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THE USE OF PULSE CODING TO DISCRIMINATE AGAINST CLUTTER

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NOTE

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THE USE OF PULSE CODING TO DISCRIMINATE
AGAINST CLUTTER*

Roger Manasse**

ABSTRACT

This paper considers the use of pulse coding (or pulse compression) in radar to obtain improved detection of targets in clutter. The effectiveness of this technique depends on the differing spatial characteristics of the target and clutter in contrast with the usual MTI which depends on the differing time-varying properties. With the assumptions of a simple clutter model and an appropriately optimized receiver, and with the aid of known results in detection theory, an expression is derived for the single-pulse detection capability of a radar operating in the presence of both clutter and additive white receiver noise. From the expression it is seen that detection performance is simply related to the spectrum of the transmitted signal and, generally speaking, improves as the bandwidth of the transmitted signal is increased. Results for clutter noise only or receiver noise only appear as special cases. The implications of these results for pulse coding (or pulse compression) in radar are discussed.

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I. Introduction

Before proceeding with a theoretical discussion of the applicability of pulse coding to obtaining improved radar detection of targets in clutter, let us consider briefly the connection between the terms "pulse coding" and "pulse compression."

For a radar receiver operating in the presence of additive white gaussian noise, modern statistical detection theory indicates that optimum receiver performance can be obtained with the aid of a linear filter which is matched to the expected radar return, a filter which has a unit impulse response which is simply a time inverted replica of the expected radar return.^{1,2,3} The signal $s(t)$ and noise are fed into a matched filter with unit impulse response $h(t) = s(T - t)$.

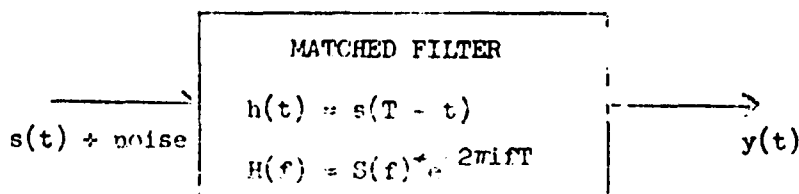


Figure 1

T is an arbitrary time delay factor chosen so that $h(t)$ satisfies the realizability condition

$$h(t) = 0 \quad \text{for} \quad t < 0$$

In the frequency domain the filter response is given by the expression shown in Fig. 1, where $H(f)$ and $S(f)$ are the Fourier transforms of $h(t)$ and $s(t)$, respectively. Then the output of the matched filter $y(t)$ is equal to the convolution of $s(t)$ with $h(t)$ plus a noise term.

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} s(z)h(t - z)dz + \text{noise} = \int_{-\infty}^{\infty} s(z)s(T - t + z)dz + \text{noise} \\
 &= \varphi(T - t) + \text{noise}
 \end{aligned}$$

where $\varphi(\tau)$ is the autocorrelation function of $s(t)$. Detection is performed at approximately time $t = T$ where the signal autocorrelation function at the output of the filter reaches its peak.

Under the assumption that the receiving filter always remains matched to a delayed replica of the transmitted signal, the signal pulse at the output of the matched filter will always be the autocorrelation function of the transmitted signal. If, for various reasons, we desire to have (in some sense) an autocorrelation function which is short compared to the transmitted pulse length, it will be necessary to code, i.e. amplitude and phase modulate, the transmitted pulse in order to increase its bandwidth appreciably beyond that for the uncoded pulse.* Pulse compression, a term taken to refer to a process whereby a relatively long low amplitude pulse is converted to a relatively short high amplitude pulse, is brought about automatically by the matched filter when the transmitted pulse has been coded. Thus, for our purposes, the terms "pulse compression" and "pulse coding" are synonymous and the terms can be used interchangeably.

* Recalling that the autocorrelation function is the Fourier transform of the signal energy spectrum, we see that the requirements on the shape of the output pulse can be expressed in terms of requirements on the shape of the signal energy spectrum.

The possibility of improving a radar's ability to discriminate against clutter through the use of pulse compression, which is the subject of this paper, was suggested to the author by Dr. Robert F. Naka of the M.I.T. Lincoln Laboratory.

II. Analysis of the Problem

Let us now consider a conventional pulsed radar in which the doppler shift on a single radar pulse is negligible: That is, the signal pulse reflected from a point target is simply a delayed and attenuated version of the transmitted pulse. On the sweep return from a single pulse, then, we completely ignore the time-varying properties of both the target and the clutter. Following the procedure used by George,⁴ the space surrounding the radar, appropriately weighted with the antenna beam pattern, may be

considered as a linear filter whose transfer characteristic is characterized by a unit impulse response function which is denoted $W(t)$.

