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AMERICAN MACHINE & FOUNDRY COMPANY

ALEXANDRIA DIVISION

ION AND ATOMIC BEAMS IN SPACE
Final Report
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ABSTRACT

Analysis of the problem of space charge dispersion of a charged ion beam indicates that the limitations imposed by this effect are quite severe. As an example of this fact, a calculation is made using protons at a velocity of one tenth that of light for a beam density of 10^6 particles per cubic centimeters at the point of origin. It is found that the beam radius increases by a factor of 8000 in a distance of 41 miles.

Beam dispersion due to random thermal velocity distribution is analyzed. It is found that this effect is not as severe as that of space charge dispersion. However, it appears that the limit of use of the beam for communications will be of the order of a few thousand miles at most. It is indicated that this limit can be extended if the particle velocity is near that of light or if the generation temperature is near absolute zero.

The effects of Rayleigh, Thompson, and Compton scattering of light by beam particles in the presence of the radiation field of the sun are analyzed. It is found that these effects are negligible in relation to those of space charge and thermal dispersion. It is concluded that radiation pressure effects are quite small.

The analysis of solar winds and particle clouds in space indicates that the beam particle mean free path is quite large so that there is no significant limitation imposed by their presence. However, these particles constitute a background of "noise" so that beam dispersion by space charge

and thermal effects will impose an adverse signal-to-noise ratio upon the information carried. Thus it appears that the presence of the particles imposes a limit upon the density reduction which can be tolerated in the beam.

The problem of neutral beam generation is discussed. The recombination coefficient is developed as a function of temperature and density and a possible mechanism for neutral beam generation is shown. It is indicated that the problems are rather severe.

The possibility of employing the "pinch effect" upon a beam in the presence of the atmosphere is discussed. It is concluded that this is not feasible since the pinch is unstable even if it occurs. Power requirements for any sustained operation are in the megawatt range.

Use of the radiation produced by impact ionization of the atmosphere by means of a beam is analyzed. This is shown to be impractical on the basis of beam power requirements.

Possible applications of the beam to propulsion, power transfer, and warfare are indicated. It is concluded that propulsion offers the best possibility. A propulsion unit based upon the use of ac power is presented.

Deviation and dispersion of the beam in electrostatic and magnetic fields in space are analyzed. Although some of the effects are fairly severe for charged beams, the limitations imposed are not as great as those already found.

The geometrical problems of aiming a beam to strike a target in the fields in space are considered. Solutions are given for uniform fields and for inverse square attractive and repulsive fields. The analysis of the magnetic case is limited to the uniform field.

The general conclusion reached is that long-range communications by the use of ion beams in space is not feasible.

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I. INTRODUCTION

In the development of the work under Contract DA-36-039 SC-78961, it was not originally anticipated that the distance limitation imposed by the dispersion of the beam due to space charge and thermal divergence would be nearly as severe as that which was found. For this reason, some of the work which was done appeared somewhat pointless in the final analysis. The section concerned with beam trajectories under the action of external fields over long distances of travel falls within this general category. In order that it may be included without interrupting the continuity of the pertinent developments it is placed at the end of the final report, although it originally appeared in the first Semi-Annual Report. In general, there is no attempt to preserve the chronological development of the work.

II. BEAM ANALYSIS

A. GRAVITATION CONSTRICTION OF AN ION BEAM

If a particle of mass, M, is exterior to a cylindrical beam of infinite length and mass per unit length, M_1, the particle is attracted as if the total beam mass were concentrated at the axis of the cylindrical beam. If the symbol a is used to represent the perpendicular distance from the point mass M to the axis of the mass cylinder, the force acting is given by

$$F = \frac{2GMM_1}{a}, \tag{1}$$

where G is the universal gravitational constant. The conditions applying are indicated in Figure 1.

The value of M_1, as represented in Figure 1, is given by

$$M_1 = \pi b^2 \rho . \tag{2}$$

The radius of the cylindrical beam is b, and the mass density is ρ. If each beam particle also has a mass M and if there are N particles per unit volume, the value of ρ is given by

$$\rho = NM \tag{3}$$

The use of Equations (2) and (3) in (1) provides

$$F = \frac{2\pi G b^2 N M^2}{a}, \quad a>b . \tag{4}$$

The force on a particle at the surface of such a beam is then

$$F = 2\pi G b N M^2, \quad a = b. \tag{5}$$

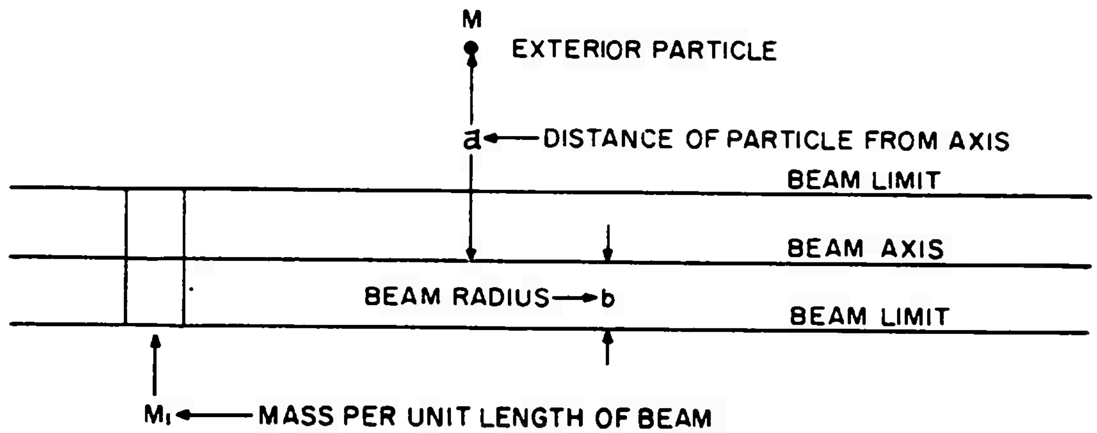

Figure 1. The force between a cylindrical beam and a mass particle.

Since an interior particle is not affected by that portion of the cylindrical beam exterior to its own radial position, such a particle experiences a force

$$F = 2\pi GrNM^2, \quad r<b, \tag{6}$$

where r is the perpendicular distance between the beam axis and the interior particle. Since r is zero for a position on the beam axis, a particle at this point experiences no net force. At all other points, there exists an attraction. The general effect of gravitational forces is that of constricting the beam.

B. ELECTROSTATIC REPULSION

In a beam composed of like charges, the effect is that of mutual repulsion. However, there is no need to make a new analysis since the form of the equation is not changed. In case the beam occupies a space with unit dielectric constant, the symbol G is replaced by this (reciprocal) value. The mass of the particle is replaced by the ionic charge q. Equation (4) then becomes

$$F = \frac{2\pi b^2 Nq^2}{a}, \quad a>b. \tag{7}$$

Equations (5) and (6) for the case of electrostatic repulsion may be written by analogy with this one. These are

$$F = 2\pi b Nq^2, \quad a = b \tag{8}$$

and

$$F = 2\pi rNq^2, \quad r<b. \tag{9}$$

C. MAGNETIC CONSTRICTION OF MOVING CHARGES

The magnetic field of a current element follows an inverse-square law similar to those applying to gravitation and electrostatics. The field associated with a cylindrical current may be written

$$H = \frac{2\pi b^2 Nqv}{a}, \quad a>b. \tag{10}$$

The force on a charge q at a distance a from the axis and traveling with a velocity v parallel to the current charges is then

$$F = \frac{2\pi b^2 Nq^2 v^2}{a}, \quad a>b. \tag{11}$$

This is seen to differ from Equation (7) only by the factor v^2. However, it is necessary to observe that Equation (7) is expressed in the electrostatic system of units and Equation (11) is in the electromagnetic system.

A particle at the surface of the beam experiences a force

$$F = 2\pi bNq^2 v^2, \quad a = b. \tag{12}$$

This is a constrictive force. In the event that the particle is in the interior of the beam, that portion of the beam exterior to the radial position of the particle has no effect upon it. In this case Equation (12) is modified to provide

$$F = 2\pi rNq^2 v^2, \quad r>b. \tag{13}$$

D. CALCULATION OF FORCES

In order that a comparison may be made between the force of space charge repulsion and the constrictive effect of the magnetic field, it is necessary to express Equation (12) in electrostatic units by means of a factor of proportionality. This factor in the case of charge is the velocity of light c. Equation (12) then becomes

$$F = 2\pi b N q^2 \frac{v^2}{c^2}, \qquad (14)$$

where it is understood that q is in esu. The difference between the force of electrostatic repulsion and the force of magnetic constriction is then

$$\Delta F = 2\pi b N q^2 \left[1 - \frac{v^2}{c^2}\right]. \qquad (15)$$

This difference becomes zero only in the event that the velocity of the beam particle is equal to the velocity of light. It follows that the beam expands under the action of space charge divergence at any particle velocity which can be attained.

As an example of the application of the equations which have been developed, a proton beam of density 10^6 is assumed. The mass of the proton is 1.67×10^{-24} grams, and its charge is 4.8×10^{-10} esu. If the velocity is taken to be 0.1c, the ratio v^2/c^2 is 0.01. Under these circumstances the factor $1-v^2/c^2$ can be neglected to an error of 1.0 percent. The force difference is approximately equal to the repulsive force given by

$$F = 2\pi \times 30 \times 10^6 \,[4.8 \times 10^{-10}]^2 = 4.34 \times 10^{-11} \text{ dynes}. \qquad (16)$$

In this equation a beam radius of 30 cm or approximately one foot has been assumed.

The ratio of the repulsive force to the force of constriction caused by the particle velocity is the inverse of the v^2/c^2 value. This ratio is then

100/1 under the assumed circumstances. Thus it appears that the constricting effect of the magnetic field will not prevent the spread of the beam.

The gravitational force on a surface particle is given by

$$F = 2\pi \times 6.67 \times 10^{-8} \times 30 \times 10^{6} \times [1.67 \times 10^{-24}]^{2} = 3.5 \times 10^{-47} \text{ dynes.} \quad (17)$$

The ratio of the force of repulsion to that of gravitational constriction is then 1.24×10^{36}. It is seen that gravitational effects are negligible in the prevention of spread of a beam.

E. SPACE CHARGE EXPANSION OF A BEAM

For an infinite cylindrical beam, the particle density is reduced as the beam radius increases. The equation is

$$\pi b_o^2 N_o = \pi b^2 N \quad (18)$$

in terms of original density and radius. The force on a surface particle is then written

$$F = M \frac{d^2 b}{dt^2} = \frac{2\pi b_o^2 q^2 N_o}{b} . \quad (19)$$

Use of the integrating factor 2db then yields the integrated form

$$\left(\frac{db}{dt}\right)^2 = \frac{4\pi b_o^2 q^2 N_o}{M} \text{Log}_e \frac{b}{b_o} , \quad (20)$$

where the constant of integration has been evaluated in terms of the original radius. The radius of the expanding beam then depends upon the integral

$$\frac{db}{\left[\text{Log}_e \frac{b}{b_o}\right]^{\frac{1}{2}}} = 2b_o q \left[\frac{\pi N_o}{M}\right]^{\frac{1}{2}} dt . \quad (21)$$

If a substitution is used in the form

$$\operatorname{Log}_e \frac{b}{b_o} = Z^2 \tag{22}$$

equation (20) becomes

$$\exp(Z^2)\, dZ = q\left[\frac{\pi N_o}{M}\right]^{\frac{1}{2}} dt. \tag{23}$$

Expansion of the exponential function and termwise integration then yields the series

$$\sum_{n=o}^{\infty} \frac{Z^{2n+1}}{(2n+1)n!} = q\left[\frac{\pi N_o}{M}\right]^{\frac{1}{2}} t. \tag{24}$$

This series is convergent for all finite values of Z.

The series has been evaluated for various values of the ratio b/b_o. These are given in Table I. The final column represents the time for a

TABLE I
SUM OF SERIES

$\frac{b}{b_o}$	Z	Sum	Time, sec
10	1.516	4.25	6.46×10^{-6}
100	2.148	27.36	4.16×10^{-5}
1000	2.628	209.37	3.18×10^{-4}
8000	3.000	1445 (approx.)	2.2×10^{-3}

proton beam of 10^6 particles per cm^3 original density to expand to the condition specified by the ratio appearing in the first column. It is seen that these times are quite limited. As an example of the application, a proton beam in which the particles travel at a velocity of 0.1c would expand 8000 diameters in traveling a distance of

$$D = 18600 \times 2.2 \times 10^{-3} = 41 \text{ miles}. \qquad (25)$$

Under the assumed conditions the original current density in the beam is 4.8×10^{-4} amp/cm^2. The power density at this point is 2.26×10^3 watts/cm^3. The accelerating voltage necessary to provide protons with the assumed velocity is 4.7×10^6 volts. At the distance of 41 miles, the reduction ratio of $\frac{1}{(8000)^2}$ must apply. The particle density is then $\frac{1}{64}$ protons/cm^3 in relation to 10^6 protons/cm^3 which applies at the point of generation. The current density is reduced to 7.5×10^{-12} amp/cm^2, and the power density is 3.53×10^{-5} watts/cm^2. Using a velocity of 0.999c will extend the range to about 4000 miles for equivalent conditions and a mass of 100 proton units improves the range only by a factor of 10. The conclusion must be that charged beams over long distances will require the use of extremely high accelerating voltages.

For extremely high velocities, equation (24) must be corrected to account for the constricting effect of the magnetic field and the relativity mass increase. Since the contraction factor is associated with the mass and the square of the contraction factor is associated with the net repulsive force as given by equation (15), equation (24) becomes

$$\sum_{n=o}^{\infty} \frac{Z^{2n+1}}{(2n+1)n!} = q\left[\frac{\pi N_o}{M}\right]^{\frac{1}{2}} \left[\frac{1 - v^2}{c^2}\right]^{\frac{3}{4}} t. \qquad (26)$$

By this equation, no expansion occurs at the velocity of light. However, practical considerations limit the attainment of such a velocity.

F. THE USE OF MIXED BEAMS

The use of beams composed of both positive and negative charges involves certain difficulties. In general, the negative charges in the beam are free electrons and the positives are composed of nuclei or ionized atoms. As an example, let it be supposed that a beam is generated from neutral hydrogen so that it consists of electrons and protons. In order that a charge may not be developed on the generating object, positive and negative charges are expelled in equal numbers. If the net space charge is to be zero along the beam, it is necessary that the electrons should be accelerated by a voltage which is reduced with respect to that applied to the protons. The required condition is

$$\frac{v_p}{M_p} = \frac{v_e}{M_e} \tag{27}$$

where the subscript p refers to the proton and e refers to the electron. In order that proton and electron may have equal velocities in the beam the voltage ratio is then 1840. Thus the transfer velocity of information is limited to the velocity of the heavier particle.

In order to indicate the difficulties associated with neutralizing the beam space charge by the method outlined, let it be assumed that the ratio of accelerating voltages is in error by 1.0%. The ratio of squared velocities is then 0.99 so that the inverse density ratio is 0.995 since the product of density and velocity is a constant for a given current and cross section. Assuming that the electrons have the larger velocity, the density of protons which remain unneutralized is $0.005\ N_p$.

It should be pointed out that in the absence of recombination, space charge divergence will not be limited to those particles in excess of neutralization. The excess protons assumed in the calculation will attract the uncombined electrons and repel the uncombined protons to cause a general spread of the whole beam as they diverge. Only in the event that recombination has occurred to form a neutral atom will a particle be free of forces causing divergence. Beam spread in this case is a function of recombination time.

In the absence of recombination, the value $0.005N_p$ must replace N_o which appears in equation (24). If the original proton density is 10^6 and the beam spreads by a linear factor of 8000 as before assumed, the corresponding time is 0.031 sec. at a velocity of 0.1c, the particles will have traveled a distance of 578 miles as compared to the 41 miles originally found.

From the analysis given it appears that the use of neutral beams is indicated. This conclusion neglects the possibility of the charged particle beam near the velocity of light. In the event that sufficient energy can be given the particle, the effect of space charge divergence can be made negligible. In theory, this makes the use of charged beams possible. However, the practical accomplishment of the fact is most difficult.

G. THERMAL VELOCITY DISPERSION

In any particle beam there is a distribution of thermal velocities oriented at random which is imposed upon the general transfer velocity of the beam particles. These thermal velocities have components along the beam and transverse to the beam axis. The ion velocity distribution along the beam axis results in varying transit times for the ions between transmitter and receiver. The ion velocity distribution transverse to the beam axis causes the beam to diverge. In the analysis of this thermal divergence the assumption will be made that the ion cloud in the beam will have a density so low that ion collisions can be neglected. Analysis of the mean free path to be given later indicates that the assumption is justified. In this case the initial ion velocities remain constant since space charge divergence is not considered. The analysis is thus limited to the consideration of neutral beams.

The initial thermal velocities of a particle cloud follow a Maxwellian velocity distribution function. For purposes of analysis a particle cloud of uniform density and cubical shape is considered. The beam axis is taken through the center of mass of the cube perpendicular to a pair of opposite faces of the cube. The divergence of the ion beam can then be stated in terms of the thermal velocity distribution with respect to the beam axis. This situation is illustrated in Figure 2.

In reference to part (b) of the figure, the number of ions with x components of velocity of magnitude v_x is determined by

$$dN_{vx} = \frac{N}{\sqrt{\pi}} \left[\frac{M}{2kT}\right]^{\frac{1}{2}} e^{-\frac{Mv_x^2}{2kT}} \, dv_x . \qquad (28)$$

The form of the curve is shown in Figure 3. The number of ions with x components of velocity less than some arbitrary value v_{xo} is given by the area

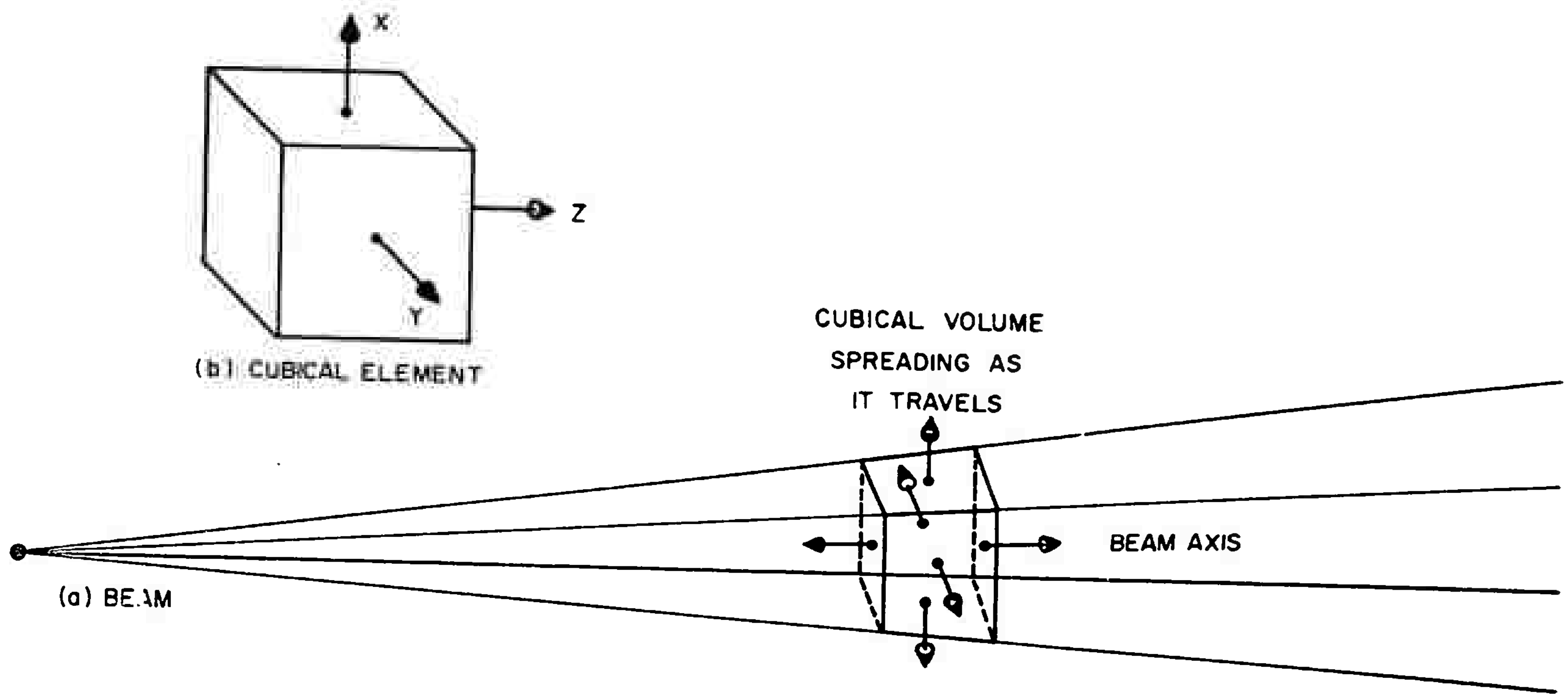

Figure 2. Thermal expansion of a cubical volume.

