] techniques has remarked that enough insight of the physical process was attained in a Monte Carlo study so that a workable analytical model could be constructed. Such outcomes give rise to the observation that good Monte Carlo is self-liquidating. [66, pp. viii-ix.]

The remainder of this chapter is devoted to the investigation of analytical models of runway landing capacity, and to an analysis of the results and consequent implications for system operation developed with these models.

3.2 An Analytical Model of Runway Landing Capacity

In this model, aircraft are considered passing through an imaginary gate in space and following a common glide path to the runway. To prevent collision between successive aircraft, a minimum distance separation dictated by maximum position uncertainty and by control system reaction times is required at the gate, and a minimum time separation dictated by maximum runway-occupancy time is required at the runway. The distance between the gate and the runway is dictated by the location of radio aids and by the distance an aircraft requires to stabilize on the final approach path.

3.2.1 Assumptions

The following assumptions are made in the analysis:

1. Aircraft arrive at the gate independently and in random sequence.
2. Aircraft land in the order in which they arrive at the entry gate (i.e., first-come-first-served, and no aircraft may be passed after it has crossed the gate).

3. Aircraft must maintain a minimum distance separation (s_o) at the gate and a minimum time separation (t_o) at the runway.
4. Aircraft maintain constant velocity from the time they enter the gate until they reach the end of the runway.
5. Aircraft are available to be landed as close to each other as separation standards permit, (i.e., the capacity situation is examined).

Figure 3.1 illustrates the problem situation for two aircraft, A_1 and A_2 . Aircraft A_1 , with velocity V_1 , has just entered the gate, located a distance m from the end of the runway. At the same time, aircraft A_2 is located a distance s_o (the minimum separation required at the gate) behind the gate and a distance

$$n = m + s_o \quad (3.1)$$

from the end of the runway. The time between landing of A_1 and A_2 must exceed a minimum landing separation time t_o , sufficient to insure that A_1 is off the runway when A_2 lands.

3.2.2 Notation

We summarize here the notation which will be used in the model. In general, chance variables are represented by capital Roman letters, and their mean values by corresponding Greek letters. A subscript "o" is used

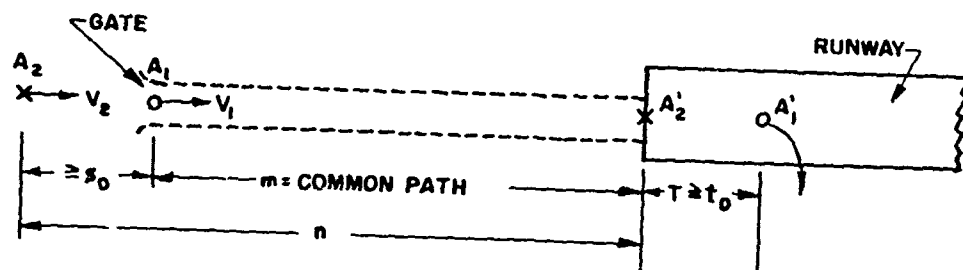


FIGURE 3.1
LANDING RUNWAY SITUATION

to denote a value set by procedural standards. The important notation is defined below:

- V_1, V_2 = the chance variable velocities of successive landing aircraft, A_2 (the aircraft with V_2) following A_1
- a = minimum of the landing-velocity distribution
- b = maximum of the landing-velocity distribution
- $\bar{V}$ = mean of the landing-velocity distribution
- $R = b - a$ = range of the landing-velocity distribution
- s_o = minimum space separation required at the entrance to the common landing path
- m = distance from the entry gate to the approach end of the runway
- n = $m + s_o$ = distance of A_2 from the runway when A_1 is at the entry gate
- t_o = minimum time separation between successive landing aircraft required at the runway
- $S_{AA}(t)$ = landing-time distribution function = $\Pr \{T_{AA} \geq t\}$
- T_{AA} = a chance variable with mean τ_{AA} representing the time interval between successive landings
- λ = $1/\tau_{AA}$ = landing capacity

3.2.3 General Formulation

There is a function of V_1 , $V^*(V_1)$, such that, if $V_2 = V^*(V_1)$ during the course of the landing, the original separation of s_0 is altered so that the separation at landing is exactly t_0 . If $V_2 > V^*(V_1)$, the time separation at landing would be less than t_0 if A_2 were allowed to proceed freely. To avoid this condition, A_2 is assumed to be held at the gate for a time sufficient to permit a landing separation of exactly t_0 . If $V_2 < V^*(V_1)$, the separation at landing is greater than t_0 , and is dictated by the original positions, m and n , and the aircraft velocities.

For any combination of velocities, V_1 and V_2 , the time separation between landings, T_{AA} , is given by:

$$\begin{aligned} T_{AA}(V_1, V_2) &= \frac{n}{V_2} - \frac{m}{V_1} \quad \text{for } V_2 \leq V^*(V_1) \\ &= t_0 \quad \text{for } V_2 \geq V^*(V_1) \end{aligned} \quad (3.2)$$

The function V^* is determined from (3.2) to be:

$$V^*(V_1) = \frac{nV_1}{V_1 t_0 + m} \quad (3.3)$$

The velocities of the landing aircraft are random variables, with a joint frequency distribution $f(V_1, V_2)$. For any specified m , s_0 , and t_0 , the average time interval between successive landings, τ_{AA} , is given by:

$$\tau_{AA} = \int_{V_2} \int_{V_1} T_{AA}(V_1, V_2) f(V_1, V_2) dV_1 dV_2 \quad (3.4)$$

3.2.3.1 Discrete Velocity Distribution

The aircraft velocity distribution could be represented by a discrete distribution based on the assumption that all aircraft of the same type fly with identical speed, and that velocity differences arise from the arrival of different aircraft types. The integration of equation (3.4) is then replaced by a summation. Denoting the relative frequency with which aircraft of velocity V_i appear at the runway by p_i ($i = 1, 2, \dots, N$), and with assumption (1) above, $f(V_i, V_j) = p_i p_j$. Then, the landing time, $T_{AA_{ij}}$, can be determined from equation (3.2), and the average landing time for the runway is given by:

$$\tau_{AA} = \sum_{j=1}^N \sum_{i=1}^N T_{AA_{ij}} p_i p_j \quad (3.5)$$

3.2.3.2 Uniform Velocity Distribution

Since aircraft do not fly at discrete speeds, particularly in the presence of varying wind conditions, it would be desirable to represent the velocity distribution by a continuous one. Furthermore, it is desirable for analytical purposes to be able to characterize the velocity distribution by a small number of parameters, and this is difficult with the particular discrete distributions characteristic of particular airports. Thus, the uniform distribution, selected because of its analytical convenience and because it is a bounded distribution, is first investigated as a representation of the aircraft velocity distribution.

It is now assumed that aircraft velocities are uniformly distributed over the continuous interval $[a, b]$. Since aircraft are assumed to arrive at the gate independently and in random sequence,

$$f(V_1, V_2) = \frac{1}{(b-a)^2} \text{ for } a \leq V_1 \leq b; a \leq V_2 \leq b \quad (3.6)$$

$$(a \neq b)$$

$$= 0 \text{ otherwise}$$

The mean landing time, $\bar{t}_{AA}$, is computed by substituting equations (3.2) and (3.6) into (3.4), and integrating over the appropriate limits. The limits are defined by the location of the $V_2 = V^*(V_1)$ line, which, as can be seen from equation (3.3), has positive slope, so that all the possible cases are covered in Figure 3.2.

The most common situation is Case A, where $V^*(a) \leq b$ and $s_o \geq bt_o$. The first inequality implies that some aircraft land with separation t_o ; the second expresses a common requirement wherein the separation distance imposed on an airplane of velocity V at the runway ($V t_o \leq bt_o$) is always less than that imposed at the gate (s_o), and is seen from equation (3.3) to be equivalent to requiring that there be a $c \leq b$ such that $V^*(c) = b$. In Case B, the gate separation is larger than the runway separation for small V_1 and smaller for large V_1 . The cross-over point is $V^*(V_e) = V_e$, or $V_e = s_o/t_o$; hence, the condition $at_o \leq s_o \leq bt_o$. Case C represents the condition where the gate separation is always less than the runway separation, and is given by the condition that $V^*(a) < a$; from equation (3.3), this

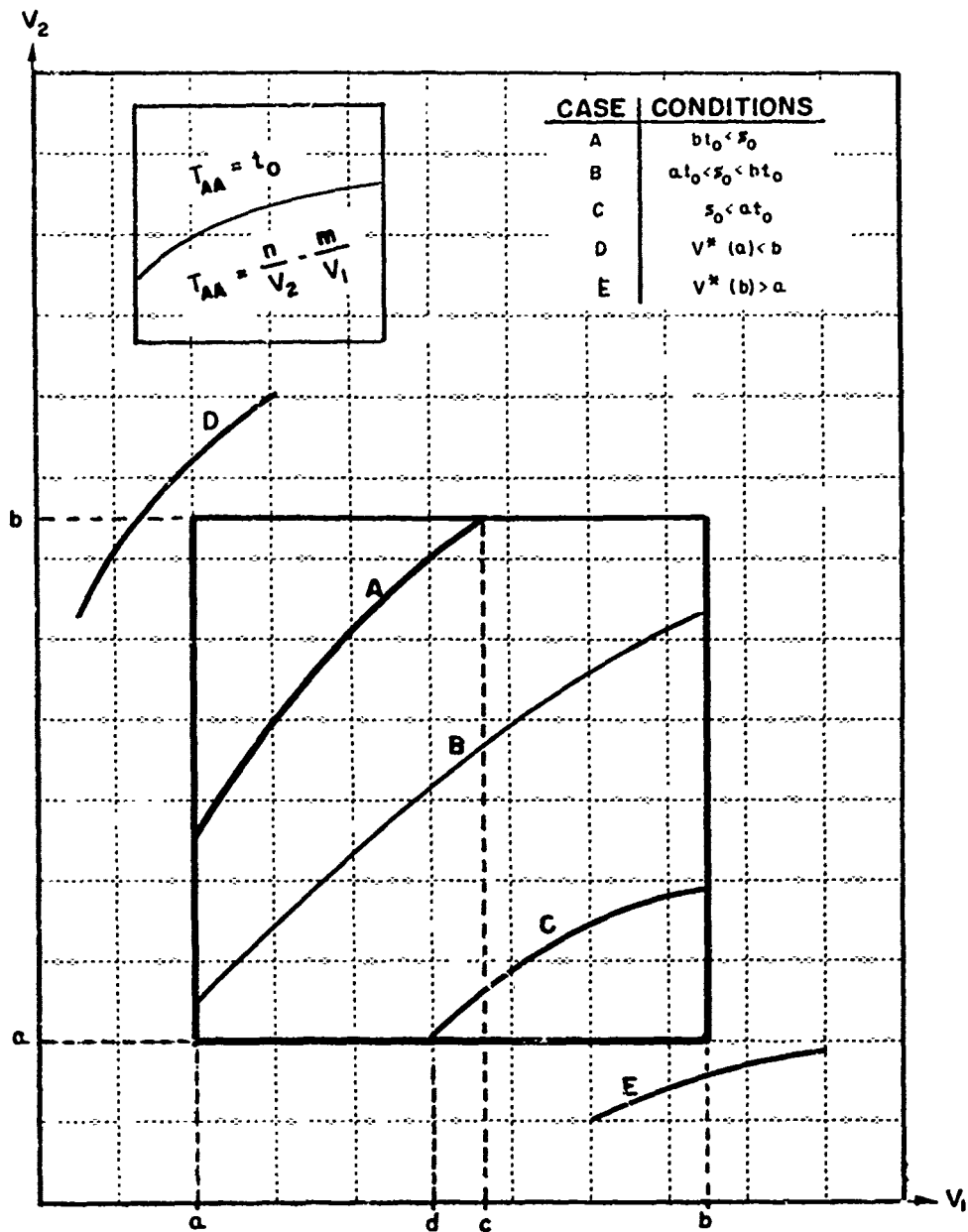


FIGURE 3.2
ILLUSTRATION OF THE CASES OF $V^*(V_1)$

is seen to be equivalent to requiring $s_o < at_o$. [Because of this condition, and since $a < b$, it is seen that another possible V^* curve (extending from $V_2 = a$ to $V_2 = b$) cannot occur.] Case D represents the condition where $T_{AA} > t_o$ for all (V_1, V_2) , so that $V^*(a) > b$. In Case E, runway separation alone dictates landing interval, and $T_{AA} = t_o$.

Development of τ_{AA} for the various cases is presented in Appendix A. The resulting expression for Cases A, B, or C is given by:

$$\begin{aligned}
 (b-a)^2 \tau_{AA}(x, y) = & (nx + mb) \log_e (x/b) \\
 & + (ma + nb) \log_e b - n(b-y) \log_e a \\
 & - (ny + ma) \log_e y + (x-y)n \log_e n \\
 & - n(x-y) bt_o(x-a) - at_o(y-a) \\
 & + (n/t_o) [(m + y t_o) \log_e (m + y t_o) \\
 & - (m + x t_o) \log_e (m + x t_o)]
 \end{aligned} \tag{3.7}$$

where the arguments x and y are given by:

$$\begin{aligned}
 x &= \text{Min } \{b, c\} \\
 y &= \text{Max } \{a, d\}
 \end{aligned}$$

where c and d are given by:

$$c = \frac{mb}{n - bt_o} \quad \text{from } V^*(c) = b \text{ in Case A}$$

$$d = \frac{ma}{n - at_o} \quad \text{from } V^*(d) = a \text{ in Case C}$$

For Case D,

$$\tau_{AA} = \frac{s_o}{b - a} \log_e \frac{b}{a}$$

In Case E,

$$\tau_{AA} = t_o$$

3.2.4 Model When Gate Separation Maintained Along Glide Path

The radar separation standards nominally require that aircraft maintain their initial separation (s_o) along the entire glide path. In the model formulated, however, it was assumed that an initial separation at the gate could be reduced as long as runway separation (t_o) is maintained. This assumption was made to express the interaction between runway and gate separations, and was based on discussion with controllers who indicated that they would not generally interfere with a situation in which space separations were being violated, but where it was clear that runway separation would be maintained.

Landing capacity under the alternate assumption that s_o is the required separation along the entire glide path can easily be calculated. In that situation, if $s_o > bt_o$ and glide-path separation always exceeds runway separation, T_{AA} is given by:

$$\begin{aligned} T_{AA} &= s_o/V_2 && \text{if } V_2 \geq V_1 \\ &= n/V_2 - m/V_1 && \text{if } V_2 \leq V_1 \end{aligned}$$

τ_{AA} is then given by:

$$(b-a)^2 \tau_{AA} = \int_a^b dV_1 \int_{V_1}^b \frac{s_o}{V_2} dV_2 + \int_a^b dV_1 \int_a^{V_1} \left(\frac{n}{V_2} - \frac{m}{V_1} \right) dV_2$$

which, when integrated, gives:

$$(b-a)^2 \tau_{AA} = [b(s_o + m) - a(s_o - m)] \log_e \frac{b}{a} - 2m(b-a)$$

3.3 Results of the Landing Capacity Analysis

Landing capacity ($\lambda = 1/\tau_{AA}$) was calculated for several values of s_o , t_o , and m for the two velocity distributions. Landing rates with the continuous distribution were computed for a wide range of values of $\bar{V}$ and R . Landing rates with the discrete distribution were calculated for the specific traffic mixes at the three major New York area airports (Idlewild, LaGuardia, and Newark).

3.3.1 Landing Capacity with the Discrete Velocity Distribution

In measuring landing capacity at a particular terminal, the relative frequency with which the terminal is used by each of the three major classes of aircraft (air carrier, military, and general aviation) was first determined from CAA data [62].

The military traffic constitutes less than 2% of the volume at each of the terminals, and is neglected. General aviation accounts for less than 10% of the operations at all terminals except LaGuardia, where it is 19%. Due to lack of specific detailed information, the general aviation component was arbitrarily assumed to be represented by an Aero Commander class and by a Learstar class aircraft, each arriving with equal probability*. The April, 1959 Airline Guide [63] provided the distribution of air-carrier types for the terminals. Typical constant approach speeds were assigned to each aircraft type based on available data [64]. The resulting landing speed distributions for the three terminals are shown in Table 3.1 and plotted in Figure 3.3.

Landing capacity (λ) is plotted in Figure 3.4 as a function of gate separation (s_0) for the discrete velocity distributions characteristic of Idlewild and LaGuardia**. The curves are plotted for three values of m

* Since general aviation constitutes a relatively minor portion of the traffic load at these terminals, particularly in IFR conditions, the results are essentially insensitive to the above assumption.

** The results for LaGuardia and Newark are essentially identical, so that only LaGuardia is explicitly discussed.

TABLE 3.1
DISTRIBUTIONS OF AIRCRAFT SPEEDS AT NEW YORK TERMINALS

Representative Aircraft Type	Approach Speed (knots)	Idlewild Percent	LaGuardia Percent	Newark Percent
Boeing Stratocruiser Britannia	151	4.6	0	0
DC-7, -7B, -7C	143	22.7	0	7.9
Boeing 707	139	4.6	0	0
DC-6, -6B	136	22.7	24.2	17.4
Viscount, Electra	130	9.1	8.3	7.2
DC-4, L-1649	123	4.6	0	0
Constellation, Super Constellation	118	15.7	11.5	15.9
Martin, Convair, Learstar	112	12.5	40.1	35.9
C-46	98	0	0	1.5
DC-3, Aero Commander	91	3.5	15.9	14.2
Average speed for Terminal (knots)	.	129.8	116.7	117.7

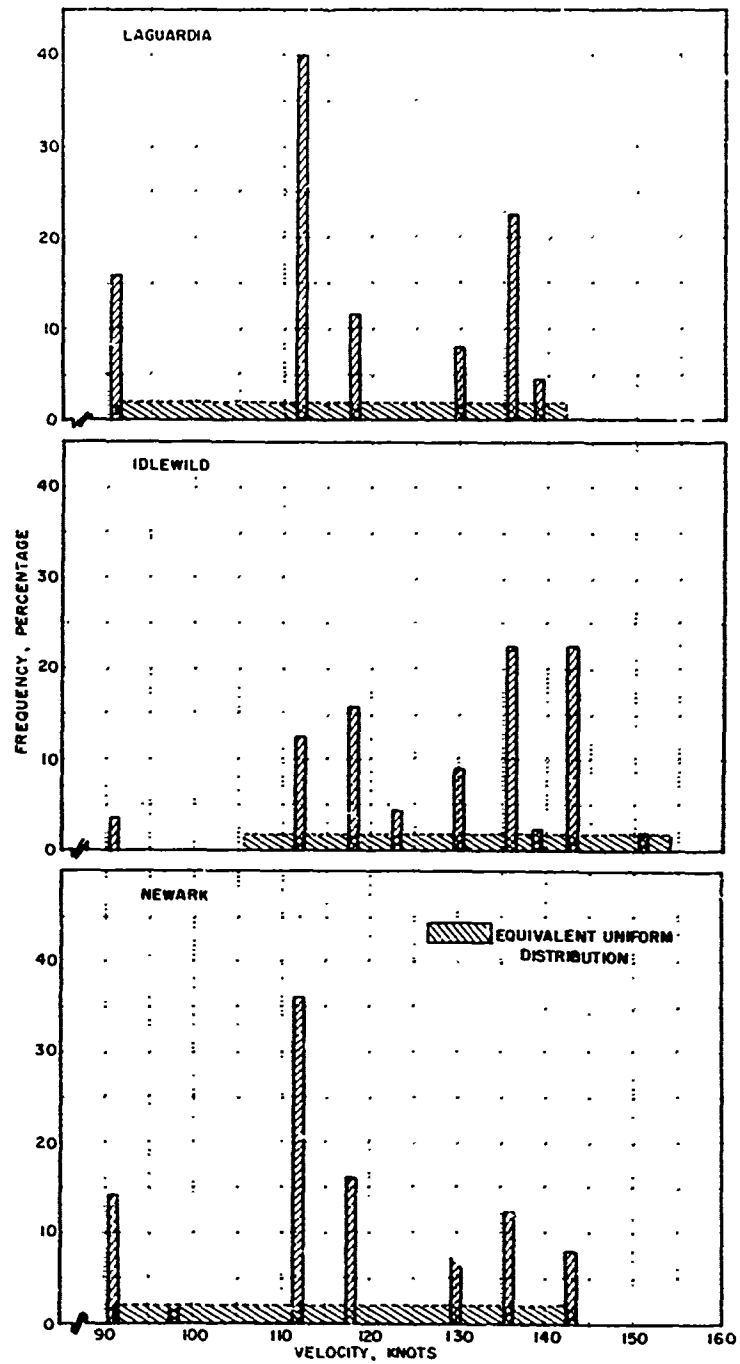


FIGURE 3.3
DISCRETE VELOCITY DISTRIBUTIONS AT NEW YORK TERMINALS

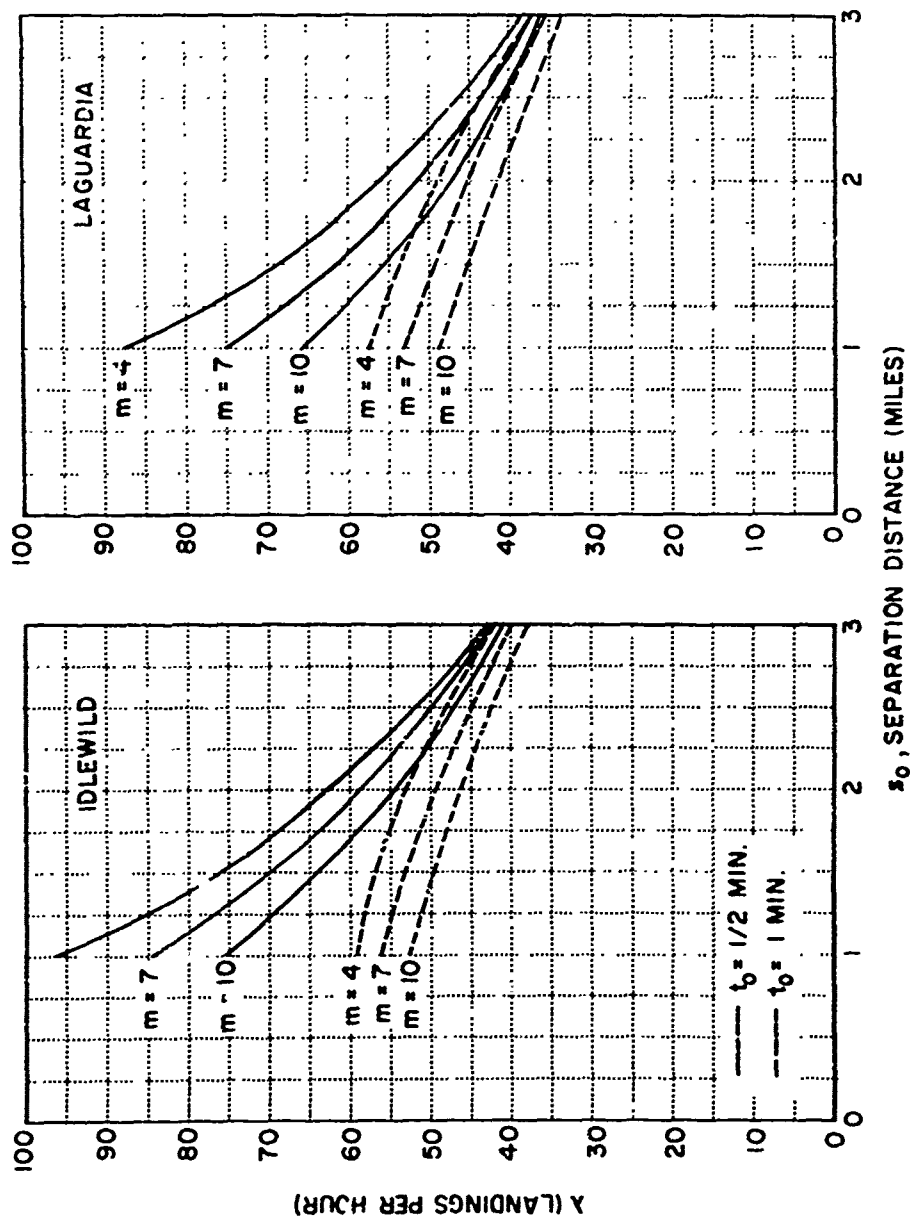


FIGURE 3.4
LANDING CAPACITY AT IDLEWILD & LAGUARDIA

(4, 7, and 10 miles) and two values of t_0 (1 minute, shown dotted, and 1/2 minute, shown solid). In the particular case most typical of current IFR operation ($s_0 = 3$ miles, $t_0 = 1$ minute, $m = 10$ miles), a landing rate of about 38 per hour is predicted for Idlewild and 33 for LaGuardia, values consistent with experience. The difference between the terminals results primarily from the higher average approach speed at Idlewild (130 knots) in comparison to LaGuardia (117 knots).

The strong influence of s_0 on landing rate is clearly shown by the figures. Gate location (m) has an effect of less than 20% over the range considered. When s_0 is 3 miles, a change in t_0 from 1 minute to 1/2 minute has a negligible effect on landing rate, and t_0 affects λ by only about ± 9 -20% even when s_0 is reduced to 2 miles. Only if it were possible to reduce s_0 close to 1 mile would a reduction of t_0 from 1 minute to 1/2 minute have a significant effect on landing rate, and the effect would be greater at Idlewild where the faster aircraft could take greater advantage of the reduction. But a major improvement in current operation evidently requires first a reduction in s_0 .

3.3.2 Landing Capacity with the Uniform Distribution

The landing capacity is examined under the assumption that the landing velocity is distributed uniformly over $[a, b]$, with mean velocity $\bar{V} = (b + a) / 2$ and range $R = b - a$.

Figure 3.5 depicts λ as a function of gate location for two average approach speeds, 110 and 130 knots. The three groups of curves on each graph represent three s_0 values; the curves within a group correspond to changes in t_0 ($t_0 = 1$ minute is shown dotted) and in R .

Comparing the two graphs, it is noted that increasing $\bar{V}$ results in a higher landing rate, primarily because of the shorter time required to close the s_0 separation. In other respects, the graphs depict similar relationships.

Examining the graph for $\bar{V} = 110$, it is noted that λ varies widely, ranging from 55 to 25 landings per hour over the range of parameter values considered. The top group of curves represents perhaps an ultimate achievement in separation standards -- 2 miles at the gate and 1/2 minute at the runway. In this case, the landing capacity is 55 landings per hour if $m = 0$, i. e., the gate is at the runway; these results contrast sharply with the potential of 120 landings per hour based on runway separation alone.

The landing capacity drops as the gate is displaced from the runway. The decrease depends critically on the velocity range. It is largest when there is a large spread in aircraft velocity, dropping 22% to a landing rate of 42 per hour if the gate is at ten miles and aircraft velocities are distributed over the 80-140 knot range. However, if a velocity range of only 100-120 knots can be achieved, by speed control or by segregation, the landing rate is effectively maintained for all gate locations to ten miles.

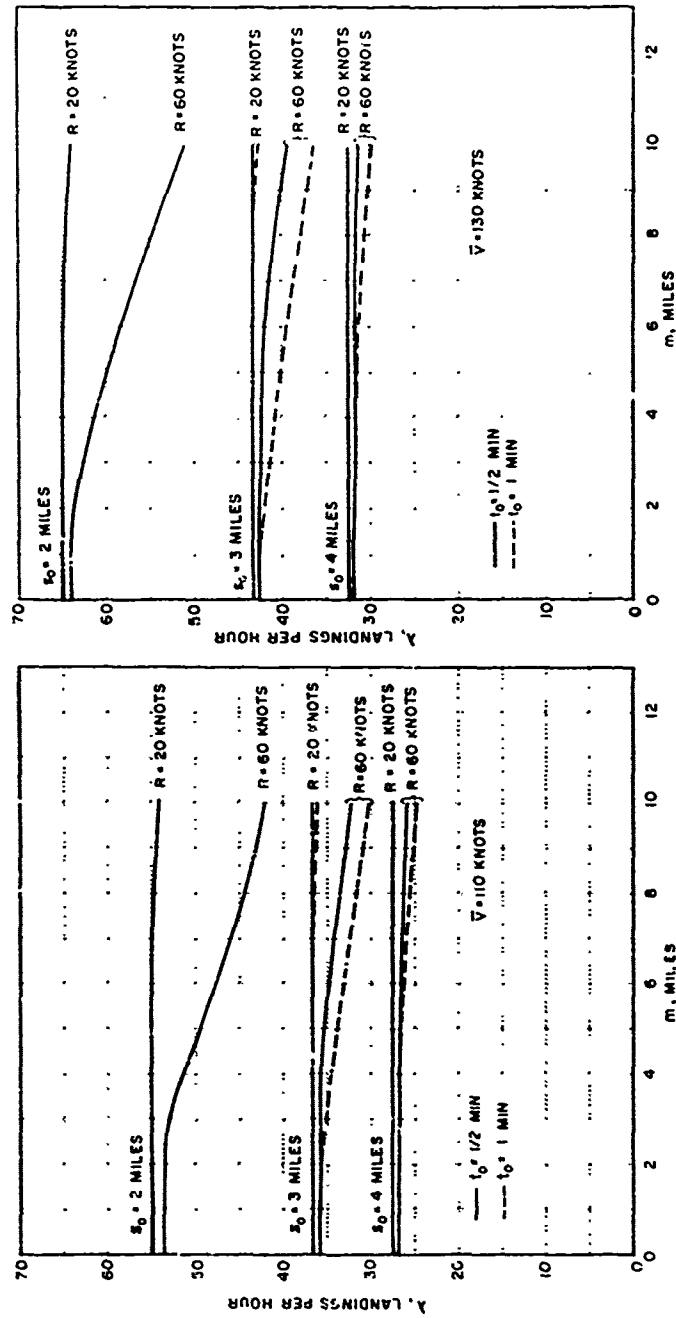


FIGURE 3.5
LANDING CAPACITY vs. COMMON PATH LENGTH FOR SEVERAL RUNWAY SEPARATION TIMES (t_0)
GATE SEPARATION DISTANCES (s_0), MEAN VELOCITIES (V), AND VELOCITY RANGES (R)

The remainder of the graph presents the landing capacity attainable with a gate separation of 3 and 4 miles, and with t_0 values of 1/2 and 1 minute. For these larger values of s_0 , the effect of t_0 is negligible: landing rate is increased by no more than 2 landings per hour when t_0 is reduced from 1 minute to 1/2 minute. Clearly, in this range, the most significant parameter is s_0 .

It can be seen from Figure 3.5 that, if the s_0 separation is fixed, a high landing rate may be maintained by reducing the spread of velocities of the aircraft arriving at a single runway (for example, by assignment of aircraft to different runways based on their performance) or by locating the gate close to the runway (i. e., shortening the common glide path) and that either approach, by itself serves the purpose. This results from the fact that while an increase in either parameter tends to create greater opportunities for long landing intervals, R by providing faster aircraft to be followed by slower ones, and m by providing a longer opportunity for the interval between a fast and a slow aircraft to open, each requires the environment provided by the other to exert its influence.

This interaction between m and R is illustrated more explicitly in the "carpet plot" of Figure 3.6. When either m or R is small (the top of the figure), the landing capacity is about 35 landings per hour and is essentially independent of the other. As both become larger, however, the landing rate drops rapidly.

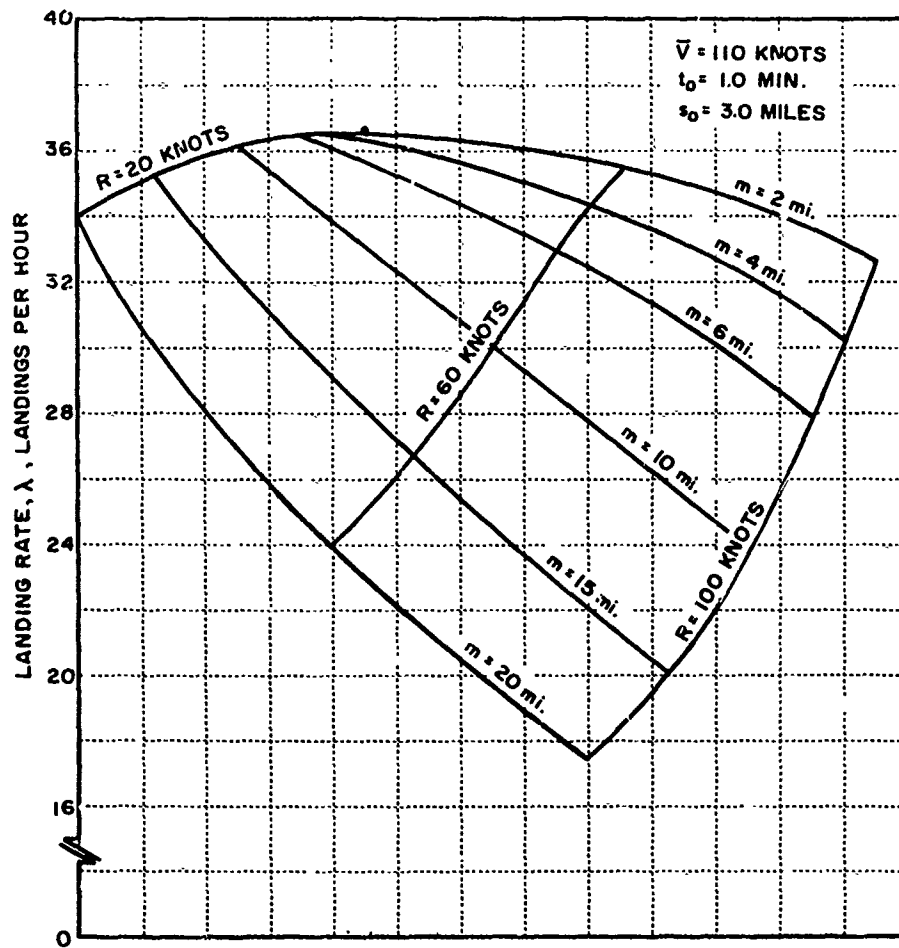


FIGURE 3.6
 EFFECTS OF GATE LOCATION (m) AND
 VELOCITY RANGE (R) ON LANDING CAPACITY

The interaction between $\bar{V}$, s_0 , and t_0 is shown more explicitly in Figure 3.7. which depicts the landing rate as a function of aircraft velocity for the limiting case of $R = 0$. When $s_0 = 3$ or 4 miles, for no velocity in the range of interest (90 - 150 knots) does a landing separation (t_0) below one minute affect landing rates. It is only when the initial gate separation (s_0) is cut below 2.5 miles that a decrease in landing separation below 1 minute provides any contribution, and then only at the extreme velocities; a gate separation less than 1.25 miles is necessary to derive any benefit from a reduction of t_0 below 1/2 minute.

The effect of velocity distribution on landing rate is shown in Figure 3.8. The landing rate is determined for typical parameters ($s_0 = 3$ miles, $t_0 = 1$ minute, $m = 6$ and 10 miles) representing current IFR operation. The landing rate clearly increases with mean velocity (since less time is required to close the initial s_0 separation) and decreases with velocity range (since T_{AA} is bounded by t_0 from below, but by $(n/a - m/b)$ from above, and a larger spread results in a higher upper bound). Comparing the two graphs, which differ in gate location (m), it is noted, as in Figure 3.6, that decreasing m inhibits the degradation in landing rate caused by velocity range.

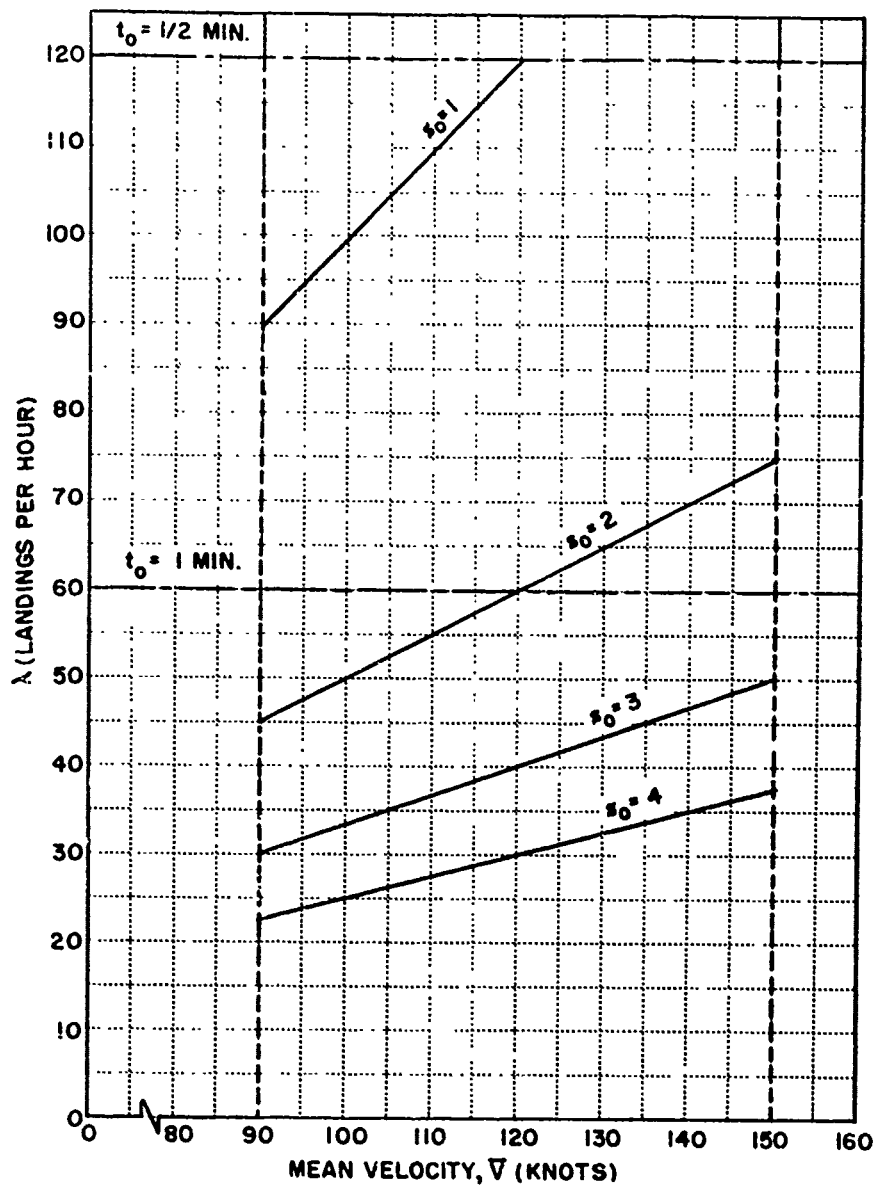


FIGURE 3.7
LANDING CAPACITY WHEN $R = 0$

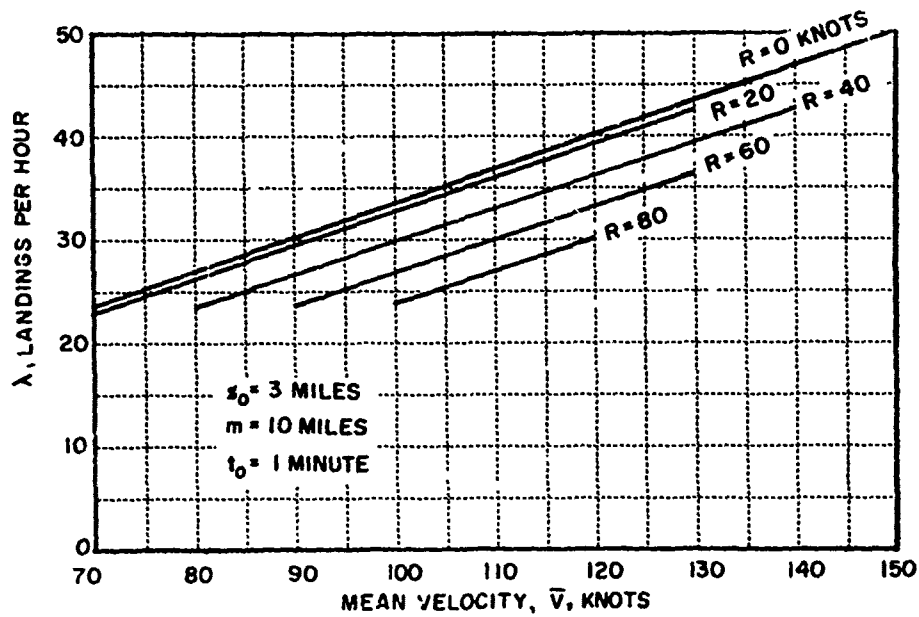
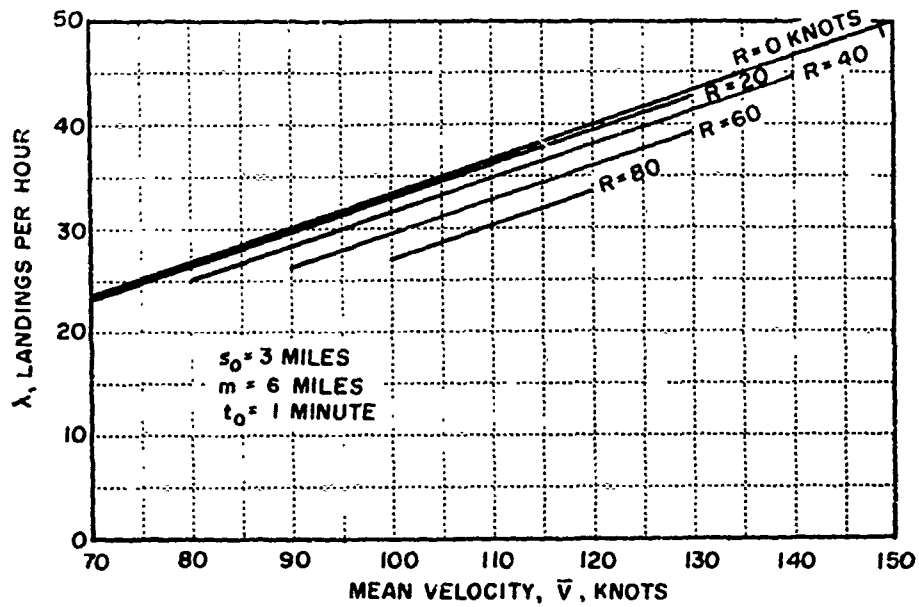


FIGURE 3.8
EFFECT OF MEAN VELOCITY & VELOCITY RANGE (R)
ON LANDING RATE

3.4 Effect of Velocity Distribution on Landing Capacity

To examine the sensitivity of landing capacity to velocity distribution, mean landing rates computed for the New York terminals were compared with those for the equivalent uniform distribution (i. e., one having the same mean and variance) for several parametric combinations. This comparison is presented in Table 3.2, and the results by the two methods are seen to agree to well within 1.5% in all the cases. Based on this evidence, and on additional evidence presented in Appendix E for a larger sample of velocity distributions with more recent data, it appears probable that the uniform distribution satisfactorily represents the approach speed distribution, at least for the determination of landing capacity.

