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The Johns Hopkins University

FOE, a Model Representing Company Actions

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by

Dorothy Kneeland Clark
Lewis E. Keefer
William W. Walton, Jr.

December 1960

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SUMMARY

PROBLEM

To develop a mathematical model representing close combat with conventional weapons at company level for use in hand-played war games depicting combat at higher echelons.

FACTS

An improved representation of company engagements was essential in developing CRO's hand-played division game and should be a useful contribution to war gaming in general.

DISCUSSION

The most significant improvement introduced in the FOE model described in this paper is the velocity factor, which represents the approach of the attacking force at a rate varying with intensity of battle. During the engagement the number of actual combatants in the forces involved build up from zero in proportion to the relative velocity with which the opposing forces are moving. Explicit treatment of each weapon type, together with the velocity factor, permits the introduction of fire from each weapon type only as it comes into range and also limits interchange of fire and resultant casualties to the areas within range at each time increment.

The equations developed for FOE cannot be solved analytically but are solved by stepwise numerical iterations. For small increments of time the variables are assumed to be constant and the values for the $t - 1$ interval are applied to the t^{th} interval to find a new set of values to be used in the next. Because of the uncertainty of input data and the level of aggregation employed it is felt that more refined techniques of numerical analysis are not warranted, and first-order differences only are considered.

FOE, incorporated with the compatible CORG tank-antitank model, is being programed for the 1103A digital computer.

FOE, A MODEL REPRESENTING COMPANY ACTIONS

INTRODUCTION

FOE is a combat model designed for use in war games. This paper describes the form in which it has thus far been developed - the depiction of close combat with conventional weapons at company level, to simplify the hand play of division games. FOE is best suited for use on a digital computer; hand play involving more than a very few weapon types is tedious.* Although the model was designed primarily for use in TACSPIEL, ORO's division-level tactical game, its usefulness is by no means limited to this setting. The general concept is, moreover, so flexible that the model might easily be adapted to represent conflict at higher levels.

The ORO war-game program includes games at several levels: division components, division, theater, and strategic, the expectation being that the less aggregated games at lower levels will provide inputs for the more highly aggregated. The Tactics Division has for some years been concerned with the first two categories. Thus CARMONETTE, which has been designed to depict company engagements by a very detailed computer simulation of interactions among company components, will provide input value for FOE, which is a much more highly aggregated mathematical representation of combat at company level. Results from plays of FOE will in turn be used as inputs for hand-played division-level games developed in TACSPIEL. The feed-in of the inputs can be handled either by drawing from a library of outcomes of company engagements compiled from computer runs of the FOE model with varied input values or by computer runs made in the course of play of the division game.

FOE obviously and intentionally aggregates combat far beyond the level in CARMONETTE; there is, for instance, no play of specific terrain features and no identification of infantry squads and platoons. It is, however, designed to permit inclusion of many of the known elements, such as target density, lethal area, fire rate, velocity of moving forces, and others, all of which affect the outcome of an engagement.

FOE avoids some of the serious defects that have plagued one or another of the numerous combat models devised in the past within and outside ORO to depict close combat. In FOE, casualty rates, instead of being drawn from historical data, are generated by the interactions within the model; the relation between weapons firing and available targets is defined so as to prevent overkill; each weapon type is treated separately so that individual effectiveness can be evaluated and ammunition expenditure recorded; movement of forces is

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*About 20 minutes of calculation on the average for one time increment of combat are required when 6 weapon types per side are included.

an integral part of the model. Differential equations to describe combat, if they are amenable to analytical solution, inevitably oversimply the factors involved and their interactions. FOE instead employs a much more flexible and inclusive method, a stepwise numerical method whereby for small increments of time the variables are assumed to be constant and the values for the $t - 1$ interval are applied to the t^{th} interval to find a new set of values to be used for the next.

Perhaps the most significant innovation in FOE is the velocity factor that represents the approach of the attacking force at a rate varying with intensity of battle. The velocity expression (described in detail in the section "Velocity Term") introduces the suppressive effect of enemy fire, often disregarded in combat models but generally acknowledged to be a major effect if not the major effect of fire on the battlefield. The term also reflects the concept that troops will not advance unless they are themselves firing and being supported by fire from their own support weapons.

The velocity factor permits representation of a build-up of engaged forces as the battle progresses. Previous models, in marked contrast to reality, have begun with total forces initially engaged and have applied attrition to these as some function of time. FOE, on the contrary, approximates the actual course of battle by starting with zero combatants and making the size of the forces engaged build up in proportion to the velocity of the moving force. The explicit treatment of each weapon type, together with the velocity factor, permits the introduction of fire from each weapon type only as it comes into range and also limits interchange of fire and resultant casualties to the areas that are within range at each time increment.*

Approximations to the actualities of combat, more acceptable for small-unit actions than for engagements involving larger groups, permit two assumptions essential to the model:

a. The relative homogeneity of a force of company size justifies the assumption that weapons and the men who serve them are uniformly distributed over an area. This assumption can be modified within the model by assigning a particular density and appropriate area within the company area, for example, to weapons of a given type or to the attacking infantry in the forward section. The relative rapidity with which men within a company can regroup to compensate for losses and disorganization permits the corollary assumption that as casualties occur immediate redistribution takes place so that the new density in the next increment is also uniform.

