USRL RESEARCH REPORT No. 53

31 July 1961

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Reprinted from the Journal of the Acoustical Society of America
Vol. 33, No. 4, pages 446-451, April 1961

632 000

Department of the Navy — Office of Naval Research

UNDERWATER SOUND REFERENCE LABORATORY

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Reprinted from THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA, Vol. 33, No. 4, 446–451, April, 1961

Cylindrical Wave Reciprocity Parameter

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(Received September 19, 1960)

A cylindrical wave reciprocity parameter, $\cancel{J_c=(2/\rho c)(R\lambda)^{\frac{1}{2}}L}$, for use in a standard reciprocity calibration when the waves between the source and receiving transducers are cylindrical is derived from both four-terminal network theory and wave propagation theory. Experimental data from calibration measurements on underwater sound line transducers are presented to prove the validity of the parameter.

INTRODUCTION

THE cylindrical wave reciprocity parameter is the parameter that should be used in a standard reciprocity calibration[1] when the shape of the waves between the source and the receiving transducers is cylindrical.

The reciprocity parameter used in conventional reciprocity calibrations of electroacoustic transducers as developed by MacLean[2] is a spherical wave reciprocity parameter, although it is not usually identified specifically as such. The transmitting response of the reciprocal transducer is defined in terms of the sound pressure produced in a spherical wave; that is, there is an inherent assumption that when the reciprocal transducer is transmitting, the hydrophone or microphone being calibrated is in a spherically divergent sound field. This requirement of spherical waves should not be confused with the additional standard requirement that a hydrophone be calibrated in a plane progressive wave sound field.[3] The two apparently contradictory requirements are not incompatible. In practice, the necessary condition is achieved by using a transmitter-to-receiver distance that is large enough so that (a) the receiver is in a spherically divergent wave or "far field," and (b) the radius of curvature of the spherical wave is large compared to the receiver size so that the sector of the wave intercepted by the receiver is essentially plane.

A plane wave reciprocity parameter has been developed by Simmons and Urick.[4] It is used in calibrations where all the transducers are plane piston radiators and the transmitter-to-receiver distance is very short so that the hydrophone is in the near field of the transmitter. Pulsed-sound measurement techniques are required to avoid standing waves between the parallel transmitter and hydrophone plane faces.

Just as the use of the spherical wave reciprocity parameter J_s requires that the transducers be effectively points, and the use of the plane wave parameter J_p requires that the transducers be effectively parallel planes, the use of the cylindrical wave parameter J_c requires that the transducers be effectively parallel lines. The parameters J_p, J_c, and J_s are thus used for wave propagation in one, two, and three dimensions, respectively.

In practice, J_c can be useful for the calibration of long-line hydrophones without requiring a long test distance or spherical wave correction factors.

The parameter J_c is defined in the usual way as the ratio of voltage receiving sensitivity to transmitting current response. The receiving sensitivity will be the standard free-field voltage sensitivity.[3] Whether the transducer is effectively a point on a spherical wave, a straight line on a cylindrical wave, or a plane on a plane wave, the ratio of output voltage to input free-field sound pressure will be the same in each case. The transmitting response, however, depends on the kind of wave being propagated and where the sound pressure is measured. As used in the definition of J_c, the transmitting current response is defined only for a transducer that can transmit cylindrical waves and only in terms of the sound pressure in the cylindrical waves.

As will be shown later, an acoustic line in a free field will transmit cylindrical waves to a maximum distance approximately equal to L^2/λ, where L is the length of the line and λ is the wavelength. If the line is n wavelengths long, the maximum distance is $n^2\lambda$ or nL.

Two different and independent methods have been used to determine the cylindrical wave reciprocity parameter. The first method is a "network" analysis in which the familiar technique of treating an electroacoustic system like a four-terminal electrical network is used. The second method is a "wave" analysis involving Fresnel integrals and the known spherical wave parameter.

