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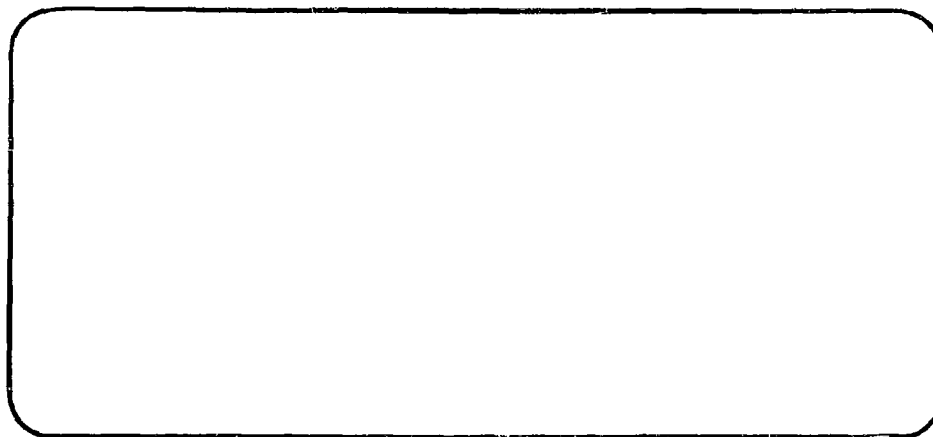
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VELOCITY RECOVERY AND SHEAR REDUCTION
IN JET-DRIVEN VORTEX TUBES

XEROX

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ABSTRACT

The laminar, two-dimensional flow of a viscous vortex, driven by tangential fluid injection in a porous cylindrical container, is considered. The jets are idealized as sources of mass and momentum; no aspect of jet mixing is considered. The tangential velocity profile in the annular region between the jet input radius and the radius of the cylinder is found analytically as a solution of the tangential momentum equation. The peripheral wall shear is evaluated with and without radial fluid injection through the cylinder wall. For a given tangential velocity in the vortex, radial fluid injection through the porous wall substantially reduces the shear.

The effect of radial fluid injection on the fraction of the jet velocity that is recovered in the vortex is found by means of a torque balance, including the torque required to accelerate the radially injected fluid to the peripheral vortex velocity. It is found that for a given total mass flow through the center of the vortex, radial fluid injection is always detrimental to the velocity recovery in the vortex; that is, for a given jet velocity, diversion of a fraction of the mass flow from tangential to radial injection always results in a reduced vortex strength. The explanation of this phenomenon is presented.

The results for zero radial fluid injection are compared with experimental data for which the flow was turbulent. It is found that the theory predicts the recovery accurately even for turbulent flow, provided an appropriate turbulent eddy viscosity is included in the evaluation of the radial Reynolds number.

CONTENTS

	Page
I. INTRODUCTION	1
II. ANALYSIS	2
III. DETERMINATION OF α	3
IV. COMPARISON WITH EXPERIMENTS	7
SYMBOLS	8
REFERENCES	9
FIGURES 1 through 4	10

I. INTRODUCTION

The use of a vortex flow as a means of containing a nuclear fuel in a gaseous fission rocket has recently been proposed in the literature.⁽¹⁾⁽²⁾ The containment process requires a high tangential mach number coupled with a small radial mass flow. This combination is difficult to achieve in hydrodynamically driven vortex tubes (i.e. vortices driven by tangential fluid injection) where it is usually necessary to supply large amounts of mass injection in order to generate strong vortices. The mechanism which limits the strength of a confined, jet-driven vortex is the peripheral wall shear, which retards the flow and removes angular momentum from the system. This loss is manifested as a reduction in the percentage of the jet velocity that is "recovered" in the vortex.

Reduction of the peripheral wall shear may enhance the jet velocity recovery in the vortex. Such a reduction can be accomplished by means of radial fluid injection through the outer wall of the vortex tube, but only at the expense of increased mass flow through the device. The purpose of the present study is to assess the effect of radial fluid injection on the wall shear in a vortex tube and to investigate the resulting influence on the jet velocity recovery which, for a given jet velocity, is equivalent to the circulation or "strength" of the vortex.