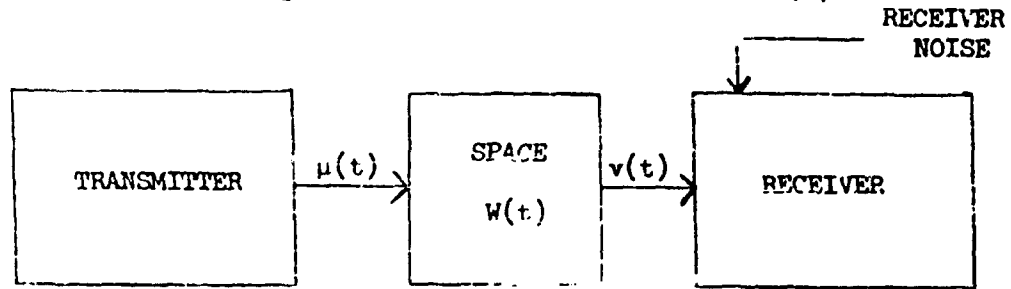


Figure 2

The transmitter generates the transmitted pulse, denoted $\mu(t)$, which is then reflected from objects in the space surrounding the radar. This process is equivalent to passing $\mu(t)$ through the filter $W(t)$ to produce the reflected waveform $v(t)$ which presents itself to the receiver along with receiver noise, where $v(t)$ is given by

$$v(t) = \int_{-\infty}^{\infty} \mu(z)W(t - z)dz$$

We can imagine, without loss of generality, that $\mu(t)$ is generated in the transmitter by sending a spike or δ -function into a filter with impulse response $\mu(t)$, and the above block diagram is equivalent to the following.

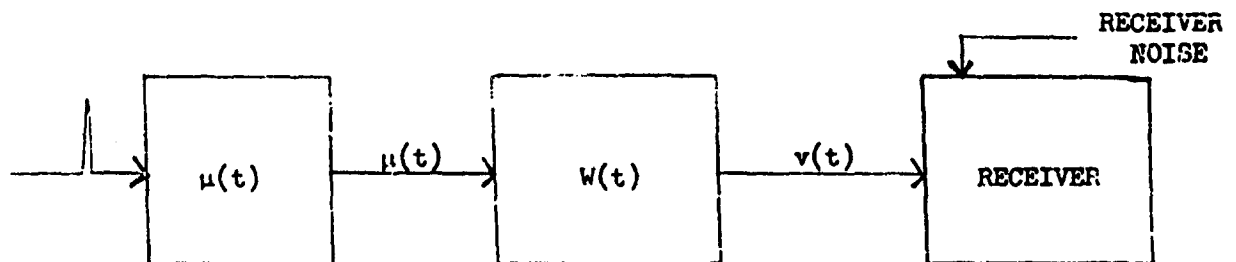


Figure 3

Because the frequency response function of the first two filters taken in series is simply the product of the frequency response functions for the separate filters, these two filters may be interchanged without affecting $v(t)$.

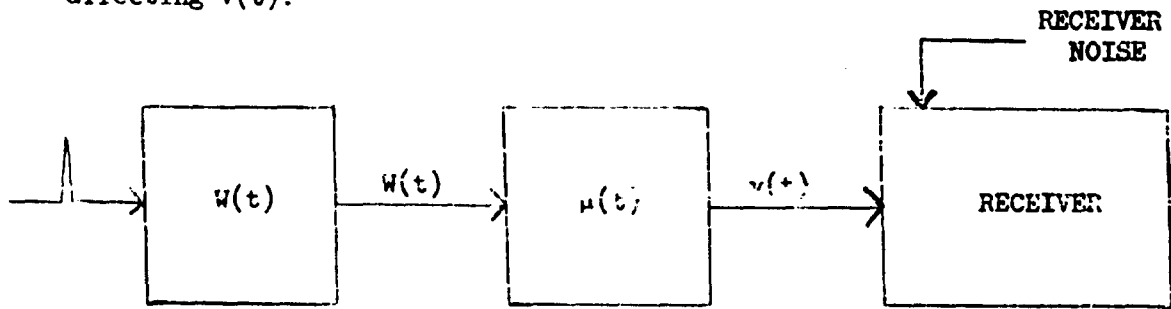


Figure 4

In order to proceed further we must assume a model for the clutter return. We assume for the purpose of analysis that the clutter consists of a large randomly distributed ensemble of very small independent point scatterers. That is,

$$W(t) = \underbrace{A\delta(t - t_s)}_{\text{Signal term}} + \underbrace{\sum_k a_k \delta(t - t_k)}_{\text{Clutter noise term}}$$

The first term is the response of the point target to a transmitted δ -function. The amplitude A is finite, corresponding to the fact that the cross section is finite, while the time delay t_s measures the range of the target. The second term represents the clutter response to a δ -function and is a sum of appropriately amplitude weighted and delayed δ -functions corresponding to the point scatterers of the clutter model.

