U.S. AIR FORCE

Project RAND

RESEARCH MEMORANDUM

STRUCTURES UNDER REPEATED BLAST LOADINGS

Paul Weidlinger*

RM-2715

March 3, 1961

The RAND Corporation

SANTA MONICA • CALIFORNIA

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Assigned to ____________________

*Consultant, The RAND Corporation

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SUMMARY

The vulnerability of a target subjected to a series of repeated shots is examined. It is shown that properly designed elasto-plastic structures are capable of surviving the cumulative effects of repeated blasts if each blast is less intense than a destructive single blast. The blast intensity of a small number of repeated shots need not be significantly smaller than the intensity of a single destructive blast to permit the survival of most structures.

The probability of kill, taking into account the cumulative effects of a series of blasts, is determined and it is shown that it is not significantly higher than the probability of kill obtained by neglecting cumulative damage.

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I. INTRODUCTION

Current methods of target analysis are, of necessity, founded on numerous simplifying assumptions. One of these is the so-called "cookie-cutter method"(1) in which the lethal radius of a weapon, with respect to a target of known hardness, is defined as that range within which the peak ambient blast overpressure is sufficient to destroy the target. In this type of analysis it is assumed that (a) any target within the lethal circle is destroyed, and that (b) no target is destroyed outside this circle.

The second assumption gives one pause: one would expect a target to suffer some damage from a single near miss or a number of them, and the target to be destroyed by a sufficient number of shots only slightly outside the lethal radius.

This memorandum examines some implications of a more realistic assumption that does consider the cumulative effects of partial damage. The vulnerability of the target structure under a given number of repeated blasts will be determined, and the effect of a series of randomly spaced near misses on the probability of kill will be investigated.

These considerations lead to the following conclusions (not derivable from the "cookie-cutter" simplification) that bear on target analysis procedures:

(a) A target will not be damaged by a series of near misses if the intensity of each individual blast is sufficiently lower than the intensity of a single destructive blast. This holds if the series' intensities are not more than one-half that of the single destructive blast. This in turn implies that the ineffective shots must impact at a range larger than 1.25 times the range of the single destructive blast.

(b) A target will be damaged by a series of near misses if at least one of the weapons impacts closer than the above-defined range. It will be shown that the effect of such damage is cumulative, and a sufficient number of near misses within this range may lead to the destruction of the target. This implies that the probability of destruction of a target by a series is higher than the computations based on the "cookie-cutter" assumptions would indicate. It will be shown, however, that the difference is negligible, and the "cookie-cutter" approximation is sufficient in most cases.

II. ASSUMPTIONS AND APPLICABILITY OF ANALYSIS

To accomplish the objectives outlined in the first section, the following assumptions are made:

1. The target structure can be represented by a single-degree-of-freedom elasto-plastic system. This approximation is usually permissible provided that a single mode of the structure gives the significant contribution to its displacement. A majority of, but not all, structures of interest are of this type. Many of these structures can be characterized by a typical elasto-plastic resistance-deflection diagram of the type shown in Fig. 1, provided that strain-hardening or instability effects are not predominant.

2. The useful service limit of a structure is reached when it undergoes a predetermined amount of permanent deformation. This deformation can be represented by the summation of a number of such deformations associated with the effects of each individual blast. This assumption will be valid for all structures, where the useful service limit is reached through small permanent deformations that do not cause destruction.

3. The elasto-plastic load-deformation relationship is not significantly changed by previous loading history. This requirement is satisfied if the previous limitation is satisfied, provided that the character of the structure is such that residual stresses are of minor significance. This requirement is satisfied by many statically determined structures.

4. The effect of the blast loading on the structure is independent of the direction of the blast. This is a customary design requirement for blast-resistant structures and can be satisfied in most instances.

5. The number of blasts is small enough to exclude fatigue phenomena. This assumption is met in all instances of practical interest.