It will be shown that although substantial reduction in wall shear is possible, it is never possible to increase the velocity recovery by diverting a fraction of the total mass flow from tangential to radial injection; that is, for a given total mass flow, maximum recovery is achieved by injecting all the fluid tangentially.

The geometry under consideration is shown in Fig. 1. Fluid is introduced by means of discrete jets or slits spaced uniformly around the periphery of the tube. The fluid is removed along the axis of the tube. The wall is considered porous so that some fluid may be injected radially. To simplify the analysis, it is assumed that the tangentially injected fluid is introduced continuously and uniformly around the circumference of a circle of radius βR , where R is the radius of the cylinder and β is less than one. The jets are idealized

as sources of mass and momentum; no aspect of the jet spreading or mixing problem is considered. In the region under study, the flow is assumed to be two-dimensional. The analysis is performed for laminar flow although there is evidence to indicate that the results can be partially extended to turbulent flow if the proper value of the eddy viscosity is included in the definition of the radial Reynolds number.

II. ANALYSIS

The equation governing the fluid motion in the annulus bounded by $r = R$ and $r = \beta R$ is, for two-dimensional, axi-symmetric flow:

$$\rho u \left(\frac{dv}{dr} + \frac{v}{r} \right) = \mu \frac{d}{dr} \left(\frac{dv}{dr} + \frac{v}{r} \right) \quad (1)$$

The solution of eqn. (1) is well known and is, for $\mu = \text{const}$:

$$v = Ar^{-1} + Br^{1 + Re_0} \quad (2)$$

where A and B are constants and

$$Re_0 = \frac{\rho u r}{\mu} = - \frac{\dot{M}}{2\pi u r} \quad (3)$$

The Reynold's number Re_0 , is based on the mass flow which is injected radially through the porous wall, and is a constant, for a given mass flow, by continuity. For flow directed radially inward, it is negative since u is defined as positive when directed radially outward.

Eq. (2) represents the solution to eq. (1) for all values of Re_0 except $Re_0 = -2$ which is a singular point of the differential equation. However, $Re_0 = -2$ is a removable singularity and will, therefore, not be considered further.

With the boundary conditions:

$$\left. \begin{array}{l} v = 0 \quad \text{at } r = R \\ \text{and} \\ v = \alpha V_j \quad \text{at } r = \beta R ; \alpha < 1 \end{array} \right\} \quad (4)$$

the constants A and B can be determined. The result is:

$$A = -BR(2 + Re_o) \quad (5)$$

and

$$B = \frac{\alpha v_j \beta R}{R(2 + Re_o) (\beta(2 + Re_o) - 1)}$$

The factor α represents the fraction of the jet velocity that is recovered in the vortex.

The shear at the wall is proportional to $\frac{dv}{dr}$ at $r = R$, which is given by:

$$\left(\frac{dv}{dr}\right)_{r=R} = \frac{\alpha v_j \beta R (2 + Re_o)}{R^2 (\beta(2 + Re_o) - 1)} \quad (6)$$

The fractional reduction in wall shear due to blowing is found by normalizing with respect to the value of $\left(\frac{dv}{dr}\right)_{r=R}$ for $Re_o = 0$ (i.e. no radial fluid injection). The result is:

$$\frac{\tau_w}{\tau_{w_0}} = \frac{\alpha}{\alpha_0} \cdot \frac{(2 + Re_o)}{2} \cdot \frac{(\beta^2 - 1)}{(\beta(2 + Re_o) - 1)} \quad (7)$$

where α_0 is the recovery with no radial mass injection.

