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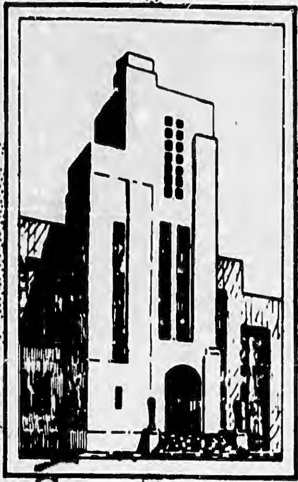

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MATHEMATICAL ANALYSIS AND DIGITAL COMPUTER SOLUTION OF NATURAL FREQUENCIES AND NORMAL MODES OF VIBRATION FOR A COMPOUND ISOLATION MOUNTING SYSTEM

by

Leon Katz

APPLIED MATHEMATICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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ABSTRACT

The mathematical analysis and solution of the natural frequencies and normal modes of vibration for a compound isolation mounting system by McGoldrick's method are discussed. The system consists of an assembly supported by a set of isolation mountings carried by a cradle which is, in turn, supported by another set of isolation mountings attached to the hull of a ship. The solution for a single, resiliently mounted, rigid body is presented as a special case of the two-body system. Instructions for obtaining computation service at the Applied Mathematics Laboratory, David Taylor Model Basin, and specifications for requisite data are included.

INTRODUCTION

The problem of determining the natural frequencies and normal modes of vibration for a single, resiliently mounted, rigid assembly was defined by R. T. McGoldrick. It was mathematically analyzed by R. P. Eddy and coded by S. Good for solution on the UNIVAC as Applied Mathematics Laboratory Problem 97-55. R. T. McGoldrick later formulated the problem for a compound isolation mounting system. An IBM-704 program was developed for solution of a compound system (Applied Mathematics Laboratory Problem 4-186) and, as explained in this report, the new program can also be used for solution of the one-body problem. Thus the new program supersedes the old.

MATHEMATICAL ANALYSIS AND SOLUTION

The physical theory leading to the development of the dynamical equations for the natural frequencies and normal modes of vibration of a compound isolation mounting system is explained in Reference 1* (and also in Reference 5). The system consists of an assembly supported by a set of isolation mountings carried by a cradle which is, in turn, supported by another set of isolation mountings attached to the hull of a ship. These dynamical equations of Reference 1 comprise a system of 12 homogeneous, simultaneous linear equations. For a given natural frequency, the unknowns in these equations are u_1, v_1, w_1, $\boldsymbol{\alpha}_1$, β_1, γ_1, u_2, v_2, w_2, $\boldsymbol{\alpha}_2$, β_2, γ_2; a solution for these unknowns comprises the components of the corresponding normal mode of vibration. The first of these 12 equations is

$$(K_{uu}^{IV} - m_1\omega^2)\, u_1 + K_{uv}^{IV}\, v_1 + K_{uw}^{IV}\, w_1 + K_{u\boldsymbol{\alpha}}^{IV}\, \boldsymbol{\alpha}_1 + K_{u\beta}^{IV}\, \beta_1 + K_{u\gamma}^{IV}\, \gamma_1$$
$$- K_{uu}^{II} u_2 - K_{uv}^{II} v_2 - K_{uw}^{II} w_2 - K_{u\boldsymbol{\alpha}}^{II}\, \boldsymbol{\alpha}_2 - K_{u\beta}^{II} \beta_2 - K_{u\gamma}^{II} \gamma_2 = 0$$

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*References are listed on page 16.

The matrix* of coefficients of the unknowns for all 12 equations is given in Figure 1. As exemplified by the above equation, each row of the matrix represents the left side of an equation; the right side is always zero.

The equations, as they now stand, are not in a convenient form for direct solution. Using matrix notation, some algebraic manipulations shall first be performed upon them to put them in standard form for eigenvalue-eigenvector analysis. Then standard, precoded subroutines for computing eigenvalues and eigenvectors can be utilized in the digital computation of the solution.

The original matrix can be regarded as a linear combination of two other matrices, one such matrix being the original matrix with all terms containing ω^2 deleted and the other matrix containing minus the coefficients of ω^2 for its elements. These two matrices, denoted by K and M, are given in Figures 2 and 3, respectively. The system of equations, in matrix notation, can now be written as

$$(K - M\omega^2)\, x = 0$$

where x is the vector of the unknowns. This is equivalent to

$$K\, x - M\omega^2\, x = 0$$

or

$$K\, x = M\omega^2\, x$$

The inverse of the M-matrix, M^{-1}, is given in Figure 4. Multiplying by M^{-1} results in

$$M^{-1}K\, x = M^{-1}M\omega^2\, x$$

$$M^{-1}K\, x = \omega^2\, x$$

This equation is in standard form for eigenvalue-eigenvector analysis. The computer program first performs computations on the input parameters (masses, moments of inertia, products of inertia, spring constants, direction cosines of axes, and coordinates of centers of gravity and of effective points of attachment) as specified by Reference 1, to determine the elements of the M^{-1} and K matrices. Next it multiplies together the two matrices M^{-1} and K, and the resulting product matrix then becomes the input to SHARE (organization of users of IBM scientific computers) subroutines** for eigenvalue and eigenvector computation. The eigenvalues are values for ω^2, from which the program computes the natural frequencies of vibration (cycles per second) by

$$f = \sqrt{\omega^2} \,/\, 2\pi$$

*Readers who are not acquainted with matrix theory or who desire a brief review should refer to Appendix A of this report.