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*The general concept of movement and build-up used in FOE was evolved by Paul Dunn as an outgrowth of his work on Project FAME while he was with ORO.

b. Strong historical evidence that much of the fire from artillery, mortars, machineguns, and rifles is directed at suspected areas rather than specific targets supports the assumption adopted in FOE that all such fire is area fire. It should be noted, however, that the tank-antitank combat model developed by CORG* promises to mesh very easily with FOE to offer a means for representing direct fire against all types of point targets.

Two other assumptions are inherent in the model in its present form. They were, however, adopted for the sake of simplicity and to facilitate testing by hand play and can be modified in computer runs if it seems desirable to do so.

c. The attacking and defending forces, though moving, are each treated as a body covering a constant magnitude of area; the relative positions of their components also do not change during an engagement.

d. Densities and casualties are stated throughout in terms of men. This is done for convenience since lethal areas and kill probabilities are customarily calculated for personnel targets. Weapons and men are equated by introducing a modifying factor for crew-served weapons to express the ratio of weapon to crew so that the density of weapons firing is converted to the density of men who are targets in a given increment.

These assumptions and the input values used are discussed in detail in later sections.

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*The CORG paper describing this tank-antitank model is to be published in final form later in 1960.

DESCRIPTION OF THE MODEL

Notation

The conventions used throughout the description of the model are:

i subscript denotes i^{th}-type weapon in a set of I weapons in force n

j subscript denotes j^{th}-type weapon in a set of J weapons in force m

ρ_{oi}, ρ_{oj}	=	initial density of i^{th}- and j^{th}-type weapons crews
$\rho_i(t), \rho_j(t)$	=	density of i^{th}- and j^{th}-type weapons crews at any time t
A_n, A_m	=	areas of the n and m forces
f_i, f_j	=	net fire rates of the i^{th}- and j^{th}-type weapons against all targets, rounds per weapon per unit time
f_{ij}	=	fire rate of the i^{th}-type weapon against the j^{th}-type weapon only
f_{ji}	=	fire rate of the j^{th}-type weapon against the i^{th}-type weapon only
L_i, L_j	=	lethal area of the i^{th}- and j^{th}-type weapons against personnel
n_{oi}, m_{oj}	=	initial number of i- and j-type weapons engaged at t = 0
$n_i(t), m_j(t)$	=	total number of i- and j-type weapons engaged at any time t
$n_i'(t), m_j'(t)$	=	total number of casualties suffered by i- and j- type weapons crews at any time t
Δt	=	one increment of battle time
t_i, t_j	=	time at which the i^{th}- and j^{th}- type weapons begin firing
t_k	=	any time point in the calculations
v_0	=	initial velocity of the attacking force relative to the defending force
$v(t)$	=	velocity at any time t of the attacking force relative to the defending force
w_n, w_m	=	widths of the n and m fronts

Build-up and Casualty Terms

Consider the two forces shown in Fig. 1 with the m force defending and the n force attacking. n has I different types of weapons uniformly distributed in area A_n and similarly m has J types of weapons in area A_m.

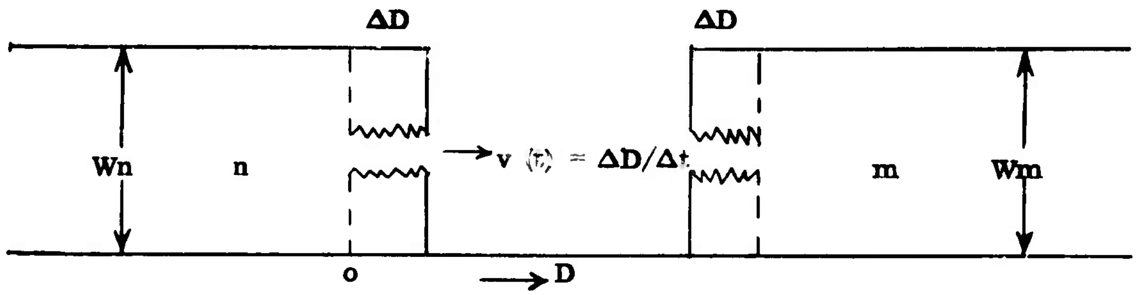

Fig. 1 - Basic FOE Concept

Initially the forces are separated by a distance D that for convenience is assumed equal to or greater than the range of the longest-range weapon. The attacking force advances a distance ΔD corresponding to one time increment so that at least one weapon type of n or m is within range and able to produce casualties on the opposing force. All these longest-range weapons within the first strip are able to reach targets located in the incremental strip ΔD of the enemy area. Initially a small proportion of the weapons are able to fire; however, as the attacker approaches the defender, more and more weapons come into play. Similarly, as a result of this movement more and more defenders' weapons become effective and there is not only a build-up of effective weapons on both sides but a corresponding build-up of available targets.

Each weapon type is assumed to fire at the range defined by Army manuals and tactical experience as the maximum effective range. It is further assumed that dispersion is such that each small increment is uniformly covered in depth by the rounds fired against it.

This assumption of uniform dispersion is recognized as the weakest link in the development of analytical models; however, it is necessary if the equations are to be kept manageable. The division of the company area into several smaller areas helps to reduce the error introduced by this assumption; nevertheless, uniformity must be assumed within the smaller areas so that the error, although reduced, is still present.

The general equations that describe the model contain two terms, one describing the build-up of weapons able to fire, which in turn become targets, and the other specifying the attrition. Since the solution is to be obtained by numerical iteration, the model was developed so as best to lend itself to this type of solution.

