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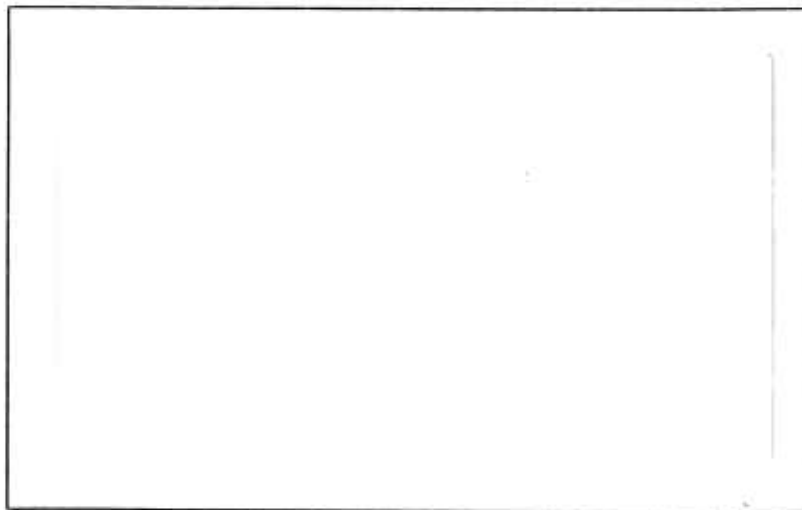
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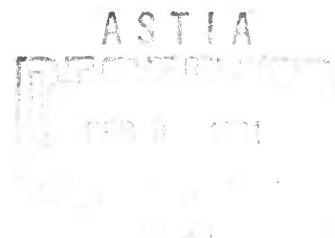
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AN ADAPTATION OF THE
MIL-STD-105B PLANS TO RELIABILITY
AND LIFE TESTING APPLICATIONS*

Technical Report No. 3

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An Adaptation of the MIL-STD-105B Plans to Reliability and Life-Testing Applications

Summary:

This paper presents a procedure, together with tables of necessary conversion ratios, for applying the MIL-STD-105B sampling-inspection plans to reliability and life-testing applications. The method assumes the Weibull distribution (including the exponential distribution) as a statistical model for item lifelength. Inspection of sample items is by attributes with life testing truncated at the end of some specified time, t . Lot quality is evaluated in terms of mean item life, μ . Both t and μ are measured from some reference time, γ .

Introduction

This paper is an extension of a previous report on work under the project (Technical Report Number 1), a report which was also presented at the Seventh National Symposium on Reliability and Quality Control¹. In Technical Report No. 1, Sobel and Tischendorf's² sampling plans were generalized from the exponential distribution as a life model to the Weibull distribution. Tables for conversion between t/μ and p' were computed for various values of β , the Weibull shape parameter. Also, tables of sampling-inspection plans were presented. The present report gives a procedure and tables for adaptation of the MIL-STD-105B³ sampling-inspection plans to reliability and life-testing applications. As in Technical Report No. 1; the underlying lifelength distribution is assumed to be of the Weibull form with the exponential included as a special case. Extensive use was made of the conversion ratios presented in the first report.

The Sampling Inspection Procedure:

The following acceptance-sampling procedure has been adopted for the adaptation considered here of the MIL-STD-105B plans for life testing:

- (1) A random sample of n items is selected from the lot.
- (2) These sample items are tested for life over some preassigned test period t units in length.
- (3) The number of test items that fail prior to time t is observed.
- (4) If the number of items failing is equal to or less than some specified acceptance number, c , the lot is accepted; if the number that fails exceeds the acceptance number, the lot is rejected.

The sample sizes and acceptance numbers used in the above procedure will be those specified by the MIL-STD-105B tables. The procedure as outlined above is for single sampling; through simple and appropriate modification, the 105B double-sampling and multiple-sampling plans may be likewise employed. It may be noted that the acceptance procedure is the same as that specified for the MIL-STD-105B plans with the single exception of the use of a test truncation time, t , (and the application, of course, to life-testing data).

The probability of acceptance for a lot under the acceptance procedure outlined depends solely upon the probability, p' , that an item will fail before the end of the test period, t . One may note from a study of the mathematics as outlined in the Appendix that if the test truncation time, t , is preassigned and if the value for β , the shape parameter, is known the probability, p' , of failure is a function only of mean item life, μ . This fact makes it possible to use this attribute acceptance procedure to evaluate lots in terms of mean item life; the operating characteristics for

any specified plan (in terms of c and n) depend only on t and μ .