**These subroutines are discussed in Appendix B.

Figure 1 – Original Matrix

$K_{uu}^{IV} - m_1\omega^2$	K_{uv}^{IV}	K_{uw}^{IV}	$K_{u\alpha}^{IV}$	$K_{u\beta}^{IV}$
K_{uv}^{IV}	$K_{vv}^{IV} - m_1\omega^2$	K_{vw}^{IV}	$K_{v\alpha}^{IV}$	$K_{v\beta}^{IV}$
K_{uw}^{IV}	K_{vw}^{IV}	$K_{ww}^{IV} - m_1\omega^2$	$K_{w\alpha}^{IV}$	$K_{w\beta}^{IV}$
$K_{u\alpha}^{IV}$	$K_{v\alpha}^{IV}$	$K_{w\alpha}^{IV}$	$K_{\alpha\alpha}^{IV} - I_x^I\omega^2$	$K_{\alpha\beta}^{IV} + I_{xy}^I\omega^2$
$K_{u\beta}^{IV}$	$K_{v\beta}^{IV}$	$K_{w\beta}^{IV}$	$K_{\alpha\beta}^{IV} + I_{xy}^I\omega^2$	$K_{\beta\beta}^{IV} - I_y^I\omega^2$
$K_{u\gamma}^{IV}$	$K_{v\gamma}^{IV}$	$K_{w\gamma}^{IV}$	$K_{\alpha\gamma}^{IV} + I_{xz}^I\omega^2$	$K_{\beta\gamma}^{IV} + I_{yz}^I\omega^2$
$-K_{uu}^{III}$	$-K_{uv}^{III}$	$-K_{uw}^{III}$	$-K_{u\alpha}^{III}$	$-K_{u\beta}^{III}$
$-K_{uv}^{III}$	$-K_{vv}^{III}$	$-K_{vw}^{III}$	$-K_{v\alpha}^{III}$	$-K_{v\beta}^{III}$
$-K_{uw}^{III}$	$-K_{vw}^{III}$	$-K_{ww}^{III}$	$-K_{w\alpha}^{III}$	$-K_{w\beta}^{III}$
$K_{uv}^{III}z^I - K_{uw}^{III}y^I - K_{u\alpha}^{III}$	$K_{vv}^{III}z^I - K_{vw}^{III}y^I - K_{v\alpha}^{III}$	$K_{vw}^{III}z^I - K_{ww}^{III}y^I - K_{w\alpha}^{III}$	$K_{v\alpha}^{III}z^I - K_{w\alpha}^{III}y^I - K_{\alpha\alpha}^{III}$	$K_{v\beta}^{III}z^I - K_{w\beta}^{III}y^I - K_{\alpha\beta}^{III}$
$K_{uw}^{III}x^I - K_{uu}^{III}z^I - K_{u\beta}^{III}$	$K_{vw}^{III}x^I - K_{uv}^{III}z^I - K_{v\beta}^{III}$	$K_{ww}^{III}x^I - K_{uw}^{III}z^I - K_{w\beta}^{III}$	$K_{w\alpha}^{III}x^I - K_{u\alpha}^{III}z^I - K_{\alpha\beta}^{III}$	$K_{w\beta}^{III}x^I - K_{u\beta}^{III}z^I - K_{\beta\beta}^{III}$
$K_{uu}^{III}y^I - K_{uv}^{III}x^I - K_{u\gamma}^{III}$	$K_{uv}^{III}y^I - K_{vv}^{III}x^I - K_{v\gamma}^{III}$	$K_{uw}^{III}y^I - K_{vw}^{III}x^I - K_{w\gamma}^{III}$	$K_{u\alpha}^{III}y^I - K_{v\alpha}^{III}x^I - K_{\alpha\gamma}^{III}$	$K_{u\beta}^{III}y^I - K_{v\beta}^{III}x^I - K_{\beta\gamma}^{III}$

Figure 2 – K-Matrix

K_{uu}^{IV}	K_{uv}^{IV}	K_{uw}^{IV}	$K_{u\alpha}^{IV}$	$K_{u\beta}^{IV}$
K_{uv}^{IV}	K_{vv}^{IV}	K_{vw}^{IV}	$K_{v\alpha}^{IV}$	$K_{v\beta}^{IV}$
K_{uw}^{IV}	K_{vw}^{IV}	K_{ww}^{IV}	$K_{w\alpha}^{IV}$	$K_{w\beta}^{IV}$
$K_{u\alpha}^{IV}$	$K_{v\alpha}^{IV}$	$K_{w\alpha}^{IV}$	$K_{\alpha\alpha}^{IV}$	$K_{\alpha\beta}$
$K_{u\beta}^{IV}$	$K_{v\beta}^{IV}$	$K_{w\beta}^{IV}$	$K_{\alpha\beta}^{IV}$	$K_{\beta\beta}^{IV}$
$K_{u\gamma}^{IV}$	$K_{v\gamma}^{IV}$	$K_{w\gamma}^{IV}$	$K_{\alpha\gamma}^{IV}$	$K_{\beta\gamma}^{IV}$
$-K_{uu}^{III}$	$-K_{uv}^{III}$	$-K_{uw}^{III}$	$-K_{u\alpha}^{III}$	$-K_{u\beta}^{III}$
$-K_{uv}^{III}$	$-K_{vv}^{III}$	$-K_{vw}^{III}$	$-K_{v\alpha}^{III}$	$-K_{v\beta}^{III}$
$-K_{uw}^{III}$	$-K_{vw}^{III}$	$-K_{ww}^{III}$	$-K_{w\alpha}^{III}$	$-K_{w\beta}^{III}$
$K_{uv}^{III}z^I - K_{uw}^{III}y^I - K_{u\alpha}^{III}$	$K_{vv}^{III}z^I - K_{vw}^{III}y^I - K_{v\alpha}^{III}$	$K_{vw}^{III}z^I - K_{ww}^{III}y^I - K_{w\alpha}^{III}$	$K_{v\alpha}^{III}z^I - K_{w\alpha}^{III}y^I - K_{\alpha\alpha}^{III}$	$K_{v\beta}^{III}z^I - K_{w\beta}^{III}y^I - K_{\alpha\beta}^{III}$
$K_{uw}^{III}x^I - K_{uu}^{III}z^I - K_{u\beta}^{III}$	$K_{vw}^{III}x^I - K_{uv}^{III}z^I - K_{v\beta}^{III}$	$K_{ww}^{III}x^I - K_{uw}^{III}z^I - K_{w\beta}^{III}$	$K_{w\alpha}^{III}x^I - K_{u\alpha}^{III}z^I - K_{\alpha\beta}^{III}$	$K_{w\beta}^{III}x^I - K_{u\beta}^{III}z^I - K_{\beta\beta}^{III}$
$K_{uu}^{III}y^I - K_{uv}^{III}x^I - K_{u\gamma}^{III}$	$K_{uv}^{III}y^I - K_{vv}^{III}x^I - K_{v\gamma}^{III}$	$K_{uw}^{III}y^I - K_{vw}^{III}x^I - K_{w\gamma}^{III}$	$K_{u\alpha}^{III}y^I - K_{v\alpha}^{III}x^I - K_{\alpha\gamma}^{III}$	$K_{u\beta}^{III}y^I - K_{v\beta}^{III}x^I - K_{\beta\gamma}^{III}$

