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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

IN

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ABSTRACT

The ratio of geostrophic drag coefficients C/C_a under various stability conditions is obtained as a function of Richardson number. A drag coefficient C_f is defined in relation to the mechanical mixing length, and its ratio to the actual drag coefficient C is related to the Richardson number. By using observational recent data on the normalized logarithmic wind shear of Monin-Obukhov one can obtain C_f/C_a.

The writer is deeply indebted to Dr. F. L. Martin (Professor of Meteorology) for his suggestion of the topic and his continued help throughout the investigation and during the preparation of this paper.

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LIST OF SYMBOLS USED

Symbol	Definition
$\bar{u}$	mean wind speed
u'	turbulent velocity fluctuation in the direction of mean wind
w'	turbulent velocity fluctuation in the vertical
l'	eddy mixing length
l	root-mean-square mixing length
τ'	eddy stress
τ_0	eddy stress in the surface layer
ρ	air density
k	Von Karman constant
u_{*a}	friction velocity in neutral surface layer
z_0	roughness parameter
τ_a	eddy stress in the neutral surface layer
u_*	friction velocity in non-neutral surface layer
β	Deacon wind parameter
C	Geostrophic drag coefficient under non-neutral condition
C_a	Geostrophic drag coefficient under neutral condition
V_g	Geostrophic wind speed
θ	potential temperature
R_o	surface Rossby number
f	Coriolis parameter
$\frac{\Delta p}{\Delta n}$	pressure gradient

Symbol	Definition
n	horizontal coordinate normal to the isobars
α	specific volume of the air
Ri_{140}	Richardson number at z = 140 cm
$z_{1,2}$	geometric mean of heights Z_1 and Z_2
S	normalized-logarithmic wind shear.
ℓ_f	mechanical turbulent mixing length
u_{*f}	friction velocity associated with ℓ_f
C_f	Geostrophic drag coefficient if ℓ_f is applied.
L'	"gradient length"
g	acceleration of gravity
K_h	eddy diffusivity coefficient for heat
K_m	eddy diffusivity coefficient for momentum
γ'	$\frac{K_h}{K_m}\gamma$
γ	constant
α'	$\frac{K_h}{K_m}\alpha$
α	constant
$\bar{S}$	integral-mean value of S.

1. Introduction.

One of the important problems in meteorology is to establish relationships between the turbulent characteristics of the lower atmosphere and the large-scale synoptic parameters. A significant turbulent characteristic is the ground drag, which enables one to specify the wind profile, the eddy diffusivity and the local energy dissipation in the surface layer, by knowing the topographical characteristics i.e., the roughness parameter of a particular place.

Lettau [7] had made an attempt to relate the drag coefficients with the static stability represented by a dimensionless parameter called Richardson number. His investigations were based on 1953 O'Neill, Nebraska, data. It is the main purpose of this paper to verify and extend his investigations using 1956 O'Neill data. It is important to note that this work deals with a relatively smooth surface, therefore the "zero-plane displacement" which is sometimes used in the adiabatic-wind profile has been neglected, a procedure which greatly simplifies the computations.

2. The Nature of the Data.

The data used in this study were obtained by the Project Prairie Grass [1] conducted near O'Neill, Nebraska during the summer 1956. Since this study makes use of some of the published data, i.e., wind speed, temperature and surface maps, it is necessary to describe some aspects of the instrumentation. The instrumentation employed was that of the mobile micrometeorological station of the Texas A&M group. The station has a slender aluminum mast supporting six anemometers at heights of 8, 4, 2, 1, 0.5 and 0.25 meters. A similar mast supported seven temperature-measuring, radiation-shielded copper-constantan thermocouple junctions at heights 8, 4, 2, 1, 0.5, 0.25 and 0.125 meters. In addition the published report of Project Prairie Grass includes sectional sea level pressure maps at times nearly synoptic with the micrometeorological data. These maps revealed some small-scale features of the circulation near O'Neill which did not appear on the larger-scale facsimile maps. The micrometeorological data periods were selected to be nearly simultaneous with the available maps. At times, linear interpolation between the available charts was used for computations of geostrophic wind speeds.

Regarding the errors of measurement, two types of temperature error occur: i.e., calibration and radiation error. An estimate for calibration error was 0.05°C. The estimated radiation error was 0.1°C with the understanding that, in daytime with clear skies, and low windspeed, all measured air temperatures overestimate the true temperature. At night with clear skies the measured temperatures would be lower than the ambient temperature. For windspeed, measurements were dependent upon different techniques of calibrations according to the windforce. All measurements used in this paper were twenty-minute means. The reader is referred to [1] for more details concerning the instrumentation.

3. Background Theory.

Extensive use will be made of Prandtl's turbulence model, because of its relative simplicity. This model is described in many standard texts. According to this theory, the following major results emerge:

$$u' = l' \frac{\partial \bar{u}}{\partial z} \tag{1}$$

$$\tau' = -\rho \overline{u'w'} = \rho l^2 \left(\frac{\partial \bar{u}}{\partial z} \right)^2 \tag{2}$$

where u', w', are the turbulent velocity fluctuations in the direction of the mean wind and in the vertical, respectively. l' is the eddy mixing length, while l is the root-mean-square mixing length. The eddy stress τ' is then considered to be very nearly a constant τ_0 within a layer called the surface layer.

In the case of the fully-rough, neutral surface layer, Rossby has shown that $l = kz$, so that (2) may be integrated to give

$$\bar{u}_a = \frac{u_{*a}}{k} \ln \frac{z}{z_0} \tag{3}$$

where $u_{*a} = \sqrt{\tau_a/\rho}$ is the friction velocity in a neutral surface layer, Z_0 is the roughness parameter and $k = 0.38$ is the Von Karman constant.

In the non-neutral surface layer, numerous theoretical profiles have been derived. However Deacon has proposed the semi-empirical relationship

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k z_0} \left(\frac{z}{z_0} \right)^{-\beta} \tag{4}$$

where the friction velocity $u_* = \sqrt{\tau_0/\rho}$ is constant in the non-neutral surface layer. The parameter β is a decreasing function of Richardson number. Actually Davidson and Barad [4] have shown that $|\beta - 1|$ increases somewhat with height. However (4) still affords a useful tool as an overall or bulk-relationship. Two useful results may be derived from (4). These are:

(i) the integrated wind profile:

$$\bar{u} = \frac{u_*}{k(1-\beta)} \left[\left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right] \tag{5}$$

(ii) the value u_* by the limiting process indicated in equation (7) below

$$u_* = k \lim_{z \to z_0} \frac{\delta u / \delta (\ln z)}{(z/z_0)^{1-\beta}} \tag{6}$$

so that

$$u_* = k \lim_{z \to z_0} \frac{\delta u}{\delta (\ln z)} \tag{7}$$

Actually the lowest level at which wind data were available was $Z = 25$ cm and Z_0 is the roughness parameter determined under the time-nearest neutral conditions. u_* may therefore be approximated by the finite-difference expression

$$u_* = \frac{0.38\,\bar{u}_{25}}{\ln 25 - \ln z_0} \tag{8}$$

Following a procedure similar to that employed by Davidson and Parad [4] in neutral cases, it may be shown that

$$\ln z_0 = \frac{\sum_{i=1}^{6} \ln z_i}{6} - \frac{\Delta(\ln z)}{6\Delta u / \sum_{i=1}^{6} u_i} \tag{9}$$

where

$$\Delta(\ln z) = \ln \frac{800}{25} + \ln \frac{400}{50} + \ln \frac{200}{100} \tag{10}$$

and

$$\Delta u = (u_6 - u_1) + (u_5 - u_2) + (u_4 - u_3) \tag{11}$$

for mean-wind measurements at 25, 50, 100, 200, 400, 800 cms.

4. Geostrophic Drag Coefficients.

Lettau [7] has defined the geostrophic drag coefficient as

$$C = u_* / V_g \tag{12}$$

For neutral stability, the corresponding drag coefficient is defined as

$$C_a = u_{*a} / V_g \tag{13}$$

Neutral wind profiles at O'Neill may be obtained by finding those cases for which the potential temperature profiles we characterized by $\partial\theta/\partial z = 0$ in the layer 25 cm - 200 cm. Altogether 15 such profiles were found. Assuming a logarithmic wind profile can be fitted to the wind data, u_{*a} of equation (3) may be obtained as

$$u_{*a} = \frac{k\Delta u}{\Delta(\ln z)} = \frac{k\Delta u}{9 \ln 2} \tag{14}$$

By analogy with flow in circular conduits, Lettau [7] has suggested a drag-coefficient C_a for neutral cases of the form

$$C_a = \frac{0.104}{\log(C_a R_o) - 2.24} \tag{15}$$

where Ro may be called the "surface Rossby number" defined as the non-dimensional expression

$$R_o = V_g / f z_o \quad , \quad V_g = \frac{\alpha}{f} \frac{\Delta p}{\Delta n} \tag{16}$$

The constants of equation (15) were obtained by Lettau by least-square methods applied to a wide variety of neutral-wind profiles for different sites.