NETWORK ANALYSIS

Figure 1 is a schematic drawing of an electroacoustic system with dimensions or boundaries such that the transmitted waves are cylindrical at a radial distance R. The system, assumed to be linear, passive, and reciprocal, is described by two linear equations:

$$p=Z_{11}U+Z_{12}I \tag{1}$$

$$E=Z_{21}U+Z_{22}I, \tag{2}$$

[1] *American Standard Procedures for Calibration of Electroacoustic Transducers, Particularly Those for Use in Water* (American Standards Associations, Inc., New York, 1957), Z24.24, Sec. 3.3.1.2.

[2] W. R. MacLean, J. Acoust. Soc. Am. **12**, 140 (1940).

[3] *American Standard Acoustical Terminology* (American Standards Association, Inc., New York, 1960), S1.1, Sec. 71.3.

[4] B. D. Simmons and R. J. Urick, J. Acoust. Soc. Am. **21**, 633 (1949).

where E is the voltage across, and I the current through, the line transducer; p is the sound pressure measured at a radial distance R or anywhere on the dashed line; U is the volume velocity emanating from the dashed line. If the dashed line is thought of as a second line transducer, U is the volume velocity being produced by the second transducer, or, in a negative sense, it is the volume velocity being absorbed by the second transducer. When waves merely pass the dashed line, U is zero. The coefficients Z_{11} and Z_{22} are acoustic and electrical impedances, respectively; Z_{12} and Z_{21} are electroacoustic transfer impedances that are equal in magnitude if the system is reciprocal. This hypothetical system is similar to that used by Wathen-Dunn[5] to derive J_s.

First, drive the electroacoustic system electrically with a current I_1. Let p_1 be the sound pressure in the cylindrical waves at a radial distance R from the center of the line transducer, or anywhere on the dashed line. Since the waves are being neither produced nor absorbed along the dashed line, U is zero, and Eq. (1) becomes

$$p_1 = Z_{12} I_1.$$

If R is taken as a standard distance, $Z_{12} = p_1/I_1$ is the transmitting current response of the line transducer, or S. Therefore,

$$Z_{12} = S. \tag{3}$$

Next, drive the electroacoustic system acoustically with a volume velocity U_2 emanating from the dashed line, and with the electrical terminals of the transducer open-circuited, so that I is zero and the voltage is E_2. Then Eq. (2) becomes

$$E_2 = Z_{21} U_2$$

or

$$Z_{21} = E_2/U_2. \tag{4}$$

Let p_2 be the free-field pressure at the transducer location when the system is driven acoustically. That is, p_2 is the sound pressure produced at radial distance R by the volume velocity emanating from the dashed line. If the numerator and denominator of Eq. (4) are multiplied by p_2, the result is

$$Z_{21} = (E_2/p_2)(p_2/U_2).$$

The ratio E_2/p_2 is the standard free-field voltage sensitivity or M. Therefore,

$$Z_{21} = M(p_2/U_2). \tag{5}$$

Since the system is reciprocal, and the sign or phase of the parameters are unimportant,

$$Z_{12} = Z_{21}. \tag{6}$$

Substituting Eqs. (3) and (5) into (6) produces

$$S = M(p_2/U_2)$$

[5] W. Wathen-Dunn, J. Acoust. Soc. Am. **21**, 542 (1949).

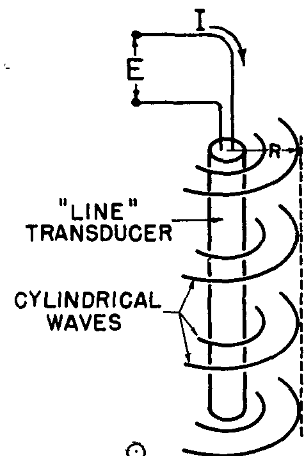

FIG. 1. Cylindrical wave electroacoustic system.

or

$$M/S = U_2/p_2.$$

Since J_c is defined as M/S,

$$J_c = M/S = U_2/p_2. \tag{7}$$

Thus, J_c is the acoustic transfer admittance between the source strength U_2 and the free-field pressure p_2 produced at distance R.