The a_k 's and t_k 's are taken to be independent random variables. Consider an interval of range, sufficiently small so that the inverse fourth power of range and other geometrical factors can be ignored across the interval. Then the t_k 's are uniformly distributed across the interval. Letting our model for clutter approach the limit in which the distribution of t_k 's over the interval is infinitely dense and the a_k 's are infinitesimal, we obtain a process which is, mathematically, exactly analogous to the shot effect.⁵ Thus, the clutter noise at the input to the filter $\mu(t)$ in Fig. 4 is equivalent to white gaussian noise.

Then $v(t)$, which is the output of the $\mu(t)$ filter and is the message presented to the radar receiver, is given by

$$v(t) = \underbrace{A\mu(t - t_s)}_{\text{Signal}} + \underbrace{n_c(t)}_{\text{Clutter noise}}$$

where $n_c(t)$ is the result of passing white gaussian clutter noise through the $\mu(t)$ filter. Because the $\mu(t)$ filter has a spectral transfer function given by $|U(f)|^2$, $n_c(t)$ is simply colored gaussian noise with power spectrum $N_c(f)$ which is proportional to the energy spectrum of the transmitted signal.* That is,

$$N_c(f) = c|U(f)|^2$$

where c is a proportionality constant which depends on the intensity of the clutter. Adding the colored noise from the clutter to the additive

* This fact has been noted by Lawson and Uhlenbeck, reference 6.

white gaussian receiver noise with noise power per cycle $N_0/2$, and noting that these two noises are independent, the result is colored gaussian noise with power spectrum $N(f)$.

$$N(f) = \frac{N_0}{2} + c|U(f)|^2$$

The problem presented to the receiver is that of detecting the returned radar signal $A_u(t - t_s)$ in the presence of colored gaussian noise. Dwork⁷ and later George⁴ have independently extended signal detectability theory to include the case of a known signal in colored gaussian noise. If $S(f)$ is the voltage spectrum of the signal and $N(f)$ is the power spectrum of the noise, they have shown that the transfer function of the optimum filter is given by

$$\frac{S^*(f)e^{-2\pi i f T}}{N(f)}$$

where T is a conveniently chosen time delay. Note that in the white noise case $N(f)$ is a constant and the above expression becomes the transfer function of the usual matched filter. The above filter is really the generalization of the matched filter to the colored noise case. The peak signal-to-noise power ratio obtained with the above filter is given by

$$(S/N)_{\text{opt}} = \int_{-\infty}^{\infty} \frac{|S(f)|^2}{N(f)} df$$

For our problem

$$S(f) = \int_{-\infty}^{\infty} A_u(t - t_s) e^{-2\pi i f t} dt = AU(f) e^{-2\pi i f t_s}$$

and

$$|S(f)|^2 = A^2 |U(f)|^2$$

Also, recalling that $N(f) = \frac{1}{2} N_0 + c|U(f)|^2$ and substituting in the above expression, we have

$$(S/N)_{\text{opt}} = A^2 \int_{-\infty}^{\infty} \frac{|U(f)|^2}{\frac{1}{2} N_0 + c|U(f)|^2} df$$

The quantity $(S/N)_{\text{opt}}$ is a good measure of the single pulse detection capability which is available from the optimum receiver.* Therefore, in order to maximize the detection capability we must choose the parameters of the radar system to maximize $(S/N)_{\text{opt}}$. The constants A and c are determined by the nature of the target, the clutter and the geometrical parameters of the system. In this discussion we assume that these parameters are not at our disposal.

*For an exactly known signal, P_D (probability of detection) versus P_F (probability of false alarm) curves given in reference 3 can be used if one replaces $2E/N_0$ by $(S/N)_{\text{opt}}$. In practice, the returned radar signal is not exactly known because of unknown parameters such as time delay. For a given P_D these unknown parameters have the effect of increasing the false alarm rate by an amount which may depend on the shape of transmitted signal waveform. However, for the level at which most radars operate the increase in false alarm rate introduced by these unknown parameters does not seriously degrade the signal detectability.

It is of great interest to determine the dependence of $(S/N)_{\text{opt}}$ on $U(f)$, that is, the dependence of the signal detectability on the transmitted signal waveform. There are several conclusions which are immediately apparent from the expression for $(S/N)_{\text{opt}}$.