2

	$K^{IV}_{u\alpha}$	$K^{IV}_{u\beta}$	$K^{IV}_{u\gamma}$	$-K^{II}_{uu}$	$-K^{II}_{uv}$	$-K^{II}_{uw}$	$-K^{II}_{u\alpha}$
	$K^{IV}_{v\alpha}$	$K^{IV}_{v\beta}$	$K^{IV}_{v\gamma}$	$-K^{II}_{uv}$	$-K^{II}_{vv}$	$-K^{II}_{vw}$	$-K^{II}_{v\alpha}$
	$K^{IV}_{w\alpha}$	$K^{IV}_{w\beta}$	$K^{IV}_{w\gamma}$	$-K^{II}_{uw}$	$-K^{II}_{vw}$	$-K^{II}_{ww}$	$-K^{II}_{w\alpha}$
	$K^{IV}_{\alpha\alpha} - I^{I}_{x}\omega^2$	$K^{IV}_{\alpha\beta} + I^{I}_{xy}\omega^2$	$K^{IV}_{\alpha\gamma} + I^{I}_{xz}\omega^2$	$K^{II}_{uv} z^{II} - K^{II}_{uw} y^{II} - K^{II}_{u\alpha}$	$K^{II}_{vv} z^{II} - K^{II}_{vw} y^{II} - K^{II}_{v\alpha}$	$K^{II}_{vw} z^{II} - K^{II}_{ww} y^{II} - K^{II}_{w\alpha}$	$K^{II}_{v\alpha} z^{II} - K^{II}_{w\alpha} y^{I}$
	$K^{IV}_{\alpha\beta} + I^{I}_{xy}\omega^2$	$K^{IV}_{\beta\beta} - I^{I}_{y}\omega^2$	$K^{IV}_{\beta\gamma} + I^{I}_{yz}\omega^2$	$K^{II}_{uw} x^{II} - K^{II}_{uu} z^{II} - K^{II}_{u\beta}$	$K^{II}_{vw} x^{II} - K^{II}_{uv} z^{II} - K^{II}_{v\beta}$	$K^{II}_{ww} x^{II} - K^{II}_{uw} z^{II} - K^{II}_{w\beta}$	$K^{II}_{w\alpha} x^{II} - K^{II}_{u\alpha} z^{II}$
	$K^{IV}_{\alpha\gamma} + I^{I}_{xz}\omega^2$	$K^{IV}_{\beta\gamma} + I^{I}_{yz}\omega^2$	$K^{IV}_{\gamma\gamma} - I^{I}_{z}\omega^2$	$K^{II}_{uu} y^{II} - K^{II}_{uv} x^{II} - K^{II}_{u\gamma}$	$K^{II}_{uv} y^{II} - K^{II}_{vv} x^{II} - K^{II}_{v\gamma}$	$K^{II}_{uw} y^{II} - K^{II}_{vw} x^{II} - K^{II}_{w\gamma}$	$K^{II}_{u\alpha} y^{II} - K^{II}_{v\alpha} x^{II}$
	$-K^{III}_{u\alpha}$	$-K^{III}_{u\beta}$	$-K^{III}_{u\gamma}$	$K^{II}_{uu} - m_2\omega^2$	K^{II}_{uv}	K^{II}_{uw}	$K^{II}_{u\alpha}$
	$-K^{III}_{v\alpha}$	$-K^{III}_{v\beta}$	$-K^{III}_{v\gamma}$	K^{II}_{uv}	$K^{II}_{vv} - m_2\omega^2$	K^{II}_{vw}	$K^{II}_{v\alpha}$
	$-K^{III}_{w\alpha}$	$-K^{III}_{w\beta}$	$-K^{III}_{w\gamma}$	K^{II}_{uw}	K^{II}_{vw}	$K^{II}_{ww} - m_2\omega^2$	$K^{II}_{w\alpha}$
$^{I} - K^{III}_{w\alpha}$	$K^{III}_{v\alpha} z^{I} - K^{III}_{w\alpha} y^{I} - K^{III}_{\alpha\alpha}$	$K^{III}_{v\beta} z^{I} - K^{III}_{w\beta} y^{I} - K^{III}_{\alpha\beta}$	$K^{III}_{v\gamma} z^{I} - K^{III}_{w\gamma} y^{I} - K^{III}_{\alpha\gamma}$	$K^{II}_{u\alpha}$	$K^{II}_{v\alpha}$	$K^{II}_{w\alpha}$	$K^{II}_{\alpha\alpha} - I^{II}_{x}\omega^2$
$^{I} - K^{III}_{w\beta}$	$K^{III}_{w\alpha} x^{I} - K^{III}_{u\alpha} z^{I} - K^{III}_{\alpha\beta}$	$K^{III}_{w\beta} x^{I} - K^{III}_{u\beta} z^{I} - K^{III}_{\beta\beta}$	$K^{III}_{w\gamma} x^{I} - K^{III}_{u\gamma} z^{I} - K^{III}_{\beta\gamma}$	$K^{II}_{u\beta}$	$K^{II}_{v\beta}$	$K^{II}_{w\beta}$	$K^{II}_{\alpha\beta} + I^{II}_{xy}\omega^2$
$^{I} - K^{III}_{w\gamma}$	$K^{III}_{u\alpha} y^{I} - K^{III}_{v\alpha} x^{I} - K^{III}_{\alpha\gamma}$	$K^{III}_{u\beta} y^{I} - K^{III}_{v\beta} x^{I} - K^{III}_{\beta\gamma}$	$K^{III}_{u\gamma} y^{I} - K^{III}_{v\gamma} x^{I} - K^{III}_{\gamma\gamma}$	$K^{II}_{u\gamma}$	$K^{II}_{v\gamma}$	$K^{II}_{w\gamma}$	$K^{II}_{\alpha\gamma} + I^{II}_{xz}\omega^2$