3.5 Estimation of Landing Capacity of an Individual Airport

The general results based on the uniform distribution may be used to estimate the landing rate for any airport. It is first necessary to estimate the mean ($\bar{V}$) and standard deviation (σ_V) of the approach speeds for the aircraft mix at the terminal. Knowing the aircraft velocity and frequency information for the airport (as in Table 3.1), $\bar{V}$ and σ_V are calculated from:

$$\bar{V} = \sum_{i=1}^N V_i P_i ; \quad \sigma_V = \sqrt{\sum_{i=1}^N V_i^2 P_i - (\bar{V})^2} \quad (3.8)$$

TABLE 3.2
COMPARISON OF LANDING RATES USING
DISCRETE AND EQUIVALENT UNIFORM DISTRIBUTIONS

Parameters			Landing Rate			
			LaGuardia		Idlewild	
			$\bar{V} = 117$ knots		$\bar{V} = 130$ knots	
			$R = 52$ knots		$R = 48$ knots	
m (miles)	s_o (miles)	t_o (min.)	Discrete	Uniform	Discrete	Uniform
4	2	1/2	56.0	56.0	62.7	63.1
		1	49.1	49.0	53.3	53.6
	3	1/2	38.2	38.3	42.6	42.8
		1	37.4	37.5	41.8	41.9
10	2	1/2	47.2	47.9	54.8	55.5
		1	41.5	41.3	45.3	46.5
	3	1/2	36.0	36.1	40.8	41.2
		1	33.3	33.6	37.9	38.2

where p_i is the relative frequency with which aircraft with approach speed V_i appear, ($i = 1, 2, \dots, N$) expressed as a fraction such that $\sum_{i=1}^N p_i = 1$.

The equivalent velocity range for the uniform distribution is determined from:

$$R = \sigma_V \sqrt{12}. \quad (3.9)$$

With this information, the average landing rate for the airport may then be read from Figure 3.8 (or similar curves for other parameter values), using $\bar{V}$ to specify the abscissa and R to interpolate between the lines of constant range.

Since $\bar{V}$ is the more significant parameter, the calculation of σ_V may often be avoided by assuming R to be between 30 and 60 knots, a range that probably holds for most major terminals. The landing rate can then probably be estimated to within 10%.

The velocities used in the above analysis are ground speed (equal to air speed in a condition of zero wind). In any computation of $\bar{V}$, only the zero-wind condition would be calculated, and the landing rate for any particular wind velocity V_w may be found by entering Figure 3.8 with $(\bar{V} - V_w)$ in place of $\bar{V}$.

The model provides an estimate of the landing capacity when the separation standards, s_o and t_o , are exactly maintained. Controllers maintain them with error, however, which is usually biased toward greater separation to insure safety, so that the capacity thus computed represents a high estimate. A more conservative estimate using other typical s_o and t_o values can easily be computed.

3.6 Increasing Landing Capacity

The results of the analysis indicate a number of means of improving the landing capacity of a runway by operating on the parameters of the system. With gate separation (s_o) in excess of three miles as at present, little can be gained by reducing runway separation (t_o) by techniques such as high-speed turnoffs. Landing rate might be increased by increasing mean landing speed ($\bar{V}$), but $\bar{V}$ is dictated by the mix of aircraft and pilot procedure, and ATC generally has little control over either. It might be possible, however, to set a minimum landing speed for the major runways.

If either the gate is close to the runway or a homogeneous class of aircraft operate at the airport, reducing either m or R provides little advantage. If both are large, however, potential for improvement exists. Gate location can be no closer than the distance corre-

sponding to the minimum time required by pilots to stabilize their aircraft on the final approach path. But since its location is often dictated by the location of radio facilities (e.g., the outer marker) rather than by a minimum path length, it can often be drawn closer to the runway.

While little can be done to alter the mix arriving at an airport, some improvement might result from a reduction in the velocity range of aircraft arriving at a particular runway achieved by segregating the aircraft by speed class onto separate runways. Such an action at a large airport would typically involve removal of the smaller aircraft from the major runways, thereby reducing the demand on these runways, but also increasing their capacity to handle the larger aircraft which are limited to the larger runways. The segregation procedures would, of course, make these runways available to the smaller aircraft whenever no queue of larger aircraft is waiting, thus avoiding the incongruous and unintended situation of a queue of smaller aircraft waiting to use the minor runway while the major one is unoccupied. A segregation decision would also have to take into account the relative demand for the two classes of service.

3.6.1 Reducing Gate Separation (s_o)

Since landing capacity is most seriously limited by the present gate separation of 3 miles, the most significant improvement could be achieved by reducing s_o . The space separation must be large enough to

compensate for the errors with which the aircraft's position and velocity are known by the control system and to allow sufficient reaction time (recognition of a dangerous situation by the controller and relay of a control message to one or both pilots) to correct an imminently dangerous situation.

If it is desired that aircraft shall never be permitted to come closer to each other than some minimum distance, d_o , then the minimum separation between two aircraft (with velocities V_1 and V_2) which a controller may tolerate is some larger value, S_{min} , given by:

$$S_{min} = E_R + (V_2 - V_1) (T_d + T_r) + d_o \quad (3.10)$$

where: E_R = radar error in locating one aircraft relative to another

T_d = controller time delay, representing the time between the existence of a separation of S_{min} and the issuance of a control order to one of the aircraft

T_r = pilot-aircraft response time, representing the time between receipt of a control order by a pilot and a response by the aircraft correcting the hazardous situation.

Thus if the error in estimating separation is E_R , in the time $(T_d + T_r)$ between development of the situation and its correction, the separation will have closed from S_{min} to d_o . A safety factor must then be imposed on

this $S_{\min}$ to account for the variability of the terms in the equation, and to make the probability of a violation of d_0 sufficiently small. The distributions of the chance variables in equation (3.10) must be known in order to establish a safety factor rationally.

Whatever safety factor is chosen, and whatever the distributions of the variables in equation (3.10), the necessary separation depends on the individual aircraft being separated; in particular, less separation is required when a slow aircraft is behind a fast one than in the reverse situation. The present value of 3 miles was set about 10 years ago as an easy value to use, and one that contains a sufficient margin of safety so that it would be satisfactory in all cases. When terminals were less congested than they are today, this excessive caution was desirable. But with the current congestion, it is important that the landing rate be increased, and this can best be done by a reconsideration of the optimum value of s_0 .

Until computer aids become available, any rule must be operationally convenient to implement. It is possible to improve acceptance rate by adopting a two-level rule on gate separation, e.g., let s_0 be the separation if $V_2 > V_1$, but use the separation s'_0 (where $s'_0 < s_0$) if $V_2 < V_1$. The resulting landing interval, τ'_{AA} , can be determined from equation (3.4) by an appropriate change in the values of T_{AA} .

3.6.2 Sequencing of Landing Aircraft

It has been assumed in the analysis that aircraft are landed in the random order in which they arrive. It is to be expected that any ordering of the landings with a view towards maximizing landing rate would improve landing capacity. Such an improvement would, of course, come at the expense of one of the most sacred of ATC principles -- first-come-first-served -- and may thus be untenable from a practical standpoint.

A thorough investigation of sequencing doctrines would require consideration of the statistics of arrival time intervals and sequences, which is beyond the scope of this study. It is possible, however, to indicate some upper bounds on the benefits to be gained from certain sequencing procedures by assuming an infinite source of aircraft waiting to land and considering that an aircraft optimally suited to the sequencing doctrine can be selected whenever desired, while bearing in mind the requirements posed by distribution of aircraft types.

It is convenient to refer first to the discrete model of aircraft arrivals, in which it is considered that M different types of aircraft arrive at the airport with landing velocity V_i ($i = 1, 2, \dots, M$) with relative probabilities p_i . A commonly-suggested sequencing plan involves grouping aircraft of like speeds, thus removing, to some extent, the effect of velocity variation. A landing sequence based on this consideration would involve k_1 landings by aircraft with velocity V_1 , followed by k_2 landings

by aircraft with velocity V_2 , and so on, followed finally by k_M landings by aircraft with velocity V_M , after which the sequence would begin again. The time between each pair of landings by aircraft of the same type is given by $\text{Max} \left\{ t_o; s_o/V_i \right\}$, while the time between the last of type i and the first of type $(i + 1)$ is given by $\text{Max} \left\{ t_o; n/V_{i+1} - m/V_i \right\}$. If the sequence is cyclical (so that $M + 1 = 1$), and k_i is made proportional to p_i (i. e., $k_i = p_i N$) to satisfy aircraft availability considerations, then the time for one cycle of the sequence of N landings is given by:

$$\tau_c = \sum_{i=1}^M (p_i N - 1) \text{Max} \left\{ t_o; s_o/V_i \right\} + \sum_{i=2}^{M+1} \text{Max} \left\{ t_o; \frac{n}{V_i} - \frac{m}{V_{i-1}} \right\} \quad (3.11)$$

The first sum in equation (3.11) represents the intra-type time, and is a function only of the average length of the run* of identical-type landings (which is N/M). The second sum represents the inter-type time, and is a function only of the sequence of types. It is thus noted that, for any specified sequence of types, the average time per landing (τ_c/N) decreases as N increases (since the first sum is proportional to N whereas the second sum is fixed), implying that long runs of identical-type landings are desirable. In the limit, the inter-type time becomes negligible, and the landing rate is maximized for this sequencing rule. Such a result, while emphasizing the

* Throughout this paper, "run" is used to mean an uninterrupted sequence of similar events; aircraft on a runway perform a landing "roll" or take-off "roll."

value of the concept of "run-of-types" sequencing, has limited operational meaning since the length of a run is dictated by the availability of aircraft of a particular type, and is necessarily short.

The problem of determining an optimum sequence of types to minimize the second term of equation (3.11) is identical to the "traveling-salesman" problem,^{*} which has been treated extensively in the literature. The traveling-salesman problem is basically one of finding a cyclical $M \times M$ permutation matrix, X , such that, for some given $M \times M$ "cost" matrix, C , the sum $\sum_{i,j=1}^M x_{ij} c_{ij}$ is a minimum. The "traveling-salesman" appellation derives from the form of the problem in which a salesman must tour M cities whose separations are represented by the elements of the C matrix (which is symmetrical in this case) such that the total distance covered is a minimum. The solution to the problem then defines the order of touring the cities. Most attempts at general formulation of a solution involve a linear programming formulation, but the solution of the linear programming problem may provide a permutation which loops before it cycles through all M elements. The general approach to recovering from this situation is to impose additional restrictions in the linear program to prevent the loops. No simple, universally-applicable solution has yet appeared.

* The author is grateful to Professor R. Conway for calling this similarity to his attention.

In examining the optimum sequence of types to minimize the second term of equation (3.11), we consider first the case where there is no limitation due to t_0 . Then, the total time for any sequence is given by $s_0 \sum 1/V_i$, and the total time is independent of sequence*.

Imposing the t_0 restriction introduces an additional time penalty $c_{ij} = (t_0 - n/V_j + m/V_i)$ for all those pairs (i, j) where $V_j > V^*(V_i)$. Thus, we note that, if some sequence exists such that the t_0 restriction is never imposed, then that sequence is an optimum sequence.

We rank the aircraft in order of their velocities such that $V_1 < V_2 < \dots < V_M$, and define the "NF" (next-fastest) sequencing rule to state that A_i lands before A_j if and only if $V_i < V_j$. We can show that, if a sequence exists that has no t_0 restrictions, then the NF sequence is such a sequence. We assume first that, for some j , $V_{j+1} > V^*(V_j)$. Then $V_i > V^*(V_j)$ for $i = j+1, \dots, M$ since V^* is an increasing function of V . Thus, in the sequence without t_0 restrictions, A_j must be followed by A_k , where $k < j$. But $V_i > V^*(V_j) > V^*(V_k)$, so that A_k will be followed by a t_0 restriction. This last problem can be avoided only by using a sequence which is the reverse of the NF sequence, but then A_M follows A_1 , and $V_M > V^*(V_1)$. Thus, if there is

* This result can be extended to the traveling-salesman problem in general: If $c_{ij} = f(Y_i) + g(Y_j)$, where Y is a characteristic of the element, and f and g are any functions, then the total time for the cycle is independent of the sequence.

a t_0 restriction in the NF sequence, then there can be no sequence without such a restriction. Consequently, the existence of an unrestricted sequence implies that the NF sequence is such a sequence.

When no sequence exists without the t_0 restriction, then the "cost" of a sequence is the sum of the t_0 penalties incurred with the sequence. Thus, solution of the traveling-salesman problem with a C matrix that has as elements:

$$c_{ij} = \text{Max} \left\{ t_0 - n/V_j + m/V_i; 0 \right\} \quad (i \neq j) \quad (3.12)$$

$$c_{ii} = \infty$$

provides an optimum sequence. The C matrix thus defined is characterized by the conditions that $c_{ij_1} \leq c_{ij_2}$ if $j_1 < j_2$ and $c_{i_1j} \geq c_{i_2j}$ if $i_1 < i_2$, i.e., the penalties increase as one moves up or to the right in the matrix. The NF sequence is seen to constitute the slant (i.e., the elements directly above the main diagonal and the element in the last row and first column) of the C matrix. (If the slant contains all zeros, then the NF sequence is seen to be optimum with no t_0 restriction.)

If $c_{i, i+1} > 0$ and $c_{i+1, i+2} > 0$, then by application of equation (3.12), it can be shown that:

$$c_{i, i+1} + c_{i+1, i+2} = c_{i, i+2} + (t_0 - s_0/V_{i+1}) \quad (3.13)$$

Thus, in the most common case, Case A of Figure 3.2, where $s_o > V_i t_o$ for all i , the penalty in an element directly above the slant is greater than the sum of the two adjacent slant elements. (If one of the two slant elements is zero, then the conclusion simply follows from the monotonicity of the c_{ij} .) Similarly, any element above the slant exceeds the sum of the two elements directly below and to the left of it. Then, we may describe any sequence $(A_1, A_{i_1}, A_{i_2}, \dots, A_{i_{M-1}})$ by $(1, A_{i_1}, \dots, A_{j_2}, \dots, A_{j_m}, \dots)$ where $j_1 < j_2 < \dots < j_m$. For this sequence, $\sum c_{ij} = c_{1j_1} + c_{j_1j_2} + \dots + c_{j_{m-1}j_m}$, since $c_{ij} = 0$ below the main diagonal. We define $c_{ij}^* = c_{i, i+1} + c_{i+1, i+2} + \dots + c_{j-1, j}$ for the subsequence $(i, \dots, j)$ in the NF sequence. Then, it follows from (3.13) and the fact that $s_o > t_o V_i$ that $c_{ij}^* \leq c_{ij}$ for all i and j , so that the NF sequence is optimum in Case A.

It can similarly be shown by expansion of equation (3.12) that when there are positive entries below the main diagonal,

$$c_{ij} + c_{jk} \leq c_{ik} + c_{kj} \quad (3.14)$$

when $i < j < k$, so that the NF sequence can also be shown to be optimal in these cases.

As an indication of the improvement that results from NF-sequencing, the ten types of aircraft shown in Table 3.1 at Idlewild were considered to be sequenced by alternative rules of random sequencing and of NF-sequencing. For $t_o = 1$ minute and $s_o = 3$ miles, the advantage of NF sequencing is 11% when $m = 7$ miles and 16% when $m = 10$ miles. Decreasing s_o , t_o , or m would reduce the savings correspondingly.

The extremes of NF sequencing may be examined by considering that a large enough number of aircraft types exist so that the distribution of types may be approximated by a continuous one. Then, it is possible to get an infinitely long NF sequence without ever experiencing the t_o limitation if the parameters follow Case A; the landing rate is then approximately $\bar{V}/s_o$, the value attained without the t_o limitation, and the best that can be achieved with the given velocity distribution and gate separator. While this process provides the longest NF sequence, it is also of interest to examine the shortest NF sequence that has no t_o limitation. This is the one in which each landing of velocity V_1 is followed by a landing of velocity $V^*(V_1)$, denoted by " V^* -sequencing." Under this procedure, every landing but the last in a sequence is followed by a t_o interval, a situation that intuitively appears desirable.

We will investigate in detail only the most common case where $s_0 > bt_0$ (Case A of Figure 3.2). The V^* -sequencing rule is uniquely defined in Case A until a landing of velocity $V \geq c$ [where $V^*(c) = b$] occurs. Since $V^*(V) > V$ for all V , V^* -sequencing obviously cannot then continue. It is then necessary to select a new initial velocity and commence another V^* -sequence.

We define a " V^* -cycle of length Q " to be a sequence of Q landings initiated by a landing with velocity $V_1 \equiv V_1^*$, followed by one of velocity $V_2^* \equiv V^*(V_1)$ which, in turn, is followed by one of velocity $V_3^* \equiv V^*(V_2^*)$, and so on, up to V_Q^* , such that each landing is followed by one of the exact velocity to provide a landing interval of exactly t_0 after starting with a space separation of s_0 , and such that $c < V_Q^* \leq b$.

The values of V_k^* ($k = 2, \dots, Q$) are all functions of V_1 as follows:

$$V_2^* = \frac{n V_1}{t_0 V_1 + m} \quad (3.15)$$

$$V_3^* = \frac{n^2 V_1}{t_0 V_1 (m + n) + m^2}$$

$$V_4^* = \frac{n^3 V_1}{t_0 V_1 (n^2 + mn + m^2) + m^3}$$

and, in general,

$$V_{k+1}^* = \frac{n^k V_1}{t_o V_1 \left(\sum_{j=0}^{k-1} n^j m^{k-j-1} \right) + m^k} \quad (k = 1, 2, \dots, Q-1) \quad (3.16)$$

a generalization that can easily be proven by induction. The sum enclosed in brackets in the denominator can be shown, by division, to be $(n^k - m^k)/(n - m)$, so that

$$V_{k+1}^* = \frac{n^k V_1 s_o}{t_o n^k V_1 + m^k (s_o - t_o V_1)} \quad (3.17)$$

The requirement that $V_Q^* \leq b$ bounds k from above, and

$$Q(V_1) = \left\lceil \frac{\log_e \left(\frac{V_1}{b} \right) \left(\frac{s_o - b t_o}{s_o - t_o V_1} \right)}{\log_e \left(\frac{m}{n} \right)} + 1 \right\rceil \quad (3.18)$$

where the square brackets represent "the greatest integer in."

We now define u_k such that, if $V_1 = u_k$, then a V^* -cycle of length k can occur with $V_k^* = b$. Thus, $u_{k+1} < V_1 \leq u_k$ implies that a V^* -cycle of length k can occur. We note that $u_1 = b$. From equation (3.17), with $V_k^* = b$, u_k is found to be:

$$u_k = \frac{\left(\frac{m}{n}\right)^{k-1} b s_o}{\left[\left(\frac{m}{n}\right)^{k-1} - 1\right] b t_o + s_o} \quad (3.19)$$

Figure 3.9 illustrates Q as a function of V_1 for $m = 4, 10$ miles, $s_o = 3, 4$ miles, $t_o = 0.5, 1$ minute, and $b = 120, 150$ knots for $V_1 \geq 70$ knots. The function has discrete jumps at the u_k values. It is noted that, for small m and small b , the cycles are short. The longest cycle, of length 8, occurs when $m = 10$, $s_o = 3$, $t_o = 1$, $b = 150$, and $a = 70$. In general, Q (and thus the opportunity for V^* -sequencing) increases with b , t_o , and m , and decreases as a or s_o increase.

The time (T_Q) to complete a V^* -cycle of length Q initiated by a landing of velocity V_1 is $(Q - 1) t_o$ plus the idle time before the beginning of the next cycle (with a landing of velocity V_1'), and is given by:

$$T_Q = (Q - 1) t_o + n/V_1' - m/V_Q^*(V_1) \quad (3.20)$$

In this time, Q landings are performed, so that the landing rate, λ_Q , for the cycle is

$$\lambda_Q = Q/T_Q \quad (3.21)$$

It can easily be shown that it is desirable to maximize the length of a V^* -cycle, and therefore, to start it with a slow aircraft. There is no advantage, however, in reducing V_1 below u_Q (where $u_{Q+1} < a$) since

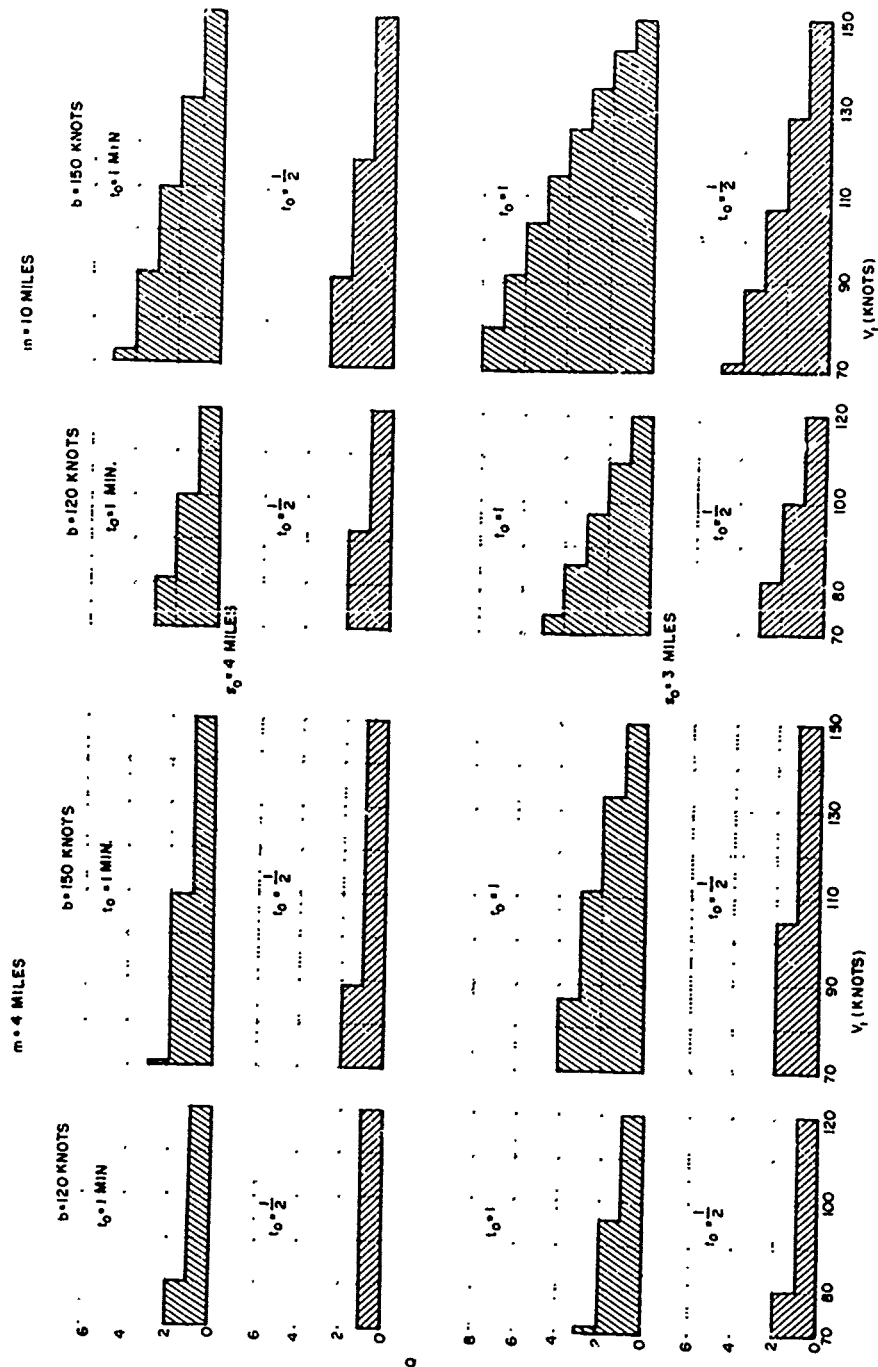


FIGURE 3.9
LENGTH OF THE V^2 -SEQUENCE (Q) AS A FUNCTION OF
 V_1, t_0, s_0, b & m

no additional length is thereby added to the cycle, and the general speed reduction increases the time for the cycle. Thus, an upper bound to the landing rate attainable under V^* -sequencing if $V_1' = V_1$ is that obtained when $V_1 = u_Q$ (and $V_Q^* = b$). The landing rate (λ_Q') can then be determined from equations (3.20) and (3.21). Table 3.3 lists this upper bound for one velocity distribution ($a = 90$, $b = 150$), and provides a comparison with λ , the landing capacity under random sequencing. It is noted that (except for the one case where $Q = 1$, so that " V^* -sequencing" is meaningless) the capacity is improved from 5% to 20% by V^* -sequencing.

To examine the effects of velocity variation, another rule on V_1 is examined. It is now assumed that $V_1 = V_1' = a$, so that $V_Q^* = V_Q^*(a)$. Thus, the length Q of each sequence is unchanged, but the average velocity of the sequence is reduced below the maximum value used in computing λ_Q' . The resulting landing rate under this assumption (λ_Q'') is also presented in Table 3.3 for comparison. It is seen here that in a majority of the cases $\lambda_Q'' < \lambda < \lambda_Q'$. Only in the last three cases, where $V_Q^*(a)$ is very close to b (and the two V^* -sequencing alternatives are very similar), is $\lambda_Q'' > \lambda$.

Table 3.3 also illustrates the landing rate with an infinite-length NF sequence, and this is seen to exceed random sequencing by less than 18%, and then only when m is large. Some improvement is found in all cases.

TABLE 3.3
LANDING CAPACITY UNDER SEQUENCING

m	s _o	t _o	Q	λ'_Q	λ'_Q	λ''_Q	λ_{NF}	V* (a)
4	3	.5	2	41.0	39.0	35.8	39.2	132.6
			3	43.2	37.7	36.8	39.2	135.7
	4	.5	1	25.0	29.4	22.5	29.4	90.0
			2	31.6	29.3	26.7	29.4	130.9
10	3	.5	3	42.0	35.9	35.7	39.2	129.7
			7	40.0	33.2	39.4	39.2	149.2
	4	.5	3	31.1	28.5	31.0	29.4	49.3
			4	29.1	27.4	28.9	29.4	149.3

a = 90 knots

b = 150 knots

The landing rates under the two V^* -sequencing rules and under random sequencing are depicted graphically in Figure 3.10 as a function of Q , the length of the sequence for the case. The longer sequences are seen to occur with the larger values of m . The improvement over random sequencing provided by optimum V^* -sequencing (λ'_Q) is indicated by the length of the solid line for each case; the dotted lines indicate the decrement of sequencing for λ''_Q . As expected, the greater improvement occurs with the longer sequences.

V^* -sequencing implies an infinite supply of aircraft waiting to be landed, a selection of V_1 by some process, which then defines the remaining $(Q-1)$ aircraft in the sequence. The selection of V_1 must meet the requirements of the distribution of arriving aircraft velocities, so that the selection of slow V_1 is more probable, since fast aircraft are included in every V^* sequence.

The sequencing concepts considered here are not presented as practical means of sequencing aircraft for landing. Rather, they are presented to indicate, by consideration of optimum situations, the potential improvement in landing capacity that might be achieved by sequencing. If such an improvement were large, then it would be desirable to study sequencing further; if small, even under the ideal conditions considered, then it might be adjudged relatively ineffective. It was noted above that the over-all improvement is not large, and that, with V^* -sequencing, even small variations from the optimum sequence

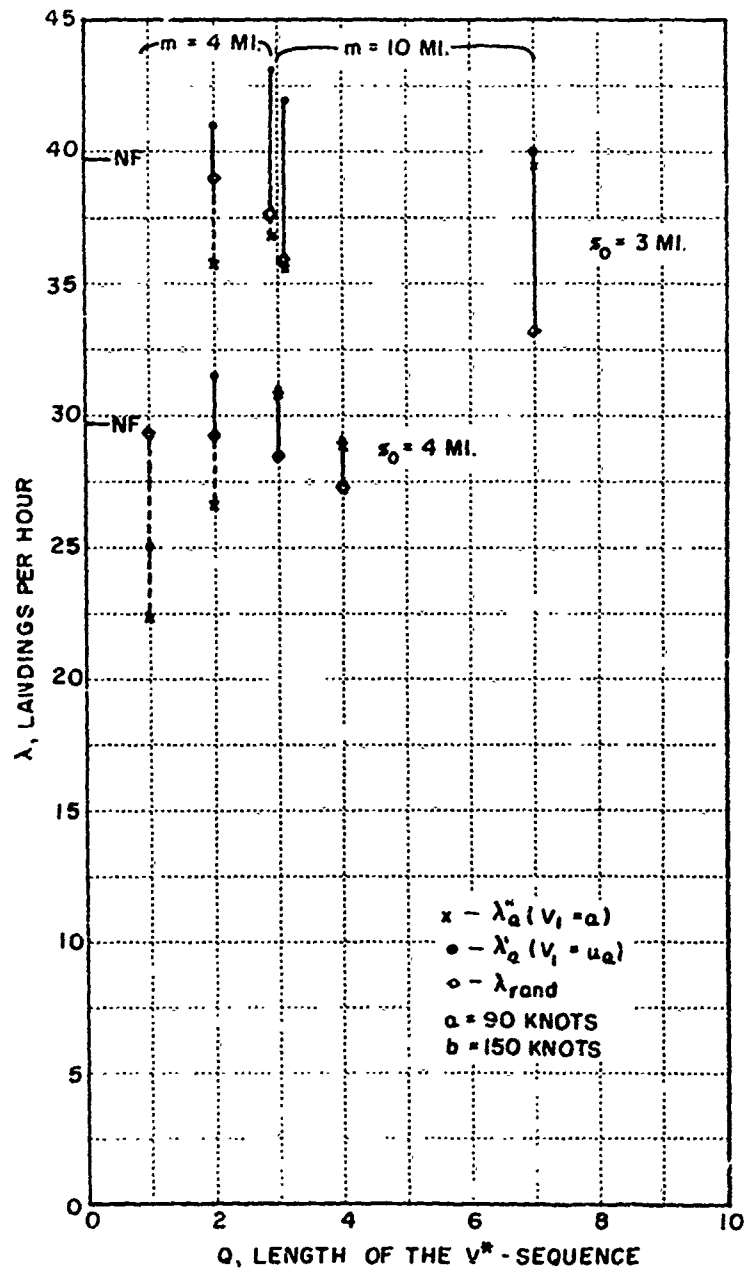


FIGURE 3.10
LANDING RATE UNDER SEVERAL SEQUENCING RULES

resulted in a degradation of landing capacity below that obtained with random sequencing (based on first-come-first-served). The failure of V^* -sequencing to provide significant improvement results from the fact that, although aircraft are dispatched to land with small separations by following each landing with one slightly faster, finally the fastest aircraft in the sequence must be followed by a slow one, introducing a long landing interval and thereby dissipating the advantage gained by the sequence of t_o -intervals. If this long interval could be used for take-offs, then V^* -sequencing could effectively improve operations rate; for improving landing rate, however, it appears relatively unpromising. NF-sequencing, which more typically represents the maximum improvement that might result from sequencing, was seen to provide an increase of less than 20%.

3.7 Queuing Considerations

Analytical investigation of landing delay has generally been based on application of the standard steady-state queuing theory models, such as those presented by Bowen and Pearcey [43] and Bell [45]. Berkowitz and Fritz [58] point out that such models are inadequate because they treat the equilibrium condition where arrival rate may not exceed service rate, whereas the important problems in ATC occur when arrival rate does exceed service rate. To correct this, they use the matrix of queue state transition probabilities to investigate the transient build-up of

delay, and find that, for Poisson arrivals and for fixed arrival and service rates, the average delay increases approximately linearly with time of arrival after the start of service.

Despite these valid objections to the use of the steady-state models for estimation of actual delay that might be experienced with a system, these models are valuable and effective tools for comparing different systems on the basis of relative delay, and for investigating queue characteristics as a function of system parameters.

3.7.1 Queue Relationships

In all investigations of landing queues, the distribution of arriving aircraft has been assumed to follow the Poisson law. This assumption has been based on observations taken at airports and reported by Bowen and Pearcey [43], Bell [45], and Berkowitz and Doering [55].

It has been shown by Kendall* [46, p. 155] that if arrivals to a queue are Poisson with average arrival rate α , and if the service time has any distribution with mean $1/\lambda$ and variance σ_s^2 , and if arrivals are served in order of their arrival, then the average waiting time in the queue is given by:

$$W_q = \frac{\rho}{2\lambda(1-\rho)} \left(1 + \sigma_s^2 \lambda^2 \right) \quad (3.22)$$

* Kendall credits Pollaczek [65] with the original development of this formula.

where $\rho = \alpha / \lambda < 1$. We introduce a quantity

$$k_s = 1 / \lambda^2 \sigma_s^2 \quad (3.23)$$

(the reciprocal of the coefficient of variation of the service-time distribution) where $k_s \equiv k$ if the service time has a k-Erlang distribution.

In terms of k_s , W_q is given by:

$$W_q = \frac{\rho}{2 \lambda (1 - \rho)} \cdot \frac{k_s + 1}{k_s} \quad (3.24)$$

It is seen from equation (3.23) that $k_s = 1$ when the service time has the exponential distribution ($\sigma_s = 1/\lambda$), and that $k_s = \infty$ when the service time is constant ($\sigma_s = 0$). It is seen from equation (3.22) that average delay is a minimum when service time is constant and the servicing operation is "well organized"; when service time is "random," and has the exponential distribution, the average delay is twice the minimum. With this intuitive characterization of the service process, the quantity k_s serves as an indication of the degree or "order" of the service process. Figure 3.11 illustrates mean queue delay in units of mean service time (i.e., $W_q \lambda$) as a function of runway utilization (ρ) for $k_s = 1, 2$, and ∞ . It is noted from equation (3.24) and Figure 3.11 that the average delay rapidly approaches the value for constant service time as k_s increases.