The change as a result of build-up of n_i weapons that are within range and thus able to fire on the m force can be found from the density of the i^{th}-type weapon and the incremental area. Δn_i represents the change in number of i^{th}-type weapons in an incremental strip[1] Δt from t_k to t_{k+1}.

$$\therefore \Delta n_i(t_{k+1}) \text{ [build-up]} = n(t_{k+1}) - n(t_k) = \rho(t_k)\Delta D(t_k) = \rho(t_k)v(t_k)\Delta t$$

For several increments the build-up of weapons can be expressed as a series beginning at the time t_i when the i^{th} weapon can open fire.

$$n_i(t_{k+1}) \text{ [build-up]} = \rho(t_k)w_n \sum_{t_p=t_i}^{t_k} v(t_p)\Delta t, \qquad \text{if } t \geq t_i$$

$$n_i(t_{k+1}) \text{ [build-up]} = 0, \qquad \text{if } t < t_i$$

where t_i is time at which i^{th}-type weapon can open fire.

For ease of development assume that the first weapon type of the m force to fire has a shorter range than that of the n force. At some later time there is then a build-up of m_j weapons able to fire on the attacking force, thus

$$m_j(t_{k+1}) \text{ [build-up]} = \rho_j(t_k)\, w_m \sum_{t_p=t_j}^{t_k} v(t_p)\Delta t, \qquad \text{if } t \geq t_j$$

$$m_j(t_{k+1}) \text{ [build-up]} = 0, \qquad \text{if } t < t_j$$

where t_j is the time at which j^{th}-type weapon can open fire.

Assuming that the effects of each of the weapon types considered can be measured in terms of lethal area, the attrition produced by an individual weapon is the product of its lethal area and the density of available targets. The expected casualties suffered during an increment Δt from t_k to t_{k+1} by m's j^{th}-type weapons as a result of firing by n's i^{th}-type weapons are

$$\Delta m'_j(t_{k+1}) \text{ [casualties]} = m'_j(t_{k+1}) - m'_j(t_k) = [\, n_i(t_{k+1}) f_{ij} L_i]\, \rho_j(t_k)\Delta t$$

where $[n_i\, t_{k+1}\, f_{ij} L_i]$ is total lethal area per unit time delivered by the i^{th}-type weapon during the interval t_k to t_{k+1} and $\rho_j\, t_k$ is instantaneous density of the j^{th}-type weapon. The densities are computed at the end of each increment by subtracting the casualty density during the increment from the density existing at the beginning of the increment. Inherent in the model is the assumption, previously mentioned, that the forces redistribute themselves after each increment. The density can be found from the following expression:

$$\rho_j(t_{k+1}) = \rho_j(t_k) - \frac{\Delta m_j'(t_{k+1})}{A_m}$$

Similarly the attrition of the n force by the j^{th}-type weapon is

$$\Delta n_i'(t_{k+1}) = \left[m_j(t_{k+1})\, f_{ji}\, L_j \right] \rho_i(t_k)\, \Delta t$$

The attrition during an interval from $(t - \Delta t)$ to t from several weapon types that have come into range can be expressed as

$$\Delta m_j'(t_{k+1}) \text{ [casualties] } = \left[\sum_{i \epsilon I} n_i(t_{k+1})\, f_{ij}\, L_i \right] \rho_j(t_k) \Delta t$$

where $i \epsilon I$ represents i summed over a set of I weapons that are within range at $t = t_k$.

Similarly

$$\Delta n_i'(t_{k+1}) \text{ [casualties] } = \left[\sum_{j \epsilon J} m_j(t_{k+1})\, f_{ji}\, L_i \right] \rho_i(t_k) \Delta t$$

where $j \epsilon J$ represents j summed over a set of J weapons that are within range at $t = t_k$.

Lethal area is a familiar concept for describing the effects of bursting munitions. Combined with the concept of uniform density of target troops it is the means customarily used for estimating expected casualties from artillery and mortar fire. Although the geometric definition of the nature of a lethal area for small-arms bullets differs from that for a bursting munition, the general concept seems equally acceptable, given the premise adopted for FOE that small-arms fire as well as artillery and mortar fire is unaimed area fire rather than aimed fire, for which, of course, kill probability is the appropriate effectiveness measure.

Determination of lethal areas for small-arms bullets is a problem that has not been extensively studied. An extremely rough approximation can be made by applying the ratio between numbers of mortar rounds and numbers of rifle bullets per casualty from historical data to reduce an average lethal area for the 81-mm mortar shell to an average lethal area for the rifle bullet. Obviously the lethal area for a bullet, like that for busting munitions, is properly defined as a variable. Among the factors upon which its values are dependent are range and the vulnerable area presented by target troops (defined by their posture and the terrain) within the space threatened by the weapon. A theoretical approach made within ORO* suggests the following expression for lethal area for a rifle bullet in unaimed fire in a flat treeless area,

$$L = \frac{a A_v}{h + a \sin i} f$$

where a = the depth of the area that the target is considered to occupy.

A_v = $P_{zi}S$, P_{zi} being the probability that a single random hit by one bullet will incapacitate a target, given the angle i formed by the ground on which the target is located and the nearly straight trajectory between the firer and the target area, and S, the presented area of target.

h = height of a target

$$f = \frac{1}{\sqrt{2\pi}} \int_{-b}^{b} e^{-x^2/2} dx$$

where b is $(h + a \sin i)/2R\sigma$, and R is the range from weapon to center of target, and σ is the vertical standard deviation of the bullet with respect to the center of the target, in radians.