Lettau [7] goes on to give values of C/C_a for different values Ri_{100}, the Richardson number at 100 cm, for the 1953 O'Neill data. He obtains a curve as shown in fig. 1.

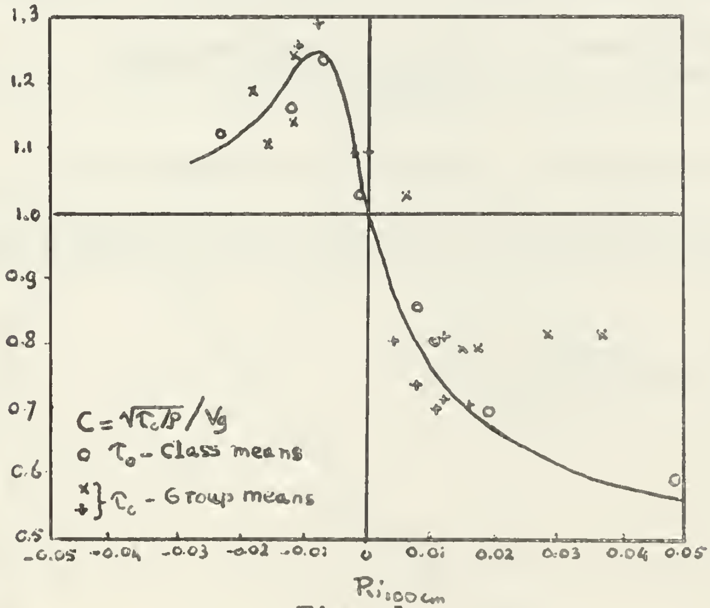

Fig. 1.

Ratio of geostrophic drag coefficient for non-adiabatic and adiabatic conditions as a function of Richardson number at Z = 100 cm, at O'Neill, Nebraska 1953 (After Lettau [7]).

It must be pointed out that C_a here represents the geostrophic drag coefficient for any surface layer of drag coefficient C, if this layer were suddenly to be converted into a neutral layer without changing V_g/fz_0.

A primary objective of this paper was to test equation (15) as a means of determining C_a based on the 1956 O'Neill data. Also application of non-neutral profile data to equations (7) and (12) afforded values of C under identical roughness conditions. The comparison of Lettau's 1953 graph of C/C_a versus Ri_{100} with that obtained by the writer at Ri_{140} (to be more exact, the Richardson number at 141.4cm) based on the 1956 O'Neill data is shown in fig. 3.

If one knows a value of $Ro = V_g/fz_0$, it is possible to solve equation (15) for C_a by numerical methods. It is shown in the Appendix that a numerical solution C_a of equation (15) may be obtained iteratively from the equation

$$y_{i+1} = y_i - \frac{y_i(2.3026 \log y_i + y_i + 6.5927 - 2.3025 \log Ro)}{1 + y_i} \qquad (17)$$

where $y_i \equiv$ i-th iterant to $0.2395/C_a$. The first estimate for C_a was obtained from Lettau's curve [7], fig. 2.

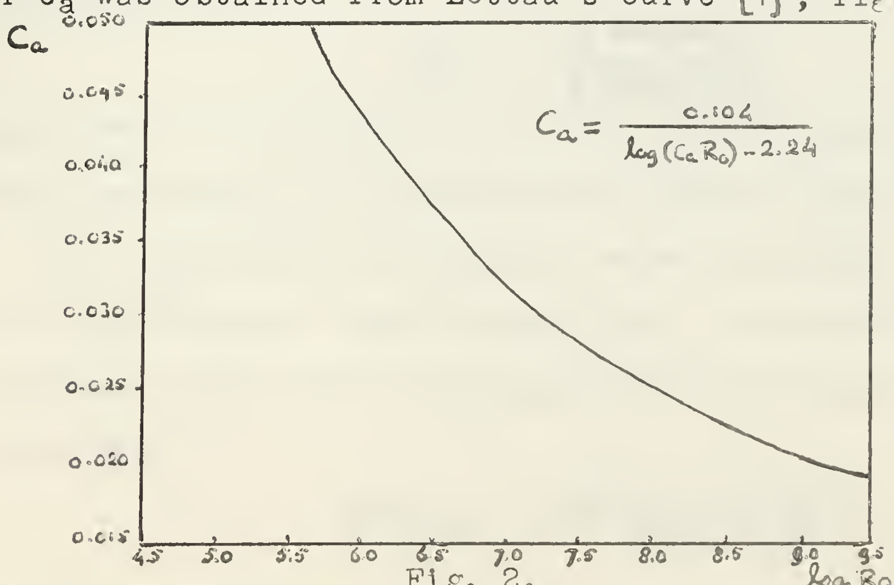

Fig. 2.

Geostrophic drag coefficient under neutral condition as a function of surface Rossby number (After Lettau [7]).

The chief inaccuracy in this approach appears to be uncertainty in the numerical constants appearing in equation(15), especially at low geostrophic wind speeds. Another source of error is the necessity in some cases, of interpolating V_g between two maps 6 or 12 hours apart in order to get the estimated value of V_g at the time of micrometeorological data-run.

A second approach to determine the relationship between C and C_a was used. Based on 15 neutral cases at O'Neill, Nebraska, values of C_a were computed using equations (13) and (14). Values of C_a were then plotted versus log Ro, and the resulting best-fitting curve compared to Lettau's, which is replotted on the same graph. The comparison will be discussed in later section.

The Richardson number has been used very extensively as a measure of stability in the micro-meteorology. Its definition is

$$Ri = \frac{g\, \partial\theta/\partial z}{\theta\left(\partial\bar{u}/\partial z\right)^2} \tag{18}$$

This number is proportional to the ratio of work done against static stability to the work done by eddy stress.

For instruments arranged at equal logarithmic spacing, as in the 1953 and 1956 O'Neill data, a convenient finite-difference form of Ri at $Z = Z_{1.2}$ has been proposed by Lettau [6]

$$Ri(z_{1.2}) = \frac{g\, z_{1.2}\, (\bar{\theta}_2 - \bar{\theta}_1)\ln 2}{T(\bar{u}_2 - \bar{u}_1)^2} \tag{19}$$

This value is understood to be applicable at the geometric mean of Z_1 and Z_2, namely $Z_{1.2} = \sqrt{Z_1 Z_2}$. $\bar{\theta}_2$ and $\bar{u}_2$ in the

1956 data are the 20-minute mean potential temperature and windspeed at level 2, and an analogous meaning applies for symbols with subscript one.

Finally, Monin and Obukhov [2] have introduced a useful non-dimensional parameter S defined by

$$S = \frac{kz\,\partial\bar{u}/\partial z}{u_*} \tag{20}$$

Note that the numerator of S has the form

$$kz\,\partial\bar{u}/\partial z = l_f\,\partial\bar{u}/\partial z \tag{21}$$

which, according to the Prandtl model, may be regarded as representing u_{*f}. u_{*f} as here defined represents that part of u_* due to turbulent fluctuations associated with the mechanical turbulent mixing length $l_f = kz$. The expression for S may then be written

$$S = \frac{u_{*f}/V_g}{u_*/V_g} = \frac{C_f}{C} \tag{22}$$

The drag coefficient defined by $C_f = u_{*f}/V_g$ is not one that can be measured directly, but is one whose value can be inferred by empirical information regarding S. This is dealt with in more detail in section 8. This interpretation of u_{*f} as $kz\,\partial\bar{u}/\partial z$ appears to have been given first by Businger [3].

5. Comparison of C/C_a for 1953 and 1956 data.

In computing C_a, Lettau's empirical formula, equation (17) was first used with the 1956 data. It was necessary to compute Z_o, using equation (9) from the time-nearest neutral wind profiles and V_g from interpolated surface charts. In computing u_* and C, equation (8) and (12) were used. The value of Ri_{140} is also included using equation (19). The results for all data-days employed are given in table 1, and the groupings of Ri_{140} in classes are given in table 2. The graph of C/C_a versus Ri_{140} is given in fig. 3, with Lettau's curve [7] superimposed. The agreement with Lettau's results is not as good as might have been expected. For instance the value $C/C_a = 1.75$ was found at $Ri_{140} = 0$ contrasted with Lettau's value $C/C_a = 1.00$ at $Ri_{100} = 0$. Moreover the maximum value of C/C_a found in this paper exceeds Lettau's maximum by a comparable factor but with $Ri_{140} = -0.055$, compared with $Ri_{100} = -0.007$, according to Lettau.