In order to allow for any desired test-time truncation value, t , and thus provide plans for general use, the tables included with this paper have been prepared in terms of the dimensionless ratio, t/μ . Actually, to give more conveniently usable figures and to work in terms adopted for the 105B plans, the ratio is given in terms of percent; $(t/\mu) \times 100$ is used. Each of the 105B plans is cataloged and described in terms of the $(t/\mu) \times 100$ ratio. These ratio values are used in the same way as the percent defective values are used in the selection and application of the 105B plans for ordinary attribute inspection. When applying the plans to a specific life-testing application, use of the ratio to convert from test time to lot mean (or vice versa) will be found quite easy. Examples of application will be given near the end of the paper.

It will be noted that it is necessary to assume a value for β , the Weibull shape parameter. For many applications this value may be known. Its magnitude may have been determined for the product in question from past life-length research data, from the results of past inspection data, or from some other source. If the value for β is not known, procedures are available for estimating this parameter (and the γ location parameter if this, too, is necessary).

The Tables of Ratios for the 105B Plans:

Tables of ratios for adapting the 105B plans to reliability and life-testing application have been prepared for each of seven values for β , the Weibull shape parameter. These seven values range from $1/2$ to $3-1/3$, covering the span commonly encountered with industrial products. A table for $\beta = 1$, which is the exponential case, is also included. These tables

will be found at the end of this paper as Tables 1-A through 1-G.

Each table lists for each 105-B Acceptable Quality Level (AQL) value the corresponding $(t/\mu) \times 100$ ratio. These matched ratios will be found in the column headings under each of the respective 105B Acceptable Quality Level values (which are in terms of 100 p' %, the acceptable percent defective). Each ratio value gives for all 105B plans of the corresponding AQL, a measure of lot quality for which the producer's risk or probability of rejection will be low. This risk of rejection will be the same as that encountered in the normal use of the 105B plans for attribute inspection. It will be recalled that this risk is not a constant value of, say .05, as in most previous tables of acceptance inspection plans, but ranges from as low as 0.01 to as high as 0.20. The risk varies with the size of the sample, which in turn varies with the size of the lot. For large lot sizes (and thus large sample sizes) the risk is relatively small; for small lot sizes it is relatively large. The specific risk value for any plan of interest may be obtained from the corresponding operating characteristic curve which will be found included with the 105B tables.

The interpretation of these matched Acceptable Quality Level ratios for life-testing and reliability use may be demonstrated by means of a specific case. Assume, for example, that $\beta = 1-2/3$ and that a 105B plan with an AQL of 4.0% is to be used. From the table of ratios for $\beta = 1-2/3$, which is Table 1-E of those included at the end of this report, it will be found that the corresponding t/μ ratio value at the AQL is 16.42. Thus lots for which $(t/\mu) \times 100 = 16.42$ are "acceptable" and the probability of acceptance will be high (the probability of rejection low). If the test period, t , is, say, 1000 hours, $(100/\mu) \times 100 = 16.42$ or $\mu = 6,090$ hours;

the mean life for the items in the lot must be 6,090 hours for it to meet the 105B acceptable quality level standards.

In the body of each table of conversion ratios will be found, for each 105B plan, ratios for which the probability of acceptance is .10. These correspond to lot tolerance per cent defective (LTPD) figures which furnish a useful measure of consumer's protection. They represent unsatisfactory lot quality values and ones for which the probability of acceptance is low. Unlike the risks associated with the AQL which vary with sample size, the risk at the LTPD quality used for the tables presented here is at .10 for all plans.

For the example cited above in which the AQL is 4.0%, suppose the Sample Size Code Letter is L. Reference to the table of conversion ratios for $\beta = 1-2/3$ shows the LTPD ratio to be 31. With the test period, t , designated as 1000 hours, the value for μ , the lot mean life may be readily determined. Substitution gives $(1000/\mu) \times 100 = 31$ or $\mu = 3,220$ hours. Thus lots whose mean life is 3220 hours or less have a probability of at most .10 of acceptance.

With the use of these complete tables of conversion ratios (tables 1-A through 1-G), suitable 105B plans may be selected in terms of either an AQL or the LTPD. If, instead, some 105B plan has been specified, its operating characteristics can be evaluated. Examples of such use will be outlined in the material that follows.

As a supplement to these tables, Table 2 has been prepared. This table gives the $(t/\mu) \times 100$ ratio at the Acceptable Quality Level for an additional number of values for β , the Weibull shape parameter. Ratios at the AQL for the β values used in Table 1 are also included. As the

Acceptable Quality Level supplies the operating characteristic of most interest in the application of 105B plans, the ratio values in this table may be all that are necessary for many applications.