1. When no clutter is present, that is when $c = 0$, $(S/N)_{\text{opt}}$ depends only on the ratio of the signal energy to noise power per cycle at the receiver. Therefore we have the well-known result that the detection capability depends only on the energy of the transmitted pulse and not on its shape.
2. In the limit where internal noise is negligible, that is where $N_0 = 0$ (or where the clutter return overpowers the receiver noise), the integrand is a constant and $(S/N)_{\text{opt}}$ depends only on the effective system bandwidth. This fact has been pointed out by George and Urkowitz.^{4,8}
In particular, the detectability does not depend on the transmitted pulse energy.

If there are no restrictions on pulse energy and pulse bandwidth, it is also clear from the above expression that $(S/N)_{\text{opt}}$ can be made as large as we please by choosing $|U(f)|^2$ to be sufficiently broad and flat versus frequency.* For the purpose of this analysis, however, the

*This conclusion depends critically on the clutter model which has been assumed. In a practical radar situation the conclusion may not hold because of the discrete or granular nature of the clutter.

requirements on the shape of the transmitted pulse can be derived by maximizing $(S/N)_{\text{opt}}$ subject to the requirements that the pulse energy and pulse bandwidth are fixed. That is,

$$\int_{-\infty}^{\infty} u(t)^2 dt = \int_{-\infty}^{\infty} |U(f)|^2 df = E \quad (\text{a finite constant})$$

and

$$U(f) = 0 \quad \text{unless} \quad f_1 \leq f \leq f_2 \quad \text{or} \quad -f_2 \leq f \leq -f_1$$

where $\Delta f = f_2 - f_1$ is called the available system bandwidth. The maximization of $(S/N)_{\text{opt}}$ is a straightforward problem in the calculus of variations which yields a very simple result. It says that the spectrum of the transmitted pulse should be flat over the available frequency band. This result is independent of the constant c , and hence is independent of what fraction of the noise is due to the receiver and what fraction is due to the clutter. Thus, a transmitted pulse which has a flat spectrum over the available system bandwidth is optimum under the assumed restrictions both at short ranges where clutter noise tends to predominate and at long ranges where thermal noise tends to predominate. Parenthetically it may be remarked that the requirements which have been derived on the spectrum of the transmitted pulse are identical to the requirements which would be derived by minimizing $\int \phi(t)^2 dt$, the integral of the squared autocorrelation function, subject to the same restrictions.

The optimized pulse energy spectrum $|U(f)|^2$ should therefore satisfy

$$|U(f)|^2 = \begin{cases} E/2\Delta f & \text{for } f \text{ in } \Delta f \\ 0 & \text{otherwise} \end{cases}$$

Substituting this in the expression for $(S/N)_{\text{opt}}$ we obtain

$$(S/N)_{\text{opt,opt}} = \frac{2A^2 E}{N_0 + cE/\Delta f}$$

The opt,opt denotes the fact that S/N has been optimized both with respect to the choice of receiving filter and shape of transmitted waveform. From this expression the desirability of having large Δf in order to minimize the effect of clutter noise and large E to minimize the effect of receiver noise is evident. For a pulse radar which is peak power limited E will be proportional to the pulse length. These two requirements, large system bandwidth and long pulse length, imply that the transmitted pulse should have large time-bandwidth product. In other words, the transmitted pulse must be coded or phase modulated to produce a time-bandwidth product substantially greater than unity. For the optimized pulse energy spectrum the returned clutter noise spectrum as well as receiver noise will be flat over the signal bandwidth and therefore the optimum receiver filter must be the usual matched filter which results from the assumption of white gaussian background noise. One signal waveform which approximately satisfies the optimum conditions derived here is a pulse with rectangular envelope and a carrier with linearly swept frequency, where the pulse length and frequency sweep are such that the time-bandwidth product of the pulse is much larger than one.

It is of interest to ask what are some of the basic limitations on the pulse coding or pulse compression technique as a means for obtaining improved detection of targets in clutter. Aside from obvious practical difficulties associated with obtaining transmitting and receiving electronic components to handle wider bandwidth signals, there are basic limitations due to the detailed properties of clutter and targets themselves. The inherent granularity of the clutter which our model does not take into account will set an upper bound on the system bandwidth Δf which can be utilized in the discrimination against clutter. For finely divided and randomly distributed types of clutter such as precipitation (and to some extent chaff) the useful system bandwidth Δf is probably quite large, but for ground clutter, which is not so well behaved because of the presence of large point scatterers (large rocks, cliffs, buildings, watertowers, etc.), the useful Δf may be much smaller. A second limitation on Δf is due to the fact that the target itself is not a point, but is actually a distributed scatterer. Reasonable target dimensions suggest a useful system bandwidth in the vicinity of 10 mc.