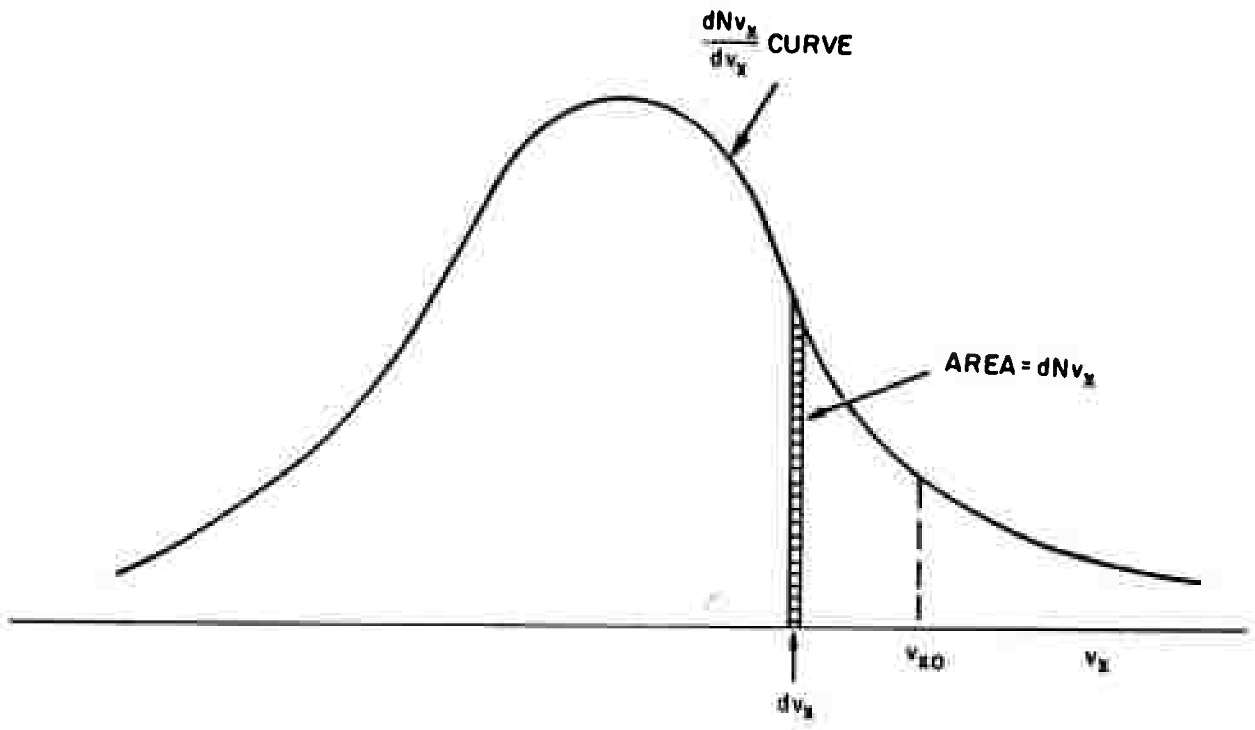

Figure 3. Maxwell–Boltzmann distribution function for a single component of particle velocity.

under the curve, the $\frac{dN_{vx}}{dV_x}$ axis and v_{xo}. This area can be determined by means of the integral

$$N_{o \to v_{xo}} = \int_{o}^{v_{xo}} dN_{vx}. \tag{29}$$

If equation (28) is used in equation (29) and the substitution

$$\frac{1}{v_m} = \left[\frac{M}{2kT}\right]^{\frac{1}{2}} \tag{30}$$

is employed in conjunction with

$$X = \frac{v_x}{v_m} \tag{31}$$

and

$$dX = \frac{dv_x}{v_m}, \tag{32}$$

the integral equation (29) may be written

$$\frac{N_{o \to x}}{N} = \frac{1}{\sqrt{\pi}} \int_{o}^{x} e^{-x^2}\, dx = \frac{1}{2}\ \mathrm{erf}(x). \tag{33}$$

It follows that the number included between the limits -x to +x is just twice this value. Therefore the equation

$$\frac{N_{-x,\ +x}}{N} = \mathrm{erf}(x) \tag{34}$$

is seen to apply.

The symbol k in equation (28) refers to Boltzmann's Constant, and T is the absolute temperature in degrees Kelvin. Other symbols are explained or have appeared in the preceding analysis. The function erf(x) is the error function which is standard to all linear probability analysis. The result given applies to the x dimension only. Since the y dimension is also transverse to the beam axis, a similar result applies to that dimension also. It follows that the fraction of the total number of ions in an x, .y cross section is

$$F = \frac{N_{\left[\begin{smallmatrix}-x, +y\\ -y, +y\end{smallmatrix}\right]}}{N} = \text{erf}\left[\frac{v_{xo}}{v_m}\right] \text{erf}\left[\frac{v_{yo}}{v_m}\right]. \tag{35}$$

Representative values of F are given in Table II. It is seen that the velocity $2v_m$ includes more than 99% of the particles.

TABLE II

Fraction of Total Number of Particles Included Between Various Velocity Limits in Units of v_m

F	$\frac{v_{\left\lvert\begin{smallmatrix}xo\\ yo\end{smallmatrix}\right\rvert}}{v_m}$
0	0
0.046	0.4
0.3647	0.6
0.5507	0.8
0.7101	1.0
0.9069	1.4
0.9532	1.6
0.9906	2.0

The cross section diameter of a beam after a time t is given by

$$d = d_o + 2\, v_{\left\lvert\begin{smallmatrix}x_o\\ y_o\end{smallmatrix}\right\rvert}\, t \tag{36}$$

using the velocity $2v_m$ in equation (36) provides

$$d = d_o + 4\sqrt{\frac{kT}{M}}\, t \tag{37}$$

where the results of equation (30) have been used. The angle subtended at the source is

$$\phi = \frac{4\sqrt{\frac{2kt}{M}}}{v} , \tag{38}$$

where v is the beam propagation velocity. If the propagation velocity is sufficiently near the velocity of light, the relativity mass increase must be considered. In this case, equation (38) becomes

$$\phi = \frac{4\sqrt{\frac{2kT}{M}}}{v} \left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{4}} , \tag{39}$$

when v is equal to the velocity of light the angle subtended is zero. Under these (hypothetical) circumstances the beam does not spread.

As an example of the application of equation (37) let it be assumed that protons at 0.1c are used at a generation temperature of 500°K. In order that conditions may be compared with those given in the analysis of space charge divergence, let it be assumed that the original beam diameter is 1.0 foot. The value of k is 1.38×10^{-16} erg/$^{\circ}$Kelvin. The time for the beam to spread to a diameter of 8000 feet is

$$t = \frac{(8000 - 1)30.48}{4\sqrt{\frac{2 \times 1.38 \times 10^{-16} \times 500}{1.67 \times 10^{-24}}}} = .212 \text{ sec.} \tag{40}$$

In this time the beam particles will have traveled a distance of 3940 miles. This is about two orders of magnitude greater than the range found for equivalent conditions considering space-charge dispersion.

The analysis was made using protons at 500°K. If the generation temperature is reduced, the spread is less severe. Since the temperature factor appears as a square root, the effect of temperature reduction is not pronounced.

In the event that the temperature is reduced by a factor of 100, the time for the beam to spread the specified amount is increased by a factor of 10. The distance then becomes 39400 miles for the specified conditions.

The analysis takes no account of space-charge divergence and thus assumes neutral beams. The calculation was made for protons but is valid for hydrogen atoms since the added electron mass is negligible in relation to the proton mass. For heavier atoms, the proper adjustment of the mass value in the analysis must be made. The angular aperture of the beam depends upon the ratio expressed by equation (38). This aperture differs for different values of temperature, mass, and velocity. However, the beam intensity in every case is proportional to the inverse square of the distance of travel of the particles. This effect limits the range over which a practical communication system will operate. Because of the limitations imposed by thermal velocity dispersion in neutral beams and space-charge dispersion in charged beams, it appears that any practical system based on the use of particle beams will be limited to relatively short distances in space.

III. COLLISION CROSS SECTION OF PARTICLES IN A RADIATION FIELD

A. RADIATION PRESSURE ANALYSIS

The analysis of the radiation cross section of particles of various sizes, shapes, and compositions has been presented in analytical and experimental form.[1] For this reason, no new analysis is necessary in this phase of the investigation. In order that the basic mechanism of the interaction between light and matter may be indicated, it is to be observed that such an interaction results in a reduction in the intensity of the light as it penetrates the region which contains the matter. This reduction is a result of the action of the two mechanisms of scattering and absorption. In the case of scattering, the light is deflected through an angle which effectively removes it from the beam. The recoil action of the intercepting particle may be such that a portion of the incident photon energy is absorbed. In this event, the scattered photon does not have the same energy as the incident photon. In certain circumstances the total energy of the incident photon may be absorbed. In terms of the remaining energy in the photon beam, both processes have resulted in the extinction of a portion of the radiation.

The equation which defines the relation between terms is

$$\text{Extinction} = \text{scattering} + \text{absorption}. \tag{41}$$

The effective cross section can be defined on the basis of this equation. The cross section equation is

$$C_{ext} = C_{sca} + C_{abs}. \tag{42}$$

The division of equation (42) by the geometrical cross section of the intercepting particle will then define a relation between efficiency factors for the different terms. The equation is written

$$Q_{ext} = Q_{sca} + Q_{abs}. \tag{43}$$

This is a relationship of pure numbers. The efficiency factor in general is defined as a number by which the geometric cross section must be multiplied in order to provide an effective cross section in the description of any particular type of interaction.

In the case of nonabsorbing spheres, the value of Q_{abs} is zero. The total value of Q_{ext} is then given by a consideration of the scattering process only. The radiation-pressure efficiency factor is then related to the scattering cross section efficiency factor by the equation

$$Q_{pr} = [1 - \cos\theta]\, Q \tag{44}$$

where θ is the angle of scattering as measured with respect to the direction of incidence of the radiation.

The value of the scattering cross section is dependent upon the parameter $x = \frac{2\pi a}{\lambda}$, where a is the radius of the spherical particle and λ is the wave length of the incident light. A graph giving relative values of Q and Q_{pr} is presented in Figure 4 for various values of the parameter. The curve, from Reference 1, is representative only. It is sufficiently exact to indicate that for a given wave length of light, the radiation-pressure efficiency factor approaches unity for spheres of large diameter and approaches zero for spheres of small diameter. The value is never in excess of 2.5. It follows

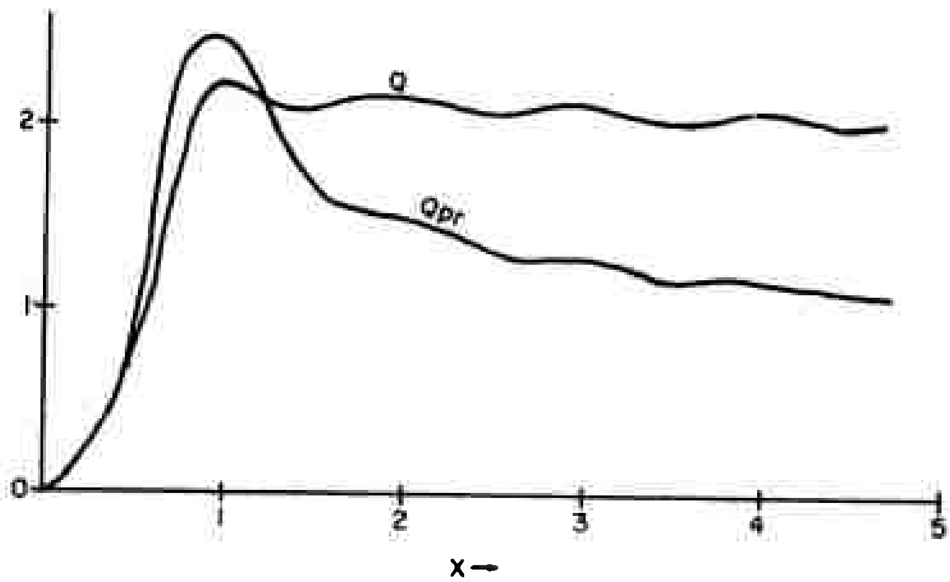

Figure 4. Efficiency factors for extinction (Q) and for radiation pressure (Qpr) for totolly reflecting spheres ($n = \infty$)

that the correct value of radiation pressure will not be as much as one order of magnitude in excess of the value provided by the use of the geometrical cross section of the particle.

Since light must be assumed not to travel within the particle, an infinite index of refraction has been postulated in the analysis. The basic conclusion is not changed by the use of a finite index of refraction. In the case of absorbing spheres, the index of refraction is complex and may be infinite or finite. The radiation pressure cross section is not much affected in any case. A typical plot is given in Figure 5. This figure is also taken from Reference 1. The curve is given for index of refraction 1.27 - 1.37j. The effect of increasing the complex index is to lower the peak value of the efficiency factor. In all cases the lower limit is zero for $x \to \infty$. In these circumstances, it appears that there is no error in the order of magnitude of the radiation-pressure calculation introduced by the use of the geometrical areas of the beam particle in the prediction of radiation-pressure effects.

B. RAYLEIGH SCATTERING

It is to be understood that the type of scattering generally referred to by the term "Rayleigh Scattering" is a particular phase of the general analysis. The Rayleigh scattering is specified by the condition

$$x = \frac{2\pi a}{\lambda} \ll 1 , \tag{45}$$

Thus it appears that the wave length is quite large in relation to the particle dimensions. The condition is satisfied, for example, in the case of visible light ($\lambda = 4000 \times 10^{-8}$ cm) falling upon protons ($2\pi a = 6.28 \times 1.4 \times 10^{-13}$ cm).

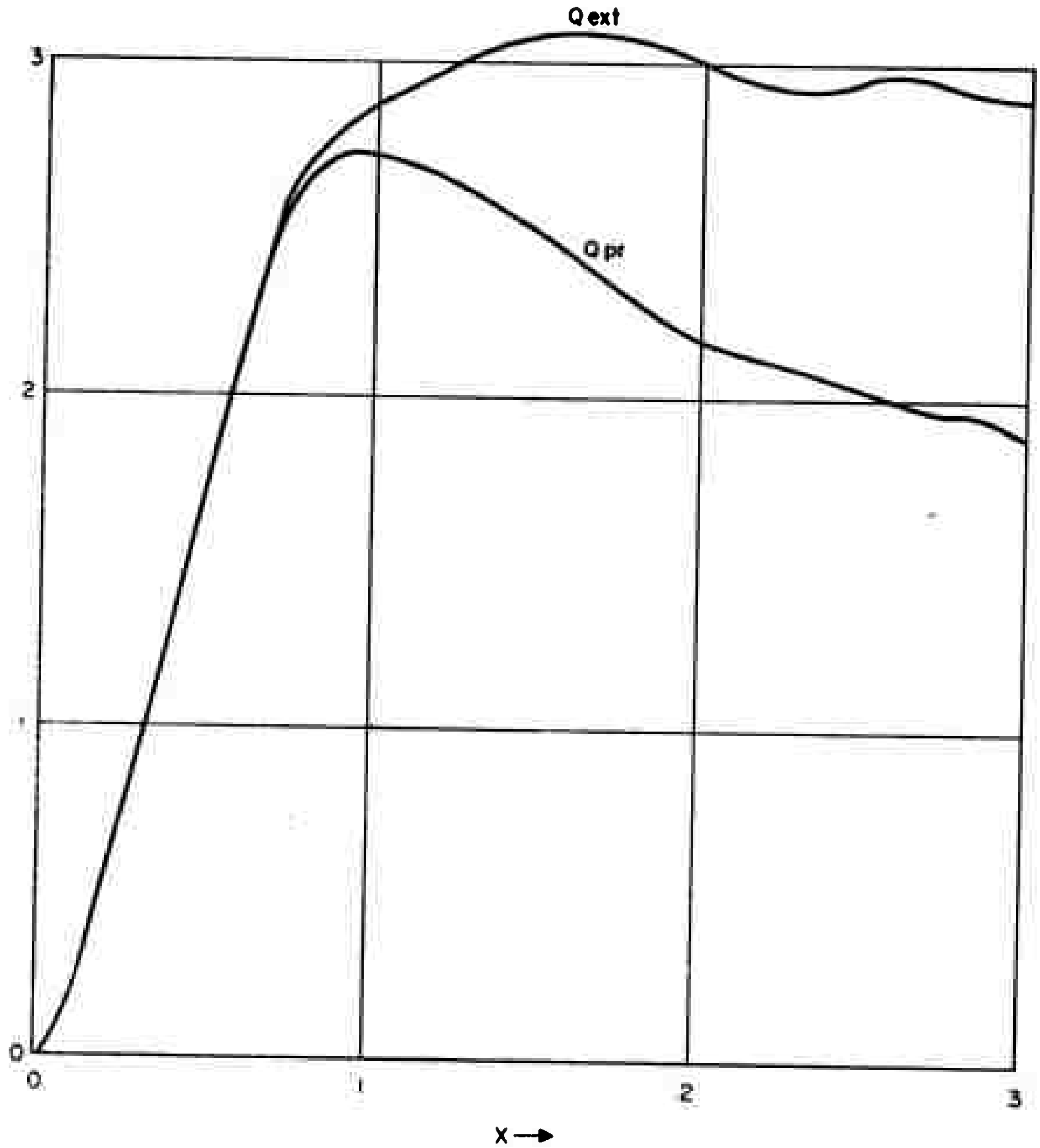

Figure 5. Efficiency factors for extinction and radiation pressure for $n = 1.27 - 1.37j$.

Using the specified values, the value of x is found to be 2.2×10^{-8}. By means of Figures 4 and 5, it appears that the effect of visible light on electron and proton beams is negligible since the particle efficiency factor is practically zero for this value of x. It is concluded that Rayleigh scattering, as specified by the condition $x \ll 1$, cannot be a factor in the determination of the limits of application of ion beams in space.

C. THOMPSON SCATTERING

The type of scattering termed "Thompson Scattering" results from the vibrational response of electrons to electromagnetic wave stimuli which results in the reradiation of a portion of the incident energy. This can be analyzed on the basis of a scattering coefficient which is given in standard texts on the subject.

An idea of the amplitude of the vibrational response of the electron can be gained by an investigation of the physical amplitude of the incident wave. This can be found by means of the observation that displacement velocities in an electromagnetic wave motion are limited to the velocity of light. The wave displacement is represented by the form

$$y = A \sin \omega t. \qquad (46)$$

The velocity of the displacement is

$$\frac{dy}{dt} = A \omega \cos \omega t. \qquad (47)$$

It is necessary that the condition

$$A\omega \cos \omega t \leq c \qquad (48)$$

should apply where c represents the velocity of light. This condition can always be satisfied by the use of the equality

$$A\omega = c \, . \tag{49}$$

Then it follows that the physical amplitude of an electromagnetic wave motion in space is given by

$$A = \frac{c}{\omega} = \frac{\lambda}{2\pi} \, . \tag{50}$$

As an example of the application of this form, the amplitude of a photon of visible light of 4000 angstroms wave length is 6.38×10^{-6} cm. It is to be expected that the amplitude of forced vibration in the electron is not in excess of this value.

The motion of vibration will not tend to deviate electrons in the beam from their original path. The reaction of the momentum of the incident light which is scattered by the particle will do so. Then it is necessary to calculate the magnitude of the expected reaction to see if it is sufficiently large to be a factor in the determination of beam limitations. The value of the scattering coefficient[2] is

$$\sigma = \frac{8\pi N e^4}{3M^2 c^4} \, , \tag{51}$$

where the electrostatic system of units is used. The symbol N represents electrons per cubic centimeter in the beam, e is the electronic charge, M is the electron mass, and c is the velocity of light.

The coefficient represents the fraction of the incident photon energy scattered per centimeter of penetration into the region of space occupied by the electrons. The intensity of the scattered photon energy in relation to the incident photon energy at any point of penetration is then given by

$$\frac{dI}{dx} = -\frac{8\pi Ne^4}{3M^2c^4} I. \qquad (52)$$

By means of the relation between radiation energy and momentum, the momentum per square centimeter per second in the scattered beam may be found. This is given by

$$p = \frac{8\pi Ne^4}{3M^2c^4} \frac{I}{c}, \qquad (53)$$

where $\frac{I}{c}$ represents the incident radiation pressure. From the analysis given, it appears that the scattering coefficient may be considered a fractional effective area in relation to the unit cross sectional area involved.

The equation relating efficiency factor to the coefficient is seen to be

$$N\pi[2.8 \times 10^{-13}]^2 Q = \frac{8\pi Ne^4}{3M^2c^4}, \qquad (54)$$

where the value 2.8×10^{-13} represents the radius of the electron in centimeters. The value of the efficiency factor Q is found to be 2.7 by means of equation (54). Using the radiation pressure value of 4.5×10^{-5} dynes/cm^2 as applying to the radiation field of the sun at the distance of the earth, the force on an electron is

$$F = 2.7 \times 3.14 \times [2.8 \times 10^{-13}]^2 \times 4.5 \times 10^{-5} = .3 \times 10^{-29} \text{ dynes}. \qquad (55)$$

The resulting acceleration of the electron is 0.033 cm/sec^2.

The analysis has been developed for the electron. The proton mass is 1840 times as large as the electron mass. The scattering coefficient as applied to the proton is less than that for the electron by a factor of 2. In the final form for force, this factor also appears as a square in the denominator. Thus, the analysis provides the fact that the effect of radiation pressure on an electron beam is sufficiently small that it cannot impose limitations of any relative severity upon the use of the beam. For protons, or heavier particles, the effect is infinitesimal in relation to that for the electron. It is to be concluded that Thompson Scattering can be neglected in the determination of the limits of use of a particle beam in space.

D. COMPTON SCATTERING

The scattering coefficient developed by Klein and Nishina[3] is given by

$$\sigma = \frac{8\pi Ne^4}{3M^2c^4} \cdot \frac{3}{4}\left\{\frac{1+\zeta}{\zeta^2}\left[\frac{2(1+\zeta)}{1+2\zeta} - \frac{1}{\zeta}\log(1+2\zeta)\right] + \frac{1}{2\zeta}\log(1+2\zeta) - \frac{1+3\zeta}{(1+2\zeta)^2}\right\}, \quad (56)$$

where ζ takes the value

$$\zeta = \frac{hf}{Mc^2}, \quad (57)$$

which is the ratio of incident photon energy to the particle rest energy. For the condition $\zeta \to 0$, the scattering coefficient reduces to the standard form for Thompson Scattering. If visible light is assumed, this condition is met and Compton Scattering does not apply to any appredicable extent. For hard gamma radiation, the expression reduced to

$$\sigma = \frac{\pi Ne^4}{hfMc^2}\left[\log\frac{2hf}{Mc^2} + \frac{1}{2}\right]. \quad (58)$$

For the condition $hf = .8Mc^2$, equation (58) takes its maximum value

$$\sigma = 1.25\frac{\pi Ne^4}{M^2c^4}. \quad (59)$$

Since this is smaller than the Thompson coefficient and is reduced with increasing photon frequency, it appears that Compton Scattering will be of negligible importance in the determination of beam deviation.