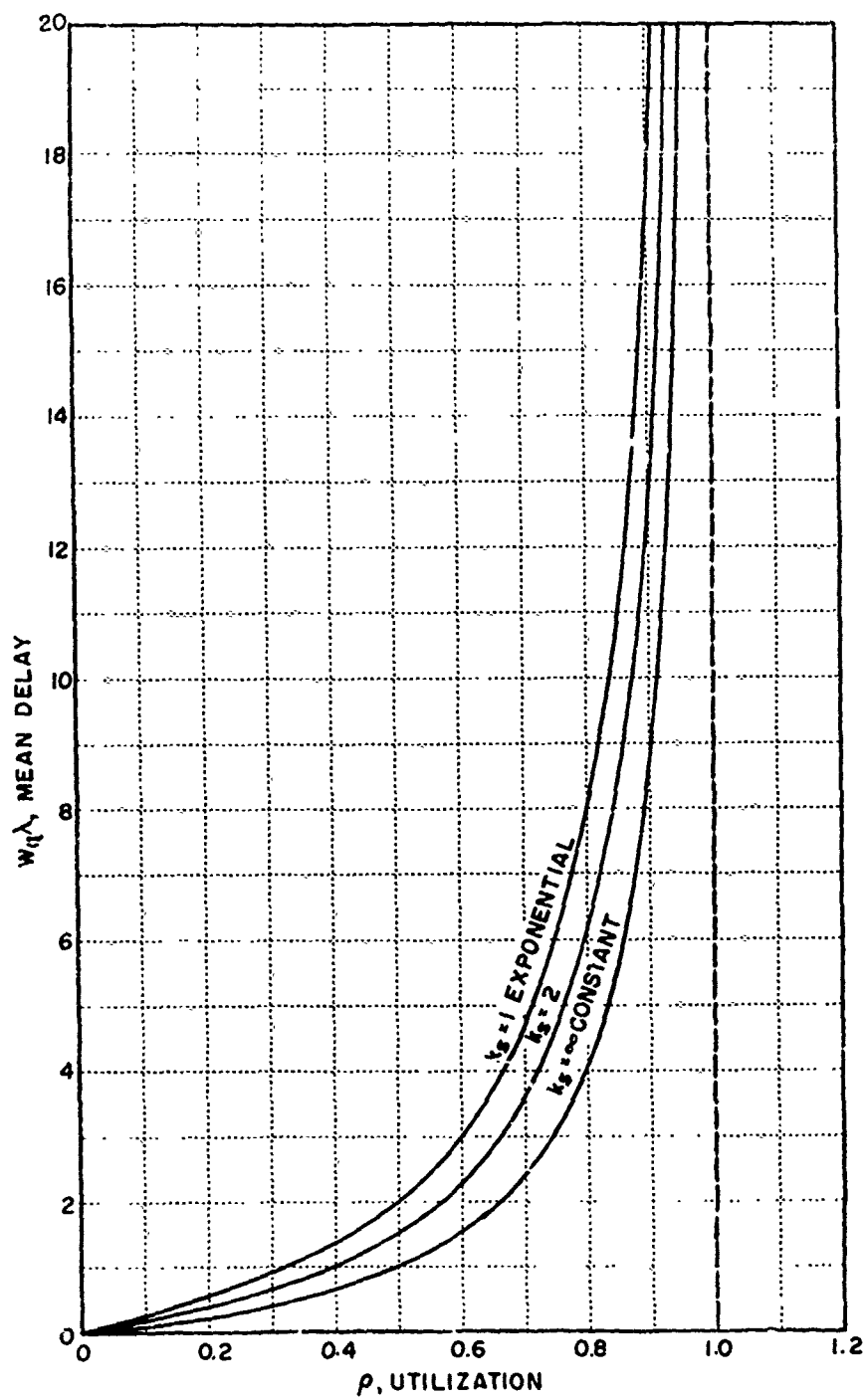


FIGURE 3.11
MEAN DELAY OF POISSON ARRIVALS AS A
FUNCTION OF RUNWAY UTILIZATION

The average queue length, L_q , is given by λW_q , or:

$$L_q = \frac{\bar{r}^2}{2(1-\rho)} \cdot \frac{(k_s + 1)}{k_s} \quad (3.25)$$

This relationship is plotted in Figure 3.12 as a function of ρ for $k_s = 1, 2$, and ∞ . Curves similar to those for λW_q are obtained, except that a sharper knee occurs.

These equations are derived on the basis of a standard queuing situation, in which a single server is occupied with a single customer for some service time while the others wait their turn to receive service. A runway does not operate in exactly that manner, since several aircraft may be descending to a landing simultaneously. It may be considered, however, that the runway is "serving" only the single aircraft closest to a landing, and service is completed at touchdown, so that the service time is then the time between landings. The delay W_q is then the average difference between an aircraft's potential landing time and the later one to which it may be displaced because of congestion. The mean queue length L_q then represents the average number of delayed aircraft waiting or landing but not being "served."

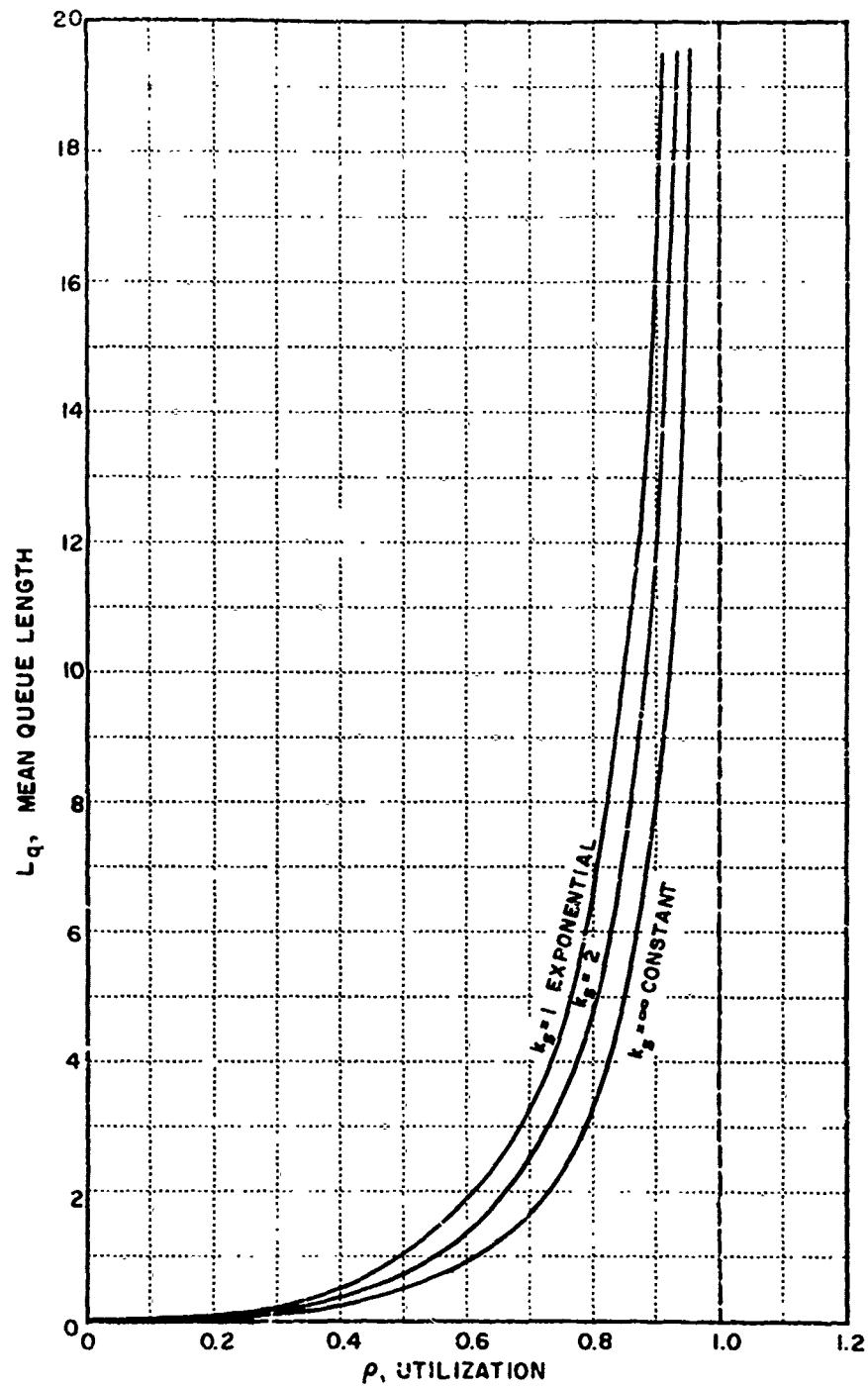


FIGURE 3.12
MEAN QUEUE LENGTH OF POISSON ARRIVALS
AS A FUNCTION OF RUNWAY UTILIZATION

3.7.2 Landing-Time Distribution

Since the assumption of Poisson arrivals seems consistent with the available data, it is necessary to determine only the mean and variance of the landing-time distribution to permit an examination of delay characteristics with equations (3.24) and (3.25) (or Figures 3.11 and 3.12). The mean landing time is given by equation (3.7). To compute the variance, the entire distribution of landing time (T_{AA}) must be derived.

We start with the assumption that V_1 and V_2 are independently, identically, and uniformly distributed with joint density function as given by equation (3.6). We change to time variables by letting:

$$T_1 = m/V_1 \text{ and } T_2 = n/V_2 \quad (3.26)$$

As shown in Appendix B, T_1 and T_2 are found to have a joint density function:

$$g(t_1, t_2) = \frac{mn}{(b-a)^2 t_1^2 t_2^2} \quad \begin{array}{l} m/b \leq t_1 \leq m/a \\ n/b \leq t_2 \leq n/a \\ (b \neq a) \end{array} \quad (3.27)$$

$$= 0 \quad \text{elsewhere}$$

Then, for $t > t_0$, the landing-time distribution is given by:

$$S_{AA}(t) = \Pr \left\{ T_{AA} \geq t \right\} = \Pr \left\{ T_2 \geq T_1 + t \right\} \quad (3.28)$$

while, for $t = t_0$, the finite probability that the landing interval is t_0 is given by:

$$\Pr \left\{ T_{AA} = t_0 \right\} = 1 - \Pr \left\{ T_2 > T_1 + t_0 \right\} \quad (3.29)$$

The resulting distribution function is developed in Appendix B and is given by:

$$S_{AA} \left\{ t \mid -\infty \leq t \leq t'_0 \right\} = 1 \quad (3.30)$$

$$S_{AA} \left\{ t \mid t'_0 < t \leq s_0/b \right\} =$$

$$\frac{1}{t^2 (b-a)^2} \left\{ t (mb - an) + t^2 (b^2 + a^2 - ab) + mn \log_e \frac{(m + at)(n - bt)}{mn} \right\}$$

$$S_{AA} \left\{ t \mid s_0/b \leq t \leq s_0/2 \right\} = \frac{n - at}{t(b-a)} + \frac{mn}{t^2 (b-a)^2} \log_e \frac{m + at}{m + bt}$$

$$S_{AA} \left\{ t \mid \frac{s_0}{a} \leq t \leq \frac{n}{a} - \frac{m}{b} \right\} = \frac{1}{t^2 (b-a)^2} \left\{ tb(n-at) - at m \right. \\ \left. - mn \log_e [(n - at)(m + bt) / mn] \right\}$$

$$S_{AA} \left\{ t \mid \frac{n}{a} - \frac{m}{b} \leq t \leq \infty \right\} = 0$$

where:

$$t'_0 = \text{Max} \left\{ t_0 ; \frac{n}{b} - \frac{m}{a} \right\}$$

To investigate the sensitivity of the landing-time distribution to the assumption of a uniform velocity distribution, the alternate discrete-velocity assumption is examined: all aircraft of type i land with identical velocity V_i and appear at random with probability p_i . The discrete landing-time distribution is then given by:

$$\Pr \left\{ T_{AA} = T_{ij} \right\} = p(V_i) p(V_j) \quad (i, j = 1, 2, \dots, n) \quad (3.31)$$

where T_{ij} is given by equation (3.2).

Using the statistics for LaGuardia and Idlewild Airports of Table 3.1, the discrete distributions are plotted in Figure 3.13 for typical values of the system parameters. These are compared with the landing-time distributions based on equivalent uniform velocity distributions having the same mean and variance. The correspondence is seen to be reasonably satisfactory, so that it appears that the entire distribution as well as the mean of the landing time is reasonably insensitive to the form of the velocity distribution. Further evidence supporting this possibility is presented in Appendix E.

Using the distribution function of equation (3.30), the variance was computed by numerical integration of:

$$\sigma_s^2 = \int t^2 dS_{AA}(t) - (\tau_{AA})^2 \quad (3.32)$$

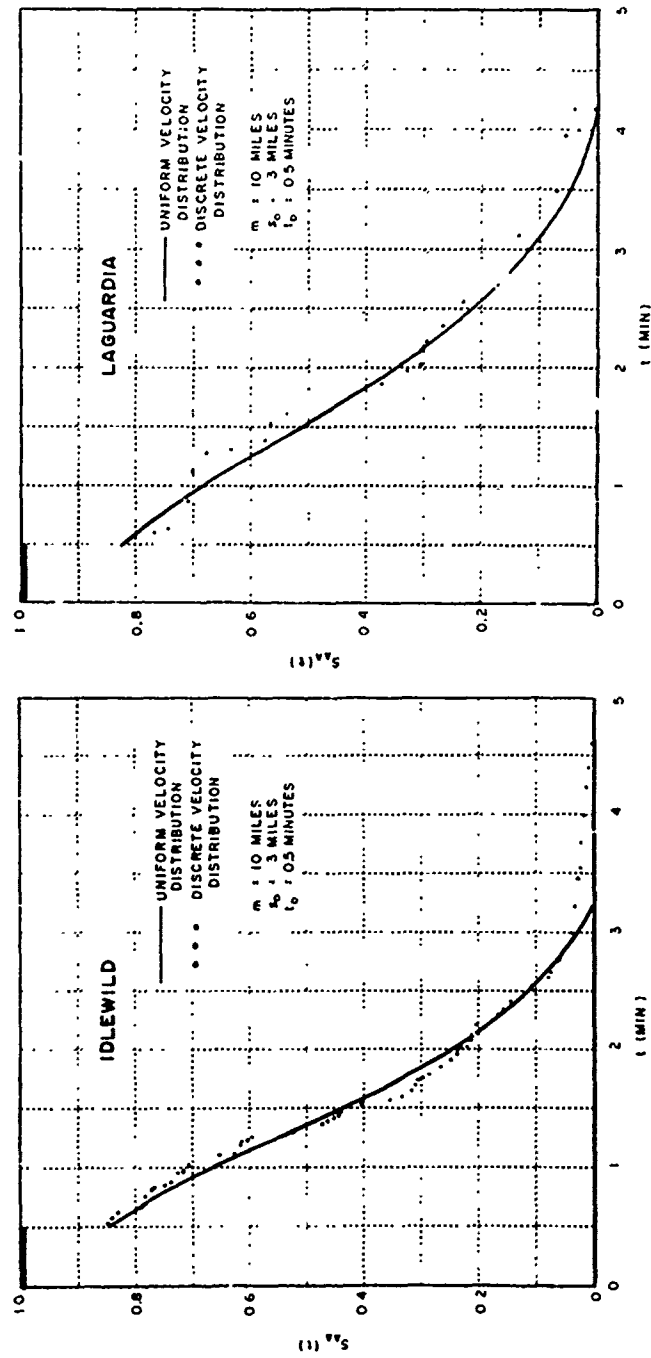


FIGURE 3.13
LANDING TIME DISTRIBUTIONS
COMPARISON OF DISCRETE & UNIFORM VELOCITY DISTRIBUTIONS

3.7.3 Parametric Investigation of Queue Characteristics

As can be seen from equations (3.24) and (3.25), all the information regarding queue characteristics for a fixed arrival rate is contained in k_s and in λ . The parametric effects on k_s represent an interaction of the effects on λ and on σ_s^2 .

Figure 3.14 illustrates the effects of m , s_o , and t_o on λ , σ_s^2 , and k_s for the uniform velocity distribution characteristic of Idlewild ($\bar{V} = 130$ knots, $R = 48$ knots). It is seen that σ_s^2 increases with s_o and m (since longer landing-time intervals occur) and decreases as t_o increases (since the discrete jump at t_o which represents a constant landing interval is larger). Since k_s is more sensitive to σ_s^2 than to λ^2 , it varies with t_o and inversely as m . Gate separation (s_o) interacts with the other parameters, as can be noted most strikingly on the first graph, where $m = 7$ miles; it is seen k_s increases with s_o for $t_o = 0.5$, has a minimum point at $s_o = 3.3$ for $t_o = 1.0$, and varies inversely as s_o when $t_o = 1.5$.

The effects of $\bar{V}$ and R on k_s are indicated on Figure 3.15. It is noted that increasing average speed or reducing speed variation increases k_s , primarily by reducing the long time intervals.

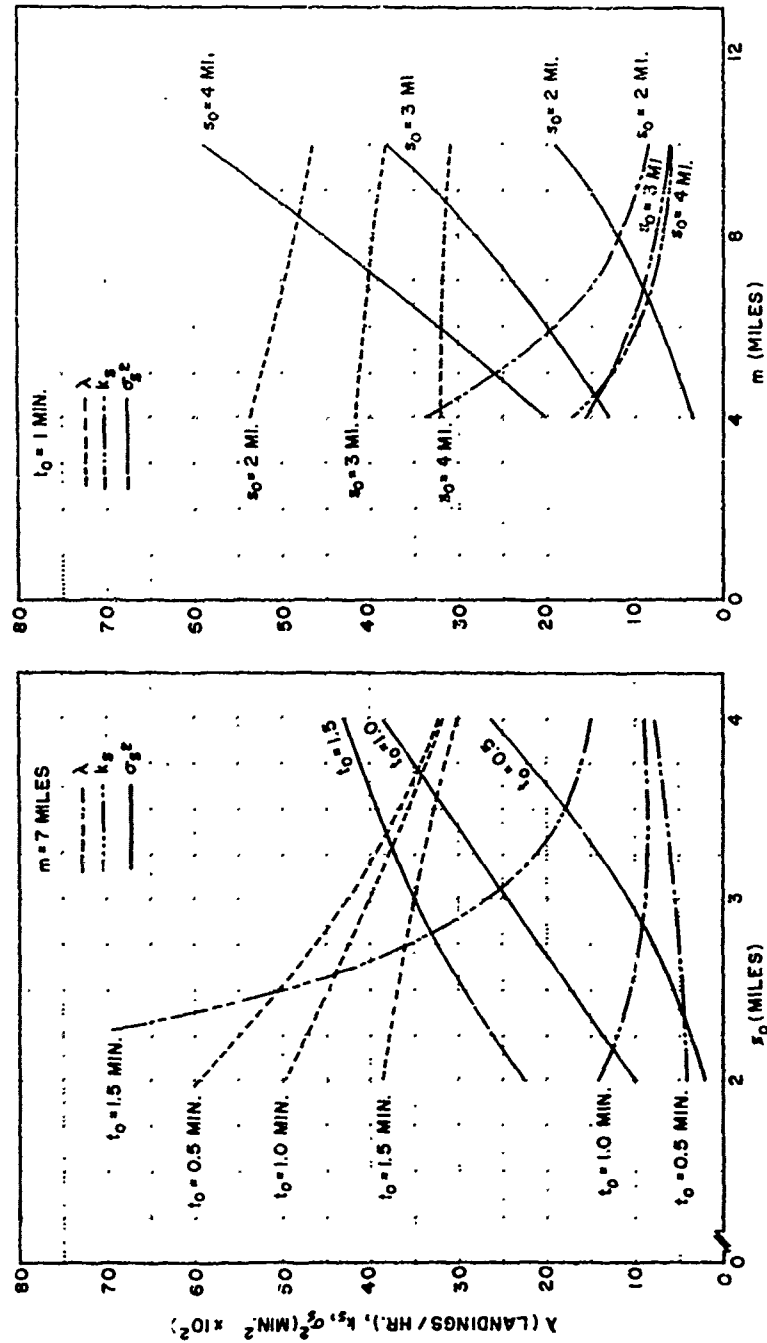


FIGURE 3.14
EFFECT OF SYSTEM PARAMETERS ON λ , σ_s^2 AND k_s
AT IDLEWILD

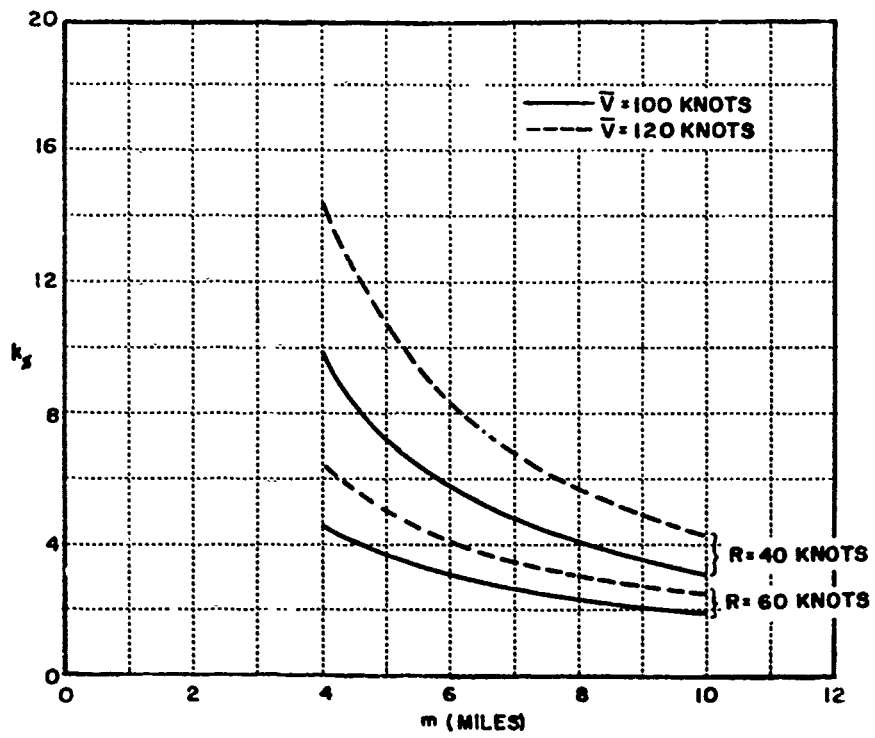


FIGURE 3.15
EFFECT OF VELOCITY DISTRIBUTION ON k_g

Knowing the value of k_s for a set of parameters, equations (3.24) and (3.25) can be used to estimate the average delay W_q and the average queue length L_q . These results are indicated on Figure 3.16 for the traffic mix at Idlewild based on an assumed arrival rate of 20 aircraft per hour. Since the value for ρ in the cases shown is well below 1.0 (and thus on the flat portion of the $L_q - \rho$ curve), L_q is seen to vary less with parametric changes (and consequent changes in λ) than does W_q . The queue measures generally reflect the variation in landing capacity. Delay incurred with greater arrival rates would be more sensitive to parametric changes characteristic of larger values of ρ .

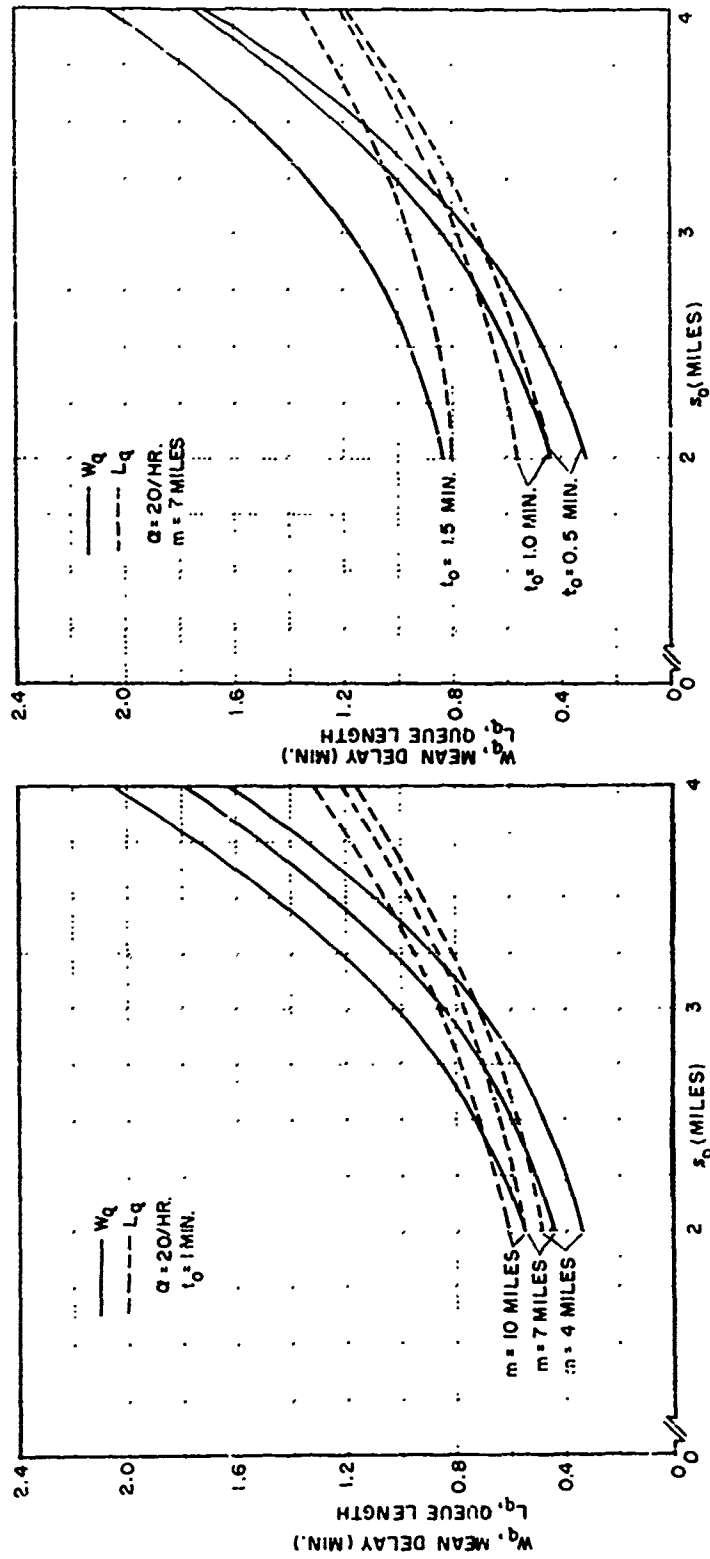


FIGURE 3.16
MEAN DELAY (W_q) AND QUEUE LENGTH (L_q) AT IDLE WILD
WHEN ARRIVAL RATE IS 20 PER HOUR

CHAPTER 4

ANALYSIS OF THE OPERATIONS CAPACITY
OF A RUNWAY USED FOR LANDINGS AND TAKE-OFFS

The previous chapter treats the problem of a runway used only for landings. It is more common for a runway to be used for both landings and take-offs. The capacity in such a situation is examined in this chapter by the formulation of an analytical model for estimating runway operations^{*} rate. The effects of the various system parameters on operations capacity is examined, and means of improving performance are investigated.

4.1 Previous Investigations of Mixed Operations

In contrast to the relatively large number of investigators who have studied the landing problem, only one other analytical effort dealing with mixed operations has been found. Galliher recently reported [67] on some work currently in progress at Airborne Instruments Laboratory under FAA contract. He is studying queuing models in which a single server (the runway) serves two arrival queues (landings and take-offs). He reports that he is considering the following alternative disciplines:

* An "operation" is a landing or a take-off.

1. First-come/First-served discipline queuing model
2. Head-of-the-line queuing discipline model
3. Pre-emptive priority discipline queuing model
4. Simulation model which permits dual servicing on final approach, i. e., arriving aircraft too widely spaced on final may have a take-off inserted betwixt arriving aircraft
5. Nonstationary analytic model with First-come/First-served queuing discipline, either random or nonrandom input, and featuring in addition nonstationary service time, if desired. [67, p. 4.]

In [67], Galliher presents the queuing formulas (developed elsewhere in the literature) for the first three models, but they all suffer from the disadvantage which led him to the simulation formulation of the fourth model, namely, that they use the parameters of the distributions of the landing and take-off service times independently, and do not consider the possibility of interposing take-offs between the landings. The formulas consequently indicate longer delays than are experienced, since they do not recognize that the single server can be serving both a landing and a take-off simultaneously. In private conversations, Galliher has indicated that he is also investigating an analytical formulation of the fourth model.

A relatively minor portion of the Franklin Institute - CAA Technical Development Center simulation effort has been devoted to mixed operations. Anderson and Vickers [57, pp. 24-6] used elementary "graphical" simulation of a situation characterized by constant take-off and constant landing intervals, and concluded that alternating operations

yields a higher operations rate than running sequences of the same operation (based on assumptions of identical aircraft, 3 miles arrival separation, 1 minute departure separation, and take-offs permitted only if an arrival is beyond 2 miles from the runway).

Berkowitz and Fritz [58] studied mixed operations in their real-time ("dynamic") simulation investigations. They report:

It was found that the average separation time between two successive landings was about 120 seconds. When a take-off intervened, the separation between the landings was about 140 seconds. The average runway separation between two successive take-offs was 65 seconds. On the basis of these figures, it seems obvious that the most efficient method for intermixing landings and take-offs when there is a backlog of both is to alternate them; a take-off requires 65 seconds when following another take-off but adds only 20 seconds when interspersed between two landings. This alternating procedure resulted in a single-runway capacity of 50 to 55 operations per hour. [58, p. 50.]

It is not clear from their discussion whether the 140 seconds resulted from the interposition of the take-off, or whether it is merely that those landing intervals into which a take-off could be interposed were naturally longer. The conclusion does appear, however, to be intuitively reasonable.

4.2 The Basic Analytical Model of Runway Operations Capacity

The model developed here for operations capacity is an extension of the model for landing capacity. In mixing operations on a runway, the practice at most airports is to perform the landings as close together as gate (s_o) and runway (t_o) separation standards permit. Controllers attempt to interpose departures between the landings whenever possible, but give the landing aircraft priority. In general, the rules require that an arriving aircraft be beyond some minimum range (r_o) from the runway in order to interpose a take-off before it; otherwise, the departing aircraft must be held until the landing aircraft has cleared the runway. If too long a take-off queue develops, then the landings may be temporarily halted while the queue is dispatched by a run of take-offs, with a minimum time interval (t_{oD}) maintained between successive departures.

A model is formulated expressing these restrictions, and providing an estimate of runway operations rate. This estimate probably represents an upper limit of performance since controllers tend to increase safety by erring conservatively in observing these standards.

4.2.1 Assumptions

In this model, the following assumptions, represented schematically in Figure 4.1, are made:

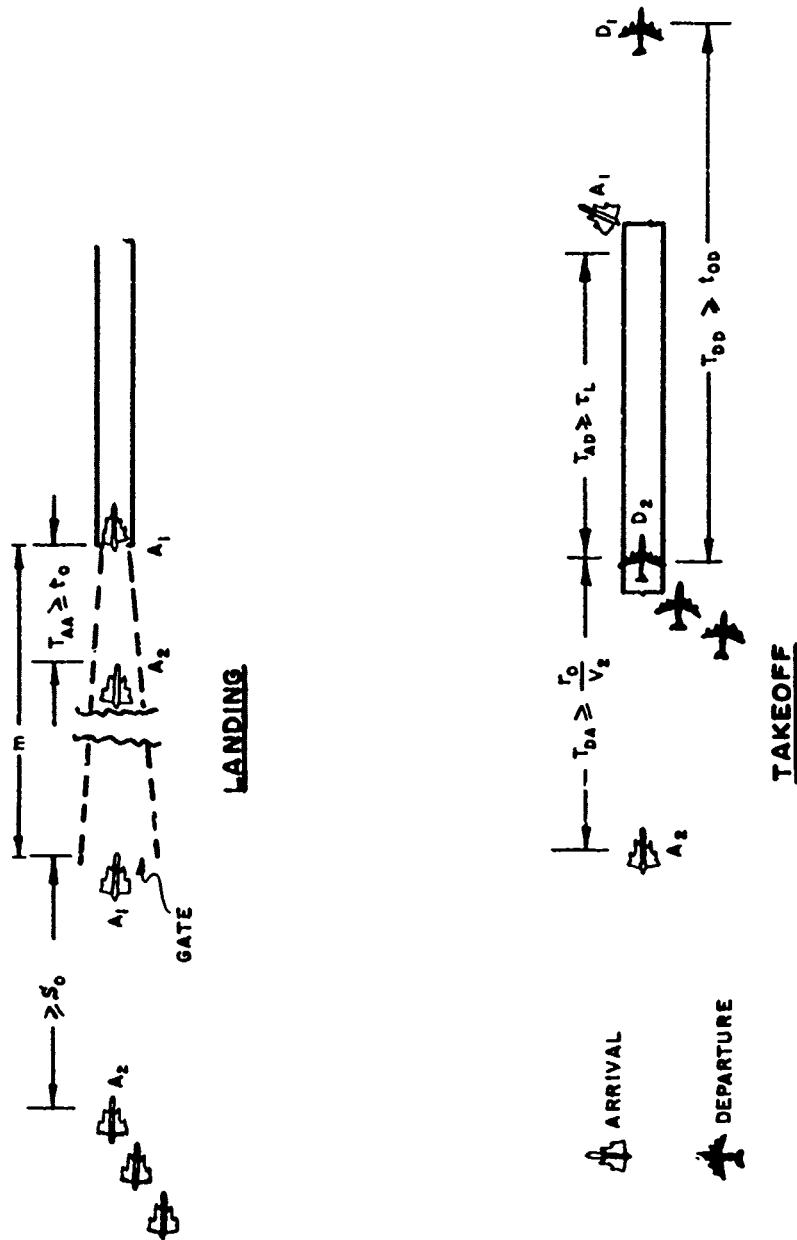


FIGURE 4.1
SCHEMATIC REPRESENTATION OF ASSUMPTIONS

1. Landing aircraft arrive at the gate independently and in random sequence.
2. Aircraft land in the order in which they arrive at the entry gate (i.e., first-come-first-served, and no aircraft may be passed after it has crossed the gate).
3. Landing aircraft must maintain a minimum distance separation (s_o) at the gate and a minimum time separation (t_o) at the runway.
4. Landing aircraft maintain constant velocity from the time they enter the gate until they reach the approach end of the runway.
5. Successive departing aircraft must maintain a minimum time separation (t_{oD}).
6. A departing aircraft may be dispatched only if a preceding landing aircraft has cleared the runway (a time τ_L after landing), and a following landing aircraft is further than some minimum distance (r_o) from the approach end of the runway.
7. A departing aircraft can take off (if so ordered) as soon as a preceding landing aircraft has cleared the runway.
8. Aircraft are available for either landing or take-off as frequently as separation standards permit (i.e., the capacity situation is examined.)

4.2.2 Notation

We summarize here the notation which will be used in this model in addition to that introduced in Section 3.2.2. The letters "A" and "D" are used here to signify arrivals and departures, respectively. The important additional notation is defined below:

r_o = minimum distance (from the approach end of the runway) of a landing aircraft to permit interposing a take-off before the landing

t_{oD} = minimum time separation required between successive take-offs

τ_L = runway-occupancy time of a landing aircraft, assumed constant

k_R = τ_L / t_o

T_{DD} = time interval between successive take-offs; when take-offs are not affected by landings, $T_{DD} = t_{oD}$

T_{AD} = time interval between the touchdown of a landing and the beginning of the roll of a following take-off; by assumption (7), $T_{AD} = \tau_L$

T_{DA} = a chance variable with mean value τ_{DA} representing the time interval between the beginning of a take-off roll and the touchdown of a following landing

τ_I = conditional expected value of the landing interval given that a departure was interposed between the two landings

μ = operations capacity (landings and take-offs per hour)

N_A = total number of landings in a sequence of N operations

N_D = total number of take-offs in a sequence of N operations

ω' = $\text{Min} \left\{ \begin{array}{l} \text{number of runs of take-offs;} \\ \text{number of runs of} \\ \text{landings} \end{array} \right\}$ in a sequence of N operations

ω = $2 \omega' / N$ = reciprocal of the average length of a run in a sequence of N operations ($0 < \omega \leq 1$)

ν_D = expected number of take-offs that can be interposed between a pair of landings without affecting the landing operation

q_k = probability of interposing at least k take-offs between a pair of landings

$q_{\bar{P}}$ = probability of interposing a take-off in a landing interval if none had been interposed in the previous interval

q_P = probability of interposing a take-off in a landing interval if one or more had been interposed in the previous interval

4.2.3 Time for a Sequence of N Operations

The time required to perform any sequence of operations depends, in general, on the structure of the sequence. Consider a particular sequence S of N operations consisting of N_A landings and N_D take-offs ($N = N_A + N_D$), with $S = \{a_0, a_1, \dots, a_N\}$

where $a_i = D$ if the i^{th} operation is a take-off
 $= A$ if the i^{th} operation is a landing

Since the time for the sequence is based on the inter-operation times, we must specify a_0 , the last operation prior to the sequence. We specify $a_0 = a_N$. Without loss of generality, let $a_0 = a_N = A$. A run of length L is a sub-sequence of L identical elements. A cycle C_j in the sequence S is a run of D 's of length h_j followed by a run of A 's of length k_j ($1 \leq h_j \leq N_D$; $1 \leq k_j \leq N_A$). The sequence S then consists of an initial sub-sequence, B_0 , of A 's ($a_0 = A$ followed by a run of k_0 A 's), ($0 \leq k_0 \leq N_A$), followed by a succession of ω' cycles, i.e., $S = \{B_0, C_1, C_2, \dots, C_{\omega'}\}$. The expectation of the time to complete a cycle, τ_{C_j} , is given by:

$$\begin{aligned} \tau_{C_j} &= \tau_{AD} + \tau_{DD} (h_j - 1) + \tau_{DA} + \tau_{AA} (k_j - 1) \\ &= \tau_L + t_{oD} (h_j - 1) + \tau_{DA} + \tau_{AA} (k_j - 1) \end{aligned} \quad (4.1)$$

The average time to complete the sub-sequence B_0 is given by:

$$\tau_{B_0} = k_0 \tau_{AA} \quad (4.2)$$

The average time to complete the sequence S, τ_N , is thus given by:

$$\begin{aligned} \tau_N &= \tau_{AA} \left(k_0 + \sum_{j=1}^{\omega'} k_j \right) + t_{oD} \left(\sum_{j=1}^{\omega'} h_j \right) + \omega' (\tau_{DA} + \tau_L - t_{oD} - \tau_{AA}) \quad (4.3) \\ &= \tau_{AA} N_A + t_{oD} N_D + \omega' (\tau_{DA} + \tau_L - t_{oD} - \tau_{AA}) \end{aligned}$$

and letting $\omega = 2\omega'/N$ (so that ω is the reciprocal of the average length of a run, $0 < \omega \leq 1$) the average time per operation, τ_M is:

$$\tau_M = \frac{\tau_N}{N} = \tau_{AA} \frac{N_A}{N} + t_{oD} \frac{N_D}{N} + \frac{\omega}{2} (\tau_{DA} + \tau_L - t_{oD} - \tau_{AA}) \quad (4.4)$$

When $N_A = N_D = N/2$, which must be the case over the long period, (4.4) becomes:

$$2 \tau_M = (\tau_{AA} + t_{oD}) + \omega (\tau_{DA} + \tau_L - t_{oD} - \tau_{AA}) \quad (4.5)$$

We note that only ω is available as a variable of choice if we desire to minimize the total time for the sequence. Thus, if

$$\tau_{DA} + \tau_L > t_{oD} + \tau_{AA}, \quad (4.6)$$

ω should be made as small as possible (approaching zero) by alternating long runs of landings with long runs of take-offs. If inequality (4.6) is reversed, then it is desired to make ω unity and to alternate operations. If both inequalities hold, the choice is immaterial.

These results show that when the "switchover time," $\tau_L + \tau_{DA}$, is smaller than the "long-run time," $\tau_{AA} + t_{oD}$, then operations should be alternated as often as possible.* The actual length of runs in practice will, of course, depend on the availability of aircraft for service.

4.2.4 Interposition of Take-Offs

4.2.4.1 Operations Rate with Interposition

During runs of landings, the time interval between successive landings is often sufficiently long to permit the interposition of one or more departures, thus reducing the number of take-offs that must be run as a group. If the average interposition rate is ω_D , then only $[N_D - \omega_D (N_A - \omega')]$ of the N_D departures consume any time, and the time for the sequence of N_A landings and N_D take-offs becomes:

* This result is a special case of results already developed in study of single-stage production, where the process here treats two "products," arrivals and departures.

(4.7)

$$\tau_N = N_A \tau_{AA} + (N_D - \nu_D N_A) t_{oD} + \omega'(\tau_L + \tau_{DA} - t_{oD} [1 - \nu_D]) - \tau_{AA}$$

if $\nu_D < N_D/N_A$, and

$$\tau_N = N_A \tau_{AA} \quad (4.8)$$

if $\nu_D \geq N_D/N_A$, in which case all take-offs can be accommodated without ever disrupting the landings.* This results in a revision of the rule of equation (4.6) to encourage longer runs to provide more opportunity for interposition.