What about the compatibility of pulse compression techniques with pulse-to-pulse integration techniques employing MTI? Provided that the pulse coding does not change from one pulse to the next, pulse phase at the output of the matched filter remains a well-defined quantity which may be compared on a pulse-to-pulse basis. Thus, in principle, pulse compression and MTI are compatible. In pulsed doppler systems range gating or sampling followed by filtering can be employed at the output of

the matched filter, but the number of range processing channels will have to correspond approximately to the effective number of resolvable range intervals. The greater the bandwidth of the transmitted pulses, the greater must be the number of range processing channels. Thus, as a practical matter for this type of data processing, pulse compression will require increased receiver complexity. MTI schemes employing two-pulse or several-pulse cancellation and integration should not have to be significantly modified if the delay lines and associated electronic components in the data processing have sufficient bandwidth to accommodate the coded pulses.

The use of MTI and pulse compression simultaneously to discriminate against clutter appears to present a fortuitous combination. For generally a type of clutter which responds poorly to one technique should respond well to the other. For example, precipitation or chaff which sometimes responds poorly to MTI because of its non-zero velocity should respond well to the pulse coding, while ground clutter which, because of its spatial properties, may respond poorly to pulse coding will respond very well to MTI.

III. Summary

The type of clutter discrimination which we have discussed here is obtained by virtue of the spatial properties of the clutter rather than its time-varying properties, as with MTI. Using a convenient model applicable to finely distributed clutter and proceeding from available results on the detection of a known signal in colored gaussian noise, we have set up an expression for the single pulse detection capability for a

radar operating in the presence of both receiver noise and clutter noise. Results for detection capability in the presence of predominantly clutter or predominantly receiver noise appear as special cases of this expression. From the expression we have seen that detection performance is simply related to the spectrum of the transmitted signal. Subject to the requirement of fixed transmitted signal energy and fixed system bandwidth, we have seen that the optimum spectrum for the transmitted pulse is one which is flat over the available system bandwidth and that this optimum is independent of the relative strength of receiver and clutter noise. For a radar whose transmitter is peak power limited, the logical result of these considerations is a pulse with large time-bandwidth product, in other words a coded pulse. We have mentioned that one method of approximately realizing the optimum pulse spectrum is a linearly swept FM pulse with rectangular envelope and large time-bandwidth product. Lastly, it has been pointed out that pulse coding and MTI are not mutually exclusive radar techniques. In fact, when used together they should form a powerful combination for the purpose of obtaining improved detection of targets in clutter.

IV. Acknowledgement.

The author is indebted to Dr. Robert F. Naka who suggested the application of pulse coding discussed here. The author is also indebted to Mr. Edwin L. Key for helpful discussions relating to this material.

BIBLIOGRAPHY

1. D. O. North, T. R. PR-6C, RCA Laboratories, Princeton, New Jersey, June 25, 1943.
2. J. H. Van Vleck and D. Middleton, "Theoretical Comparison of the Visual, Aural and Meter Reception of Pulsed Signals in the Presence of Noise," Jour. App. Phys. 17, p. 940, 1946.
3. W. W. Peterson, T. G. Birdsall, and W. C. Fox, "Theory of Signal Detectability," Parts I and II, T. R. No. 13, Electronic Defense Group, University of Michigan, 1953. Also available in 1954 Symposium on Information Theory, Trans. I.R.E., PGIT-4, September, 1954.
4. T. S. George, "Fluctuations of Ground Clutter Return in Airborne Radar Equipment," Inst. Elec. Eng. (London), Monograph No. 22, 1952.
5. S. O. Rice, "Mathematical Analysis of Random Noise," B. S. T. J. Vol. 23, pp. 282-332, and Vol. 24, pp. 46-156, 1945-46. Also available in Selected Papers on Noise and Stochastic Processes, N. Wax (editor), pp. 133-294, Dover Publications, 1954.
6. J. L. Lawson and G. E. Uhlenbeck, Threshold Signals, Radiation Laboratory Series, Vol. 24, p. 297 (McGraw-Hill, New York, 1950).
7. B. M. Dwork, "Detection of a Pulse Superimposed on Fluctuation Noise," Proc. I.R.E. 38, p. 771, July, 1950.
8. H. Urkowitz, "Filters for Detection of Small Radar Signals in Clutter," Jour. App. Phys. 24, No. 8, p. 1024, August, 1953.

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