A simple form that seems applicable to unaimed fire is

$$L \rightarrow \frac{a A_v}{\sqrt{2\pi} R\sigma}$$

when the ratio of the angular height of the target to the vertical standard deviation approaches zero, hence b approaches zero, and $f \rightarrow 2b/\sqrt{2\pi}$.

Perhaps fortuitously, a ratio from historical data and what seemed reasonable values put in the previous expression both resulted in a value of 11 ft² for a lethal area of defender's rifle bullet at a 170-yd range. A lower value would of course be used to describe the effectiveness of attacker's bullets as a function of well-protected defenders.

Examination was also made of the possible use of the more customary kill probability to measure small-arms effectiveness, modified to fit the FOE conditions of small-arms fire against targets uniformly distributed, assuming very low probability values appropriate to unaimed fire. Because the lethal-area concept appears to be a more accurate description of area fire, these

*Unpublished paper by Theodore E. Sterne.

elements were not included in the model, but because they may be of interest to combat-model designers they are described in detail in the appendix.

Velocity Term

As has been indicated, build-up and casualties in FOE are functions of the velocity with which the attacking force moves relative to the velocity of the defender, and velocity during the engagement is in turn dependent on the intensity of battle. Velocity is here defined as the average velocity of the center of mass of the entire attacking force. The expression adopted in FOE assumes that the attackers' velocity during battle is determined by the amount of fire they are able to deliver and the amount they are receiving. The time-varying velocity during the engagement can be expressed as

$$v(t_{k+1}) = v_0 \frac{\sum_{i \epsilon I} n_i(t_{k+1})f_i}{\sum_{i \epsilon I} n_i(t_{k+1})f_i + \sum_{j \epsilon J} n_j(t_{k+1})f_j}$$

where v_0 represents some initial unopposed rate of advance and I and J are the sets of all i- and j-type weapons within range.

The equations of the model are solved by introducing the appropriate term as each weapon comes into range. A small increment of time Δt is selected, and calculations are made for each increment. Until the first weapon type comes into range, casualties are of course zero and the attacking force proceeds at its initial velocity. The depth of the first build-up increment is measured by the distance traveled in that time at initial velocity; its density is the initial density of the force. Casualties from enemy fire on the first build-up increment are computed, and from these a new density is secured for the following period. A new velocity for the second interval is determined from the number of weapons firing in the first increment and their corresponding fire rates; this velocity in turn determines the depth of the build-up area for the second interval. Trial hand calculations run through the model indicate that the immediate reaction to the introduction of fire from an additional j-type weapon is marked decrease in velocity followed by a leveling off in subsequent increments around the new and slower rate—a result that seems to accord well with reality.

Breakpoints

The foregoing step calculations are repeated until the game ends, either when the front lines meet or when a breakpoint is reached. The following breakpoints can be used in the model, the first that occurs ending the game: a reduction in velocity to a rate that indicates that the attack has lost its impetus; some casualty percentage that indicates inability of the depleted unit to continue its mission, force ratios indicating defeat or a stalemate and discontinuance of the attack.

PROPOSED MODIFICATIONS OF THE MODEL

Suggestions have been made with regard to increasing the realism of the model. The modifications listed below would convert, to variables, elements presently treated as constants:

a. Vary the areas occupied by the two forces by allowing either the length or depth to shrink as the engagement progresses.

b. Vary the fire rates in proportion to density of targets.

c. Allow lethal areas to vary with range and with velocity.

d. Represent suppression of defenders' fire by incoming attackers' rounds by the expression

$$\frac{K}{K + \sum_{i=1}^{i} n_i f_i}$$

where K = the degree of protection afforded by defenders' positions.

Because the model consists simply of the iteration of steps containing simple arithmetical operations and does not suffer from the limitations imposed by having to hold an analytical expression within the bounds of ready solution, there is a strong temptation to seek refinement by elaborating its components, as for example by providing for variation in input values during a single run. The temptation is the stronger in that any one of such elaborations can easily be added to a computer program. Such elaborations in sum may, however, quickly overflow even a large computer without adding much to the realism of the representation in view of the wide margins of doubt surrounding most input data. Appendix A illustrates clearly the complexities that develop from attempts to develop precise treatments of even simple concepts. It is our belief that the temptation to elaborate should be firmly resisted until it is apparent that the additional element will exercise a strong influence on the outcome and that it has a sound rational basis and can be given reasonably reliable values.

RATIONALE OF ASSUMPTIONS

There is plenty of evidence from historical data, the results of tests, and the judgment of experienced combat officers to support the premise that by far the greater part of the rounds from artillery and mortars are expended in area fire intended to harass, interdict, neutralize, and suppress enemy activity rather than to destroy specific targets by precision adjustment. In FOE, therefore, effects from bursting munitions are measured simply as lethal area times enemy density and are considered to occur uniformly over the area occupied by a given weapon type, the lethal area chosen being that appropriate to the average target vulnerability. The fact is no less well authenticated that small-arms fire in combat is also for the most part area fire intended to keep heads down, and unaimed except as fire may be directed at small areas regarded as suspicious but ill-defined as to size or content or at targets glimpsed but no longer visible at the moment of fire. ORO's recent best estimate of the usual percentage of total small-arms fire that can rightly be considered aimed is 20 percent. This assumption that all fire is area fire, of course, makes consideration of target acquisition unnecessary. The relatively small amount of fire from infantry direct-fire weapons on point targets will, as has been said, be introduced in FOE through incorporation of the CORG tank-antitank model.