IV. MEAN FREE PATH CALCULATIONS

A. MEAN FREE PATH OF PHOTONS AND PARTICLES

The analysis presented to this point has been concerned with the deviation of the beam as a whole. As a matter of fact, the interaction of photons and particles occurs on a statistical basis. If the photon is sufficiently energetic and the interaction sufficiently strong, the final effect is that of removing the particle from the beam. The analysis of the general average of radiation pressure assumed to apply uniformly to all beam particles will not provide a value of beam attenuation with distance caused by random photon impacts. Under the circumstances a knowledge of mean free path of particles in a photon field is necessary.

The process of attenuation of a beam penetrating a field in which interactions occur is an exponential function of the distance of penetration. The attenuation constant is the reciprocal of the mean free path[4]. If this concept is applied to photons in an electron field, the photon mean free path is given by

$$L = \frac{3M^2 c^4}{8\pi N e^4}, \tag{60}$$

which is the reciprocal of the Thompson Scattering coefficient. If an electron density of 10^6 is assumed, the mean free path corresponding to this value is 1.5×10^{18} cm.

It is now necessary to find the mean free path of an electron in a photon field. The mean free path of particles of one kind in a distribution of particles of another kind is given by[4]

$$L_1 = \frac{1}{\pi\left[\frac{\sigma_1 + \sigma_2}{2}\right]^2 N_2 \sqrt{\frac{v^2 + c^2}{v}}} . \tag{61}$$

The mean free path of particles of the second kind in a distribution of particles of the first kind is

$$L_2 = \frac{1}{\pi\left[\frac{\sigma_1 + \sigma_2}{2}\right]^2 N_1 \sqrt{\frac{v^2 + c^2}{c}}} . \tag{62}$$

In the particular application, it is to be understood that v represents the electron velocity and c represents the photon velocity. The photon mean free path is L_2 and the electron mean free path is L_1. The photon density is represented by N_2 and the electron density by N_1. The quantity $\frac{\sigma_1 + \sigma_2}{2}$ represents the radius of the sphere of exclusion of the particles and photons. When the ratio of L_1 to L_2 is taken, the simple form

$$\frac{L_1}{L_2} = \frac{v}{c}\,\frac{N_1}{N_2} \tag{63}$$

is found. The use of equation (60) in equation (63), with the proper interpretation, provides

$$L_1 = \frac{3M^2 c^3 v}{8\pi N_2 e^4} . \tag{64}$$

It is seen that the electron density does not appear in the final form.

If the photon field is predominantly composed of visible light, a value of the photon density N_2 can be estimated using a particular wave length of radiation. The radiation pressure in the photon field is 4.5×10^{-5} dynes/cm^2. In energy units this is equivalent to ergs/cm^3. Very obviously the energy of an average photon multiplied by the photon density will equal the photon-energy density. The use of a wavelength of 4000×10^{-8} cm for the average photon in the radiation field then yields

$$N_2 = \frac{4.5 \times 10^{-5} \times 4000 \times 10^{-8}}{6.624 \times 10^{-27} \times 3 \times 10^{10}} = 9.06 \times 10^{6}. \qquad (65)$$

The use of this value in equation (64) with an electron velocity of 0.5c provides a mean free path of 10^{17} cm.

The Compton Scattering coefficient is equal to the Thompson coefficient for relatively large wavelengths, but it is reduced to lower values for shorter wavelengths. The use of the Compton coefficient in equation (64) for the given value of velocity will result in equal or longer mean free paths. If the proton or heavier particle mass is used, the mean free path is increased in proportion to the square-of-the-mass ratio with respect to the electron. Since the mean free path is quite large in all cases for any appreciable particle velocity, it appears that the effect of Rayleigh, Thompson, and Compton scattering will not impose any significant attenuation upon the beam. The attenuation to be expected can now be calculated.

B. ATTENUATION, IONIZATION, AND SELF-COLLISION

The reciprocal of the mean free path is in the units of impacts per centimeter. Since it can be expected that an impact will remove an electron from the beam, the reduction in intensity of the electron beam in the photon field is given by

$$-dI = I \frac{8\pi N_2 e^4}{3M^2 c^3 v} \, dx. \qquad (66)$$

The integrated form is

$$I = I_o \exp\left[-\frac{8\pi N_2 e^4}{3M^2 c^3} \frac{x}{v}\right]. \qquad (67)$$

The use of the relation

$$t = \frac{x}{v} \tag{68}$$

and the evaluation of the coefficient then provides

$$I = I_o \exp[-1.5 \times 10^{-7} t]. \tag{69}$$

It follows that any attenuation introduced in any reasonable transit time is negligible.

In the use of atomic or molecular beams in the neutral state, any ionization which occurs must constitute a reduction in the beam intensity as measured in terms of neutral particles. Since photon interaction is able to cause ionization, the magnitude of this effect must be considered. The photon ionization process is basically a Compton effect. In general, it appears that the Compton coefficient is not significantly less than the Thompson coefficient except for gamma radiation. This would constitute a very small percentage of the total radiation spectrum. Thus it appears that the ionization occurring with time of travel would not be large. Since the mean free path for larger mass units is increased by the square of the relative mass, as shown by equation (64), it appears that any ionization caused by photons in the field would impose no limitations on the use of the beam.

The mean free path of a particle in a distribution of like particles is given by[4]

$$L = \frac{1}{\sqrt{2}\pi\sigma^2 N}. \tag{70}$$

In the process of transfer from generator to collector, a certain number of collisions between particles can be expected to occur, because of the random thermal velocity distribution imposed upon the transfer velocity of the beam. Since the mean free path depends upon the particle density in the beam, the chance of a collision becomes less as the beam spreads with distance. If the mean free path calculation is based upon the particle density at the beam origin, it will provide a lower limit of mean free path to be expected under any conditions of beam particle transfer. If electrons at a density of 10^6 are assumed and the diameter of 5.6×10^{-13} is used, the mean free path is 7.3×10^{17} cm. Since the calculation is based upon a reference traveling with the average velocity of transfer of the beam, the possibility of a self-collision because of thermal velocity distribution is very remote. For a density of 10^{16}, the mean free path is 7.3×10^7 at the beam origin and increases rapidly as the beam spreads with distance. The mean free path for protons is even greater.

For atomic beams the particle diameter is of the order of 10^{-8} cm. The mean free path for a particle density of 10^6 is then of the order of 10^9 cm. However, since the mean free path is described in relation to a reference moving with the average transfer velocity of the beam particles, even in this case the effect of self-collision is negligible. It is necessary to conclude that the process of self collision plays an extremely minor role in the determination of the limits of application of any type of beam.

V. SOLAR WINDS AND PARTICLE CLOUDS

A. THE NATURE OF THE PROBLEM

The four effects of the interference between beam particles and field particles are: (1) Elastic scattering, (2) Excitation, (3) Ionization, and (4) Radiation. In addition, the possibility of charge transfer must be considered. The discussion of theory and measurement of total cross section for accelerating voltages of the order of kilovolts[5] indicates that the maximum efficiency factor occurs at voltages much less than this value. The maximum value of the efficiency factor is about 20. At the voltages considered for the transmission of ion beams in space, the efficiency factor is not greatly in excess of unity. Then, in terms of the proper order of magnitude, it is sufficient to use the geometrical area of the particle in the calculation of mean free path.

The distribution of matter in space is not known with any degree of certainty. For purposes of calculation, it may be assumed that 10^3 particles/cm^3 is a reasonable average. The use of a particle diameter of the order of 10^{-8} then gives a mean free path value of the order of 10^{13} cm. On the basis of this fact it appears that attenuation of the beam due to the interaction between beam particles and space particles will not be severe.

There is another point to be considered in relation to particles in space. It is to be observed that there is no fundamental difference between the particles in the solar winds and the particles in the beam. Then, the particles in the solar winds will constitute a background of "noise". Any information which is carried by the beam must be significantly greater in power than the random noise caused by the reception of space particles in order that the message may be received. It appears reasonable that the number of beam particles received per second of time should be maintained at a level of about ten times the number of space particles. Since the velocity of the beam particles may be made quite high in relation to the general average of the space particle velocities, the particle density need not be maintained at a higher value than that which applies to space in order that information may be received. However, since the beam spreads under the action of space charge divergence and thermal dispersion, a definite limitation in distance is imposed by the requirement that the signal must be maintained at a value higher than the noise.. It is most doubtful that the transfer of information on the basis of individual particle count can be accomplished unless a properly oriented directional counter is used.

B. DETAILS OF PARTICLE INTERACTION

In the consideration of the interaction of solar particles with beam particles, there is some disagreement about the nature and level of activity of solar corpuscular radiation. However, it seems that the best information to date is given by Parker[6]. According to this analysis, the solar particles are a continuous flow of approximately equal amounts of protons and electrons with less than one percent neutral particles. The intensity of the flow is dependent upon the variation of solar activity which is in general within the limits termed "quiet" and "active" sun.

For quiet sun conditions the average particle velocity is about 500 km/sec with a particle density of the order of 10^2 particles/cm^3. For active sun, the respective values are v = 1500 km/sec and N = 10^4/cm^3. It is clear that relativistic effects will not have to be considered. If a proton beam in this particle field is assumed, the two types of interactions to be considered are proton-proton and proton-electron. The conditions of scattering are those of elastic collisions and inelastic radiative recombinations for p-p and p-e interactions respectively. The resultant effect will be an attenuation of the beam passing through the solar corpuscular radiation and a bending of the beam in the direction of the solar wind. The attenuation of the beam is dependent on the mean free path of the beam proton in a mixture of protons and electrons. If the diameters are σ_1 and σ_2, the equation for the mean free path is

$$L_{12} = \frac{1}{\sqrt{2}\pi N_1 \sigma_1^2 + \pi N_2 \sigma_a^2 \sqrt{\frac{\bar{v}_1^2 + \bar{v}_2^2}{\bar{v}_1}}}, \tag{71}$$

where $\sigma = \frac{\sigma_1 + \sigma_2}{2}$ and N_1 and N_2 are the respective densities of protons and electrons. The symbols $\bar{v}_1$ and $\bar{v}_2$ represent the respective average velocities of the protons and electrons in the mixture. The mean free path of electrons in an electron beam penetrating the mixture is

$$L_{21} = \frac{1}{\sqrt{2}\pi N_2 \sigma_2^2 + \pi \sigma_a^2 N \sqrt{\frac{\bar{v}_1^2 + \bar{v}_2^2}{\bar{v}_2}}}. \tag{72}$$

In the case of solar winds it is to be observed that the densities of protons and electrons are the same. Also, the velocities $\bar{v}_1$ and $\bar{v}_2$ are about equivalent. Use of the values for proton and electron diameters then yields a mean free path value of the order of 10^{20} cm. On the basis of this fact, it is to be concluded that if the solar radiation particles are protons and electrons as assumed, interaction between beam particles and radiation particles is about seven orders of magnitude less than that found assuming particles of molecular diameter of the order of 10^{-8} cm. Under the circumstances no limitation is imposed upon the beam by the presence of solar particles. This conclusion neglects the possible effect of space magnetic and electric fields. Since the mean free path value given is that for active sun, the limitations imposed are the most severe to be expected.

C. PROTON LOSS BY RECOMBINATION

Bates[7] and Chapman[8] have calculated total recombination cross sections for capture of electrons by protons in the range of electron energies applicable to the problem. The coefficient of recombination found to apply is 10.6×10^{-12} recombinations per cubic centimeter per second. Assuming active sun conditions, the electron density of 10^4. Using the recombination coefficient and a beam density of 10^6, the recombination rate is

$$\frac{dN}{dt} = \mu N_1 N_2 = 10.6 \times 10^{-2} . \qquad (73)$$

The percentage rate of change of density in the beam is then 10^{-5} percent decrease per second or one in 10^7 per second. Thus it appears that recombination is of negligible importance in the determination of beam limitations.

D. BEAM DEVIATION BY SOLAR WINDS

The effect of acceleration of charged particles by solar particles has been examined by Bierman[9, 10]. The acceleration of beam protons by solar particles will be due almost entirely to solar electrons since the effective collision cross section for electrons exceeds the gas kinetic cross section for protons by a factor of about 10^2. By setting up equations for force on particles due to the associated electric and magnetic vectors, it is possible to express the equations in the form of Ohm's law and thereby relate the probability that a particular type of collision will occur to the electrical conductivity. The magnetic intensity is assumed to be negligible. The acceleration of beam protons can then be approximated by

$$\frac{dv_p}{dt} = \gamma_{pe} \frac{M_e}{M_p} v_e = \frac{e^2}{\sigma M_p} N_e v_e \, , \tag{74}$$

where v_p and v_e are velocities of the beam protons and solar electrons respectively. The respective masses are M_p and M_e; γ_{pe} is probability of collision between the respective particles; e is the electronic charge; N_e is the electron density; and σ is the electrical conductivity.

Biermann[9] gives an order of magnitude value of conductivity of 10^{12} esu based on observed acceleration of cometary ions. For active sun conditions, $N_e = 10^4$ and $v_e = 6.2 \times 10^7$. Using the proton mass of 1.67×10^{-24}, equation (74) then provides an acceleration value of 8.5×10^4 cm/sec^2. For quiet sun conditions, this value is divided by a factor of 10^2.

If the force of the solar wind is transverse to the beam, the transverse velocity established in any time t is

$$v_p = \frac{e^2}{\sigma M_p} N_e v_e t \,, \tag{75}$$

and the distance is

$$S = \frac{1}{2} \frac{e^2}{\sigma M_p} N_e v_e t^2 . \tag{76}$$

In one second of time at a transfer velocity of 0.1c, a beam particle will advance 1.86×10^4 miles. In the same time the particle will experience a transverse displacement of 0.26 miles under the action of the accelerating forces resulting from transverse solar winds for active sun conditions. It appears that this is not a limiting factor considering the limitations already imposed by space charge and thermal dispersion.

VI. NEUTRAL BEAM GENERATION

A. MECHANICS OF THE PROBLEM

Analysis to this point indicates that a neutral beam provides an advantage over a charged beam due to the fact that beam dispersion with distance is less severe. However, the problem of the generation of a high energy neutral beam is quite complex. The only reasonable approach at present is that of ionization of the neutral material, acceleration of the resulting ions, and recombination after the acceleration is complete.

The recombination of ions by shooting the beam through a neutral gas or through a cloud of oppositely charged ions is not feasible in the present case. Recombination is quite limited in interaction in which a high velocity difference exists between the ions. Therefore, it will be necessary that both positive and negative ions should have the same final velocity. This can be accomplished only by the separation of ions and the application of separate accelerating voltages to the two types.

The ionization of the neutral material can be accomplished by presently accepted techniques. However, it must be remembered that the efficiency of the ionization process is so low that the power required for ionization will be relatively high. Separation can be accomplished by magnetic or electrical means. Acceleration of the separated charges by their respective accelerating voltages then provides the high final velocity which is desired.

A possible approach to the recombination problem is that of maintaining the charged particles in circular paths which are coincident at one point. The circular paths are maintained by the action of magnetic fields upon the ionic charges. When neutralization occurs, the magnetic fields are no longer active so that the neutral particles leave the system in a direction tangential to the two circular orbits at the point of coincidence. The basic features of the system are shown in Figure 6.

As an example of the application of such a mechanism, let it be supposed that a neutral beam of 10^6 particles/cm^3 is to be generated. The assumed velocity is 0.1 times the velocity of light. The radius of the beam at the point of origin is assumed to be one foot. In order that such a beam should be generated, the number of recombinations per second is given by

$$10^6 \times 3 \times 10^9 \times 3.14 \times (30.48)^2 = N_r. \tag{77}$$

This is approximately 8.75×10^{18} recombinations per second.

Recombination takes place within a limited volume at the junction of the two circular trajectories. If a radius of one foot is assumed, the length of the recombination volume is roughly three feet. This provides a recombination volume which is of the order of 10^6 cm^3.

It is assumed that the ion densities are the same on the two circular paths. The recombination coefficient is pressure and temperature dependent. For low pressures, such as those applying here, a coefficient of 10^{-13} is reasonable. Assuming this value, the equation

$$\frac{8.75 \times 10^{18}}{10^6} = 10^{-13} N^2 \tag{78}$$

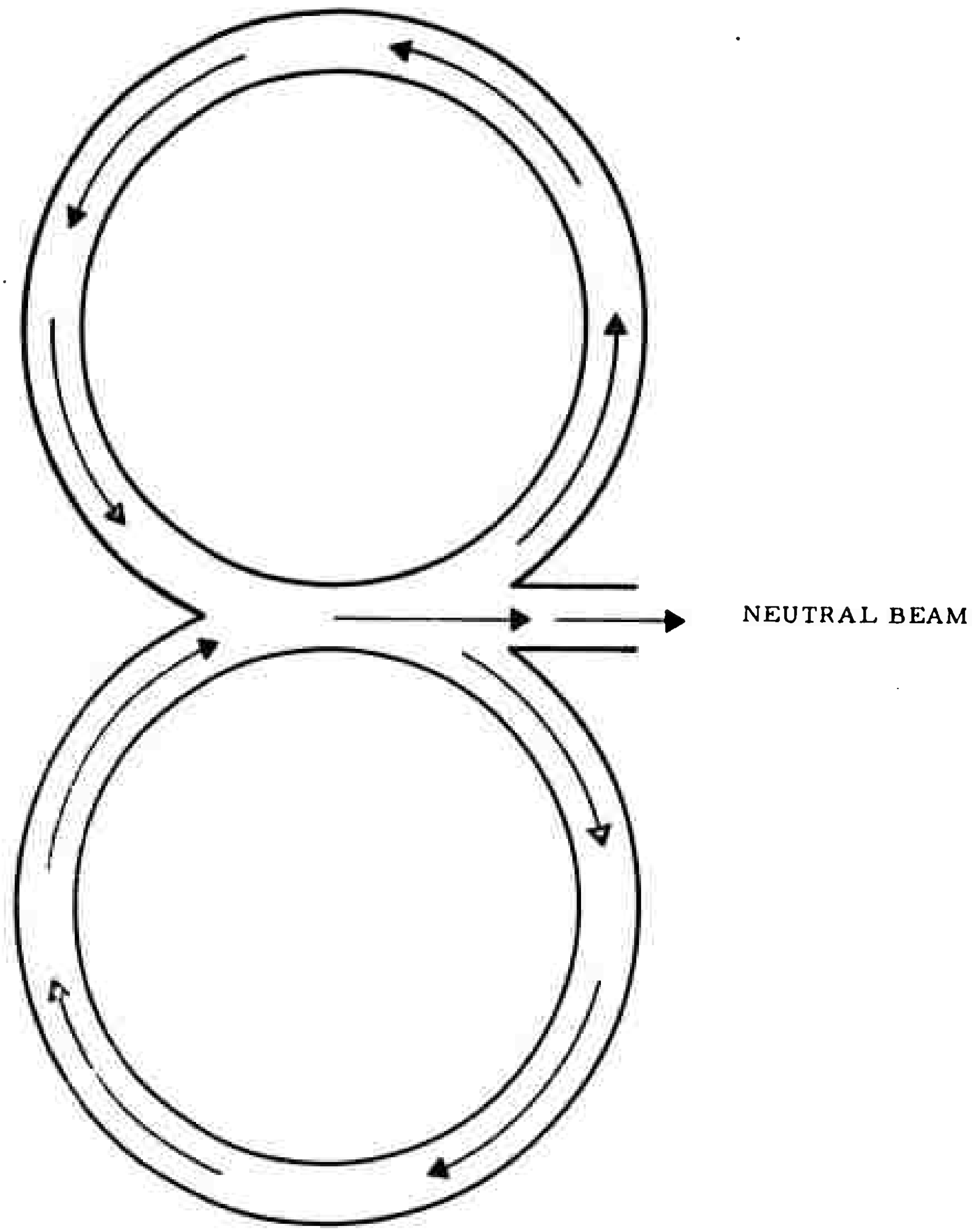

Figure 6. Recombination apparatus for neutral beam generation.

applies. The left member of the equation describes the number of recombinations per second per unit volume. The right member is the product of the recombination coefficient and the two equal ion densities. The ion density in a single orbit is then found to be 9.2×10^{12}. It follows that under the given circumstances the ion density is about 10^7 times the neutral beam density. Considering the density of a gas under standard conditions of temperature and pressure is about 3×10^{19} particles/cm^3, it is seen that the stated ion density is less than this by a factor of about 3×10^6.

The conclusion to be reached is that the proposed method of neutral beam formation is feasible as far as the required relative densities are concerned. Carefully shaped guiding fields must be provided to keep the ions from striking the walls in the circular paths. Further, the velocities of the two types of ions must be equal to each other, and necessarily equal to the desired beam velocity. This may be somewhat difficult to accomplish.

If the negative ions are free electrons and the positive ions are atomic nuclei, the difference in mass is such that the condition of equal velocities after acceleration is quite difficult to attain. The least mass difference in this case would be provided by the use of protons and electrons. The ratio of accelerating voltages to provide equal velocities is the ratio of the masses. It follows that the accelerating voltage for the proton is 1840 times that of the electron of equal velocity. The magnetic field ratio to provide equal path radii is also 1840. The difficulty of maintaining voltage and magnetic field relationships to the necessary limits of tolerance is such that the effective operation of such a mechanism is doubtful. If the relative velocity between the two particles has any significant value, the rate of recombination is reduced.

Electron attachment to neutral atoms occurs. In fact, this is the predominant effect in many interactions. Therefore, it appears feasible to form negative ions by electron attachment and then to accelerate the negative ions thus formed. This can be accomplished by permitting the electrons formed by the ionization process to diffuse into a volume occupied by neutral gas molecules. A separation of the heavy negatives from the light ones can then be accomplished and the heavy negative ions can be accelerated to the final high velocity. Since the negative and positive ions then have masses differing only by the order of the electron mass, equal accelerating voltages and equal magnetic fields can be employed for the two types of ions.

