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Scientific Report 1

DETERMINATION OF THE PERCENT OF TIME THE GROUND IS VISIBLE FROM AN AIRCRAFT FLYING ABOVE CLOUDS

Prepared for:

GEOPHYSICS RESEARCH DIRECTORATE AIR FORCE RESEARCH DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND BEDFORD, MASSACHUSETTS

By: Roy H. Blackmer, Jr. John Harllee

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STANFORD RESEARCH INSTITUTE

MENLO PARK, CALIFORNIA

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SRI Project 3245

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ABSTRACT

This report summarizes work done on Objective 3 of Contract AF 19(604)-7312.

Equations were derived to express the percent of time the ground is visible from an aircraft flying above various conditions of cloudiness. To permit rapid solution of the equations over a wide range of cloud dimensions and aircraft height, nomographic charts were constructed.

The assumptions made to simplify the equations, and the intended applications of the equations, are discussed.

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I INTRODUCTION

Present synoptic and climatological cloud data are given in terms of total amount of sky covered with clouds. This information is not sufficient for all purposes. When planning aerial photographic missions, for example, the distribution of the clouds over the sky is as important as the total amount of clouds. A search of meteorological literature to find information on typical distributions of clouds and especially to find typical dimensions of cloud elements, shows that little work has been done in this area.

This report describes one phase of a program of detailed studies of clouds. The program is concerned with making detailed studies of the distribution and dimensions of clouds and determining frequency of visual contact with the ground through various cloud distributions, both synoptic and climatological.

As a first step toward methods of determining the percent of time the ground is visible from an aircraft flying above various cloud distributions, simple equations have been derived to express visual contact with the ground in terms of various cloud and non-cloud parameters. The equations illustrate the minimum cloud information necessary to the problem, and solution of the equations using various hypothetical cloud dimensions will permit an evaluation of the accuracy with which typical cloud dimensions must subsequently be determined.

To permit rapid solution of the equations over a wide range of cloud dimensions, nomographic charts have been constructed. This report presents the equations which were derived and the nomographic charts which were constructed for solution of the equations.

II EQUATIONS

A. Derivation

A basic equation was derived expressing the percent of time the ground is visible in terms of pertinent cloud and non-cloud parameters. The problem was treated in only two dimensions in order to obtain as simple an equation as possible. Also, it was assumed that the clouds are rectangular in cross section and that two adjacent clouds have identical vertical extent. The cloud parameters considered in formulating the equation were:

- h - height of cloud base (feet)
- T - vertical extent of clouds (feet)
- S - distance between clouds (nautical miles)
- W - width of clouds (nautical miles).

Non-cloud parameters necessary to the equation were:

- H - height of vehicle (feet)
- α - angle of view (degrees)
- F - field of view (degrees)
- β - 1/2 field of view $\left(\frac{F}{2}\right)$ (degrees)

Figure 1 illustrates these various parameters.

The equation is based on the fact that at a constant velocity the ratio of two time intervals is equal to the ratio of the distances traveled during the time intervals. Thus, the percent of time the ground is visible may be expressed in terms of the distance along the flight paths that ground is visible.

The basic equation derived with these assumptions and considerations is: *

$$P = 100 - 100 \left\{ \frac{(H - h) [\text{Cot}(\alpha + \beta) - \text{Cot}(\alpha - \beta)] + T\text{Cot}(\alpha - \beta) + W}{S + W} \right\} \quad (1)$$

where P = percent of time the ground is visible.

It should be stressed that this equation expresses the percent of time some ground may be seen, and does not distinguish between a completely cloud-free beam and the condition that some cloud will be in the field of view. The ground is considered visible when the beam is not completely cloud-filled.

This basic equation must be replaced by a second equation when certain relationships (see Appendix A) exist between S, T, α, and β. This second equation is:

$$P = 100 \left(\frac{H - h}{S + W}\right) \left[\frac{S}{T} - \text{Cot}(\alpha + \beta)\right] \quad . \qquad (2)$$

Technically, a third equation is necessary when the angle of view is within β of 90 degrees. However, as shown in Appendix A, the change in P with α at these large angles is so small that a third equation is not considered necessary.

B. Discussion

It is of interest to note that the height of the cloud base only enters the equation as a measure of the vertical distance from the aircraft to the top and bottom of the cloud, that is:

H - h = vertical distance from aircraft to bottom of cloud

H - h - T = vertical distance from aircraft to cloud top.

* See Appendix A for details of the formulation of the equation.

Thus, the height of the cloud base alone does not affect the percent of time the ground is visible.

Examination of Eq. (1) shows that the term $\mathrm{Cot}(\alpha - \beta)$ becomes infinity when $\alpha = \beta$, causing P to become infinity at this angle. Before the critical angle is reached, however, Eq. (2) must be used. This second equation goes to minus infinity at $\alpha = \beta$, but we are not interested in values less than zero.

The shape of the curves described by these two equations (and also a third equation for large values of α) is shown in Fig. 2. The values of the other parameters used in solving the equation for this illustration are indicated on the figure. Lines have been drawn across the figure to designate the ranges over which Eqs. (1) and (2) are valid in this example. The range depends on S, T, α, and β. That is, no ground is visible if $\mathrm{Cot}(\alpha + \beta) \geq S/T$. The transition from Eq. (1) to Eq. (2) occurs when $\mathrm{Cot}(\alpha - \beta) = S/T$. In this example $S = 5$ nautical miles = 30,400 ft and $T = 20{,}000$ ft. Thus, $S/T = 1.52$ which equals the cotangent of 33.3 degrees. With $\beta = 5°$ the transition from Eq. (1) to Eq. (2) will be where

$$\alpha - 5 = 33.3° \quad \text{or} \quad \alpha = 38.3°$$

and no ground will be visible at an angle less than

$$\alpha + 5 = 33.3° \quad \text{or} \quad \alpha = 28.3° \quad .$$

A simple method will be presented later whereby one can readily determine which equation to use, or whether no ground is visible.

C. Application

The assumptions and restrictions used in deriving these equations may appear to limit their usefulness for application to actual cloud distributions. It is ultimately planned, however, to use these equations in the preparation of frequency distributions of the percent of time the ground is visible. The climatological cloud data which must be used for the frequency distributions are those tabulated on USWB forms WBAN 10A and 10B. These tabulations do not contain measurements of S, T, or W, and the tabulated values of h are sparse when h is greater than about 10,000 ft.

It is necessary, then, to make basic measurements of cloud dimensions from pictorial cloud data. Examination of the photographic records will, insofar as possible, be made only for situations where WBAN-10 observations are available. Thus, the observer's estimate of N and cloud type will be known and the pictures will provide information on S, W, and possibly T.

In order to determine the sensitivity of P to the various cloud dimensions and thus establish the accuracy with which dimensions of various cloud types need be known, numerous solutions of the equations are desirable. For example, if everything else is constant, what effect do differences of 1000, 2000, or 5000 feet in cloud thickness have on the percent of time the ground will be seen? To permit rapid solution of the equations over a wide range of cloud parameters without resorting to expensive automatic computing devices, nomographic charts were constructed. In addition to charts for the solution of Eqs. (1) and (2), three other nomographic charts were also constructed which have considerable usefulness under certain conditions. These nomographic charts, together with instructions in their use, are discussed in Sec. III.