Example (1)

For a simple example of application, consider a receiving inspection case for which incoming lots of a product are to be tested for lifelength by sampling. From past experience with the product it has been determined that the life distribution can be expected to follow the Weibull form with a value for β , the shape parameter, of approximately $1-1/3$. The value for γ , the location parameter, is expected to be 0. The MIL-STD-105B plans are to be employed. A test period for the sample items of 200 hours and an Acceptable Quality Level in terms of percent defective (as used in the Standard) of 1.5% have been more or less arbitrarily selected for use. The size of incoming lots is 5,000 items. Inspection Level II, the one for normal use, will seemingly be appropriate. Single sampling is to be employed. The acceptance procedure for the above conditions and the resulting operating characteristics must be determined.

Reference to Table III of MIL-STD-105B shows that for a lot size of 5,000 items and for Inspection Level II, Sample Size Code Letter M is designated for ordinary inspection. Reference is next to Table IV-A of the Standard, the master table for normal single-sampling inspection. Here it will be found that for Sample Size Code Letter M and for an Acceptable Quality Level of 1.5% the sample size is 225 items, the acceptance number 8 items, and the rejection number 9. The acceptance-rejection procedure will thus be the following: (a) draw at random from the submitted lot a sample of 225 items and place them on life test for 200 hours, (b) determine the number of items that have failed by the end of this test period, (c) if

If the number failing is 8 or less, accept the lot. If the number is 9 or more, reject it.

The operating characteristics of this plan can be determined from information included in the Tables of $(t/\mu) \times 100$ Ratios included as part of this report. For this example reference will be to Table 1-D, the table of ratios for $\beta = 1-1/3$. Examination of the two lines of Acceptable Quality Level values across the top of this table shows that for an Acceptable Quality Level in terms of p' (%) of 1.5, the Acceptable Quality Level in terms of $(t/\mu) \times 100$ is 4.69. With this latter ratio, and with the value for t , the test period length of 200 hours, the value for μ , the mean item life for the lot, can be determined. Thus:

$$(t/\mu) \times 100 = 4.69 \quad (\text{AQL})$$

$$(200/\mu) \times 100 = 4.69$$

$$\mu = 4,260 \text{ hours.}$$

One now knows that the operation of the plan is such that if the mean item life for the lot is 4,260 hours or more the probability that it will be accepted is high. (A rough value for this probability may be found from the operating characteristic curves in the Military Standard. For an AQL of 1.5 and for Code Letter M one may note the probability is approximately .97.) The Acceptable Quality Level is thus 4,260 hours.

The ability of the plan to protect the consumer may be measured by the lot mean life for which the probability of acceptance is low. The tables of $(t/\mu) \times 100$ ratios included in this report include ratios at the Lot Tolerance Percent Defective (LTPD) quality level, the level at which the probability of acceptance is .10. These ratios will be found in the body of the tables. Reference to the same table, Table 1-D for $\beta = 1-1/3$, gives an LTPD ratio of 13 for Sample Size Code Letter M and an AQL of 1.5.

Computations similar to those previously made give:

$$(t/\mu \times 100 = 13 \quad (\text{LTPD}))$$

$$(200/\mu) 100 = 13$$

$$\mu = 1,540 \text{ hours.}$$

One now knows if the mean life for the items in the submitted lot is 1,540 hours or less, the probability of it being accepted is at most .10; the probability of its rejection is at least .90.

The operating characteristics in hours as computed above apply also (with the same values) if a double-sampling or a multiple-sampling plan for the same Sample Size Code Letter and AQL value is employed instead of single-sampling plan. For double-sampling in this application, the data for the plan will be found in Table IV-B of the Military Standard. The first sample size will be 150 items. These sample items would be tested for 200 hours. If 5 or fewer items failed within this time, the lot would be accepted; if 14 or more failed, it would be rejected. If from 6 to 13 failed, a second sample of 300 items would be selected and tested for 200 hours. If the total number failing (in the first and second samples combined) is 13 or less, the lot would be accepted, if it is 14 or more, the lot would be rejected.

Example (2)

For a second example, consider an acceptance-inspection by sampling application for which the following achievements are desired: (a) If the mean item life for the lot is 20,000 hours or more the probability of acceptance will be high. Lots of this mean life or greater are considered "acceptable." (b) If the mean item life for the lot is 6,000 hours or less, the probability of acceptance will be low - .10. A test period of

500 hours will be employed. It is expected that the item life distribution will be of the Weibull form with a value for β , the shape parameter of $3/4$. The value for the location parameter, γ , is zero.