III. DETERMINATION OF α

The velocity recovery in the vortex, α , is determined by equating the jet input torque to (minus) the algebraic sum of the fluid torques existing at $r = \beta R$. Thus, it is necessary to know the velocity profile in the inner region, $r < \beta R$, as well as the velocity profile in the outer annulus determined above. For the purpose of the following calculation the approximate solution of Einstein and Li⁽³⁾ will be assumed for the inner flow. That is:

$$\frac{v'}{r'} = \frac{K}{r'^2} + (1 - K) r'^{Re_i} ; r'_e < r' < 1 \quad (8)$$

where: $v' = \frac{v}{\alpha V_j}$, $r' = \frac{r}{\beta R}$ and

$$K = \frac{\left[Re_i + 2 (1 - e^{Re_i/2}) \right]}{Re_i \left(1 - e^{Re_i/2} r'_e \right) + 2 (1 - e^{Re_i/2})} \quad (9)$$

The radial Reynolds number Re_i is based on the total mass flow through the interior of the vortex, which is equal to the sum of the radially injected mass flow and the mass flow entering through the jets. r'_e is the dimensionless radius of the exit hole at the center of the vortex. Once again $Re_i = -2$ is a removable singularity of the governing differential equation.

Using the above assumption, the torque exerted by the inner flow at $r = \beta R_-$ can be computed. The calculations follows:

$$T_i = (-2\pi r^2 \gamma)_r = \beta R_- = \left[-2\pi r^3 \mu \frac{d}{dr} \left(\frac{v}{r} \right) \right]_r = \beta R_-$$

and using eq. (8):

$$T_i = 2\pi \mu \alpha V_j \beta R \left[2K - Re_i (1 - K) \right] \quad (10)$$

It is to be noted that this torque acts in the same direction as the torque exerted by the jets, i.e. it tends to accelerate the outer fluid. Thus, neglecting this component of the torque (as was done, for example, in Ref. (1)) is conservative, in that a smaller value of α will be predicted.

From the calculation of section II, the torque exerted by the outer fluid at $r = \beta R_+$ is found to be:

$$T_o = 2\pi \mu \alpha V_j \beta R \frac{\left(2 + Re_o \beta^{(2 + Re_o)} \right)}{\left(\beta^{(2 + Re_o)} - 1 \right)} \quad (11)$$

It is easily verified that this torque is numerically equal to the retarding torque exerted by the wall plus the increase in angular momentum experienced by the radially injected fluid in moving from $r = R$ to $r = \beta R$.

The jet input torque is proportional to the mass flow introduced by the jets and the velocity difference $(1 - \alpha)V_j$. Explicitly:

$$T_j = -2\pi\mu(1 - \alpha)V_j\beta R(\text{Re}_i - \text{Re}_o) \quad (12)$$

Setting:

$$T_j = -(T_i + T_o)$$

and solving for α yields:

$$\alpha = \left\{ 1 + \frac{1}{\text{Re}_i(1 - \gamma)} \left[\frac{2 + \gamma\text{Re}_i\beta^{(2 + \gamma\text{Re}_i)}}{(\beta^{(2 + \gamma\text{Re}_i)} - 1)} + (2 + \text{Re}_i)K - \text{Re}_i \right] \right\}^{-1} \quad (13)$$

where $\gamma (= \text{Re}_o/\text{Re}_i)$ represents the fraction of the total mass flow that is injected radially through the porous wall.

For no radial mass injection, $\gamma = 0$ and, in this case,

$$\alpha_o = \frac{\text{Re}_i}{\left(\frac{2}{\beta^2 - 1}\right) + (2 + \text{Re}_i)K} \quad (14)$$

Eq. (14) is plotted in Fig. 2 for several values of β . The solid curves are for $r'_e = 0.1$ and the dashed for $r'_e = 0.5$. It is seen that α_o is a very weak function of r'_e . This stems from the fact that the function K is relatively insensitive to the value of r'_e and, in fact, approaches unity exponentially as Re_i becomes increasingly negative. The limit $K = 1$ corresponds to potential flow in the vortex and is a valid approximation as long as $\text{Re}_i < -3$. For this case, eq. (14) can be simplified somewhat and may be reduced to:

$$\alpha_o = \frac{\text{Re}_i}{a + \text{Re}_i} \quad ; \quad a = \frac{2\beta^2}{\beta^2 - 1} \quad (15)$$