B. THE RECOMBINATION COEFFICIENT

The significance of the recombination coefficient in its relation to the problem of neutral beam generation is such that a detailed analysis seems desirable. Recombination of ions in a volume element in a field-free region with positive and negative ions is determined by the equation

$$\frac{dN}{dt} = -\mu N_- N_+ , \tag{79}$$

where N_- and N_+ represent the respective densities of negative and positive ions and μ is defined as the coefficient of recombination. The physical natures of the recombination processes are the determining factors in the description of μ.

Two theories have been advanced to explain the recombination process. That of Langevin[4, 11] relates μ to the ion mobilities k- and k+; and that of J. J. Thompson[12] relates μ to the random thermal motions of the gas ions and molecules. The latter process appears more applicable to the present analysis since it is applicable to conditions existing in gases under reduced pressures[13]. Thompson's equation for the recombination coefficient is

$$\mu = \pi r^2 \rho \sqrt{\overline{v_-^2} + \overline{v_+^2}}\,, \tag{80}$$

where r is the distance at which the thermal kinetic energy of one ion in the field of the other is equal to the potential energy existing between them. This is

$$\frac{e^2}{r} = \frac{3}{2} kT\,. \tag{81}$$

The values v_- and v_+ are root mean square thermal velocities for the respective ions and ρ is the probability of a recombination at the distance r.

The desired application is that of recombination of positive and negative ions of equal masses as assumed for the neutral beam generator. The use of

$$v_- = v_+ = \sqrt{\frac{3kT}{M}} \tag{82}$$

and equation (81) in equation (80) yields

$$\mu = \frac{4}{9} 6^{\frac{1}{2}} \pi \frac{e^4}{(kT)^{3/2}} \frac{1}{M^{\frac{1}{2}}} \rho\,, \tag{83}$$

where e is the ionic charge, k is Boltzmann's Constant, T is absolute temperature in degrees Kelvin, and M is the ion mass. The probability ρ as developed by Thompson is

$$\rho = \frac{4}{3}\left[\frac{2r}{L}\right] \tag{84}$$

in terms of the distance r and the mean free path L. This is valid for low-pressure conditions in which the mean free path is quite large in relation to r.

The equation

$$\frac{2r}{L} = 0.81 \left[\frac{273}{T}\right]^2 \left[\frac{p}{760}\right] \frac{L_a}{L} \tag{85}$$

is found to apply[4]. The ratio $\frac{L_a}{L}$ is the mean free path in the neutral gas over that which applies in the ionized gas. The value of the ratio from experiment is about 5.0. The pressure p by kinetic theory is

$$p = N_o kT, \tag{86}$$

where N_o is the sum of the two ion densities in the present application. The use of the relation 760 mm Hg = 10^6 dynes/cm^2 then provides

$$\mu = 1.38 \frac{e^4 N_o}{k^{2} M^{1/2} T^{5/2}}. \tag{87}$$

The analysis indicates that the coefficient increases directly as the ion density and inversely as the absolute temperature raised to the 5/2 power. Thus, it appears that a low temperature and a high density is desirable. Most unfortunately, high density values are somewhat hard to manage in the generation and control of ions. Also, temperatures in the neighborhood of absolute zero are difficult to maintain except in outer space conditions.

It is now possible to conclude whether or not the assumption of a recombination coefficient of 10^{-13}, as used in the calculation of neutral beam generation, was realistic. The ionic charge is 4.8×10^{-10} esu. The value of k is 1.38×10^{-16} and the proton mass is 1.67×10^{-24}. The temperature to provide the recombination coefficient is then $3800^{o}K$. This is quite high but may not be unrealistic if the original ionization is produced by an electric arc such as that used in the production of a plasma. A lower temperature of operation will increase the coefficient significantly above that used in the calculation.

Figure 7 indicates the temperature dependence of μ for the density $N_o = 10^9$. Other density values will determine curves of similar shapes offset from this one. The same data can be presented by using the density as a variable and giving the temperature a fixed value. Figure 8 shows the variation of μ with density for a temperature of $100^{o}K$. A family of similar curves can then be found by taking other fixed temperature values.

There are certain disadvantages to the use of a neutral beam as a carrier of information. Electromagnetic modulation would be most difficult considering the recombination mechanism as a function of time and space in the gas. Therefore, it appears that information must be modulated into the beam by mechanical means after neutralization has been accomplished. Another disadvantage, as noted before, is the existing particle density in space. Since such a beam would most likely be detected by means of a pressure sensitive device, the noise level resulting from space particles must be of a much lower intensity.

The idea occurs to use a similar configuration as a means of developing a neutral beam propulsion system. The most obvious advantage is the

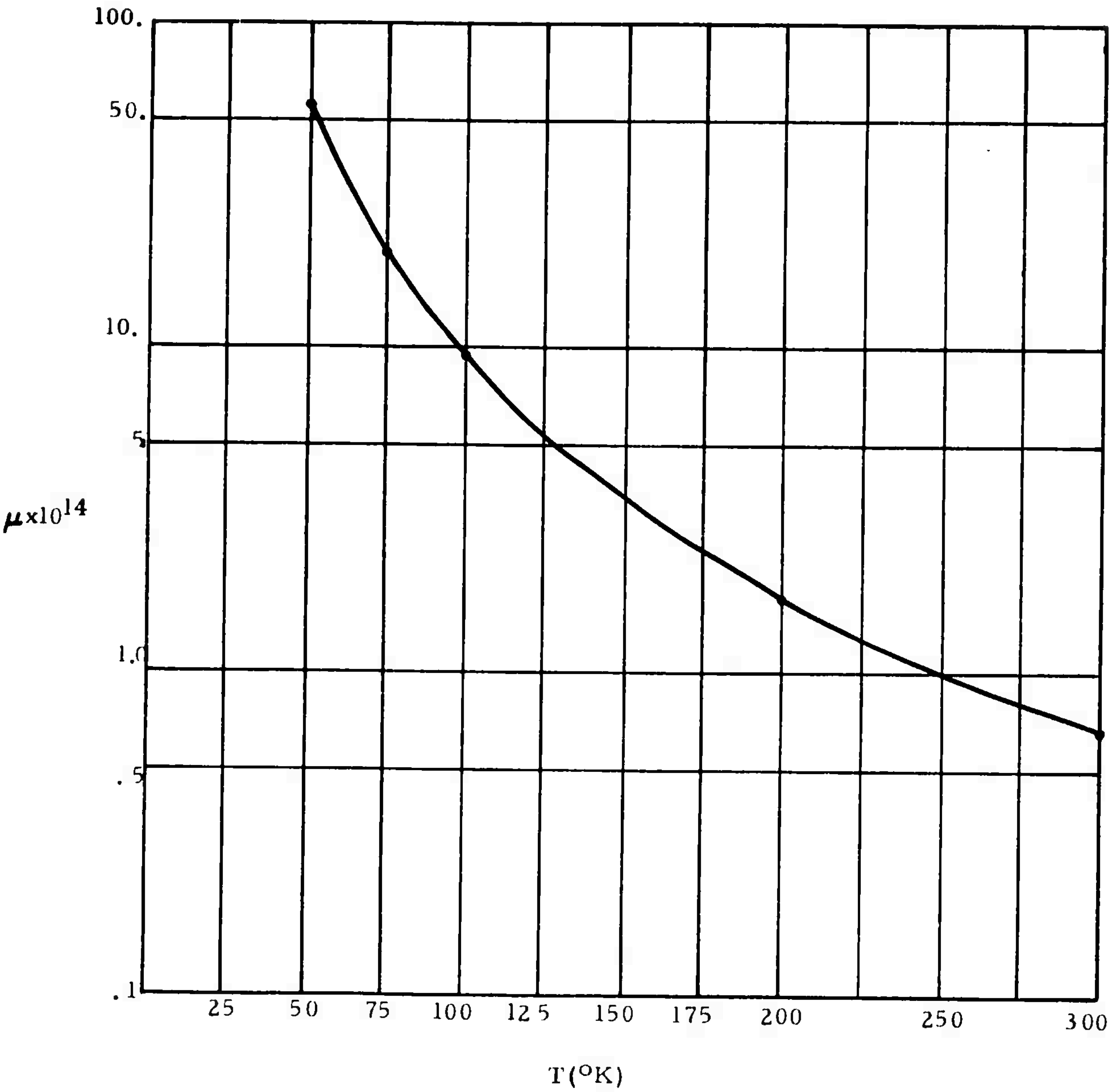

Figure 7. Theoretical curve of μ vs T(°K) for $N_0 = 10^9 cm^{-3}$.

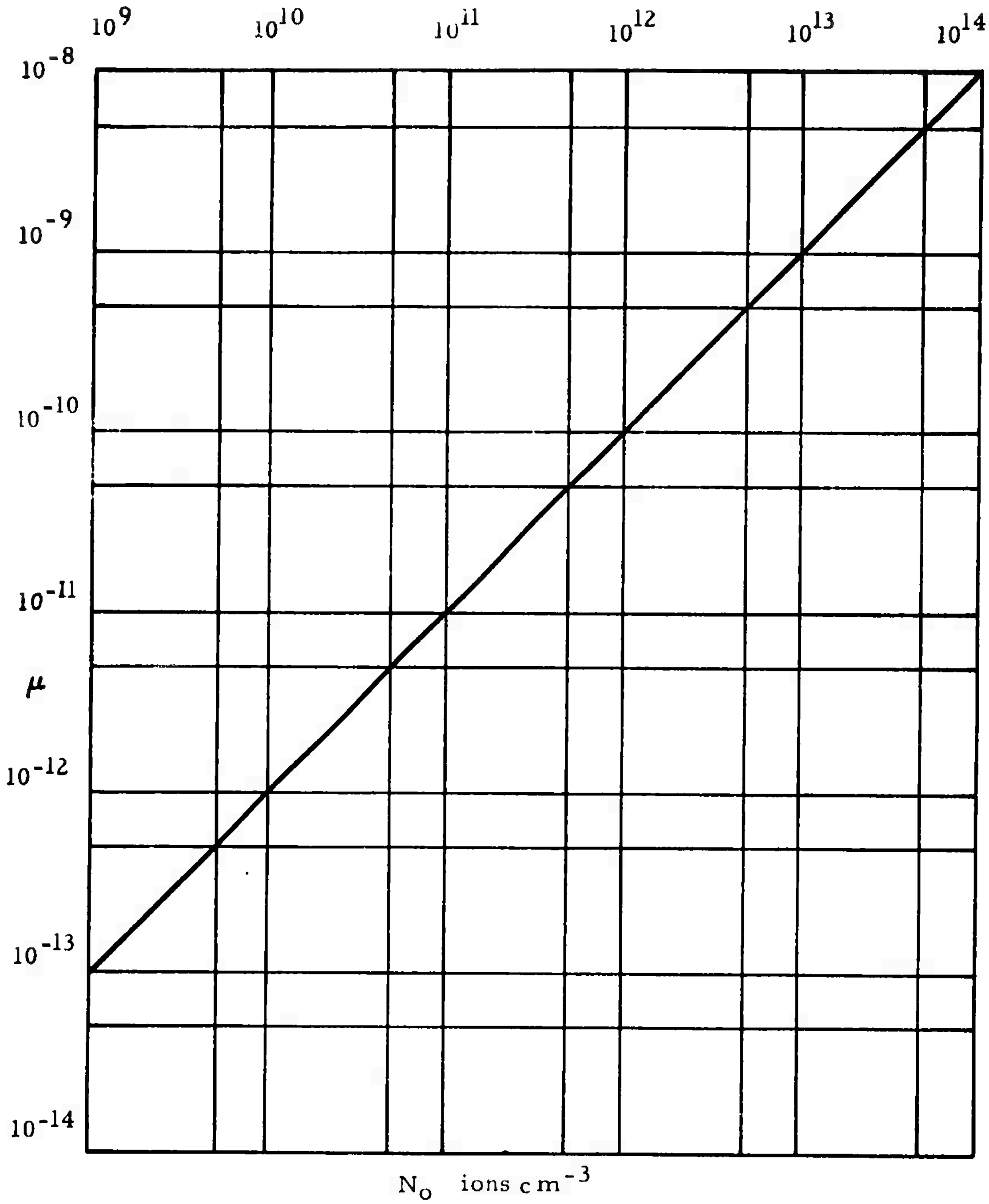

Figure 8. Theoretical values of μ vs N_0 for $T=100°K$ (a logarithmic plot.)

absence of electrostatic charge on the vehicle and in space. This permits the full utilization of the specific impulse of the fuel without losses occasioned by attractive forces induced by the presence of unneutralized space charge. The use of heavier ions would impart greater momentum than hydrogen ions by the relative mass increase. However, the recombination coefficient would be reduced by the square root of the relative mass appearing in the denominator.

VII. BEAM TRANSMISSION IN THE ATMOSPHERE

A. THE PINCH EFFECT

The problem of space charge and thermal divergence in a beam emphasizes the necessity of some mechanism to overcome the effect. It is considered that the presence of a tenuous gaseous medium may provide an advantage over that of empty space in this respect. If the medium is in the ionized state, the presence of these ions in addition to those of the beam may introduce forces which tend to prevent dispersion. This is generally termed the "pinch effect". It has been the subject of fairly extensive investigations both in theory and in experiment.

In the study of conduction of electricity through gases, two main divisions are apparent. For currents up to about 10^2 amperes, the characteristics of the discharge are the standard striations and dark spaces forming a steady luminous structure. Another aspect appears for currents in excess of this value[14]. Studies have been made in the range from 10^2 to 10^6 amperes. The main physical characteristic of these high currents is the well established phenomenon of pinching of the discharge. It is found that the passage of the current constrains the carrying gas into the form of a tube with the axis in the direction of the current flow. The surface of the conduction volume is maintained without external mechanical or electrical constraints.

It must be indicated that for the type of conduction employed, the effect is to be expected. The current is established on the basis of a high-potential breakdown of the medium and subsequent condenser discharge across it. Under these circumstances it is necessary that the gas itself should be the only source of carrier charges. Since it was electrically neutral before ionization, equal numbers of positive and negative charges are present after ionization takes place. With such a high current as that postulated, it is not possible that electron attachment to neutral molecules should occur to any measurable extent. It follows that the negative carriers are free electrons. The positive carriers are molecules from which electrons have been removed by the ionization process. Considering the relative masses of the two types of carriers, the current consists of high velocity transfer of electrons in one direction with a much lower velocity transfer of positive charges in the other. This condition tends to approximate metallic conduction in which the positive charges are bound and only the negatives are free to flow. It follows that a net constrictive magnetic field is established by the flow.

It is not possible that a condition of stability with a definite beam radius should be established. The force of constriction acting on the surface varies inversely as the plasma radius. Any slight decrease in radius increases the force on the plasma. Actually, the condition is made worse by the consideration that any reduction in radius must be accompanied by an increase in the velocity of the beam particle. If a cylindrical beam is assumed, the force of constriction is provided by

$$F = 2\pi q^2 v^2 rN, \tag{88}$$

where q is the particle charge and v is its velocity. The symbol r represents the beam radius and N represents the ion particle density. The fact that the discharge constitutes a series current requires the condition

$$I = \pi r^2 N q v = \pi r_o^2 N_o q v_o . \tag{89}$$

If the current is assumed constant, the use of equation (89) in equation (88) provides

$$F = k \frac{v}{r} , \tag{90}$$

where k represents the constant quantities present. If r is reduced with v constant, the constrictive force is increased. However a reduction in r also requires an increase in the velocity v. Therefore, the force is increased more than that which applies to the inverse first power of the radius. In any case, a reduction in radius increases the constrictive force to cause a further reduction.

Under the circumstances indicated in the analysis, any random disturbance of the beam surface is sufficient to cause the beam to be disrupted. It appears that in the limit the beam must pinch itself off by its own self-constrictive property. As a matter of fact, any such self-constriction must in the limit become so severe that the particle flow is retarded by mutual interference in the constriction. When this occurs, a reduction in the constrictive force with a consequent increase in beam radius must take place. In the event that there exists no mechanism whereby the particles are again accelerated, unlimited expansion must occur.

It must be emphasized that there exists a basic difference between the ionization of a medium by a high potential gradient existing between condenser plates and the ionization by means of a beam of charged particles penetrating the medium from an external source. In the case of a condenser discharge, equal numbers of positive and negative ions are generated. These are accelerated differentially by the gradient because of the difference in mass. However, the conducting plasma is essentially neutral just as it is in metallic conduction. In this case, the constrctive effect may predominate to overcome any tendency of the beam to diverge. Further, there exists the necessary gradient to accelerate the charges again after the first constriction has occurred to its limit. Thus, it should be found that alternate conditions of constriction and expansion should occur with time at a point in the discharge path. The condition of constriction should be associated with an intense radiation of energy from the beam.

In the event that the neutral medium is ionized by a beam of particles of one type of charge only instead of an arc discharge, the plasma is no longer neutral. There exists an excess of one type of charge by the amount of the charge introduced. Another basic difference is that there exists no high potential gradient to accelerate the charges formed by ionization. Further, the absence of a gradient requires that a beam particle should advance through the medium at the expense of its own momentum which is gradually lost in the ionizing process. There exists no mechanism whereby the velocity of the particle can be increased along the path after it has lost its momentum. Under these circumstances there will exist one point of constriction in the beam at most. In the event that it exists at all, it will be found quite near the beam source. Beyond this point of possible constriction there will exist a region of unlimited expansion. Thus, it does not appear feasible to depend upon the self-constricting action of a beam over any appreciable distance in the presence of a medium.

There is experimental[14] as well as theoretical evidence to support the simple analysis given for the condenser discharge. Considering the magnitude of the current and the available means of generating it, any such discharge is of very short duration. The experiment was conducted using cylindrical discharge tubes ranging in diameter from 5 to 30 cm and length-to-diameter ratios of about 5. Gas pressure in the range of 10^{-3} mm to several mm of mercury were used. The inductance of the assembly was about 10^{-1} microhenry. The condenser banks used ranged from 10 to 10^2 microfarads with energies in the kilojoule range and voltages up to tens of kilovolts. Transient currents up to 10^6 amperes resulted on discharge through the gas.

It was found that the nature of the discharge was influenced by the rate of current increase. This rate was in the range of 3×10^{10} to 3×10^{11} amp/sec. The time to reach a maximum was about 5 μ sec, and the time of current flow was about 20 μ sec. Representative voltage and current curves are shown in Figure 9.

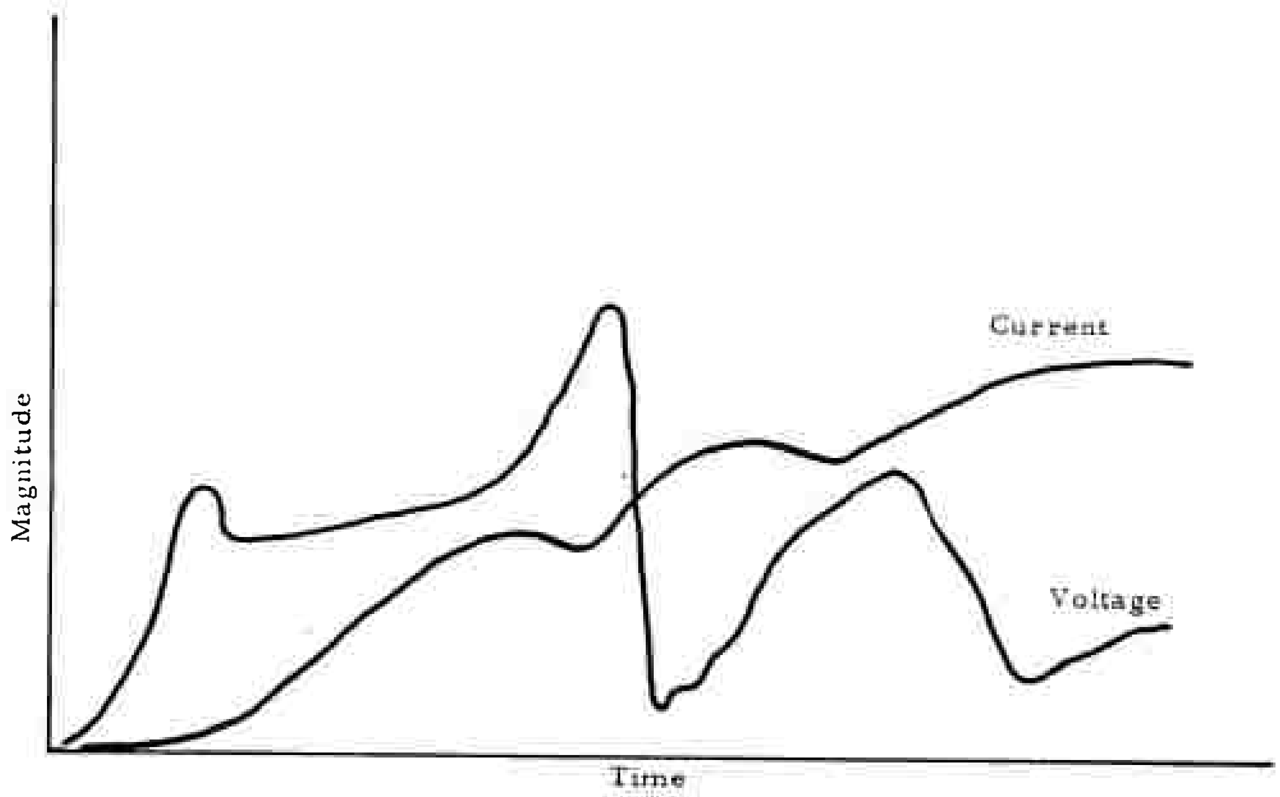

Figure 9. Voltage and current during discharge.

The discharge is a source of light. The conduction volume is in the form of a luminous cylinder whose radius varies periodically with time, the light intensity being highest when the radius is least. The temperature of the gas during the pinch may be as high as 10^6 degrees due to the nearly adiabatic compression. Careful study indicates that the cylindrical surface of the conduction volume is smooth up to the time of the first pinch. Beyond this time various modes of oscillation are displayed. Thus it is shown experimentally that the high current discharge is inherently unstable beyond the time of the first pinch.