If $N_D = N_A = N/2$, then the operations rate** μ is N/τ_N , and is given by:

(4.9)

$$\mu = \frac{2}{(1 - \omega) \{ \tau_{AA} + t_{oD} (1 - \nu_D) \} + \omega (\tau_L + \tau_{DA})} \quad \text{if } \nu_D < 1$$

$$\mu = 2\lambda = 2/\tau_{AA} \quad \text{if } \nu_D \geq 1$$

*

It should be noted here that, with interposition,
 $\omega' \leq \text{Min} \{ N_A, N_D - \nu_D N_A \}$.

**

While an operations rate $\mu(N_D, N_A)$ can be computed for any combination (N_D, N_A) , the one we use to characterize a runway is the steady-state one based on an equal number of landings and take-offs.

The common operational situation is characterized by a long run of landings (albeit with interposition of departures which do not affect the landing schedule nor interrupt the run) followed by a long run of the waiting take-offs. This is represented by small values of ω , so we treat in the basic model only the limiting case, $\omega = 0$.

4.2.4.2 Interposition Rate

It is now necessary to determine ν_D to arrive at the operations rate. Figure 4.2 depicts longitudinal distance from the approach end of the runway as a function of time for the successive aircraft using the runway. It can be seen from Figure 4.2 and assumptions (5) and (6) that, in order to interpose a take-off (D_1) in the interval between the landing of A_1 and A_2 , the following two requirements must be satisfied:

1. r_o requirement : The preceding landing aircraft (A_1) must be off the runway, and the succeeding landing aircraft (A_2) must be beyond r_o when the departure (D_1) is ready to roll at a time τ_L after A_1 touches down.
2. t_{oD} requirement: a time interval of at least t_{oD} must have elapsed since the last take-off (D_0).

The r_o requirement assures that the landing interval is long enough to permit an interposed take-off; the t_{oD} requirement relates the current situation to the last previous interposition.

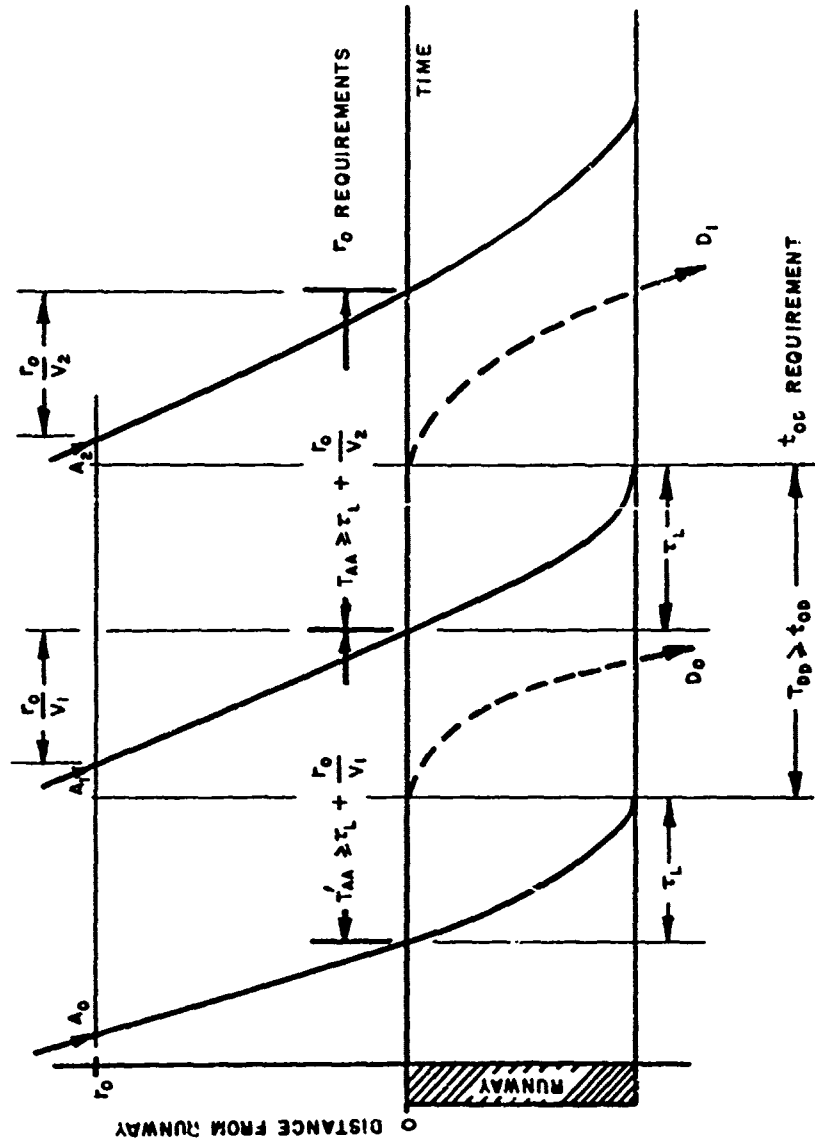


FIGURE 4.2
 t_{00} AND τ_0 REQUIREMENTS

The t_{OD} requirement enters only if there was an interposition in the previous interval (between A_0 and A_1) and we assume that an interposition in the second previous interval (before A_0) would not be inhibiting. For an interposition to have any effect two intervals later, it would be necessary that t_{OD} exceed $(r_o/b + t_o + \tau_L)$, the minimum possible time between two successive interpositions that are separated by two landings. Since no values of t_{OD} larger than two minutes are considered, this situation is not likely to occur.

4.2.4.2.1 Interposition Probabilities

To determine the probability of interposing a take-off in a landing interval, we define the following probabilities:

q_1 = probability that at least one interposition occurs in an interval

$q_{\bar{p}}$ = conditional probability that at least one interposition occurs in an interval, given that there was no interposition in the previous interval (i.e., the r_o requirement is met)

q_p = conditional probability that at least one interposition occurs in an interval, given that there was interposition in the previous interval (i.e., both the r_o and t_{OD} requirements are met); thus, $q_p \leq q_{\bar{p}}$

A stochastic process can be defined on the successive landing intervals with states "interposition occurs (I)" and "interposition does not occur ($\bar{I}$)". This process is Markovian, since the state depends only on the previous interval, and has the transition matrix:

	I	$\bar{I}$
I	q_P	$1 - q_P$
$\bar{I}$	$q_{\bar{P}}$	$1 - q_{\bar{P}}$

Each state is recurrent (since the chain is finite) and aperiodic (even if $q_P = 0$ since $I \bar{I} I$, $I \bar{I} \bar{I} I$, etc., are possible even if $I I$ is not), and therefore ergodic. It then follows, as shown by Feller [68, p. 325], that a limiting probability q_1 exists, and is given by:

$$q_1 = q_{\bar{P}} (1 - q_1) + q_P q_1 \quad (4.10)$$

Solving (4.10) for q_1 gives:

$$q_1 = q_{\bar{P}} / (1 + q_{\bar{P}} - q_P), \quad (4.11)$$

leaving only $q_{\bar{P}}$ and q_P to be determined.

We see that (4.11) has the properties intuitively expected. If the t_{OD} requirement is always satisfied when the r_O criterion is satisfied, then $q_P = q_{\bar{P}}$, and $q_1 = q_{\bar{P}}$. If $q_P = 0$, i.e., interposition is never possible in successive intervals, then $q_1 \longrightarrow 0.5$ if $q_{\bar{P}}$ is large, and $q_1 \longrightarrow q_{\bar{P}}$ if $q_{\bar{P}}$ is small and the situation calling for successive interpositions rarely occurs.

As can be noted in Figure 4.2, the r_o requirement in the $A_1 - A_2$ landing interval may be stated as

$$T_{AA} > \tau_L + r_o/V_2 \quad (4.12)$$

Ordinarily, to permit an interposition, T_{AA} must be larger than t_o ,

$$T_{AA} = n/V_2 - m/V_1 > t_o, \quad (4.13)$$

so that $q_{\overline{P}}$ is given by:

$$q_{\overline{P}} = \Pr \left\{ (n - r_o) / V_2 - m/V_1 > \tau_L \right\} \quad (4.14)$$

It is often possible to interpose more than one take-off between a pair of landings. If more than one take-off can be interposed, the t_{oD} requirement is automatically satisfied, so that the probability of interposing k take-offs in a landing interval is given by:

$$q_k = \Pr \left\{ \frac{n - r_o}{V_2} - \frac{m}{V_1} > \tau_L + (k-1) t_{oD} \right\} \quad (k = 2, \dots, K) \quad (4.15)$$

where K is the maximum possible number of interpositions dictated by the maximum landing interval, and is given by:

$$K = \left[(1/t_{oD}) \left\{ (n-r_o)/a - m/b - \tau_L \right\} + 1 \right]$$

where the square brackets represent "the greatest integer in."

If we denote the distribution function of equation (3.30) by $S_{AA}(t \mid m, n, t_o)$, then we note that:

$$q_{\overline{P}} = S_{AA}(\tau_L \mid m, n-r_o, 0) \quad (4.16)$$

$$q_k = S_{AA}(\tau_L + (k-1)t_{oD} \mid m, n-r_o, 0) \quad (k = 2, \dots, K) \quad (4.17)$$

The t_{oD} requirement applies only when an interposition had occurred in the previous interval. The requirement may be stated as:

$$T_{D_0 A_1} + T_{A_1 A_2} - r_o/V_2 \geq t_{oD} \quad (4.18)$$

where $T_{D_0 A_1}$ represents the time between the beginning of the last previous departure (D_0) and the touchdown of the following landing (A_1). The requirement of (4.18) is necessary to assure that the (A_1, A_2) interval is long enough so that the interposed departure may begin before A_2 reaches r_o , and that a time t_{oD} has elapsed since the previous departure.

Estimation of the probability of satisfying (4.18) requires some information on the value (or distribution) of T_{DA} . It is assumed for this basic model that the last previous interposition occurred as early as

possible in the previous interval, i. e., a time τ_L after A_0 lands.*

With this assumption,

$$T_{D_0 A_1} = T_{A_0 A_1} - \tau_L$$

and the t_{oD} requirement becomes:

$$T'_{AA} + T_{AA} - r_o/V_2 - \tau_L > t_{oD} \quad (4.19)$$

where T'_{AA} is the landing time separation in the previous (A_0, A_1) interval.

Exact evaluation of the probability of satisfying equation (4.19) requires a cumbersome triple integration over the three velocities involved. This is circumvented by replacing T'_{AA} by τ_I , its conditional expected value given that the r_o requirement has been satisfied. (The expression for τ_I is derived in Appendix C.) With this substitution, the t_{oD} requirement becomes:

* This assumption minimizes the t_{oD} restriction, and thus provides an upper bound on q_P . It is perfectly satisfied when there had been no interposition in the second previous interval, since the departure in the previous interval (D_0) would have had no reason to wait longer than τ_L after the landing of A_0 ; such a situation is probable when t_{oD} is large. If t_{oD} is small, then the t_{oD} requirement is easily met, interposition is controlled by the r_o requirement, and the assumption is appropriate; this is always the case if $t_{oD} \leq \tau_L + r_o/b$. Section 4.3.4 contains the development for an alternative assumption that the interposition in the previous interval occurs as late as possible, thereby providing a lower bound on q_P .

$$T_{AA} - \frac{r_o}{V_2} = (n - r_o) / V_2 - m / V_1 > \tau_L + (t_{oD} - \tau_I) \quad (4.20)$$

This is similar to the r_o requirement in equation (4.14), and differs only by inclusion of the term in parentheses on the right side of the above inequality. Thus, if $\tau_I \geq t_{oD}$, then the t_{oD} requirement is satisfied a fortiori when the r_o requirement is satisfied, and

$$q_P = q_{\overline{P}} \quad (4.21)$$

If $\tau_I < t_{oD}$, then the t_{oD} requirement is the more severe, $q_P < q_{\overline{P}}$, and:

$$q_P = S_{AA} (\tau_L + t_{oD} - \tau_I \mid m, n - r_o, 0) \quad (4.22)$$

We are now able to compute $\mathcal{V}_D$. Equations (4.21) or (4.22) provide q_P and (4.16) provides $q_{\overline{P}}$, and these are substituted into (4.11) to provide q_1 . Equation (4.17) is used to compute q_k ($k = 2, \dots, K$). Since the q 's represent probabilities of at least k interpositions, $(q_k - q_{k+1})$ provides the probability of exactly k interpositions, and $\mathcal{V}_D$, the expected number of interpositions, is given by:

$$\mathcal{V}_D = \sum_{k=1}^K k (q_k - q_{k+1}) = \sum_{k=1}^K q_k \quad (4.23)$$

Operations rate μ can now be computed from (4.9).

4.2.4.2.2 Interposition Probabilities When $a < r_o / (t_o - \tau_L) < b$

The case of interposition where $T_{AA} > t_o$ was just considered. It is also possible, although not probable, for the r_o requirement to be satisfied when $T_{AA} = t_o$, making the requirement of relation (4.12):

$$V_2 \geq r_o / (t_o - \tau_L)^* \quad (4.24)$$

This case represents the situation where t_o is sufficiently large to permit interposing a landing if V_2 is fast enough so that it arrives at r_o late in the t_o interval, and consumes only a small portion of the interval in its passage from r_o to the runway. The necessary condition for this case (denoted as Case II) is:

$$r_o / (t_o - \tau_L) < b \quad (4.25)$$

The inequality in equation (4.14) may be rewritten as:

$$V_2 < \frac{(n - r_o) V_1}{V_1 \tau_L + m} \equiv \tilde{V}(V_1) \quad (4.26)$$

and the r_o requirement is satisfied if V_2 satisfies the conditions of relations (4.24), (4.26), or both.

Expressions for $q_{\overline{P}}$ and τ_I for this case are derived in Appendix D. With these, the remainder of the analysis is identical to that of the previous section.

* Since t_o is the maximum runway occupancy time plus a safety factor, $(t_o - \tau_L) \geq 0$.

4.3 Extensions of the Basic Operations Capacity Model

The model outlined above provides a means for adequately estimating the operations capacity of a runway, and all later numerical computations are performed with it. Extensions to it are desirable -- particularly in the removal of some of the limiting assumptions -- and several are treated in this section.

4.3.1 Treatment of T_{DD} and T_{AD} as Discrete Chance Variables

In the basic model, the time between take-offs (T_{DD}) and the interval between touchdown of a landing and the beginning of a following take-off roll (T_{AD}) have been treated as constants t_{oD} and τ_L . They may also be treated as chance variables, and both can probably best be approximated by discrete distributions.

The on-runway landing time, T_{AD} , may be considered a function of the runway exit used by the landing aircraft and, for a limited group of aircraft types, may be considered to be relatively independent of aircraft type. Thus, assuming that a runway has w exits with associated occupancy times τ_{L_i} and probability of utilization p_{L_i} ($i = 1, 2, \dots, w$), then a value of q_{k_i} ($k = 1, 2, \dots, K$) must be computed for each τ_{L_i} , and the final set of q_k 's to be used in equation (4.23) is given by:

$$q_k = \sum_{i=1}^w p_{L_i} q_{k_i} \quad (k = 1, 2, \dots, K) \quad (4.27)$$

Similarly, T_{DD} may assume several values. Since the separation standards specify one, two, or three minutes separation between successive departures depending on the distance they fly before their courses diverge, a discrete distribution is also indicated here. Thus, T_{DD} may take the values t_{oD_j} ($j = 1, 2, 3$) with associated probabilities p_{D_j} reflecting the course structure of traffic from the terminal. The values of p_{D_j} should also reflect deviations from the first-come-first-served priority rule; thus, if policy dictates that whenever two aircraft following identical departure courses for more than five minutes - requiring three-minute separation - arrive for take-off successively, a departure to a different course shall be interposed between them, then $p_{D_3} = 0$.

We might then assume that T_{AD} and T_{DD} are independent, and that each is independent of the landing interval T_{AA} . The independence of T_{AD} and T_{DD} and of T_{AA} and T_{DD} follows from the independence of the departure and arrival sequences. The independence of T_{AD} and of T_{AA} follows from the selection of runway exit independent of landing speed (an assumption that clearly breaks down over a wide performance range).

Then, letting:

$$q_{ijk} = \Pr \left\{ \text{interposing } k \text{ landings} \mid T_{DD} = t_{oD_j} ; T_{AD} = \tau_{L_i} \right\} \quad (4.28)$$

$$(i = 1, 2, \dots, w; j = 1, 2, 3; k = 1, 2, \dots, K)$$

we find the set of q_k 's from:

$$q_k = \sum_{j=1}^3 \sum_{i=1}^w P_{L_i} P_{D_j} q_{ijk} \quad (k = 1, 2, \dots, K) \quad (4.29)$$

A more realistic - and more complex - model would express the correlation that exists between runway-occupancy time, T_{AD} , and aircraft type as represented by V_1 in T_{AA} . This correspondence would be expressed in equations (4.14), (4.15), and (4.20) by replacing τ_L by T_{AD} , and using the joint distribution of V_1 and T_{AD} to find the values of q_k .

4.3.2 Operations Rate for $\omega > 0$

In the basic model, only the case of $\omega = 0$ was considered. While this reasonably represents the typical operational situation (characterized by long planned runs of operations), the normal situation is more generally characterized by some value of $\omega > 0$ where the runs are finite. Analysis of this situation involves the inter-operation-type times, T_{AD} and T_{DA} . Accepting $T_{AD} = \tau_L$ as a satisfactory representation, τ_{DA} remains to be determined. If it can be assumed that:

1. the time duration of a run of take-offs is always greater than the maximum landing interval ($n/a - m/b$),

2. the controller can dispatch the first landing of the next run from the stack with sufficient precision so that it arrives at r_o just as the last departure begins its take-off roll (thereby operating to the limit of the separation standards),

then:

$$T_{DA} = r_o / V_1, \quad (4.30)$$

where V_1 is the velocity of the first arrival after the run of take-offs.

Assuming V_1 to have the uniform distribution over $[a, b]$, we find that τ_{DA} , the expected value of T_{DA} , is:

$$\tau_{DA} = \frac{r_o}{b-a} \log_e \left(\frac{b}{a} \right) \quad (4.31)$$

Substituting this value into equation (4.9) provides the operations rate for any value of ω .

Assumption (2) above can easily be modified by substituting any greater value for r_o to represent the controller's inability to perfectly position the landing aircraft.

Assumption (1) is imposed to prevent the interaction between the last of a run of landings and the first of a following run. Ignoring the effect of the interaction and retaining assumption (2) above, the time between touchdown of these two landings when separated by a run of h departures is given by:

$$\tau_L + (h - 1) t_{oD} + r_o/V_1 \quad (4.32)$$

Thus, a sufficient condition on the minimum length of a run of take-offs for assumption (1) to hold is determined by making the maximum time between landings ($r/a - m/b$) less than the time of (4.32), and is given by:

$$h \geq \frac{1}{t_{oD}} \left\{ \left(\frac{n - r_o}{a} \right) - \frac{m}{b} \right\} + 1 \quad (4.33)$$

Using typical data ($m = 7$, $s_o = 3$, $r_o = 2$, $a = 90$, $b = 150$, $t_{oD} = 1$), a run of at least three take-offs between every run of landings (not considering interposed take-offs as a "run") is seen to be sufficient to assure the validity of the assumption.

If the assumption is not satisfied, then it is necessary, in the application of equation (4.9), to specify the relative frequency (p_h) of all runs of take-offs having length less than h^* , where h^* is defined as the least integer greater than the right side of (4.33).

For each such run length, a mean time τ_{DA_h} must be determined accounting for the interaction between successive runs of landings. The interaction exists when V_n (the last landing of the previous run) is fast and V_1 is slow, so that imposition of gate separation results in a long landing interval. Specifically, interaction occurs when:

$$\tau_L + r_o/V_1 + (h - 1) t_{oD} < n/V_1 - m/V_n \quad (4.34)$$

which can be described by:

$$V_1 < \frac{(n - r_o) V_n}{m + V_n t_h} \quad (4.35)$$

where:

$$t_h = \tau_L + (h - 1) t_{oD} \quad (4.36)$$

Equation (4.35) is of the same form as that for $\tilde{V}$ in equation (4.26) so that the methods of Appendix C for determining τ_I may be used to determine τ_{DA_h} ($h = 1, 2, \dots, h^* - 1$). Then, for any specified value of ω and associated p_h , equation (4.9) for operations rate is used with τ_{DA}' substituted for τ_{DA} , where:

$$\tau_{DA}' = \sum_{h=1}^{h^*-1} p_h \tau_{DA_h} + \left(1 - \sum_{h=1}^{h^*-1} p_h\right) \tau_{DA}^* \quad (4.37)$$

where τ_{DA}^* is the value given by equation (4.31).

It is to be noted that τ_{DA}' is the value of τ_{DA} to be used in equation (4.6) for the selection of an optimum run-length doctrine.

4.3.3 Interposition of Landings between Take-Offs

Although it has less operational meaning than its converse, it is possible to consider the interposition of landings between take-offs during runs of take-offs. We assume initially that long runs are performed.

Letting $\mathcal{V}_A$ represent the expected number of landings that can be interposed between a pair of take-offs, and using $\mathcal{V}_D$ as defined previously, we find that the time for the sequence of N operations (comprising N_A landings and N_D take-offs) depends on the interposition rates as follows:

i. If $\mathcal{V}_i N_j \geq N_i$ and $\mathcal{V}_j N_i < N_j$ ($i = A$ or D ; $i \neq j$), then all i 's can be accommodated between the j 's and need not be run separately, so that:

$$\tau_N = N_j \tau_{jj} \quad (4.38)$$

2. If $0 \leq \mathcal{V}_i < N_i/N_j$ for $i = A$ and D ($i \neq j$), then runs of both operations must be scheduled. N_A landings and N_D take-offs must be performed. But since $\mathcal{V}_A N_D$ landings can be interposed within the runs of take-offs, they require no time, and only $(N_A - \mathcal{V}_A N_D)$ landings must be scheduled. Since take-offs can be interposed among these landings, it is necessary to schedule only

$$N_D - \mathcal{V}_D (N_A - \mathcal{V}_A N_D) = N_D (1 + \mathcal{V}_D \mathcal{V}_A) - \mathcal{V}_D N_A \quad (4.39)$$

take-offs. But this reduces the number of landings that can be interposed, so that:

$$N_A - \mathcal{V}_A [N_D (1 + \mathcal{V}_D \mathcal{V}_A) - \mathcal{V}_D N_A] = (N_A - \mathcal{V}_A N_D) (1 + \mathcal{V}_D \mathcal{V}_A) \quad (4.40)$$

landings must be scheduled. Continuing this process, the factor that appears as $(1 + \mathcal{V}_D \mathcal{V}_A)$ in expressions (4.39) and (4.40) becomes the geometric series:

$$1 + (\nu_A \nu_D) + (\nu_A \nu_D)^2 + \dots = 1/(1 - \nu_A \nu_D) \quad (4.41)$$

The time to perform the sequence of N operations with ω' cycles then becomes:

$$\begin{aligned} \tau_N = & [(N_D - \omega') - \nu_D (N_A - \omega')] \frac{t_{oD}}{1 - \nu_A \nu_D} \\ & + [(N_A - \omega') - \nu_A (N_D - \omega')] \frac{\tau_{AA}}{1 - \nu_A \nu_D} + \omega' (\tau_L + \tau_{DA}) \end{aligned} \quad (4.42)$$

Operations rate is N/τ_N , and can easily be determined for any N_A and N_D , and for $N_A = N_D = N/2$ in particular.

An analysis similar to that for the take-off interposition is required to determine the landing interposition rate ν_D . Since it is desired to interpose the landings without affecting the take-offs, it is necessary, for landing interposition to be possible, that:

$$t_{oD} \geq \tau_L + r_o/b \quad (4.43)$$

where the right side of the inequality represents the minimum time a landing aircraft might "claim" the runway (including its claim during its flight from r_o).

The necessary condition for being able to interpose h landings in a take-off interval is:

$$t_{oD} > \tau_L + r_o/b + t_o (h - 1) \quad (4.44)$$

and is rarely met for $h > 1$, so that only the case of a single interposition is considered. It is first assumed that a controller has sufficiently precise control to deliver the arriving aircraft to r_o at the moment the departing aircraft begins its take-off roll (consistent with landing separation standards).

With these assumptions, the occurrence of interposition in an interval depends only on the aircraft desiring to land in that interval and on the time of the last previous interposition. Since there is no interaction between arrivals separated by more than $(n/a - m/b)$, only the previous

$$k^* = \left[\frac{n/a - m/b}{t_{oD}} + 1 \right] \quad (4.45)$$

take-off intervals need be considered, for an interposition in an interval previous to the k^{*th} would have no effect. The square brackets in (4.45) represent "the greatest integer in."

We now define the following probabilities:

$$p_k = \Pr \left\{ \begin{array}{l} \text{interposition occurs in an interval / the last} \\ \text{previous interposition occurred in the } k^{\text{th}} \text{ previous} \\ \text{interval} \end{array} \right\} \quad (k = 1, 2, \dots, k^*, k^* + 1)^*$$

$$q_k = 1 - p_k$$

$$P_{A_1} = \Pr \left\{ \text{interposition occurs in an interval} \right.$$

As with take-off interposition, a Markov process can here be defined on the successive take-off intervals, this time with $(k^* + 1)$ ergodic states, where the process is in state k if the last previous interposition had occurred in the k^{th} previous interval. This process has the transition matrix:

$$\begin{aligned} p_{kj} &= p_k \quad (k = 1, \dots, k^* + 1; j = 1) \\ &= q_k \quad (k = 1, \dots, k^*; j = k + 1) \\ &= q_{k^* + 1} \quad (k = k^* + 1; j = k^* + 1) \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{4.46}$$

The probability of an interposition, P_{A_1} , is the probability that the process is in state 1. Letting U_k ($k = 1, 2, \dots, k^* + 1$) be the steady state probability that the system is in state k , then P_{A_1} is given by:

* $(k^* + 1)$ represents all intervals prior to the k^{th} .

$$P_{A_1} = \sum_{k=1}^{k^*+1} p_k U_k \quad (4.47)$$

where:

$$U_1 = P_{A_1} \quad (4.48)$$

$$U_k = P_{A_1} \left(\prod_{i=1}^{k-1} q_i \right) \quad (k = 2, \dots, k^*)$$

$$\begin{aligned} U_{k^*+1} &= P_{A_1} \prod_{i=1}^{k^*} q_i + U_{k^*+1} q_{k^*+1} \\ &= P_{A_1} \left(\frac{\prod_{i=1}^{k^*} q_i}{1 - q_{k^*+1}} \right) \end{aligned}$$

Letting:

$$U_k = A_k P_{A_1} \quad (4.49)$$

where A_k is the factor in parentheses in (4.48), and using the condition that:

$$U_2 = 1 - P_{A_1} - \sum_{k=3}^{k^*+1} U_k \quad (4.50)$$

we find from equation (4.47).

$$P_{A_1} = \frac{P_2}{(p_2 - p_1) + \sum_{k=3}^{k^*+1} A_k (p_2 - p_k)} \quad (4.51)$$

It is now necessary to determine only the p_k values. As in the take-off situation, two requirements must be met:

r_{oA} requirement: the landing, arriving at r_o when the first departure begins its roll, must be off the runway by a time t_{oD} later in order not to interrupt the take-off operations

T_{AA} requirement: sufficient separation must have been provided from the last previous landing.

The r_{oA} requirement refers only to the velocity of the landing to be interposed, and is given by:

$$\tau_L + r_o/V_1 < t_{oD} \quad (4.52)$$

or:

$$V_1 > r_o / (t_{oD} - \tau_L) \quad \text{when} \quad \tau_L < t_{oD} \quad (4.53)$$

The T_{AA} requirement depends on when the last previous landing occurred. If the last previous landing (A_0) was in the k^{th} previous take-off interval, then interposition is possible only if :

$$T_{AA} = \text{Max} \left\{ n/V_1 - m/V_0 ; t_o \right\} < (k-1) t_{oD} \quad (4.54)$$

These requirements are depicted in the time-coordinate illustration of Figure 4.3, where $T_1 = n/V_1$ and $T_0 = m/V_0$. A landing can be interposed only when T_0 and T_1 fall into the cross-hatched area and thus satisfy both the requirements. The probability of interposing a landing, P_k , is given by :

$$P_k = \iint_{C_k} f(t_0, t_1) dt_0 dt_1 \quad (4.55)$$

where the integration is performed over the cross-hatched area, C_k , and the density function $f(t_0, t_1)$ is given by equation (3.27).

It can be seen from Figure 4.3 that, when:

$$(k-1) t_{oD} < (n/r_o) (t_{oD} - \tau_L) - m/a \quad (4.56)$$

satisfaction of the r_o requirement implies satisfaction of the T_{AA} requirement a fortiori, while, when :

$$(k-1) t_{oD} > (n/r_o) (t_{oD} - \tau_L) - m/b \quad (4.57)$$

the reverse implication holds.

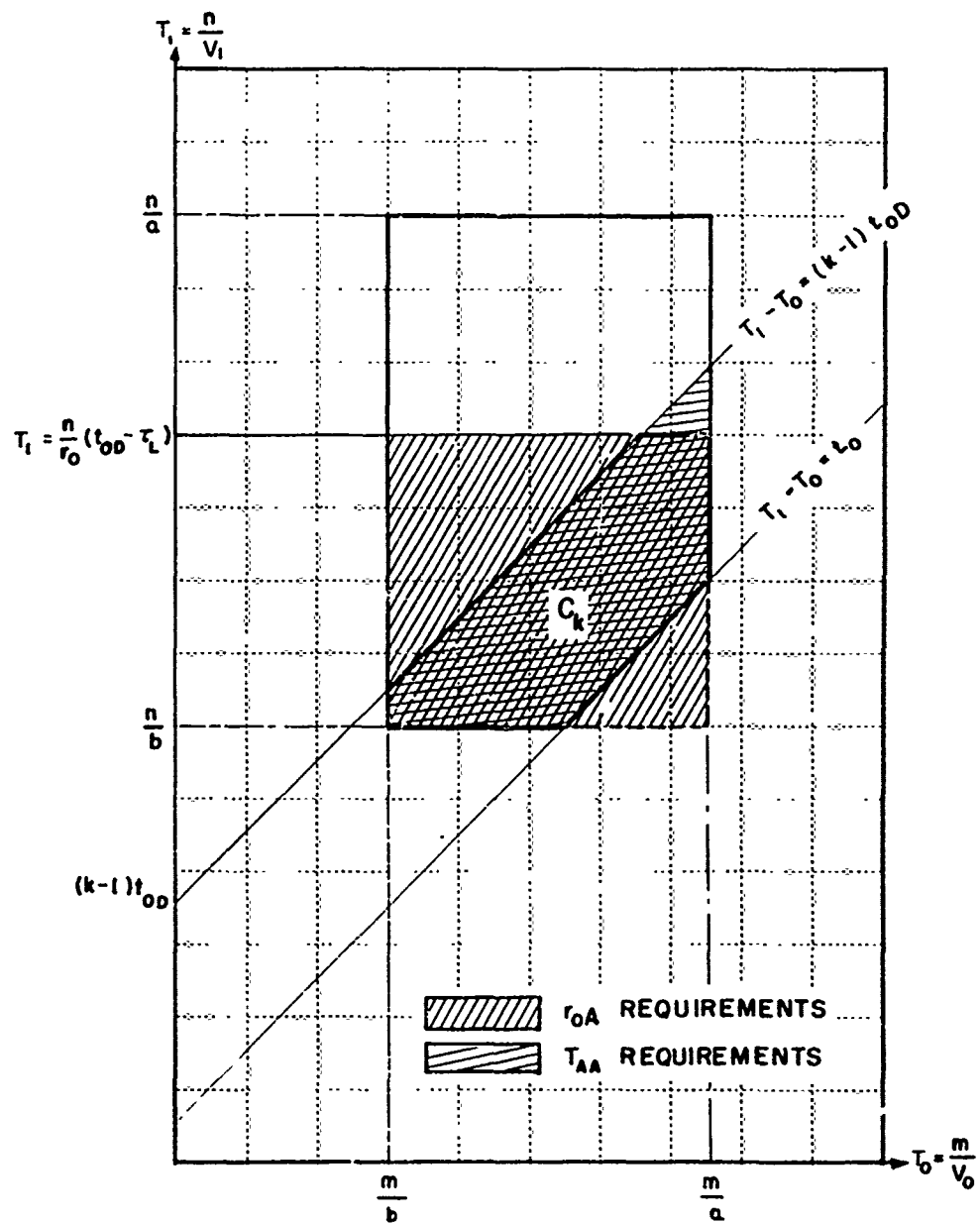


FIGURE 4.3
REQUIREMENTS FOR INTERPOSING A LANDING

Equation (4.55) may be used to compute the set of p_k , which, with (4.48) and (4.49) may be substituted into (4.51) to compute P_{A_1} .

Since only one interposition is permitted,

$$\mathcal{V}_A = P_{A_1} \quad (4.58)$$

4.3.4 Conservative Estimate of q_P

In computing q_P , the probability of being able to perform an interposition when one was performed in the previous landing interval, it was assumed that the interposition in the previous interval occurred as early as possible in that interval. Such an assumption tends to produce a high estimate of $\mathcal{V}_D$, since it minimizes the effect of the t_{oD} requirement in the following interval.

An alternative assumption that the interposition occurred as late as possible in the previous interval would provide a lower bound to q_P . Under this alternative, the departure in the previous interval would begin its take-off roll at the time A_1 is at r_o , a time r_o/V_1 before A_1 lands. In this case, the t_{oD} requirement becomes:

$$r_o/V_1 + T_{AA} - r_o/V_2 > t_{oD} \quad (4.59)$$

while the r_o requirement remains as in expression (4.12).

We now transform to time coordinates, where:

$$T_2 = n/V_2 \text{ and } T_1 = m/V_1 \quad (4.60)$$

so that:

$$T_{AA} = \text{Max} \left\{ T_2 - T_1; t_o \right\} \quad (4.61)$$

In the case where $T_{AA} > t_o$, the t_{oD} requirement then becomes:

$$T_2 \geq \left(\frac{n}{n-r_o} \right) t_{oD} + \left(\frac{n(m-r_o)}{m(n-r_o)} \right) T_1 \quad (4.62)$$

and the r_o requirement becomes:

$$T_2 \geq \frac{n \tau_L}{n-r_o} + \left(\frac{n}{n-r_o} \right) T_1 \quad (4.63)$$

These requirements are depicted in (T_1, T_2) space in Figure 4.4, along with the requirement that $T_2 - T_1 > t_o$. It is noted from expressions (4.62) and (4.63) that the boundaries of the inequalities have slopes that are respectively less than and greater than unity. The probability q_p is given by the integration over the cross-hatched region of Figure 4.4, using the density function of equation (3.27). If the boundaries do not intersect in the region of positive probability, then one requirement implies the other a fortiori.

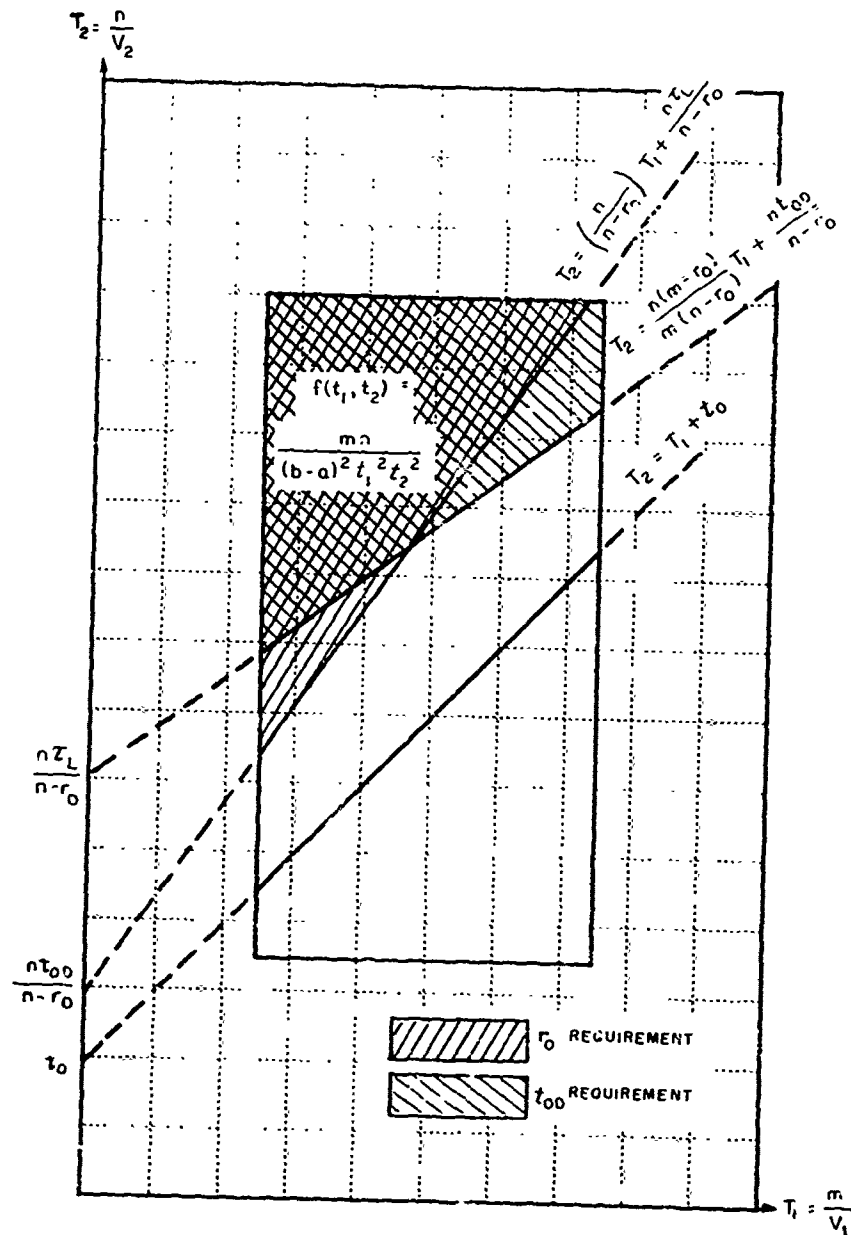


FIGURE 4.4
 r_0 AND t_{00} REQUIREMENTS FOR LOWER BOUND ON q_p

It may also be possible for one or both of the requirements to be satisfied when $T_{AA} = t_o$. In that case, the appropriate boundary would cross the line $T_2 - T_1 = t_o$. The probabilities in this case are found by letting $T_{AA} = t_o$ in expressions (4.12) and (4.59). The r_o requirement then becomes:

$$T_2 \leq \frac{(t_o - \tau_L) n}{r_o} \quad (4.64)$$

and the t_{oD} requirement becomes.

$$T_2 \geq \frac{n}{m} T_1 - \frac{n}{r_o} (t_{oD} - t_o) \quad (4.65)$$

Figure 4.5 illustrates the situation where both requirements can be satisfied when $T_{AA} = t_o$. (The case where only one can be so satisfied is merely a special case of this more general one.) The regions defining the requirements are indicated by hatched lines.