III NOMOGRAPHIC CHARTS *

Five nomographic charts were constructed.** In addition to charts for the two equations described previously, three auxiliary charts were constructed. One of these auxiliary charts will be discussed first, since it is used to determine whether Eq. (1) or Eq. (2) should be used for given values of S, T, and α. This chart is shown in Fig. 3. (In the construction of this chart, and the charts for Eqs. (1) and (2), a value of $\beta = 5°$ has been used.) This chart is a combination nomographic chart and graph. The graph has S/T as ordinate and α as abscissa. The graph is divided into three areas by the lines

$$S/T = \mathrm{Cot}(\alpha - 5)$$

and

$$S/T = \mathrm{Cot}(\alpha + 5) \quad .$$

The areas have then been labeled with the appropriate equation to be used with given values of S/T and α.

The nomographic chart portion of Fig. 3 is for determining the ratio S/T from given values of S and T. The chart is designed so that a straightedge placed across the given values of S and T intersects the y axis of the graph at a point S/T. The numerical value of S/T is not of interest, hence the ordinate of the graph is not labeled. What is of interest is how the ratio S/T compares with $\mathrm{Cot}(\alpha + 5)$ and $\mathrm{Cot}(\alpha - 5)$. Therefore, when the point representing S/T is found, it can be projected across the graph to the intersection with the lines mentioned above, or to an intersection with a given value of α. Detailed steps for use of this chart are given on the page facing the chart.

** A detailed discussion of nomographic charts is given in Appendix B.

The nomographic chart for Eq. (1) is shown in Fig. 4. Because of the numerous parameters in the equation on which this chart is based, a number of successive collineations are required to arrive at the final answer. When making these collineations it is frequently not necessary to mark the point found, but rather use this point as a pivot point for the next operation. Often a scale graduated in one parameter serves as an index line during intermediate steps. This is possible because the numerical answer is not necessary in the intermediate steps. Also, to have graduated scales serve as index lines permits the chart to have fewer lines and larger, more legible scales in a smaller space. The detailed instructions facing Fig. 4 specify whether the point found on a scale is an index point or whether the given values of the variables on the scale should be used.

Although the first attempt to use this chart may be discouraging, a few additional attempts will result in surprising proficiency. In fact, within a very short time it is possible to solve the equation on the chart in the same time it takes to find the difference between $\mathrm{Cot}(\alpha + 5)$ and $\mathrm{Cot}(\alpha - 5)$ from trigonometric tables.

The nomographic chart for Eq. (2) is shown in Fig. 5. The instructions for use of this chart are on the page facing the figure.

The two remaining nomographic charts are much less complicated than those for Eqs. (1) and (2). The chart illustrated in Fig. 6 is for the special case $\beta = 0$. Note that neither H nor h enters into the solution.

The final chart (Fig. 7) is for solving for either H or β in the expression

$$W = 2(H - h - T) \tan \beta \quad .$$

This expression was derived for the purpose of determining what width of cloud would fill a vertically pointing beam of varying diameter, which depends on the height of the vehicle above the cloud top.

These five nomographic charts permit rapid accurate solutions of the equations for which they were constructed. They contain, of course, the assumptions and limitations of the equations. It is hoped that future research on clouds will make it possible to label areas of the charts with cloud types. For example, if typical thicknesses can be determined for the various types of clouds, the thickness scale can be marked with cloud types. Similarly, typical values of S and W may be determined for various reported total sky covers and these can be marked on the charts.

No mention has been made yet of the treatment of multiple cloud layers. These layers could be each treated independently and the percent of time the ground is visible determined for each layer. These figures would then have to be weighted by the probability that cloud elements in the separate layers would coincide.

IV CONCLUSIONS

The simplest possible equation was formulated to express the percent of time the ground is visible from an aircraft in terms of pertinent cloud and non-cloud parameters. To obtain a simple expression the problem was treated in only two dimensions; it was assumed that two adjacent clouds were identical in vertical extent and that the clouds were rectangular in cross section. With these assumptions it was found that the important parameters were:

(1) Width of cloud elements

(2) Spacing of cloud elements

(3) Vertical extent of clouds

(4) Height of aircraft above the cloud base

(5) Height of aircraft above the cloud top.

The available cloud data permit the direct determination of only one of the five items above (Item 4), then only if the cloud base is low. It is necessary, therefore, to make basic measurements of cloud width, spacing, and thickness, which will then permit an evaluation of the validity of the assumptions used in the equations. The fact that one use of the equations is to aid in preparing frequency distributions of the percent of time the ground is visible tends to increase the validity of the assumptions, since the climatological data which must be used do not contain even gross estimates of the cloud parameters.

Nomographic charts were prepared to facilitate rapid, accurate solution of the equations over a wide range of cloud dimensions. Scales on these charts will eventually be labeled with cloud types if subsequent investigation proves this step feasible. Concurrently with quantitative estimates of typical cloud dimensions, solutions of the equations using the nomographic charts will be made to aid in determining the accuracy with which cloud dimensions must be known.