To evaluate U_2/p_2, the pressure in a cylindrical wave must be analyzed. It is given by[6,7]

$$p = AH_0^{(2)}(kR) = A[J_0(kr) - jN_0(kR)],$$

where k is the wave number $2\pi/\lambda$, R is the radial distance, A is a constant, $H_0^{(2)}$ is a Hankel function of zero order and second kind, J_0 is a Bessel function of zero order and first kind, and N_0 is a Neumann function of zero order and first kind. As kR becomes large,[6,7]

$$H_0^{(2)}(kR) \xrightarrow[kR\to\infty]{} (2/\pi kR)^{\frac{1}{2}}. \tag{8}$$

Figure 2 is a plot of both $H_0^{(2)}$ and the approximation.

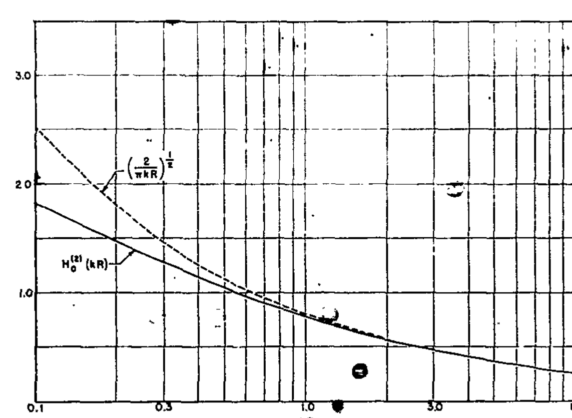

FIG. 2. Hankel function and its approximation for large values of the argument.

[6] Philip M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), 2nd ed., p. 298.

[7] Leo L. Beranek, *Acoustic Measurements* (John Wiley & Sons, Inc., New York, 1949), pp. 57–62.

For $kR>0.35$ the approximation error is less than 10%. For $kR>3.0$ the error is less than 1%. Thus, Eq. (8) is a good approximation if R is greater than about $\lambda/2$, and

$$p=A(2/\pi kR)^{\frac{1}{2}}. \tag{9}$$

Morse[6] evaluates the constant A as

$$A=\pi^2 f\rho a u, \tag{10}$$

where f is frequency, ρ is the density of the medium, a is the radius of the cylindrical radiator, and u is the linear velocity of the cylindrical radiator. The radius a is assumed to be small compared to a wavelength.

No assumption has been made as to how the electroacoustic system was driven acoustically or as to the source of U_2. It is assumed now that the system is driven by an imaginary line transducer located on the dashed line of Fig. 1 with an infinitely small radius a and a surface linear velocity u. This assumption in no way affects the development of Eq. (7).

Substituting (10) in (9) produces

$$p=\pi a f\rho u(\lambda/R)^{\frac{1}{2}}. \tag{11}$$

The volume velocity U_2 is related to the linear velocity u by

$$U_2=2\pi a L u, \tag{12}$$

where L is the length of the imaginary line transducer, and also the length of the real line transducer.

Substituting (11) and (12) in (7) produces

$$\begin{aligned} J_c=U_2/p_2&=(2\pi a L u)/[\pi a f\rho u(\lambda/R)^{\frac{1}{2}}] \\ &=(2L/\rho f)(R/\lambda)^{\frac{1}{2}}=(2L/\rho c)(R\lambda)^{\frac{1}{2}}, \end{aligned}$$

where c is the speed of sound.

WAVE ANALYSIS

The second approach to the problem of determining the cylindrical wave reciprocity parameter involves analysis of the transmission of sound from a line source to a line receiver, as in Fig. 3. The two line transducers are of equal length, placed parallel to each other and separated by a distance R. When R is sufficiently large, the pressure at the receiver is inversely proportional to the distance. At this large distance the source and

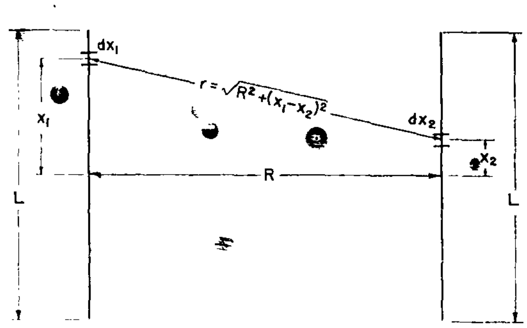

FIG. 3. Line source and a line receiver, both of length L, separated by distance R.