Each requirement individually is seen to be satisfied in the entire T_1, T_2 - space except for a wedge-shaped portion surrounding the line $T_2 - T_1 = t_o$. The corners of the wedges may easily be obtained from the equations for the boundaries. They both lie on the line $T_2 - T_1 = t_o$. The point labeled (1), the minimum value of (T_1, T_2) for which the r_o requirement is limiting is:

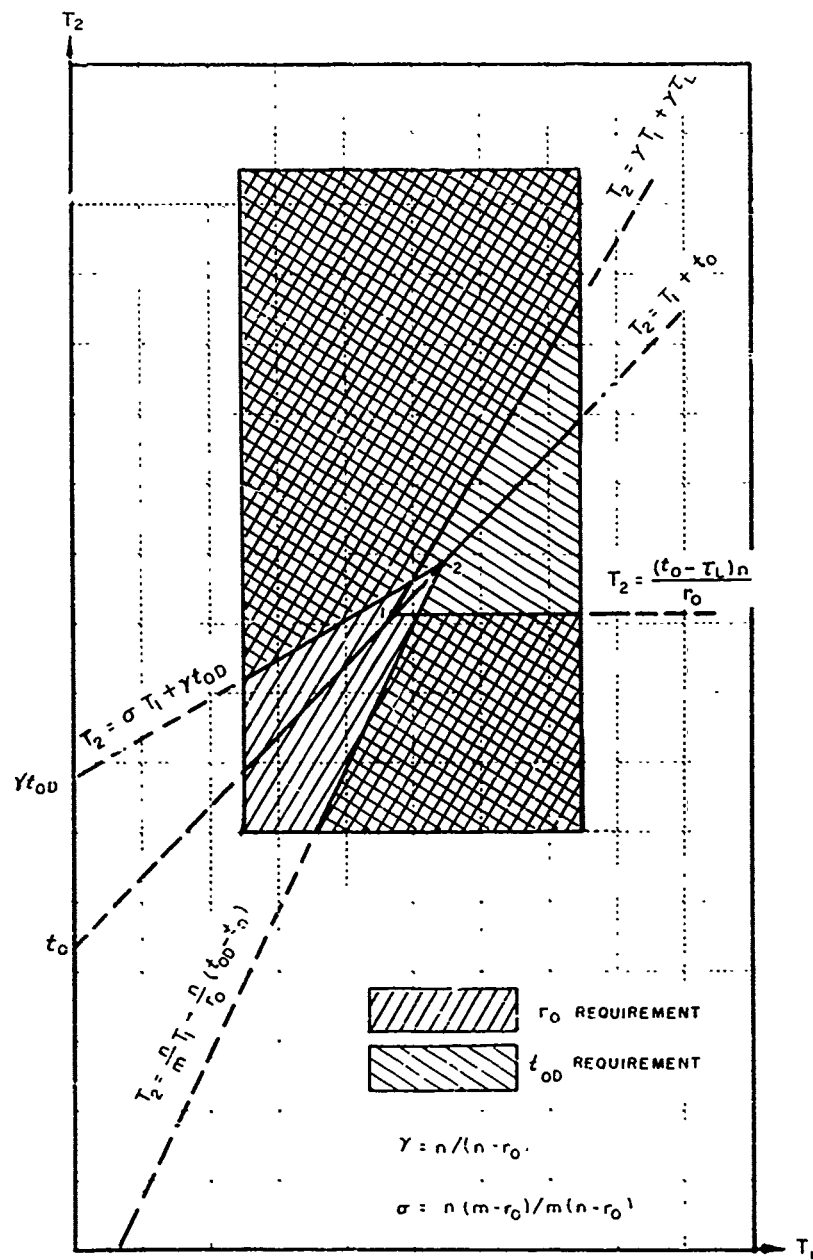


FIGURE 4.5
REQUIREMENTS FOR LOWER BOUND ON q_p WHEN $T_{AA} = t_0$

$$\left(\frac{t_o (n-r_o) - n \tilde{t}_L}{r_o}, \quad n \frac{(t_o - \tilde{t}_L)}{r_o} \right)$$

and the point labeled (2), the maximum value of (T_1, T_2) for which the t_{oD} requirement is limiting is:

$$\left(\frac{mn t_{oD} - m t_o (n-r_o)}{s_o r_o}, \quad \frac{mn t_{oD}}{s_o r_o} \right)$$

4.3.5 Operation of Multiple Runways

While the models developed explicitly treat only single runways, it is possible to extend them to apply to some multiple-runway situations. Major airports often operate with two equivalent parallel runways sufficiently separated (3000 feet separation is currently required) so that they can be used independently. This case can be treated with the methods already developed.

The decision must first be made on the distribution of the traffic load between the two runways. In general, to minimize delay, it is a desirable policy to divide the load (in terms of time) equally among the two runways, since delay is a convex function (i.e., non-negative second derivative) of runway utilization in all normal queuing situations. If the load is divided unequally, the additional delay incurred by the additional utilization on one runway would exceed the delay avoided by decreasing the utilization on the other.

If the policy on length of runs called for by relation (4.6) dictates large ω and short runs, then frequent alternation of operations would be performed on both runways, and the runways would be operated similarly at a rate per runway equal to that for a single runway. If the policy calls for small ω and long runs, then it would probably be expeditious to schedule on one runway only the single operation that requires the greater time (thereby creating the case of $\omega = 0$), interposing as many of the other operations as possible, and performing the remainder of the operations on the other runway.

Considering that only take-offs are interposed, and that N_A landings and N_D take-offs must be performed, then the operation performed on the first runway (R_1) is the one requiring the longer time $N_i' \tau_{ii}$ ($i = A$ or D), where N_i' is given by:

$$N_i' = (N_i - \omega') - \bigvee_i (N_j - \omega') \quad (i \neq j) \quad (4.66)$$

Without loss of generality, we may assume that:

$$N_A' \tau_{AA} > N_D' \tau_{DD} \quad (4.67)$$

so that R_1 is devoted primarily to landings. Then, to equalize the time load on the two runways, N_{A1} landings are assigned to the first runway (making possible the interposition of $\bigvee_D N_{A1}$ take-offs on that runway) such that:

$$N_{A1} \tau_{AA} = [(N_D - \omega' N_{A1} - \omega') - \omega' (N_A - N_{A1} - \omega')] t_{oD} \quad (4.68)$$

$$+ (N_A - N_{A1} - \omega') \tau_{AA} + \omega' (\tau_{DA} + \tau_L)$$

The right side of (4.68) expressing the time consumed by the $(N_A - N_{A1})$ landings and $(N_D - \omega' N_{A1})$ take-offs performed on R_2 follows directly from equation (4.7). Solving for N_{A1} gives:

$$N_{A1} = \frac{N_D t_{oD} + N_A \tau_{AA} + \omega' (\tau_{DA} + \tau_L)}{2 \tau_{AA}} \quad (4.69)$$

Letting $N_A = N_D = N/2$, the time to perform the N operations is $N_{A1} \tau_{AA}$, so that the average operations rate per runway is given by:

$$\mu = \frac{2}{(1 - \omega) [(1 - \omega') t_{oD} + \tau_{AA}] + \omega (\tau_{DA} + \tau_L)} \quad (4.70)$$

which is identical to the operations rate for the single runway, an expected result since neither runway hinders the other.

If operational conditions are such that interposition is not possible on R_1 , and all take-offs (as well as additional landings) must be performed on R_2 , then the average operations rate per runway is found to be:

$$\mu = \frac{2 - \nu_D t_{oD} / \tau_{AA}}{(1 - \omega) \{ (1 - \nu_D) t_{oD} + \tau_{AA} \} + \omega (\tau_{DA} + \tau_L)} \quad (4.71)$$

which is less than that for the single runway by the extra term in the numerator because of the interposed take-offs lost from R_1 .

When two runways are not parallel, they are often used by performing take-offs on one and landings on the other. Since operations on one inhibit those on the other, their combined operations capacity is less than that for two independent runways. The operations capacity may be estimated using the relations for a single runway, expressing the effect of landings on one as an inhibition of take-offs on the other. An equivalent runway-occupancy time, τ'_L , represents the time the landing aircraft delays a waiting departure. In the case of the single runway, this is the entire runway-occupancy time. If the runways do not intersect, but their extended centerlines intersect on the windward side of the airport (as in Figure 4.6), then it may be desired to hold the take-off as long as the landing is still on the runway (and $\tau'_L = \tau_L$) or at least for some lesser time until the arrival is committed to a landing. If the centerlines intersect on the leeward side, and if it is desired to hold the take-off until the arrival crosses the runway, then $\tau_L = 0$; if the departure may be sent as soon as the arrival crosses the intersection, or some other point short of the runway, then τ'_L would take an appropriate negative value. If the

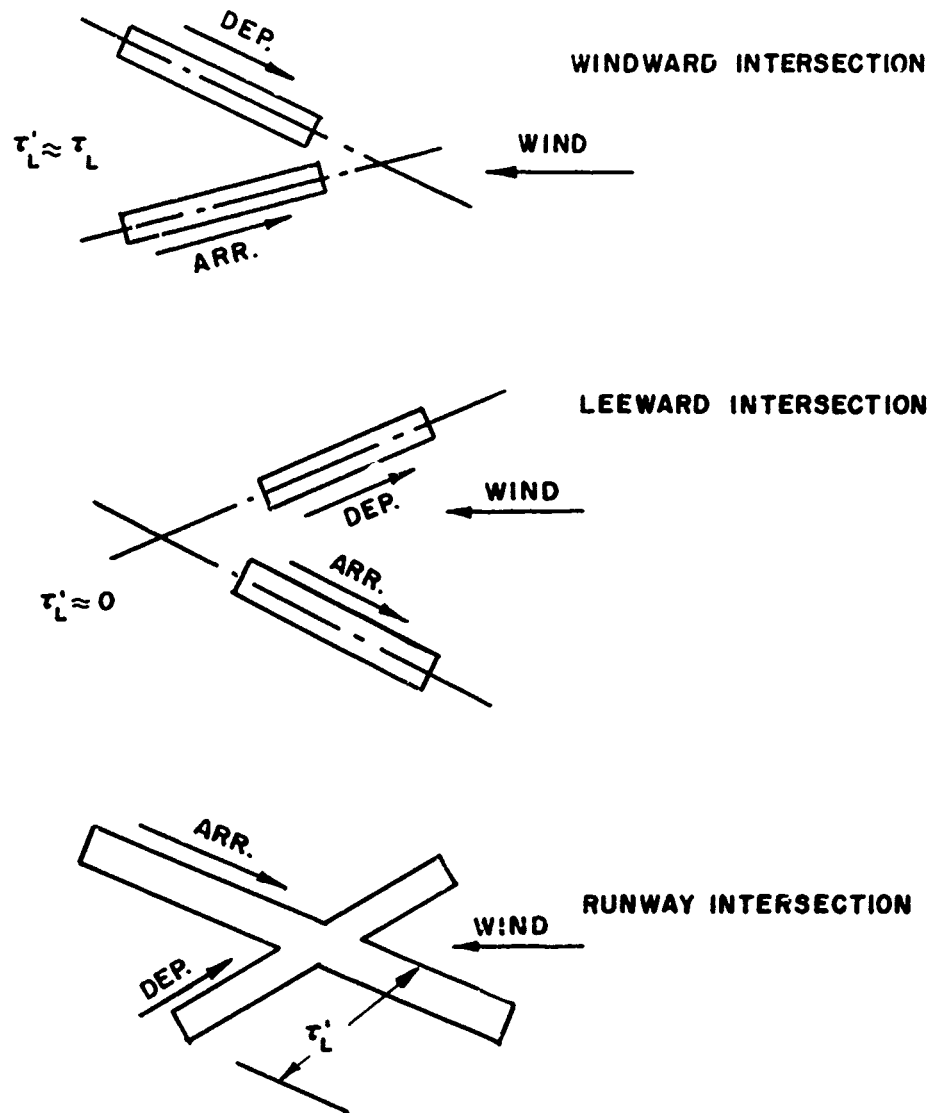


FIGURE 4.6
TWO NON-PARALLEL RUNWAYS

runways physically intersect, then τ_L' would be the time for the landing aircraft to cross the intersection point. The values of τ_L' so determined may be used in place of τ_L in the models developed in this chapter to estimate the operations capacity of intersecting runways operated in this manner.

4.4 Results of the Operations Capacity Analysis

4.4.1 Parametric Analysis

Using the basic model described in Section 4.2, computations were performed to investigate the effect of system parameters on interposition rate (ν_D) and operations rate (μ). Results were computed for all combinations of the following parameters at the indicated levels.

1. mean velocity of the landing aircraft ($\bar{V}$) (100, 120 knots)
2. range of the landing-velocity distribution (R) (40, 60 knots)
3. length of the common landing path (m) (4, 10 miles)
4. minimum distance separation at the common path gate (s_o) (2, 3, 4 miles)
5. minimum time separation between landings at the runway (t_o) (0.5, 1.0 minutes)
6. ratio of runway-occupancy time (τ_L) to t_o (k_R) (0.5, 1.0)
7. minimum take-off time separation (t_{oD}) (1, 2 minutes)

8. closest location of a landing to permit interposing a take-off (r_o) (2, 4 miles)*

All the results of these computations are tabulated in Appendix F. Some illustrative examples are discussed in this section to indicate some of the parametric effects.

Both m and R influence system performance similarly. Increasing either increases the variation in the landing time, particularly by introducing long landing intervals. Thus, to reduce the number of cases discussed, these two parameters may be considered together. Of the m, R combinations studied, two are presented here: Case Max ($m = 10$ miles, $R = 60$ knots), the combination that produces maximum landing interval variation, and Case Min ($m = 4$ miles, $R = 40$ knots), the combination that produces minimum landing interval variation.

4.4.1.1 Effects on Interposition Rate (ν_D)

In Figure 4.7, interposition rate is plotted against s_o for the case where $t_{oD} = 1$ minute and $r_o = 2$ miles (typical of radar requirements). It is seen that ν_D increases with s_o since the resulting longer landing interval provides additional opportunity for interposition.

* When $r_o = 4$ miles, only combinations with $t_o = 0.5$ minutes were calculated.

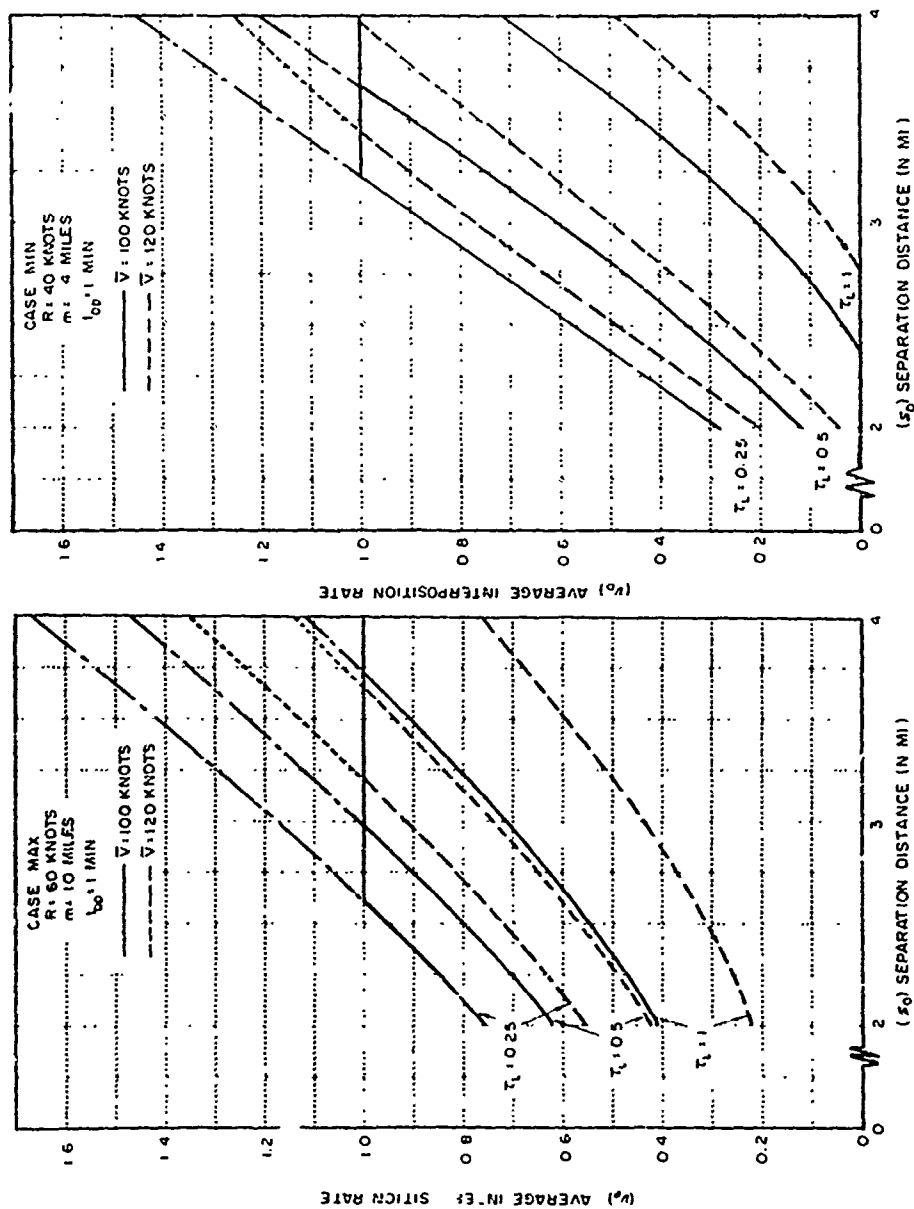


FIGURE 4.7
INTERPOSITION RATE ($\bar{u}_0$) vs. SEPARATION DISTANCE (s_0)

Similarly, $\mathcal{U}_D$ decreases as $\bar{V}$ and $\mathcal{C}_L$ increase. The two graphs of Figure 4.7 compare the effect of variation in landing interval, and it is noted that larger variation (Case Max) provides more opportunity for interposition. The cases presented consider $t_o \leq 1 = t_{oD}$, and no effect of t_o is noted; large t_o values would increase $\mathcal{U}_D$ by increasing the landing interval. Increasing r_o reduces the slope of the curves and moves their abscissa intercept to the right. Increasing t_{oD} reduces the slope of the curves. A high-speed turn-off on the runway would reduce $\mathcal{C}_L$, and increase the interposition rate.

We note that, with the exception of t_o (which has little effect on either rate) all the factors that affect landing rate have an opposite effect on interposition rate. These conflicting effects are resolved in operations rate, in which the complex interactions among the system parameters are most significant.

4.4.1.2 Effects on Operations Rate ($\mathcal{U}_D$)

Operations rate is plotted against s_o in Figure 4.8 for the case of $\bar{V} = 100$ knots, $t_o = 0.5$ minutes, and $r_o = 4$ miles. By comparing Case Max with Case Min, we note that, when $s_o = 2$ miles, Case Min yields a higher operations rate than Case Max. As s_o increases, however, the effect of interposition rate on operations rate manifests itself. In Case Min, $\mathcal{U}_D = 0$ for all s_o less than some value between 3

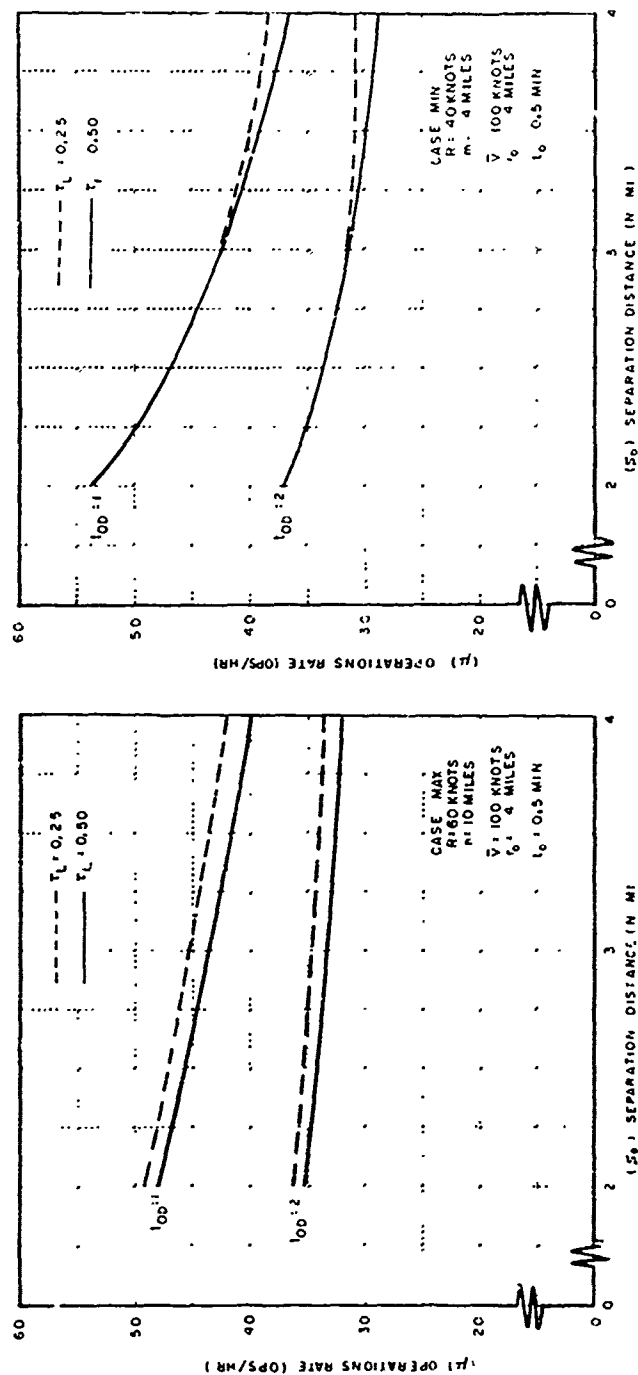


FIGURE 4.8
 OPERATIONS RATE (μ) vs. SEPARATION DISTANCE (x_0)
 WHEN $\tau_0 = 4$ MILES

and 4 miles, consequently, μ drops rapidly as s_o increases, reflecting the landing rate penalty. In Case Max, however, interposition is possible even for $s_o = 2$ miles, so that the decline in landing rate is partially compensated by an increase in ν_D , and the degradation with s_o is less severe. We note that, for $3 \leq s_o \leq 4$, a higher operations rate is achieved in Case Max than in Case Min, so that the apparently undesirable variation in landing velocity becomes desirable when it can be utilized to achieve greater interposition.

The effects of the other parameters may be roughly estimated. Runway occupancy time (τ_L) affects operations rate through its effect on interposition rate, and variation in τ_L from 0.25 to 0.50 minutes is seen to affect operations rate by about 5% in the cases shown (τ_L has no effect when $\nu_D = 0$, e.g., for $s_o \leq 3$ in Case Min). In the cases shown, halving t_{oD} from 2 to 1 minute increases operations rate by about 50%, while increasing mean landing speed from 100 to 120 knots improves operations rate by about 10%.

An even more striking effect is noted in Figure 4.9 in which operations rate is plotted against s_o for the case of $r_o = 2$ miles, the other parameters remaining the same as in Figure 4.8. The upper right portion of each graph is the function

$$\mu(s_o) = 2 \lambda(s_o)$$

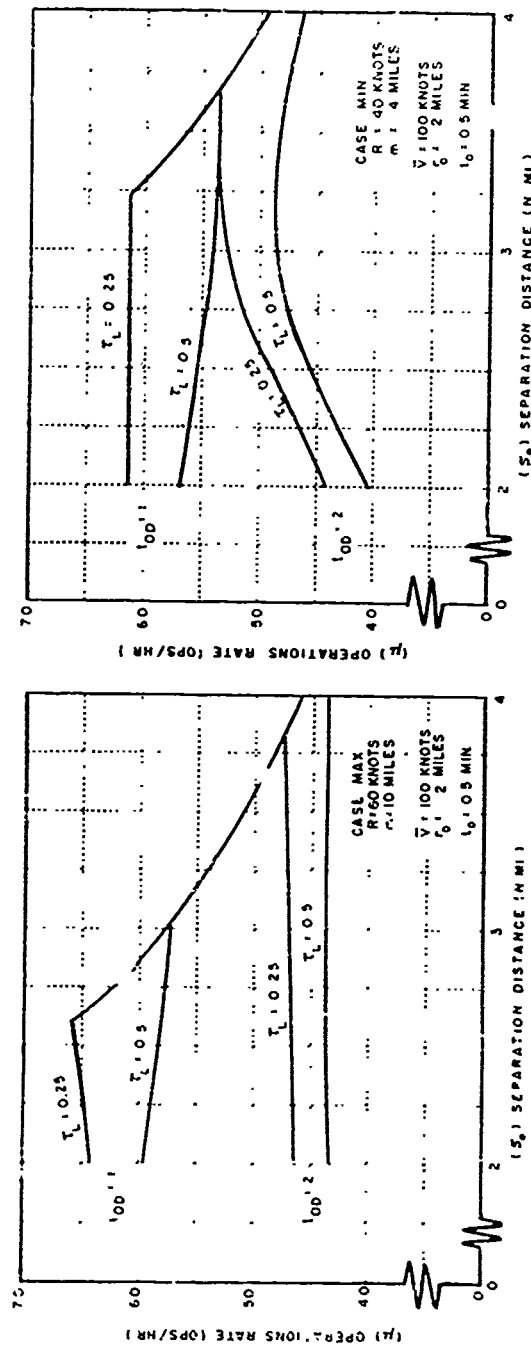


FIGURE 4.9
OPERATIONS RATE (μ) vs. SEPARATION DISTANCE (S_0)
WHEN $c_0 = 2$ MILES

depicting the situations where $\lambda_D \geq 1$, in which case the operations rate is twice the landing rate, and follows the typical decrease of landing rate with s_o . It is seen that, in some cases (e.g., $t_{OD} = 2$ minutes, Case Min) the advantage of interposition is sufficiently great that operations rate increases with s_o . In Case Max, where $t_{OD} = 2$ minutes, we note that the effects of s_o on landing rate and on interposition rate cancel, and operations rate is essentially independent of s_o .

4.4.2 Operations Capacity at Idlewild and LaGuardia

The operations capacity model may be used to study the effect of the various parameters at individual airports based on the traffic mix characteristic of that airport. The results of such an investigation might be an indication of the most effective direction for obtaining improved operations rate or an indication of an optimum combination of parameters. Values of landing rate, interposition rate, and operations rate were computed for all combinations of the following parameters at the indicated levels for the specific $\bar{V}$ and R characteristic of Idlewild and LaGuardia airports:

1. length of the common landing path (m) (4, 7, 10 miles)
2. required distance separation at the common path gate (s_o)
(2, 3, 4 miles)
3. required time separation at the runway (t_o) (0.5, 1.0, 1.5 minutes)

4. ratio of runway-occupancy time to t_o (k_R) (0.5, 0.75, 1.0)
5. take-off time separation (t_{oD}) (1, 2 minutes)
6. closest location of a landing to permit interposing a take-off (r_o) (2, 4 miles)

The results are tabulated in Appendix G, and some of the highlights are discussed here.

Figure 4.10 illustrates the landing rate at LaGuardia for the case where $r_o = 4$. Because of the severity of the r_o restriction, interposition is rarely possible, and operations rate is limited primarily by the take-off interval, t_{oD} . Runway characteristics (t_o and k_R) have relatively little effect on operations rate. The effect of s_o is slightly greater when $t_{oD} = 1$ minute than when $t_{oD} = 2$ minutes since, in the latter case, operations rate is more tightly restricted by t_{oD} , while the effect of s_o on landing rate is more readily reflected in operations rate when t_{oD} is small.

Figure 4.11 illustrates the operations rate at Idlewild when $r_o = 2$ miles, a value more representative of operation at radar-equipped airports, and indicates the variety of the parameter interactions in determining operations rate. When $t_{oD} = 1$ minute, it is noted that increases in s_o are advantageous when runway occupancy time is shorter than t_o (the solid curves), whereas it is disadvantageous when the landing

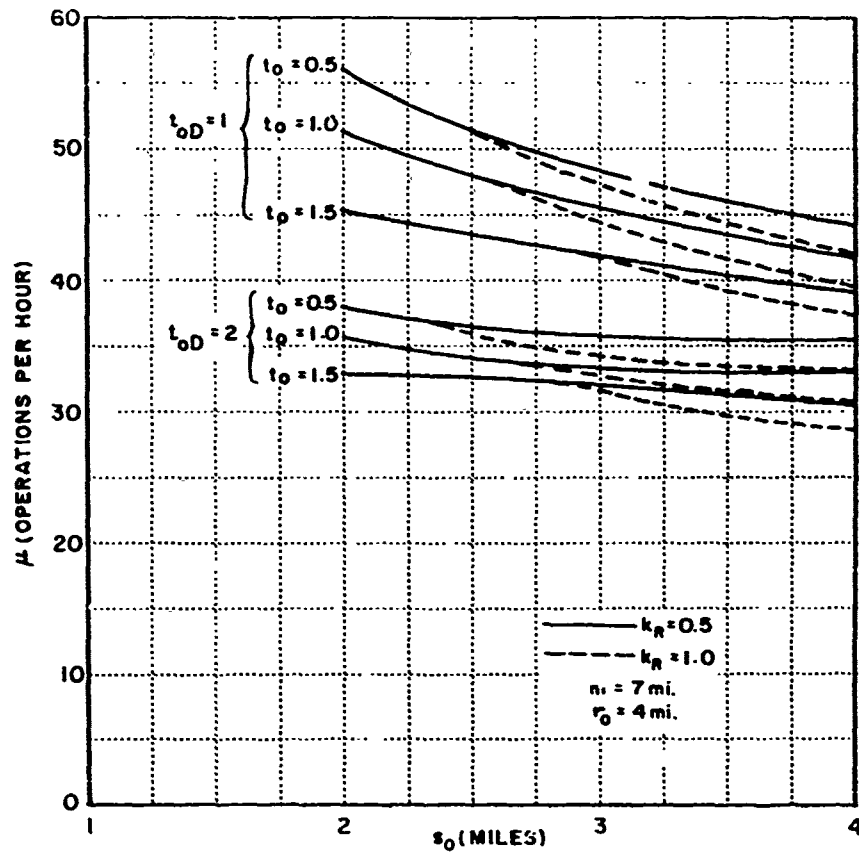


FIGURE 4.10
OPERATIONS RATE (μ) AT LAGUARDIA
WHEN $r_0 = 4$ MILES

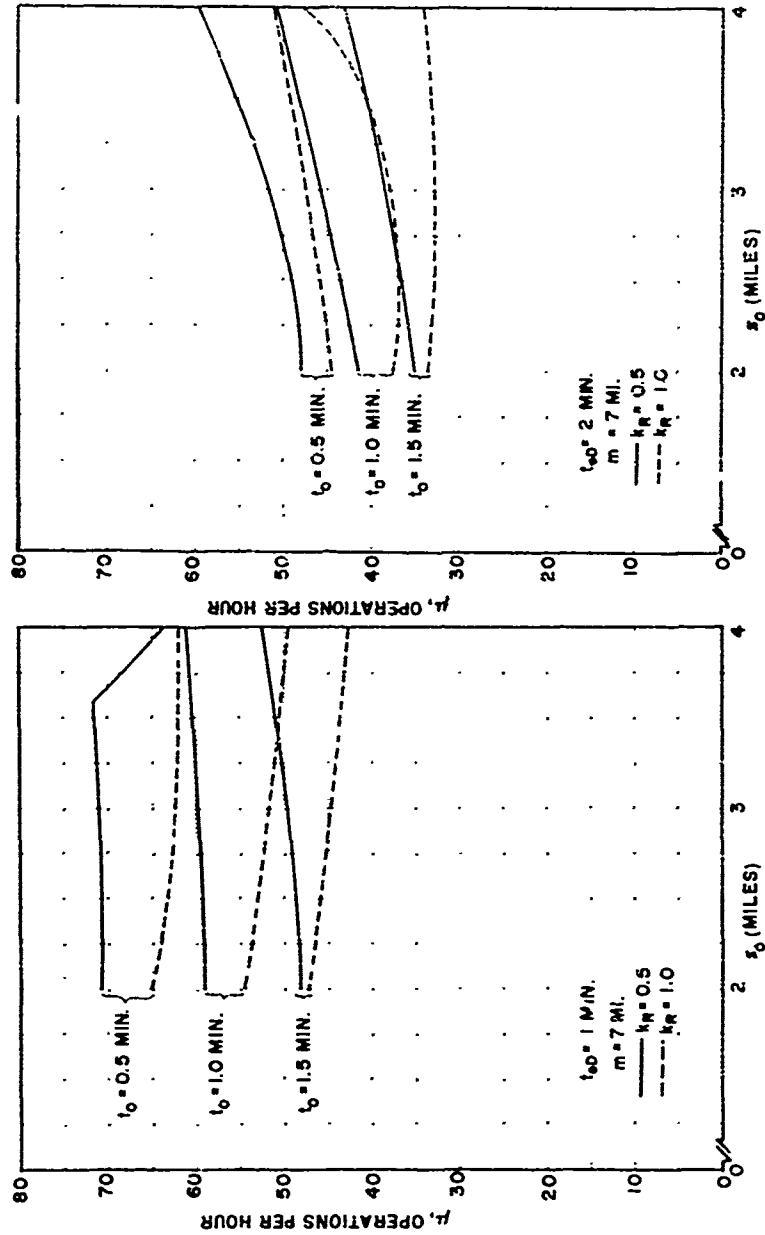


FIGURE 4.11
OPERATIONS CAPACITY AT IDLEWILD WHEN $t_0 = 2$ MILES

aircraft occupies the runway during the entire t_o interval (the dashed curves). Thus, increasing the opportunity for interposition by increasing s_o is desirable only when the interposition can be used effectively and when the increased interposition rate is sufficiently great to compensate for the reduction in landing rate. When $t_{oD} = 2$ minutes, interposition is sufficiently effective to result in an increase in operations rate with s_o in almost all cases.

Figure 4.12 illustrates the data of Table 4.1, which indicates the effects of the various time separations on landing and operations rates at Idlewild and LaGuardia. The values of r_o , m , s_o , and k_R are fixed at the typical values indicated. It is noted that reduction of either t_o or t_{oD} over the ranges indicated provide appreciable improvement in potential operations capacity of the runway. Reducing t_o from 1.5 to 0.5 increases μ by 40 - 43%, while a reduction in t_o from 1.0 to 0.5 increases μ by 17 - 19%. In contrast, it is noted that comparable reductions in t_o increase landing capacity by 17 - 21% and 11 - 14% respectively. It thus appears that operations capacity can be appreciably increased by installation of high-speed turnoffs which would reduce t_o and τ_L although no such comparable increase is obtained in landing capacity. The improvement results from the additional opportunity provided for interposing take-offs between landings when the landing aircraft can be removed from the runway earlier.

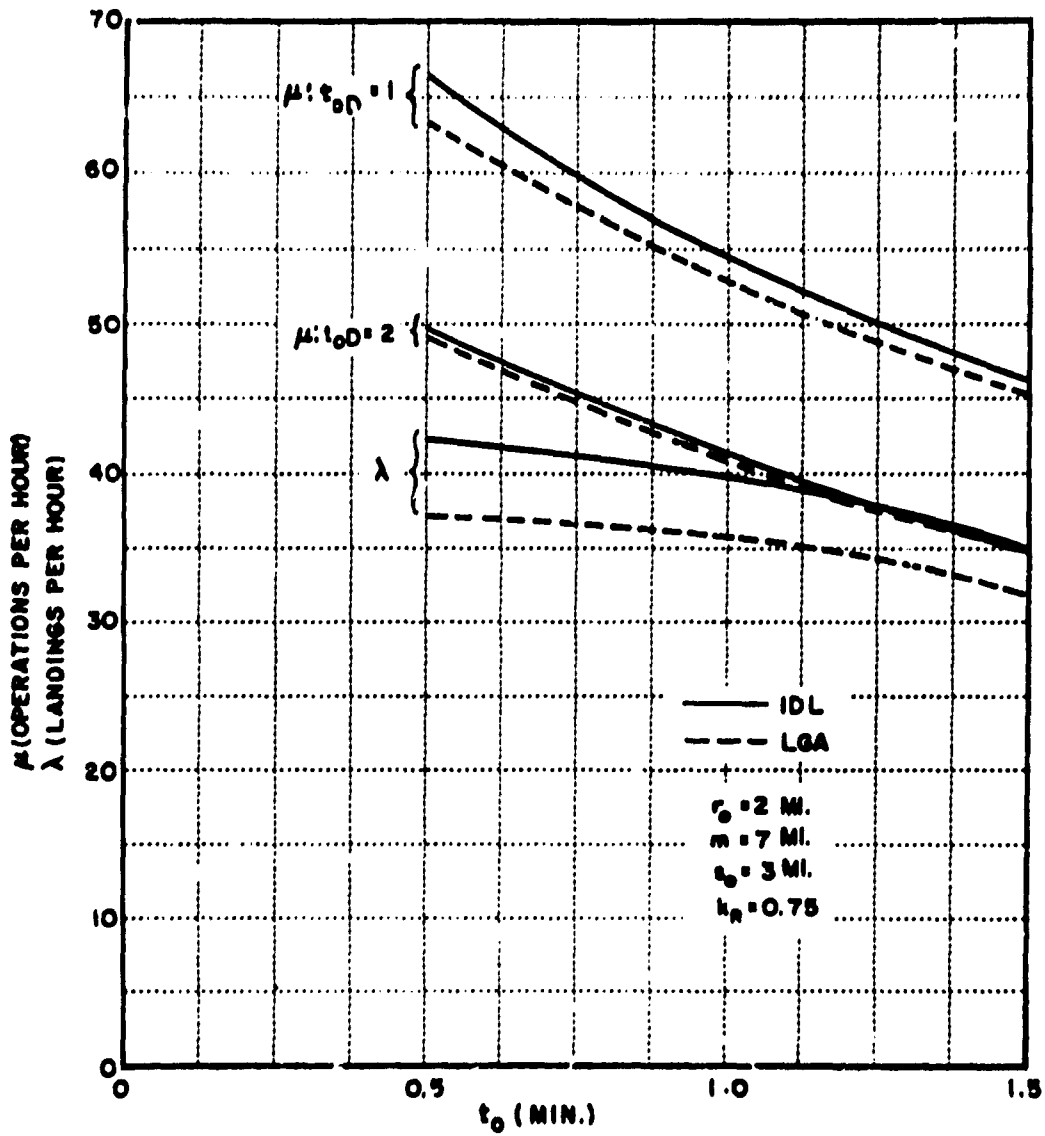


FIGURE 4.12
 TYPICAL CURRENT OPERATIONS AND LANDING CAPACITY
 AT IDLEWILD AND LAGUARDIA

TABLE 4.1
 RUNWAY PERFORMANCE MEASURES AT IDLEWILD* AND
 LAGUARDIA** FOR SEVERAL VALUES OF t_o AND t_{oD}

$$m = 7 \text{ miles}$$

$$k_R = \tau_L / t_o = .75$$

$$r_o = 2 \text{ miles}$$

$$s_o = 3 \text{ miles}$$

Terminal	t_o (min)	t_{oD} (min)	λ (landings/hr.)	λ_D	μ (operations/hr.)
Idlewild	.5	1	42.4	.61	66.5
		2	"	.50	49.7
	1	1	40.1	.30	54.6
		2	"	.30	41.4
	1.5	1	35.1	.13	46.6
		2	"	.13	34.8
LaGuardia	.5	1	37.5	.71	63.5
		2	"	.58	49.3
	1	1	35.6	.42	52.9
		2	"	.37	40.7
	1.5	1	32.0	.22	45.3
		2	"	.21	34.8

$$* \bar{V} = 129.8 \text{ knots} \quad R = 47.8 \text{ knots}$$

$$** \bar{V} = 116.7 \text{ knots} \quad R = 51.4 \text{ knots}$$

Reduction in take-off separation provides a similarly large increase in operations rate - halving t_{OD} increases λ by 29 - 34%. This might be accomplished by sequencing of departures to assure that separate routes are followed by successive aircraft or by maintaining close radar control on the departure routes.