The dependence of the plasma radius on the time was inferred by considering the tube a circuit element. The equation is written

$$V = \frac{d}{dt} [LI] + R, \tag{91}$$

where V is voltage, L is inductance, I is current, and R is resistance. The inductance is not constant with time so that the equation may be expanded and written in the form

$$\frac{dL}{dt} + R = \frac{\left(V - L \frac{dI}{dt}\right)}{I} . \tag{92}$$

All quantities on the right of this equation are subject to measurement so that a numerical value may be arrived at. Both resistance and rate of change of inductance must be related to the rate of change of the radius. By making "certain plausible assumptions", Cole[5] finds that the radius of the discharge tube varies as shown in Figure 10.

Since the high voltage discharge does not simulate the conditions existing in the process of ionization by the high energy particle beam, a

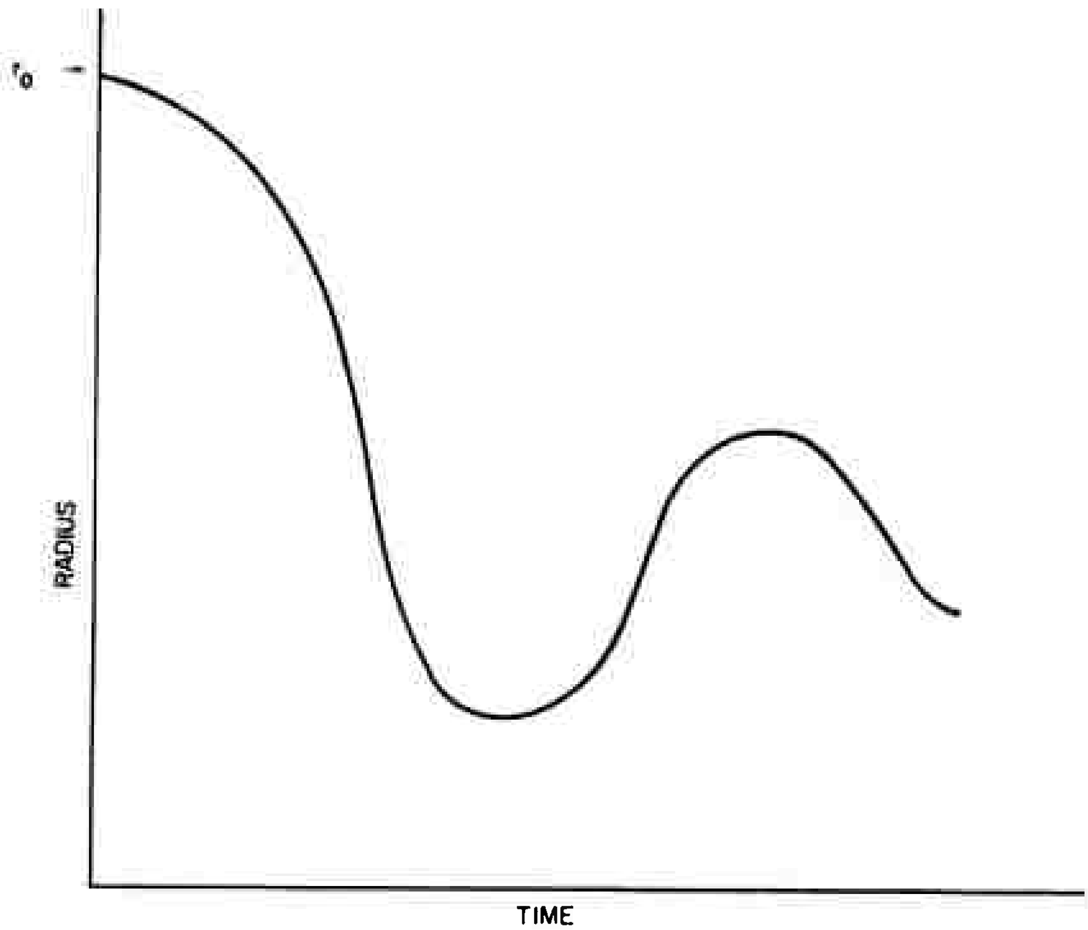

Figure 10. Variations of discharge radius.

more detailed mathematical analysis will not be given. However, it appears that at the beginning of the conduction period, the "skin effect" in which a current sheath is confined to the surface of the conducting column is quite pronounced. This sheath grows progressively thicker and migrates toward the axis to form the first neck. Beyond this time the oscillatory nature of the flow tends to destroy the surface conduction effect.

It appears that the only possibility of existence of the pinch effect in a particle beam in the presence of a medium depends upon the difference in mobility of the positive and negative ions formed by the impact process. Electrons removed by impact can be expected to diffuse out of the ionization region at a much more rapid rate than the much heavier positive ions formed. If the beam ions are negative with relatively high forward velocities, the conditions necessary to produce a pinch effect exist in the event that the beam current is of sufficient magnitude to cause it. The pinch radius can be expected to vary with time in an oscillatory manner. This will tend to group the beam particles into bunches like a string of sausages (Sausage Instability). In the event that forces in the pinch are not symmetrically distributed, the beam may be deflected to alternate sides in a regular pattern to simulate a sinusoidal effect (Kink Instability). This is somewhat analogous to the shedding of vortices by an object in aerodynamic flow.

If the beam particles are positive, the diffusion of the electrons released by the ionization process may be retarded by the presence of the net positive charge. If the electrons are lost, conditions whereby a pinch may occur are not present. Under the circumstances it is doubtful that a pinch will occur in a positive beam. However, the pinch effect is far from desirable in beam propagation since the radiation emitted in the pinch constitutes an energy loss. The ultimate effect is that of retarding the beam and increasing its relative spread. For this reason, it is concluded that self-containment of a beam by this mechanism is not feasible.

B. POWER CONSIDERATIONS

Under the circumstances, it is not possible to give an exact analysis of power requirements. However, the power required to maintain ionization over an extended distance is quite large. As an example, let it be supposed that an ionizing beam of particles penetrates into a region containing a gas at 10^{-3} mm of mercury. This provides a particle density at $0^{\circ}C$ equal to 4×10^{13}. A beam radius of one foot is assumed with an axial length of one mile. The volume considered is about 4.7×10^{8} cm^3, as a result. Assuming an ionization energy of 15 ev per molecule, the initial energy to produce complete ionization in this column is

$$E = 4.7 \times 10^{8} \times 4 \times 10^{13} \times 15 \times 1.6 \times 10^{-12} \times 10^{-7} \text{ Joules.} \qquad (93)$$

This provides the ionization energy of 4.5×10^{4} Joules per mile. However, it must be remembered that recombination takes place so that energy must be supplied continuously at a certain rate to maintain ionization. This rate may be found if the recombination coefficient is known. For the assumed conditions a coefficient of the order of magnitude 10^{-11} is not unreasonable.

If this value is assumed to apply, the recombination rate is given by

$$\frac{dN}{dt} = -10^{-11} (4 \times 10^{13})^2. \qquad (94)$$

The calculated value is then 1.6×10^{16}. The power input to maintain ionization is

$$p = 1.6 \times 10^{16} \times 4.8 \times 10^{8} \times 15 \times 1.6 \times 10^{-12} \times 10^{-7} \text{ Joules/sec.} \qquad (95)$$

This is a value of 1.8×10^{7} watts/mi. Under the circumstances, it is seen that a power source in the megawatt range is required to maintain ionization in an atmosphere of any appreciable density and over any reasonable distance.

C. RECOMBINATION RADIATION AS A MEANS OF COMMUNICATION

In the problem of ion beam communication between a satellite and a ground station, the presence of the atmosphere must be considered. It has been suggested that ion beam particle impact with atmospheric molecules may be used as a means of generating radiation which can be received by the ground station. Modulation of the beam intensity will modulate the intensity of the electromagnetic radiation so that information can be conveyed. Thus, a communication link may be established from space to ground.

Ground to space communication may be established on the same basis. An ion beam originating on earth cannot be expected to penetrate through the entire height of the atmosphere to be received by a satellite. However, the process of atmospheric ionization by impact of the beam particles and the subsequent recombination of the atmospheric ions will provide electromagnetic radiation which can be received by the satellite.

Radiation may be emitted without ionization. The ionization potential of a gas is generally of the order of 15 volts as a minimum. If a recombination occurs with a transition to the ground state in a single step, the recombination energy must be emitted as a single photon. Since the photon energy must equal the ionization energy, the wavelength of the emitted radiation is much shorter than that of visible light. On the other hand, if orbital transitions occur without ionization, the resulting radiation is usually in the visible range. Therefore, it is necessary to assume some average condition in order that a calculation may be made. It is assumed that the average can be represented by the mid-range of visible radiation. The corresponding wavelength is 5500 angstroms. The use of this value then provides the energy furnished by a single molecule in the form

$$E_1 = hf = 6.623 \times 10^{-27} \times \frac{3 \times 10^{10}}{5.5 \times 10^{-5}} = 3.4 \times 10^{-12} \text{ ergs.} \qquad (96)$$

This is a value of about two electron volts. It is seen to be significantly below the ionizing energy of 15 electron volts.

When radiation occurs by recombination or orbital transition, the process is random. Therefore, the resultant radiation spreads in a spherical pattern. The intensity of the radiation received at a remote point varies inversely as the square of the distance from the source. This is represented by the equation

$$E_r = \frac{I_o}{4\pi r^2}, \qquad (97)$$

where I_o represents the radiant intensity of the source. In the present application it is assumed that the distance from the ground station to the point of generation of the radiation in the upper atmosphere is 100 miles or 1.6×10^{7} cm. If the received power density is 10^{-8} watts/cm^{2}, the intensity of the source is found to be 32.6×10^{6} watts. It must be understood that this power must be supplied by the beam. The use of equation (96) then provides the number of photons per second generated at the source. This is given by

$$N \times 3.4 \times 10^{-12} \times 10^{-7} = 32.6 \times 10^{6}. \tag{98}$$

The value found is 9.6×10^{25}.

Considering the limited efficiency of the energy transformation by impact processes, it is doubtful that more than one photon per beam particle within the useful energy range can be expected. If this assumption is made, it follows that 9.6×10^{25} beam particles must be generated each second. This is about 160 gram molecules per second. For a propulsion system this may be a reasonable value. It is not reasonable for a communication system.

On the basis of the power requirements indicated by the calculation, it is to be concluded that the proposed method of communication is not feasible. The use of maser or laser receivers may bring the power considerations within a more reasonable range. However, it is quite doubtful whether the results will justify the effort in view of the difficulties involved. It is certain that there exist more dependable methods of communication in which the power requirements are much less severe. The same conclusion is applicable to the reverse problem of transmitting from a ground station to a satellite by means of radiation generated by a particle beam.

VIII. SHORT RANGE COMMUNICATIONS

There may be some advantage in the use of an electron beam for communication over short distances in space. For example, a communication link may be established between members of a work crew assembling a space platform. The spread of the beam limits the distance over which communications can be maintained. This limitation in distance is in itself an advantage where secrecy is desired. There is no possibility of interception of the short-range information by an unfriendly power.

The series of equation (24) has been evaluated for values of the ratio of the expanded radius to the original radius equal to 10, 100, and 1000 for electron beams. Corresponding values of the series are found to be 4.25, 27.36 and 209.37, respectively. The time for an electron beam to spread by a factor of 10 is given by

$$t = \frac{4.25}{4.8 \times 10^{-10}} \left[\frac{9.1 \times 19^{-28}}{3.14 \times 10^{6}}\right] = 1.5 \times 10^{-7} \text{ sec}, \qquad (99)$$

if an original beam density of 10^6 is assumed. For a transfer velocity of 0.1 c, the distance of travel for this condition is

$$D = 3 \times 10^{9} \times 1.5 \times 10^{-7} = 450 \text{ cm}. \qquad (100)$$

The accelerating voltage to provide this velocity is

$$V = \frac{9.1 \times 10^{-28} \times 9 \times 10^{18}}{2 \times 4.8 \times 10^{-10}} \times 300 = 2700 \text{ volts}. \qquad (101)$$

Equation (100) indicates that the beam spreads to 10 diameters in a distance of 450 cm under the assumed conditions of density and velocity. This assumes a small diameter beam with particles moving parallel to each other at the point of origin. For example, a beam of 1.0 cm diameter will be 10 cm in diameter at this distance from the source. Since the distance is less than 15 feet, the spread is rather severe. Even so, a collector of 10 cm diameter at the proper location will intercept the full power of the beam.

Since only the sum of the series is changed in the calculation, the time for the beam to spread to 100 diameters is given by

$$t = \frac{27.36}{4.25} \times 1.5 \times 10^{-7} = 9.65 \times 10^{-7} \text{ sec.} \tag{102}$$

Assuming the conditions of density and velocity to be unchanged, this provides a distance

$$D = 9.65 \times 10^{-7} \times 3 \times 10^{9} = 2900 \text{ cm.} \tag{103}$$

If the assumed beam diameter at the point of origin is 1 cm, a collector of 100 cm diameter will be required at this distance to intercept the full power of the beam. The distance is about 95 feet.

The time for the beam to spread to 1000 diameters is given by

$$t = \frac{209.37}{4.25} \times 1.5 \times 10^{-7} = 73.8 \times 10^{-7} \text{ sec.} \tag{104}$$

The corresponding distance under the assumed circumstances is

$$D = 73.8 \times 10^{-7} \times 3 \times 10^{9} = 22140 \text{ cm.} \tag{105}$$

This is a distance of about 725 feet. The beam density at this point is reduced by a factor of 10^{-6}. However, a collector (such as a metallized space suit) of reasonable size can be expected to intercept a sufficient percentage of the beam power to make communications possible.

In order that a link may be established between two individuals, it seems necessary that both should emit electrons simultaneously. In this manner a return circuit is established. Saturation of the collector with the consequent repulsion of the modulated beam and loss of information transfer is avoided. In fact, all members of the space crew would work with their beam generators operating at all times. The resultant space current would be mutually shared and information imposed by voice modulation would be heard by all who received a sufficient percentage of the beam energy.

Another advantage of such a system is that radio frequency channels would not be used. When the overcrowding of the electromagnetic spectrum is considered, this is not a minor point.

IX. ION BEAM APPLICATIONS

From the analysis given in the present report it appears that one of the basic characteristics of an ion beam is that of high power. For this reason it is evident that applications other than that of communications may be made. The possible application to the generation of thrust has been mentioned. In addition, the transfer of power over short distances in space appears possible. The possible application as a weapon also appears. However, limitations imposed by the spread of the beam must be considered in any application in which this is a factor. The suggested applications are all dependent upon the same basic mechanism which is used to introduce power into a particle beam. Such a mechanism may be based upon the design of a unit for the production of thrust.

Electric power is generated and used most advantageously in alternating current form. Then, as a matter of convenience and simplicity of design, it appears that the application of electric power to the generation of thrust can be best accomplished by the use of alternating current. The basic necessity is that of a conversion of electrical power to thrust power. Although this requirement differs in detail from the power conversion provided by the electrical transformer, the most logical solution to the problem can be given on the basis of such an approach. Therefore, the modification of the electrical transformer to apply to the generation of thrust is proposed.

When neutral matter is ionized, both positive and negative charges are present in equal numbers. In order that positive net thrust may be maintained for an indefinite period of time, it is necessary to eject both positive and negative charges in equal numbers at any given time. In theory, even this is not sufficient since the greater mobility of the electrons will cause them to move out of the vicinity of the space vehicle at a higher velocity than that which applies to the positive charges for a given accelerating potential. The predominance of the positive charges in the less remote vicinity of the vehicle then induces an attracting charge upon it so that the net thrust is reduced. Although the best solution, by theory, is the recombination of charges either before or after ejection, the practical accomplishment of the neutralization is most difficult. The effect of the induced charge may be removed in any case by the use of a small auxiliary unit which ejects negatives only. The small net positive charge then maintained on the space vehicle will be sufficient to overcome the effect of the positive space charge. The necessity of charge recombination is thus removed.

As a matter of fact, the use of the auxiliary unit may not be necessary under actual operating conditions in space. Considering the presence of the charged particles in the solar winds it appears that any charge on the space vehicle with respect to its surroundings would be neutralized by collection of charges from this source. If this is the case, the neutralization of the beam is rendered immaterial and the full thrust of the unit is realized.

In the design of the thrust unit, it is desired that both positive and negative charges should be useful in providing thrust. For this reason the accelerating unit is made in the shape of a U tube. The mixture of positive and negative ions formed from the neutral propellant is fed into the unit at the bend of the U tube. An alternating voltage is developed between the ends of the tube. The voltage on one half of the cycle will be of such a polarity that positive charges will be expelled at one port of the U tube and negatives at the other. When the voltage between the ends of the tube reverses to correspond to the other part of the cycle, the polarity of the charges expelled at a given port is reversed to correspond. However, thrust is generated on each half cycle and at each of the two ports.

The explanation of the mechanism is given in reference to Figure 11. In order to analyze the conditions applying, the analogy between the magnetic and electric circuits is used. If, for example, the U tube is wrapped as a solenoid, the establishment of a changing electrical current around the periphery will cause a changing magnetic field to be generated in the interior of the tube and oriented along its axis. In analogy with this condition, it is possible to interchange the magnetic and electric effects. If charges are present within the tube a changing magnetic field around its periphery will generate a current oriented along the axis of the tube. Even in the absence of interior charges, a voltage oriented along the axis of the tube will be developed by the changing magnetic field around the periphery.

As indicated in the figure, the magnetic field concentric with the tube axis is established by means of a solenoid wrapped around the tube. This field is equivalent to that which would apply in the case of an electric current flowing along the axis in the interior of the tube. A varying current

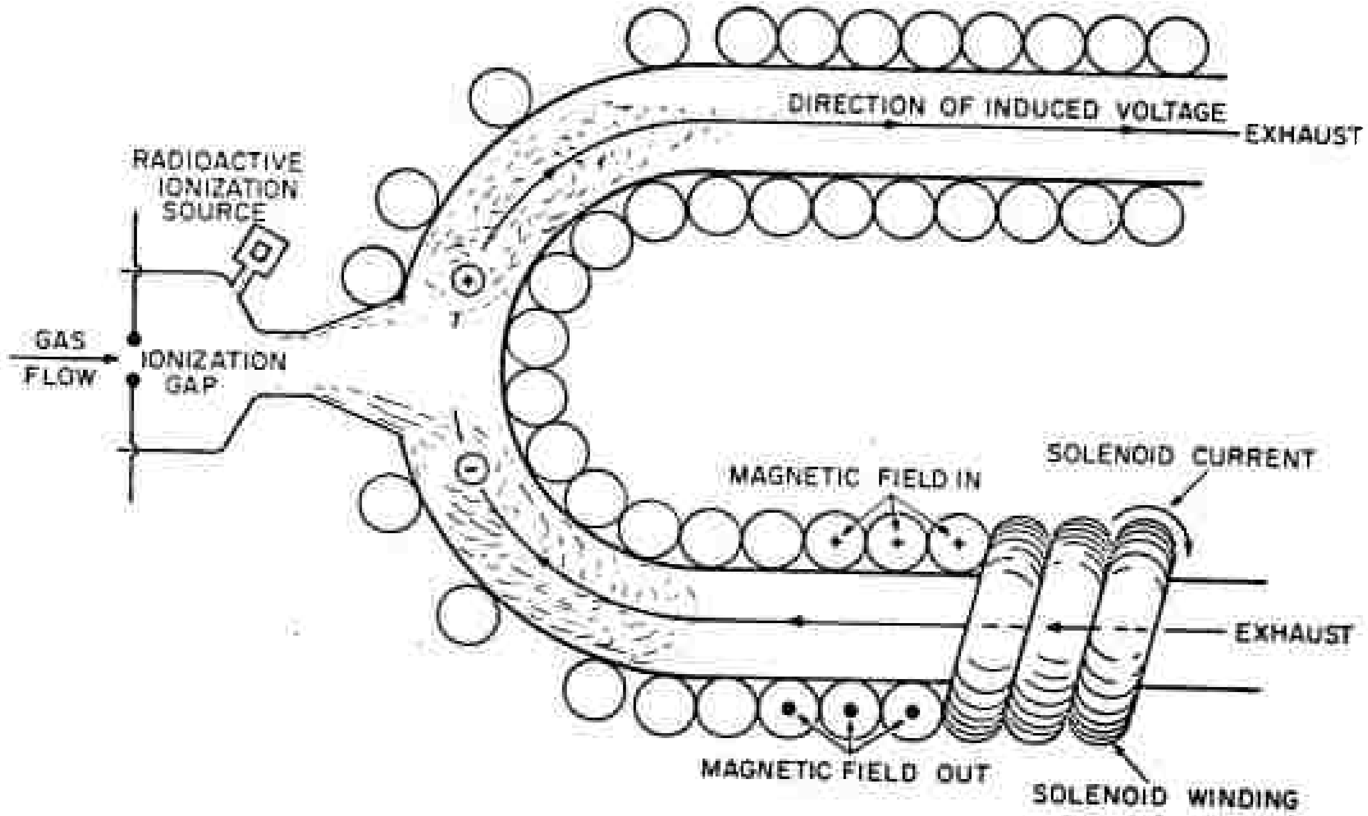

Figure II. Details of solenoid wrapped U-tube.

in the solenoid windings will cause corresponding variations in the magnetic field in the solenoid core which is concentric to the U tube axis. These variations in the concentric magnetic field will generate a voltage oriented along the U tube axis and acting between its ends.

The magnitude of the induced voltage can be calculated from the equation

$$V = 10^{-8} \frac{d\phi}{dt}, \tag{106}$$

where ϕ represents the magnetic flux. If N is used to represent the number of wraps of the solenoid around the U tube, n represents the number of turns of wire per unit length on the solenoid, A represents the cross sectional area of the solenoid, $I = I_o \sin \omega t$ represents the solenoid current, and μ is the magnetic permeability, the value of flux linkage is

$$\phi = 4\pi N n \mu A \frac{I_o \sin \omega t}{10}. \tag{107}$$

The peak voltage is then found to be

$$V_o = \frac{8\pi^2 N n \mu A f I_o}{10^9}, \tag{108}$$

by use of the equality $\omega = 2\pi f$.