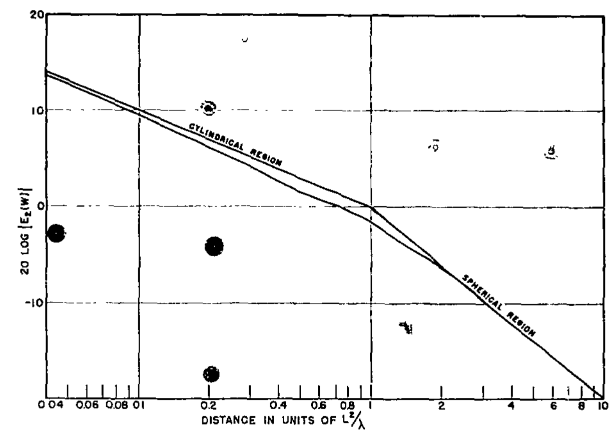

FIG. 4. Logarithmic plot of the magnitude of $E_2(W)$ and its asymptotic expressions as a function of distance.

the receiver are effectively point sources, and the spherical wave reciprocity parameter J_s can be used for reciprocity calibrations. For shorter test distances, the appropriate reciprocity parameter is derived in terms of the pressure at the receiving line as a function of separation distance. Each element of the line source radiates to each element of the line receiver. The element of pressure on dx_2 on the receiving line that is due to dx_1 on the line source is

$$dp=P_0(1/r)e^{-ikr}dx_1, \tag{13}$$

where P_0 is the reference level and k is the wave number $2\pi/\lambda$. The pressure at x_2 is computed by integrating over L, the length of the line source:

$$p(x_2)=P_0\int_{-L/2}^{L/2}(1/r)e^{-ikr}dx_1. \tag{14}$$

Similarly, the average free-field pressure $\langle p(R)\rangle_{\text{av}}$ observed by the line receiver is computed by integrating $p(x_2)$ over the length of the receiving line and dividing by this length, so that

$$\langle p(R)\rangle_{\text{av}}=(P_0/L)\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(1/r)e^{-ikr}dx_1dx_2. \tag{15}$$

With the usual Fresnel approximations, Eq. (15) leads to

$$\langle p(R)\rangle_{\text{av}}=P_0(\lambda/L)E_2(W), \tag{16}$$

where $E_2(W)$ is a complex second Fresnel integral $C_2(W)-jS_2(W)$, $W=2L(2\lambda R)^{-\frac{1}{2}}$. Details of this integration are shown in the Appendix. Figure 4 is a plot of the magnitude of E_2 as a function of distance R in units of L^2/λ. At long distances, the magnitude of E_2 drops approximately 6 db as the distance is doubled; this is, of course, the spherical wave region. At short distances, the drop is about 3 db as the distance is doubled; this is the cylindrical wave region. The transition from the cylindrical wave region to the spherical wave region occurs at $R=L^2/\lambda$. This is, of course, a gradual transition.

The asymptotic values of Eq. (16), as is shown in the Appendix, are

$$\langle p(R)\rangle_{av} \approx P_0(\lambda/R)^{\frac{1}{2}} \text{ for short distances,} \quad (17)$$

$$\approx P_0(L/R) \text{ for long distances,} \quad (18)$$

so that the expression for the transmitting current response $S(R)$ is

$$S'(R)=\langle p(R)\rangle_{av}/I=(P_0/I)(\lambda/R)^{\frac{1}{2}} \text{ for short distances}$$

$$S(R)=(P_0/I)(L/R) \text{ for long distances.}$$

At long distances, the ratio of the free-field voltage sensitivity to the transmitting current response is J_s, the spherical wave reciprocity parameter. Similarly, at short distances, this ratio is J_c, the cylindrical wave reciprocity parameter. With the earlier condition that the free-field voltage sensitivity is the same for spherical and cylindrical wave conditions, the following expressions can be written:

$$J_c=M/S'(R)$$

$$J_s=M/S(R)$$

$$J_c=J_s[S(R)/S'(R)]=J_sL(R\lambda)^{-\frac{1}{2}}=(2L/\rho c)(R\lambda)^{\frac{1}{2}},$$

which is the same as the result that was obtained by the network analysis method.