Possible sources of error lie in the values of V_g, which affect the values of C and C_a given by equations (12) and (15) respectively, as well as in the value of Z_0. The values of Z_0 determined here ranged from 0.35cm to 3.1 cm, as contrasted with values quoted by Blackadar et al [2] of $Z_0 = 0.6$ cm for profiles in approximately the same data-period.

TABLE I - Values of C/C_a by equations (3), (12), (15).

Date CST	Z_o (cms)	u_* (cm/sec)	V_g (kts)	C	C_a	C/C_a	Ri_{140}
10 Jul 1956							
1305	1.10	24.67	13.60	0.036	0.034	1.08	-0.46
1405	1.10	35.85	12.80	0.056	0.034	1.65	-0.17
1505	1.10	31.24	12.00	0.052	0.034	1.52	-0.28
1530	1.10	30.99	15.64	0.040	0.033	1.20	-0.25
1605	1.10	35.75	11.60	0.053	0.034	1.54	-0.20
1705	1.10	35.00	10.80	0.065	0.035	1.87	-0.05
1805	1.10	44.00	10.52	0.084	0.035	2.41	-0.02
2105	1.10	21.03	12.00	0.035	0.034	1.02	0.06
23 Jul 1956							
0930	3.14	49.26	19.44	0.051	0.037	1.38	-0.15
1105	3.14	42.85	19.00	0.045	0.037	1.22	-0.21
1205	3.14	46.69	18.00	0.052	0.037	1.39	-0.16
1305	3.14	52.92	17.00	0.062	0.038	1.66	-0.09
1605	3.14	52.55	16.20	0.065	0.038	1.72	-0.14
1705	3.14	59.69	16.00	0.075	0.038	1.97	-0.06
1805	3.14	67.38	15.80	0.085	0.038	2.25	-0.02
2130	3.14	31.03	9.52	0.059	0.040	1.47	0.04
2305	3.14	31.86	7.52	0.085	0.042	2.01	0.03
25 Jul 1956							
0205	1.49	2.15	4.80	0.009	0.040	0.22	0.10
0405	1.49	12.66	9.60	0.026	0.037	0.72	0.05
0805	1.49	50.49	24.00	0.042	0.034	1.22	-0.10
0905	1.49	52.24	26.00	0.040	0.032	1.24	-0.01
1230	0.70	58.87	32.00	0.038	0.029	1.27	-0.02
1405	0.70	65.24	46.00	0.028	0.028	1.02	-0.02
1605	0.70	62.80	54.00	0.023	0.027	0.85	-0.01
2330	0.70	48.72	44.80	0.022	0.028	0.78	0.00
6 Aug 1956							
1805	0.35	29.53	10.60	0.056	0.030	1.84	0.00
2105	0.35	11.65	10.80	0.022	0.030	0.71	0.07
2205	0.35	25.00	11.00	0.045	0.030	1.50	0.01

TABLE 1 - (Cont'd)

Date CST	Z_o (cms)	u_*(cm/sec)	V_g (kts)	C	C_a	C/C_a	Ri_{140}
7 Aug 1956							
0405	3.06	57.72	11.88	0.097	0.039	2.47	0.01
0805	3.06	63.14	12.88	0.098	0.039	2.52	-0.04
0905	3.06	109.10	13.08	0.167	0.039	4.30	-0.02
1105	3.06	69.66	13.48	0.103	0.039	2.67	-0.08
1330	0.62	59.79	15.84	0.075	0.031	2.44	-0.04
1605	0.62	64.83	15.00	0.086	0.031	2.77	-0.01
1805	0.62	36.02	15.76	0.046	0.031	1.47	-0.01
2205	0.62	23.87	14.00	0.034	0.031	1.09	0.05
2205	0.62	9.47	12.40	0.015	0.032	0.48	0.13
27 Aug 1956							
1230	0.55	42.96	26.48	0.032	0.029	1.12	-0.07
1505	0.55	38.09	24.80	0.031	0.029	1.06	-0.02
1705	0.55	40.08	25.20	0.032	0.029	1.10	-0.02
1930	0.55	1.79	15.92	0.002	0.031	0.07	0.15
2205	0.55	2.98	26.60	0.002	0.029	0.07	0.04
29 Aug 1956							
2135	1.53	24.09	7.20	0.067	0.038	1.75	0.03

TABLE 2 - Class means of Ri_{140} vs C/C_a

Ri_{140}	-0.43	-0.37	-0.31	-0.25	-0.19	-0.12	-0.06	0.00	0.06	0.13
C/C_a	1.08	-	1.52	1.20	1.45	1.44	2.04	1.72	1.11	0.26

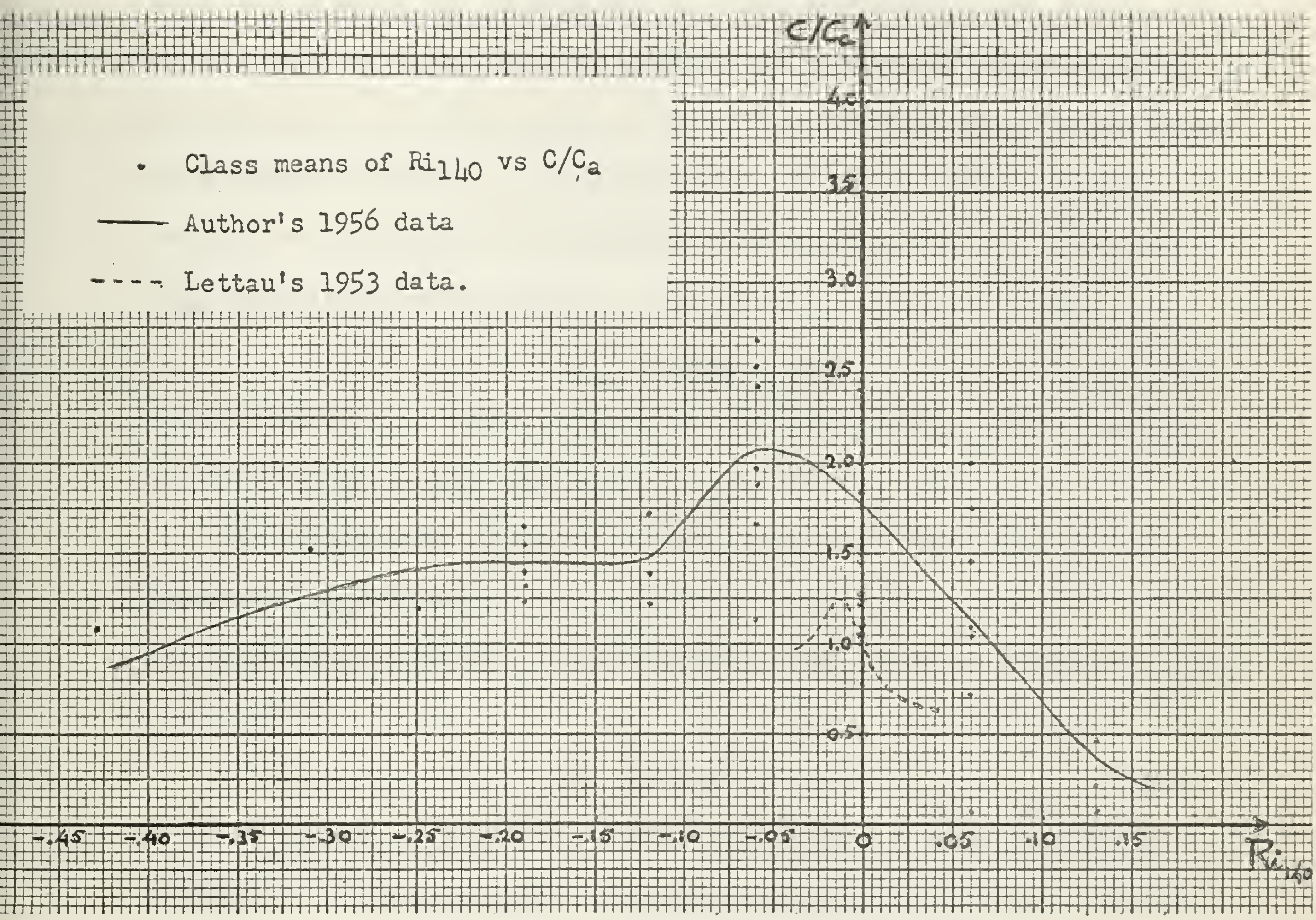

Fig. 3

Ratio of geostrophic drag coefficient for non-adiabatic and adiabatic conditions as a function of Richardson number at Z = 140 cm, at O'Neill, Nebraska 1956.