	$K^{IV}_{u\alpha}$	$K^{IV}_{u\beta}$	$K^{IV}_{u\gamma}$	$-K^{II}_{uu}$	$-K^{II}_{uv}$	$-K^{II}_{uw}$	$-K^{II}_{u\alpha}$
	$K^{IV}_{v\alpha}$	$K^{IV}_{v\beta}$	$K^{IV}_{v\gamma}$	$-K^{II}_{uv}$	$-K^{II}_{vv}$	$-K^{II}_{vw}$	$-K^{II}_{v\alpha}$
	$K^{IV}_{w\alpha}$	$K^{IV}_{w\beta}$	$K^{IV}_{w\gamma}$	$-K^{II}_{uw}$	$-K^{II}_{vw}$	$-K^{II}_{ww}$	$-K^{II}_{w\alpha}$
	$K^{IV}_{\alpha\alpha}$	$K^{IV}_{\alpha\beta}$	$K^{IV}_{\alpha\gamma}$	$K^{II}_{uv} z^{II} - K^{II}_{uw} y^{II} - K^{II}_{u\alpha}$	$K^{II}_{vv} z^{II} - K^{II}_{vw} y^{II} - K^{II}_{v\alpha}$	$K^{II}_{vw} z^{II} - K^{II}_{ww} y^{II} - K^{II}_{w\alpha}$	$K^{II}_{v\alpha} z^{II} - K$
	$K^{IV}_{\alpha\beta}$	$K^{IV}_{\beta\beta}$	$K^{IV}_{\beta\gamma}$	$K^{II}_{uw} x^{II} - K^{II}_{uu} z^{II} - K^{II}_{u\beta}$	$K^{II}_{vw} x^{II} - K^{II}_{uv} z^{II} - K^{II}_{v\beta}$	$K^{II}_{ww} x^{II} - K^{II}_{uw} z^{II} - K^{II}_{w\beta}$	$K^{II}_{w\alpha} x^{II} - K$
	$K^{IV}_{\alpha\gamma}$	$K^{IV}_{\beta\gamma}$	$K^{IV}_{\gamma\gamma}$	$K^{II}_{uu} y^{II} - K^{II}_{uv} x^{II} - K^{II}_{u\gamma}$	$K^{II}_{uv} y^{II} - K^{II}_{vv} x^{II} - K^{II}_{v\gamma}$	$K^{II}_{uw} y^{II} - K^{II}_{vw} x^{II} - K^{II}_{w\gamma}$	$K^{II}_{u\alpha} y^{II} - K$
	$-K^{III}_{u\alpha}$	$-K^{III}_{u\beta}$	$-K^{III}_{u\gamma}$	K^{II}_{uu}	K^{II}_{uv}	K^{II}_{uw}	$K^{II}_{u\alpha}$
	$-K^{III}_{v\alpha}$	$-K^{III}_{v\beta}$	$-K^{III}_{v\gamma}$	K^{II}_{uv}	K^{II}_{vv}	K^{II}_{vw}	$K^{II}_{v\alpha}$
	$-K^{III}_{w\alpha}$	$-K^{III}_{w\beta}$	$-K^{III}_{w\gamma}$	K^{II}_{uw}	K^{II}_{vw}	K^{II}_{ww}	$K^{II}_{w\alpha}$
$- K^{III}_{w\alpha}$	$K^{III}_{v\alpha} z^{I} - K^{III}_{w\alpha} y^{I} - K^{III}_{\alpha\alpha}$	$K^{III}_{v\beta} z^{I} - K^{III}_{w\beta} y^{I} - K^{III}_{\alpha\beta}$	$K^{III}_{v\gamma} z^{I} - K^{III}_{w\gamma} y^{I} - K^{III}_{\alpha\gamma}$	$K^{II}_{u\alpha}$	$K^{II}_{v\alpha}$	$K^{II}_{w\alpha}$	$K^{II}_{\alpha\alpha}$
$- K^{III}_{w\beta}$	$K^{III}_{w\alpha} x^{I} - K^{III}_{u\alpha} z^{I} - K^{III}_{\alpha\beta}$	$K^{III}_{w\beta} x^{I} - K^{III}_{u\beta} z^{I} - K^{III}_{\beta\beta}$	$K^{III}_{w\gamma} x^{I} - K^{III}_{u\gamma} z^{I} - K^{III}_{\beta\gamma}$	$K^{II}_{u\beta}$	$K^{II}_{v\beta}$	$K^{II}_{w\beta}$	$K^{II}_{\alpha\beta}$
$- K^{III}_{w\gamma}$	$K^{III}_{u\alpha} y^{I} - K^{III}_{v\alpha} x^{I} - K^{III}_{\alpha\gamma}$	$K^{III}_{u\beta} y^{I} - K^{III}_{v\beta} x^{I} - K^{III}_{\beta\gamma}$	$K^{III}_{u\gamma} y^{I} - K^{III}_{v\gamma} x^{I} - K^{III}_{\gamma\gamma}$	$K^{II}_{u\gamma}$	$K^{II}_{v\gamma}$	$K^{II}_{w\gamma}$	$K^{II}_{\alpha\gamma}$