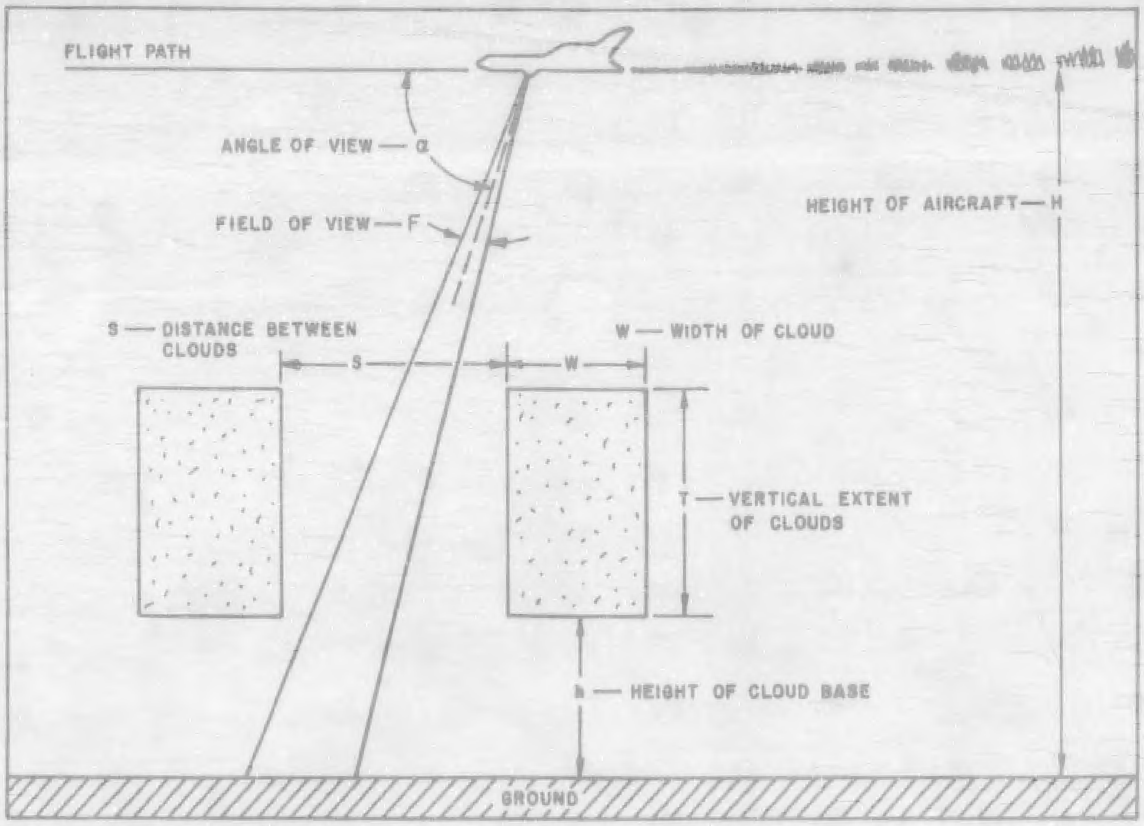

FIG. 1

IDENTIFICATION OF PARAMETERS USED IN EQUATIONS

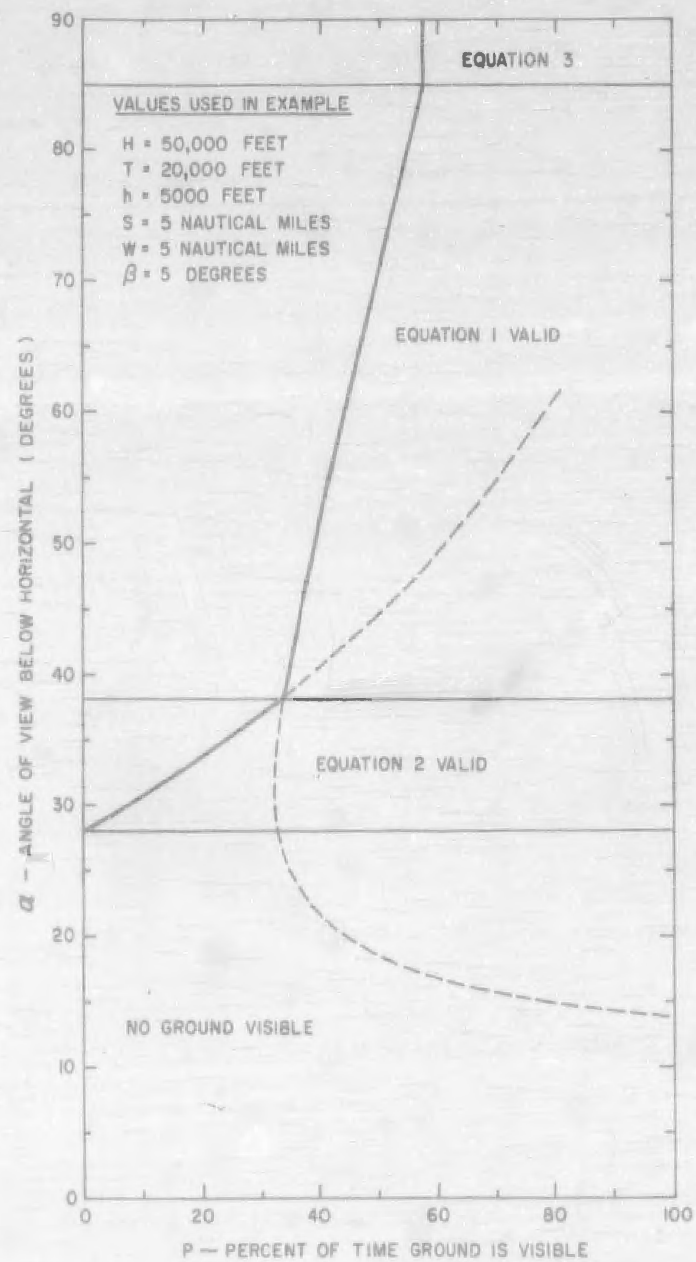

FIG. 2

CURVES DESCRIBED BY EQUATIONS

Note: The nomographic charts on the following pages have been reduced in size for inclusion in this report. Larger reproductions (18 by 24 inches) can be obtained from the Aerophysics Group of Stanford Research Institute. Address your request to:

Mr. Roy H. Blackmer, Jr.
Aerophysics Group
Bldg. 404A
Stanford Research Institute
Menlo Park, California

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INSTRUCTIONS FOR USE OF FIG. 3

(A) Place a straightedge across the given values of S and T, and mark where the straightedge intersects the y axis of the graph.

(B_1) Find the point on the graph defined by the values of S/T and the given value of α. The area in which the point lies is labeled with the appropriate equation to be used to find P.

(B_2) To find the range of angles over which no ground is visible, or to find where Eq. (1) or Eq. (2) should be used, project the point on the y axis horizontally and note the value of α at the intersections with the two curves on the graph.

EXAMPLE: Let S = 7 miles, T = 20,000 feet, and α = 25 degrees. Use of these values in following the steps for Fig. 3 shows that Eq. (2) should be used. Also, for angles less than 20 degrees, no ground is visible. Equation (2) is valid from 20 degrees to 30 degrees, and at angles greater than 30 degrees, Eq. (1) is valid.

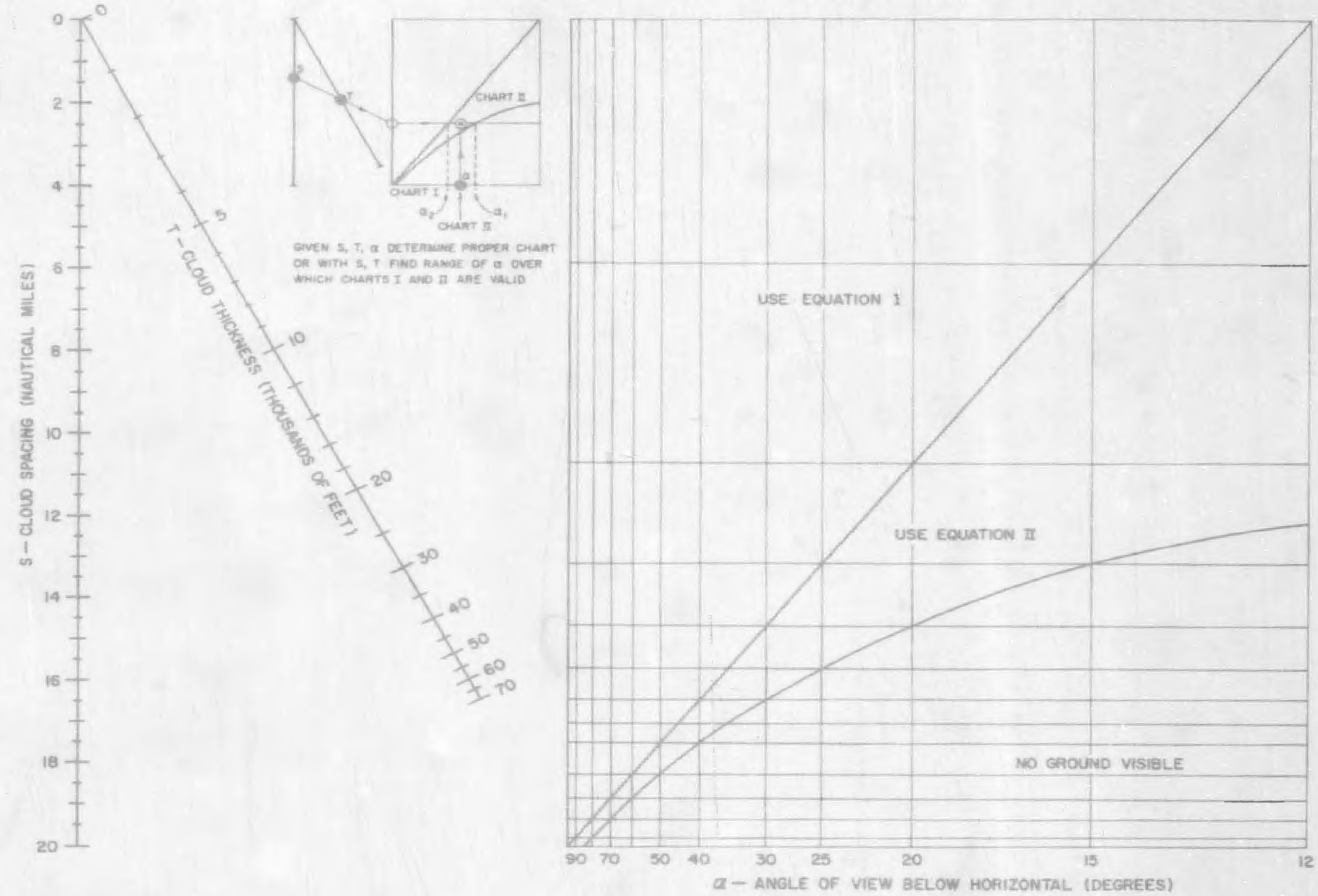

FIG. 3

NOMOGRAPHIC CHART FOR DETERMINING WHICH EQUATION IS VALID

INSTRUCTIONS FOR USE OF FIG. 4

Note: It is helpful to follow through the instructions once or twice using a clear plastic straightedge on the small insert on the figure. It should be noted that there are two W scales and two α scales. This was necessary because these parameters both enter the equation twice. When solving the equation, the same value of W must be used on both W scales, and the same value of α must be used on both α scales.