6. Revised formula for C_a

Hitherto C_a has been computed by using Lettau's empirical function for C_a, equation (15). Since Lettau himself realized that the numerical constants in (15) were not very exact [7], an attempt is made here to obtain an improved relationship between C_a and Ro by using only 1956 O'Neill data. The first step again consists in computing Z_0 for the time-nearest neutral wind profile according to equation (9). As before, u_{*a} results from equation (14) with $k = 0.38$, so that

$$u_{*a} = 0.061\,\Delta u_a \tag{23}$$

Equation (13) is then applied to determine C_a. The results of the computations are shown in table 3.

An empirical function for C_a was then obtained by the following procedure:

Let $\log Ro = x$ and $C_a = y$ and suppose the desired function has the form

$$y = be^{cx} \tag{24}$$

where y means the desired function, while b and c are constants to be determined. Taking the log of (24) one obtains

$$\ln y = \ln b + cx \tag{25}$$

To simplify the notation, let $\ln y = Y$ and $\ln b = a$, and therefore the desired function becomes

$$Y = a + cx \tag{26}$$

Using the least-squares method, the normal equations are

$$\begin{gathered} \bar{Y} - a - c\bar{x} = 0 \\ \sum_{i=1}^{m} Y_i x_i - a\sum_{i=1}^{m} x_i - c\sum_{i=1}^{m} x_i^2 = 0 \end{gathered} \tag{27}$$

Substituting the values of observations (table 3) in the normal equations one obtains

$$-3.51 - a - 7.21c = 0$$

$$-38190-108.28a - 785.18c = 0$$

from which, it follows that

$$a = -0.85 \quad \text{or} \quad b = 0.427$$

$$c = -0.369$$

Substituting these constants in (24), and further by changing y to C_a, x to log Ro, the expression for C_a can be obtained

$$C_a = 0.427 \exp\left[-0.369 \log Ro\right] \qquad (28)$$

The graph of equation (28) is shown in fig. 4. compared with Lettau's graph. From fig. 4 it is evident that the revised formula for C_a tends to underestimate the drag at larger values of Rossby number, and overestimate it at smaller Ro (log Ro < 7.1). It should be noted, however, that there was considerable scatter of the observations relative to the line of best fit.

If one assumes that equation (28) is a universal formula for geostrophic drag coefficient in middle and higher latitudes under neutral conditions, one can then write

$$u_a = \frac{u_{*a}}{k} \ln \frac{z}{z_o} = \frac{C_a V_g}{k} \ln \frac{z}{z_o} \qquad (29)$$

with C_a given by (28). Hence, adiabatic wind speed at level Z is obtainable as a function of V_g, Z_o, Z and latitude.

TABLE 3. - Results of computations for C_a

Adiabatic hours	Z_o cm	u_* (cm/sec)	Vg (kts)	$y_i = C_a$	$x_i = \log R_o$	$Y_i = \ln y_i$	$Y_i x_i$	x_i^2
10 July 1956								
1905 CST	1.10	36.66	11.00	0.067	6.707	-2.703	-18.1	44.89
23 July 1956								
1905 CST	3.14	31.48	14.00	0.045	6.355	-3.101	-19.1	40.45
25 July 1956								
0605 CST	1.49	29.59	16.80	0.035	6.759	-3.352	-22.6	45.70
1805 CST	0.70	57.34	64.00	0.018	7.670	-4.017	-30.7	58.83
6 Aug 1956								
1905	0.35	24.79	10.80	0.046	7.192	-3.079	-22.1	51.70
7 Aug 1956								
0605 CST	3.06	36.42	12.20	0.058	6.313	-2.847	-17.9	39.82
1905 CST	0.62	32.94	14.00	0.044	7.084	-3.124	-22.1	50.13
27 Aug 1956								
1705 CST	0.55	40.69	25.20	0.032	7.368	-3.442	-25.4	54.32
29 Aug 1956								
1930 CST	1.53	36.72	32.00	0.032	8.025	-3.817	-30.6	64.48
11 July 1956								
0605 CST	0.11	14.64	14.00	0.020	7.848	-3.912	-30.6	61.62
24 July 1956								
0605 CST	0.49	27.57	20.00	0.028	7.431	-3.576	-26.5	55.20
1905 CST	0.81	23.79	21.20	0.022	7.124	-3.817	-27.1	50.69
26 July 1956								
1805 CST	0.81	37.52	40.00	0.019	7.398	-3.963	-29.3	54.76
8 Aug 1956								
1905 CST	0.37	32.64	32.00	0.020	7.645	-3.912	-29.8	58.52
28 Aug 1956								
1805 CST	0.71	27.57	32.00	0.017	7.360	-4.075	-30.0	54.17
Sums					108.279	-52.737	-381.9	785.18
Average					7.210	-3.51		

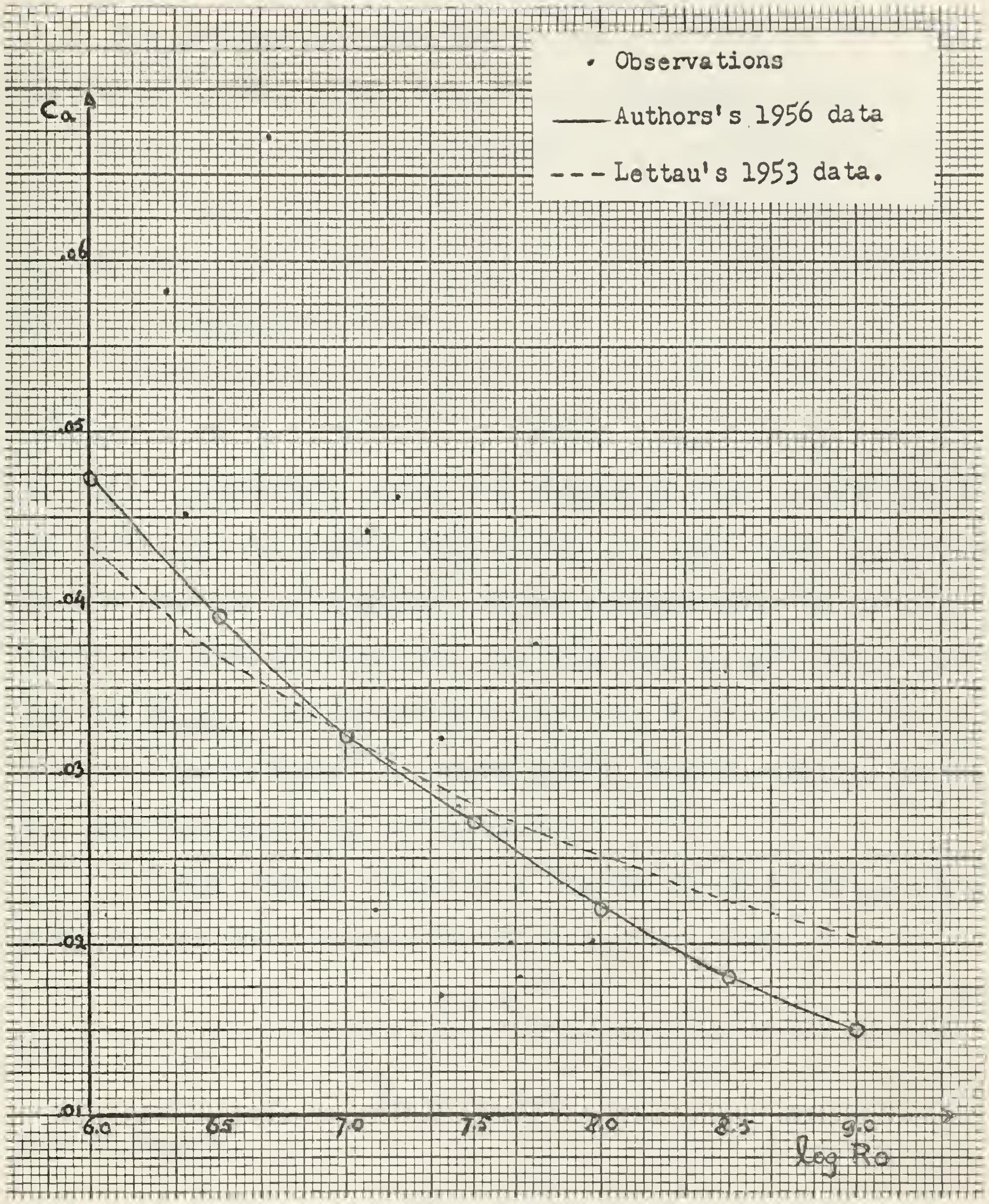

Fig. 4

Geostrophic drag coefficient under neutral conditions as a function of surface Rossby number.