	$-K^{II}_{uu}$	$-K^{II}_{uv}$	$-K^{II}_{uw}$	$-K^{II}_{u\alpha}$	$-K^{II}_{u\beta}$	$-K^{II}_{u\gamma}$
	$-K^{II}_{uv}$	$-K^{II}_{vv}$	$-K^{II}_{vw}$	$-K^{II}_{v\alpha}$	$-K^{II}_{v\beta}$	$-K^{II}_{v\gamma}$
	$-K^{II}_{uw}$	$-K^{II}_{vw}$	$-K^{II}_{ww}$	$-K^{II}_{w\alpha}$	$-K^{II}_{w\beta}$	$-K^{II}_{w\gamma}$
^{2}	$K^{II}_{uv} z^{II} - K^{II}_{uw} y^{II} - K^{II}_{u\alpha}$	$K^{II}_{vv} z^{II} - K^{II}_{vw} y^{II} - K^{II}_{v\alpha}$	$K^{II}_{vw} z^{II} - K^{II}_{ww} y^{II} - K^{II}_{w\alpha}$	$K^{II}_{v\alpha} z^{II} - K^{II}_{w\alpha} y^{II} - K^{II}_{\alpha\alpha}$	$K^{II}_{v\beta} z^{II} - K^{II}_{w\beta} y^{II} - K^{II}_{\alpha\beta}$	$K^{II}_{v\gamma} z^{II} - K^{II}_{w\gamma} y^{II} - K^{II}_{\alpha\gamma}$
^{2}	$K^{II}_{uw} x^{II} - K^{II}_{uu} z^{II} - K^{II}_{u\beta}$	$K^{II}_{vw} x^{II} - K^{II}_{uv} z^{II} - K^{II}_{v\beta}$	$K^{II}_{ww} x^{II} - K^{II}_{uw} z^{II} - K^{II}_{w\beta}$	$K^{II}_{w\alpha} x^{II} - K^{II}_{u\alpha} z^{II} - K^{II}_{\alpha\beta}$	$K^{II}_{w\beta} x^{II} - K^{II}_{u\beta} z^{II} - K^{II}_{\beta\beta}$	$K^{II}_{w\gamma} x^{II} - K^{II}_{u\gamma} z^{II} - K^{II}_{\gamma\beta}$
	$K^{II}_{uu} y^{II} - K^{II}_{uv} x^{II} - K^{II}_{u\gamma}$	$K^{II}_{uv} y^{II} - K^{II}_{vv} x^{II} - K^{II}_{v\gamma}$	$K^{II}_{uw} y^{II} - K^{II}_{vw} x^{II} - K^{II}_{w\gamma}$	$K^{II}_{u\alpha} y^{II} - K^{II}_{v\alpha} x^{II} - K^{II}_{\alpha\gamma}$	$K^{II}_{u\beta} y^{II} - K^{II}_{v\beta} x^{II} - K^{II}_{\beta\gamma}$	$K^{II}_{u\gamma} y^{II} - K^{II}_{v\gamma} x^{II} - K^{II}_{\gamma\gamma}$
	$K^{II}_{uu} - m_2 \omega^2$	K^{II}_{uv}	K^{II}_{uw}	$K^{II}_{u\alpha}$	$K^{II}_{u\beta}$	$K^{II}_{u\gamma}$
	K^{II}_{uv}	$K^{II}_{vv} - m_2 \omega^2$	K^{II}_{vw}	$K^{II}_{v\alpha}$	$K^{II}_{v\beta}$	$K^{II}_{v\gamma}$
	K^{II}_{uw}	K^{II}_{vw}	$K^{II}_{ww} - m_2 \omega^2$	$K^{II}_{w\alpha}$	$K^{II}_{w\beta}$	$K^{II}_{w\gamma}$
$^{II}_{\gamma} y^I - K^{III}_{\alpha\gamma}$	$K^{II}_{u\alpha}$	$K^{II}_{v\alpha}$	$K^{II}_{w\alpha}$	$K^{II}_{\alpha\alpha} - I^{II}_x \omega^2$	$K^{II}_{\alpha\beta} + I^{II}_{xy} \omega^2$	$K^{II}_{\alpha\gamma} + I^{II}_{xz} \omega^2$
$^{II}_{\gamma} z^I - K^{III}_{\beta\gamma}$	$K^{II}_{u\beta}$	$K^{II}_{v\beta}$	$K^{II}_{w\beta}$	$K^{II}_{\alpha\beta} + I^{II}_{xy} \omega^2$	$K^{II}_{\beta\beta} - I^{II}_y \omega^2$	$K^{II}_{\beta\gamma} + I^{II}_{yz} \omega^2$
$^{II}_{\gamma} x^I - K^{III}_{\gamma\gamma}$	$K^{II}_{u\gamma}$	$K^{II}_{v\gamma}$	$K^{II}_{w\gamma}$	$K^{II}_{\alpha\gamma} + I^{II}_{xz} \omega^2$	$K^{II}_{\beta\gamma} + I^{II}_{yz} \omega^2$	$K^{II}_{\gamma\gamma} - I^{II}_z \omega^2$

	$-K^{II}_{uu}$	$-K^{II}_{uv}$	$-K^{II}_{uw}$	$-K^{II}_{u\alpha}$	$-K^{II}_{u\beta}$	$-K^{II}_{u\gamma}$
	$-K^{II}_{uv}$	$-K^{II}_{vv}$	$-K^{II}_{vw}$	$-K^{II}_{v\alpha}$	$-K^{II}_{v\beta}$	$-K^{II}_{v\gamma}$
	$-K^{II}_{uw}$	$-K^{II}_{vw}$	$-K^{II}_{ww}$	$-K^{II}_{w\alpha}$	$-K^{II}_{w\beta}$	$-K^{II}_{w\gamma}$
	$K^{II}_{uv} z^{II} - K^{II}_{uw} y^{II} - K^{II}_{u\alpha}$	$K^{II}_{vv} z^{II} - K^{II}_{vw} y^{II} - K^{II}_{v\alpha}$	$K^{II}_{vw} z^{II} - K^{II}_{ww} y^{II} - K^{II}_{w\alpha}$	$K^{II}_{v\alpha} z^{II} - K^{II}_{w\alpha} y^{II} - K^{II}_{\alpha\alpha}$	$K^{II}_{v\beta} z^{II} - K^{II}_{w\beta} y^{II} - K^{II}_{\alpha\beta}$	$K^{II}_{v\gamma} z^{II} - K^{II}_{w\gamma} y^{II} - K^{II}_{\alpha\gamma}$
	$K^{II}_{uw} x^{II} - K^{II}_{uu} z^{II} - K^{II}_{u\beta}$	$K^{II}_{vw} x^{II} - K^{II}_{uv} z^{II} - K^{II}_{v\beta}$	$K^{II}_{ww} x^{II} - K^{II}_{uw} z^{II} - K^{II}_{w\beta}$	$K^{II}_{w\alpha} x^{II} - K^{II}_{u\alpha} z^{II} - K^{II}_{\alpha\beta}$	$K^{II}_{w\beta} x^{II} - K^{II}_{u\beta} z^{II} - K^{II}_{\beta\beta}$	$K^{II}_{w\gamma} x^{II} - K^{II}_{u\gamma} z^{II} - K^{II}_{\gamma\beta}$
	$K^{II}_{uu} y^{II} - K^{II}_{uv} x^{II} - K^{II}_{u\gamma}$	$K^{II}_{uv} y^{II} - K^{II}_{vv} x^{II} - K^{II}_{v\gamma}$	$K^{II}_{uw} y^{II} - K^{II}_{vw} x^{II} - K^{II}_{w\gamma}$	$K^{II}_{u\alpha} y^{II} - K^{II}_{v\alpha} x^{II} - K^{II}_{\alpha\gamma}$	$K^{II}_{u\beta} y^{II} - K^{II}_{v\beta} x^{II} - K^{II}_{\beta\gamma}$	$K^{II}_{u\gamma} y^{II} - K^{II}_{v\gamma} x^{II} - K^{II}_{\gamma\gamma}$
	K^{II}_{uu}	K^{II}_{uv}	K^{II}_{uw}	$K^{II}_{u\alpha}$	$K^{II}_{u\beta}$	$K^{II}_{u\gamma}$
	K^{II}_{uv}	K^{II}_{vv}	K^{II}_{vw}	$K^{II}_{v\alpha}$	$K^{II}_{v\beta}$	$K^{II}_{v\gamma}$
	K^{II}_{uw}	K^{II}_{vw}	K^{II}_{ww}	$K^{II}_{w\alpha}$	$K^{II}_{w\beta}$	$K^{II}_{w\gamma}$
$^{III}_{\gamma} y^I - K^{III}_{\alpha\gamma}$	$K^{II}_{u\alpha}$	$K^{II}_{v\alpha}$	$K^{II}_{w\alpha}$	$K^{II}_{\alpha\alpha}$	$K^{II}_{\alpha\beta}$	$K^{II}_{\alpha\gamma}$
$^{III}_{u\gamma} z^I - K^{III}_{\beta\gamma}$	$K^{II}_{u\beta}$	$K^{II}_{v\beta}$	$K^{II}_{w\beta}$	$K^{II}_{\alpha\beta}$	$K^{II}_{\beta\beta}$	$K^{II}_{\beta\gamma}$
$^{III}_{\gamma} x^I - K^{III}_{\gamma\gamma}$	$K^{II}_{u\gamma}$	$K^{II}_{v\gamma}$	$K^{II}_{w\gamma}$	$K^{II}_{\alpha\gamma}$	$K^{II}_{\beta\gamma}$	$K^{II}_{\gamma\gamma}$