(a) Lay straightedge across given values of S and W_2 (see Note, and see Line labeled a on inset). Mark index Point 1, where straightedge intersects h, T scale, for future use.

(b) Lay straightedge across given values of T and α_1 (see Note, and see Line b). Place pencil at Index Point 2, where straightedge intersects S scale.

(c) Pivot straightedge on Point 2 to value of W_1 (see Line c) and mark Index Point 3, where straightedge intersects h, T scale.

(d) Lay straightedge across given values of h and H (Line d).

(e) Place pencil at Index Point 4, where straightedge intersects W_1, W_2 scales, and pivot straightedge to value of α_2 (Line e).

(f) Move pencil to Index Point 5 (H scale) and pivot to Index Point 3 marked in Step (b) (Line f) on H, T scale. Move pencil to Index Point 6 on S scale and pivot straightedge to Index Point 1 marked in Step (a) on H, T scale. The answer is now at the point where the straightedge crosses the P scale.

EXAMPLE: When

S = 5 nautical miles	α = 40 degrees
W = 5 nautical miles	h = 5,000 feet
T = 20,000 feet	H = 50,000 feet

then P = 33 percent.

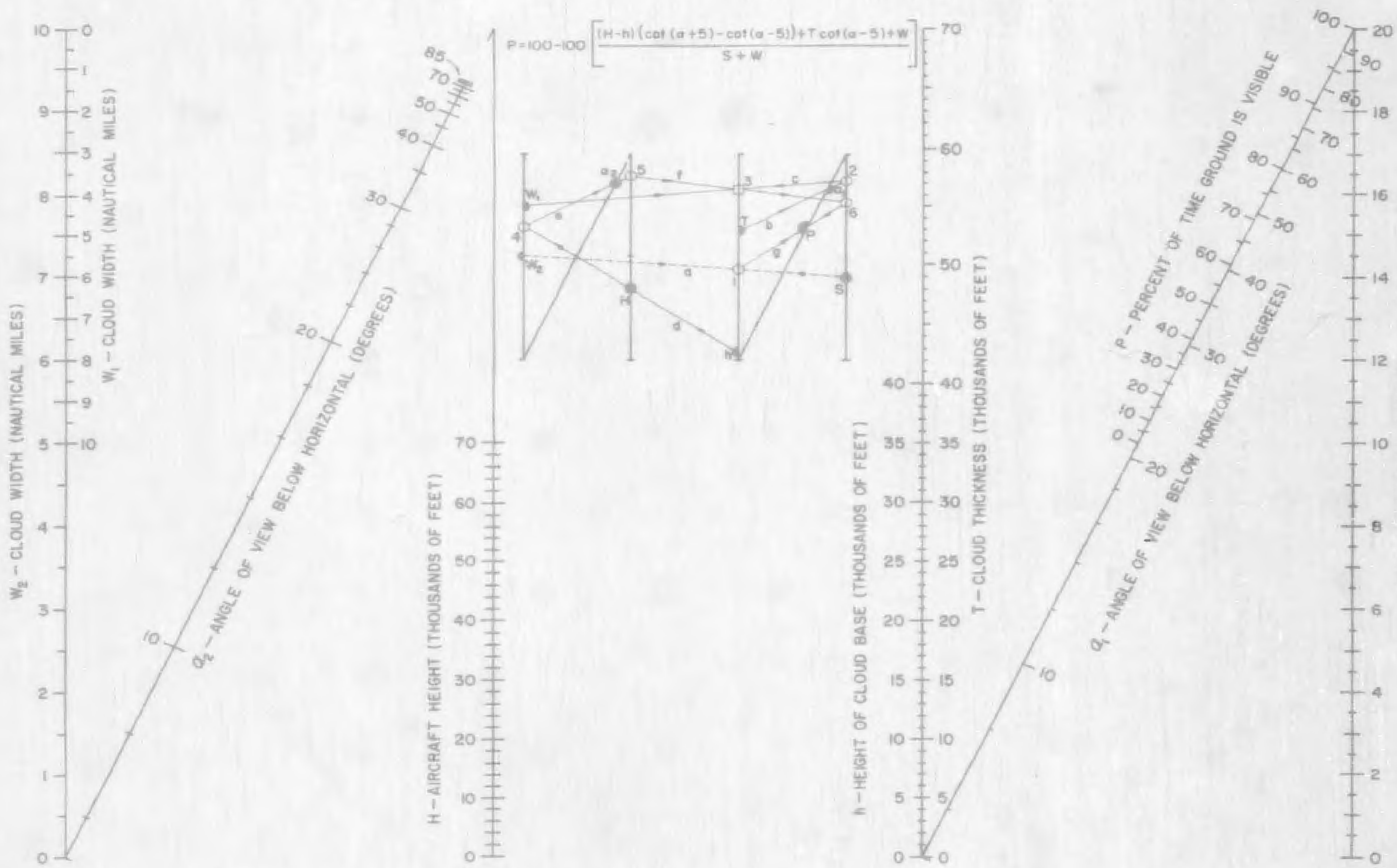

FIG. 4
NOMOGRAPHIC CHART FOR SOLUTION OF EQ. 1

INSTRUCTIONS FOR USE OF FIG. 5

Note: It is helpful to follow through the instructions once or twice using a clear plastic straightedge on the small inset on the figure.

(a) Lay straightedge across a given value of S_1 and W (Line a). Mark Index Point 1 where straightedge intersects h scale for future use.

(b) Lay straightedge across given value of h and H (Line b).

(c) Put pencil at Index Point 2, where straightedge intersects P, W scale, and pivot straightedge back to Index Point 1 marked in Step (a) (Line c). Mark Index Point 3 where straightedge crosses index line for future reference.

(d) Lay straightedge across given values of S_2 and T (Line d).

(e) Put pencil at Index Point 4 where straightedge intersects P, W scale and pivot straightedge to given value of α (Line e).

(f) Move pencil to Point 5, where straightedge intersects h scale, and pivot straightedge to Point 3 marked in Step (c) (Line f). The answer is given where the straightedge intersects the P scale.

EXAMPLE: When

S = 5 nautical miles	α = 35 degrees
W = 5 nautical miles	h = 5,000 feet
T = 20,000 feet	H = 50,000 feet

then P = 24 percent.

