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Determination of Parameters in an Empirical Function for Build-up Factors for Various Photon Energies

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by
Arthur B. Chilton
Dennis Holoviak*
Lawrence K. Donovan

* Bureau of Yards and Docks

OBJECT OF THE TASK

The objective is to develop mathematical formulae, graphs and tables based on available scientific data on nuclear shielding for inclusion in a shielding manual on nuclear defense construction.

ABSTRACT

In the computation of gamma-ray attenuation, it is desired to use a simple expression for build-up factors. In this note a simple analytical expression was used for Dose Build-up Factors from a radioactive isotropic point source. The parameters of the expression were determined by the method of "least squares" to obtain an optimum fit to experimental data for build-up factors for aluminum at various photon energies.

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INTRODUCTION

Goldstein and Wilkins (Reference 1), using the method of moments devised by Spencer and Fano (Reference 2), have obtained build-up factors for use in computation of gamma-ray attenuation.

The build-up factor has been defined as the ratio of some property of the photon beam (i.e. intensity, number of photons, energy flux or biological dose) when the effects of all quanta are included to that obtained when only the uncollided flux is considered.

Of the various materials studied in relation to gamma-ray attenuation, which are of practical interest in shielding problems, aluminum most closely approximates the atomic numbers of air, earth, and concrete. For this reason our calculations on the build-up factor constant are based on the table values of build-up factors for aluminum. Limited gamma-ray energies of 0.5 to 10.0 Mev are covered.

We are concerned herein with Dose Build-up Factors for a point isotropic source. It is desired to use a simple analytical expression for the same, with parameters to be determined to obtain an optimum fit to the Goldstein and Wilkins data. Various expressions are usable; however, in this note we shall use an expression suggested by Berger and Spencer, (Reference 3).

$$B(r) = 1 + a\,r\,e^{br} \quad \dots\dots\dots\dots\dots\dots\dots (1)$$

where r is the number of mean-free-paths. (A mean-free-path of material is defined as that thickness of material necessary to reduce the intensity of incident radiation by 1/e of its initial value by energy absorption). We shall match the above expression to the build-up data provided by Goldstein and Wilkins from 1 through 10 mean-free-paths.

It is desired to evaluate a and b for the various photon energies. To accomplish this, the technique known as the method of "least squares" will be employed. It may be of some use to sketch the method.

METHOD OF LEAST SQUARES

The method of least squares can be easily applied after the equation is reduced to a linear form. This can be accomplished by using logarithmic operations. Taking the logarithm of equation (1), we have,

$$\ln \left[B(r) - 1 \right] = \ln (a\, r\, e^{br}) \qquad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

rearranging and setting the left side equal to zero,

$$0 = \ln a + b r + \ln r - \ln \left[B(r) - 1 \right] \qquad \ldots\ldots\ldots\ldots \quad (3)$$

Since the tabulated values of $B(r) - 1$ exist for any $r = 1, 2 \ldots 10$, the above expression is linear in the terms $\underline{a}$ and $\underline{b}$.

We obtain from the tables approximate values for the terms $\underline{a}$ and $\underline{b}$ and denote these by a_o and b_o. Equation (3) then becomes:

$$V_i = \ln a_o + b_o r_i + \ln r_i - \ln \left[B(r) - 1 \right] \qquad \ldots\ldots\ldots \quad (4)$$

Since a_o and b_o are relatively accurate approximations, they will almost satisfy equation (1) with the value of V_i being very near zero. As a_o and b_o become more accurate, $V_i \longrightarrow 0$. Thus V_i is a good indicator of the accuracy of a_o and b_o and for this reason is termed the "variance." For each value of r_i, $r_i = r_1, r_2 \ldots r_n$, V_i will assume corresponding values of $V_i = V_1, V_2, \ldots, V_n$. It follows mathematically that the most exact values of $\underline{a}$ and $\underline{b}$ occur if the variance is minimized or if the following conditions are satisfied;

$$\frac{\partial}{\partial a} \sum_{i=1}^{n} V_i^2 = \sum_{i=1}^{n} V_i \frac{\partial V_i}{\partial a} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\frac{\partial}{\partial b} \sum_{i=1}^{n} V_i^2 = \sum_{i=1}^{n} V_i \frac{\partial V_i}{\partial b} = 0 \qquad \ldots\ldots\ldots\ldots\ldots \quad (6).$$