Figure 3 – M-Matrix

m_1					
	m_1				
		m_1			
			I_x^I	$-I_{xy}^I$	$-I$
			$-I_{xy}^I$	I_y^I	$-I$
			$-I_{xz}^I$	$-I_{yz}^I$	I

Figure 4 – M^{-1}-Matrix

m_1^{-1}					
	m_1^{-1}				
		m_1^{-1}			
			$D^I\left[I_y^I I_z^I - (I_{yz}^I)^2\right]$	$D^I\left[I_z^I I_{xy}^I + I_{xz}^I I_{yz}^I\right]$	$D^I\left[I_y^I I_x\right.$
			$D^I\left[I_z^I I_{xy}^I + I_{xz}^I I_{yz}^I\right]$	$D^I\left[I_x^I I_z^I - (I_{xz}^I)^2\right]$	$D^I\left[I_x^I I_y\right.$
			$D^I\left[I_y^I I_{xz}^I + I_{xy}^I I_{yz}^I\right]$	$D^I\left[I_x^I I_{yz}^I + I_{xy}^I I_{xz}^I\right]$	$D^I\left[I_x^I I_y\right.$

$$D^I = \left[I_x^I I_y^I I_z^I - 2 I_{xy}^I I_{yz}^I I_{xz}^I - I_y^I (I_{xz}^I)^2 - I_z^I (I_{xy}^I)^2 - I_x^I (I_{yz}^I)^2\right]^{-1}$$

$$D^{II} = \left[I_x^{II} I_y^{II} I_z^{II} - 2 I_{xy}^{II} I_{yz}^{II} I_{xz}^{II} - I_y^{II} (I_{xz}^{II})^2 - I_z^{II} (I_{xy}^{II})^2 - I_x^{II} (I_{yz}^{II})^2\right]^{-1}$$

I^I_x	$-I^I_{xy}$	$-I^I_{xz}$					
$-I^I_{xy}$	I^I_y	$-I^I_{yz}$					
$-I^I_{xz}$	$-I^I_{yz}$	I^I_z					
			m_2				
				m_2			
					m_2		
						I^{II}_x	
						$-I^{II}_{xy}$	
						$-I^{II}_{xz}$	

$^{I}\left[I^I_y I^I_z - (I^I_{yz})^2\right]$	$D^I\left[I^I_z I^I_{xy} + I^I_{xz} I^I_{yz}\right]$	$D^I\left[I^I_y I^I_{xz} + I^I_{xy} I^I_{yz}\right]$					
$^{I}\left[I^I_z I^I_{xy} + I^I_{xz} I^I_{yz}\right]$	$D^I\left[I^I_x I^I_z - (I^I_{xz})^2\right]$	$D^I\left[I^I_x I^I_{yz} + I^I_{xy} I^I_{xz}\right]$					
$^{I}\left[I^I_y I^I_{xz} + I^I_{xy} I^I_{yz}\right]$	$D^I\left[I^I_x I^I_{yz} + I^I_{xy} I^I_{xz}\right]$	$D^I\left[I^I_x I^I_y - (I^I_{xy})^2\right]$					
			m_2^{-1}				
				m_2^{-1}			
					m_2^{-1}		
						$D^{II}\left[I^{II}_y I^{II}_z - (I^{II}_{yz})^2\right]$	D
						$D^{II}\left[I^{II}_z I^{II}_{xy} + I^{II}_{xz} I^{II}_{yz}\right]$	D
						$D^{II}\left[I^{II}_y I^{II}_{xz} + I^{II}_{xy} I^{II}_{yz}\right]$	D

2

	m_2					
		m_2				
			m_2			
				I_x^{II}	$-I_{xy}^{II}$	$-I_{xz}^{II}$
				$-I_{xy}^{II}$	I_y^{II}	$-I_{yz}^{II}$
				$-I_{xz}^{II}$	$-I_{yz}^{II}$	I_z^{II}

$\ldots I^{I}_{xy}I^{I}_{yz}]$						
$\ldots I^{I}_{xy}I^{I}_{xz}]$						
$\ldots I^{I}_{xy})^2]$						
	m_2^{-1}					
		m_2^{-1}				
			m_2^{-1}			
				$D^{II}\left[I_y^{II}I_z^{II} - (I_{yz}^{II})^2\right]$	$D^{II}\left[I_z^{II}I_{xy}^{II} + I_{xz}^{II}I_{yz}^{II}\right]$	$D^{II}\left[I_y^{II}I_{xz}^{II} + I_{xy}^{II}I_{yz}^{II}\right]$
				$D^{II}\left[I_z^{II}I_{xy}^{II} + I_{xz}^{II}I_{yz}^{II}\right]$	$D^{II}\left[I_x^{II}I_z^{II} - (I_{xz}^{II})^2\right]$	$D^{II}\left[I_x^{II}I_{yz}^{II} + I_{xy}^{II}I_{xz}^{II}\right]$
				$D^{II}\left[I_y^{II}I_{xz}^{II} + I_{xy}^{II}I_{yz}^{II}\right]$	$D^{II}\left[I_x^{II}I_{yz}^{II} + I_{xy}^{II}I_{xz}^{II}\right]$	$D^{II}\left[I_x^{II}I_y^{II} - (I_{xy}^{II})^2\right]$

3

The eigenvectors are the normal modes of vibration; these are scaled by the program so that the magnitude of the largest component is unity.

THE ONE-BODY PROBLEM

The theory leading to the development of the dynamical equations for the natural frequencies and normal modes of vibration for a single, resiliently mounted, rigid body (having six degrees of freedom) is explained in Appendix 5 of Reference 2. These dynamical equations comprise a system of six homogeneous, simultaneous, linear equations. The matrix of coefficients for these six equations is identical to the upper left 6×6 submatrix of the matrix of coefficients for the two-body problem given in Figure 1, the data for the single body and set of mountings of the one-body problem being regarded as data for the cradle and lower mountings of the two-body problem.