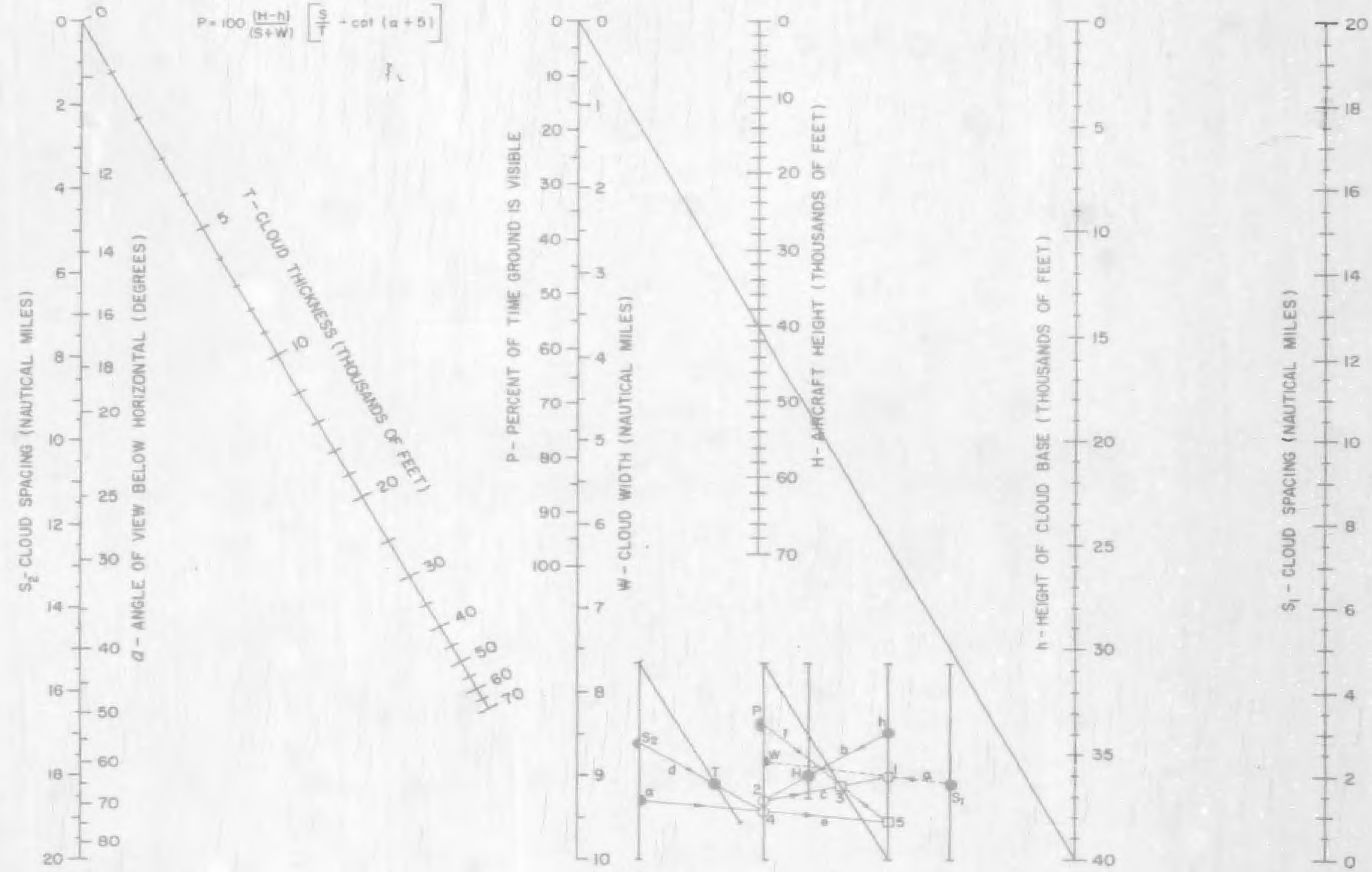

FIG. 5

NOMOGRAPHIC CHART FOR SOLUTION OF EQ. 2

INSTRUCTIONS FOR USE OF FIG. 6

Note: It is helpful to follow through the instructions once or twice using a clear plastic straightedge on the small inset on the figure.

(a) Lay straightedge across given values of S and W_1 (Line a). Mark Index Point 1 where index line is intersected by straightedge.

(b) Lay straightedge across given values of T and α (Line b).

(c) Put pencil point where straightedge intersects index line and pivot straightedge to value of W on W_2 scale (Line c). Move pencil to Index Point 3 where straightedge intersects S scale.

(d) Pivot straightedge to Index Point 1 marked in Step (a) (Line d). Read answer on P scale.

EXAMPLE: When

$S = 5$ nautical miles

$W = 5$ nautical miles

$T = 20{,}000$ feet

$\alpha = 40$ degrees

then $P = 10$ percent.

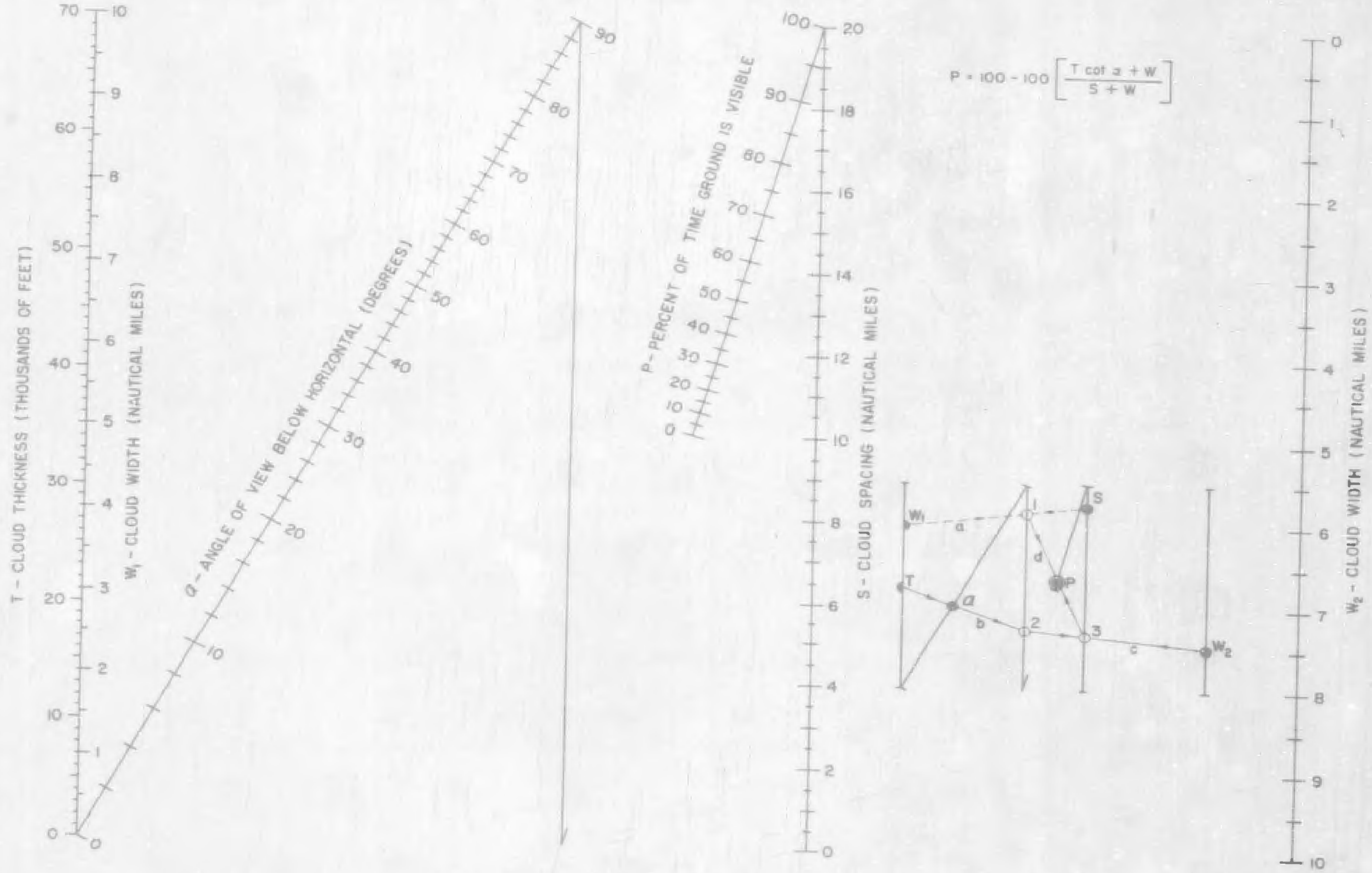

FIG. 6

NOMOGRAPHIC CHART FOR SOLUTION OF EQ. 1 WITH A ZERO DEGREE FIELD OF VIEW

INSTRUCTIONS FOR USE OF FIG. 7

<u>To find H, given T, h, W, and F</u>

(a) Lay straightedge across given values of h and T (Line a). Mark Index Point 1 on W scale.