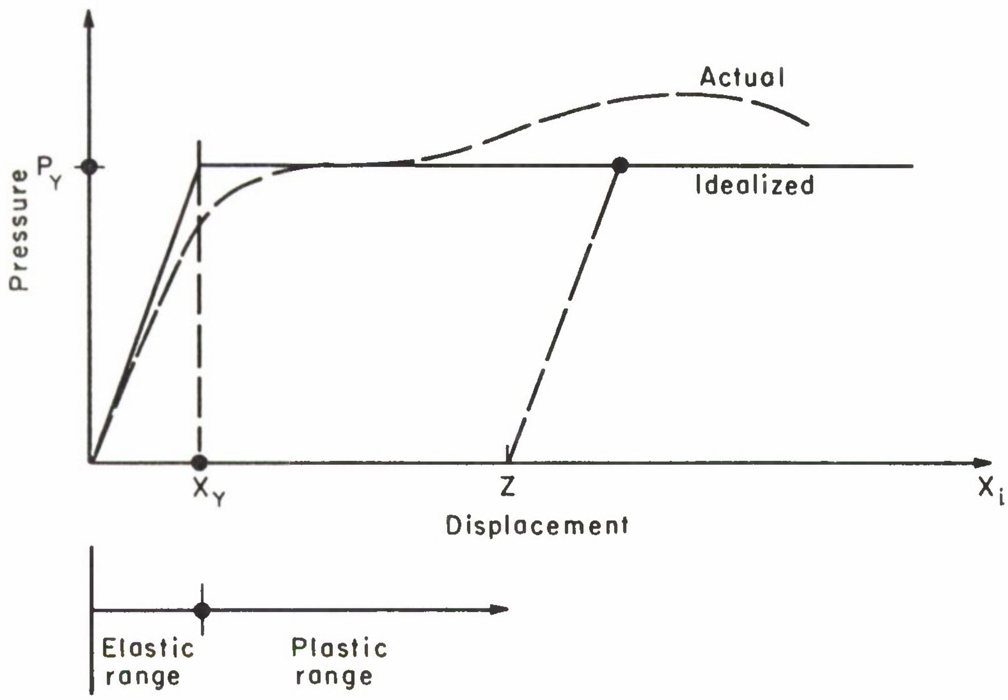

Fig. 1—Pressure versus displacement

6. The time interval between blasts is larger than the duration of the blast pressure and larger than the elasto-plastic response time of the structure. Inasmuch as the response time of most structures of average spans (about 50 ft) is shorter than the blast duration, the second part of the above requirement will be met if the first part is satisfied. The first part of the requirement implies that the blast pressure has significantly decayed before the next shock occurs.

7. The distribution of the points of impact around an assumed point target can be described by a known probability density function. This assumption is fundamental in current target analysis procedures.

Obviously, these assumptions limit the validity of the conclusions to the extent to which the model approximates the behavior of the real structure. These limitations, however, are not severe and are in accordance with current methods used in engineering analysis.

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III. PHYSICAL DAMAGE CRITERIA

It is customary to measure the hardness of a target by the maximum peak overpressure that it is able to withstand without significant impairment of its usefulness. The blast pressure intensity is a meaningful measure only if two additional data are provided--the yield of the weapon, and damage criteria.

For high-yield weapons (5 MT or more), information regarding the yield may be dispensed with because the decay time of the blast pressure is usually long compared to the response time of the usual military and civilian structures; therefore the blast pressure can be replaced by a step pulse of indefinitely long duration (Fig. 2). The dynamic response of the structure then is independent of the yield and is influenced only by the peak intensity of the blast pressure.

Structures of steel or reinforced concrete respond approximately in an elasto-plastic manner; i.e., on initial loading, up to the yield load, or pressure p_y, the displacements are linear and recoverable. Beyond the pressure p_y and the corresponding displacement x_y, displacements are independent of the load and are irrecoverable. Such an idealized pressure displacement relationship is shown in Fig. 1.

The usefulness of such structures may be impaired by large, inelastic (permanent) displacements long before total collapse or destruction occurs. Therefore, the ability to absorb work inelastically without failure is of great importance. This ability, called ductility (z) is measured by the magnitude of the allowable permanent deformations x_p in terms of the