A simultaneous solution of (5) and (6) will yield the optimum values of $\underline{a}$ and $\underline{b}$. Equation (4) can now be expressed as:

$$V_i = \alpha + \beta\, r_i + \gamma\, (r_i) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Where $\alpha = \ln a$

$$\beta = b$$

$$\gamma(r_i) = \ln r_i - \ln\left[\beta(r_i) - 1 \right]$$

Thus $\frac{\partial V_i}{\partial \alpha} = 1$ and $\frac{\partial V_i}{\partial \beta} = r_i$. The normal equations are:

$$\sum_{i=1}^{n} V_i \frac{\partial V_i}{\partial \alpha} = 0 \qquad \ldots\ldots\ldots (8)$$

$$\sum_{i=1}^{n} V_i \frac{\partial V_i}{\partial \beta} = 0 \qquad \ldots\ldots\ldots (9)$$

Substituting in equations (8) and (9) we have:

$$\sum_{i=1}^{n} \left[\alpha + \beta r_i + \gamma(r_i) \right] \cdot 1 = 0 \qquad \ldots\ldots\ldots (10)$$

$$\sum_{i=1}^{n} \left[\alpha + \beta r_i + \gamma(r_i) \right] \cdot r_i = 0 \qquad \ldots\ldots\ldots (11)$$

OR

$$\sum_{i=1}^{n} \alpha + \sum_{i=1}^{n} \beta r_i + \sum_{i=1}^{n} \gamma(r_i) = 0 \qquad \ldots\ldots\ldots (12)$$

$$\sum_{i=1}^{n} \alpha r_i + \sum_{i=1}^{n} \beta r_i^2 + \sum_{i=1}^{n} r_i \gamma(r_i) = 0 \qquad \ldots (13)$$

For values of $r_1, \ldots, r_5 = 1, 2, 4, 7, 10$, equations (12) and (13) become:

$$5\alpha + 24\beta + \sum_{i=1}^{5} \gamma(r_i) = 0 \qquad \ldots\ldots\ldots (14)$$

$$24\alpha + 170\beta + \sum_{i=1}^{5} r_i \gamma(r_i) = 0 \qquad \ldots\ldots\ldots (15)$$

In order to solve the above equations simultaneously for α and β, it is necessary to compute $\sum_{i=1}^{5} \gamma(r_i)$ and $\sum_{i=1}^{5} r_i \gamma(r_i)$ for each value of source energy, E. The tabulation of this data is enclosed in Table 1.

SAMPLE CALCULATIONS

For a source energy of E=1.0 Mev, the addition of equations (14) and (15) yields:

$$\therefore \beta = \frac{24 \sum_{i=1}^{5} \gamma(r_i) - 5 \sum_{i=1}^{5} r_i \gamma(r_i)}{274} \quad \ldots\ldots\ldots\ldots (16)$$

From Table 1, for a source energy of 1.0 Mev, $\sum_{i=1}^{5} \gamma(r_i) = -1.7463$ and $\sum_{i=1}^{5} r_i \gamma(r_i) = -12.5026$.

$$\beta = b = \frac{24\,(-1.7463) - (5)\,(-12.5026)}{274} = 0.0752 \quad \ldots . (17)$$

Substituting these results into equation (14) and solving for α, we have

$$\alpha = \frac{-24\,(0.07519) - (-1.7463)}{5} = -0.011647 = \ln \underline{a}$$

$$\therefore \underline{a} = 0.988$$

RESULTS

Complete solutions for a and b for all the photon energies considered are listed in the table below. A plot of a and b for energy from 0.5 Mev to 10 Mev is given in Figure 1.

E	0.5	1.0	2.0	3.0	4.0	6.0	8.0	10.0
a	1.288	0.988	0.742	0.634	0.527	0.416	0.336	0.274
b	0.112	0.0752	0.041	0.0197	0.0114	0.0072	0.006	0.0072

CHECK

Included in Table 2 is a comparison of the Goldstein and Wilkins values of build-up factors on which this work is based versus the values obtained by calculating build-up factors using the values of a and b above.

REFERENCES

1. AEC NYO-3075, "Calculations of the Penetration of Gamma Rays", H. Goldstein and J. E. Wilkins, Jr. (1954).

2. J. Res. NBS 46, "Penetration and Diffusion of X-Rays", L. V. Spencer and U. Fano, (1951), 446.

3. NBS 4942, "Effects of Boundries and Inhomogeneites of the Penetration of Gamma Radiation," M. J. Berger.

TABLE I

COMPUTATION OF $\sum_{i=1}^{5} \gamma(r_i)$ and $\sum_{i=1}^{5} r_i \gamma(r_i)$ for