As an example of the magnitude of voltage which may be developed, let it be assumed that the U tube is 5 feet in axial length wrapped with a solenoid one inch in diameter. In this case, N is 60 and A is 5.06 cm^2. A peak current of 10 amperes is not unreasonable. The value of n depends upon the size of the wire used. Certainly 5 turns per centimeter is not an excessive value. The permeability may be assumed of the order of 1000 with a frequency also 1000 cps. The use of these values in equation (108) provides a peak voltage value of 1200 volts. This is not large in relation to the voltage required for ion beam communication. The specific impulse imparted to protons is 4.9×10^4.

The accelerating voltage can be made higher than that found by increasing any or all of the factors involved in the equation. It appears to be no problem to increase the peak voltage by one or two orders of magnitude. The calculated value was given as an example only. It appears that the physical size of the unit need not be excessive. The power can be made quite large depending upon the cross section of the U tube and the ion density in the beam. The advantage of the alternating current system in relation to the direct current system in terms of weight and complexity of the electrical power supply is quite obvious.

Suggested ionization sources are the plasma jet and the Tesla coil. Also a residual ionization may be provided by the location of a radioactive source adjacent to the gas flow. If the density of the gas in the accelerator tube is of a sufficiently high value that ion impacts with neutral molecules can be expected, the accelerating voltage may be made high enough to cause a cascade ionization. This is a problem for further investigation. The point to be considered is that of ionization efficiency. It is not desirable to expend more than the minimum energy necessary to ionize the gas.

Another possible application is that of a power transfer between units in space. Over limited distances, the spread of the emitted ion beams would not be excessive. A mechanism based upon the U tube concept could then be used as a receiver of the transmitted power to transform it back into conventional electrical power. This mechanism would act in a manner which is the exact reverse of conditions applying to the transmitter.

Since a large amount of power can be carried by a beam, the possibility of its use as a weapon is obvious. However, the calculations made indicate that the effective range of such a weapon is limited by the space charge dispersion in the beam. Since this effect imposes a limit upon the use of the beam as a weapon and as a means of power transfer, it appears that the most promising application is to the development of thrust for propulsion in space. Although the thrust level is small so that the unit could not be used as a primary thrust source, its application to the expansion or adjustment of an established orbit is obvious.

X. DEVIATION AND DISPERSION IN SPACE FIELDS

A. THE UNIFORM ELECTRIC FIELD

Dispersion effects discussed in preceding sections must be considered self-dispersion in the sense that no external agency acted on the beam. There are other dispersion effects associated with velocity differences in the presence of field forces acting on the beam. It was concluded that one third of the thermal velocity distribution was oriented along the beam as an average effect. Since this is imposed upon the transfer velocity, there exists a transfer velocity distribution. Field forces acting on the beam will deflect particles having different velocities by different amounts. As a result of this fact, both deviation and dispersion of the beam will occur in the presence of field forces even in the absence of transverse thermal velocity effects.

In the analysis of the effect of a uniform electric field, it is assumed that the original direction of the beam is along the x axis as indicated by Figure 12. The beam is composed of charged particles which are affected by the field. Field orientation is such that a component acts at right angles to the beam and another component opposes the beam in the negative x direction.

The components of acceleration are given by

$$\frac{d^2 y}{dt^2} = \frac{qE_y}{M} , \tag{109}$$

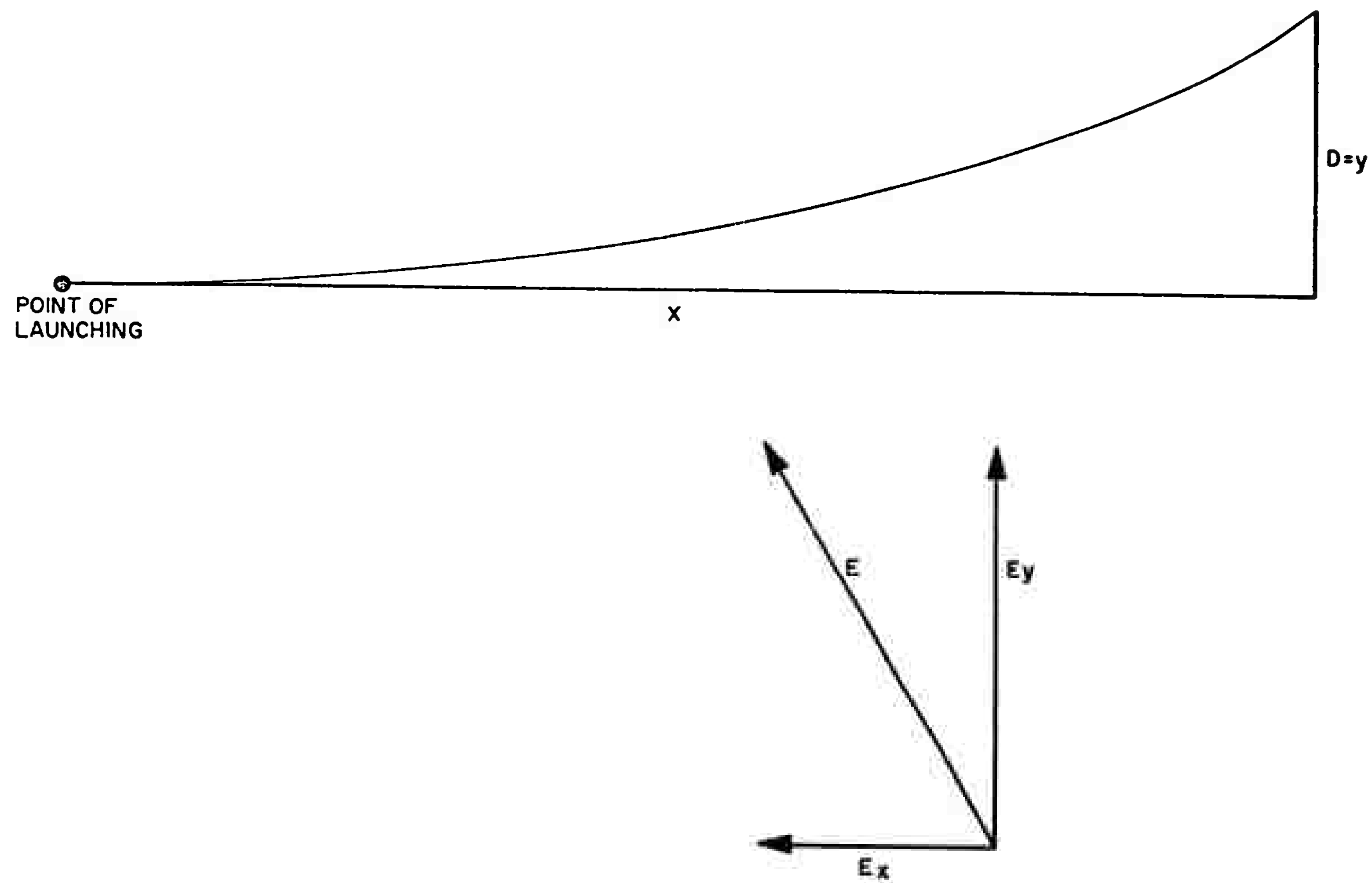

Figure 12. Beam deviation in a field with a component opposing the velocity.

and

$$\frac{d^2x}{dt^2} = -\frac{qE_x}{M}. \tag{110}$$

The symbol q represents the ionic charge and M is the beam particle mass. E_x and E_y represent x and y field components respectively. Two integrations for each equation provide the values

$$y = \frac{qE_y}{2M} t^2 \tag{111}$$

and

$$x = v_o t - \frac{qE_x}{2M} t^2, \tag{112}$$

where v_o is the original beam transfer velocity in the positive x direction. The value of t from equation (112) when used in equation (111) yields

$$y = D = \frac{ME_y v_o^2}{2qE_o^2} \left\{1 - \sqrt{1 - \frac{2xqE_x}{M_o v^2}}\right\}^2, \tag{113}$$

where it is understood that D represents the deviation of the beam. In the event that the component of E along the x axis aids the motion, equation (113) becomes

$$D = \frac{ME_y v_o^2}{2qE_x^2} \left\{1 - \sqrt{1 + \frac{2xqE_x}{M_o v_o^2}}\right\}^2. \tag{114}$$

If the value of E_x is zero, a new solution is necessary. This is given by making the substitutions $E_x = 0$, $E_y = E$ in equations (109) and (110). Two integrations provide

$$y = \frac{qE}{2M} t^2 \tag{115}$$

and

$$x = v_o t \, . \tag{116}$$

The elimination of t then yields

$$y = D = \frac{qEx^2}{2Mv_o^2} \, . \tag{117}$$

Equation (117) is useful in providing a general analysis of beam transmission requirements. For a given transverse field E and a given ion charge q, the deviation is directly proportional to the square of the transmission distance x and inversely proportional to the beam particle energy. It follows that for a fixed beam acceleration voltage there is no advantage in the use of light particles. The deviation is thus seen to be inversely proportional to the accelerating voltage.

As an example of the application of equation (117), let it be assumed that protons at 0.1c are used in a transverse electric field of 10^{-6} dynes/esu. Suppose an original beam radius of one foot and a limit of detection due to beam spread to be attained when the radius is 8000 feet. If the aim is assumed to be perfect, the deviation of the beam must be limited to 8000 feet in order to be intercepted by a detector. The distance at which this condition is met is given by

$$x = \sqrt{\frac{2 \times 1.67 \times 10^{-24} \times 9 \times 10^{18} \times 8000 \times 30.48}{4.8 \times 10^{-10} \times 10^{-6}}} = 1.24 \times 10^{8} \text{ cm.} \tag{118}$$

This is about 770 miles.

For a field having a component of force against the beam, the limit of advance is given by the condition that the radical in equation (113) should be zero. If the field is 10^{-8} dynes/esu and oriented at an angle of 45° against the beam, the value of x for this condition is 1.37×10^{5} miles. The corresponding deviation is also 1.37×10^{5} miles since $E_y = E_x$ in this case.

Equation (114) indicates that there is no theoretical limit on the distance of travel of a beam when the x component of the field aids the motion. However, deviation becomes quite large for long distances. For the specified field at an angle of 45° with respect to the beam, with the x component aiding the motion, the deviation is found to be 10^{4} miles in a distance of travel of 1.37×10^{5} miles.

Dispersion effects can be analyzed by considering the differential of deviation with respect to velocity. Differentiating equation (117) as a function of velocity yields

$$\Delta D = \frac{qEx^2 \Delta v_o}{Mv_o^3} = D \cdot \frac{2\Delta v_o}{v_o} . \tag{119}$$

The basic mechanism affecting the beam velocity is that of thermal velocity distribution. Neglecting effects transverse to the beam, more than 99% of the particles will lie within the velocity limits given by $v_o \pm 2\sqrt{\frac{2kT}{M}}$. In this event the beam dispersion due to this cause is

$$\Delta D = \frac{8D}{v_o}\sqrt{\frac{2kT}{M}} , \tag{120}$$

for the velocity distribution in a transverse field.

The dispersion due to velocity differences in the transverse field will be small relative to the deviation of the beam. For example, if protons are generated at 500°k and translated at a velocity of 0.1c, using a deviation of 8000 feet in equation (120) yields a dispersion value of 6.12 feet. This is negligible relative to the values of space charge dispersion and dispersion due to thermal velocities transverse to the beam. It is to be concluded that the dispersion due to the thermal velocity distribution along the beam axis is quite small.

In the general case of equations (113) and (114), it is best to calculate two separate deviations for the velocity extremes given and to take the difference of these results. The same final conclusion is to be expected in these cases as that found for the transverse field.

B. THE RADIATION PRESSURE FIELD

For a uniform radiation field, the equations for deviation and dispersion due to radiation pressure can be obtained from the electrical equations by replacing the electric field components by the corresponding pressure components. In the case of a uniform radiation pressure oriented to have a component opposing the beam, the equation of deviation is

$$D = \frac{M p_y v_o^2}{2 A p_x^2} \left\{ 1 - \sqrt{1 - \frac{2 A x p_x}{M v_o^2}} \right\}^2 . \tag{121}$$

For a pressure component aiding the beam, the equation becomes

$$D = \frac{M p_y v_o^2}{2 A p_x^2} \left\{ 1 - \sqrt{1 + \frac{2 A x p_x}{M v_o^2}} \right\}^2 . \tag{122}$$

The value of A in these equations is the effective particle cross section upon which the radiation pressure acts. Radiation pressure components in the x and y directions are represented by p_x and p_y respectively. For a field transverse to the beam, the equation becomes

$$D = \frac{Apx^2}{2Mv_o^2} , \tag{123}$$

where the subscript applying to the pressure symbol has been dropped. The dispersion in the case of a transverse field is given by

$$\Delta D = \frac{Apx^2}{Mv_o^3} \Delta v_o = D \cdot \frac{2\Delta v_o}{v_o} \tag{124}$$

in analogy with equation (119).

An idea of the magnitude of radiation pressure effects can be derived by the use of equation (123). The product of pressure p and area A is the deflecting force and x the distance to the target from the point of origin of the beam. If protons at 0.1c are used with a particle radius of 1.4×10^{-13} cm and a mass of 1.67×10^{-24} grams are used in a transverse field of 4.5×10^{-5} dynes/cm^2 with a target distance of 10^{14} cm, the resulting deviation is found to be 920 cm. From this calculation, it is seen that the effect of radiation pressure in any region except in the near vicinity of the sun is negligible. It should be noted that the value of radiation pressure used in the calculation is that which applies at the distance of the earth in the radiation field of the sun.

C. THE INVERSE SQUARE REPULSIVE FIELD

The path of a particle in an inverse square field will be a conic section. If the eccentricity of the conic is represented by the symbol ϵ, the equation of the path in polar coordinates (r, θ) is given by

$$r = \frac{\epsilon p}{1 - \epsilon \cos(\alpha - \theta)} , \tag{125}$$

where p and α are constants. The values of these constants may be determined by means of the equations

$$\epsilon p = \frac{M(r_o v_o \sin \theta_o)^2}{-Qq}, \tag{126}$$

$$\epsilon^2 - 1 = \frac{M^2 (r_o v_o \sin \theta_o)^2}{Q^2 q^2} \left\{ v_o^2 + \frac{2Qq}{Mr_o} \right\}, \tag{127}$$

and

$$\alpha = \theta_o - \cos^{-1} \left\{ \frac{1}{\epsilon} \left[1 - \frac{\epsilon p}{r_o} \right] \right\}. \tag{128}$$

The values of r_o and θ_o are those corresponding to the coordinates of the point of origin of the beam. The velocity v_o represents the original velocity of the beam particles, q is the particle charge, and M is its mass. The symbol Q represents the charge upon the object generating the field. Repulsion between the field and the beam is assumed. The orientation is shown in Figure 13.

By means of the figure, the equations

$$x = r_o \cos \theta_o - r \cos \theta \tag{129}$$

and

$$D = r \sin \theta - r_o \sin \theta_o \tag{130}$$

may be written. Equation (125) may be transformed to read

$$\epsilon p = r - r \epsilon \cos \theta \cos \alpha - r \epsilon \sin \theta \sin \alpha \tag{131}$$

by elimination of the denominator and expansion of the cosine function. When the coordinates r and θ are eliminated between (129), (130), and (131), there results a quadratic equation in $D + r_o \sin \theta_o$. Solving this quadratic and making use of the identity

$$r_o \epsilon \cos \theta_o \cos \alpha + r_o \epsilon \sin \theta_o \sin \alpha = r_o - \epsilon p, \tag{132}$$

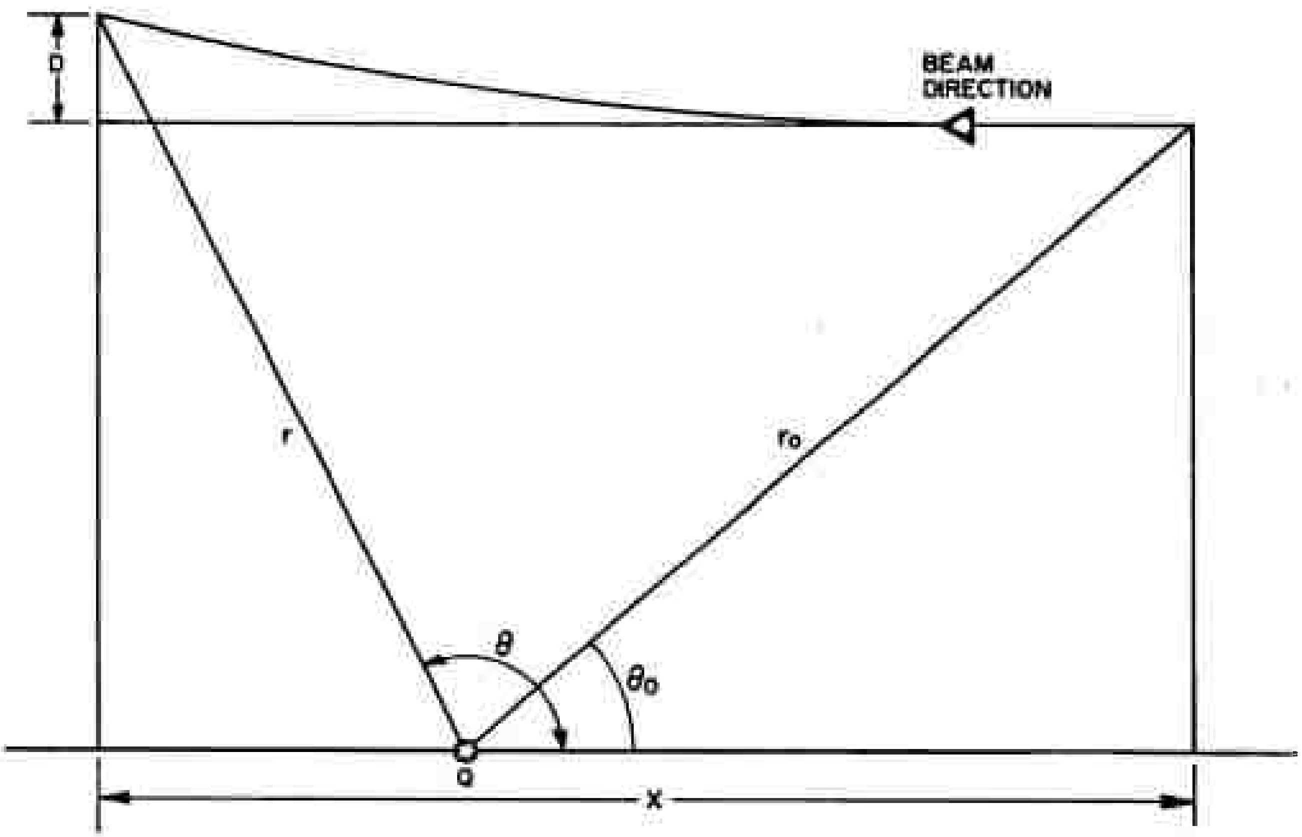

Figure 13. Deviation in an inverse square repulsive field.

the value of D is found to be given by

$$D = [1 - \epsilon^2 \sin^2 \alpha] \{\epsilon \sin\alpha (r_o - x\epsilon\cos\alpha) - r_o \sin\theta_o + [\epsilon^2 p^2 + 2\epsilon^2 p\cos\alpha(r_o\cos\theta_o - x) - (1-\epsilon^2)(r_o\cos\theta_o - x)^2]^{\frac{1}{2}}\}. \quad (133)$$

This is an exact equation, but quite difficult to use in the form presented.

If the value of x is taken quite large in relation to r_o, equation (133) reduces to the form

$$D = \frac{-x[\epsilon^2 \sin\alpha\cos\alpha + \sqrt{\epsilon^2 - 1}]}{1 - \epsilon^2 \sin^2\alpha}. \quad (134)$$

If ϵ^2 is large in relation to unity and α is sufficiently near 90°, the equation reduces to

$$D = \frac{x}{\epsilon}. \quad (135)$$

As an example of the application of the equations, let it be assumed that protons at 0.1c are to be used. Let it be assumed also that the beam is launched at a distance of 8×10^8 cm above the center of the earth (about 1000 miles altitude) at right angles to the radius vector in a radial field of 10^{-6} dynes/esu. The value of r_o is then 8×10^8 and θ_o is 90°. From the assumptions used, the value of Q is $10^{-6}(8 \times 10^8)^2 = 64 \times 10^{10}$ esu. Using the proton mass and charge, the eccentricity is found to be 40.15. The value of ϵp is 3.13×10^{10} and α is $\pi/2$ radians or 90°. The conditions are thus satisfied for the application of equation (135) if x is of the order of 10^{10} or larger. The deviation of the beam in this distance under the assumed circumstances is 2.5×10^8 cm or about 1250 miles.

The axial dispersion of the beam in this distance may be estimated by the observation ϵ is proportional to v_o^2 under the assumed conditions.

As a result of this fact, equation (120) is applicable to a reasonable degree of approximation. The dispersion of the beam in the given distance, assuming a generation temperature of 500° Kelvin is then found to be 1.9×10^5 cm or 1.18 miles.

The dispersion due to transverse thermal velocities is greatly in excess of this value. The time of 3.3 sec is necessary for the beam to travel 10^{10} cm at 0.1c. The application of equation (37), neglecting d_o, gives d as 23.5 miles. Under the conditions indicated, the dispersion due to transverse velocity components is 20 times the dispersion due to axial velocity differences.