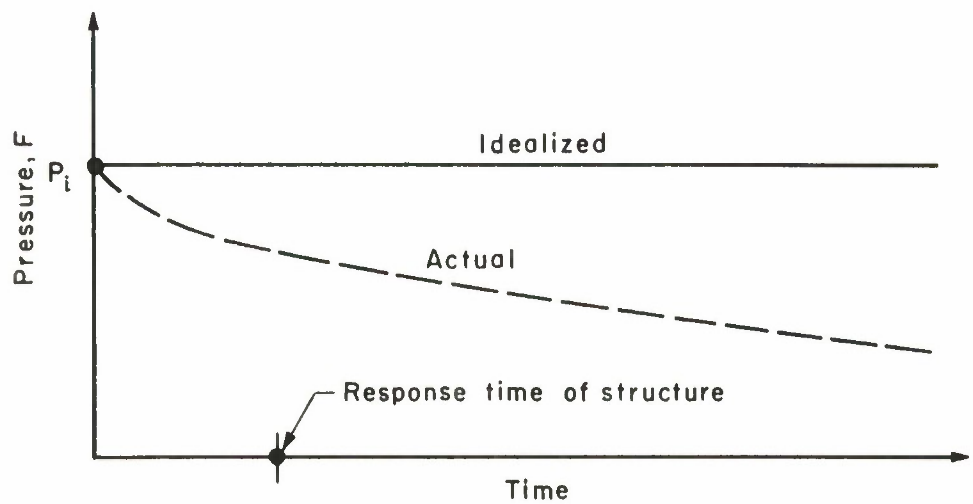

Fig. 2—Blast pressure versus time

maximum elastic deformation x_y, and therefore

$$z = \frac{x_p}{x_y} \quad * \tag{1}$$

The nature and purpose of both the structure and its contents determine the maximum allowable ductility. Therefore, physical damage criteria of specific structures can be specified adequately by a value of z.

<u>Physical Damage Criteria</u>

$z = 0$	No damage, since only elastic, i.e., recoverable deformations occur.
$z \leq 10$	Useful service limit of structures where jamming of movable parts (doors, elevators, etc.) may impair operation.
$z \leq 30$ to 50	Useful service limit where lesser requirements do not apply, and provided that structural integrity can be maintained.
$z > 50$	Serious damage or collapse.

These criteria are of necessity empirical and approximate. Fortunately, as will be shown later, vulnerability and probability of kill estimates are relatively insensitive to variations of the value of the allowable ductility, for $z > 3$. The selection of a particular value of z depends on specific details of the structure. It also depends on whether this value is used in computations involving the design of a blast-resistant structure or in target analysis. In the latter case a higher value of z is selected to guarantee the serious impairment of the target at that value; in the former case a low value is used to insure the continued usefulness of the structure. The ductility value that can be assigned to a specific structure is also limited by

*The ductility is sometimes defined by

$$\frac{x_p + x_y}{x_y} = z + 1.$$

the physical characteristics of the material. Steel structures under certain circumstances are capable of preserving their structural integrity under large permanent deformations at values of $z \leq 50$. The ductility of reinforced concrete members is greatly affected by the amount of steel reinforcement used and other design details. Heavily reinforced flexural members may fail at $z < 3$, while under-reinforced sections may reach ductility factors of about 30 or even higher. This assumes that the sections are properly reinforced to preclude shear and diagonal tension failures. Reinforced concrete compression members behave in a brittle manner and are designed for z equal to about 1.3.

IV. DAMAGE MECHANISM

Consider an elasto-plastic structure with a static yield capacity p_y, that is subjected to step pressure pulses (Fig. 2) of varying intensities $p_i (i = 1, 2, \ldots n)$, measured in units of p_y. All pressure applications produce permanent deformations, $x_i \geq 0$, which are therefore additive.

The useful service limit is reached when (from Eq. 1)

$$\sum_{1}^{n} x_i = z \tag{2}$$

where the non-dimensional permanent displacement, x_i, is measured in units of x_y, and z is selected according to the criteria described previously.

Reference 2 shows that by equating the internal strain energy with the work done by a step pressure pulse on an elasto-plastic, single-degree-of-freedom structure, one can obtain the simple relationship

$$p_i = \frac{1 + 2x_i}{2(1 + x_i)} \tag{3}$$

A series consisting of n blasts will effectively destroy the target if

$$\sum_{1}^{n} x_i \geq z$$

For this to occur it is sufficient that there is at least one permanent deformation, x_k, such that

$$x_k = \infty$$

By Eq. 3, this implies that there must be at least one destructive blast, p_k, such that

$$p_k \geq 1 \tag{4}$$

On the other hand, a series of n blasts will leave a target undamaged if

$$\sum_{1}^{n} x_i = 0$$

For this to occur, it is necessary that all

$$x_i (i = 1, 2, \ldots n) = 0$$

and by Eq. 3, this implies that all blasts

$$p_i (i = 1, 2, \ldots n) \leq 1/2 \qquad (5)$$

Since Eq. 5 remains valid for $n \rightarrow \infty$, we conclude that a target remains undamaged under a very large number of shots if none of the blasts has more than half the intensity of a single destructive shot. The intensity of the single destructive shot must be at least equal to the static yield capacity of the structure.