7. A revised formula for C/C_a

The revised formula for C_a will be used in this section to compute C/C_a, still using the previously tabulated values of geostrophic winds and C (table 3). The results are shown in table 4 and the resulting values of C/C_a then plotted versus Ri_{140} as shown in fig. 5. The procedure of fitting the curve is shown below.

After plotting the values of C/C_a, a further restriction is placed on the empirical function C/C_a: namely that $C/C_a=1$ at $Ri=0$. Moreover, since the shapes of the curves for C/C_a in unstable and stable conditions are different (according to the scatter of points), it is assumed that in stable conditions the desired function has the form $y = e^{ax}$ where $y = C/C_a$ and $x = Ri_{140}$. In unstable condition, y is assumed to be of the form $y = 1 + k_1 x e^{k_2 x}$ where $y = C/C_a$ and $x = -Ri_{140}$, a, k_1, k_2, are constants to be determined.

(i) For stable conditions (table 5), with $Ri_{140} = x$ and $C/C_a = y$, if the assumed function has the form

$$y = e^{ax} \tag{30}$$

then

$$\ln y = ax$$

and if $\ln y = Y$

$$Y = ax$$

By the method of least-squares, the normal equation will be

$$\sum_{i=1}^{m} Y_i x_i - a \sum_{i=1}^{m} x_i^2 = 0 \tag{31}$$

Solving for a,

$$a = -4.159$$

Substituting this constant in (30) and changing y to C/C_a x to Ri_{140}, the expression for C/C_a is obtained as

$$C/C_a = \exp\left[-4.159\, Ri_{140}\right] \tag{32}$$

TABLE 4. - Computations of C/C_a using equation (28) for C_a

Date CST	log Ro	p = -0.369 log Ro	e^p	C_a	C	C/C_a	Ri_{140}
10 July 1956							
1305	6.799	-2.51	0.081	0.0348	0.0363	1.045	-0.46
1405	6.773	-2.50	0.082	0.0350	0.0560	1.600	-0.17
1505	6.745	-2.49	0.083	0.0360	0.0521	1.445	-0.28
1530	6.860	-2.53	0.080	0.0342	0.0396	1.160	-0.25
1605	6.730	-2.48	0.084	0.0359	0.0530	1.475	-0.20
1705	6.699	-2.47	0.085	0.0363	0.0648	1.785	-0.05
1805	6.687	-2.46	0.085	0.0363	0.0838	2.320	-0.02
2105	6.745	-2.49	0.083	0.0360	0.0350	0.975	0.06
23 July 1956							
0930	6.500	-2.40	0.091	0.0388	0.0507	1.305	-0.15
1105	6.487	-2.39	0.092	0.0393	0.0451	1.145	-0.21
1205	6.464	-2.38	0.093	0.0397	0.0519	1.305	-0.16
1305	6.439	-2.37	0.093	0.0397	0.0623	1.570	-0.09
1605	6.418	-2.36	0.094	0.0401	0.0649	1.615	-0.14
1705	6.413	-2.36	0.094	0.0401	0.0746	1.860	-0.06
1805	6.407	-2.36	0.094	0.0401	0.0853	2.150	-0.02
2130	6.230	-2.30	0.100	0.0427	0.0591	1.385	0.04
2305	6.084	-2.22	0.109	0.0465	0.0850	1.830	0.03
25 July 1956							
0205	6.215	-2.29	0.101	0.0431	0.0090	0.209	0.10
0405	6.516	-2.40	0.091	0.0388	0.0264	0.680	0.05
0805	6.721	-2.47	0.085	0.0363	0.0421	1.160	-0.10
0905	6.948	-2.56	0.077	0.0329	0.0402	1.220	-0.01
1230	7.366	-2.72	0.066	0.0282	0.0368	1.305	-0.02
1405	7.523	-2.77	0.063	0.0269	0.0284	1.055	-0.02
1605	7.593	-2.80	0.061	0.0260	0.0233	0.895	-0.01
2330	7.512	-2.77	0.063	0.0269	0.0218	0.810	0.00
6 Aug 1956							
1805	7.188	-2.65	0.071	0.0303	0.0557	1.835	0.00
2105	7.196	-2.66	0.070	0.0299	0.0216	0.722	0.07
2205	7.204	-2.66	0.070	0.0299	0.0454	1.515	0.01
7 Aug 1956							
0405	6.295	-2.32	0.098	0.0418	0.0970	2.320	0.01
0805	6.330	-2.33	0.097	0.0415	0.0979	2.360	-0.04
0905	6.337	-2.34	0.096	0.0410	0.1666	4.060	-0.02
1105	6.350	-2.34	0.096	0.0410	0.1032	2.510	-0.08
1330	7.111	-2.62	0.073	0.0312	0.0755	2.420	-0.04
1605	7.087	-2.61	0.074	0.0316	0.0864	2.730	-0.01
1805	7.108	-2.65	0.071	0.0306	0.0437	1.540	-0.01
2005	7.057	-2.60	0.074	0.0316	0.0341	1.069	0.05
2205	7.000	-2.56	0.074	0.0329	0.0153	0.465	0.13

TABLE 4. (Cont'd)

Date CST	log Ro	$p = -0.369 \log R_o$	e^p	C_a	C	C/C_a	Ri_{140}
27 Aug 1956							
1230	7.390	-2.73	0.065	0.0278	0.0325	1.170	-0.07
1505	7.361	-2.72	0.066	0.0282	0.0307	1.090	-0.02
1705	7.368	-2.72	0.066	0.0282	0.0318	1.125	-0.02
1930	7.168	-2.64	0.071	0.0303	0.0023	0.076	0.15
2205	7.392	-2.72	0.066	0.0282	0.0022	0.078	0.04
29 Aug 1956							
2135	6.377	-2.35	0.095	0.0406	0.0669	1.640	0.03

TABLE 5. - C/C_a in stable conditions

i	$Ri_{140} = x_i$	$C/C_a = y_i$	$\ln y_i = Y_i$	$Y_i x_i$	x_i^2
1	0.057	0.975	-0.025	-0.001	0.003
2	0.036	1.385	0.326	0.012	0.001
3.	0.035	1.830	0.604	0.021	0.001
4.	0.099	0.209	-1.565	-0.155	0.010
5.	0.051	0.680	-0.386	-0.020	0.003
6.	0.067	0.722	-0.326	-0.022	0.004
7.	0.014	1.515	0.415	0.006	0.000
8.	0.008	2.320	0.842	0.007	0.000
9.	0.054	1.069	0.068	0.004	0.003
10.	0.127	0.465	-0.766	-0.010	0.016
11.	0.153	0.076	-0.274	-0.042	0.023
12.	0.040	0.078	-0.248	-0.010	0.002
13.	0.031	1.640	0.495	0.015	0.001
Sums				-0.283	0.068

(ii) For unstable conditions (table 6), the scatter of points of C/C_a versus Ri_{140} suggests a curve of the form

$$y = 1 + k_1 x e^{k_2 x} \qquad (33)$$

where $-Ri_{140} = x$ and $C/C_a = y$. Again C/C_a has been forced to approach unity as Ri approaches zero. If we let $y-1 = \varphi$ and then take natural logs

$$\ln \varphi = \ln k_1 + \ln x + k_2 x \qquad (34)$$

$$\ln \frac{\varphi}{x} = \ln k_1 + k_2 x$$

Let $\ln \frac{\varphi}{x} = Y$ and $\ln k_1 = a$, and substitute in (34)

$$Y = a + k_2 x$$

The normal equations are

$$\bar{Y} - a - k_2 \bar{x} = 0$$

$$\sum_{i=1}^{m} Y_i x_i - a \sum_{i=1}^{m} x_i - k_2 \sum_{i=1}^{m} x_i^2 = 0 \qquad (35)$$

Solving for k_1 and k_2

$$k_1 = 30.364 \;, \quad k_2 = -13.324$$

Substituting these constants in (33) and replacing x by $-Ri_{140}$, y by C/C_a, gives

$$C/C_a = 1 - 30.364\, Ri_{140} \exp\left[13.324\, Ri_{140}\right] \qquad (36)$$

Note that C/C_a of equation (36) may be maximized for $Ri_{140} = \frac{1}{k_2} = -0.075$, for which $(C/C_a)_{max}$ is 1.839. This is close to the maximum of the fitted curve of fig. 5. Lettau's graph of C/C_a has been included in fig. 5 and it may be noted that the computations of this paper are in general agreement with those of Lettau, although his data did not span as large a range of Ri values. The results indicate that $C/C_a > 1$ in unstable conditions, whereas $C/C_a < 1$ in stable conditions.