Except for the size of their respective matrices, the solution of the one-body problem and the solution of the two-body problem are identical in terms of matrix theory. Furthermore, because the M-matrix is a quasi-diagonal matrix,* the inverse of the upper left 6×6 submatrix of M is the upper left 6×6 submatrix of M^{-1}. Also, because the partitioning* of a 12×12 matrix into 6×6 submatrices is a partitioning in which all diagonal submatrices are of identical order, the upper left 6×6 submatrix of $M^{-1}K$ is the product of the upper left 6×6 submatrix of M^{-1} times the upper left 6×6 submatrix of K. Thus the coding which computes the solution for the two-body problem can also be utilized to compute the solution of the one-body problem. The program reads in the data and sets up two 12×12 matrices, just as it does for a two-body problem. However, after the two matrices are multiplied together, only the upper left 6×6 submatrix of the product matrix is used in the computation of the eigenvalues and eigenvectors.

DATA REQUIRED FOR PROGRAMMED COMPUTING AND SUGGESTED FORMATS

The following data must be furnished for each compound isolation mounting system: (Axes set I has its origin at the center of gravity of the cradle. Axes set II has its origin at the center of gravity of the assembly. These two sets of axes must be parallel to each other and may have any convenient orientation. In particular, they need not be principal axes of inertia.)

Identifying Information:

This information (maximum of 350 characters including spaces between words and punctuation marks) will appear on the edited output page to distinguish this compound system from others.

---

*These topics of matrix theory are not discussed in Appendix A of this report. A theoretical treatment can be found in Reference 3.

Cradle Data:

Mass, $m_1 = \dfrac{\text{weight in lb}}{386.4 \text{ in./sec}^2}$.

Moments of inertia I_x^I, I_y^I, I_z^I in units of lb-in.-sec^2 with respect to axes set I.

Products of inertia I_{xy}^I, I_{yz}^I, I_{xz}^I in units of lb-in.-sec^2 with respect to axes set I.

Coordinates x^I, y^I, z^I in inches of center of gravity of cradle with respect to axes set II.

Assembly Data:

Mass, $m_2 = \dfrac{\text{weight in lb}}{386.4 \text{ in./sec}^2}$.

Moments of inertia I_x^{II}, I_y^{II}, I_z^{II} in units of lb-in.-sec^2 with respect to axes set II.

Products of inertia I_{xy}^{II}, I_{yz}^{II}, I_{xz}^{II} in units of lb-in.-sec^2 with respect to axes set II.

Coordinates x^{II}, y^{II}, z^{II} in inches of center of gravity of assembly with respect to axes set I.

Data for Each Lower Mounting (joining cradle and hull of ship):

Axial spring (stiffness) constant k_a in lb/in.

Radial spring (stiffness) constant k_r in lb/in.

Direction cosines $\cos \phi_x$, $\cos \phi_y$, $\cos \phi_z$ of mounting axis with respect to positive directions of the axes sets.

Coordinates x_c, y_c, z_c in inches of effective point of attachment with respect to axes set I.

Data for Each Upper Mounting (joining assembly and cradle):

Axial spring (stiffness) constant k_a in lb/in.

Radial spring (stiffness) constant k_r in lb/in.

Direction cosines $\cos \phi_x$, $\cos \phi_y$, $\cos \phi_z$ of mounting axis with respect to positive directions of the axes sets.

Coordinates x_c, y_c, z_c in inches of effective point of attachment with respect to axes set I.

Coordinates x_a, y_a, z_a in inches of effective point of attachment with respect to axes set II.

IDENTIFYING INFORMATION: ____________________

____

____

____

____

CRADLE	m_1 ________		
	I_x^I ________	I_y^I ________	I_z^I ________
	I_{xy}^I ________	I_{yz}^I ________	I_{xz}^I ________
	x^I ________	y^I ________	z^I ________
ASSEMBLY	m_2 ________		
	I_x^{II} ________	I_y^{II} ________	I_z^{II} ________
	I_{xy}^{II} ________	I_{yz}^{II} ________	I_{xz}^{II} ________
	x^{II} ________	y^{II} ________	z^{II} ________

Suggested Format

MOUNTINGS

The data for each mounting are entered in a separate column.

Indicate "Upper" or "Lower"							
k_a							
k_r							
$\cos \phi_x$							
$\cos \phi_y$							
$\cos \phi_z$							
x_c							
y_c							
z_c							

upper mountings only	x_a							
	y_a							
	z_a							

Suggested Format

The following data must be furnished for each single-body system:

1. Identifying information (maximum of 350 characters including spaces between words and punctuation marks).

2. Mass, the three moments of inertia, and the three products of inertia of the body.

3. For each mounting, the same data as required for lower mountings of a compound system.

REQUESTING COMPUTATION SERVICE

Requests for computation using this program should be addressed to: Technical Director, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C. Only about 4 minutes of computer time are required for each compound mounting system. However, the need for scheduling computer time and for transcribing the input data onto punched cards in the form required by the program will ordinarily result in several days elapsed time between receipt of data and forwarding of results.

ACKNOWLEDGMENT

The author is indebted to Mr. L. Kenton Meals, who patiently analyzed the cryptic passages of References 6 and 7.

APPENDIX A

INTRODUCTORY MATRIX THEORY

The purpose of this section is to acquaint readers who are not familiar with matrix theory with the fundamental principles necessary for understanding the mathematical analysis given in this report. For a more complete treatment of the principles of matrix theory, see, for example, Reference 4.

PRELIMINARY DEFINITIONS

A *matrix* is a rectangular array of numbers. These numbers are called *elements* of the matrix. A *row* of a matrix is comprised of all elements in the same horizontal alignment. A *column* of a matrix is comprised of all elements in the same vertical alignment. If a matrix has m rows and n columns, it is of *order* $m \times n$. A matrix of order $n \times 1$ is also called a *vector* with n components. If the number of rows and the number of columns in a matrix are equal, the matrix is said to be a *square matrix.*

The 3×3 matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

will be used to illustrate various fundamentals of matrix theory. All such fundamentals apply to square matrices of any order. Some—but not all—of these fundamentals apply also to matrices which are not square. Such fundamentals are not identified, since all matrices (which are not vectors) used in the solution of the vibration problems discussed in this report are square matrices.

It is conventional to number the rows of a matrix from top to bottom and to number the columns from left to right. Note that the subscripts of an element indicate the row and column, respectively, in which it lies; for example, a_{13} lies in the first row and third column.