The effects of a series can be further clarified by considering the improbable case of a series consisting of n blasts of equal intensity, p_n, resulting in identical permanent deformations, x. If the cumulative effect of the permanent deformation is to be

$$nx = z \qquad (6)$$

the required intensity, p_n, by Eq. 3 is

$$p_n = 1 - \frac{n}{2(n + z)} \qquad (7)$$

which in case of a single blast, p_1 (Eq. 7) reduces to

$$p_1 = 1 - \frac{1}{2(1 + z)} \qquad (7a)$$

Equation 7 can be used to determine the intensities that are required for various numbers of repeated identical shots to produce a total permanent deformation, z. This relationship is shown on Fig. 3, which also shows that for higher values of z, the required intensities decrease rather slowly with increasing numbers of shots.

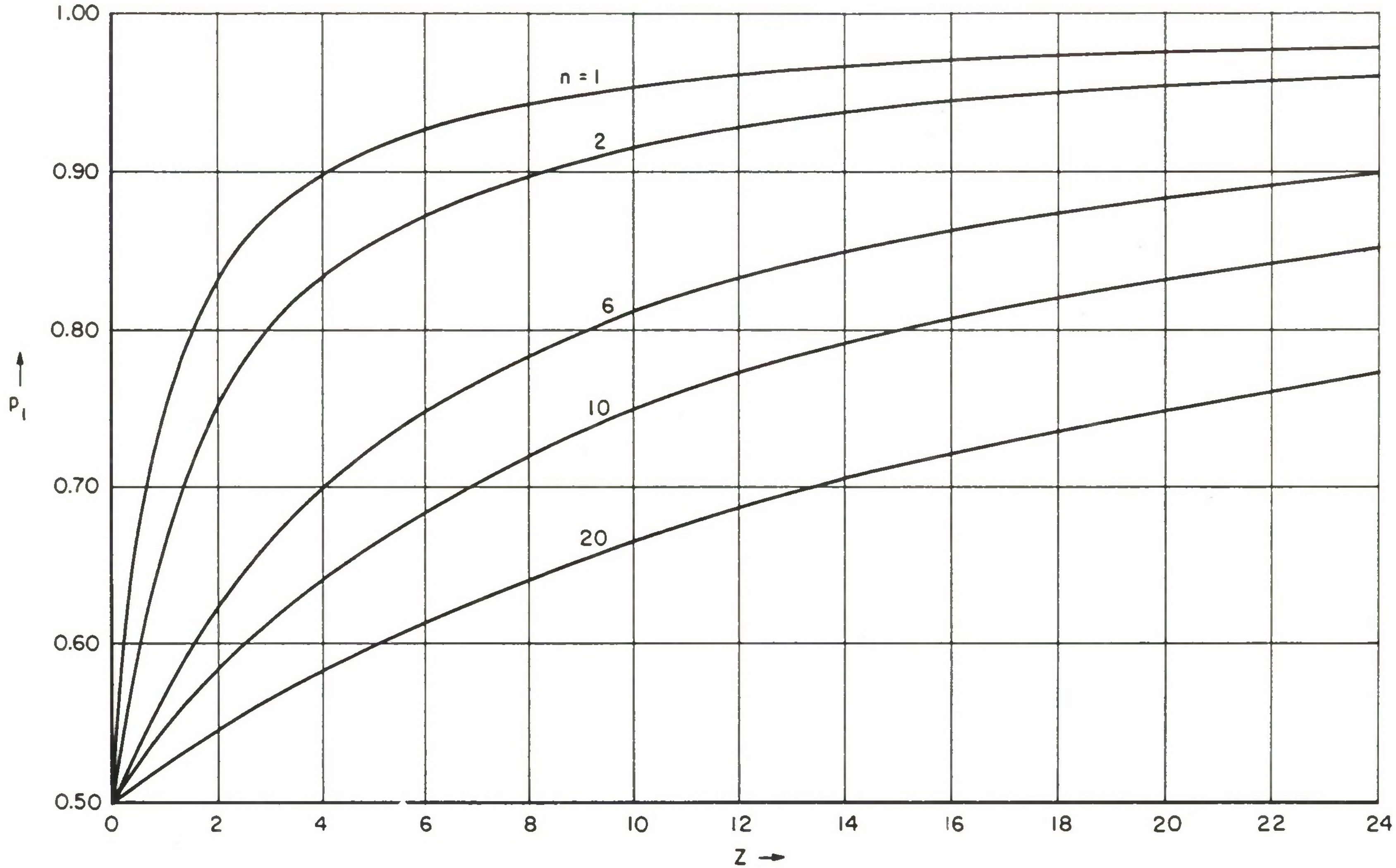

Fig. 3—Blast intensities, p_1 (for n repeated equal blasts), versus allowable ductility

V. THE LETHAL RADIUS

At a target, the peak blast intensity of a nuclear weapon is approximated by the following formula, in which the elements, a strong shock, a point source, and solution of a spherical blast wave[3] are considered:

$$r = c p_o^{-1/3} \tag{8}$$

where r is the distance from the target; p_o, the peak ambient blast overpressure; and c, a constant that depends on the weapon yield.*

Introducing the non-dimensional pressure intensity as

$$p_1 = \frac{p_o}{p_y}$$

into Eq. 7a and combining with Eq. 8, one can find the lethal radius, R_z (see Fig. 4), associated with the ductility z by

$$R_z = c p_y^{-1/3} \left(\frac{2 + 2z}{1 + 2z}\right)^{1/3} \tag{9}$$

One also can define a radius of vulnerability, R_o, beyond which a target remains unaffected by impact of weapons of given yield. This is obtained by letting z = 0 in Eq. 9, so that

$$R_o = 2^{1/3} c p_y^{-1/3} \tag{10}$$

Similarly, one defines the radius of destruction, R_∞, such that a target is destroyed by a weapon within this range. Letting $z \rightarrow \infty$ we obtain

$$R_\infty = c p_y^{-1/3} \tag{11}$$

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*This approximation is valid at high pressure levels. At lower pressures better agreement is obtained by introducing an additive constant to the right side of Eq. 8. An empirical fit for low pressure ranges is given in Appendix II of Ref. 1.