The condition $J_s=J_c$ corresponds to the transition distance $R=L^2/\lambda$.

MEASUREMENTS

A series of measurements was made to support experimentally the validity of the cylindrical wave parameter. Three underwater sound line transducers, each 84 cm long, were calibrated from 5 to 25 kc, using both a near-field cylindrical wave reciprocity technique and a conventional far-field comparison with a standard. Figure 5 shows the results of both calibrations of one of the transducers. The agreement between these two calibrations is good with maximum differences of about 1 db. One possible source of error for the near-field calibration is the determination of the effective length of the acoustic line. The poor boundary conditions for the long test distances required for the far-field calibration are also a possible source of error, particularly at the lower frequencies. Data from the calibrations of

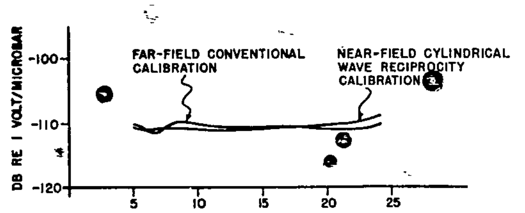

FIG. 5. Free-field voltage sensitivity of a line hydrophone.

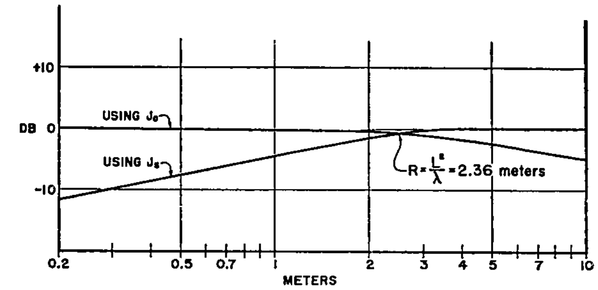

FIG. 6. Measured relative sensitivity of a line hydrophone at 5 kc as a function of projector-to-hydrophone measurement distance.

the other two transducers show essentially the same agreement.

A series of reciprocity calibrations as a function of test distance is shown in Fig. 6. This is relative receiving sensitivity of one transducer, at one frequency. The same measurement data were used for both curves in the figure. One curve was computed by using J_c in the reciprocity calibration equations, the other curve was computed by using J_s. At short distances (less than 2 m for this case) cylindrical J gives consistent and accurate results; at long distances (greater than 4 m) spherical J gives essentially the same result. Between 2 and 4 m, the region of transition from cylindrical to spherical waves, both parameters yield inaccurate results. The maximum discrepancy, 0.7 db, appears at the distance L^2/λ, or 2.36 m in this case. The calculated value of this discrepancy is the square root of the ratio of the magnitude of Eq. (16) to Eq. (17) in the cylindrical wave region or to Eq. (18) in the spherical wave region. The discrepancy appears in Fig. 4 as one-half the decibel difference between the curve and the straight line approximations. The value at the crossover distance L^2/λ, which should be the maximum value, is calculated as -0.78 db. This value agrees well with the experimental data of Fig. 6. The expected discrepancies of lesser magnitude predicted by Fig. 4 for the cylindrical wave region were not observed in this set of experimental data. The crossover point is proportional to frequency so that at 25 kc it would be at almost 12 m thus illustrating the long distances needed for conventional spherical wave reciprocity calibration of line transducers.

Measurements on other line transducers have yielded equally satisfactory results for reciprocity calibrations at short distances by the use of cylindrical J. In particular, one stave ($4\frac{1}{2}$ wavelengths long and $\frac{1}{2}$ wavelength wide) of a sonar transducer was calibrated by self-reciprocity impedance measurements[8] using short test distances. The free-field voltage sensitivity computed by J_c showed good agreement with the sensitivity determined from conventional far-field measurements.