The $(t/\mu) \times 100$ ratio at the AQL will be

$$(500/20,000) \times 100 \text{ or } 2.5 .$$

The $(t/\mu) \times 100$ ratio at the LTPD will be

$$(500/6,000) \times 100 \text{ or } 8.3 .$$

With these values, Table 1-B giving the $(t/\mu) \times 100$ ratios for $\beta = 3/4$ may be scanned to determine the appropriate MIL-STD-105B plan. One may note in this table that an AQL of 6.5 (in percent defective) corresponds to a $(t/\mu) \times 100$ ratio of 2.29. This is the closest value available for the desired ratio value of 2.5. The column under the 6.5 AQL value heading may next be scanned to find a close approximation to the desired value of 8.3 for the LTPD. A value, 8.2 is found corresponding to Sample Size Letter K.

Thus any MIL-STD-105B plan with Sample Size Code Letter K and for an Acceptable Quality Level of 6.5 will give approximately the desired operating characteristics for the specified test period of 500 hours. For single sampling, for example, the sample size will be 110 items and the acceptance number 12 as indicated in the MIL-STD-105B tables.

Example (3)

The procedure to be followed for cases in which the Weibull location parameter, γ , is not zero but is of some other known value may be illustrated by outlining a third example. The method to be followed in this case is to simply subtract the value for γ from the value for t , the test time to get t_0 , and from μ to get μ_0 . These transformed values t_0 and μ_0 are then used for all $(t/\mu) \times 100$ computations. The solution obtained in terms of

t_0 and μ_0 can then be converted back to original values by simply adding the value for γ to each.

Consider, for example, an application for which a single-sampling plan with n equal to 35 and c equal to 1 has been specified. This corresponds to a plan with Sample Size Letter H and an AQL (in terms of p') of 1.5% in the 105B collection. Item life is measured in terms of cycles of operation. Protection against lots for which the average item life is less than 5,000 cycles is required. From experience with this product it has been determined that the Weibull distribution applies and that a value for γ of 2,000 cycles and a value for β of 2 can be expected. The problem is to determine a test time, t , in cycles that will enable the plan to meet the above requirement for consumer's protection. A related problem is to find whether the plan so determined will give adequate producer protection. It has been determined that a mean item life of 16,000 can reasonably be produced by a competent supplier.

The first step toward a solution is to convert the required lot mean life, μ , to a transformed value, μ_0 . This new value, μ_0 , is $\mu - \gamma$ or 5,000 - 2,000 which is 3,000 cycles. Next, from Table 1-F which gives conversion ratios for use when $\beta = 2$, one finds that for Sample Size Letter H and an AQL in p' (%) of 1.5, the ratio at the LTPD Quality Level is 38 and at the AQL it is 13.87. Since the plan is to be determined in terms of consumer's needs, the next step is to use the LTPD ratio to determine t_0 . Thus $(t_0/\mu_0) \times 100 = 38$ or $(t_0/3,000) \times 100 = 38$. From this it is determined that t_0 must equal 1140. By adding the value for γ (which is 2,000) to this latter figure, the required test time in absolute terms, t , of 3,140 cycles is obtained.

The related question of reasonableness of this plan for the producer may be answered by substitution of the test time just determined in the ratio for the AQL. The relationship is that $(t_o/\mu_o) \times 100 = 13.87$ or that $(1140/\mu_o) \times 100 = 13.87$. From this a value for μ_o of 8,240 cycles is obtained. This is then converted to original terms by the addition of the value for γ of 2,000 cycles. This gives an Acceptable Quality Level of 10,240 cycles. This is well below the level considered reasonable so no hardship will be imposed on the supplier.

Appendix - The Relationship Between p' and t/μ Under the Weibull Model

(The material in this appendix can be found in expanded form in Technical Report No. 1¹. It is presented here for completeness)

The probability of an item failing prior to some specified time x is given by the cumulative distribution. If the lifelength, X has a Weibull distribution, this probability is,

$$P \{ X \leq x \} = 1 - e^{-\left(\frac{x-\gamma}{\eta}\right)^\beta} \quad \text{for } x \geq \gamma; \alpha, \beta > 0.$$

= 0, otherwise.

Here η (sometimes referred to as the characteristic life) is the scale parameter; β , the shape parameter and γ , the location parameter.