To determine the effect of radial fluid injection on the recovery, and to evaluate the wall shear, it is necessary to examine the function (α/α_o) . In the region of validity of the approximation $K = 1$, this can be written as:

$$\left(\frac{\alpha}{\alpha_0}\right) = \frac{Re_1 + a}{Re_1 + \left(\frac{b}{b-1}\right) \frac{(2 + \gamma Re_1)}{(1 - \gamma)}} \quad (16)$$

where:

$$b = \beta(2 + \gamma Re_1)$$

Eq. (16) may be rewritten in the form:

$$\left(\frac{\alpha}{\alpha_0}\right) = 1 + \frac{a - \left(\frac{b}{b-1}\right) \frac{(2 + \gamma Re_1)}{(1 - \gamma)}}{Re_1 + \left(\frac{b}{b-1}\right) \frac{(2 + \gamma Re_1)}{(1 - \gamma)}} \quad (17)$$

It can be shown that the second term on the right hand side of eq. (17) is always negative when $0 \leq \gamma \leq 1$, $0 < \beta \leq 1$ and $Re_1 < 0$, so that (α/α_0) is always less than one. In order to show this, it is necessary and sufficient to prove that the function

$$\frac{2\beta^2/(1 - \beta^2)}{x\beta^x/(1 - \beta^x)}$$

is less than or equal to one whenever $-\infty < x \leq 2$ and $0 < \beta \leq 1$. This can be seen by noting that the function in the denominator of the above expression is a monotonically decreasing function of x .

Eq. (17) is plotted in fig. 3 for two values of β and several values of γ . The shear stress from eq. (7) is shown in fig. 4 with (α/α_0) evaluated from eq. (17).

At first glance, it may seem rather surprising that radial fluid injection is detrimental to the recovery even though it produces a considerable reduction in the shear stress at the wall. However, this may be understood by noting that for fixed values of Re_1 and V_j , increasing γ implies decreasing jet momentum available to drive the vortex. In addition, some of the remaining input momentum is required to accelerate the radially injected fluid to the peripheral vortex velocity. Apparently, the decrease in retarding torque exerted

by the wall is not sufficient to offset these two "losses".

As a matter of interest, and for computational purposes it may be noted that

$$\lim_{Re_1 \rightarrow \infty} \left(\frac{\alpha}{\alpha_0} \right) = 1 - \gamma$$

independent of the value of β .

IV COMPARISON WITH EXPERIMENTS

There is some existing experimental data on jet-driven vortices of the type analyzed here. ⁽⁴⁾⁽⁵⁾ Experimentally determined values of α_0 were taken from Ref. (4) and (5) and the points are shown on Fig. 2. The vortices in these experiments were turbulent and the corresponding Reynolds numbers were computed using an experimentally determined value of eddy viscosity. The geometrical values of β varied from 0.88 to 0.95. Apparently the "effective" value of β was somewhat less than the geometrical value, which may be the result of unsymmetrical jet spreading. In any case, considering the uncertainty involved in evaluating the eddy viscosity, the agreement with the analysis must be considered satisfactory, indicating that the analytical results may be applied to turbulent flow provided suitable estimates can be made of the turbulent eddy viscosity and effective injection radius.

SYMBOLS

a	function of β defined in eq. (15)
A	constant
b	function defined in eq. (16)
B	constant
K	function defined in eq. (9)
$\dot{m}_r$	mass flow introduced radially through porous wall
r	radial coordinate
R	radius of porous cylinder
Re	radial Reynolds number ($= \frac{\rho u r}{\mu}$)
T	torque
u, v	radial and tangential velocity components
v_j	jet injection velocity
x	$(2 + \gamma Re_i)$
α	fraction of the jet velocity recovered in the vortex ($= \frac{v(\beta R)}{v_j}$)
α_o	velocity recovery with no radial mass injection
β	ratio of injection radius to tube radius
γ	fraction of total mass flow that is injection radially ($= \frac{Re_o}{Re_i}$)
μ	viscosity
τ	shear stress
τ_{w_o}	wall shear with no radial mass injection
Superscript	
'	normalized with respect to conditions at $r = \beta R$.
Subscripts	
e	refers to exit conditions
i	refers to conditions for $r < \beta R$
o	refers to conditions in outer annulus, $r > \beta R$
w	refers to conditions at $r = R$