It is noted that the difference in the aircraft characteristics of the two airports affect operations capacity less than they affect the landing capacity, primarily as a result of the cancellation of the opposite effects on λ_D and λ . For similar reasons, changes in m would have little effect on operations rate. The effect of r_0 , not indicated in Figure 4.12 would be large, but only the radar case of $r_0 = 2$ miles is presented since all major terminals are radar-equipped.

In addition to an indication of relative effects, the data may be used to roughly estimate the absolute value of operations capacity. Using a value of t_0 slightly greater than 1 minute and a value of t_{OD} of about 1.5 minutes, an operations capacity of 40-45 operations per hour is noted, consistent with current experience although slightly higher since it is assumed that the separation standards are exactly adhered to.

CHAPTER 5

CONCLUSIONS

The investigations reported herein make it possible to delineate some of the significant factors affecting airport capacity, and to draw some conclusions regarding potential means for increasing that capacity. These are reported in this chapter. Since the applicability of these conclusions is limited by the extent to which the physical situation being studied is realistically represented by the assumptions of the model, some of the major assumptions are discussed to indicate the extent of the models' limitations for general applicability. The chapter is concluded with some discussion of the directions in which this research may be extended.

The capacity of a runway and the factors affecting capacity depend on whether that runway is used for only a single operation or for both landings and take-offs. It appears that when interposition is possible the operations rate for the airport is maximized when operations can be mixed on the runways in use, thereby retaining the advantages resulting from interposition.

It is also necessary, in operating the runway, to decide whether long runs of landings and take-offs should be performed (with interposition wherever possible) or whether operations should be alternated by intentionally spacing the landings so that a take-off can be inserted in every landing interval. Such a decision can be made based on the criterion of relation (4.6) and the variations of it discussed in Chapter 4. It can often be expected that the decision will dictate intentional alternation of operations, but this is not always the case. When the take-off interposition rate is high, more may be lost by intentionally handicapping the landing rate to provide alternation of operations than if the take-offs that could not be interposed were performed as a separate run.

5.1 Improving Landing Capacity

When considerations dictate the use of a single runway for landings only, then it is desired to maximize the landing capacity of that runway. This can most effectively be achieved by a reduction in the distance separation required at the beginning of the common landing path (s_0). The current separation standard of 3 miles should be examined to determine whether it can be reduced without endangering safety; it might be possible to reduce s_0 in only those cases where a slow aircraft follows a fast one, thereby reducing the extremely long landing intervals. Separation might be reduced by increasing the effective precision of radar observation either through more frequent observation of relative aircraft positions or through more precise radars.

Landing rate could be appreciably increased by increasing the mean of the landing velocity ($\bar{V}$), or by decreasing its spread (R). But, by tradition and policy, little manipulation is possible with these parameters. It would probably be difficult to convince pilots to fly faster than they desire, since they generally want as long a time as possible for the transition (after breaking out of the clouds) from instrument to visual flying before touchdown. It might be possible to speed up the slow aircraft, which deteriorate landing capacity most seriously, and which are probably most capable of operating at higher speeds since they already have the longest transition time. Requiring all aircraft using the airport to adhere to some pre-determined minimum speed is an alternative technique for changing $\bar{V}$ and R .

If R is large, then advantage would accrue from shortening the length of the common landing path (m). This might be achieved by relocation of a radio facility (e.g., the outer marker) which all aircraft must overfly or by a change in standard procedures, keeping these changes consistent with the minimum distance required for stabilization on the final approach.

The minimum landing separation (t_0) represents the maximum runway-occupancy time that might be expected, and could be decreased by construction of high-speed turnoffs. The landing capacity

advantage that would result from such construction appears to be relatively small when t_0 has a value of 1 minute considered typical of current operation and the required gate separation is 3 miles. Where larger t_0 values exist or when the gate separation is reduced, the advantage would be correspondingly greater.

Based on a preliminary examination, sequencing of arrivals in an order other than first-come-first-served appears to offer little advantage in terms of landing capacity, particularly since substantial gain would be required to overcome the traditional resistance to priorities.

5.2 Increasing Operations Capacity

When landings and take-offs are performed on separate runways, or when the two types of operation are otherwise performed independently, the operations rate is improved by improving either individually, and the conclusions of the previous section would all apply, in addition to techniques for reducing the time interval between take-offs.

The more common situation is the one in which a runway is used for both operations, and it is possible to interpose take-offs between landings. All parameters that affect landing rate (except t_0) affect landing rate and interposition rate oppositely, so that the effects of any parameter on operations rate depend strongly on the values of the other parameters.

Consequently, the situation must be examined individually for each case to determine these effects. Under certain circumstances, for instance, even an increase of gate separation, which would adversely affect landing rate, may increase operations rate by providing greater opportunity for interposition of take-offs. The data of Appendices F and G may be used to cover many cases of interest.

Operations capacity is significantly affected by the location of r_0 , the point outside of which a landing aircraft must be to permit interposing a take-off before the landing. If r_0 is large, then interposition will rarely be possible if landings are run as close as possible, and separate runs of take-offs must be conducted. Since r_0 is determined largely by the runway-occupancy time of the departing aircraft, it is possible that high-speed turn-ons might provide some advantage if they could appreciably reduce the time of the take-off roll. Since the take-off must be held until the runway is clear, however, this possibility appears to be limited. Furthermore, the turn-on may introduce other operational problems (e. g., steering difficulties) during the high-speed run on the turn-on. In those cases where $r_0 = 4$ miles (i. e., airports not equipped with radar), the reduction of r_0 to 2 miles is another of the many advantages associated with the introduction of radar.

The operations rate is clearly affected by the minimum time interval between successive take-offs. These values are currently set by operational procedures, which should be re-examined to determine if these separations may also be reduced without compromising safety. Radar departure control provides opportunity for closing the departure separations. Where ANC separations of 1, 2, or 3 minutes (based on respective courses) are required, the first-come-first-served separation should be over-ruled when two aircraft having identical courses arrive successively; an aircraft intending to operate on a different course should be interposed between the two.

Reduction of runway occupancy time by techniques such as high-speed turnoff, while of little merit for raising landing capacity, can appreciably increase operations capacity when the runway is used for mixed operations by clearing the landing aircraft from the runway earlier, thus providing greater opportunity for interposition of a take-off. Problems of long time for dissipation of turbulence behind some aircraft must be considered in evaluating the minimum interval between operations on the same runway.

Sequencing procedures, which also appear to provide little advantage in a landing-only situation, might contribute appreciably to operations capacity. The long landing intervals that appear to inevitably develop when some of the landing intervals are made short can be used for interposition of take-offs, thus making the procedures practical.

5.3 Limitations of the Models

In the practice of operations research, it has been found that system models are valuable devices for obtaining an understanding of the operation of a system and as research devices for performing some preliminary investigations into the system's operation without the expense and difficulty of interfering with an operating system or constructing a designed system. A "system model" is basically a mathematical representation of the relationships among the parameters characterizing the system. Formulation of a model requires first a specification of a set of postulates or assumptions regarding the manner in which the system operates, and then involves mathematical manipulation of these primary relationships. A certain minimum set of assumptions is necessary to provide a starting point for the analysis; additional assumptions are imposed for analytical convenience and to make the analysis tractable, or to emphasize in the model specific interactions in the system which are of particular interest.

The ability to extrapolate to the real world the results derived with the model is obviously limited by the extent to which the model's assumptions represent the real world. The assumptions are always an idealized abstraction of reality, and are never exactly satisfied in any complex operating system, particularly when human behavior is involved. It is thus necessary for anyone contemplating application of a model, or

the conclusions drawn from a model, to critically examine the correspondence between the real world he is studying and abstract world of the model, and to determine the effect of their differences on the decisions to be made. He must decide whether the assumptions of the model are adequately satisfied and the results may be applied, perhaps with some slight modification, or whether the model is inapplicable and an alternative formulation is necessary to describe his particular situation.

In this section, some of the more important assumptions of the models developed in this study are discussed. The discussion indicates the reasons for some of these assumptions, the extent to which they may differ from reality, and the author's opinion of the effects of these differences on the major conclusions derived from the models. Modification of the models to conform more closely to reality are indicated.

5.3.1 Use of the Rectangular Velocity Distribution

Most of the numerical results generated in this study have been based on the assumption of a uniform velocity distribution of arriving aircraft. The actual velocity distribution is never exactly uniform, nor does it follow the discrete distribution considered here as an alternative. Rather, the true distribution is probably a continuous one that resembles the discrete distribution with variation about the specified velocity of the aircraft types.

The velocity distribution enters in the determination of landing capacity, queuing delay measures, and interposition rate. It has been shown (in Appendix E and in Section 3.4) that the difference in landing capacity between the uniform and the discrete distributions for a reasonable sample of distributions and system parameter values is very small, so that it appears probable that conclusions derived using the uniform distribution would hold for the actual distribution.

In the queuing analysis, the first two moments of the landing-time distribution enter the mean delay and queue length equations. The first moment (mean landing time) has been shown to be relatively independent of the form of the distribution. The second moment would probably also be relatively independent of the distribution forms, since both the first and second moments of the velocity distribution are equated in forming the uniform distribution. Consequently, the author believes that the delay estimates are sensitive to the first two moments of the velocity distribution and relatively insensitive to its form.

The entire landing-time distribution is used only in the estimation of interposition rate, and then only at a few points (rarely are more than three points used). It can be seen from the data presented in Appendix E and in Section 3.7.2 that, while the agreement is not as striking as with mean landing rate, the landing-time distribution with the uniform and

discrete distributions agree reasonably well, the probability S_{AA} rarely differing by as much as 0.1. Percentage errors are greatest in the right tail, but the absolute differences are the more important, and these rarely exceed 0.05 in the tail, so that a relatively small error is introduced into the computation of interposition rate. A better fit might be obtained by fitting a three-parameter family of distributions, such as the triangular distribution, in which the third parameter would provide a measure of skewness of the distribution.

On the basis of these considerations, the author believes that the results are relatively insensitive to the assumption of the uniform velocity distribution, and would be little changed if the actual distribution were known and used. The results are sensitive, however, to the mean and, to a lesser extent, the variance of the actual velocity distribution, as can be seen from the parametric investigations, so that it is important that these be known correctly.

The slight errors caused by the use of the uniform distribution are, in practice, compensated by the convenience of characterizing the velocity distribution by two physically-meaningful parameters as well as the analytical convenience of working with the uniform distribution.

5.3.2 Runway-Occupancy Time Assumption

Runway-occupancy time is represented in the operations capacity model by its mean value, τ_L . In reality, runway occupancy time varies stochastically about this mean value, and depends on the runway exit used, on the aircraft type, and on individual differences among pilots. A more realistic representation of runway-occupancy time would be as a discrete chance variable, as suggested in Section 4.3.1, in which the runway exit used would be the chance variable, and the runway occupancy time would be fixed for each exit. An even better representation would involve use of the joint distribution $f(V_1, T_{AD})$ of aircraft landing velocity and runway-occupancy time to determine the indicated probabilities in equations (4.14), (4.15), and (4.20). The analysis would then require the determination of the joint distributions (which would probably be done empirically) and integration over these distributions (which would probably be performed numerically).

These changes would affect the results of only the operations capacity analysis and would probably tend to reduce the interposition and operations rates. The very short landing intervals (where interposition does not occur in the present model) are associated with the situation in which a slow aircraft is the first of the pair (A_1). These slow aircraft generally have short runway-occupancy times, but the reduction below the

average would probably do little to improve interposition rate since the landing interval is small and a reduction even to zero runway-occupancy time may not permit an interposition. The long intervals, on the other hand, occur when A_1 is a fast aircraft, which is likely to have a long runway-occupancy time. Since interposition generally can occur in the long interval, the increase in the runway-occupancy time may occasionally prevent an interposition. Thus, a bias appears towards reducing the interposition rates when the joint distribution of V_1 and T_{AD} is introduced. The author conjectures that the effect of this change is not large, particularly in examination of parametric effects, since it would probably serve occasionally to eliminate at most a single interposition in a long landing interval.

5.3.3 Representation of the Controller

In formulating a model of any system in which human performance plays a major role, representation of the human operators is generally the most difficult and the least valid portion of the model. In the models developed here, the intention is to represent the controller basically by the set of standards by which he supposedly operates. He is considered to perform with perfect judgment in ordering aircraft onto the common landing path with appropriate spacing; he is considered able to place A_2 exactly s_0 miles behind A_1 [when $V_2 \leq V^*(V_1)$] or, when $V_2 > V^*(V_1)$, an exactly-computed distance depending on the values of

V_1 and V_2 such that the time separation between the landings is exactly t_0 . The capacity thus determined represents an ideal performance, and represents an upper bound to attainable performance when the standards are obeyed. At many airports, the controllers tend to operate conservatively and to install additional separation to account for their errors, and their rates would be lower than those shown here. Often, however, particularly at the larger airports at times of peak demand, the controllers tend to aim for specified separations, but to tolerate separations that go below the standards if it appears certain that no dangerous situation is likely to develop. It may even be that, under saturation conditions, the controllers actually aim for a separation less than the standards, since they recognize the conservatism in the standards and are faced with the immediate problem of moving a large number of waiting aircraft onto and off the runways.

Most other analyses of the landing problem consider the effect of wave-offs as reducing the landing rate. A landing aircraft is waved off if it appears that the runway will be occupied (generally by the preceding arrival) when it would touch down. If t_0 represents the true maximum runway-occupancy time, and if t_0 separation is always maintained between successive landings, then there is no wave-off problem. In specifying values for t_0 , however, it more precisely represents a practical maximum, which is occasionally exceeded, and the controller often may violate t_0 . Thus, the actual landing rate would be lower by the effect

of the waved-off aircraft which do not land but which consume time in the landing sequence. If a wave-off rate is known, this may be expressed easily in the models. Otherwise, one may be computed using the joint distribution of landing separation and runway-occupancy time, which would have to be determined experimentally under saturation conditions.

5.3.4 Computation of Operations Capacity

Computations using the operations capacity model were performed only for the case of $\omega = 0$. Section 4.3.2 indicates the analytical procedure for dealing with the case of $\omega > 0$, and requires specification of the frequency of certain short run lengths. In conversations, controllers have indicated that they do attempt to space the landings as close together as possible, allowing take-offs to be interposed wherever possible, and that it is not until a reasonably long queue of take-offs has developed that the Tower asks Approach Control to discontinue the landings while the take-offs are dispatched. The alternate procedure of requiring a greater spacing between landings to allow more frequent take-off interposition is also used, however; it might be studied with the $\omega = 0$ model by using an increased value of s_0 .

The assumption is made that interpositions in the second-previous landing interval would not affect an interposition. This assumption can be violated (and then only with small probability) in only a small

number of possible situations in which parameters assume extreme values and which are of little practical concern. These cases could be treated with the techniques for the k -state Markov process developed in Section 4.3.3.

The most complex process investigated here is the one of interposing take-offs between landings when both the t_{oD} requirement and the r_o requirement must be satisfied, i.e., a take-off had been interposed in the previous interval. Consequently, the assumptions made in computing q_p , most of which were made for analytical convenience, are the most tenuous ones in the models. Further investigation is thus probably warranted in applying the model to those situations where interposition of take-offs is frequently limited by the need for maintaining separation between departures.

The assumptions that the take-off in the previous interval occurred as early as possible tended to over-estimate q_p and, consequently, ν_D . Use of the more conservative assumption of Section 4.3.4 could provide an under-estimate of q_p . (If results with the two differ only slightly, then the choice of assumption is immaterial.) Substitution of the more conservative assumption would reduce operations capacity, and by reducing interposition rate, might make less attractive those parameter changes which tend to increase operations capacity through increases in interposition rate. This effect results from the

increasing slope of the curve of operations rate as a function of interposition rate. A more complete analysis would require determination and use of the actual distribution of T_{DA} .

5.4 Extensions of the Research

This study has produced an analytical tool which can be used to study quantitatively an area that has produced much emotional debate -- the landing and operations capacity of airports and means for improving them. With this tool, it is possible for anyone who accepts the inherent assumptions to insert his own numbers to determine the capacity resulting from those values. Less easily, he might change some of the assumptions while keeping the basic framework of the model.

Extension of the research reported herein might be aimed in two primary directions: application of the models (with the extensions indicated, if necessary) to specific operational situations at individual airports, and theoretical extensions of the model.

Application would require data gathering in the specific situation being studied to obtain estimates of the parameters needed in the model. The model aids in this respect by indicating the important data to be collected. In the process of gathering the data, the assumptions inherent in the model should be examined to ascertain whether they apply

to the particular situation being studied; if not, variations in the model might be necessary. The model could then be used to determine the airport's capacity, which would be compared to the peak demands predicted for the future to determine if capacity must be increased to meet demand. Some alternative techniques for raising capacity could be evaluated with the model.

The model could also be used in the current fast-time ATC simulation efforts. The analytical formulation helps to delineate some of the most significant factors in terminal operation, which should be expressed in the simulation. In a simulation of the enroute system, in which the terminals are expressed only in terms of their operations rates, the model may be included in the simulation to compute the capacities of the individual airports.

Theoretical extension lies in a more complete formulation of the model to provide better estimates of capacity under varying conditions and in the use of the service rates derived with the model in queuing studies to determine delay measures. The landing capacity model should be extended to a consideration of the effect of controller error in spacing the aircraft at the runway gate, and to consider the effect of these spacing errors and stochastic runway-occupancy time on the reduction in landing rate through wave-offs. The operations capacity model should be

extended along the lines indicated in Section 4.3. Computation of q_P , particularly for large values of t_{oD} , could be improved by determining the distribution of T_{DA} for the last interposition in the previous interval. Expression of T_{DD} and T_{AD} , particularly the latter, as chance variables would provide a more complete model of operations capacity. Computations should be performed with values of $\omega > 0$. More complete examination should be given to the problem of multiple runways. Queuing models with a single server (the runway), multiple queues (landings and take-offs), and appropriate priority rules concerning the interaction of the queues, particularly the interposition of take-offs between landings, would have to be defined to arrive at estimates of delay, thereby permitting an economic assessment of improvements in capacity.

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APPENDIX A

DETERMINATION OF MEAN LANDING INTERVAL τ_{AA}

Referring to Figure 3.2, we consider first Case A, where

$s_o > b t_o$. Since

$$V^*(V_1) = n V_1 / (m + V_1 t_o), \quad (A-1)$$

there is some c ($a < c < b$) such that $V^*(c) = b$. The value of c is then given by:

$$c = mb / (n - b t_o) \quad (A-2)$$

Using equation (3.2) for $T_{AA}(V_1, V_2)$ and equation (3.6) for $f(V_1, V_2)$, and substituting these into equation (3.4) yields the following integral:

$$\begin{aligned} \tau_{AA} (b - a)^2 = & \int_a^c dV_1 \int_a^{V^*(V_1)} (n/V_2 - m/V_1) dV_2 \\ & + \int_c^b dV_1 \int_a^b (n/V_2 - m/V_1) dV_2 + \int_a^c dV_1 \int_{V^*(V_1)}^b t_o dV_2 \end{aligned} \quad (A-3)$$

Performing the integration with elementary methods results in:

$$(b - a)^2 \tau_{AA} = (nc + mb) \log_e (c/b) \quad (A-4)$$

$$+ (ma + nb) \log_e (b/a)$$

$$+ (c - a) n \log_e n - (n - t_o b) (c - a)$$

$$+ (n/t_o) [(m + a t_o) \log_e (m + a t_o) - (m + c t_o) \log_e (m + c t_o)]$$

In Case B, where $a t_o \leq s_o \leq b t_o$, the integral becomes:

$$(b - a)^2 \tau_{AA} = \int_a^b dV_1 \int_a^{V^*(V_1)} (n/V_2 - m/V_1) dV_2$$

$$+ \int_a^b dV_1 \int_{V^*(V_1)}^b t_o dV_2 \quad (A-5)$$

which, when integrated, yields:

$$(b - a)^2 \tau_{AA} = (ma + nb) \log_e (b/a) \quad (A-6)$$

$$+ (b - a) n \log_e n - (b - a) (n - t_o b)$$

$$+ (n/t_o) [(m + a t_o) \log_e (m + a t_o) - (m + b t_o) \log_e (m + b t_o)]$$

In Case C, where $s_o < a t_o$, there is some d ($a \leq d \leq b$) such that $V^*(d) = a$, which is given by

$$d = ma / (n - a t_o) . \quad (A-7)$$

τ_{AA} is then given by:

$$(b - a)^2 \tau_{AA} = (b - a)^2 t_o + \int_d^b d V_1 \int_a^{V^*(V_1)} (n/V_2 - m/V_1 - t_o) d V_2 \quad (A-8)$$

which integrates to:

$$\begin{aligned} (b - a)^2 \tau_{AA} = & n (b - d) \log_e (n/a) + (nb + ma) \log_e b \\ & - (nd + ma) \log_e d - (b - d) (n - a t_o) \\ & + (b - a)^2 t_o + \frac{n}{t_o} [(m + d t_o) \log_e (m + d t_o) \\ & - (m + b t_o) \log_e (m + b t_o)] \end{aligned} \quad (A-9)$$

All three of the above cases can be covered in a single equation

$$\begin{aligned} (b - a)^2 \tau_{AA} (x, y) = & (nx + mb) \log_e (x/b) \\ & + (ma + nb) \log_e b - n (b - y) \log_e a \\ & - (ny + ma) \log_e y + (x - y) n \log_e n \\ & - n (x - y) - b t_o (x - a) - a t_o (y - a) \\ & + (n/t_o) [(m + y t_o) \log_e (m + y t_o) - (m + x t_o) \log_e (m + x t_o)] \end{aligned} \quad (A-10)$$

where the arguments x and y are given by:

$$x = \text{Min} \{b, c\} \quad (\text{A-11})$$

$$y = \text{Max} \{a, d\} \quad (\text{A-12})$$

In Case D where $V^*(a) > b$, then $T_{AA} > t_o$ for all (V_1, V_2) , and τ_{AA} is given by:

$$(b - a)^2 \tau_{AA} = \int_a^b dV_1 \int_a^b (n/V_2 - m/V_1) dV_2 \quad (\text{A-13})$$

which integrates to:

$$\tau_{AA} = \frac{s_o}{b-a} \log_e \frac{b}{a}$$

In Case E, where $V^*(b) < a$, then $T_{AA} = t_o = \tau_{AA}$ for all (V_1, V_2) .

APPENDIX B

DERIVATION OF LANDING-TIME DISTRIBUTION $S_{AA}(t)$

The time interval between successive landings is a chance variable whose value is given by equation (3.2). The landing speeds of successive aircraft are assumed to be independently distributed with identical uniform distribution given by equation (3.6). The landing-time distribution is derived by first transforming to time variables:

$$T_2 = n/V_2 \quad \text{and} \quad T_1 = m/V_1 \quad (\text{B-1})$$

The Jacobian of the transformation is given by:

$$\begin{vmatrix} -n/t_1^2 & 0 \\ 0 & -m/t_2^2 \end{vmatrix} = \frac{mn}{t_1^2 t_2^2} \quad (\text{B-2})$$

The T's are monotonic (decreasing) functions of the V's, so that the density function of the T's is given by

$$\begin{aligned} f(t_1, t_2) &= \frac{mn}{(b-a)^2} \cdot \frac{1}{t_1^2 t_2^2} & \begin{matrix} n/b \leq t_2 \leq n/a \\ m/b \leq t_1 \leq m/a \\ (b \neq a) \end{matrix} & (\text{B-3}) \\ &= 0 \text{ elsewhere} \end{aligned}$$

The distribution function

$$S_{AA}(t) = \Pr \left\{ T_{AA} \geq t \right\} = \Pr \left\{ T_2 \geq T_1 + t \right\} \quad (B-4)$$

is the complement of the cumulative distribution function. It can be determined by integration over the (T_1, T_2) distribution. Figure B-1 illustrates the various possible values of the line $T_2 - T_1 = t_0$ in (t_1, t_2) - space [with the case designations corresponding to those shown in Figure 3.2 for (V_1, V_2) - space]. For any case, the entire probability below the line is concentrated at the line giving the positive $\Pr \left\{ T_{AA} = t_0 \right\}$ so that $S_{AA}(t)$ must be computed only for $t > t_0$.

The computation is illustrated for Case A, as depicted in Figure B-2. It is seen that any $t_0 \leq t \leq (n/a - m/b)$, is represented by a 45° line and $S_{AA}(t)$ is the area above the line, as illustrated by the dotted line and shaded area in Figure B-2. The $S_{AA}(t)$ function is continuous but must be defined piecewise. Thus:

$$S_{AA} \left\{ t \mid s_0/a \leq t \leq n/a - m/b \right\} \quad (B-5.1)$$

$$= \frac{mn}{(b-a)^2} \int_{m/b}^{n/a-t} dt_1 \int_{t_1+t}^{n/a} \frac{1}{t_1^2 t_2^2} dt_2$$

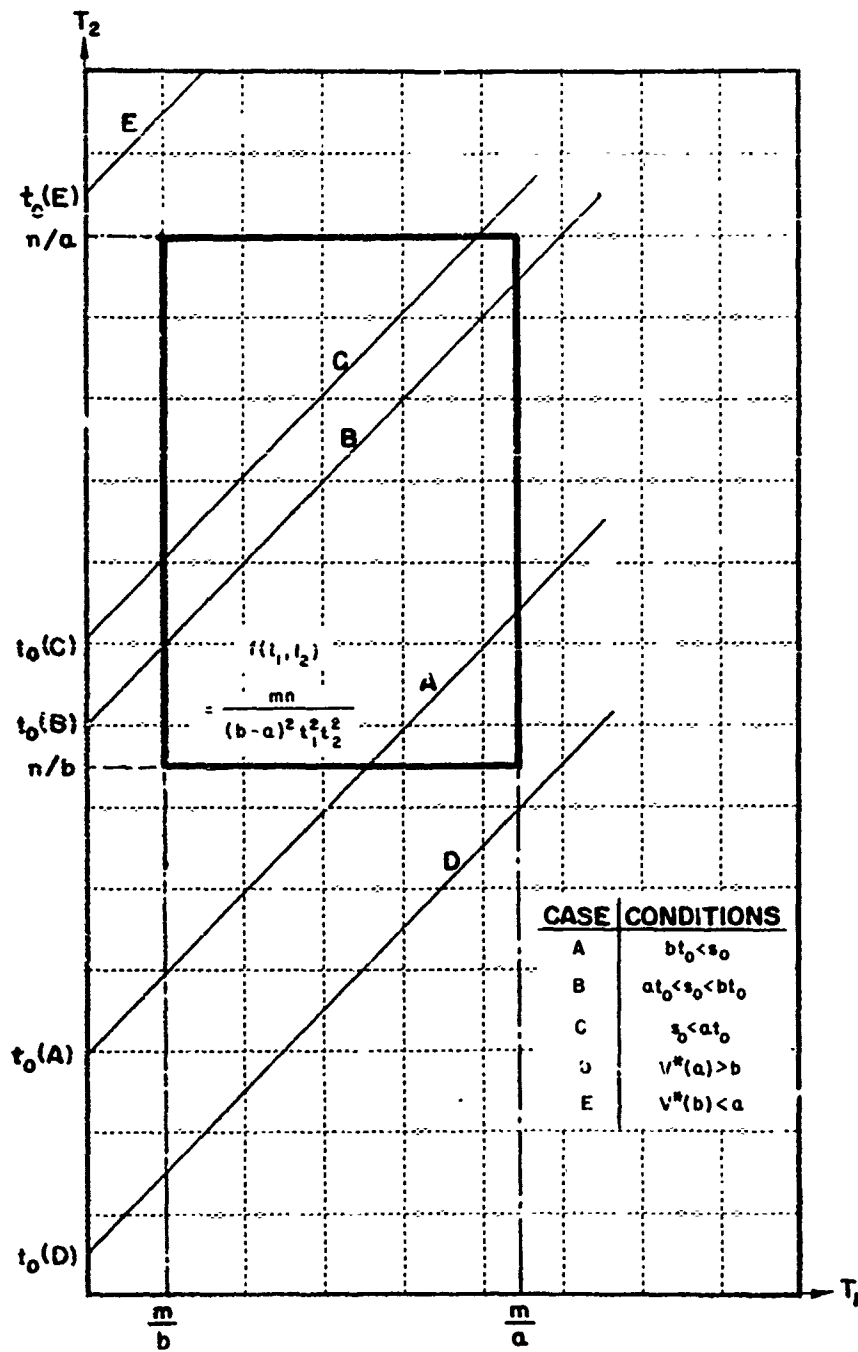


FIGURE B-1
FIVE CASES IN (T_1, T_2) SPACE

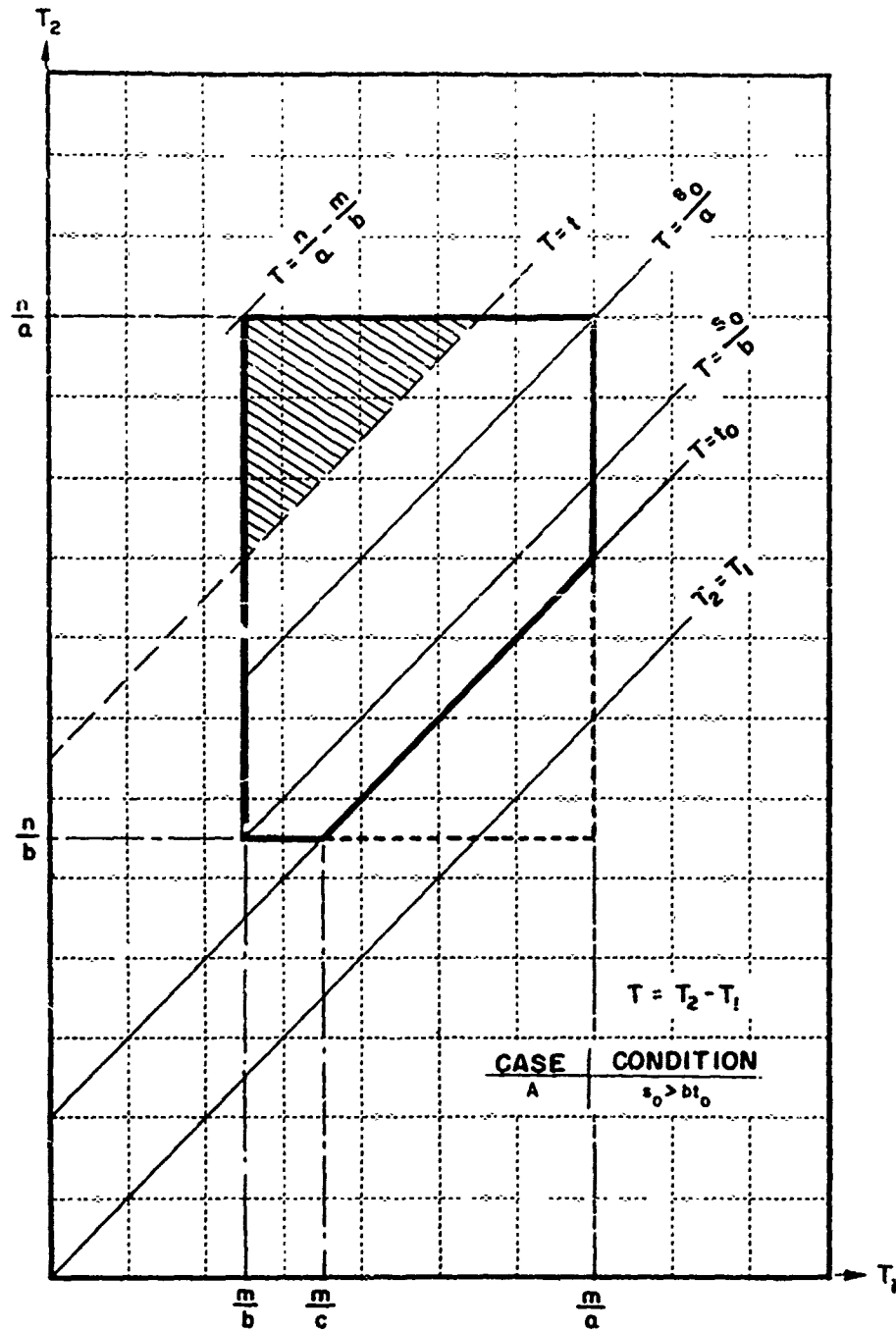


FIGURE B-2
DETERMINATION OF $S_{AA}(t)$ FOR CASE A

$$S_{AA} \left\{ t \mid s_o/b \leq t \leq s_o/a \right\} = S_{AA} (s_o/a) \quad (B-5.2)$$

$$+ \frac{m n}{(b-a)^2} \int_{m/b}^{m/a} dt_1 \int_{t_1+t}^{t_1+s_o/a} \frac{1}{t_1^2 t_2^2} dt_2$$

$$S_{AA} \left\{ t \mid t_o < t \leq s_o/b \right\} = S_{AA} (s_o/b) \quad (B-5.3)$$

$$+ \frac{m n}{(b-a)^2} \int_{m/b}^{n/b - t} dt_1 \int_{n/b}^{t_1+s_o/b} \frac{1}{t_1^2 t_2^2} dt_2$$

$$+ \frac{m n}{(b-a)^2} \int_{n/b - t}^{m/a} dt_1 \int_{t_1+t}^{t_1+s_o/b} \frac{1}{t_1^2 t_2^2} dt_2$$

Using elementary methods to perform the integration indicated, $S_{AA}(t)$ is determined, for Case A, to be:

$$S_{AA} \left\{ t \mid -\infty \leq t \leq t_o \right\} = 1 \quad (B-6.1)$$

$$S_{AA} \left\{ t \mid t_o < t \leq s_o/b \right\} = \quad (B-6.2)$$

$$\frac{1}{t^2 (b-a)^2} \left\{ [mb - an + t(b^2 + a^2 - ab)] t + mn \log_e \frac{(m + at)(n - bt)}{mn} \right\}$$

$$S_{AA} \left\{ t \mid s_o/b \leq t \leq s_o/a \right\} = \frac{n - at}{t(b-a)} + \frac{mn}{t^2 (b-a)^2} \log_e \frac{m + at}{m + bt} \quad (B-6.3)$$

$$S_{AA} \left\{ t \mid \frac{s_o}{a} \leq t \leq \frac{n}{a} - \frac{m}{b} \right\} = \frac{1}{t^2 (b-a)^2} \left\{ tb(n-at) - at m \right. \\ \left. - mn \log_e \left[\frac{(n - at)(m + bt)}{mn} \right] \right\} \quad (B-6.4)$$

$$S_{AA} \left\{ t \mid \frac{n}{a} - \frac{m}{b} \leq t \leq \infty \right\} = 0 \quad (B-6.5)$$

As can be noted in Figure B-1, the function (B-6.3) is meaningless for Case B since $s_o < b t_o$. Similarly in Case C, (B-6.3) and (B-6.4) are meaningless. The distribution is given in both of these cases by the valid portion of (B-6), using t_o as the lower bound on t in the last portion. In Case D, $T_{AA} \geq n/b - m/a > t_o$, so that t_o must be replaced by t'_o , where:

$$t'_o = \text{Max} \left\{ t_o; n/b - m/a \right\}$$

In Case E, T_{AA} takes the constant value t_o .

APPENDIX C DERIVATION OF τ_I

The r_o requirement for interposing a take-off in a landing interval is given by:

$$T_{AA} > \tau_L + r_o/V_2 \quad (C-1)$$

It is necessary to determine τ_I , the average landing interval given that the r_o requirement is satisfied. We consider first only the case (Case I) where

$$r_o / (t_o - \tau_L) > b \quad (C-2)$$

which is a necessary and sufficient condition for:

$$T_{AA} = \frac{n}{V_2} - \frac{m}{V_1} > t_o \quad (C-3)$$

for all (V_1, V_2) that permit interposition. In this case, the r_o requirement is:

$$(n - r_o) / V_2 - m/V_1 > \tau_L. \quad (C-4)$$

This is met for:

$$V_2 < \tilde{V}(V_1) \quad (C-5)$$

where:

$$\tilde{V}(V_1) = \frac{(n - r_o) V_1}{V_1 \tau_L + m} \quad (C-6)$$

The $\tilde{V}$ function is of the same form as V^* , and could occur in the corresponding five cases shown in Figure 3.2. The desired τ_I is the expected value of $(n/V_2 - m/V_1)$ in the region below and to the right of the appropriate curve. In Case ID, $\tau_L = \tau_{AA}$, and in Case IE, $\tau_I = \tau_{AA} = t_o$. In the remaining cases, τ_I is determined by integration. Thus, in Case IA,

$$\tau_I = \frac{\int_a^e dV_1 \int_a^{\tilde{V}(V_1)} \left(\frac{n}{V_2} - \frac{m}{V_1} \right) dV_2 + \int_e^b dV_1 \int_a^b \left(\frac{n}{V_2} - \frac{m}{V_1} \right) dV_2}{\int_a^e dV_1 \int_a^{\tilde{V}(V_1)} dV_2 + \int_e^b dV_1 \int_a^b dV_2} \quad (C-7)$$

where $\tilde{V}(e) = b$. The denominator in (C-7) is the normalizing factor equal to $q_{\bar{P}}$ of equation (4.17). The integrations are performed by elementary methods, and yield, for the numerator N_I :

$$N_I = ne \log_e [(n - r_o) e/a] - na \log_e (n - r_o) \quad (C-8)$$

$$- n G(e) + n G(a) - \frac{m(n - r_o)}{\tau_L} \log_e \left(\frac{\tau_L e + m}{\tau_L a + m} \right) \\ + ma \log_e (e/a) + n(b-e) \log_e (b/a) - m(b-a) \log_e (b/e)$$

$$\text{where } G(x) = [(\tau_L x + m) / \tau_L] \log_e (\tau_L x + m)$$

The denominator integrates to:

$$q_{\overline{P}} = [(n - r_o) / \tau_L - a] [e^{-a}] \quad (C-9)$$

$$- \frac{m(n - r_o)}{\tau_L^2} \log_e \frac{\tau_L e + m}{\tau_L a + m} + (b - e)(b - a)$$

Similar expressions can be derived for Cases IB and IC.