Although the three transducer reciprocity measurements were made with three transducers of the same

[8] G. A Sabin, J. Acoust. Soc. Am. 28, 705 (1956).

length, this is not a requirement of the calibration. The length L used in the theory applies specifically to the length of the reciprocal transducer. The transducer used only as a sound source may, in any type of reciprocity calibration, have any size or configuration provided it insonifies the other two transducers with the same effectively plane sound field. The length of the third transducer or hydrophone may be smaller than L, and, within the assumed condition of uniform cylindrical waves, be only a point; however, the calibration of a short line or point hydrophone by a cylindrical wave reciprocity method offers no advantage over a conventional spherical wave reciprocity calibration. More important, the cylindrical wave from the reciprocal transducer would not, in practice, be perfectly uniform. The sound pressure at a single point would probably deviate from the theoretical uniform pressure much more than would the average pressure along a line. Consequently, the calibration of a point hydrophone would be subject to larger errors than that of a long line hydrophone.

CONCLUSION

It is interesting to compare the spherical, cylindrical, and plane wave reciprocity parameters:

$$J_s=(2/\rho c)(R\lambda)$$
$$J_c=(2/\rho c)(R\lambda)^{\frac{1}{2}}L$$
$$J_p=(2/\rho c)(R\lambda)^0A.$$

The factor $(R\lambda)^0$ is, of course, superfluous, and is not normally included in the expression for J_p. It has been added here to illustrate that the power of $R\lambda$ in each expression is the same as the power of R in the spreading loss function for each kind of wave. That is, the sound pressure spreading loss is proportional to R for spherical waves, to $R^{\frac{1}{2}}$ for cylindrical waves, and to $R^0=1$ for plane waves.

The effective size of the transducer is finite for cylindrical waves and plane waves, and the finite dimensions appear as L, the length of the line transducer, and A, the area of the plane transducer.

All the parameters contain the constant $2/\rho c$.

Thus, not only has the cylindrical wave reciprocity parameter been developed by two independent methods and been verified by measurements, but it is consistent with the spherical and plane wave parameters as well.

APPENDIX

If the usual Fresnel approximations, $r=R$ for magnitude, and $r=R+(x_1-x_2)^2/2R$ for phase, are made in Eq. (15), the result is

$$\langle p(R)\rangle_{\mathrm{av}}=(P_0/LR)$$
$$\times\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\exp[-jk(x_1-x_2)^2/2R]dx_1dx_2. \quad (19)$$

This integral has the form

$$I=A\int_{-b}^{b}\int_{-a}^{a}f(\eta-\xi)d\eta d\xi. \quad (20)$$

If

$$\int f(x)dx=g(x)$$

and

$$\int g(x)dx=h(x),$$

then

$$I=A[h(a+b)+h(-a-b)-h(a-b)-h(-a+b)]. \quad (21)$$

In particular, if $a=b$, then

$$I=A[h(2a)+h(-2a)-2h(0)]. \quad (22)$$

In the case being considered, as is shown below by series representation, h is an even function so that $h(2a)=h(-2a)$ and $h(0)=0$. Then

$$I=2Ah(2a). \quad (23)$$

TABLE I. Computation of the function $E_2(W)=C_2(W)-jS_2(W)$.

W	$C_2(W)$	$S_2(W)$	$\lvert E_2(W)\rvert$
0.0	0.0000	0.0000	0.0000
0.1	0.0050	0.0000	0.0050
0.2	0.0200	0.0002	0.0200
0.3	0.0450	0.0010	0.0450
0.4	0.0798	0.0034	0.0799
0.5	0.1243	0.0081	0.1246
0.6	0.1781	0.0168	0.1789
0.7	0.2403	0.0308	0.2422
0.8	0.3095	0.0517	0.3138
0.9	0.3841	0.0811	0.3926
1.0	0.4616	0.1200	0.4769
1.1	0.5390	0.1687	0.5648
1.2	0.6132	0.2269	0.6538
1.3	0.6812	0.2926	0.7414
1.4	0.7404	0.3629	0.8245
$2^{\frac{1}{2}}$	0.7480	0.3731	0.8359
1.5	0.7898	0.4339	0.9011
1.6	0.8301	0.5010	0.9696
1.7	0.8640	0.5606	1.0300
1.8	0.8966	0.6105	1.0847
1.9	0.9326	0.6516	1.1377
2.0	0.9766	0.6868	1.1940
2.1	1.0298	0.7223	1.2577
2.2	1.0913	0.7632	1.3317
2.3	1.1551	0.8138	1.4130
2.4	1.2148	0.8730	1.4960
2.5	1.2653	0.9356	1.5736
2.6	1.3071	0.9945	1.6424
2.7	1.3459	1.0446	1.7037
2.8	1.3882	1.0862	1.7626
2.9	1.4398	1.1258	1.8277
3.0	1.4988	1.1706	1.9018