D. THE INVERSE SQUARE ATTRACTIVE FIELD

If the field is attractive rather than repulsive, the equations for the trajectory constants are

$$\epsilon p = \frac{M(r_o v_o \sin\theta_o)^2}{Qq}, \tag{136}$$

$$\epsilon^2 - 1 = \frac{M^2 (r_o v_o \sin\theta_o)^2}{Q^2 q^2} \left\{ v_o^2 - \frac{2Qq}{M r_o} \right\}, \tag{137}$$

and

$$-\alpha = \theta_o - \cos^{-1}\left\{\frac{1}{\epsilon}\left[1 - \frac{\epsilon p}{r_o}\right]\right\}. \tag{138}$$

This final equation reverses the sign of α in equation (128) but it must be observed that the value of ϵp as given by equation (136) has been reversed in sign also with respect to that of equation (126) so that the value of α as given by equation (138) is not equivalent to that found in equation (128). The sign of the final term in the brackets in equation (137) is also reversed with respect to that appearing in equation (127). Equation (133) is still valid in form, but because of the reversals in sign of some of the terms involved, it is best written

$$D = [1 - \epsilon^2 \sin^2 \alpha]^{-1} \Big\{ \epsilon \sin \alpha (r_o - x \epsilon \cos \alpha) - r_o \sin \theta_o + \epsilon p \Big[1 + \frac{2}{p} \cos \alpha (r_o \cos \theta_o - x) - \frac{(1 - \epsilon^2)}{\epsilon^2 p^2} (r_o \cos \theta_o - x)^2 \Big]^{\frac{1}{2}} \Big\}. \qquad (139)$$

In this form the reversals in sign are not obscured by the squaring of negative terms under the radical. The values of $r_o \theta_o$, ϵ and x are positive by hypothesis. For a repulsive field α is positive and ϵp is negative so that p is negative. For an attractive field, α is negative and ϵp is positive so that p is positive. It should also be observed that the value of $r_o \cos \theta_o - x$ is negative if x is large. The deviation calculated for the case of attraction should be negative by the definitions used. However, its absolute value is correct in either case.

E. THE GRAVITATIONAL FIELD

Gravitational deviation will always be due to attraction. Since the field is inverse square, no new calculation is necessary. Equations (136) and (137) must be replaced by the equations

$$\epsilon p = \frac{r_o^2 v_o^2 \sin \theta_o}{G M_f} \qquad (140)$$

and

$$\epsilon^2 - 1 = \frac{r_o^2 v_o^2 \sin^2 \theta_o}{G^2 M_f^2} \left\{ v_o^2 - \frac{2 G M_f}{r_o} \right\}, \qquad (141)$$

where G is the universal gravitational constant and M_f is the mass which establishes the field. The ion mass does not appear. Equations (138) and (139) apply without modification except for the fact that the results of equations (140) and (141) are now to be used in them to provide the value of gravitational deflection.

If a velocity of 0.1c is assumed for v_o for the conditions $r_o = 8 \times 10^8$ cm and $\theta_o = \pi/2$, the use of the values $G = 6.67 \times 10^{-8}$ and $M_f = 5.97 \times 10^{27}$ in equation (141) provides a value of eccentricity of 1.81×10^7. It appears by equation (135) that the gravitational deviation will be small for any distance over which the beam can be used. This is to be expected since gravitational forces are small. The mass used in the calculation is the mass of the earth.

F. MAGNETIC FIELD ANALYSIS

In general, the magnetic field will have a component of force along the beam and another transverse to the beam. The beam follows a helical path with the axis of the helix oriented along the field lines. Thus, it is seen that ion beams, when transmitted over long distances, have a tendency to follow magnetic lines in space. Since the deviation is caused only by the transverse field component, the following analysis is limited to the supposition that the field is transverse to the beam. As a matter of further simplicity, it is assumed that the field is uniform over the path.

The conditions of the problem are presented in Figure 14. From the figure, the value of the deviation is $r(1 - \cos\theta)$ and the value of the projected path length is $r \sin\theta$. From the relation

$$\cos\theta = \frac{\sqrt{r^2 - x^2}}{r}, \tag{142}$$

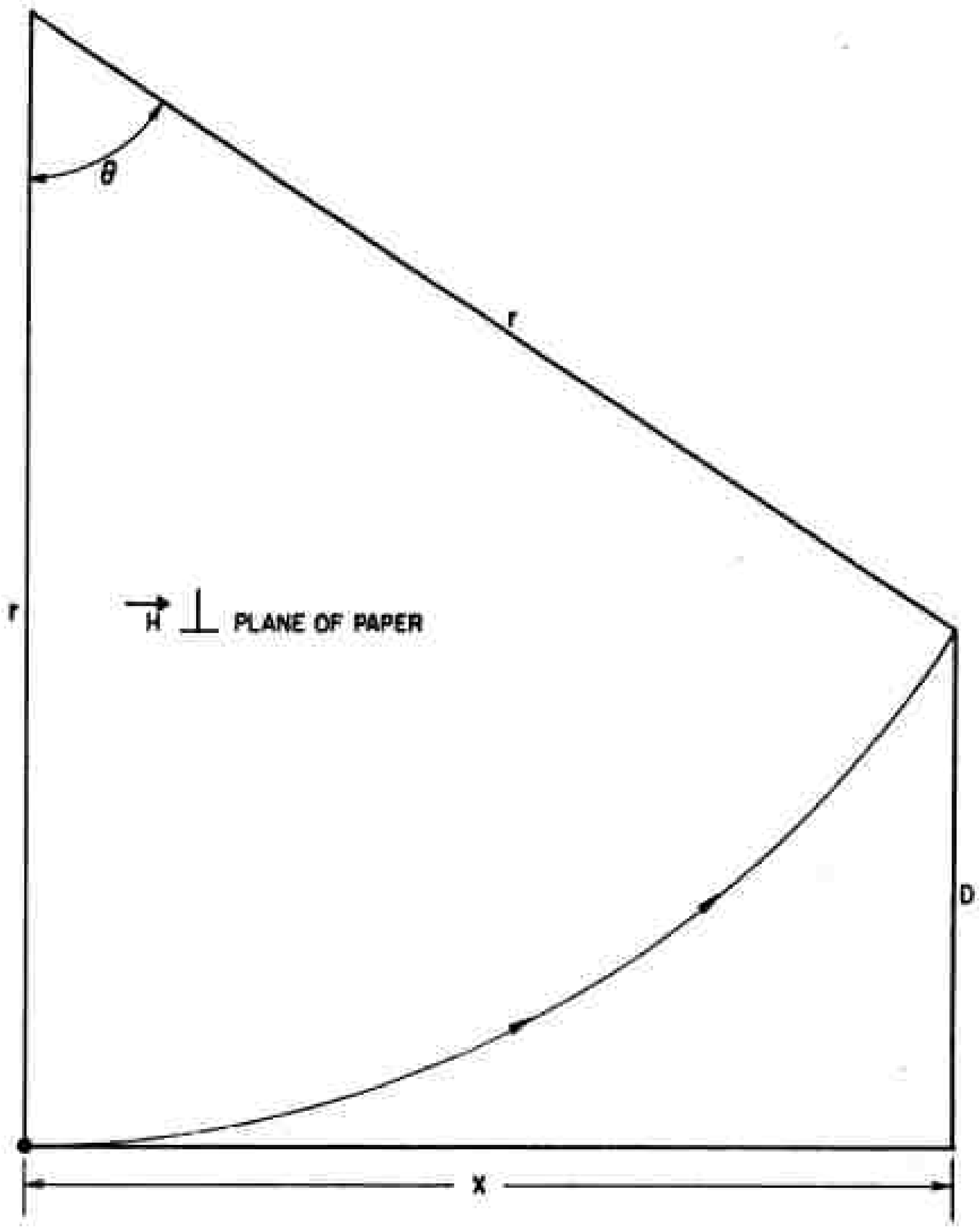

Figure 14. Magnetic deviation in a transverse field.

the deviation is written

$$D = r - \sqrt{r^2 - x^2} . \qquad (143)$$

In a magnetic field, the radius of motion of a charged particle is

$$r = \frac{Mv_o}{Hq} , \qquad (144)$$

where M is particle mass, v_o is velocity, and q is charge. The symbol H represents the field. The use of equation (144) in equation (143) provides

$$D = \frac{M v_o}{Hq} \left\{1 - \sqrt{1 - \frac{H^2 q^2 x^2}{M^2 v_o^2}} \right\} , \qquad (145)$$

where all units are expressed in the electromagnetic system. For a small distance or a large velocity, the value of D is given in approximate form by

$$D = \frac{Hqx^2}{2Mv} . \qquad (146)$$

The value of the beam dispersion caused by transfer velocity differences imposed by thermal effects in the general case is given by

$$\Delta D = \frac{M}{Hq} \left\{1 - \left|1 - \frac{H^2 q^2 x^2}{M^2 v^2}\right|^{-\frac{1}{2}} \right\} \Delta v . \qquad (147)$$

For the limiting case as given by equation (146) the dispersion is

$$\Delta D = \frac{Hqx^2}{2Mv_o^2} \Delta v_o = D \cdot \frac{\Delta v_o}{v_o} . \qquad (148)$$

The space magnetic field established by the motions of charged particles in the solar winds has been estimated as high as 10^{-6} gauss. This is subject to extreme variations at times of high solar activity. As an example of the effect of such a field on a proton beam at a transfer velocity of 0.1c transverse to the field, the following calculation is presented. Let it be assumed that a deviation of 3×10^5 cm is sufficient to cause the target to be missed. The distance to provide this limit under the assumed circumstances is found to be

$$x = \sqrt{\frac{2 \times 1.67 \times 10^{-24} \times 3 \times 10^{9} \times 3 \times 10^{5}}{10^{-5} \times 1.6 \times 10^{-20}}} = 1.37 \times 10^{8} \text{ cm} \qquad (149)$$

by the application of equation (146). This is less than 870 miles. It is indicated that the use of neutral beams may be necessary to avoid excessive deviations in the magnetic fields of space. If this is done, the effect of space electrostatic fields will be eliminated as well. This fact also applies to the space charge divergence of the beam.

G. MAGNETIC STORMS

Magnetic storms cause fluctuations in the magnetic fields of space. These changes will induce fluctuations in the beam deviation. The time derivative of equation (145), using the magnetic field as a variable, is

$$\frac{dD}{dt} = \frac{M v_o}{H^2 q}\left[1 - \left(1 - \frac{H^2 q^2 x^2}{M^2 v_o^2}\right)^{-\frac{1}{2}}\right]\frac{dH}{dt}. \qquad (150)$$

For a large value of v_o or a small value of x, this reduces to

$$\frac{dD}{dt} = \frac{qx^2}{2Mv_o}\ \frac{dH}{dt}. \qquad (151)$$

The effect of magnetic storms on beam dispersion may be evaluated by the derivative of ΔD, assuming the field variable with time. The time derivate of equation (147) for this condition is

$$\frac{d[\Delta D]}{dt} = -\frac{M}{H^2 q}\left\{1 - \frac{\left(1 - \frac{2H^2 q^2 x^2}{M^2 v_o^2}\right)}{\left[1 - \frac{H^2 q^2 x^2}{M^2 v_o^2}\right]^{3/2}}\right\}\frac{dH}{dt}\ \Delta v_o. \qquad (152)$$

In the event that v_o is large or x is small, this becomes

$$\frac{d[\Delta D]}{dt} = \frac{qx^2}{2Mv_o^2} \frac{dH}{dt} \Delta v_o . \quad (153)$$

As an example of the application of magnetic field analysis, equation (145) provides a maximum value for x as required by the fact that the radical must be real. The limiting case is determined by the condition in which the radical becomes zero. Physically, the value of x thus determined is the radius of the circle which the beam will traverse under these conditions. Thus, a beam can be transmitted around an object in a uniform magnetic field. For protons traveling at a velocity of 0.1c in a field of 10^{-6} gauss, the limiting value of x for the assumed conditions is 3.13×10^{11} cm. As seen by equation (145), the value of the deviation for this condition is also equal to 3.13×10^{11} cm.

H. BEAM WIDTH

The calculation of beam width, determined by transfer velocity differences in the beam as it penetrates the magnetic field, is facilitated by the use of geometry as shown in Figure 15. From that figure it is seen that the beam width w is determined by the relation

$$(r_1 + w)\cos(\theta_1 - \theta_2) + (r_1 - r_2)\cos\theta_2 = r_2 . \quad (154)$$

The two radii shown are represented by r_1 and r_2, and the corresponding central angles are given by θ_1 and θ_2. The two circles are determined by particle velocity limits in the beam. Since the velocity differences are small in relation to the average velocity, equation (154) becomes

$$w = (r_2 - r_1)(1 - \cos\theta_2 \quad (155)$$

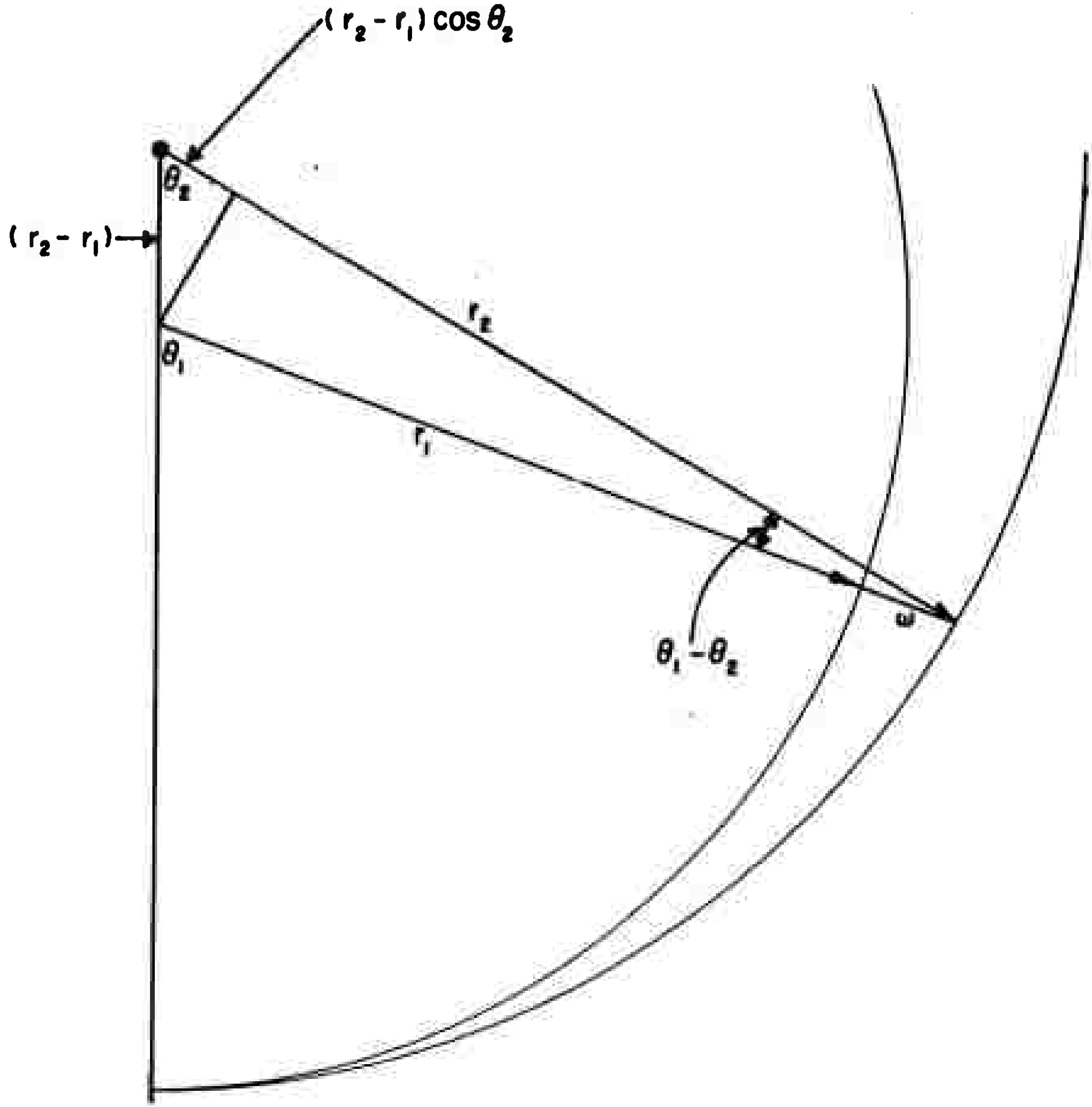

Figure 15. Beam spread due to velocity distribution in a magnetic field.

by use of the fact that the angle $\theta_1 - \theta_2$ is quite small in any practical case. The difference in radii can be expressed in terms of the velocity difference by the application of equation (144). When this is done, the beam width is written

$$w = \frac{M}{Hq} [1 - \cos \theta] \Delta v_o . \tag{156}$$

For the condition $\theta = \pi/2$, using protons at 500°K in a field of 10^{-6} gauss, the beam width is found to be about 750 miles. The value of x corresponding is 1.95×10^6 miles for $v_o = 0.1c$. From this analysis it appears that the beam spread due to this effect is negligible in any distance over which the beam can be used.

XI. THE PROBLEM OF STRIKING A TARGET

A. THE UNIFORM FIELD

The problem of aiming a beam to strike a target at a distance must be considered. The spread of the beam will compensate for minor errors in aiming, but it cannot compensate for deviations produced by fields unless allowance is made in some manner. Since deviations can be predicted, corrections to be applied to compensate for them can also be predicted. Thus, deviations may prove advantageous under certain circumstances such as transmitting around objects.

Figure 16 indicates conditions applying in the case of a uniform radiation pressure field with a component of the field opposing the particle motion. The problem is that of striking a target at a distance x. If it is assumed that the pressure components and target distance are known, the correction angle α can be determined to allow for the deviation to be expected. Since the x component of pressure opposes the beam, the acceleration components are

$$\frac{d^2x}{dt^2} = -\frac{Ap_x}{M} \tag{157}$$

and

$$\frac{d^2y}{dt^2} = \frac{Ap_y}{M}. \tag{158}$$

The symbols p_x and p_y represent the x and y components of pressure respectively. The symbol A represents the effective particle area on which the pressure acts, and M represents the particle mass. The corresponding

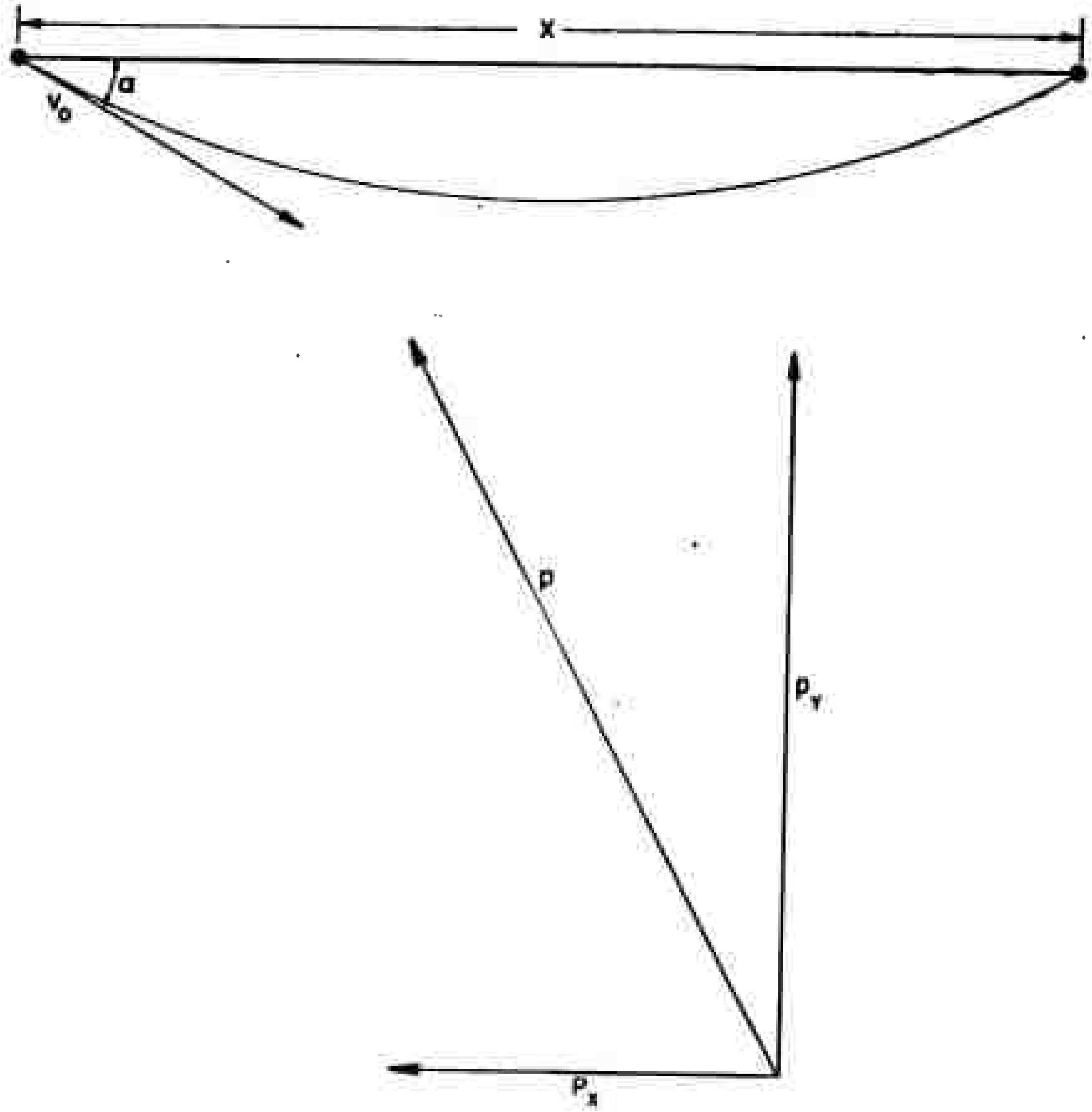

Figure 16. Correction for deviation in a uniform field with a component opposing the velocity.

velocities are

$$\frac{dx}{dt} = v_o \cos\alpha - \frac{Ap_x}{M} t \tag{159}$$

and

$$\frac{dy}{dt} = \frac{Ap_y}{M} - v_o \sin\alpha \, . \tag{160}$$

considering initial conditions imposed by the original velocity v_o of the beam particle acting at the angle α. These equations integrate into

$$x = v_o t \cos\alpha - \frac{Ap_x}{2M} t^2 \tag{161}$$

and

$$y = \frac{Ap_y}{2M} t^2 - v_o t \sin\alpha \, , \tag{162}$$

respectively. The angle α represents the angle between the target direction and the initial velocity v_o which must apply in order that the beam may strike the target.