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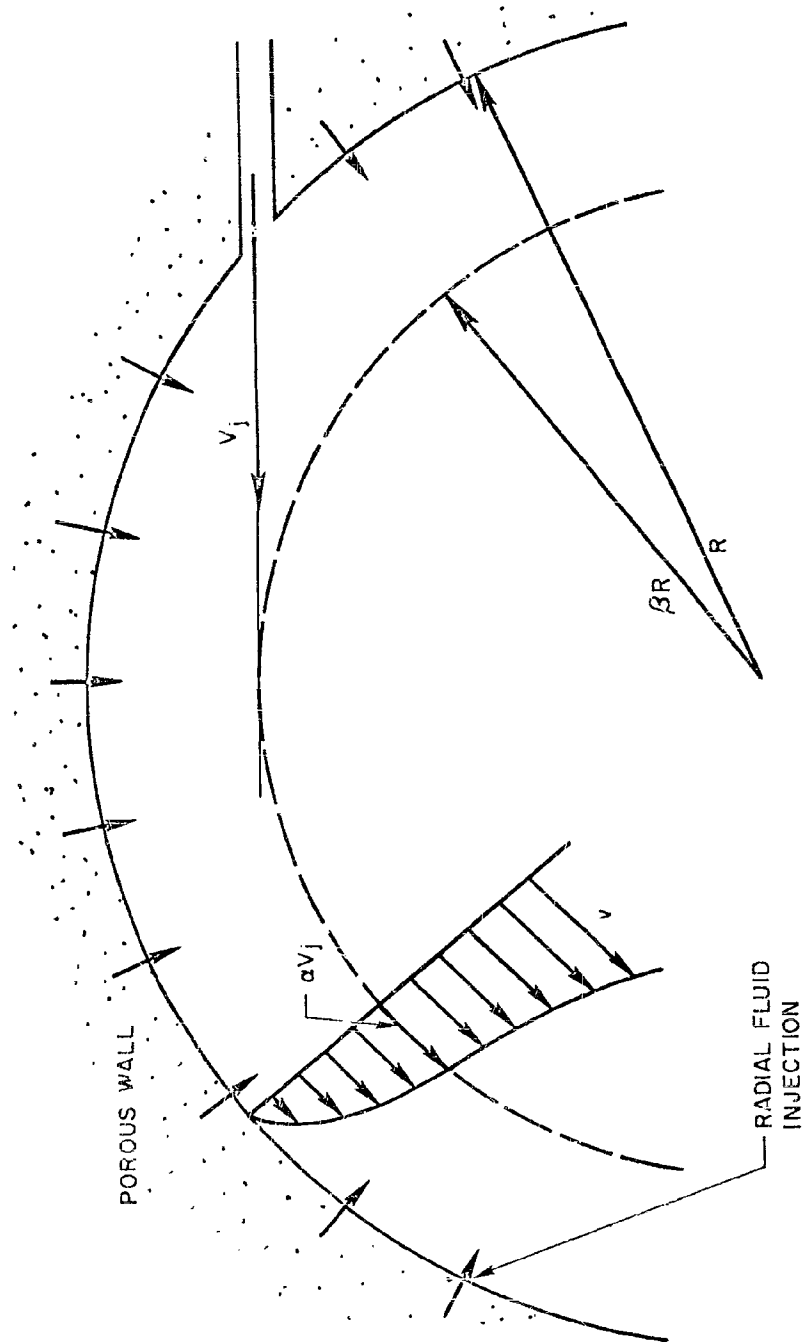


FIGURE 1, JET-DRIVEN VORTEX TUBE GEOMETRY.