The mean (in excess of γ) of the Weibull distribution is,

$$\mu = \eta \Gamma \left(\frac{1}{\beta} + 1 \right)$$

If p' represents the probability of an item failing prior to some test time t (in excess of γ), then,

$$p' = 1 - e^{-(t/\eta)^\beta} = 1 - e^{-\left[\frac{t}{\mu} \cdot \Gamma\left(\frac{1}{\beta}\right)\right]^\beta}$$

$$\text{or } \frac{t}{\mu} = \frac{[-\ln(1-p')]^{1/\beta}}{\Gamma\left(\frac{1}{\beta} + 1\right)}$$

which establishes the relationship between p' and t/μ under the Weibull Model.

Bibliography

1. Goode, Henry P. and Kao, John H.K., "Sampling Plans Based on the Weibull Distribution," to be published in the Proceedings of the Seventh National Symposium on Reliability and Quality Control, 1961.
2. Sobel, Milton and Tischendorf, J.A., "Acceptance Sampling with New Life Test Objectives," Proceedings of the Fifth National Symposium on Reliability and Quality Control in Electronics, 1959, pp. 108-118.
3. Military Standard - 105B, "Sampling Procedures and Tables for Inspection by Attributes," United States Government Printing Office, 1959

Table of $(t/\mu) \times 100$ Ratios for $\beta = 1/2$ [illegible]

TABLE 1-C

Table of $(t/\mu) \times 100$ Ratios for $\beta = 1$

Sample Size Code Letter	Acceptable Quality Level (AQL)														
	$p'(\%)$	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
	(t/μ) $\times 100$	.015	.035	.065	.10	.15	.25	.40	.65	1.01	1.51	2.53	4.08	6.72	10.54
A															
B															
C															
D															
E															
F															
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I															
J															
K															
L															
M															
N															
O															
P															
Q															

 $(t/\mu) \times 100$ at LTPD Quality Level
 $[P(A) = .10]$

Table of $(t/\mu) \times 100$ Ratios for $\beta = 1 \frac{1}{3}$

[illegible]

TABLE 1-E
Table of (t/ μ)x100 Ratios for $\beta = 1 \frac{2}{3}$

[illegible]

TABLE 2
Table of $(t/\mu) \times 100$ Ratios
at the Acceptable Quality Level (AQL)

β	Acceptable Quality Level - p' (%)													
	0.015	0.035	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10.0
1/3	56×10^{-12}	72×10^{-11}	46×10^{-10}	17×10^{-9}	56×10^{-9}	26×10^{-8}	11×10^{-7}	46×10^{-7}	17×10^{-6}	59×10^{-6}	27×10^{-5}	11×10^{-4}	51×10^{-4}	19×10^{-3}
1/2	11×10^{-7}	61×10^{-7}	21×10^{-6}	50×10^{-6}	11×10^{-5}	31×10^{-5}	80×10^{-5}	21×10^{-4}	51×10^{-4}	11×10^{-3}	32×10^{-3}	83×10^{-3}	.23	.56
3/4	67×10^{-5}	21×10^{-4}	47×10^{-4}	84×10^{-4}	14×10^{-3}	29×10^{-3}	53×10^{-3}	.10	.18	.31	.62	1.18	2.29	4.18
1	15×10^{-3}	35×10^{-3}	65×10^{-3}	.10	.15	.25	.40	.65	1.01	1.51	2.53	4.08	6.72	10.54
1-1/8	41×10^{-3}	88×10^{-3}	.15	.22	.32	.50	.76	1.18	1.73	2.49	3.94	6.02	9.37	13.98
1-1/4	94×10^{-3}	.18	.30	.43	.59	.89	1.30	1.92	2.71	3.75	5.67	8.31	12.38	17.74
1-1/3	.15	.28	.44	.61	.83	1.22	1.73	2.50	3.45	4.69	6.91	9.88	14.36	20.12
1-1/2	.31	.55	.83	1.11	1.45	2.04	2.80	3.87	5.16	6.77	9.55	13.13	18.31	24.71
1-2/3	.57	.94	1.37	1.78	2.26	3.08	4.07	5.46	7.08	9.07	12.33	16.42	22.15	29.01
2	1.38	2.11	2.88	3.57	4.37	5.64	7.14	9.12	11.31	13.87	17.95	22.79	29.25	36.63
2-1/2	3.32	4.68	5.98	7.11	8.36	10.27	12.39	15.06	17.90	21.08	25.90	31.35	38.28	45.82
3-1/3	7.94	10.24	12.32	14.03	15.84	18.47	21.27	24.62	28.03	31.68	36.98	42.69	49.57	56.73
4	12.21	15.10	17.62	19.62	21.71	24.68	27.76	31.35	34.93	38.68	44.01	49.59	56.18	62.85
5	18.72	22.17	25.10	27.36	29.67	32.87	36.12	39.81	43.40	47.09	52.21	57.45	63.46	69.44