It is to be observed that the value of y is zero at the target. The corresponding value of the time as found from equation (162) for this condition is

$$t = \frac{2Mv_o \sin\alpha}{Ap_y} \, . \tag{163}$$

When this value is used in equation (161), there results

$$x = \frac{2Mv_o^2 \sin\alpha}{Ap_y^2} \{p_y \cos\alpha - p_x \sin\alpha\} \, . \tag{164}$$

An explicity value for α may be found by substituting $\sqrt{1 - \sin^2\alpha}$ for $\cos\alpha$, isolating the radical and squaring the resulting expression. The equation found by this procedure is a quadratic in $\sin^2 \alpha$, which may be written in the form

$$\left[\frac{p_x^2}{p_y^2} + 1\right] \sin^4 \alpha - \left[1 - \frac{xAp_x}{Mv_o^2}\right] \sin^2 \alpha + \frac{x^2 A^2 p_y^2}{4M^2 v_o^2} = 0 . \qquad (165)$$

The solution is

$$\sin^2 \alpha = \tfrac{1}{2}\left[\frac{p_x^2}{p_y^2} + 1\right]^{-1} \left\{1 - \frac{xAp_x}{Mv_o^2} - \left[1 - \frac{2xAp_x}{Mv_o^2} - \frac{x^2 A^2 p_y^2}{M^2 v_o^4}\right]^{\frac{1}{2}}\right\}, \qquad (166)$$

where the negative sign has been chosen for the radical to satisfy the requirement that α must approach zero as x approaches zero.

In the event that the x component of the radiation pressure aids the motion of the beam rather than opposing it, there is no need to develop a new solution to the problem. A change in the sign governing p_x in equation (166) is sufficient.

The case of the uniform electrostatic field can be handled in the same manner as that applied to the radiation pressure analysis. For a field in which the x component opposes the motion of the beam particles, the substitution of E_x, E_y and q for p_x, p_y, and A respectively can be made in equation (166). The equation then becomes

$$\sin^2 \alpha = \tfrac{1}{2}\left[\frac{E_x^2}{E_y^2} + 1\right]^{-1} \left\{1 - \frac{xqE_x}{Mv_o^2} - \left[1 - \frac{2xqE_x}{Mv_o^2} - \frac{x^2 q^2 E_y^2}{M^2 v_o^4}\right]^{\frac{1}{2}}\right\}. \qquad (167)$$

In the event that the x component of the field aids the particle motion, the sign of E_x must be reversed in equation (167).

Under the circumstances the electric fields in space may be of sufficient magnitude to prevent the beam from striking a target. It is to be observed that the value of $\sin^2 \alpha$, as given by equation (167), cannot be negative or complex. In order that it may not be complex, the maximum permissible value of x is given by setting the radical in the equation equal to zero. When this is done and the resulting equation is solved for x, the value found is

$$x = \frac{Mv_o^2}{qE_y}\left[\sqrt{\frac{E_x^2}{E_y^2} + 1} - \frac{E_x}{E_y}\right]. \tag{168}$$

In a field of 10^{-6} dynes/esu, oriented at an angle of 45^o with respect to the beam, the use of protons at a velocity of 0.1c results in the value of $x = 114{,}000$ miles. The use of this value in equation (167) then gives $\alpha = 22.7$ degrees.

B. THE INVERSE SQUARE FIELD

The analysis for an inverse square repulsive field is made in reference to Figure 17. It is always possible to choose the reference frame in such a way that the target line is parallel to the x axis. The target distance x is given by

$$x = r_0 \cos \theta_0 - r \cos \theta. \tag{169}$$

The y distance is the same at target and launch point. The equation

$$r_0 \sin \theta_0 = r \sin \theta \tag{170}$$

is also seen to apply. The equation of the trajectory is

$$r = \frac{\epsilon p}{1 - \epsilon \cos (\theta - \alpha)} . \tag{171}$$

With the elimination of the denominator and the expansion of the cosine function, this equation may be written

$$r = \epsilon p + \epsilon r \cos \theta \cos \alpha + \epsilon r \sin \theta \sin \alpha . \tag{172}$$

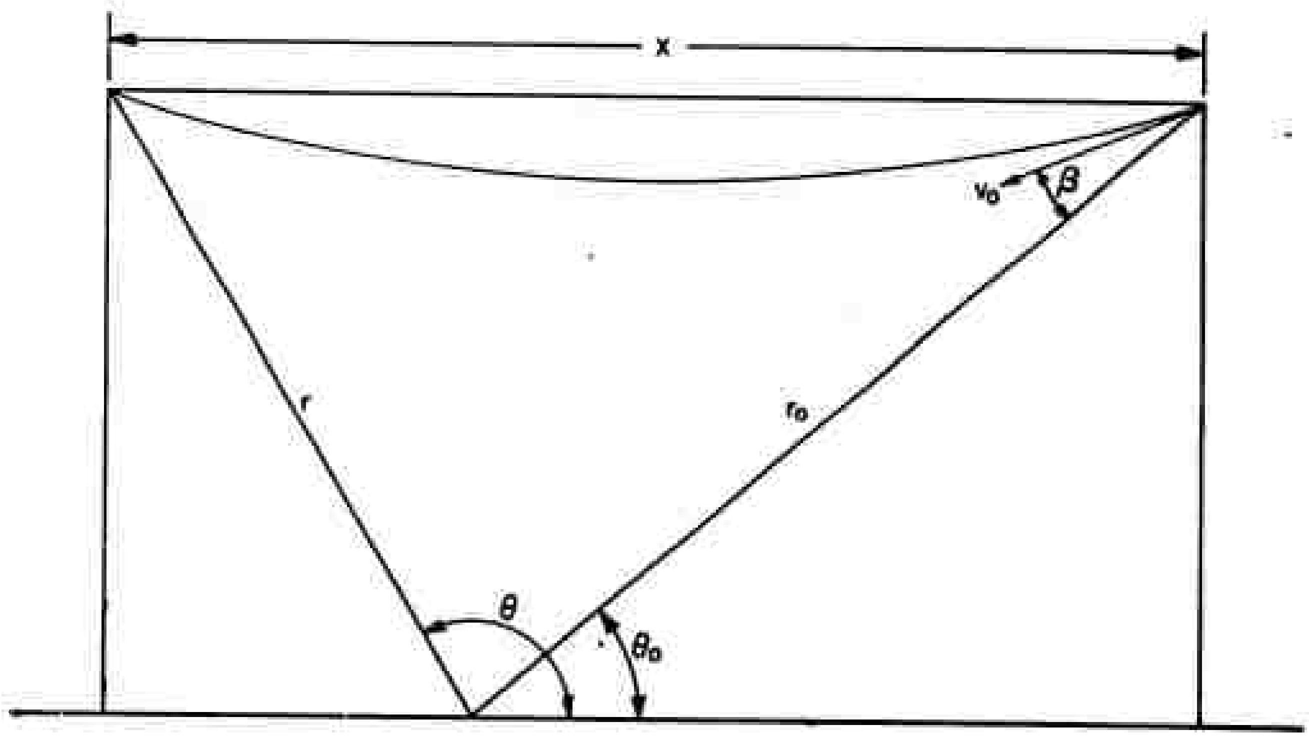

Figure 17. Correction for deviation in an inverse square repulsive field.

The use of equations (169) and (170) in equation (172) then provides

$$r = \epsilon p + \epsilon \cos\alpha\,[\dot{r}_0 \cos\theta_0 - x] + \epsilon r_0 \sin\alpha \sin\theta_0 \,. \tag{173}$$

By means of the figure, the equation

$$r^2 = r_0^2 - 2r_0\, x \cos\theta_0 + x^2 \tag{174}$$

is seen to apply. When equation (173) is squared and set equal to equation (174) and the resulting equation is simplified by the use of the relation

$$r_0 = \frac{\epsilon p}{1 - \epsilon\cos(\theta_0 - \alpha)}\,, \tag{175}$$

it is found that the value of $\cos\alpha$ may be expressed in the form

$$\cos\alpha = \frac{r_0 - \sqrt{r_0^2 - 2r_0\, x\cos\theta_0 + x^2}}{\epsilon x}\,. \tag{176}$$

The use of equation (174) in equation (176) then provides

$$\cos\alpha = \frac{r_0 - r}{\epsilon x}\,. \tag{177}$$

Equation (175) may be solved for α in the form

$$\alpha = \theta_0 - \cos^{-1}\left\{\frac{r_0 - \epsilon p}{\epsilon r_0}\right\}, \tag{178}$$

and equation (177) may be written

$$\alpha = \cos^{-1}\left[\frac{r_0 - r}{\epsilon x}\right]. \tag{179}$$

The elimination of α between the two equations then provides

$$\theta_0 = \cos^{-1}\left[\frac{r_0 - r}{\epsilon x}\right] + \cos^{-1}\left[\frac{r_0 - \epsilon p}{\epsilon r_0}\right]. \tag{180}$$

Taking the cosine of both sides and expanding the resulting cosine function on the right yields

$$\cos\theta_0 = \left[\frac{r_0 - r}{\epsilon x}\right]\left[\frac{r_0 - \epsilon p}{\epsilon r_0}\right] - \left[1 - \left(\frac{r_0 - r}{\epsilon x}\right)^2\right]^{\frac{1}{2}}\left[1 - \left(\frac{r_0 - \epsilon p}{\epsilon r_0}\right)^2\right]^{\frac{1}{2}}, \tag{181}$$

where the identity involving the sine and cosine functions has been used. In order to remove the fractional powers appearing, equation (181) may be written

$$\left[1 - \left(\frac{r_0 - r}{\epsilon x}\right)^2\right]\left[1 - \left(\frac{r_0 - \epsilon p}{\epsilon r_0}\right)^2\right] = \left\{\left[\frac{r_0 - r}{\epsilon x}\right]\left[\frac{r_0 - \epsilon p}{\epsilon r_0}\right] - \cos\theta_0\right\}^2. \quad (182)$$

This collects into the equation

$$\left(\frac{r_0 - r}{x}\right)^2 + \left(\frac{r_0 - \epsilon p}{r_0}\right)^2 - 2\left(\frac{r_0 - r}{x}\right)\left(\frac{r_0 - \epsilon p}{r_0}\right)\cos\theta_0 - \epsilon^2 \sin^2\theta_0 = 0. \quad (183)$$

The value of the eccentricity is given by

$$\epsilon^2 = \frac{(M r_0 v_0^2 + 2Qq)\, M r_0 v_0^2 \sin^2\beta + Q^2 q^2}{Q^2 q^2}, \quad (184)$$

for a repulsive field. The equation

$$\frac{r_0 - \epsilon p}{r_0} = \frac{M r_0 v_0^2 \sin^2\beta + Q q}{Q q} \quad (185)$$

also applies.

The use of equations (184) and (185) in equation (183) then results in a quadratic equation in $\sin^2\beta$ which may be solved in the form

$$\sin^2\beta = \left\{x M r_0 v_0^2 \sin^2\theta_0 + 2Qq\cos\theta_0\,(r_0 - r - x\cos\theta_0)\right.$$
$$+ \sin\theta\,[x^2 M^2 r_0^2 v_0^4 \sin^2\theta_0 + 4xQq\cos\theta\,\{(M r_0 v_0^2 + 2Qq)(r_0 - r)$$
$$\left. - x M r_0 v_0^2 \cos\theta_0\} - 4Q^2 q^2 \{(r_0 - r)^2 + x^2\cos^2\theta_0\}]^{\frac{1}{2}}\right\}[2x M r_0 v_0^2]^{-1}. \quad (186)$$

The value of r may be eliminated by means of equation (174). The positive sign for the radical was chosen since the launch angle β must be equal to θ_0 for the condition $Qq = 0$. The angle of correction with respect to the line of sight is given by $\theta_0 - \beta$.

As an example of the application of equation (186), suppose that $\theta_0 = 60^\circ$ and both r and r_0 are equal to 8×10^8 cm. A repulsive field of 10^{-8} dynes/esu is assumed at the point of launching so that $Q = 64 \times 10^{10}$ esu. Assuming protons at 0.1c and a path distance of 8×10^8 as found from equation (174), equation (186) provides a value $\sin^2 \beta = .737$. The corresponding value of β is 59°. The correction angle is then $60 - 59 = 1^\circ$ as a final result.

For a condition of electrostatic attraction rather than repulsion, there is no need to make a new calculation. The charge combination Qq as it appears in the equations is then replaced by -Qq. The correction angle in this case is given by $\beta - \theta_0$ in order that a positive value may be obtained. For gravitational attraction, the charge combination Qq is replaced by the gravitational product $-GMM_f$. When this substitution is made, the equations are found to be independent of the particle mass. Since the gravitational potential is quite small, a first order approximation reduces the equation for the angle β to

$$\sin^2 \beta = \sin^2 \theta_0 . \tag{187}$$

Thus it appears that the gravitational correction is negligible for any appreciable particle velocity unless extreme distances are to be used.

C. MAGNETIC FIELD ANALYSIS

The general case of a magnetic field which is variable along the path cannot be handled in a reasonably simple manner. Even in the case of a uniform field, the analysis over an extended distance is somewhat difficult. The path of the beam will be a helix with the particles spiraling along the magnetic field lines. An analysis is made in reference to Figure 18.

There are two velocity components in the general case. The velocity component in the direction of the field is not modified by the spiral motion. The velocity component which is transverse to the field is changed in direction but not in magnitude.

The target distance is assumed known and is given as a resultant of x and y in the form

$$R^2 = x^2 + y^2 . . \tag{188}$$

The value of y in terms of time and the component of particle velocity v_a, along the field lines is

$$y = v_a t . \tag{189}$$

The length of the arc S, shown in the figure, is given by

$$S = v_t t = 2 r\left(\frac{\phi}{2}\right), \tag{190}$$

where v_t is the component of particle velocity transverse to the field lines. The x axis of the reference frame lies in the plane of v_a and R. The y axis is in the direction of v_a, parallel to the field lines. The position of the x axis is then made definite by drawing it perpendicular to the y axis as specified.

The beam velocity vector lies in the plane of v_a and v_t. An angle θ as measured with respect to v_a is determined by the equation

$$\tan \theta = \frac{v_t}{v_a} . \tag{191}$$

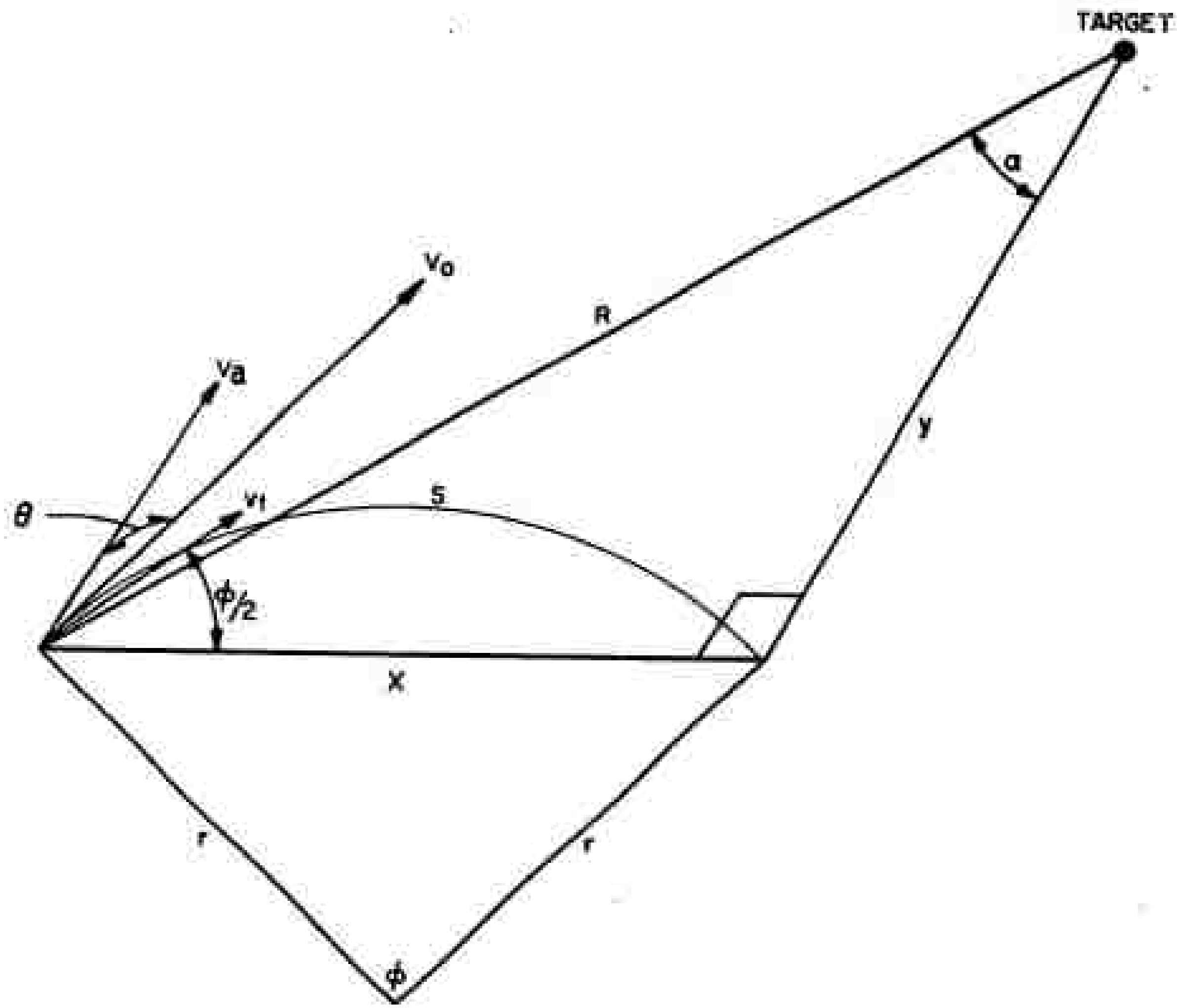

Figure 18. Correction for magnetic deviation in the general case of a uniform field.

The use of equations (189), (190), and (191) then provides

$$y = \frac{2r}{\tan\theta}\left(\frac{\phi}{2}\right). \tag{192}$$

When equation (192) is used in conjunction with the value of x in the form $2r \sin \phi/2$ in equation (188), there results

$$R^2 = 4r^2 \left\{\frac{\left(\frac{\phi}{2}\right)^2}{\tan^2\theta} + \sin^2\frac{\phi}{2}\right\}. \tag{193}$$

The circular radius of the particle path in the magnetic field is given by

$$r = \frac{Mv_0 \sin\theta}{H q}. \tag{194}$$

This value may be substituted in equation (193). The resulting equation is written

$$R^2 = \frac{4M^2 v_0^2}{H^2 q^2}\left\{\left(\frac{\phi}{2}\right)^2 \cos^2\theta + \sin^2\left(\frac{\phi}{2}\right)\sin^2\theta\right\}. \tag{195}$$

In order to make the solution definite, it is necessary to know the orientation of R in space. If the ion source and the target are both specified and the direction of the magnetic field lines in space are known, the plane of the reference frame can be determined as the plane of R and the field line through the ion source. Thus, it appears that the angle α, which R makes with respect to the field lines, is also fixed. Considering the geometry of the system, it follows that the angle α is given by

$$\tan\alpha = \frac{x}{y} = \frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}}\tan\theta. \tag{196}$$

When this is used in equation (195), the result may be reduced to

$$\frac{R^2 H^2 q^2}{4 M^2 v_o^2} = \frac{\phi}{2}^2 \cos^2 \theta \sec^2 \alpha . \tag{197}$$

The value of $\cos^2 \theta$, as developed from equation (196), is

$$\cos^2 \theta = \frac{\sin^2 \frac{\phi}{2}}{\frac{\phi}{2}^2 \tan^2 \alpha + \sin^2 \frac{\phi}{2}} . \tag{198}$$

When equation (198) is used in equation (197), there results

$$\frac{\frac{\phi}{2}^2 \sin^2 \frac{\phi}{2} \sec^2 \alpha}{\frac{\phi}{2}^2 \tan^2 \alpha + \sin^2 \frac{\phi}{2}} = \frac{R^2 H^2 q^2}{4 M^2 v_o^2} . \tag{199}$$

For relatively short distances in the magnetic fields of space, the angle $\frac{\phi}{2}$ will be small. If this is the case, equation (199) may be approximated in the form

$$\frac{\phi}{2} = \frac{R H q}{2 M v_o} . \tag{200}$$

If the distance R is assumed to be 10^{10} cm with the angle α equal to 45^o, the use of protons at 0.1c in a field of 10^{-6} gauss provides a value of ϕ equal to 1.83 degrees. The use of this value in equation (198) then yields a value of θ equal to 45 degrees. The direction in which the beam must be aimed in order to strike the target under the given circumstances is completely specified.

XII. CONCLUSIONS

Upon examination of the effects of scattering of ion beams by photons and solar particles, it is concluded that, although these effects provide some limitations as to the use of ion beams for communications, they are by no means as severe as the beam dispersion due to self-contained electrostatic fields and thermal velocity distributions. In the approach to a quantitative analysis of the effects of phenomena discussed in this report, the most detrimental conditions were assumed to exist. In all cases where the theory is only accurate for equilibrium conditions, they are assumed since only order of magnitude effects can be expected under the somewhat nebulous knowledge of outer space conditions. It is concluded that the use of ions as an effective means of communication must be limited to relatively short distances within the solar system, as for example, between satellites or space ships where transfer of information could be effected at short ranges without extreme interference. Further, the possibility of utilization of the exceptional ability of power transfer by this means should be examined with the concurrent problems of reception and storage for maintenance of suitable artificial environments in space. It is concluded that the possibility of an effective short-range weapons system can be examined with pertinent applications to future use in space. Finally, the possibility of a practical ion propulsion system may well be determined by consideration of recombination principles with resultant experimental configurations for effecting recombinations. In view of the above conclusions, it may be of further advantage to examine certain types of experimental apparatus with an eye to future development and testing under actual space conditions.

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