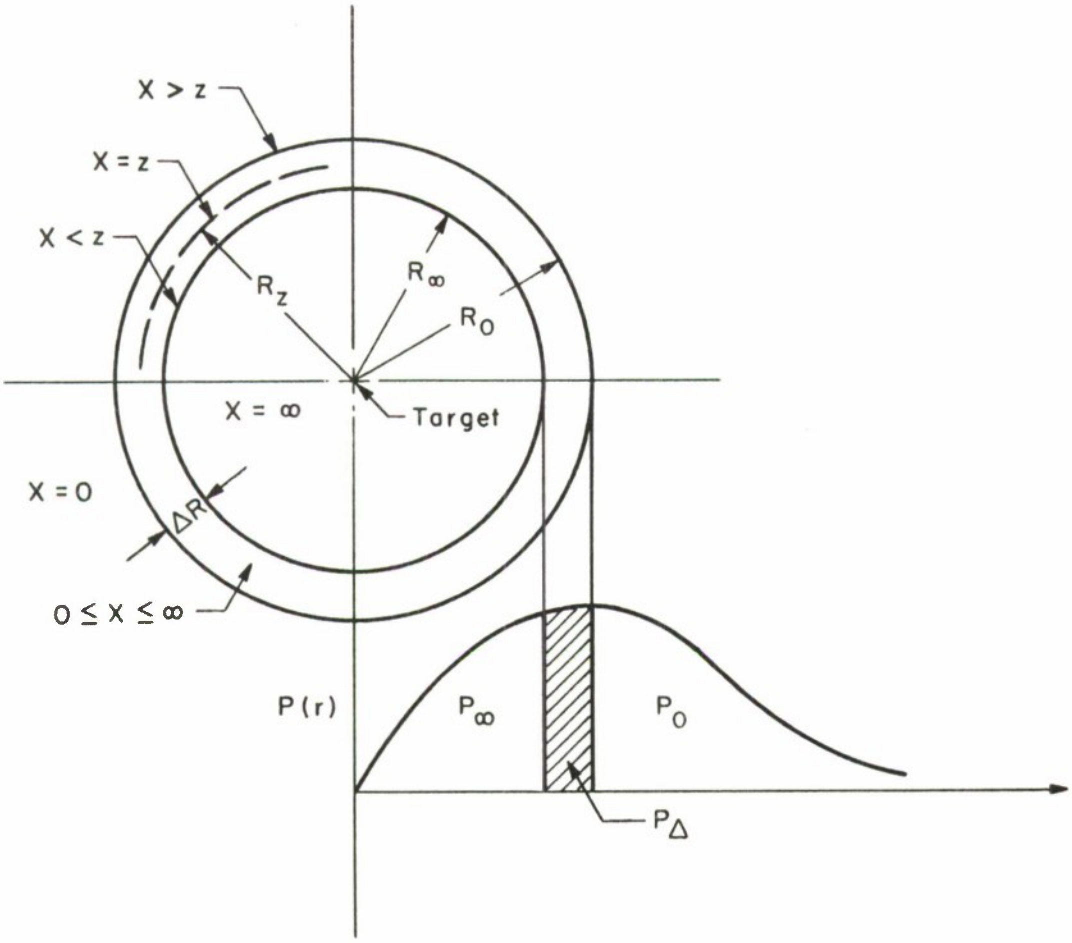

Fig. 4 — Relationships of radius of destruction, R_∞, lethal radius, R_z, and radius of vulnerability, R_0

By comparing Eqs. 10 and 11, it is noted that if a target is destroyed at the range R_∞, it remains intact at the range

$$R_o = 2^{1/3} R_\infty$$

Inspection of Eq. 9 also shows that for $z > 10$

$$R_z - R_\infty \leq 0.015 R_\infty$$

The cumulative effect of permanent deformations can be felt only within a circular annulus of width

$$\Delta R = R_o - R_\infty = 0.26 R_\infty \tag{12}$$

where the relationship between the range, $R_\infty \leq r_i \leq R_o$, and the permanent deformation, $0 \leq x_i \leq \infty$, is given by

$$r_i(x_i) = R_o \phi(x_i) \tag{13}$$

where

$$\phi(x_i) = \left[\frac{1 + x_i}{1 + 2x_i}\right]^{1/3} \tag{13a}$$

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VI. PROBABILITY OF KILL

Let the random distribution of the points of impact around the target be defined by a probability density function $p(r)$. The probability that a permanent deformation (caused by a blast occurring in the interval r and $r + dr$) is in the interval x and $x + dx$ is given by the probability density function $\gamma_1(x)$ such that

$$\gamma_1(x)dx = p\left[r(x)\right]dr \tag{14}$$

where the random variable r is a function of x by Eq. 13. The domain of the function $\gamma_1(x)$ is $0 < x < \infty$, and it corresponds to the domain $R_\infty < r < R_o$ of the function $p(r)$. Consequently, the probabilities

$$P_\infty = P(r < R_\infty) = \int_0^{R_\infty} p(r)dr \tag{15}$$

and

$$P_o = P(r > R_o) = \int_{R_o}^{\infty} p(r)dr \tag{16}$$

must be added as discrete values at $x = \infty$ and $x = 0$ to the function $\gamma_1(x)$ to obtain a complete probability density function $g_1(x)$.

The function $g_1(x)$ corresponding to the domain $0 \leq r \leq \infty$ is therefore expressed as

$$g_1(x) = P_o\delta(x) + \gamma_1(x) + P_\infty\,\delta\,(x - \infty) \tag{17}$$

where δ is the Dirac delta function[4] defined by

$$\int_{-\infty}^{+\infty}\delta(x)dx = 1 \text{ and } \delta(x) = 0 \text{ for } x \neq 0$$

and the function $\gamma_1(x)$ is an incomplete probability function such that

$$\int_0^{\infty} \gamma_1(x)dx = \int_{R_o}^{R_\infty} p(r)dr < 1 = 1 - (P_o + P_\infty) \tag{18}$$

by Eqs. 14, 15, and 16.