TABLE 6 - C/C_a in Unstable Conditions

i	$x_i = -Ri_{140}$	$y_i = C/C_a$	$\varphi_i = y_i - 1$	φ_i / x_i	$\ln \frac{\varphi_i}{x_i} = Y_i$	x_i^2	$Y_i x_i$
1	0.463	1.045	0.045	0.097	-2.333	0.214	-1.080
2	0.165	1.600	0.600	3.630	1.289	0.027	0.213
3	0.280	1.445	0.445	1.590	0.464	0.078	0.130
4	0.251	1.160	0.160	0.638	-0.449	0.063	-0.113
5	0.199	1.475	0.475	2.390	0.871	0.040	0.173
6	0.053	1.785	0.785	14.700	2.688	0.003	0.142
7	0.017	2.320	1.320	77.600	4.352	0	0.074
8	0.147	1.305	0.305	2.080	0.737	0.022	0.108
9	0.210	1.145	0.145	0.690	-0.371	0.044	-0.078
10	0.157	1.305	0.305	1.940	0.663	0.025	0.104
11	0.091	1.570	0.570	62.600	4.137	0.008	0.376
12	0.141	1.615	0.615	4.360	1.472	0.020	0.208
13	0.058	1.860	0.860	14.800	2.695	0.003	0.156
14	0.022	2.150	1.150	52.300	3.957	0	0.087
15	0.103	1.160	0.160	1.550	0.438	0.011	0.045
16	0.011	1.220	0.220	20.000	2.996	0	0.033
17	0.017	1.305	0.305	17.950	2.888	0	0.049
18	0.019	1.055	0.055	2.890	1.061	0	0.020
19	0.042	2.360	1.360	32.400	3.478	0.002	0.146
20	0.077	2.510	1.510	19.600	2.976	0.006	0.229
21	0.043	2.420	1.420	33.000	3.500	0.002	0.151
22	0.011	2.730	1.730	157.300	5.058	0	0.056
23	0.010	1.540	0.540	54.000	3.989	0	0.040
24	0.075	1.170	0.170	22.700	3.122	0.006	0.234
25	0.024	1.090	0.090	3.750	1.322	0.001	0.032
26	0.024	1.125	0.125	5.200	1.649	0.001	0.040
Sum	2.710				52.649	0.576	1.575

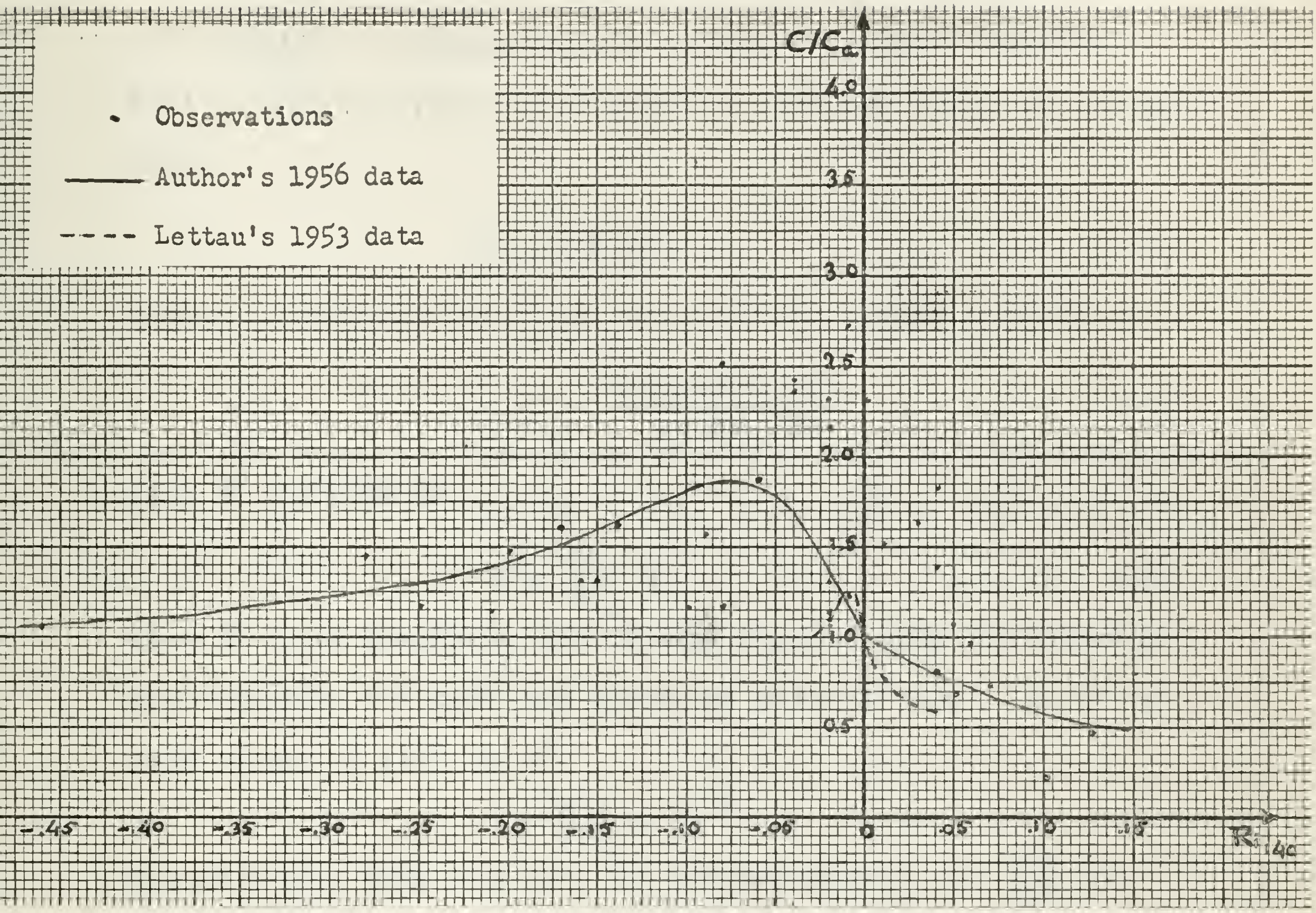

Fig. 5.

Ratio C/C_a as a function of Richardson number at $Z = 140$ cm, at O'Neill, Nebraska 1956 with C_a given by equation (28).

8. The normalized—logarithmic wind shear S

It was explained in section 4 that if one knows $S = \frac{kz\,\partial\bar{u}/\partial z}{u_*}$, one may obtain directly C_f/C, or u_{*f}/u_*. Ellison [2] has given a prediction equation for S in unstable cases

$$S^4 + \frac{\gamma' z}{L'} S^3 = 1 \qquad (37)$$

where Z is height and L' is the gradient length defined as

$$L' = -\frac{u_* \theta\, \partial\bar{u}/\partial z}{kg\, \partial\theta/\partial z} \qquad (38)$$

with γ' defined as

$$\gamma' = \frac{K_h}{K_m}\gamma \qquad (39)$$

here γ is a constant, and K_h and K_m are eddy diffusivity coefficients for heat and momentum respectively. Dividing both sides of (37) by S^4

$$1 + \frac{\gamma' z}{L' S} = S^{-4}$$

Substituting (20) and (18) in the expression for $\frac{\gamma' z}{L' S}$ and solving for S

$$S = (1 - \gamma' R_i)^{-1/4} \qquad (40)$$

Blackadar et al [2] suggest the value $\gamma' = 18$ for unstable conditions. However in stable conditions, they find that equation (40) gives no better approximation than the Monin-Obukhov "log + linear" wind profile defined by

$$S = 1 + \frac{\alpha}{L'} z, \quad \alpha = \text{constant} \qquad (41)$$

which is ordinarily valid for Richardson numbers near one. Again, it may be verified that (41) is equivalent to an equation

$$S = 1 + \alpha' Ri \tag{42}$$

Blackadar et al [2] have tabulated values of α' at O'Neill for various layers. For the layer 1-2 m under stable conditions, the best-fitting value of the constant was $\alpha' =$ [illegible] which will be used for the stable cases in this paper.

The computations for S are shown in table 7. The values of S are presumed applicable at the height assigned to Ri, that is at $Z = \sqrt{2}$ m or $Z = 141.4$ cm. The curve of S is shown in fig. 6. The result indicates that $S < 1$ in unstable cases, and $S > 1$ in stable cases. Whether S in stable conditions levels off with a further increase of Ri, is not certain yet, since the value chosen for α' may not be valid for large positive values of Ri.