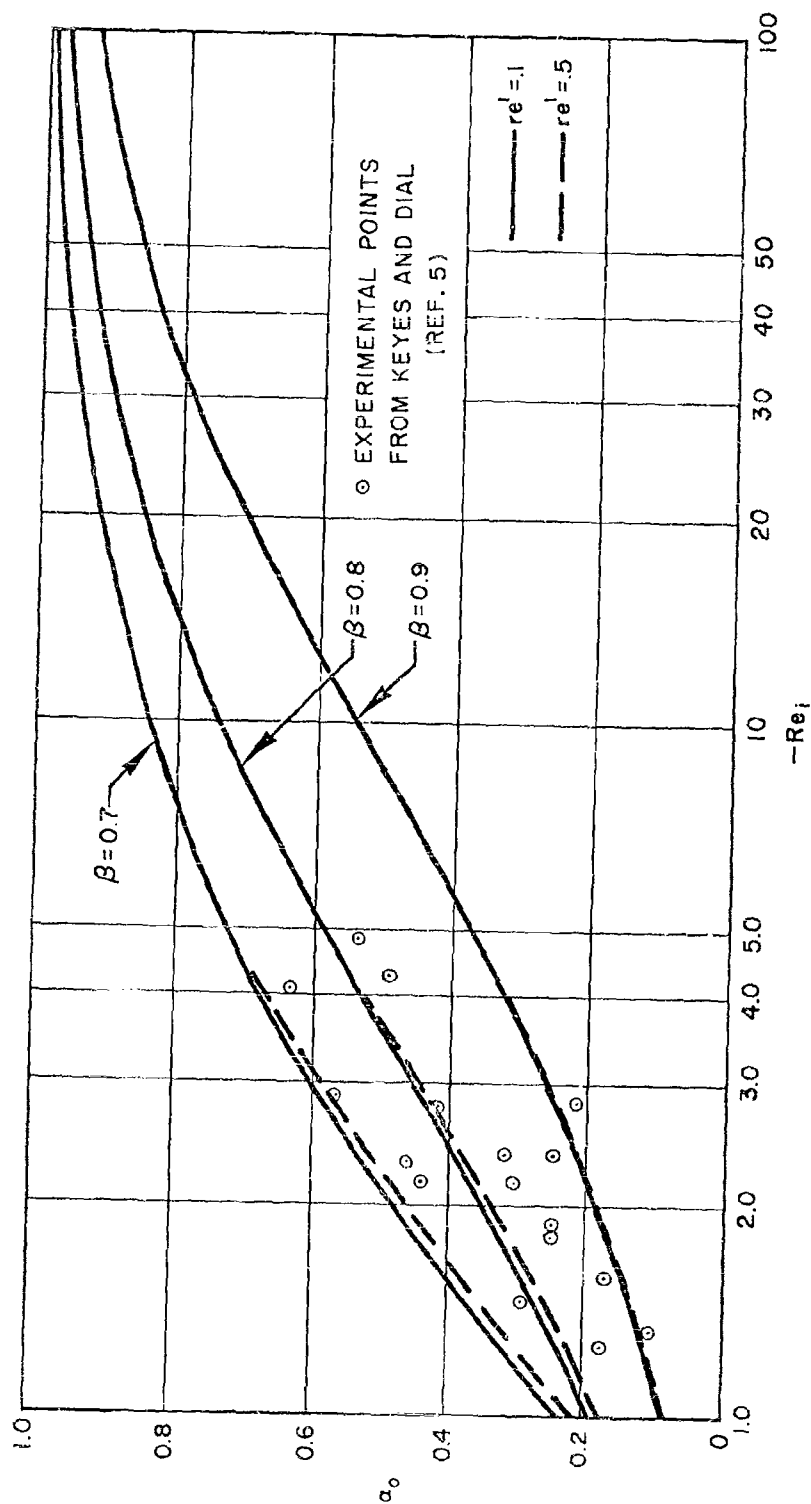


FIGURE 2. VELOCITY RECOVERY WITH NO RADIAL FLUID INJECTION.