All three cases can be covered by the general expression:

$$\tau_I(x, y) = N_I(x, y) / q_{\overline{P}}(x, y) \quad (C-10)$$

where:

$$N(x, y) = n x \log_e \frac{(n - r_o) x}{a} - n y \log_e \frac{(n - r_o) y}{a} \quad (C-11)$$

$$- n G(x) + n G(y) - \frac{m(n - r_o)}{\tau_L} \log_e \left(\frac{\tau_L x + m}{\tau_L y + m} \right)$$

$$+ m a \log_e \frac{x}{y} + n(b - x) \log_e \frac{b}{a} - m(b - a) \log_e \frac{b}{x}$$

$$q_{\overline{P}}(x, y) = \left(\frac{n - r_o}{\tau_L} - a \right) (x - y) - \frac{m(n - r_o)}{\tau_L^2} \log_e \frac{\tau_L x + m}{\tau_L y + m} \quad (C-12)$$

$$+ (b - x)(b - a)$$

where:

$$x = \text{Max} \{e, b\}$$

$$y = \text{Max} \{z, f\}$$

$$e = \frac{b m}{n - r_o - a \tilde{\tau}_L} \quad (\text{from } \tilde{V}(e) = b)$$

$$f = \frac{a m}{n - r_o - a \tilde{\tau}_L} \quad (\text{from } \tilde{V}(f) = a)$$

Thus, in Case IA, $\tilde{\tau}_I = \tilde{\tau}_I(e, a)$

in Case IB, $\tilde{\tau}_I = \tilde{\tau}_I(b, a)$

in Case IC, $\tilde{\tau}_I = \tilde{\tau}_I(b, f)$

APPENDIX D

DERIVATION OF $q_{\overline{P}}$ AND $\widetilde{\mathcal{C}}_I$ WHEN $a < r_o / (t_o - \widetilde{\mathcal{C}}_L) < b$

The expression for $\widetilde{\mathcal{C}}_I$ in Case I [$b < r_o / (t_o - \widetilde{\mathcal{C}}_L) \equiv A$] is developed in Appendix C. If $A \leq a$, then $q_{\overline{P}} = 1$ and $\widetilde{\mathcal{C}}_I = \widetilde{\mathcal{C}}_{AA}$. In Case II ($a < A < b$), interposition can occur in two ways by (V_1, V_2) being in either of the shaded regions of Figure D-1 represented by the inequalities:

$$V_2 \leq \frac{(n - r_o) V_1}{V_1 \widetilde{\mathcal{C}}_L + m} \equiv \widetilde{V}(V_1) \quad (D-1)$$

$$V_2 \geq r_o / (t_o - \widetilde{\mathcal{C}}_L) \equiv A \quad (D-2)$$

It is necessary to find the size of the shaded regions (for $q_{\overline{P}}$) and the expected value of T_{AA} in the shaded regions (for $\widetilde{\mathcal{C}}_I$).

Both equalities are just satisfied at :

$$V_1 = \hat{V} = \frac{m r_o}{n (t_o - \widetilde{\mathcal{C}}_L) - r_o t_o} \quad (D-3)$$

We notice that $V^* (\hat{V}) = A$, and, by differentiation, that:

$$\widetilde{V}'(\hat{V}) > V^{*'}(\hat{V}) > 0 \quad (D-4)$$

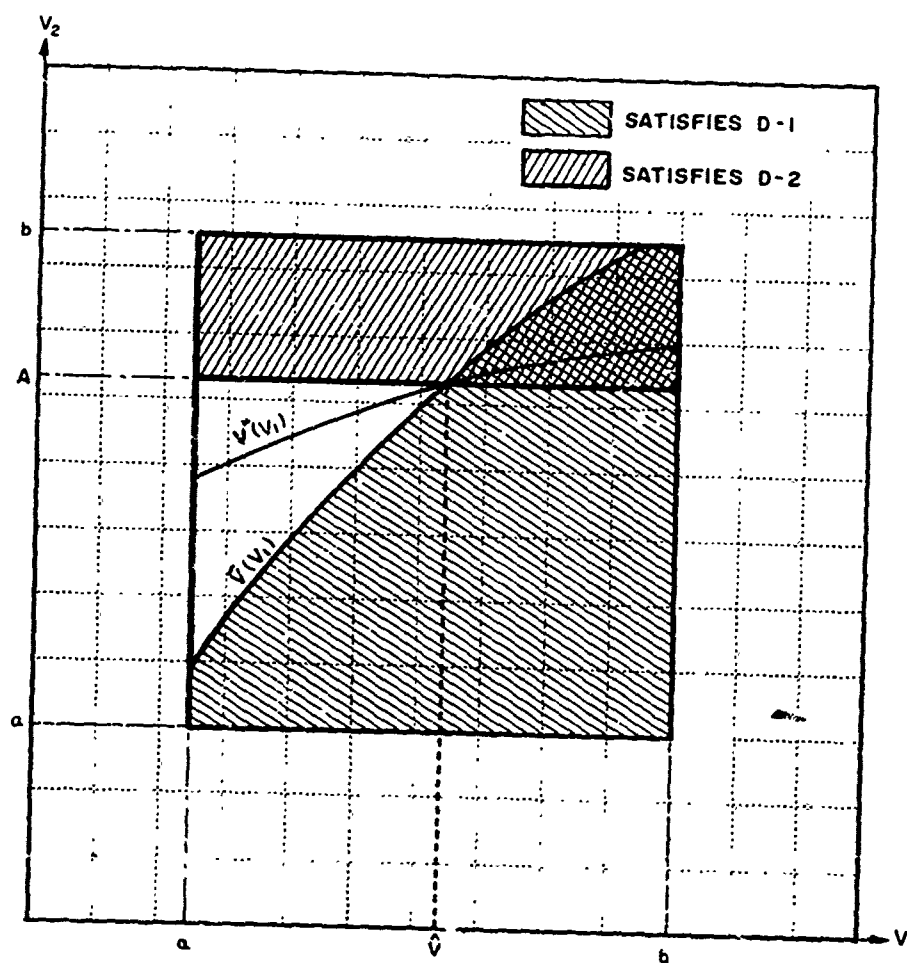


FIGURE D-1
THE r_0 REQUIREMENT WHEN $a < r_0/(t_0 - \tau_L) < b$

Thus, since there is a single intersection,

$$V^* (\hat{V}) = \tilde{V} (\hat{V}) \quad (D-5)$$

$$\tilde{V} (V_1) < V^* (V_1) \quad \text{for } V_1 < \hat{V} \quad (D-6)$$

$$V^* (V_1) < \tilde{V} (V_1) \quad \text{for } \hat{V} < V_1$$

If $\hat{V} > b$ or if $\hat{V} < a$, then the two r_o requirements are non-overlapping. Then $q_{\overline{P}}$ is given by:

$$q_{\overline{P}} = q_{\overline{P}_I} + A / (b - a) \quad (D-7)$$

where $q_{\overline{P}_I}$ is the value given in Appendix C for Case I. The value of τ_I is given by:

$$\tau_I = \frac{N_I + \tau_{AA} (A, b) (b - a)^2}{q_{\overline{P}}} \quad (D-8)$$

where N_I is the numerator of τ_I for Case I, and where $\tau_{AA} (A, b)$ is the value of τ_{AA} computed for $a = A$.

Consideration of the various situations possible when $a \leq \hat{V} \leq b$ is aided by reference to the cases of V^* in Figure 3.2. When V^* follows Case A, the requirements (D-4, 5, 6) and the impossibility of the case $\tilde{V} (a) < a$ and $\tilde{V} (b) > b$ permit a $\tilde{V}$

function only at Case A* [i. e., $\hat{V}(V_1) > V_1$] for $a \leq V_1 \leq b$. When V^* follows Case B, the $\hat{V}$ function may be any of Cases A, B, or C. When V^* is Case C, then only Case C is possible for $\hat{V}$. If V^* is of Case D, then Case I (of Appendix C) is applicable. If V^* is of Case E and $a < A < b$, then interposition is possible only by meeting the r_o requirement (D-2) and $q_{\overline{P}} = A / (b-a)$ and $\tau_I = t_o$.

The computation of τ_I in Case II for $a < \hat{V} < b$ and V^* in Cases A, B, or C is similar to that performed in Appendix C for Case I. We illustrate the process for V^* of Case A or B and $\hat{V}$ of Case C, as depicted in Figure D-2. We compute N_{II} and $q_{\overline{P}}$ by integration over the shaded area:

$$N_{II} = (b-a)^2 \tau_{AA} - \int_a^{\hat{V}} dV_1 \int_{V^*(V_1)}^A t_o dV_2 \quad (D-9)$$

$$- \int_a^{\hat{V}} dV_1 \int_a^{V^*(V_1)} \left(\frac{n}{V_2} - \frac{m}{V_1} \right) dV_2 + \int_f^{\hat{V}} dV_1 \int_a^{\hat{V}(V_1)} \left(\frac{n}{V_2} - \frac{m}{V_1} \right) dV_2$$

* The cases of $\hat{V}$ appear similar in V_1, V_2 - space to those for V^* , with the conditions being described in terms of $s_o - r_o$ and τ_L instead of s_o and t_o .

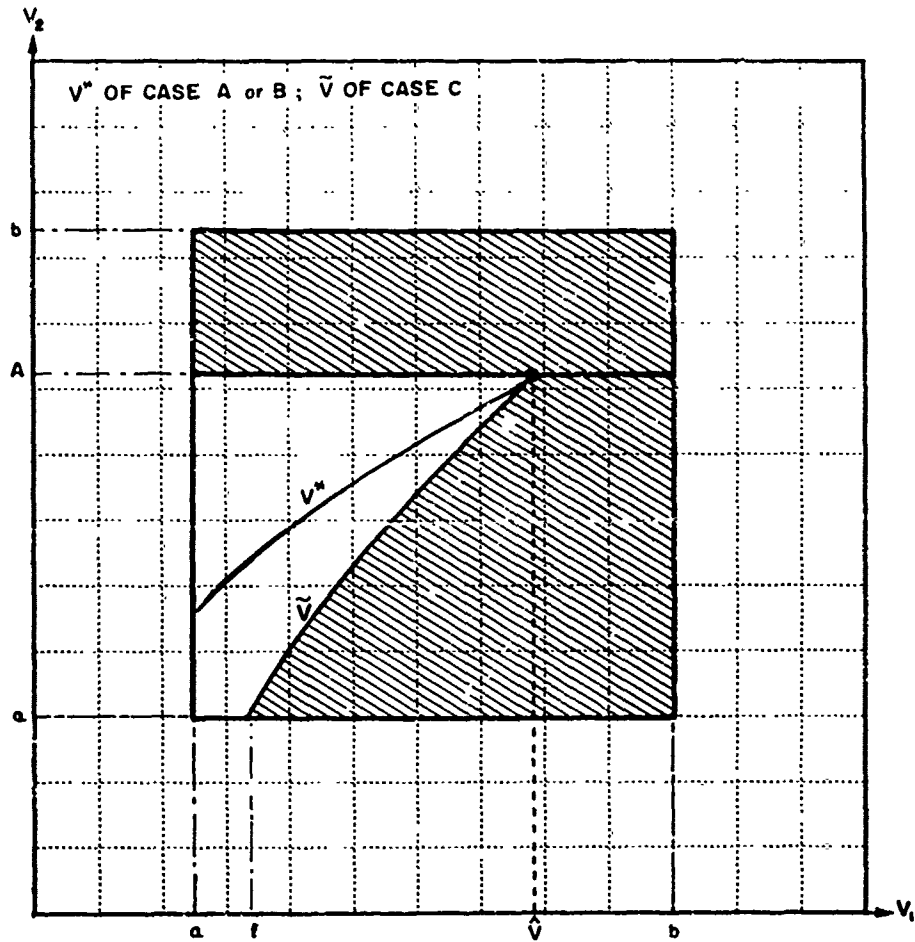


FIGURE D-2
 A PARTICULAR CASE OF THE τ_0 REQUIREMENT

$$q_{\overline{P}} = (b - a) (b - A) + (b - \hat{V}) (A - a) + \int_f^{\hat{V}} dV_1 \int_a^{\hat{V}(V_1)} dV_2 \quad (D-10)$$

Integrating, and generalizing for all Case II situations, yields:

$$\tau_I = N_{II}(x, y) / q_{\overline{P}}(x, y) \quad (D-11)$$

where:

$$\begin{aligned} N_{II}(x, y) = & (b - a)^2 \tau_{AA} - n [G_1(\hat{V}) - G_1(x) - G_2(\hat{V}) \\ & + G_2(y)] + n \left[\hat{V} \log_e \left(\frac{n-r_o}{n} \right) + y \log_e \left(\frac{ny}{a} \right) \right. \\ & \left. - x \log_e \left(\frac{(n-r_o)x}{a} \right) \right] - \frac{m(n-r_o)}{\tau_L} \log_e \frac{\tau_L \hat{V} + m}{\tau_L x + m} \\ & + ma \log_e \left(\frac{y}{x} \right) - \left(A - \frac{n}{t_o} \right) (\hat{V} - y) t_o - t_o (A-a)(y-a) \end{aligned}$$

$$q_{\overline{P}}(x, y) = (b - A) (b - a) + (b - V) (A - a)$$

$$\begin{aligned} & + \left(\frac{(n-r_o)}{\tau_L} - a \right) (\hat{V} - x) \\ & - \frac{(n-r_o)m}{\tau_L^2} \log_e \left(\frac{\tau_L \hat{V} + m}{\tau_L x + m} \right) \end{aligned}$$

where

$$G_1(z) = \left(\frac{\tau_L z + m}{\tau_L} \right) \log_e (\tau_L z + m)$$

$$G_2(z) = \left(\frac{t_0 z + m}{t_0} \right) \log_e (\tau_L z + m)$$

The arguments x and y take the values

$$x = \text{Max} \{a, f\}$$

$$y = \text{Max} \{a, d\}$$

where $f = \frac{am}{n - r_0 - a \tau_L}$ (from $\tilde{V}(f) = a$)

$$d = \frac{am}{n - t_0 a}$$
 (from $V^*(d) = a$)

Thus, if

V^* is Case A or B and $\tilde{V}$ is Case A or B, $\tau_I = \tau_I(a, a)$

V^* is Case B and $\tilde{V}$ is Case C, $\tau_I = \tau_I(f, a)$

V^* is Case C and $\tilde{V}$ is Case C, $\tau_I = \tau_I(f, d)$

APPENDIX E

INVESTIGATION OF THE RELATIONSHIP BETWEEN DISCRETE VELOCITY DISTRIBUTIONS AND THEIR UNIFORM EQUIVALENTS

This appendix provides further discussion of the general relationship between the discrete velocity distributions and the corresponding uniform distributions having identical mean and variance. The most recent data available on aircraft performance characteristics and on the distribution of aircraft types at several airports is used.

Distributions of only air carrier aircraft are considered since airline aircraft constitute the largest proportion of the traffic at major airports during IFR weather, and since data on the distribution of types of air carrier arrivals are most readily available. Data were obtained for the following seven airports:

1. Washington National (DCA)
2. Newark (EWR)
3. New York International Idlewild (IDL)
4. Los Angeles International (LAX)
5. LaGuardia (LGA)
6. Chicago Midway (MDW)
7. Chicago O'Hare (ORD)

The distribution of aircraft types arriving at each of these airports was determined from the March 15, 1960 Airline Guide [69]. These distributions are presented in Table E-1. For each aircraft type, the speed along the glide path (also shown in Table E-1) was determined from a recent report prepared for the FAA by Fairchild Corporation [70], and from supplementary available information. The resulting velocity distributions (based on an assumption that the landing speeds of all aircraft of a given type are equal to the specified value) at the seven airports are presented in Table E-2 and depicted in Figure E-1.

Using these data, the characteristics of the equivalent uniform distributions were determined, and are tabulated at the bottom of Table E-2. Landing rates were determined for each of the airports under the alternative assumptions of uniform and discrete velocity distribution. The resulting landing rates at each airport for several parameter values are shown in Table E-3; the correlation between the two velocity distributions is depicted in Figure E-2, where the solid lines enclosing all the points but one represent deviations corresponding to differences of $\pm 2\%$. The average absolute difference in all 56 points is only 0.26 landings per hour, representing an error of about 0.6%. The maximum difference is 0.93, or 2.4%. The fit here is seen to be exceptionally good, and to be retained despite the differences in the shape of the original discrete distributions.

TABLE E-1

DISTRIBUTION OF AIRCRAFT TYPES ARRIVING AT AIRPORTS
 (Data from March 15, 1960 Airline Guide)

Aircraft Type	Velocity (knots)	DCA	EWR	IDL	LAX	LGA	MDW	ORD
Boeing 707	135			25	32			20
Britannia	135			2				
Stratocruiser	120			1			1	
Constellation	130	43	24	22	23	42	53	4
Comet	125			1				
Convair	115	23	29	9	13	14	39	22
DC-3	95	15	10		5	14	69	6
DC-4	110	16		6			6	6
DC-6	130	28	13	18	33	40	45	14
DC-6B	130	20	8	31	40	6	18	11
DC-7	125	21	5	13	24		24	11
DC-7B	125	26	18	9	4		8	4
DC-7C	130	14		12	2		8	
DC-8	145			12	13			15
F-27	100	3			11		5	
Electra	145	42	18	38	8	47	50	2
Martin 404	110	44	22	8	6	13	16	2
Viscount	125	55	25	23	1	43	46	7

TABLE E-2

VELOCITY DISTRIBUTIONS AT AIRPORTS

(Data from March 15, 1960 Airline Guide)

Velocity (knots)	DCA (%)	EWR (%)	IDL (%)	LAX (%)	LGA (%)	MDW (%)	ORD (%)
95	4.29	5.81	0	2.33	6.39	17.78	4.84
100	0.86	0	0	5.12	0	1.29	0
110	17.14	12.79	6.09	2.79	5.94	5.67	6.45
115	6.57	16.86	3.91	0.05	6.39	10.05	17.74
120	0	0	0.44	0	0	0.26	0
125	29.14	27.91	20.00	13.49	19.64	20.10	17.74
130	30.00	26.16	36.08	45.57	40.18	31.96	23.39
135	0	0	11.74	14.88	0	0	16.13
145	12.00	10.47	21.74	9.77	21.46	12.89	13.71
$\bar{V}$ (knots)	124.2	123.1	131.0	127.7	127.9	121.7	126.3
R (knots)	41.6	41.4	32.8	38.4	44.4	53.6	42.6
a (knots)	103.4	102.4	114.6	108.5	105.7	94.9	105.0
b (knots)	145.0	143.8	147.4	146.9	150.1	148.5	147.6

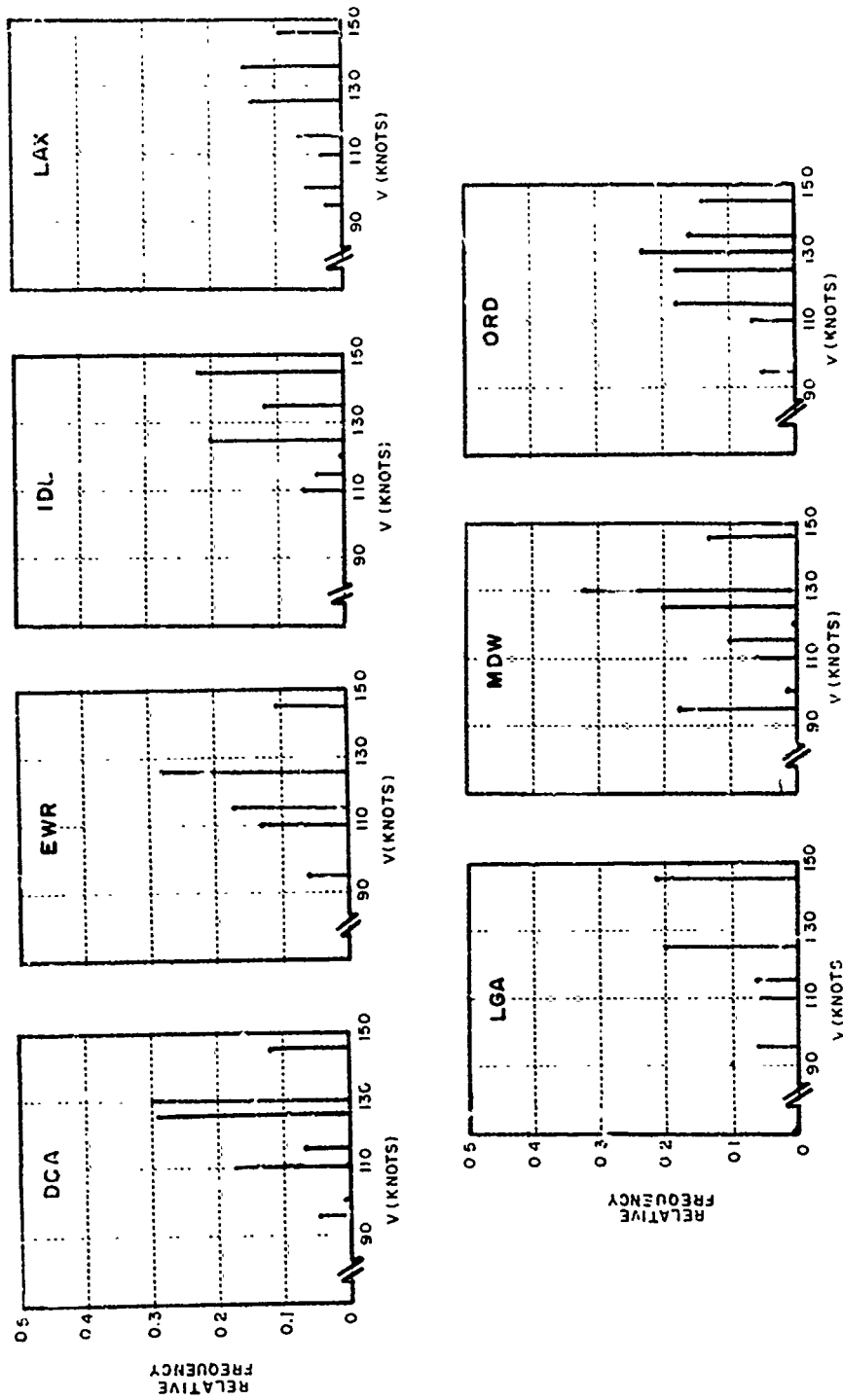


FIGURE E-1
VELOCITY DISTRIBUTIONS AT SEVEN AIRPORTS

TABLE E-3
COMPARISON OF LANDING RATES AT SEVEN AIRPORTS USING
UNIFORM AND DISCRETE VELOCITY-DISTRIBUTION ASSUMPTION

Airport	Distribution	(10, 3, 1) ^a	(10, 3, 0.5)	(10, 2, 1)	(10, 2, 0.5)	(4, 3, 1)	(4, 3, 0.5)	(4, 2, 1)	(4, 2, 0.5)
MDW	Discrete	34.4	37.0	42.3	48.9	38.7	39.8	50.1	57.7
	Uniform	34.8	37.5	42.5	49.8	38.8	39.8	50.2	58.1
ORD	Discrete	37.9	40.4	46.8	55.3	41.1	41.7	53.5	61.6
	Uniform	38.0	40.7	46.7	55.4	41.2	41.8	53.3	61.9
DCA	Discrete	37.4	39.8	46.7	54.4	40.5	41.0	53.2	60.8
	Uniform	37.4	38.9	46.2	54.5	40.5	40.9	52.9	60.8
IDL	Discrete	41.1	43.1	51.3	60.8	43.2	43.4	56.5	64.0
	Uniform	41.2	42.2	51.2	61.2	43.3	43.5	56.2	65.1
LGA	Discrete	37.9	40.4	47.0	55.0	41.4	42.1	53.6	61.6
	Uniform	38.3	41.1	46.9	55.8	41.6	42.2	53.5	62.6
EWR	Discrete	37.2	39.4	46.4	54.1	40.1	40.6	53.0	60.1
	Uniform	37.1	39.5	45.9	54.0	40.1	40.6	52.6	60.3
LAX	Discrete	38.9	40.9	48.9	56.9	41.7	42.4	54.8	62.4
	Uniform	39.0	41.4	48.2	57.2	41.8	42.2	54.3	62.8

^a The triplet at the top of each column indicates the (m [miles], s₀ [miles], t₀ [minutes]) combination for the column.

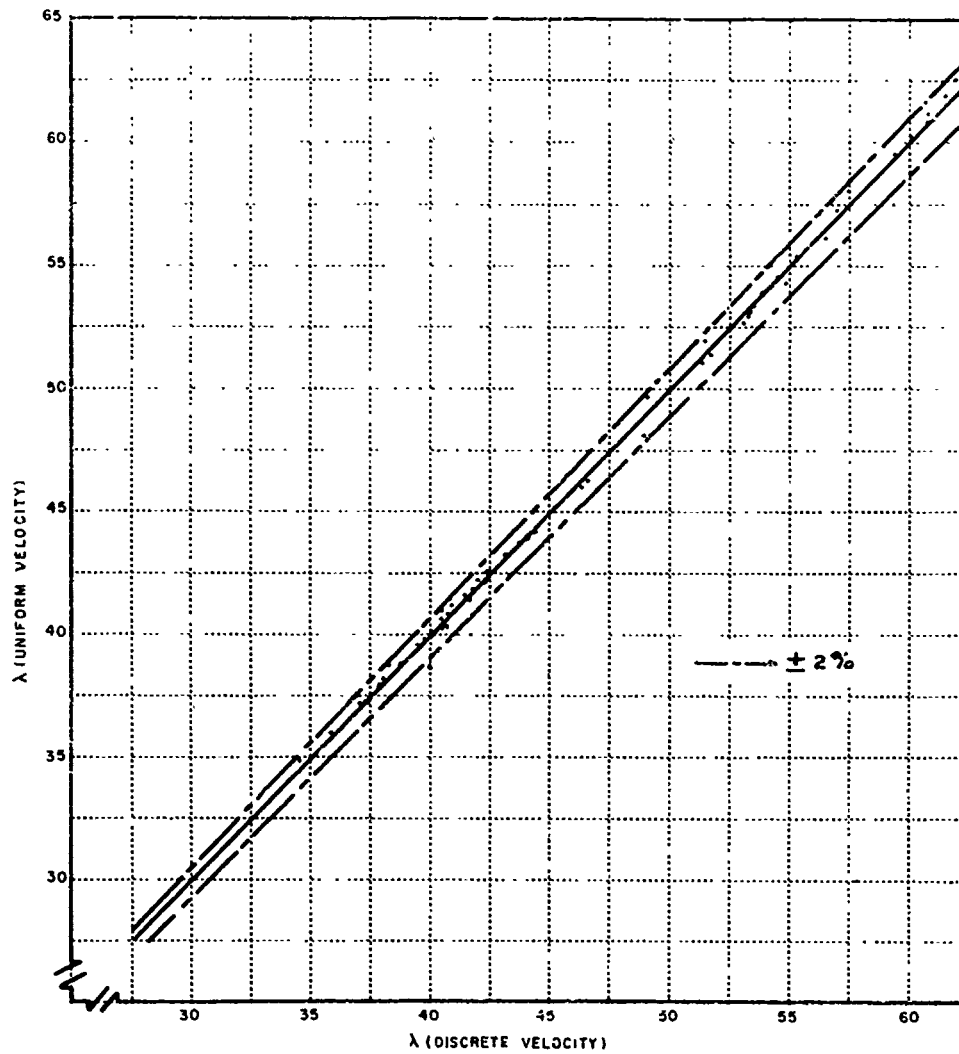


FIGURE E-2
CORRELATION BETWEEN LANDING CAPACITY AS COMPUTED USING
UNIFORM AND DISCRETE VELOCITY DISTRIBUTIONS

As would be expected, the fit over the entire distribution is less exact than in only the mean. These data are shown in Figures E-3.1 - E-3.7 for the cases of $m = 7$ and 10 miles, $s_0 = 3$ miles, and $t_0 = 0.5$ minutes. (The curves for larger t_0 values can be determined merely by displacing the discrete jump to the t_0 value desired.) Here again, despite the basic differences in the discrete distributions, the fit using the uniform distribution seems to be reasonably satisfactory.

Consideration of the two-point discrete distribution aids in examining the extent of the correspondence between the landing rates based on the uniform and discrete distributions. We consider that there are two aircraft types of velocities v_1 and v_2 , which occur with probabilities p and $(1 - p)$. The mean ($\bar{V}$) and range (R) of the distribution are given by:

$$\bar{V} = v_1 p + v_2 (1 - p) \quad (\text{E-1})$$

$$R^2 = 12 \sigma_V^2 = 12 (v_2 - v_1)^2 p (1 - p) \quad (\text{E-2})$$

or, for a fixed $\bar{V}$ and R :