TABLE 7. - Results of computations for S.

Date CST	Ri_{140}	Vg (kts)	$S_{(140)}$
10 July 1956			
1305	-0.46	13.60	0.572
1405	-0.17	12.80	0.708
1505	-0.28	12.00	0.638
1530	-0.25	15.64	0.652
1605	-0.20	11.60	0.683
1705	-0.05	10.80	0.850
1805	-0.02	10.52	0.935
2105	0.06	12.00	1.515
23 July 1956			
0930	-0.15	19.44	0.724
1105	-0.21	19.00	0.676
1205	-0.16	18.00	0.715
1305	-0.09	17.00	0.784
1605	-0.14	16.20	0.729
1705	-0.06	16.00	0.836
1805	-0.02	15.80	0.918
2130	0.04	9.52	1.326
2305	0.03	7.52	1.313
25 July 1956			
0205	0.10	4.80	1.888
0405	0.05	9.60	1.457
0805	-0.10	24.00	0.769
0905	-0.01	26.00	0.958
1230	-0.02	32.00	0.936
1405	-0.02	46.00	0.929
1605	-0.01	54.00	0.960
2330	0.00	44.80	1.033
6 Aug 1956			
1805	0.00	10.60	0.994
2105	0.07	10.80	1.605
2205	0.01	11.00	1.123
7 Aug 1956			
0405	0.01	11.88	1.076
0805	-0.04	12.88	0.868
0905	-0.02	13.08	0.939
1105	-0.08	13.48	0.804
1330	-0.04	15.84	0.867
1605	-0.01	15.00	0.958
1805	-0.01	15.76	0.960
2005	0.05	14.00	1.490
2205	0.13	12.40	2.147

TABLE 7. (Cont'd)

Date CST	Ri_{140}	V_g (kts)	$S_{(140)}$
27 Aug 1956			
1230	-0.07	26.48	0.808
1505	-0.02	24.80	0.915
1705	-0.02	25.20	0.914
1930	0.15	15.92	2.375
2205	0.04	26.60	1.362
29 Aug 1956			
2135	0.03	7.20	1.278

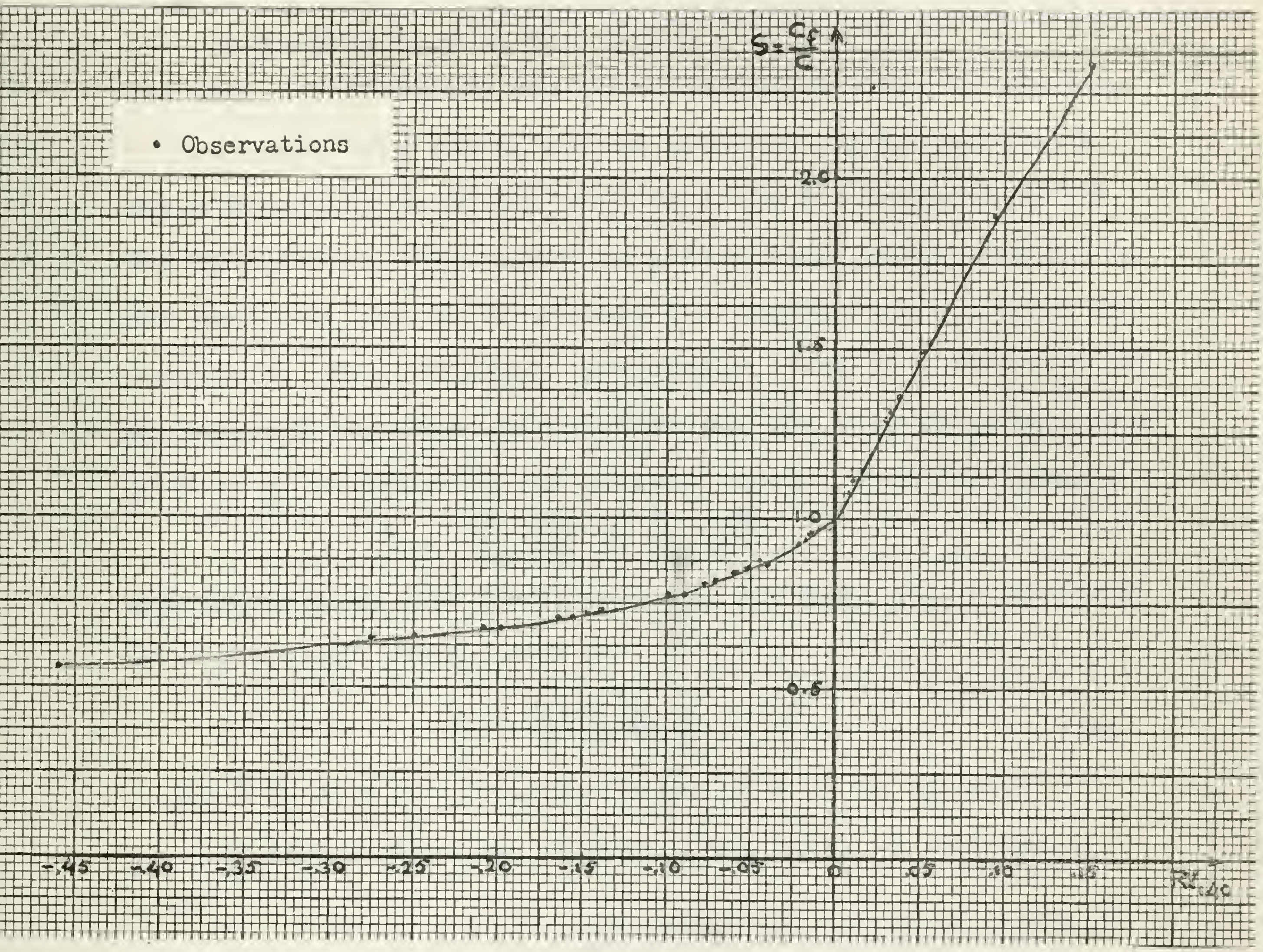

Fig. 6.

The normalized logarithmic wind shear S as a function of Richardson number at Z = 140 cm based on 1956 O'Neill, Nebraska data.

9. An alternative method of determination of u_*

In section 7, C/C_a was computed by equations (8), (12), and (28). While there may be considerable error in assuming a functional form for C_a of the type derived in section 6, there is also a possibility of error in the computation of u_* by equation (8). Hence an alternative computation of u_* was employed, one which employs the expressions for S of the section 8. This approach essentially leads to a value of u_* in terms of the wind-profile $u(z)$ and the integral-mean value of S in a sub-layer of the surface layer. Thus for example

$$S = \frac{kz}{u_*}\frac{\partial \bar{u}}{\partial z} = \frac{k}{u_*}\frac{\partial \bar{u}}{\partial \ln z}$$

and integrating from $Z = Z_0$ to $Z = Z$, assuming u_* is constant in the surface layer leads to

$$u = \frac{u_*}{k}\bar{S}\ln\frac{z}{z_0} \qquad (43)$$

where $\bar{S}$ is the integral-mean value of S up to level Z.

As in the previous section S was taken in accordance with

$$S = (1 - 18\,Ri(z))^{-1/4} \qquad \text{unstable cases}$$

$$(44)$$

$$S = 1 + 9\,Ri(z) \qquad \text{stable cases}$$

Although Priestly [8] has raised a question regarding the the validity of a constant u_* with height in stable conditions, it was assumed that a surface layer at the depth of at least 2m exists for both stable and unstable cases.

Values of S according to equation (44) were plotted using Ri (Z) for values of Z = 5, 35, 70, 140, 280cms. The lowest value of height corresponds to a level at the geometric mean of the roughness parameter Z_0 and $Z = 25$ cm, using the known values of $\partial u/\partial z$ and $\partial \theta/\partial z$ for this layer. The graphs of S versus $\ln z$ was plotted and the integral-mean value $\bar{S}$ extracted. Fig. 7 shows an example of an unstable and stable case which occurred at 1905 and 2105 CST, respectively on 10 July 1956. This permits one to solve for u_* by means of

$$u_* = \frac{k\, u(200)}{\bar{S} \ln(200/z_0)} \tag{45}$$

The values of u_* computed by equation (45) are displayed in table 8 along with the values of u_*, listed earlier in table 1. The absolute average error is approximately 10%, although the algebraic error is 3%.

Note that the ratio C_f/C_a may be obtained as the product

$$\frac{C_f}{C_a} = \frac{C_f}{C}\,\frac{C}{C_a}$$

$$= S\,\frac{u_*/V_g}{C_a} \tag{46}$$

The value of S as a function of Ri(Z) may be obtained from equation (44). Values of u_* may be obtained from equation (45); and C_a may be determined from equation (28).