SOURCE ENERGY

r_i	B - 1	ln(B-1)	$\ln r_i$	$\gamma(r_i)$	$r_i\gamma(r_i)$	$\sum_{i=1}^{5}\gamma(r_i)$	$\sum_{i=1}^{5} r_i\gamma(r_i)$
			E = 0.5 Mev				
1	1.37	0.31481	0.0000	-0.3148	- 0.3148		
2	3.24	1.17557	0.69315	-0.4824	- 0.9648		
4	8.47	2.1336	1.38629	-0.7473	- 2.9892	-3.9511	-25.1122
7	20.5	3.0201	1.94591	-1.0742	- 7.5194		
10	37.9	3.6350	2.30259	-1.3324	-13.3240		
			E = 1.0 Mev				
1	1.02	0.0198	0.0000	-0.01980	- 0.0198		
2	2.31	0.8373	0.69355	-0.14410	- 0.2882		
4	5.57	1.7175	1.38629	-0.3312	- 0.3248	-1.7463	-12.5026
7	12.1	2.4933	1.94591	-0.5474	- 3.8318		
10	20.2	3.0064	2.30259	-0.7038	- 7.0380		
			E = 2.0 Mev				
1	0.75	-0.8786	0.0000	+0.2877	0.2877		
2	1.61	0.47623	0.69315	0.21692	0.4338		
4	3.62	1.28647	1.38629	0.0998	0.3992	0.5110	0.2083
7	7.05	1.9531	1.94591	-0.0072	- 0.0504		
10	10.9	2.3888	2.30259	-0.0862	- 0.8620		
			E = 3.0 Mev				
1	0.64	-0.44629	0.0000	0.4463	0.4463		
2	1.32	0.27763	0.69315	0.4155	0.8310		
4	2.78	1.02245	1.38629	0.3638	1.4552	1.8022	7.5721
7	5.14	1.63710	1.94591	0.3088	2.1616		
10	7.65	2.03480	2.30259	0.2678	2.6780		

TABLE I (cont)

r_i	B - 1	ln(B-1)	$\ln r_i$	$\gamma(r_i)$	$r_i\gamma(r_i)$	$\sum_{i=1}^{5}\gamma(r_i)$	$\sum_{i=1}^{5} r_i\gamma(r_i)$
			E = 4.0 Mev				
1	0.53	-0.63488	0.0000	0.6349	0.6349		
2	1.08	0.07696	0.69315	0.6162	1.2324		
4	2.22	0.79751	1.38629	0.5888	2.3532	2.9280	13.4302
7	4.01	1.38879	·1.94591	0.5571	3.8997		
10	5.88	1.77160	2.30259	0.5310	5.3100		
			E = 6.0 Mev				
1	0.42	-0.867500	0.0000	0.8675	0.8675		
2	0.85	-0.16252	0.69315	0.8557	1.7114		
4	1.70	0.53063	1.38629	0.8557	3.4228	4.2071	19.8012
7	3.00	1.11841	1.94591	0.8275	5.7925		
10	4.49	1.50185	2.30259	0.8007	8.0070		
			E = 8.0 Mev				
1	0.34	-1.07881	0.0000	1.0788	1.0788		
2	0.68	-0.38566	0.69315	1.0788	2.1576		
4	1.37	0.31481	1.38629	1.0715	4.2859	5.3061	25.1429
7	2.45	0.89609	1.94591	1.0498	7.3486		
10	3.58	1.27536	2.30259	1.0272	10.2720		
			E = 10.0 Mev				
1	0.28	-1.27297	0.0000	1.2730	1.2730		
2	0.55	-0.59784	0.69315	1.2910	2.5820		
4	1.12	0.11333	1.38629	1.2730	5.0920	6.3022	29.8556
7	2.01	0.69813	1.94591	1.2478	8.7346		
10	2.96	1.08519	2.30259	1.2174	12.1740		

TABLE II

CALCULATED VERSUS GIVEN BUILD-UP FACTORS

	$r_1=1$	$r_2=2$	$r_3=4$	$r_4=7$	$r_5=10$
			E = 0.5 Mev		
B-CALCULATED	2.44	4.23	9.07	20.80	40.50
B-TABLE	2.37	4.24	9.47	21.50	38.90
			E = 1.0 Mev		
B-CALCULATED	2.07	3.30	6.35	12.72	21.90
B-TABLE	2.02	3.31	6.57	13.10	21.20
			E = 2.0 Mev		
B-CALCULATED	1.77	2.61	4.50	7.92	12.20
B-TABLE	1.75	2.61	4.62	8.05	11.90
			E = 3.0 Mev		
B-CALCULATED	1.65	2.32	3.76	6.11	8.72
B-TABLE	1.64	2.32	3.78	6.14	8.65
			E = 4.0 Mev		
B-CALCULATED	1.54	2.09	3.22	5.00	6.91
B-TABLE	1.53	2.09	3.22	5.01	6.88
			E = 6.0 Mev		
B-CALCULATED	1.42	1.84	2.71	4.06	5.47
B-TABLE	1.42	1.85	2.70	4.06	5.49
			E = 8.0 Mev		
B-CALCULATED	1.34	1.68	2.38	3.45	4.57
B-TABLE	1.34	1.68	2.37	3.45	4.58
			E = 10.0 Mev		
B-CALCULATED	1.27	1.56	2.13	3.02	3.94
B-TABLE	1.28	1.55	2.12	3.01	3.96