INPUTS

The explicit input values required in the model are few and, aside from velocity and numbers of weapons and densities, are held fixed through the play. Each weapon type opens fire at its maximum effective range, fires at a fixed rate, and has a fixed lethal area for each target type—the selected values representing an average of the varying values that might actually occur in the course of a battle under the particular conditions premised.

Many factors that in other close-combat models are played explicitly or accounted for by adjustments in final results are introduced in FOE through their influence on the input values selected. Initial densities are, for example, determined by assumptions concerning the composition of the opposing forces, their missions, and the degree of nuclear threat. Initial velocity is affected by terrain and weather. The rate of fire (rounds per minute per weapon) selected for defenders' rifles may involve assumptions that only one-fourth of the riflemen within range actually fire and that defenders fire more rounds than attackers. Lethal areas are selected to fit assumptions concerning weapon range, attackers' and defenders' posture (percentage erect, crouching, prone, or in foxholes) and cover provided by terrain. This approach of course involves the assumption that these implicit factors do not vary during a specific engagement. Such an assumption is more easily justified for a company-sized engagement, which presumably lasts only a few hours and covers a relatively small area, than for combat at higher levels. It seems reasonable to suppose that within these limits terrain and weather will not vary greatly, ammunition supply will not usually become a serious problem, and in the majority of cases the engagement will run its course without the interposition of new orders from higher headquarters.

It seems convenient to assume that action described by the model starts at the opening range for tanks placing direct fire on infantry (about 2000 m) and, unless a breakpoint occurs, continues until the front line of attacking troops reaches the defenders' front line. To account for preparatory fire delivered before a planned attack, the resultant attrition can be reflected in the initial density assigned to defenders' forces. Subsequently, throughout the battle, fire from artillery and larger mortars enters the model as a fixed number of rounds per minute, some percentage of which is delivered against each weapon type in the enemy company area with a lethal area appropriate to the target type. It is assumed that these weapons are outside the area of company engagements and will not suffer attrition from company action and that a predetermined level of support fire will be maintained by regimental and division controls outside the scope of this model.

PROPOSED USE OF THE MODEL

It is proposed to build, from computer runs with various inputs, a library of outcomes of company engagements that can be used to lessen the detailed play on TACSPIEL's division games. Runs will be necessary to reflect variation in at least the following factors: initial densities, initial force ratios, rates of fire, the degree of man-made protection allowed the defenders (which affects lethal areas), and terrain (which affects initial velocity as well as lethal areas). Insertion of the CORG tank-antitank model will introduce further variations. About 100 runs should suffice to show to which factors the model is sufficiently sensitive to warrant further runs with additional changes in these input values. Computer time for one run with the CORG model incorporated will probably be 10 to 15 min. and may be outweighed by the time necessary to introduce new input values for the next run. The system for handling these changes must therefore be very carefully developed.

The mere accumulation of a collection of computer runs representing varied inputs does not ensure to the players of a division game a complete store of outcomes of company engagements that can be applied directly without further evaluation. Obviously it is impossible to provide enough runs to cover all possible variations in input values. Even a small number of runs serve, however, to indicate the range of outcomes and provide a guide to players as to outcomes they can assume to be reasonable in a specific situation that may have been engendered in the division game.

Obviously, too, a division battle is something more than the sum of the individual-company engagements. The course of a given company battle depends to a varying degree on the presence and action of, at least, the companies adjacent to it. These interactions are not depicted by FOE in its present form but must be considered by the players of the division game in making use of FOE outcomes.

Furthermore, and perhaps even more significantly, the progress of a company engagement depends on the type, amount, and timeliness of support from weapons — ground and air — under the control of echelons higher than company. In this model such support is represented simply by a fixed rate of fire delivered on the whole area, whereas in actuality the outcome of a company engagement is often determined by the arrival or nonarrival of an artillery concentration or airstrike at the crucial time on a specific target; this is brought about by the interaction of factors mostly outside the scope of the company. Such episodes can, of course, be introduced into this model by using the Monte Carlo technique, but in a division game they should occur as a result of play and not on a preset probability basis.

It may be that an intermediate model aggregating combat at battalion or regimental level will prove necessary before a division game drawing on "canned" outcomes of its lower-echelon engagements can be played with an acceptable degree of assurance that important interactions have not been omitted. Whether this model or some other be used for that higher level of aggregation, the collection of outcomes from company engagements appears to be an essential first step toward supplying inputs for more aggregated play.

A library of FOE outcomes might also be used to determine ground-action results in games that are designed primarily to explore such areas as intelligence, communications, or logistics, and in which some representation of combat is desired at a minimum expenditure of time and effort.

Another way of using FOE in more highly aggregated games is to run the company engagements on the computer when they occur in the course of play, using the inputs dictated by the parent game as it progresses. Such a method is probably preferable to the employment of canned outcomes but is feasible only if the computer is located near the game rooms and is available whenever needed — ideal conditions that do not often occur.