The target will be rendered useless by a series of n weapons if the condition

$$\sum_1^n x_i \geq z \tag{19}$$

is fulfilled. The probability of the occurrence of this event is given by

$$P\left(\sum_1^n x_i \geq z\right) = 1 - \int_0^z g_n(\xi)d\xi \tag{20}$$

where

$$g_k(z) = \int_0^z g_1(\xi)g_{k-1}(z - \xi)d\xi \tag{21}$$

is the k^{th} convolution of the function $g_1(\xi)$, and in which $g_o(\xi) = \delta(\xi)$.*

We also denote the k^{th} convolution of the function $\gamma_1(x)$ by

$$\gamma_k(z) = \int_0^z \gamma_1(\xi)\gamma_{k-1}(z - \xi)d\xi \tag{22}$$

and define for completeness

$$\gamma_o(\xi) = \delta(\xi) \tag{23}$$

Substituting successively Eq. 22 into 17 into 21, and remembering that

$$\int_0^z \delta(\xi)\delta(z - \xi)d\xi = \delta(z) \tag{24}$$

the convolution integral of Eq. 21 becomes

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*Eq. 21 may be established by induction.[5]

$$g_k(z) = \sum_{k=0}^{k=n} C_{n,k} P_o^k \gamma_{n-k}(z) \tag{25}$$

where $C_{n,k}$ is the k^{th} binomial coefficient of an n^{th} power expansion. Introducing Eq. 25 into Eq. 20, the probability of kill by a series consisting of k blasts, taking into account the cumulative effects of permanent deformations, is given by

$$P\left(\sum_1^n x_i \geq z\right) = 1 - \sum_{k=0}^{k=n} C_{n,k} P_o^k \int_o^z \gamma_{n-k}(\xi) d\xi \tag{26}$$

For comparison we also find the probability of kill when the cumulative effect on the permanent deformations is not taken into account. This condition of kill requires that there is at least one x_k such that

$$x_k \geq z$$

which requirement is also equivalent to that of the cookie-cutter condition, namely that

$$r_k \leq R_z$$

This probability is given by

$$P_n\left(r_k \leq R_z\right) = 1 - \left(\int_{R_z}^{\infty} p(r) dr\right)^n \tag{27}$$

But since

$$\int_{R_z}^{\infty} p(r) dr = P_o + \int_o^z \gamma_1(\xi) d\xi \tag{28}$$

Eq. 27 can be written as

$$P_n\left(r_k \leq R_z\right) = 1 - \sum_{k=0}^{k=n} C_{n,k} P_o^k \left(\int_o^z \gamma_1(\xi) d\xi\right)^{n-k} \tag{29}$$

and we note that Eq. 26 is of a form similar to Eq. 29 except that the $(n - k)^{th}$ power of the first convolution integral in the latter is replaced by the $(n - k)^{th}$ convolution integral in the former. Both equations are identical for the case, $n = 1$, and also for all values of n, if $z = \infty$ or $z = 0$.

VII. NUMERICAL VALUES

We consider now the customary assumption that the points of impact around the target are defined by the circular Gaussian distribution

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \tag{30}$$

where σ is the standard derivation. Using Eq. 15 and introducing the parameter

$$\alpha = \frac{1}{2}\left(\frac{R_o}{\sigma}\right)^2 \tag{31}$$

where $R_o = 2^{1/3} R_\infty$, one obtains

$$P_\infty = 1 - e^{-\frac{\alpha}{2^{2/3}}} \tag{32}$$

In a similar fashion, the use of Eq. 16 and the parameter α results in

$$P_o = e^{-\alpha} \tag{33}$$

As seen from Eq. 18, the probability P_Δ that a weapon impacts within the interval $R_o - R_\infty$ is therefore given by

$$P_\Delta = e^{-\frac{\alpha}{2^{2/3}}} - e^{-\alpha} \tag{34}$$

Figure 5 shows P_Δ versus α, and we note that

$$P_{\Delta_{max}} = 0.1685 \text{ at } \alpha = 1.249$$

and that $P_\Delta < .10$ outside the interval

$$1/2 < \alpha < 3$$

Inserting Eqs. 13, 30, and 31 into Eq. 14, the incomplete probability density function is given by