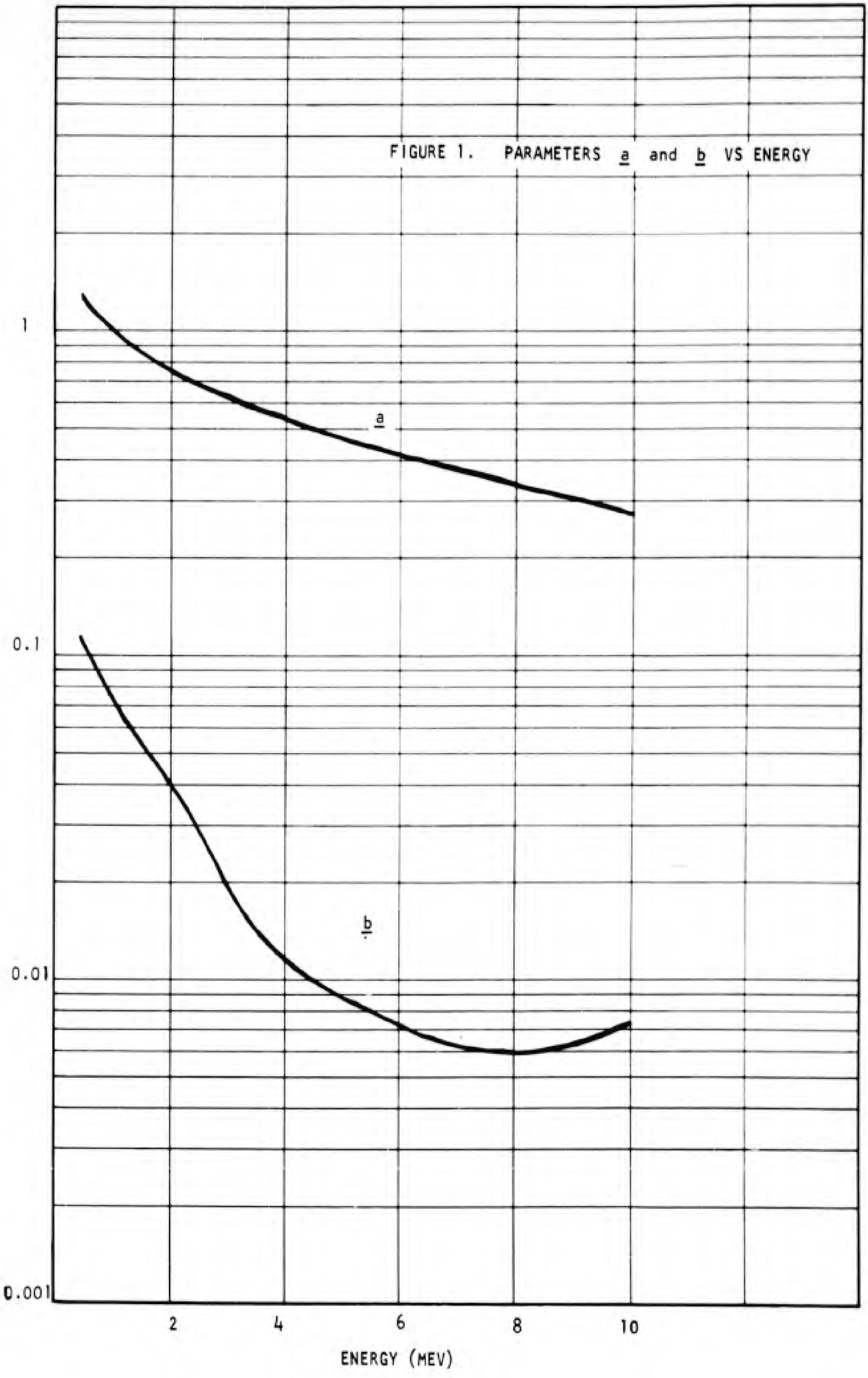

FIGURE 1. PARAMETERS a and b VS ENERGY
a
b
1
0.1
0.01
0.001
2
4
6
8
10
ENERGY (MEV)