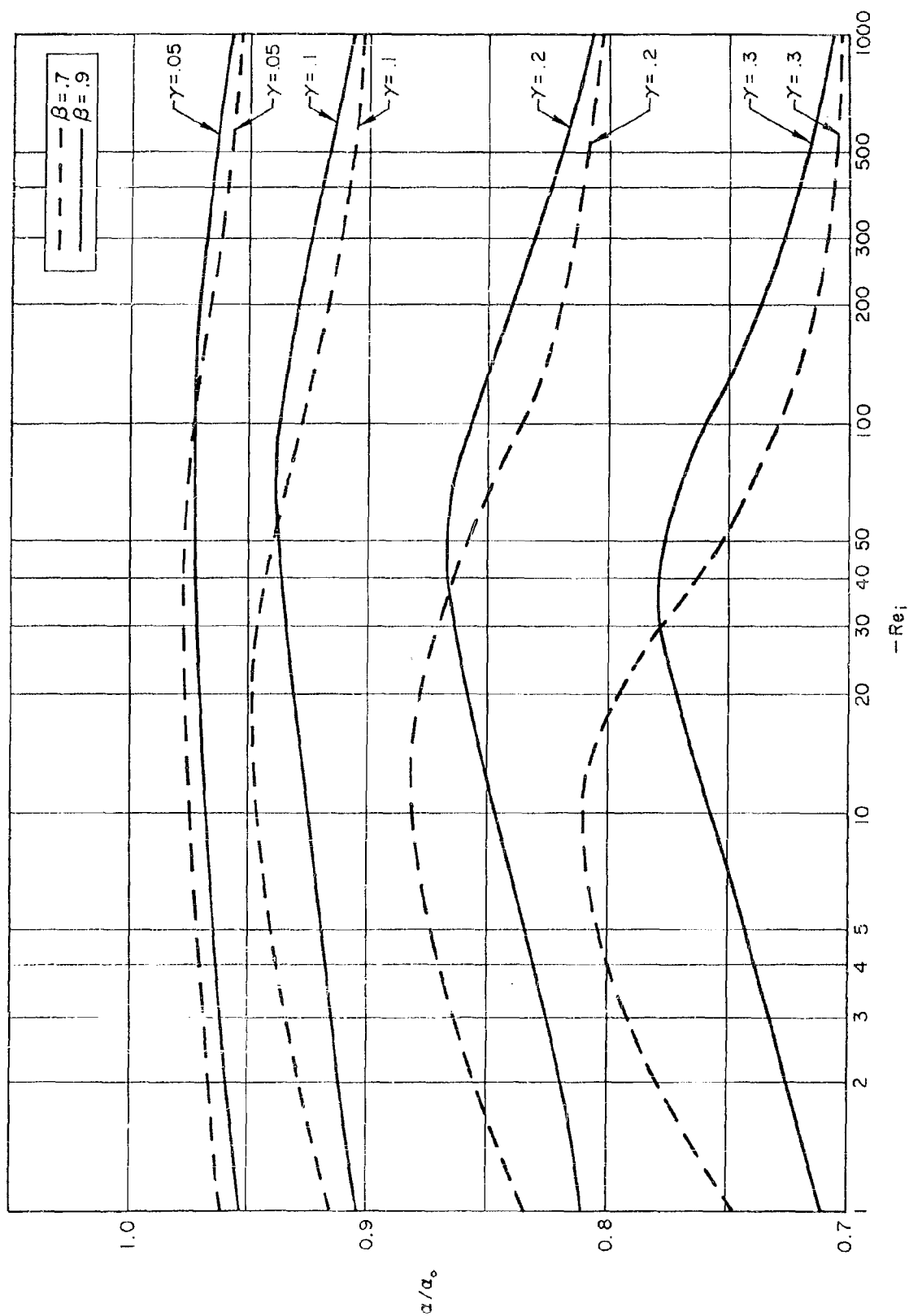


FIGURE 3. VELOCITY RECOVERY WITH RADIAL FLUID INJECTION.