$$\gamma_1(x) \equiv \frac{d}{dx} e^{-\alpha\phi^2(x)} \tag{35}$$

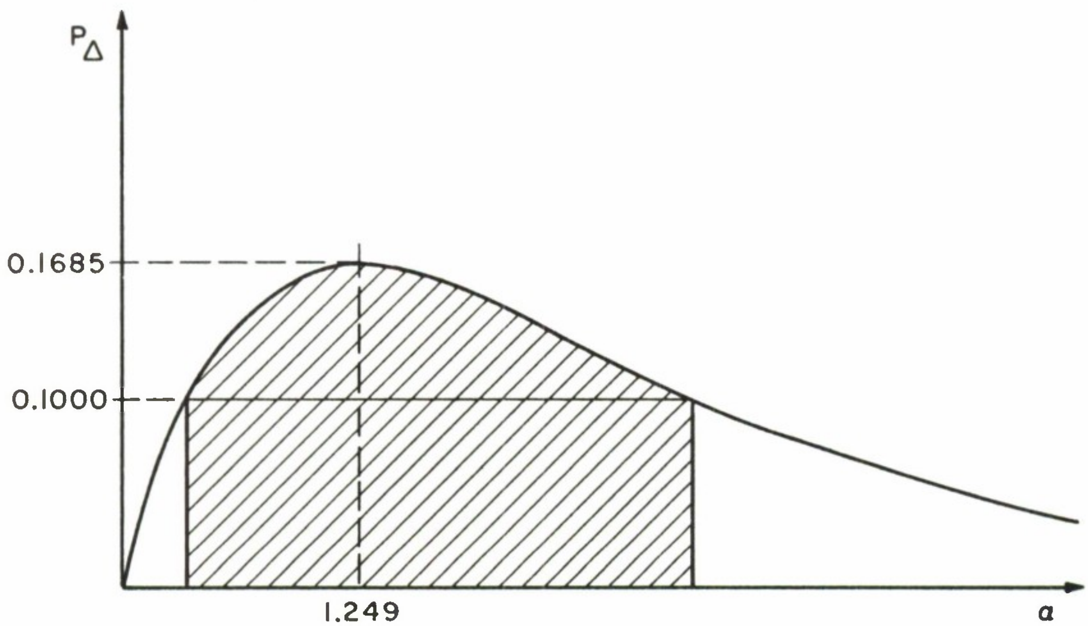

Fig. 5 — P_Δ versus a

where $\phi(x)$ is from Eq. 13a. Equation 20 can now be evaluated by iterated numerical integrations and the values obtained are compared with those of Eq. 29, which is obtainable analytically in its original form given by Eq. 27.

Table 1 gives a comparison of probabilities computed with the parameters $\alpha = 1(R_o = \sqrt{2\sigma})$ and $z = 2.0(R_z = 0.84R_o$, by Eq. 13).

Table 1

COMPARISON OF PROBABILITIES WITH AND WITHOUT THE CONSIDERATION OF CUMULATIVE DAMAGE

n (Number of weapons)	$P\left(\sum_{1}^{n} x_i \geq 2\right)$ (Eq. 26)	$P_n(r_k < 0.84R_o)$ (Eq. 29)
1	.5051	.5051
2	.7668	.7550
3	.8932	.8787
4	.9519	.9400
5	.9785	.9703
...	...	...
∞	1.0000	1.0000

Table 1 indicates that the difference in the probabilities computed by the two approaches is negligible. Since the values are identical for $n = 1$ and $n = \infty$, the maximum difference occurs for a specific value of n, which in this case is $n = 3$. Similarly, both probabilities are equal for $z = 0$ and $z = \infty$; consequently, the maximum difference is expected at a particular value of z. Probabilities computed by Eqs. 26 and 29 for other values of α and z show similar small differences due to accumulated damage.

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VIII. CONCLUSIONS

It has been shown that structures designed to resist a single blast of specified peak intensity are capable of resisting a larger number of blasts, provided that the peak intensity of each blast is less than the design assumption for a single blast.

If such blasts of lower than design intensity are considered as near misses, the probability of kill due to a series of n shots can be computed. It is found, however, that the probability computed with these assumptions does not give significantly different results from computations based on the customary cookie-cutter approximations.

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in brief

This study examines the vulnerability of a target subjected to a series of blasts. It is shown that properly designed elasto-plastic structures are capable of surviving the cumulative effects of repeated blasts if each blast is less intense than a destructive single blast. The probability of kill, taking into account the cumulative effects from a series of blasts, is determined and is not significantly higher than the probability of kill obtained by neglecting cumulative damage.

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