Appendix A

TREATMENT OF P_k FOR SMALL ARMS IN FOE CONTEXT

DEFINITION OF P_k

If kill (hit x conditional kill) probability rather than lethal area is used in FOE to measure small-arms effectiveness, the basic expression for casualties becomes

$$\Delta m'_j(t_{k+1}) = \left[n_i(t_{k+1}) f_{ij} K_i \right] \left[\rho_j(t_k) w_m \sum_{t_p = t_i}^{t_k} v(t_p) \quad \Delta t \right] \Delta t$$

where K_i is kill probability of the i^{th} weapon and

$\rho_j(t_k) w_m \sum_{t_p = t_i}^{t_k} v(t_p) \Delta t$ is the number of men who are within range of the i^{th} weapon at a given time t_k.

The use of kill probability carries with it the assumption that actual targets are acquired and aimed at (although these may be for the most part small suspected areas rather than men clearly seen) and that each incremental build-up provides additional targets as well as weapons to fire on them. The kill probability K_i in the previous expression is well defined for one weapon firing against one target. The employment of such a value in this expression, however, results in overkill, as the following simple but not impossible case illustrates. Assume that two men each fire one bullet with a kill probability of 1.0 at two men, then

$$m'_j(t_{k+1}) = \left[n_i(t_{k+1}) f_{ij} K_i \right] \left[\rho_j(t_k) w_m \sum_{t_p = t_i}^{t_k} v(t_p) \Delta t \right] \Delta t = \left[2 \cdot 1 \cdot 1 \right] 2 = 4$$

a result that is absurd. A possible solution is to insert a limit so that

$\Delta m'_j(t_{k+1})$ cannot exceed $\rho_j(t_k) w_m \sum_{t_p = t_i}^{t_k} v(t_p) \Delta t$, but this is a palliative and does not get to the root of the difficulty.

Ideally the problem should be solved through combinatorial analysis. In the case of two weapons firing against one target the actual kill probability is twice the product of the probability that one man misses and the other hits plus the probability that both hit. If P_1 is the probability of a kill by weapon 1 and P_2 is the probability of a kill by weapon 2, then

$$P\left[\text{kill by } 1+2\right] = P_1 + P_2 - P_1 P_2.$$

A rigorous continuation of a combinatorial analysis would lead to a general expression for the total kill probability of n weapons against one target. However, if several targets are introduced, the combinatorial analysis not only becomes complex but questions arise for which there are no satisfactory answers; for example, how many weapons may be assumed to engage one target when many targets are available. The complexity of analysis for a company-sized engagement obviously precludes this approach.

LIMITATION OF TARGETS

Limitation by Increments

A substitute solution was developed consisting of two compatible and essential parts, each of which in a different way limits the targets on which a weapon can fire. The first involves recognition that all targets within a series of incremental build-ups are not within range of all engaged weapons; the second recognizes that all weapons within range will not with equal probability fire at any target within range.

Figure A1 illustrates the first type of limitation.

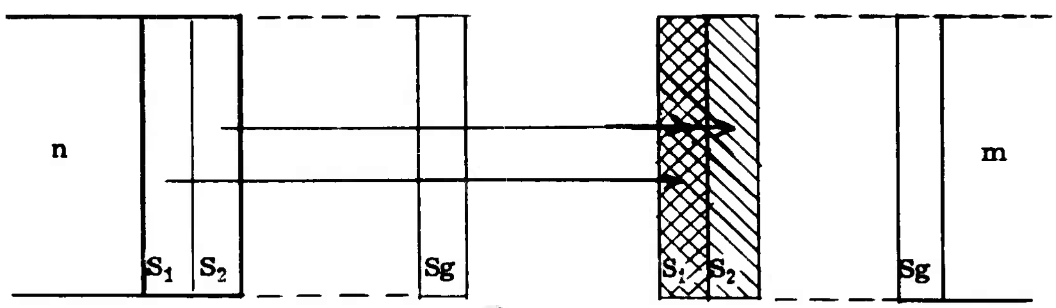

Fig. A1—m_j Target Areas within Range of n_i Weapons in Succeeding Increments

During the first increment (S_1 in depth) when weapons of type n_i come into range, they can reach all the build-up in S_1 of targets consisting of weapons of type m_j. During the second interval, however, only half the n_i weapons can fire on all targets (in the build-ups S_1 and S_2 where $S_1 = S_2$) and the other half can reach only half the targets (those in S_1). Thus the total engaged of n_1 weapons firing and m_j targets fired on is

$$\tfrac{1}{2} n_i \cdot 1\, m_j + \tfrac{1}{2}\, n_i \cdot \tfrac{1}{2} m_j = \tfrac{3}{4} n_i m_j$$

Similarly for the third interval one-third of the weapons can fire on all the targets in the build-up area, one-third on two thirds of the targets in the build-up area, and one-third on one-third of the targets, thus

$$\tfrac{1}{3} n_i \cdot 1\, m_j + \tfrac{1}{3} n_i \cdot \tfrac{2}{3}\, m_j + \tfrac{1}{3} n_i \cdot \tfrac{1}{3} m_i = \tfrac{4}{6} n_i m_j$$

and for the fourth interval:

$$\tfrac{1}{4} n_i \cdot 1\, m_j + \tfrac{1}{4} n_i \cdot \tfrac{3}{4} m_j + \tfrac{1}{4}\, n_i \cdot \tfrac{2}{4} m_j + \tfrac{1}{4} n_i \cdot \tfrac{1}{4} m_j = \tfrac{5}{8} n_i m_j$$

A continuation of this process leads to a geometric series for the coefficient of $n_i\, m_j$, the total at each increment being

$$\frac{g+1}{2g}$$

where g is number of increments.*

Introduction of this term into the model necessitates separate bookkeeping for each type of small-arms weapon per increment, since differences in range mean differing numbers of increments to compute.