(b) Lay straightedge across given values of W and F (Line b).

(c) Place pencil at Index Point 2 on h scale and pivot straightedge to Index Point 1 (Line c). Read answer on H scale.

<u>Example:</u> When

T = 10,000 feet
h = 5,000 feet
W = 5 nautical miles
F = 45 degrees

then

H = 51,000 feet.

<u>To find F, given T, h, W, and H</u>

(a) Lay straightedge across given values of h and T (Line a).

(b) Place pencil at Index Point 1 on W scale and pivot straightedge to given value of H (Line b).

(c) Move pencil to Index Point 2 on h scale and pivot straightedge to value of W (Line c). Read answer on F scale.

<u>Example:</u> When

T = 10,000 feet
h = 5,000 feet
W = 5 nautical miles
H = 40,000 feet

then

F = 63 degrees.

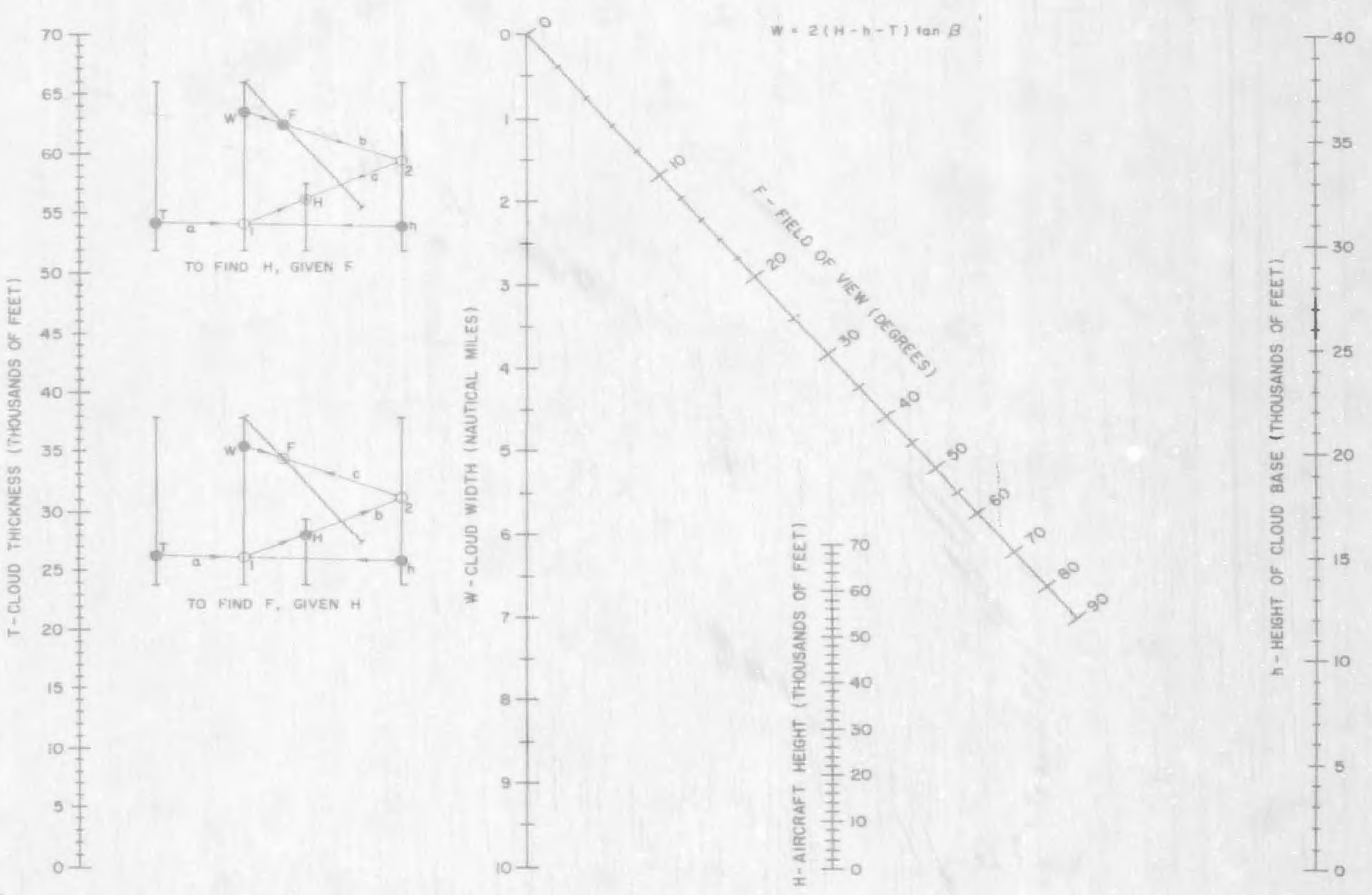

FIG. 7

NOMOGRAPHIC CHART RELATING CLOUD WIDTH AND VERTICAL FIELD OF VIEW

APPENDIX A

DERIVATION OF EQUATIONS

APPENDIX A

DERIVATION OF EQUATIONS

Given a cloud layer such that two adjacent clouds are equal in height above ground and in vertical extent, and are rectangular in vertical cross section along the path of flight of an aircraft at constant velocity and altitude, the percent of time that the ground can be seen at any angle α, with any field of view F, is developed below.

Since at a constant velocity the ratio of two time intervals is equal to the ratio of the distances traveled during the time intervals, it is only necessary to find the distance the aircraft travels during which ground is visible and divide by the total distance (or some unit distance) to obtain the desired ratio.

Figure A-1 illustrates the geometry of obscuration of the ground by clouds. Reference to the figure will enable the following derivation to be more easily visualized.

(1) Let the airplane be at point P_1 with an α such that the trailing edge of the beam has just entered the first cloud.

(2) With a constant α, let the airplane move to a point P_2 such that the leading edge of the beam is just emerging from the first cloud.

(3) Still with the same α, let the airplane move to a point P_3 where the trailing edge of the beam is just entering the second cloud.

The airplane has thus completed one cycle through cloud and clear air between cloud.

Along the distance P_1P_2 the ground is completely obscured. This distance may be expressed as:

$$P_1P_2 = \Delta P = Z + W - Y$$

By simple trigonometry it may be shown that

$$Y = (H - h - T)\,\mathrm{Cot}(\alpha - \beta)$$

$$Z = (H - h)\,\mathrm{Cot}(\alpha + \beta)$$

and W is cloud width

Thus by substitution

$$\Delta P = (H - h)\,\mathrm{Cot}(\alpha + \beta) - (H - h - T)\,\mathrm{Cot}(\alpha - \beta) + W$$

$$= (H - h)\,[\mathrm{Cot}(\alpha + \beta) - \mathrm{Cot}(\alpha - \beta)] + T\,\mathrm{Cot}(\alpha - \beta) + W$$

The distance P_1P_3 is $S + W$ which will be considered a unit distance.