The main purpose of this last discussion is to emphasize that C_f is not equal to C_a. Listed below in table 8 are some computed values of C_f/C_a.

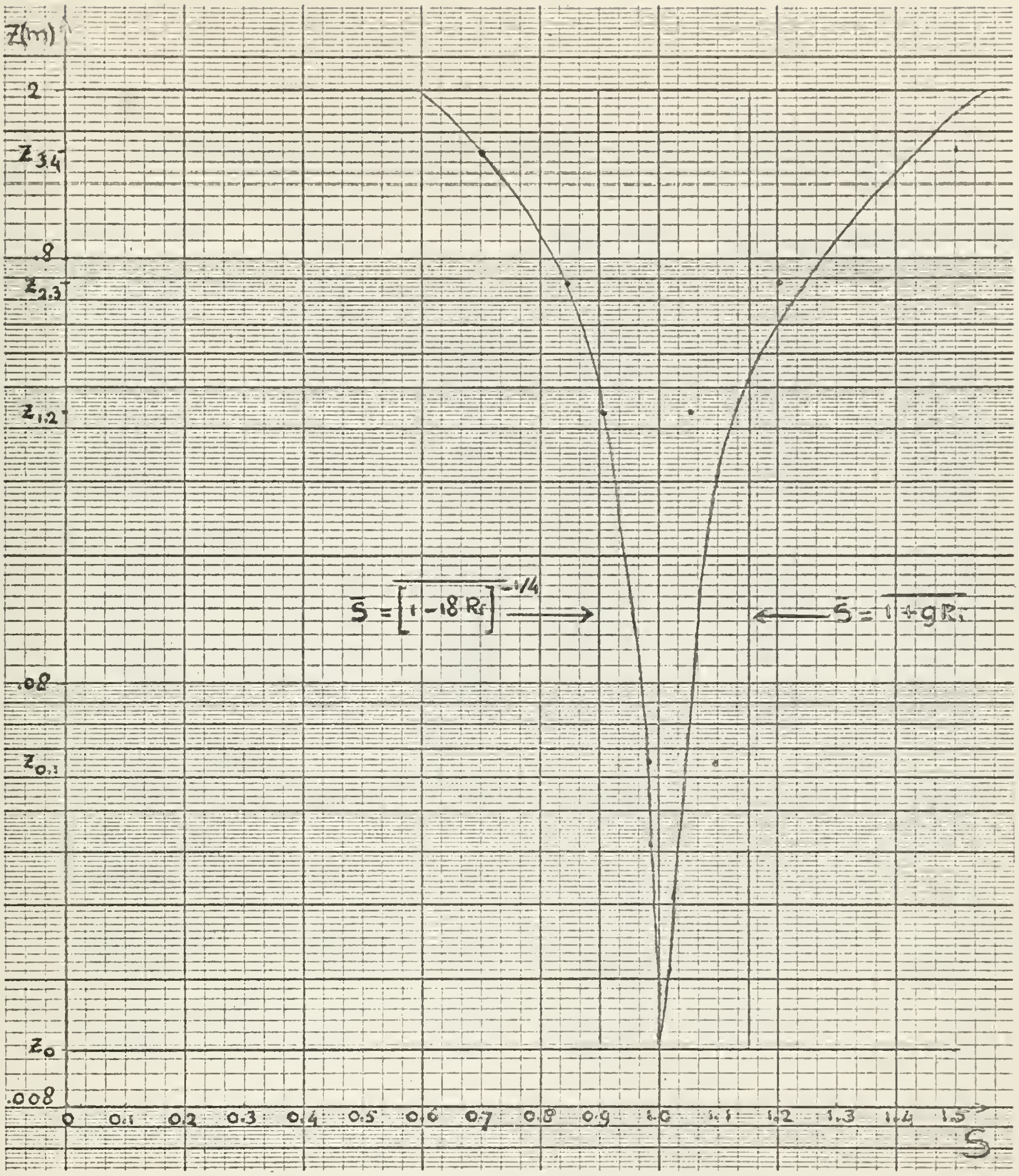

Fig. 7

The normalized logarithmic wind shear S as a function of $\ln z$ for unstable and stable conditions for 10 July 1956.

TABLE 8. C_f/C_a in unstable and stable cases

Date CST	Ri_{140}	$\overline{S}$	u_* (cm/sec)	$(C_f/C_a)_{140}$	u_* (cm/sec) from (3)
10 July 1956					
1305	-0.46	0.572	25.1	0.605	24.7
1405	-0.17	0.900	35.0	1.106	35.9
1505	-0.28	0.885	31.3	0.925	31.2
1605	-0.20	0.900	31.6	1.038	35.8
1705	-0.05	0.950	34.8	1.515	35.0
1805	-0.02	0.970	43.0	2.100	44.0
2105	0.06	1.150	20.5	1.435	21.0
23 July 1956					
1105	-0.21	0.850	35.7	0.648	42.9
1205	-0.16	0.880	38.7	0.773	46.7
1305	-0.09	0.885	44.3	1.025	52.9
1605	-0.24	0.885	42.6	0.956	52.6
1705	-0.06	0.950	46.7	1.215	59.7
2305	0.04	1.115	27.1	2.030	31.8
25 July 1956					
0405	0.05	1.100	16.9	1.325	12.7
6 Aug 1956					
2105	0.07	1.200	12.1	1.210	11.7
7 Aug 1956					
2005	0.05	1.100	22.6	1.517	23.9
27 Aug 1956					
2205	0.04	1.00	10.6	0.386	7.98

10. Summary and conclusions.

A functional relationship between C/C_a and Ri was obtained by using an expression for C_a suggested by Lettau. Better agreement was obtained after making some revisions in Lettau's formula for C_a. An expression for C_a has the practical significance that one can get directly the ratio of a windspeed at some anemometer height to a surface geostrophic wind by knowing the roughness parameter.

By using the revised expression for C_a, a new functional relationship between C/C_a and Ri was obtained. This function has different analytical forms in unstable and stable conditions.

Another way of obtaining C/C_a was based upon computation of u_* using Blackadar's results. By this means a wind-profile expression was obtained which varies with stability. From this new procedure one can get the value of C_f/C_a without using C/C_a.

APPENDIX

Computation for C_a using Lettau's formula

The original equation is

$$C_a = \frac{0.104}{\log(C_a R_o) - 2.24}$$

By cross multiplying

$$\log(C_a R_o) = 2.24 + 0.104/C_a$$

or

$$C_a R_o = 10^{(2.24 + 0.104/C_a)}$$

$$= 173.8 \times 10^{0.104/C_a}$$

then

$$\frac{C_a R_o}{173.8} = \exp\left(\frac{0.104 \times 2.303}{C_a}\right)$$

$$= \exp(0.2395/C_a)$$

Let $0.2395/C_a = y$ and substituting in the above equation

$$F(y) = \frac{R_o}{173.8}\left(\frac{0.2395}{y}\right) - e^y = 0$$

$$= \frac{0.2395/173.8\, R_o - y e^y}{y} = 0$$

Since $y \neq 0$

$$f(y) = 0.2395/173.8\, R_o - y e^y = 0 \qquad (47)$$

Taking the logs to the base 10

$$\log\left(\frac{0.2395}{173.8} R_o\right) = \log y + 0.4343 y \qquad (48)$$

Let $\log\left(\frac{0.2395}{173.8} R_o\right) = K =$ constant, and replace log y by $0.4343 \ln y$ and substitute in (48)

$$f(y) = 0.4343 \ln y + 0.4343 y - K = 0 \qquad (49)$$

To get the root of equation (49), we will use the

Newton-Raphson method

$$y_{i+1} = y_i - \frac{f(y_i)}{f'(y_i)} \tag{50}$$

Since $f'(y) = 0.4343/y + 0.4343$, therefore by substituting this in (50) we obtain

$$y_{i+1} = y_i - \frac{0.4343 \ln y_i + 0.4343 y_i - K}{0.4343/y_i + 0.4343}$$

Substituting $\log\left(\frac{0.2395}{173.8} R_o\right)$ for K and divide the fraction by $\frac{0.4343}{0.4343}$,

$$y_{i+1} = y_i - \frac{\ln y_i + y_i - 1/0.4343 \log\left(\frac{0.2395}{173.8} R_o\right)}{1/y_i + 1}$$

Changing $\ln y_i$ to 2.3026 log y_i

$$y_{i+1} = y_i - \frac{2.3026 \log y_i + y_i + 6.5927 - 2.3025 \log R_o}{1/y_i + 1} \tag{51}$$

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