This analysis as previously mentioned assumes that the distance advanced in each time increment is equal, whereas in FOE the attackers' velocity varies with intensity of battle. An exact solution taking account of the distance advanced in each increment is possible but the running independent tab for each weapon in each increment becomes much more complex. The magnitude of the error introduced by the approximation described above has not been sufficiently investigated to determine which method should be used.

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*That this holds only until the last weapons have all targets within range does not matter, since the game stops when the front lines meet, and the depths of opposing companies will always exceed the ranges of small arms.

The casualty expressions using the approximation are, in general terms,

$$\Delta m'_j(t_{k+1}) = \left[\sum_{i\epsilon I} n_i(t_{k+1}) f_{ij} K_i\right]\left[\rho_j(t_k) w_m \sum_{t_p = t_j}^{t_k} v(t_p)\,\Delta t\right] \frac{g - g_j + 2}{2(g - g_j + 1)}\,\Delta t$$

and

$$\Delta n'_i(t_{k+1}) = \left[\sum_{j\epsilon J} m_j(t_{k+1}) f_{ji} K_j\right]\left[\rho_i(t_k) w_n \sum_{t_p = t_i}^{t_k} v(t_p)\,\Delta t\right] \frac{g - g_i + 2}{2(g - g_i + 1)}\,\Delta t$$

where g is total number of increments g_i, g_j are increment numbers at time that i^{th} or j^{th} weapon comes into range, and I, J are sets of all i- and j-type weapons within range.

The approach in which each increment is considered independently leads to the following general equations

$$\Delta m'_j(t_{k+1}) = \left[w_n \sum_{i\epsilon I} \rho_i(t_k) f_{ij} K_i\right] w_m \rho_j(t_k) \sum_{h = g_j}^{g} \left[s_h \sum_{p = h}^{g} s_p\right]\Delta t$$

$$\Delta n'_i(t_{k+1}) = \left[w_m \sum_{j\epsilon J} \rho_j(t_k) f_{ji} K_j\right] w_n \rho_i(t_k) \sum_{h = g_i}^{g} \left[s_h \sum_{p = h}^{g} s_p\right]\Delta t$$

where s is the distance advanced in an increment.

Limitation by Arc of Fire

The second factor limiting the targets on which small-arms fire is brought can be handled as an arc of fire that confines fire from each weapon to the area in front of it. The assumption in FOE of a fixed range per weapon type suggests the concept illustrated in Fig. A2 of a very small arc of fire against the first incremental build-up that gradually widens as more and more targets come into range. Thus a weapon in the first build-up increment can fire on the entire shaded area in Fig. A2a, but a weapon in the second increment can fire only on the darker area. Theoretically the angle of fire might vary from 0 deg when the weapon first comes into range to 180 deg when the lines of the two forces meet. In practice, however, the angle probably reaches and remains at some rather small value (as shown in Fig. A2b), determined by convenience in handling the weapon, the individual's normal field of vision, and training doctrine. One might assume a somewhat wider angle of coverage for the stationary defender whose sole task is to seek targets among the advancing attackers than for the attacker who is also concerned with making his way forward over the approach terrain.

To compute the number of targets within an arc of fire it is necessary to compute the area of the opposing force within range of the weapon and multiply this by enemy density. The most precise way involves a different computation for each small-arms-weapon type for each increment.

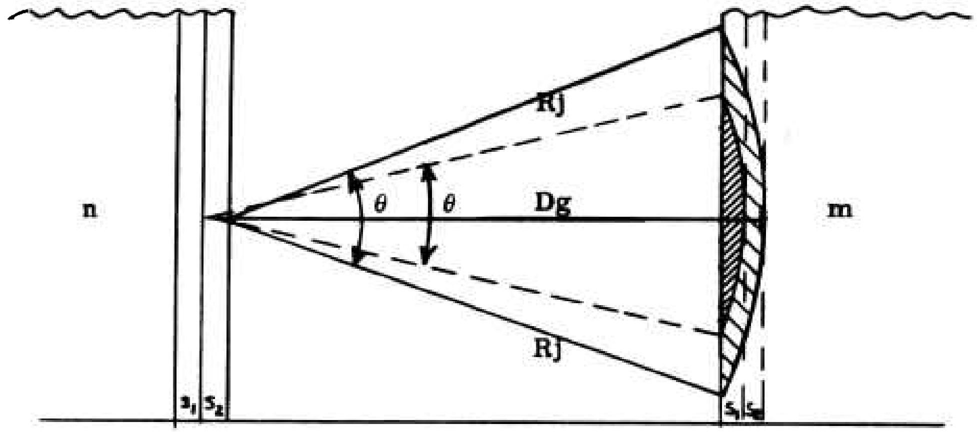
An expression for the angle of fire during an arbitrary increment g is a function of the distance separating the forces and the range of the specific weapon under consideration. It is easily seen from Fig. A2a that

$$\theta_g = 2 \cos^{-1} \frac{D_g(t_k)}{R_i} \quad \text{(for an n type-i weapon)}$$