Now let the time to travel the distance ΔP be Δt and the time to travel the unit distance $S + W$ be t. The percent of time the ground is obscured will then be $100\frac{\Delta t}{t}$. We are interested in the percent of time the ground is visible, which will be

$$P = 100 - 100\frac{\Delta t}{t}$$

Substituting distance for Δt and t we obtain

$$P = 100 - \frac{100\Delta P}{S + W}$$

Substituting ΔP expressed in terms of the other parameters, the basic equation is

$$P = 100 - 100\left\{\frac{(H - h)[\mathrm{Cot}(\alpha + \beta) - \mathrm{Cot}(\alpha - \beta)] + T\,\mathrm{Cot}(\alpha - \beta) + W}{S + W}\right\} \quad (1)$$

It should be stressed that this equation expresses the percent of time some ground may be seen and does not distinguish between a completely cloud free beam and the condition that some cloud will be in the field of view. The ground is considered visible when the beam is not completely cloud filled.

No ground will be visible if the following relationship between S, T, α, and β exists.

$$\mathrm{Cot}(\alpha + \beta) \geq \frac{S}{T}$$

With values satisfying this inequality (if the beam is not larger than the cloud cross section) any portion of the beam that breaks out of the first cloud will be obscured by the next cloud.

If S, T, α, and β are such that

$$\mathrm{Cot}(\alpha + \beta) \leq \frac{S}{T} \leq \mathrm{Cot}(\alpha - \beta)$$

then the leading edge of the beam will intercept the next cloud when first emerging from the first cloud but at a point P_2 such that the distance

$$P_1P_2 = (H - h)\,\mathrm{Cot}(\alpha + \beta) - (H - h - T)\,\frac{S}{T} + W$$

some portion of the beam will reach the ground unintercepted by cloud.

The equation when this condition [that the ratio S/T lies between the values of $\mathrm{Cot}(\alpha + \beta)$ and $\mathrm{Cot}(\alpha - \beta)$] obtains is, therefore,

$$P = 100 - 100\left[\frac{(H - h)\,\mathrm{Cot}(\alpha + \beta) - (H - h - T)\,\frac{S}{T} + W}{S + W}\right]$$

This equation may be reduced considerably by expanding the term $(H - h - T)\frac{S}{T}$.

If this term is written as $\frac{(H - h)\,S}{T} - T\frac{S}{T}$ the equation may be written

$$P = 100\left\{1 - \left[\frac{(H - h)\mathrm{Cot}(\alpha + \beta) - (H - h)\frac{S}{T} + S + W}{S + W}\right]\right\}$$

Now two terms contain (H - h), and S + W appears in both the numerator and denominator. Combining terms gives

$$P = 100\left(\frac{H - h}{S + W}\right)\left[\frac{S}{T} - \mathrm{Cot}(\alpha + \beta)\right] \qquad (2)$$

This equation and the previous equation do not completely cover the entire range of angles. An additional limitation is caused by the behavior of the cotangent function at angles near 90°. When the angle of view is in the range

$$(\alpha - \beta) < 90° < (\alpha + \beta)$$

a third equation must technically be used. This equation is

$$P = 100\left\{\frac{S + (H - h - T)\left[\tan(\alpha + \beta - \frac{\pi}{2}) + \tan(\beta + \frac{\pi}{2} - \alpha)\right]}{S + W}\right\}$$

The usefulness of this equation becomes doubtful when the changes in magnitude of the term containing the tangents is examined. Let this term be $\psi(\alpha)$ thus

$$\psi(\alpha) = \tan(\alpha + \beta - \frac{\pi}{2}) + \tan(\beta + \frac{\pi}{2} - \alpha)$$

With a β of 5° the value of $\psi(\alpha)$ varies with α as follows:

α	$\psi(\alpha)$
85	0.17633
86	0.17584
87	0.17546
88	0.17519
89	0.17503
90	0.17498

This very small change in $\psi(\alpha)$ results in a change of P of only 0.05%. With smaller values of β the range of α over which the equation applies is reduced. As a result, this third equation will not be considered further.

The special case when $\beta = 0$ was investigated. By either an independent derivation or simplification of Eq. 1 it may be shown that with $\beta = 0$

$$P = 100 - 100 \left[\frac{T \operatorname{Cot} \alpha + W}{S + W} \right]$$

An additional equation was derived for determining what width of cloud would just fill a vertical field of view. The derivation of this equation is readily apparent from Fig. 9. It may be seen from the figure that

$$\tan \beta = \frac{W/2}{(H - h - T)}$$

or $$W = 2(H - h - T) \tan \beta$$

This equation may be used to determine what height H would permit a given field of view F to see around clouds or to determine what F is necessary when flying at a given height over clouds of a given width.