Then

$$\int \exp(-jkx^2/2R)dx$$

$$=\int \cos(kx^2/2R)dx-j\int \sin(kx^2/2R)dx$$

$$=\alpha[C(x/\alpha)-jS(x/\alpha)], \quad (24)$$

where

$$\alpha=(\lambda R/2)^{\frac{1}{2}}$$

$$C(u)-jS(u)=\int_0^u \cos(\pi t^2/2)dt-j\int_0^u \sin(\pi t^2/2)dt$$

$$\alpha\int[C(x/\alpha)-jS(x/\alpha)]dx=\alpha^2[C_2(x/\alpha)-jS_2(x/\alpha)] \quad (25)$$

$$C_2(u)-jS_2(u)=\int_0^u C(t)dt-j\int_0^u S(t)dt.$$

By using Eq. (23) with Eq. (19), the equivalent of Eq. (16) is obtained:

$$\langle p(R)\rangle_{\text{av}}=P_0(\lambda/L)[C_2(L/\alpha)-jS_2(L/\alpha)]. \quad (26)$$

The expressions

$$\begin{aligned} C_2(u)&=uC(u)-(1/\pi)\sin\tfrac{1}{2}\pi u^2 \\ S_2(u)&=uS(u)+(1/\pi)(\cos\tfrac{1}{2}\pi u^2-1) \end{aligned} \quad (27)$$

given by Magnus and Oberhettinger[9] can be used for computations involving the function in Eq. (26). These expressions lead to the values given in Table I, which were computed for values of $C(u)$ and $S(u)$ in the range $u=0$ to $u=3.0$.

For large values of u, $C_2(u)$ becomes approximately $u/2$ and similarly $S_2(u)$ becomes approximately $u/2$. Thus, when u is large, $|E_2(u)|\approx u/\sqrt{2}$, which leads to the result expressed in Eq. (17) for short distances. For small values of u, the series must be written for $C_2(u)$ and $S_2(u)$, as follows:

$$\begin{aligned} C_2(u)&=\tfrac{1}{2}u^2-(\tfrac{1}{2}\pi)^2\frac{u^6}{2\cdot 3!\cdot 5}+(\tfrac{1}{2}\pi)^4\frac{u^{10}}{2\cdot 5!\cdot 9} \\ &\quad+\cdots(-1)^{n-1}(\tfrac{1}{2}\pi)^{2(n-1)}\frac{u^{4n-2}}{2(2n-1)!(4n-3)} \\ S_2(u)&=\tfrac{1}{2}\pi\frac{u^4}{2\cdot 2!\cdot 3}-(\tfrac{1}{2}\pi)^3\frac{u^8}{2\cdot 4!\cdot 7}+(\tfrac{1}{2}\pi)^5\frac{u^{12}}{2\cdot 6!\cdot 11} \\ &\quad+\cdots(-1)^{n-1}(\tfrac{1}{2}\pi)^{2n-1}\frac{u^{4n}}{2(2n)!(4n-1)}. \end{aligned} \quad (28)$$

These series result from integrating twice the cosine and sine series, respectively, for the argument $\cdot\frac{1}{2}\pi u^2$. When u is sufficiently small, $|E_2(u)|=\frac{1}{2}u^2$, which leads to the result in Eq. (18) for long distances.

[9] W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Functions of Mathematical Physics*, translated from the German by John Wermer (Chelsea Publishing Company, New York, 1954), p. 96.

UNCLASSIFIED

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