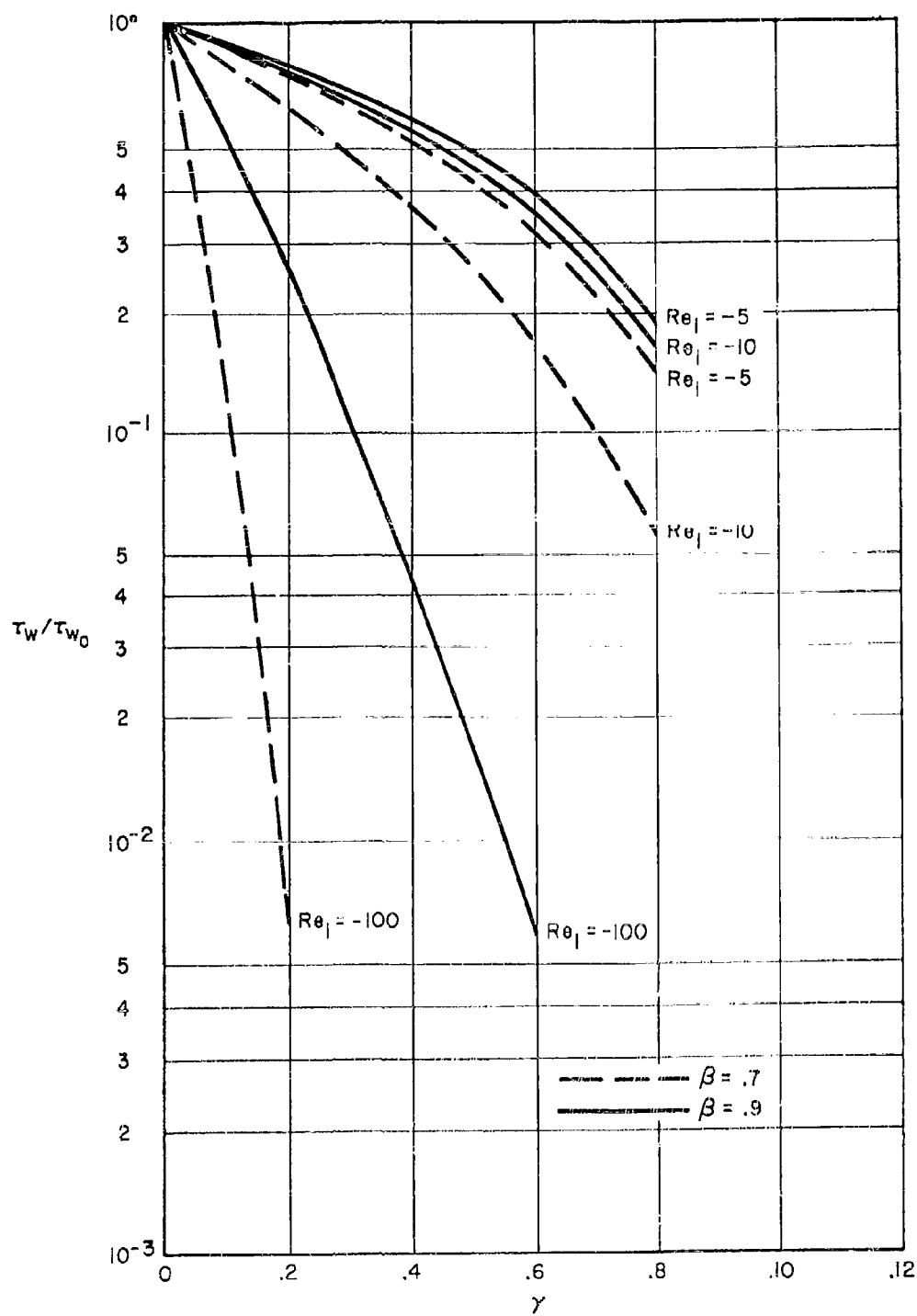


FIGURE 4. PERIPHERAL WALL SHEAR AS A FUNCTION OF RADIAL FLUID INJECTION.

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