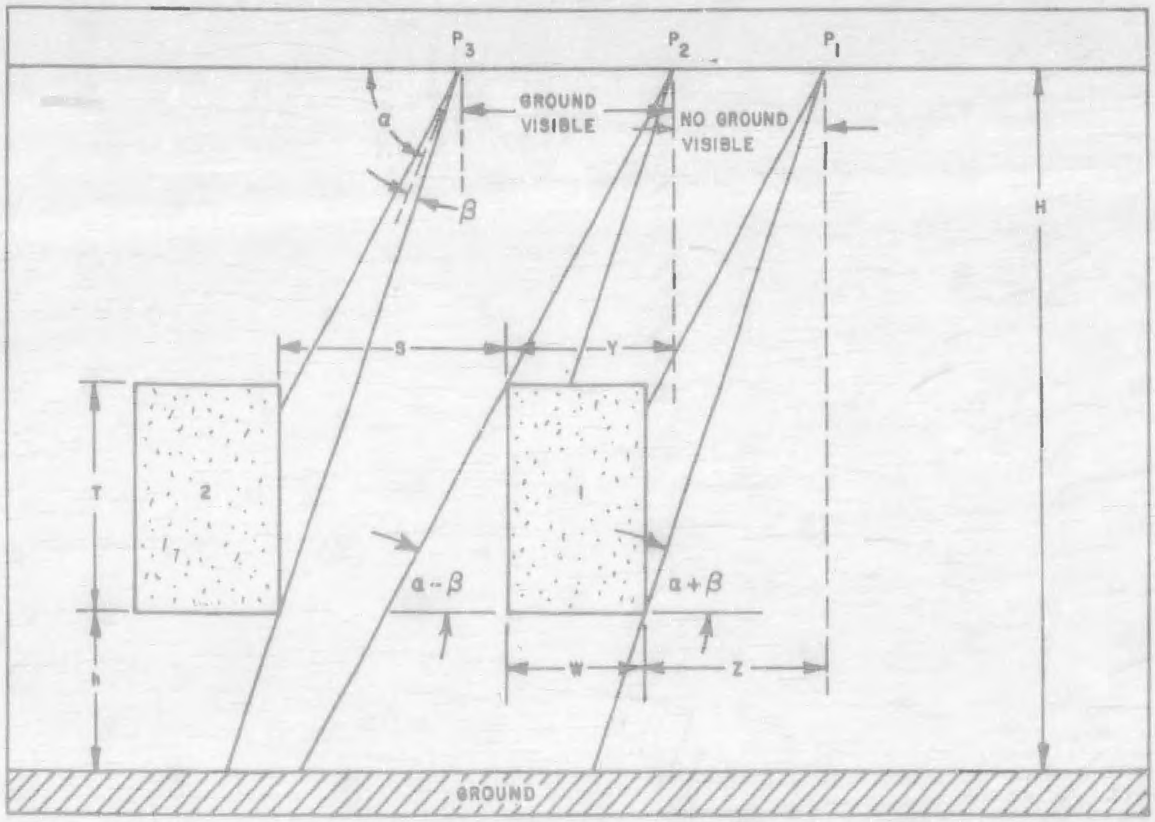

FIG. A-1

GEOMETRY OF OBSCURATION WITH A SLANTING BEAM

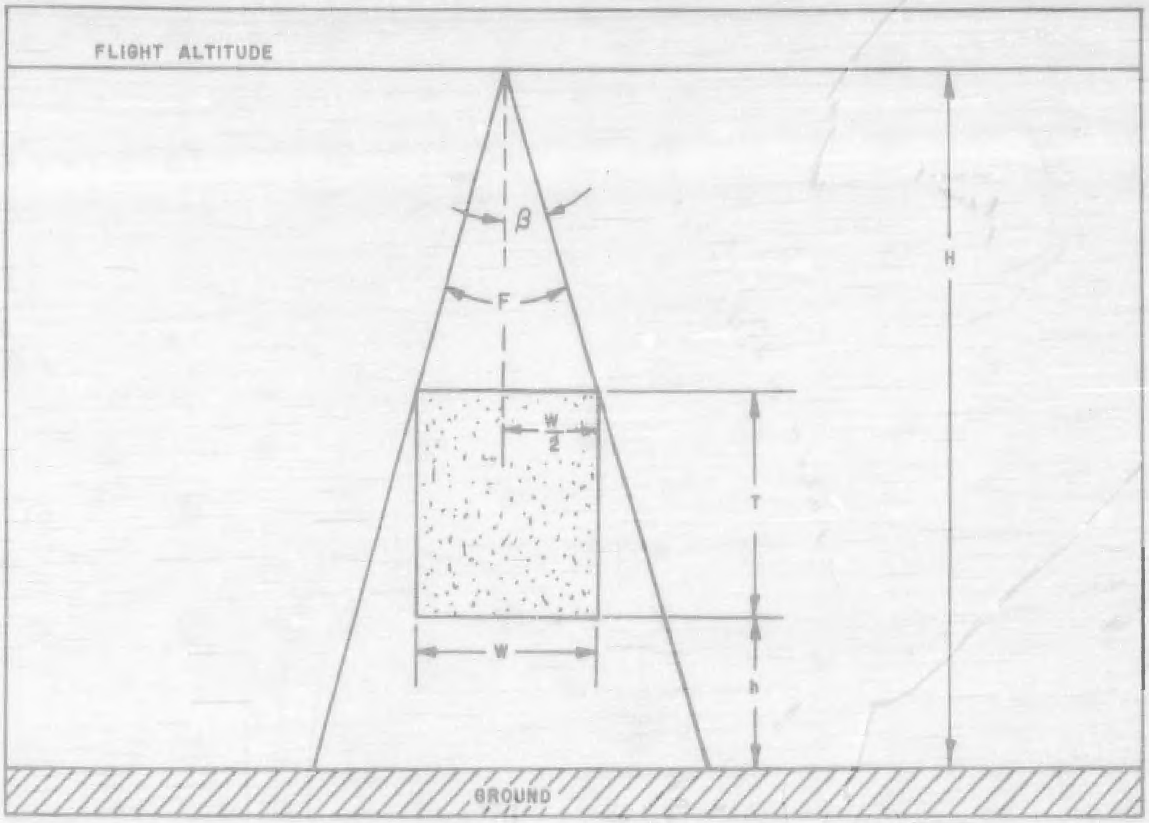

FIG. A-2

GEOMETRY OF OBSCURATION WITH A VERTICAL BEAM

APPENDIX B

DISCUSSION OF NOMOGRAPHIC CHARTS

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DISCUSSION OF NOMOGRAPHIC CHARTS

A lengthy discussion on nomographic charts seems unnecessary, but because of the number of variables in the equations, and the resulting seemingly complicated procedures required to reach an answer, some discussion of the charts would seem helpful.

A nomographic chart is a chart constructed so that a straight line through two graduated curves intersects a third graduated curve at a value which satisfies an equation relating the three variables. When an equation contains more than three variables, it must be separated into a series of equations, each of which has only three variables. Thus an equation such as

$$W + X + Y = Z$$

must be separated into two equations such as

$$W + X = A$$

$$A + Y = Z$$

The solution of the first is then used as a new variable in the second.

The equations derived in this report contain numerous variables. They must therefore be reduced as illustrated above, and numerous successive solutions must be performed on the nomographic chart to arrive at the final answer. Consider the equation for $\beta = 0$:

$$P = 100 - 100 \left[\frac{T \text{ Cot } \alpha + W}{S + W} \right] .$$

This equation may be rewritten in the form

$$R = \frac{T \text{ Cot } \alpha + W}{S + W}$$

where R is the ratio of the length along which ground is obscured to the unit length, or

$$R = \frac{L_o}{L_T} \quad .$$

Then $L_o = T \text{ Cot } \alpha + W$

$L_T = S + W \quad .$

If we now write

$T \text{ Cot } \alpha = X$

then

$L_o = X + W$

and the equation has been reduced to a series of equations, each with only three variables. To solve the equation, then, the following steps are taken, where each step consists of drawing a line through values of two variables to read an answer on a third scale. Reference to the instructions facing Fig. 6 shows how these steps are performed on the nomographic chart:

step (a) $S + W = L_T$

step (b) $T \text{ Cot } \alpha = X$

step (c) $X + W = L_o$

step (d) $L_o/L_T = R$ [R can be graduated in P since $P = 100 (1 - R)$] .

It is not necessary on the nomographic chart to have scales graduated in numerical values of X, L_V, and L_T. Instead, it is sufficient merely to mark a point on an index line where the line through two parameters crosses the third line. By careful arrangement of the chart it is possible to have a scale graduated in one parameter serve as an index line for the relationship between two other parameters. For example, on Fig. 6 the index line for L_V is coincident with the scale for S, and the index lines for both X and L_T are coincident. This permits a chart with larger, more legible, scales to be constructed in a smaller space.

When β is not zero, the variables H and h must be included on the nomogram, increasing the number of collineations necessary for solution. To avoid making the charts unduly complicated, a value of F = 10 (β = 5) degrees was assumed. This avoided the necessity of having both an α and a β scale and the necessity of solving

$$\text{Cot}(\alpha + \beta) = \frac{1 - \text{Cot}\,\alpha\ \text{Cot}\,\beta}{\text{Cot}\,\alpha + \text{Cot}\,\beta}$$

and

$$\text{Cot}(\alpha - \beta) = \frac{1 + \text{Cot}\,\alpha\ \text{Cot}\,\beta}{\text{Cot}\,\alpha - \text{Cot}\,\beta} \quad .$$

With $\beta = 5°$, we can write Eq. (1) as

$$P = 100\,(1 - R)$$

where

$$R = \frac{(H - h)\,\varphi\,(\alpha) + T\,\text{Cot}(\alpha - 5) + W}{S + W}$$

and

$$\varphi\,(\alpha) = \text{Cot}(\alpha + 5) - \text{Cot}(\alpha - 5) \quad .$$

The expression for R must be reduced to a series of equations with three variables in the same manner as for the case with $\beta = 0$. This was done as follows:

$$S + W = L_T$$

$$T \operatorname{Cot}(\alpha - 5) = X$$

$$X + W = a$$

$$H - h = b$$

$$b \times \varphi(\alpha) = c$$

$$c + a = L_o$$

$$\frac{L_o}{L_T} = R$$ but R is again graduated in P.

Comparison of the above series of three variable equations with the instructions for solution of Eq. (1) (see Fig. 4) will show how these steps are followed in the order shown to arrive at the final answer.

The equations for the nomographic charts shown in Figs. 5 and 7 were treated in a similar manner.

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