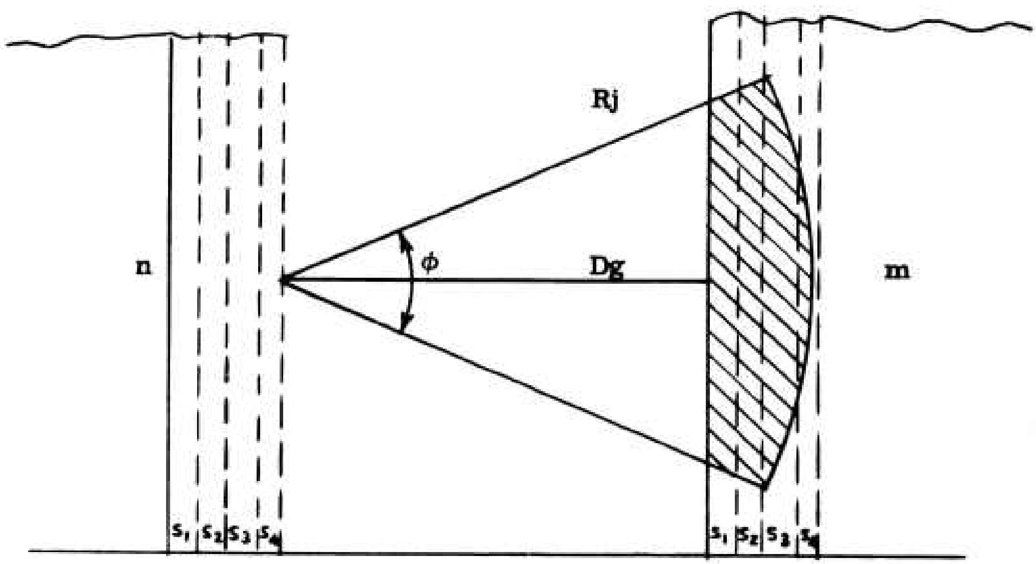
where $D_g(t_k)$ is the distance from the g^{th} increment to the forward edge of the opposing force at time t_k, and R_i is the range of the i^{th} weapon. The area of fire can then be shown to be

$$A_{gi}(t_k) = R_i^2 \cos^{-1} \frac{D_g(t_k)}{R_i} - D_g(t_k) \sqrt{R_i^2 - D_g^2} \quad (1)$$

where $\cos^{-1}\left[D_g(t_k)/R_i\right]$ is measured in radians.



a. Change in Arc of Fire by Increment when $\theta \leq \phi$



b. Change in Arc of Fire by Increment when $\theta > \phi$

Fig. A2—Effect of Succeeding Increments on Arc of Fire

As pointed out previously the angle of fire will reach a limiting value beyond which it will not vary with the distance separating the forces. If the limiting angle of θ is called φ_i for the i^{th} -type weapon (See Fig. A2b), then when $\theta \geq \varphi_i$,

$$A_{gi}(t_k) = \tfrac{1}{2}\varphi_i R_i - 2 \tan \tfrac{1}{2}\varphi_i \tag{2}$$

where φ is measured in radians.

Thus when the angle of fire is less than the limiting value φ_i, Eq. 1 is used for the area of fire. Correspondingly, Eq. 2 is used when θ reaches its limiting value φ_i.

Less sophisticated solutions that have the virtue of simplicity are perhaps acceptable approximations. An average arc of fire may be assumed in which the area covered increases with each increment but the angle does not; or the area covered might be approximated by a narrow rectangle that does not vary in width but increases in depth in each increment by the distance advanced.

Incorporation of this arc-of-fire concept into the model results in the following expressions in the case of fire on m_j targets:

The number of j targets for one i weapon in the g^{th} -incremental slice at time t_k to t_{k+1} is

$$m_{jg}(t_{k+1}) = \rho_j(t_k)A_{gi}(t_k)$$

where $A_{gi}(t_k)$ is given by either Eq. 1 or 2.

Casualties inflicted by one i weapon in one time increment on $m_{jg}(t_{k+1})$ targets are

$$f_{ij}K_i m_{jg}(t_{k+1})\Delta t = f_{ij}K_i\,\rho_j(t_k)A_{gi}(t_k)\Delta t$$

Casualties inflicted by all weapons of one type in slice g in one time increment are

$$n_{ig}(t_{k+1})f_{ij}K_i\rho_j(t_k)A_{gi}(t_k)\Delta t = w_n s_g\;\rho_i(t_k)f_{ij}K_i\;\rho_j(t_k)A_{gi}(t_k)\Delta t$$

Casualties inflicted by all types of i weapons in the g^{th} slice in one time increment are

$$\Delta m'_j(t_{k+1}) \text{ [casualties] } = \sum_i w_n s_g \rho_i(t_k) f_{ij} K_i \rho_j(t_k) A_{gi}(t_k) \Delta t =$$

$$w_n s_g \rho_j(t_k) \sum_g \rho_i(t_k) f_{ij} K_i A_{gi}(t_k) \Delta t$$

Casualties inflicted by all types of i weapons in all g slices in one time increment are

$$\Delta m'_j(t_{k+1}) \text{ [casualties] } = \sum_i w_n s_g \rho_j(t_k) \sum_i \rho_i(t_k) f_{ij} K_i A_{gi}(t_k) \Delta t =$$

$$w_n \rho_j(t_k) \sum_g s_g \sum_i \rho_i(t_k) f_{ij} K_i \sum_g A_{gi}(t_k) \Delta t$$