$$v_1 = \bar{V} - R' / K_p \quad (\text{E-3})$$

$$v_2 = \bar{V} + R' / K_p \quad (\text{E-4})$$

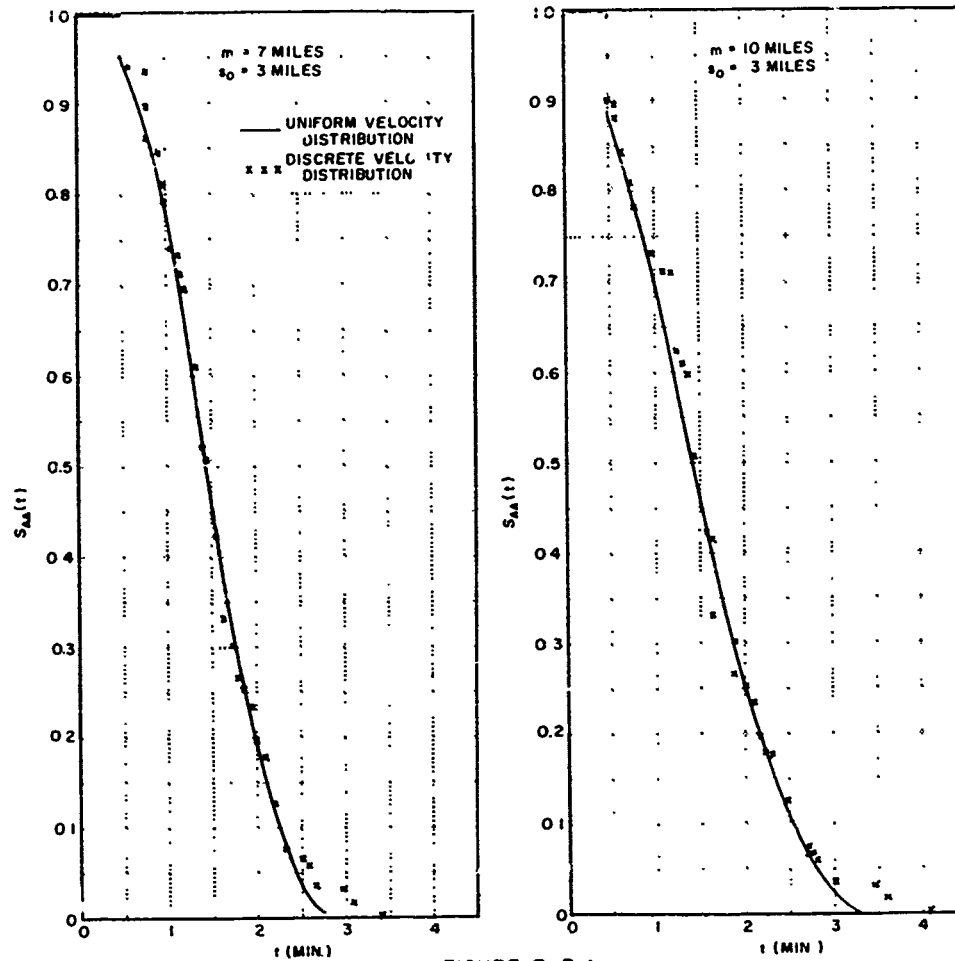


FIGURE E-3.1
LANDING-TIME DISTRIBUTION AT DCA
 $\bar{V}=124.2$ KNOTS $R=41.6$ KNOTS

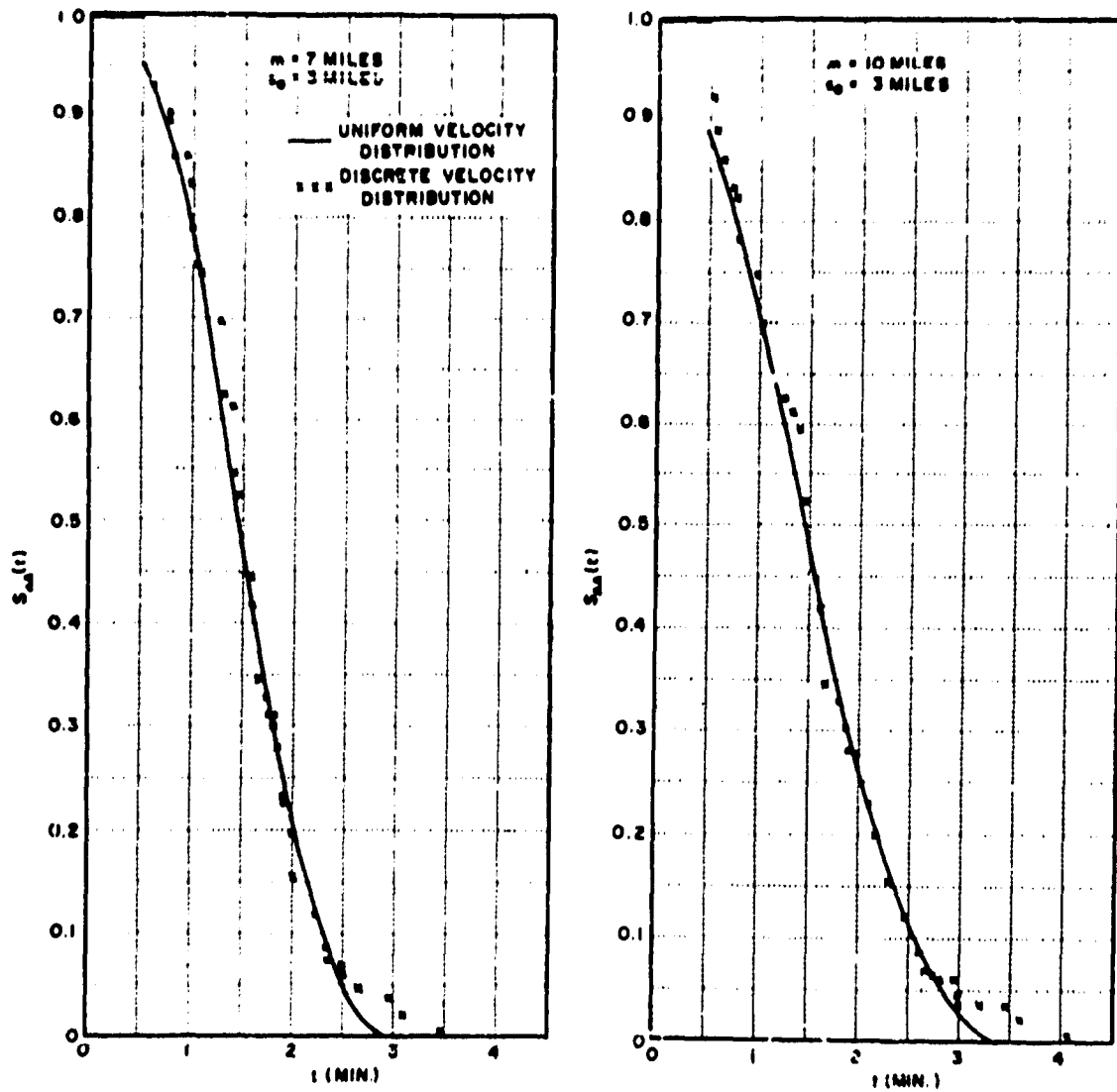


FIGURE E-3.2
LANDING-TIME DISTRIBUTION AT EWR
 $V = 123.1$ KNOTS $R = 41.4$ KNOTS

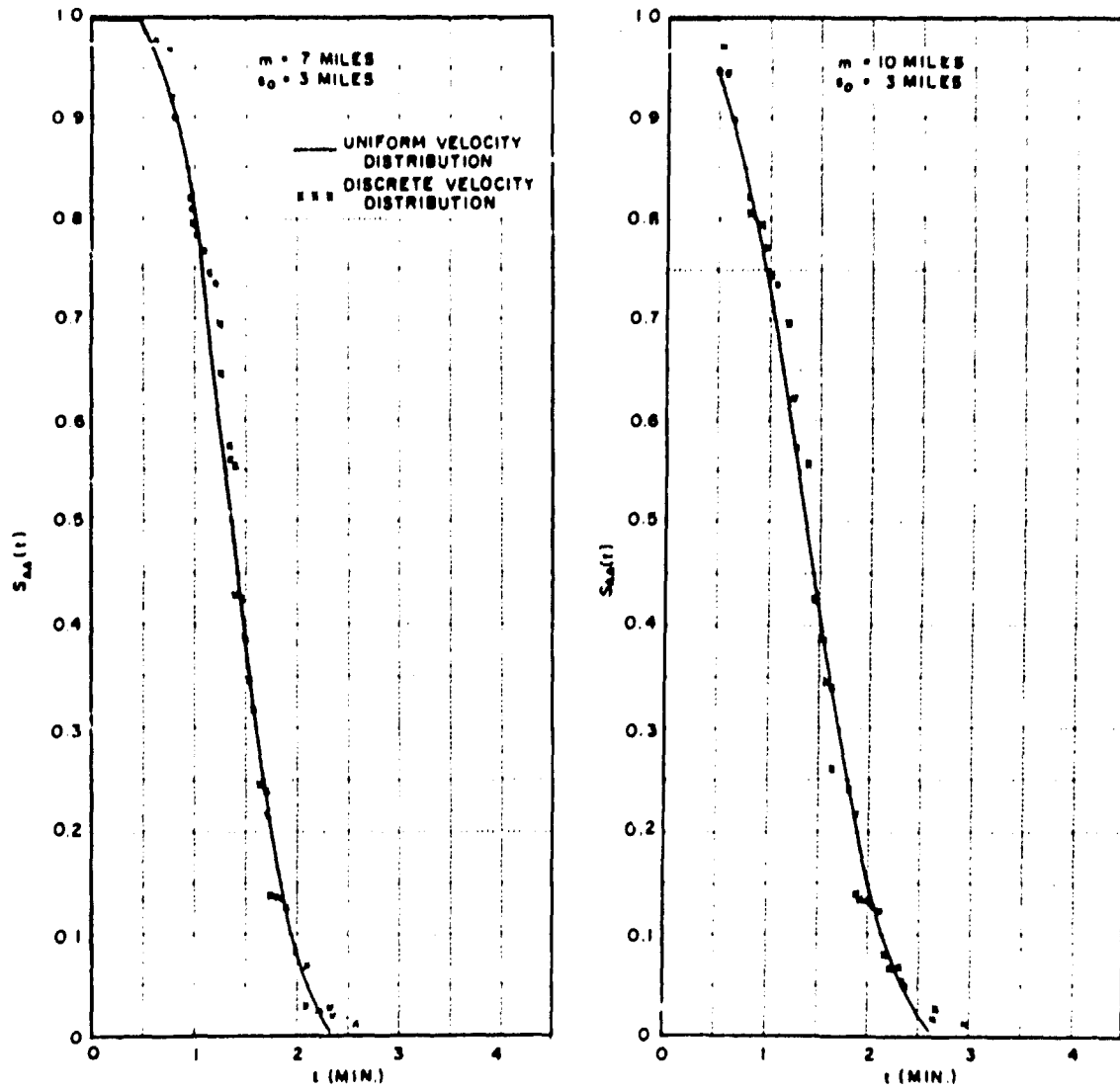


FIGURE E-3.3
LANDING-TIME DISTRIBUTION AT IDL
 $V = 131.0$ KNOTS $R = 32.7$ KNOTS

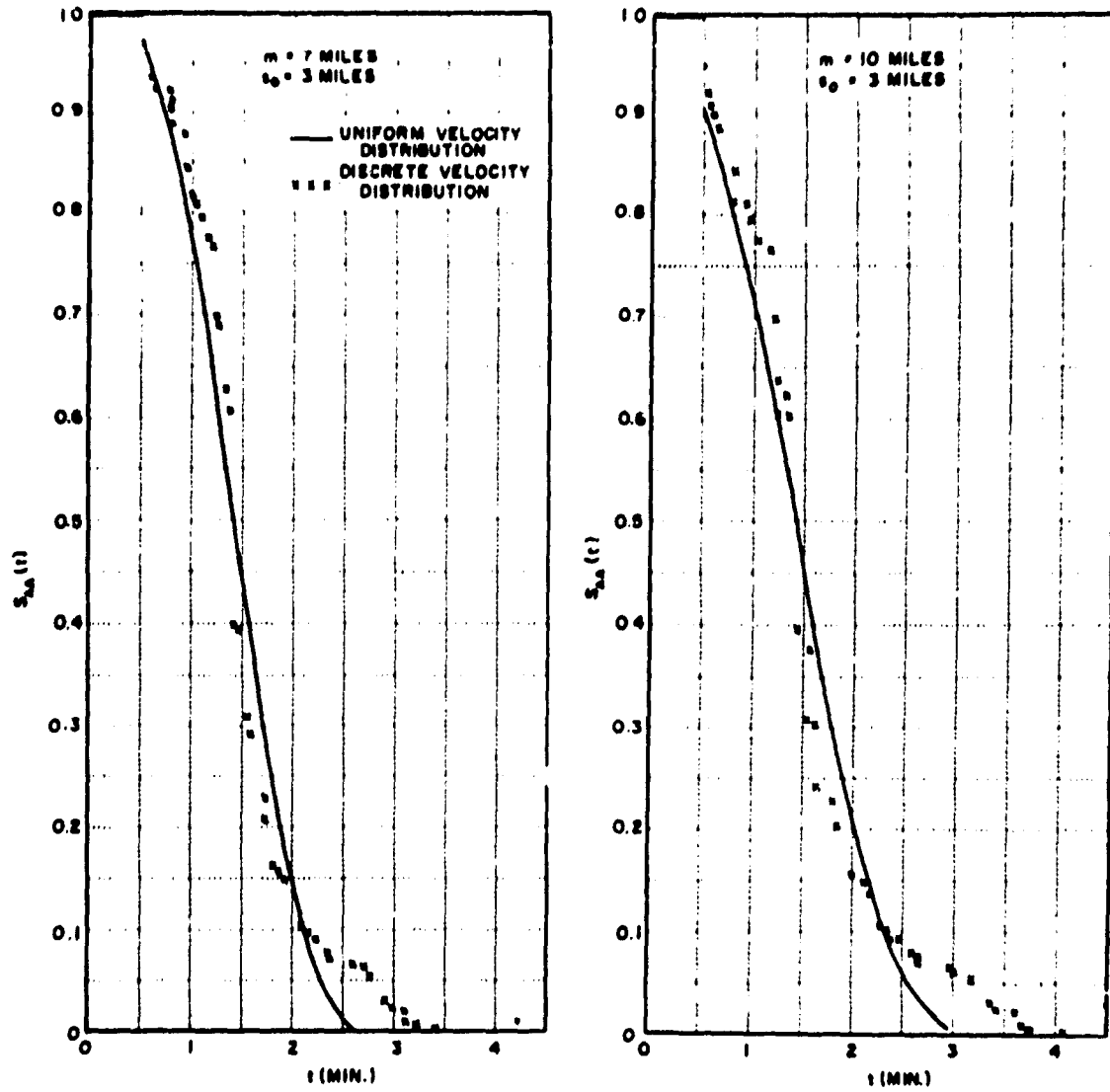


FIGURE E-3.4
LANDING-TIME DISTRIBUTION AT LAX
 $V = 127.7$ KNOTS $R = 38.4$ KNOTS

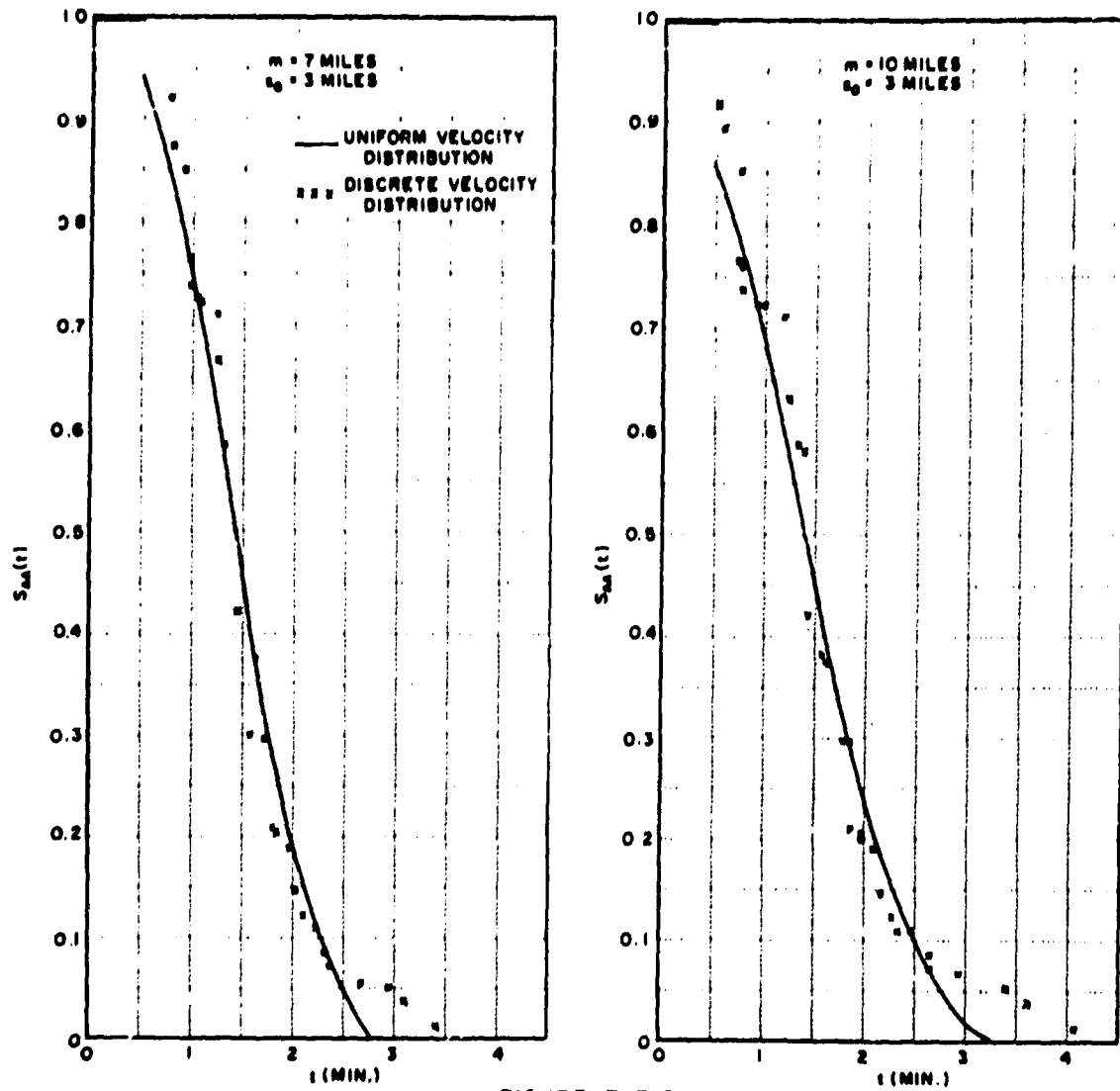


FIGURE E-3.5
LANDING-TIME DISTRIBUTION AT LGA
 $V = 127.9$ KNOTS $R = 44.5$ KNOTS

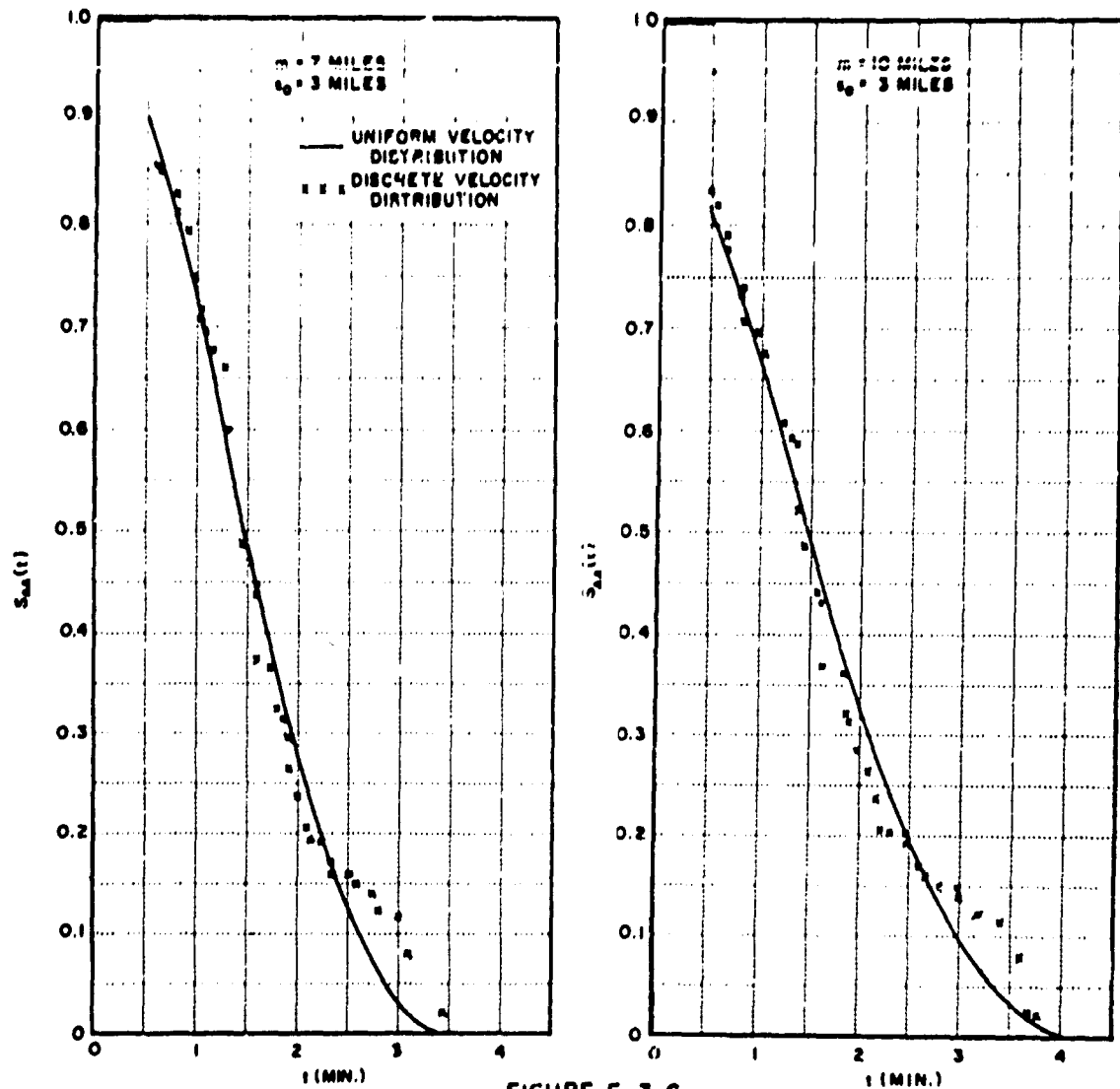


FIGURE E-3.6
LANDING-TIME DISTRIBUTION AT MDW
 $V = 121.7$ KNOTS $R = 53.6$ KNOTS

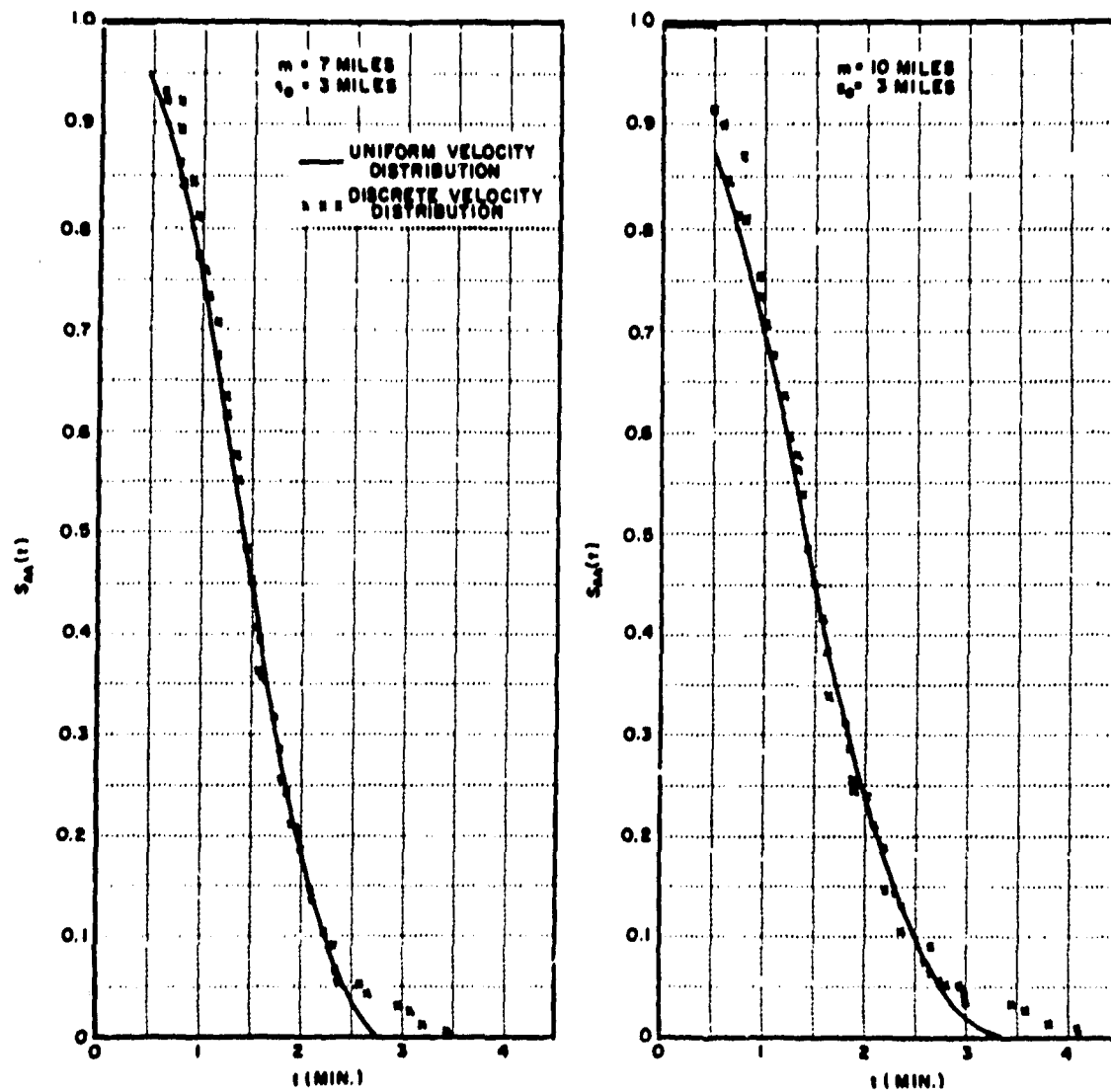


FIGURE E-3.7
LANDING-TIME DISTRIBUTION AT ORD
 $V = 126.3$ KNOTS $R = 42.5$ KNOTS

where $R' = R / \sqrt{12}$

$$K_p = \sqrt{(1-p)/p}$$

Ignoring at first the t_c restriction, we note that:

$$\tau_{AA} = \sum_{j=1}^2 \sum_{i=1}^2 \left(\frac{n}{v_i} - \frac{m}{v_j} \right) p_i p_j = \frac{s_o p}{v_1} + \frac{s_o (1-p)}{v_2} \quad (E-5)$$

Making substitutions for v_1 and v_2 from equations (E-3) and (E-4), it is found after manipulation, that:

$$\tau_{AA} = \frac{s_o (\bar{V} \gamma + R')}{(\bar{V}^2 - R'^2) \gamma + R' \bar{V}} \quad (E-6)$$

where:

$$\gamma = \frac{\sqrt{p(1-p)}}{2p-1} \quad (E-7)$$

Further manipulation yields:

$$\tau_{AA} = \frac{s_o}{\bar{V}} \cdot \frac{1}{(1-\eta^2) + \frac{\eta^3}{\gamma + \eta}} = \frac{s_o}{\bar{V}} K(\eta, \gamma) \quad (E-8)$$

where:

$$\eta = R' / \bar{V} = R / \sqrt{12} \bar{V} \quad (E-9)$$

The distribution's moments higher than the second affect τ_{AA} only through γ . We note from equation (E-7) that the function, $\gamma(p)$, is zero for $p = 0$, drops rapidly to $-\infty$ at $p = 0.5^-$, becomes $+\infty$ at $p = 0.5^+$, and then returns to zero at $p = 1.0$; in the vicinity of $p = 0$, $\gamma^2 \approx p$.

It is necessary to restrict our consideration to values of p at least greater than η , since smaller ones would lead to unrealistically low values of v_1 . We note, from examination of maximum R and minimum $\bar{V}$ values, that $\eta < 0.2$ in all practical cases, and that a more typical value is $\eta = 0.1$. Figure E-4 illustrates the second factor of equation (E-8) plotted as a function of p for these two values of η . It is seen that, in the typical case, even the second moment of the velocity distribution affects the landing time by only 1% (the $1 - \eta^2$ term), and the perturbation due to γ is negligible for all reasonable values of p . Even in the extreme case, the exaggerated effects of both η and γ influence the mean landing time by less than 6%, while the effect of γ in the reasonable range of p (from 0.3 to 0.8) is less than 1%.

The t_0 limitation would affect the above conclusion slightly. If $s_0 V > t_0$ for $V = v_1, v_2$, then the t_0 restriction could apply only in the case of A_2 following A_1 . τ_{AA} is then given by:

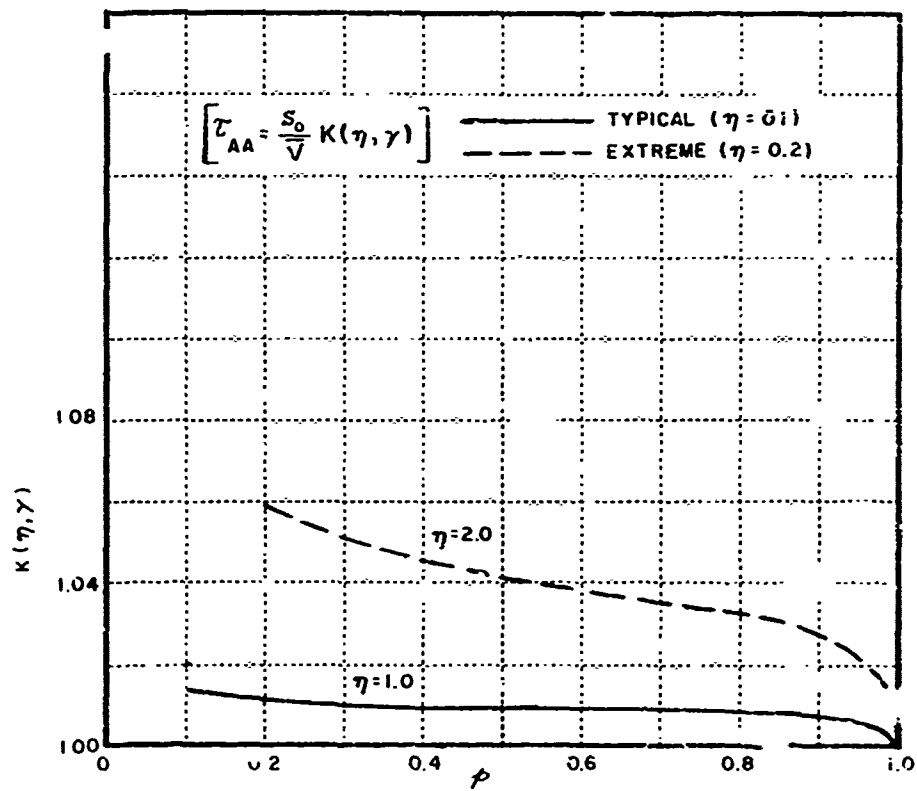


FIGURE E-4
EFFECT OF HIGHER MOMENTS OF TWO-POINT VELOCITY
DISTRIBUTION ON LANDING TIME

(E-10)

$$\tau_{AA} = \frac{s_o}{\bar{V}} \frac{1}{(1-\rho^2) + \frac{\rho^3}{\gamma + \rho}} + p(1-p) \text{Max} \left\{ 0; t_o - \frac{n}{v_2} + \frac{m}{v_1} \right\}$$

where the second term represents the " t_o penalty," i.e., the addition to the mean landing time that results from the t_o restriction. When $t_o = 1$ min., $\bar{V} = 120$ knots, $R = 12 \sqrt{12} = 41.6$ knots, the t_o penalty takes the following values as a function of p :

.104 minutes when $p = 0.1$

.085 minutes when $p = 0.5$

.069 minutes when $p = 0.9$

These values are also small compared to a typical value of $s_o/\bar{V} = 1.5$ minutes.

The investigations discussed in this appendix indicate the relative insensitivity of the landing capacity to the assumed form of the landing-velocity distribution other than that represented by the first two moments of that distribution, thereby justifying the use of the analytically-convenient rectangular distribution as an approximation to the real distribution.

APPENDIX F
 RUNWAY PERFORMANCE MEASURES AS A FUNCTION OF

$$\bar{V}, R, r_0, m, t_0, R_R, C_{OD}, S_0$$

LANDING RATES (λ), INTERPOSITION RATES (ν_0), AND OPERATIONS RATES (μ)

$$\bar{V} = 100 \text{ KNOTS} \quad R = 40 \text{ KNOTS}$$

r_0 (MILES)	m MILES	t_0 MIN.	R_R r_L/t_0	t_{OD} MIN.	$\Delta_0 = 2 \text{ MILES}$			$\Delta_0 = 3 \text{ MILES}$			$\Delta_0 = 4 \text{ MILES}$		
					λ (ℓph)	ν_0	μ (oph)	λ (ℓph)	ν_0	μ (oph)	λ (ℓph)	ν_0	μ (oph)
2	4	0.5	0.5	1	48.8	0.28	61.4	32.9	0.87	61.3	24.8	1.46	49.5
			↓	2	↓	0.25	44.0	↓	0.77	52.6	↓	1.00	49.5
		1.0	↓	1	↓	0.12	56.9	↓	0.61	54.1	↓	1.22	49.5
			↓	2	↓	0.12	40.1	↓	0.56	48.8	↓	0.92	48.5
		1.0	0.5	1	45.1	0.12	54.3	32.7	0.61	53.8	24.7	1.22	49.3
			↓	2	↓	0.12	38.8	↓	0.58	48.4	↓	0.92	46.4
		1.0	↓	1	↓	0	51.4	↓	0.21	45.7	↓	0.72	44.2
			↓	2	↓	0	36.0	↓	0.21	35.1	↓	0.64	38.1
		1.5	0.5	1	37.5	0.03	46.7	31.2	0.38	47.0	24.6	0.98	48.6
			↓	2	↓	0.03	33.8	↓	0.38	37.8	↓	0.80	42.1
		1.0	↓	1	↓	0	46.1	↓	0.02	41.4	↓	0.30	38.2
			↓	2	↓	0	33.3	↓	0.02	31.0	↓	0.30	31.2
	10	0.5	0.5	1	43.0	0.53	64.3	31.7	0.98	62.6	24.5	1.50	49.0
			↓	2	↓	0.11	46.8	↓	0.70	48.3	↓	0.99	48.7
		1.0	↓	1	↓	0.39	60.0	↓	0.79	57.0	↓	1.28	49.0
			↓	2	↓	0.31	43.4	↓	0.58	44.0	↓	0.87	44.4
		1.0	0.5	1	38.3	0.39	55.0	30.1	0.79	54.5	24.0	1.28	48.0
			↓	2	↓	0.31	40.7	↓	0.58	42.5	↓	0.87	43.5
		1.0	↓	1	↓	0.19	50.6	↓	0.47	47.6	↓	0.88	45.9
			↓	2	↓	0.18	37.3	↓	0.36	36.7	↓	0.63	37.1
		1.5	0.5	1	32.8	0.28	47.1	27.9	0.62	47.3	23.2	1.08	46.4
			↓	2	↓	0.24	35.8	↓	0.46	37.1	↓	0.75	38.8
		1.0	↓	1	↓	0.08	43.6	↓	0.25	41.4	↓	0.55	39.5
			↓	2	↓	0.08	32.7	↓	0.21	32.2	↓	0.41	31.9
4	4	0.5	0.5	1	48.8	0	53.8	32.9	0	42.5	24.7	0.28	38.1
			↓	2	↓	0	37.2	↓	0	31.4	↓	0.28	31.0
		1.0	↓	1	↓	0	53.8	↓	0	42.5	↓	0.12	36.4
			↓	2	↓	0	37.2	↓	0	31.4	↓	0.12	28.7
		10	0.5	0.5	43.0	0.06	51.3	31.7	0.22	44.9	24.5	0.53	41.0
			↓	2	↓	0.06	36.6	↓	0.20	34.3	↓	0.41	33.0
	10	1.0	↓	1	↓	0.02	50.6	↓	0.14	43.6	↓	0.39	39.2
			↓	2	↓	0.02	35.9	↓	0.14	33.2	↓	0.31	31.4

LANDING RATES (λ), INTERPOSITION RATES (ν_D), AND OPERATIONS RATES (μ) $\bar{V} = 100$ KNOTS $R = 60$ KNOTS

τ_0 (MILES)	m MILES	t_0 MIN.	A_P $\bar{t}_L/t_0$	t_{OD} MIN.	$\Delta_0 = 2$ MILES			$\Delta_0 = 3$ MILES			$\Delta_0 = 4$ MILES		
					λ (ℓph)	ν_D	μ (oph)	λ (ℓph)	ν_D	μ (oph)	λ (ℓph)	ν_D	μ (oph)
2	4	0.5	0.5	1	45.9	0.36	61.7	32.1	0.90	60.8	24.2	1.48	48.5
			↓	2		0.34	45.7		0.70	58.0		1.01	48.5
		1.0	1.0	1		0.22	57.5		0.68	54.7		1.25	48.5
			↓	2		0.22	39.1		0.55	43.4		0.89	44.6
		1.5	0.5	1	41.3	0.22	53.8	31.2	0.68	53.4	24.1	1.25	48.2
			↓	2		0.22	37.4		0.55	42.5		0.89	44.3
		2.0	1.0	1		0.06	50.2		0.32	46.1		0.79	43.4
			↓	2		0.06	35.6		0.30	36.1		0.60	36.5
		2.5	0.5	1	34.9	0.13	46.3	29.1	0.48	46.5	23.6	1.02	47.2
			↓	2		0.13	34.7		0.41	37.0		0.75	39.4
		3.0	1.0	1		0	44.2		0.13	40.9		0.44	38.6
			↓	2		0	32.3		0.13	31.5		0.36	31.4
	10	0.5	0.5	1	36.9	0.76	34.2	28.7	1.18	57.3	23.0	1.67	46.0
			↓	2		0.52	46.4		0.77	47.0		1.04	46.0
		1.0	1.0	1		0.62	59.9		1.01	57.3		1.47	46.0
			↓	2		0.43	43.4		0.67	43.7		0.93	43.7
		1.5	0.5	1	32.9	0.52	54.5	26.9	1.01	53.7	22.2	1.47	44.3
			↓	2		0.43	40.4		0.67	41.5		0.93	42.5
		2.0	1.0	1		0.41	49.7		0.72	47.7		1.12	44.3
			↓	2		0.29	35.9		0.49	36.9		0.72	36.7
		2.5	0.5	1	28.8	0.51	46.6	24.7	0.86	46.7	21.1	1.29	42.2
			↓	2		0.36	35.6		0.57	36.5		0.83	37.7
		3.0	1.0	1		0.25	42.4		0.49	40.9		0.81	39.6
			↓	2		0.19	32.4		0.34	32.1		0.54	31.8
4	4	0.5	0.5	1	45.9	0	52.0	32.1	0.06	42.7	24.2	0.36	38.5
			↓	2		0	36.3		0.06	32.0		0.34	31.7
		1.0	1.0	1		0	52.0		0.01	42.0		0.22	36.9
			↓	2		0	36.3		0.01	31.2		0.22	29.8
	10	0.5	0.5	1	36.9	0.20	49.4	28.7	0.43	45.0	23.0	0.76	42.0
			↓	2		0.16	36.2		0.31	34.5		0.52	33.6
		1.5	1.0	1		0.14	48.3		0.33	43.5		0.62	41.2
			↓	2		0.12	35.4		0.25	33.3		0.43	32.1

LANDING RATES (λ), INTERPOSITION RATES (ν_D), AND OPERATIONS RATES (μ) $\bar{V} = 120$ KNOTS $R = 40$ KNOTS

r_0 (MILES)	m MILES	t_0 MIN.	A_R τ_i/t_0	t_{0D} MIN.	$\Delta_0 = 2$ MILES			$\Delta_0 = 3$ MILES			$\Delta_0 = 4$ MILES		
					λ (ℓph)	ν_D	μ (oph)	λ (ℓph)	ν_D	μ (oph)	λ (ℓph)	ν_D	μ (oph)
2	4	0.5	0.5	1	58.9	0.20	66.0	3.6	0.78	69.2	30.1	1.26	60.2
			↓	2		0.17	44.1		0.56	50.2		1.00	59.4
		1.0	↓	1		0.04	60.6		0.50	59.5		1.02	60.2
			↓	2		0.03	40.0		0.42	45.0		0.92	55.1
		1.0	0.5	1	52.1	0.04	56.8	39.3	0.50	59.1	29.7	1.02	59.4
			↓	2		0.03	38.8		0.42	44.6		0.92	55.1
		1.0	↓	1		0	55.8		0.07	48.8		0.49	47.5
			↓	2		0	38.1		0.07	35.4		0.49	39.5
	10	0.5	0.5	1	39.7	0	47.8	35.9	0.23	49.2	29.4	0.76	52.8
			↓	2			34.2		0.23	37.4		0.75	47.3
		1.0	↓	1			47.8		0	44.9		0.10	40.8
			↓	2			34.2		0	32.7		0.10	31.3
		0.5	0.5	1	53.2	0.40	63.3	38.7	0.80	68.7	29.6	1.26	59.3
			↓	2		0.33	48.5		0.63	52.4		0.89	53.5
		1.0	↓	1		0.25	64.1		0.60	61.5		1.03	59.3
			↓	2		0.25	45.6		0.50	47.0		0.76	48.0
	4	0.5	0.5	1	45.5	0.25	58.2	36.5	0.60	58.7	29.1	1.03	58.3
			↓	2		0.25	42.5		0.50	45.4		0.76	47.3
		1.0	↓	1		0.09	53.8		0.27	50.7		0.61	49.0
			↓	2		0.09	38.2		0.26	38.5		0.50	39.2
		1.5	0.5	1	36.8	0.16	48.5	32.7	0.42	49.6	27.8	0.81	51.1
			↓	2		0.16	36.3		0.37	38.8		0.63	41.4
		1.0	↓	1		0.01	45.8		0.10	43.8		0.30	41.9
			↓	2		0.01	33.2		0.10	33.0		0.27	33.2
4	4	0.5	0.5	1	58.9	0	59.5	39.6	0	47.7	30.1	0.20	42.9
			↓	2			39.8			34.2		0.20	33.4
		1.0	↓	1			59.5			47.7		0.04	40.6
			↓	2			39.8			34.2		0.04	30.6
	10	0.5	0.5	1	53.2	0.01	55.8	38.7	0.14	49.8	29.6	0.40	45.7
			↓	2		0.01	38.6		0.14	36.8		0.36	37.0
		1.0	↓	1		0	56.4		0.07	48.4		0.25	43.3
			↓	2		0	38.4		0.07	35.2		0.25	34.0

LANDING RATES (λ), INTERPOSITION RATES (ν_D), AND OPERATIONS RATES (μ) $\bar{V} = 120$ KNOTS $R = 60$ KNOTS

r_0	m	t_0	A_R	t_{0D}	$\Delta_0 = 2$ MILES			$\Delta_0 = 3$ MILES			$\Delta_0 = 4$ MILES		
(MILES)	MILES	MIN.	t_L/t_0	MIN.	λ (lph)	ν_D	μ (oph)	λ (lph)	ν_D	μ (oph)	λ (lph)	ν_D	μ (oph)
2	4	0.5	0.5	1	56.3	0.28	67.3	39.0	0.77	68.0	29.4	1.26	58.7
				2		0.24	46.3		0.61	51.8		0.93	55.0
		1.0	1.0	1		0.14	62.1		0.4	59.3		1.03	58.7
				2		0.13	42.7		0.49	46.9		0.82	50.0
		1.0	0.5	1	48.6	0.14	57.2	37.7	0.51	57.8	29.3	1.03	58.5
				2		0.13	40.4		0.49	46.0		0.82	49.8
		1.0	1.0	1		0	53.8		0.17	49.7		0.53	47.7
				2		0	36.0		0.17	36.9		0.49	39.1
		1.5	0.5	1	38.4	0.04	47.6	33.9	0.31	48.9	28.4	0.78	51.4
				2		0.04	34.4		0.31	38.1		0.67	43.3
		1.0	1.0	1		0	46.8		0.02	43.6		0.21	41.3
				2		0	33.7		0.02	32.2		0.21	32.5
	10	0.5	0.5	1	46.8	0.55	69.4	35.9	0.92	68.7	28.5	1.35	56.9
				2		0.42	49.2		0.56	51.0		0.90	52.0
		1.0	1.0	1		0.42	64.3		0.74	62.2		1.14	56.9
				2		0.32	45.5		0.54	46.4		0.79	47.5
		1.0	0.5	1	40.3	0.42	57.8	33.2	0.74	58.1	27.4	1.14	54.7
				2		0.32	42.1		0.54	44.0		0.79	46.0
		1.0	1.0	1		0.22	52.8		0.44	50.8		0.77	49.4
				2		0.19	38.6		0.34	38.4		0.55	38.8
		1.5	0.5	1	33.6	0.31	48.4	29.7	0.58	49.2	25.7	0.95	50.2
				2		0.25	36.6		0.43	38.0		0.67	40.1
		1.0	1.0	1		0.39	44.6		0.24	43.2		0.48	42.0
				2		0.09	33.3		0.20	32.2		0.35	33.1
4	4	0.5	0.5	1	56.3	0	58.1	39.0	0.01	47.5	29.4	0.28	43.5
				2		0	39.2		0.01	34.2		0.28	34.5
		1.0	1.0	1		0	58.1		0	47.3		0.14	41.3
				2		0	39.2		0	33.9		0.14	31.8
	10	0.5	0.5	1	46.8	0.10	55.1	35.9	0.28	50.2	28.5	0.55	46.9
				2		0.10	38.9		0.24	37.6		0.42	36.7
		1.0	1.0	1		0.06	54.0		0.19	48.4		0.42	44.6
				2		0.06	38.0		0.17	36.1		0.32	34.7

APPENDIX G RUNWAY PERFORMANCE MEASURES AT LAGUARDIA AND IDLEWILD LANDING RATES (λ), INTERPOSITION RATES (ν_0), AND OPERATIONS RATES (μ)

$R_0 = 2$ LAGUARDIA ($\bar{V} = 116.7$ KNOTS $R = 51.4$ KNOTS)

m	t_0	A_R	$A_0 = 2$ MILES						$A_0 = 3$ MILES						$A_0 = 4$ MILES					
			$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.	
			λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0	λ (mph)	ν_0
4	.50	.50	55.9	0.28	66.3	0.22	45.7	0.38	57.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63
	.75	.75	0.18	0.18	63.3	0.15	43.6	0.65	62.4	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57
	1.00	1.00	0.11	0.11	61.1	0.10	41.8	0.51	58.3	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51
	1.00	.50	49.0	0.11	56.8	0.10	39.7	0.37	57.4	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51
	1.00	.75	0.02	0.02	54.5	0.02	37.7	0.30	52.2	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30
	1.00	1.00	0.00	0.00	54.0	0.00	37.2	0.15	49.0	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15
	1.50	.50	38.7	0.02	47.4	0.02	33.2	0.34	46.9	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34
	1.50	.75	0.00	0.00	47.0	0.00	33.3	0.10	45.1	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10
	1.50	1.00	0.00	0.00	47.0	0.00	33.8	0.01	43.6	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01
	1.50	.50	52.0	0.19	67.8	0.23	48.0	0.37	67.3	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37
7	.50	.50	55.9	0.28	66.3	0.22	45.7	0.38	57.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63
	.75	.75	0.18	0.18	63.3	0.15	43.6	0.65	62.4	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57
	1.00	1.00	0.11	0.11	61.1	0.10	41.8	0.51	58.3	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51
	1.00	.50	49.0	0.11	56.8	0.10	39.7	0.37	57.4	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51
	1.00	.75	0.02	0.02	54.5	0.02	37.7	0.30	52.2	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30
	1.00	1.00	0.00	0.00	54.0	0.00	37.2	0.15	49.0	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15
	1.50	.50	38.7	0.02	47.4	0.02	33.2	0.34	46.9	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34
	1.50	.75	0.00	0.00	47.0	0.00	33.3	0.10	45.1	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10
	1.50	1.00	0.00	0.00	47.0	0.00	33.8	0.01	43.6	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01
	1.50	.50	52.0	0.19	67.8	0.23	48.0	0.37	67.3	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37
10	.50	.50	55.9	0.28	66.3	0.22	45.7	0.38	57.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63	52.0	0.63
	.75	.75	0.18	0.18	63.3	0.15	43.6	0.65	62.4	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57	48.9	0.57
	1.00	1.00	0.11	0.11	61.1	0.10	41.8	0.51	58.3	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51	47.0	0.51
	1.00	.50	49.0	0.11	56.8	0.10	39.7	0.37	57.4	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51	46.3	0.51
	1.00	.75	0.02	0.02	54.5	0.02	37.7	0.30	52.2	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30	39.8	0.30
	1.00	1.00	0.00	0.00	54.0	0.00	37.2	0.15	49.0	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15	36.3	0.15
	1.50	.50	38.7	0.02	47.4	0.02	33.2	0.34	46.9	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34	38.0	0.34
	1.50	.75	0.00	0.00	47.0	0.00	33.3	0.10	45.1	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10	33.7	0.10
	1.50	1.00	0.00	0.00	47.0	0.00	33.8	0.01	43.6	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01	32.0	0.01
	1.50	.50	52.0	0.19	67.8	0.23	48.0	0.37	67.3	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37	51.7	0.37

LANDING RATES (λ), INTERPOSITION RATES (ν_0), AND OPERATIONS RATES (μ) $\bar{v}_0 = 4$ LAGUARDIA ($\bar{v} = 116.7$ KNOTS $R = 51.4$ KNOTS)

m	t_0 (MIN)	t_0/t_0	$\Delta_0 = 2$ MILES						$\Delta_0 = 3$ MILES						$\Delta_0 = 4$ MILES					
			$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.		$t_{0D} = 2$ MIN.		$t_{0D} = 1$ MIN.	
			λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0	λ (EPH)	ν_0
1	.50	.50	55.9	0	39.0	0	38.2	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
7	.50	.50	55.9	0	39.0	0	38.2	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
10	.50	.50	55.9	0	39.0	0	38.2	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11
	.75	.75	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	1.00	57.9	0	39.0	0	39.0	0	46.7	0	33.6	0	33.6	0	28.7	0.26	42.5	0.26	28.7	0.26
	1.00	.50	49.0	0	37.2	0	37.4	0	46.1	0	33.3	0	33.3	0	28.7	0.11	40.2	0.11	28.7	0.11