ADDITION AND SUBTRACTION

The sum of two matrices is formed by adding the elements occupying the same relative positions. Thus

$$A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \\ a_{31} + b_{31} & a_{32} + b_{32} & a_{33} + b_{33} \end{pmatrix}$$

Subtraction of matrices is similar to addition except that the "–" sign replaces the "+" sign throughout.

MULTIPLICATION

The element in the intersection of the ith row and jth column of the product matrix AB is formed by summing the products of the respective elements of the ith row of A and the jth column of B. Thus

$$AB =$$

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{\;1} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{pmatrix}$$

Matrix multiplication is distributive with respect to addition, i.e.,

$$(A + B) \cdot C = A \cdot C + B \cdot C$$

The multiplication of a matrix times a vector is of special interest. Let

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Then by applying the rule for multiplication of matrices,

$$Ax = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{pmatrix}$$

Thus by regarding the components of the vector x as unknowns, the matrix notation Ax can be used to represent the left side of a system of simultaneous linear equations.

UNIT MATRIX

A *unit matrix* is a square matrix in which all elements on the diagonal which extends from the upper left corner to the lower right corner are 1's and all elements not on this diagonal are 0's. Unit matrices are so called because of the property that

$$IW = W$$

and

$$WI = W$$

where W is an arbitrary matrix and I is a unit matrix.

INVERSE OF A MATRIX

The *inverse* of a matrix is a matrix which when multiplied by the original matrix results in the unit matrix. Thus, if M is a matrix and M^{-1} denotes the inverse matrix,

$$MM^{-1} = I$$

and

$$M^{-1}M = I$$

EIGENVALUES AND EIGENVECTORS

Consider a system of simultaneous linear equations such as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= tx_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= tx_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= tx_3 \end{aligned}$$

In this system, the a_{ij}'s are known constants and the x_i's are the unknowns. Also, t is unknown.

The system can be rewritten as

$$\begin{aligned} (a_{11}-t)\,x_1 + a_{12}\,x_2 \quad + a_{13}\,x_3 &= 0 \\ a_{21}\,x_1 \quad + (a_{22}-t)\,x_2 + a_{23}\,x_3 &= 0 \\ a_{31}\,x_1 \quad + a_{32}\,x_2 \quad + (a_{33}-t)\,x_3 &= 0 \end{aligned}$$

It is obvious that for any value of t,

$$x_1 = 0 \qquad x_2 = 0 \qquad x_3 = 0$$

is a (trivial) solution. If the system is to have a solution other than the trivial solution, it is necessary that the determinant

$$\begin{vmatrix} a_{11} - t & a_{12} & a_{13} \\ a_{21} & a_{22} - t & a_{23} \\ a_{31} & a_{32} & a_{33} - t \end{vmatrix} = 0$$

for otherwise Cramer's Rule implies that the solution must be the trivial solution. Expanding the determinant results in a polynomial equation in t; the roots of this equation are called *eigenvalues*. Each eigenvalue can be substituted as a value for t in the original system of equations; these equations can now be solved for x_1, x_2, x_3. Such a solution is called an *eigenvector* corresponding to the eigenvalue which was substituted for t. If X_1, X_2, X_3 is a solution of the system of equations corresponding to an eigenvalue, then for any constant

k, it is obvious that kX_1, kX_2, kX_3 is also a solution. Thus an eigenvector is not unique but may be scaled by any convenient scale factor, preserving the ratios between its components.

By letting x denote the vector of the unknowns x_1, x_2, x_3, the original system of simultaneous equations can be written using matrix notation:

$$Ax = tx$$

This standard form is always implicitly contained in references to the eigenvalues and eigenvectors of a matrix A.

APPENDIX B

MATHEMATICAL METHODS OF THE SUBROUTINES

The eigenvalues of $M^{-1}K$ (which is not a symmetric matrix) are computed by a package of subroutines which first reduce the matrix to a "nearly triangular" matrix N, through a sequence of elementary, similarity transformations E_k. A nearly triangular matrix here is one in which all elements which are located two or more positions to the left of (or down from) an element on the principal diagonal are zero. The transformations which produce these zeroes are effected by adding (subtracting) a multiple of one row to another row and subtracting (adding) a multiple of one column from another column. Thus $N = (\prod_k E_k) M^{-1} K (\prod_k E_k^{-1})$, and elements $n_{ij} = 0$ for $i - j > 1$. The motivation for these transformations is that a matrix obtained by similarity transformations has the same eigenvalues as the original matrix, and the determinant of a nearly triangular matrix can be computed with relative ease.

The determinant $|N - \omega^2 I|$ set equal to zero yields an algebraic equation in ω^2. This algebraic equation is solved by an iterative method of Muller, which is analogous to a well-known iterative technique, the chord method (or *regula falsi*). In the chord method, a straight line is passed through two points on the graph of the equation whose roots are desired; the root of the equation of this line is then taken to be the next iterant. In Muller's method, a parabola is passed through three points on the graph of the equation whose roots are desired and the nearer root of the quadratic equation of this parabola is taken as the next iterant. (For a complete description of Muller's method, see Reference 6.)

The evaluations of the determinant which are required when iterating towards a root are accomplished by adding (or subtracting) a multiple of one row to another to triangularize the determinant; then the product of all elements on the principal diagonal is computed.

To compute an eigenvector associated with an eigenvalue, the latter is subtracted from each element on the principal diagonal of the matrix, $M^{-1}K$. The matrix is then triangularized. The validity of triangularization is immediately seen by regarding the matrix as an abbreviated representation of (the left members of) a system of simultaneous homogeneous equations. Thus each equation has one fewer unknown than the preceding equation. To reduce the probability of inaccuracies due to accumulated round-off errors, the right members of the equations are set equal to the last element on the diagonal of the triangular matrix before the unknowns are found by back substitution; the justification for this is contained in Reference 7.

The following SHARE subroutines are used to compute the eigenvalues and eigenvectors: RWEIGN, RWNTRI, RWGRT, RWDETN, and RWVCTR.

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1	The Waugh Equipmt Co, New York, Attn: Dr. N.H. Roy
1	Westinghouse Elec Corp, Electronics Div, Radar Equipmt Engin, Baltimore, Attn: Mr. H.C. Simons
1	York Corp, Washington, D.C.
1	Technical Operations, Inc, Burlington, Mass. Attn: Mr. F. Everett Reed
1	Gibbs & Cox, Inc, New York

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MATHEMATICAL ANALYSIS AND DIGITAL COMPUTER SOLUTION OF NATURAL FREQUENCIES AND NORMAL MODES OF VIBRATION FOR A COMPOUND ISOLATION MOUNTING SYSTEM, by Leon Katz. Jan 1961. ii, 17p. illus., refs.
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