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THE MEASUREMENT OF UNSTEADY PRESSURES IN WIND TUNNELS

by

E. L. DAVIS Jr.

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THE MEASUREMENT OF UNSTEADY PRESSURES IN WIND TUNNELS

by

Eugene L. Davis, Jr.

This Report was presented at the Pressure Measurements Meetings, sponsored by the Agard Wind Tunnel and Model Testing Panel, held from 24th to 28th March, 1958, in London

SUMMARY

Practical solutions to the problem of accurately measuring unsteady pressures in wind tunnels are described, with emphasis on the response of pressure systems, calibration techniques and equipment, and wind-tunnel instrumentation. Basic guides for the selection of a pressure-gage-volume-connecting-tubing system are given. A cam-type pulsator calibrator with a sinusoidal pressure variation up to ±3 lb/in.^{2} and a frequency range up to 5,000 c/sec is described. The minimum number of pressure gages required for lift and moment measurements is discussed. A brief comment on the interpretation of pressure fluctuations in terms of velocity fluctuations is given.

SOMMAIRE

L'auteur en exposant des méthodes pratiques permettant de résoudre le problème de la mesure précise en soufflerie de pressions instationnaires, et examine tout particulièrement les questions portant sur le temps de réponse des appareils de mesure de pressions, les techniques et appareillage d'étalonnage et les instruments utilisés en soufflerie. Il indique les principes fondamentaux du choix d'un système manomètre - tuyautage de liaison et décrit l'emploi comme appareil étalon d'un pulsomètre du type à came caractérisé par des variations de pression sinusoïdales jusqu'a ± 13 livres par pouce carré et par une gamme de fréquences allant jusqu'à 5 000 périodes par seconde. En conclusion il traite du nombre minimum de jauges de pression nécessaires aux mesures de la portance et des moments et commente de façon sommaire sur l'interprétation des variations de pression en fonction des variations de vitesse.

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NOTATION

A	a constant coefficient
a	tubing radius
B	a constant coefficient
C	specific heat of fluid
c	velocity of sound in a cylindrical tube
F	a function
f	frequency, cycles per second
H	total pressure
i	$\sqrt{-1}$
J_n	Bessel function of first kind of order n
k	complex wave number, $i\omega \div$ propagation velocity
L	lift
l	tubing length
n	summation index
P	mean pressure
p	excess pressure
T	averaging time interval, seconds
t	time, seconds
U	mean velocity
V	volume
v	unsteady velocity
x	linear distance
β	complex thermal wave number
γ	specific-heat ratio
Δ	condensation
δ	a linear distance

ϵ	a response parameter
η	absolute viscosity
κ	thermal conductivity
Λ	bulk modulus of elasticity
ρ	mean density
σ	complex shear wave number
ω	frequency, radians per second

Subscripts

g	pressure gage volume
i	input
o	mean value
t	tube
v	constant volume

THE MEASUREMENT OF UNSTEADY PRESSURES IN WIND TUNNELS

Eugene L. Davis, Jr.*

1. INTRODUCTION

Unsteady pressure measurements are important in many areas of wind tunnel research and testing, including such activities as dynamic-stability research, flutter research, helicopter-blade research, engine-inlet pulsation studies, blast-wave and shock-tube measurements, and blowdown-tunnel measurements.

The measurement of unsteady pressures emphasizes problems of instrumentation and data reduction entirely different from the problems of static-pressure measurements. The frequency response of the pressure gage and of auxiliary instrumentation is of paramount importance; calibration techniques and equipment are different; and the data representation is usually in the form of time histories or complex frequency plane plots. Despite the different emphasis, however, the practical solution to these problems can be simple and straightforward. Experience at the Langley Aeronautical Laboratory of the N.A.C.A. (National Advisory Committee for Aeronautics) over the last ten or twelve years has shown that the problem of accurately measuring unsteady pressures can be solved without resorting to complex or expensive equipment. The purpose of this paper is to review some of the principal aspects of the problem and of the solution.

Until recent years the principal problem in unsteady pressure measurements was the development of a suitable pressure gage. Unsteady pressures are often associated with severe environmental conditions, such as wide temperature variations, high vibratory accelerations, and, in the case of rotating devices such as helicopter blades, high static accelerations. The effects of connecting tubing on the pressure-gage response dictate that the pressure gage be mounted close to the point of measurement; the pressure gage must be relatively unaffected by the severe environmental conditions, must have a sufficiently high frequency response, and must be small enough to be mounted in small models or in thin airfoils. Criteria for the pressure gage for unsteady pressure measurements have been given by Patterson[1] and by Molyneux and Ruddlesden[2].

Although the importance of the pressure gage for unsteady pressure measurements cannot be overemphasized, pressure gages as such will not be discussed in this paper. Pressure gages developed specifically for unsteady pressure measurements have been described in the literature[1-3]. The effects of connecting tubing on the frequency response of the pressure gage may be avoided by the use of flush-diaphragm pressure gages; the useful frequency range of the flush diaphragm pressure gage is limited only by the mechanical resonances of the diaphragm and associated mechanical parts. The use of flush diaphragm pressure gages is not always possible nor always desirable. Measurements near the leading and trailing edges of thin airfoils require the use of connecting tubing because of the physical size of currently available flush-diaphragm pressure gages; differential pressure measurements between two surfaces such as the

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top and bottom of an airfoil are conveniently made by using a single pressure gage with connecting tubing to each surface. The connecting tubing must be selected, however, with proper regard for the frequency response and phase shift requirements of the specific research problem.

In wind-tunnel-model installations, the physical size and frequency range of interest is usually such that viscous and thermal effects on the frequency response of the pressure-gage-volume-connecting-tubing system predominate. Apparently no adequate treatment of the frequency response of a pressure system in this range has appeared in the literature; for this reason, basic guides for the selection of the pressure gage-connecting tubing systems are given.

The response characteristics of a pressure-gage-volume--connecting-tubing system cannot be calculated to the degree of precision required in an instrument. The possibility of non-linearities always exists in a pressure system; one of the purposes of the calibration is to determine whether the non-linear effects of excess pressure amplitude and turbulence are important. The calibration must be made over the complete frequency range of interest at the excess pressure amplitudes and local mean densities expected during the research tests. Special calibration equipment and techniques are described herein.

In the study of force and moment coefficients on an oscillating body experimental results are compared with theory, but data reduction and data interpretation problems are difficult because wind-tunnel turbulence and local separation effects introduce noise and second-order non-linear effects into the measurements. Simple analog procedures for integrating pressure distributions to obtain linearized in-phase and quadrature lifts and moments are invaluable aids in overcoming these problems. Some simple analog procedures are briefly described.

2. PRESSURE-SYSTEM RESPONSE

A pressure-gage-volume-connecting-tubing system is represented schematically in Figure 1 by a tubing of length l and radius a connected to a gage volume Vg. The instantaneous ratio of the gage-volume pressure p_g to the input pressure p_i is defined as the dynamic response of the system; for a sinusoidal input pressure the dynamic response is a complex function of frequency and may be represented by a magnitude and a phase angle.

Typical wind-tunnel-model pressure-gage-volume-connecting-tubing installations are shown in Figure 2. Experience indicates that the frequency range of interest is usually less than 1,000 c/sec, with the range of greatest interest being less than 100 or 200 c/sec. Because of the physical size of typical models, and because of the frequency range of interest, a typical installation is made up of tubing lengths ranging from 0.1 in. to 6 in., of tubing diameters ranging from 0.015 in. to 0.125 in., and gage volumes from 0.01 cu in. to 0.1 cu in. In these ranges viscous and thermal effects predominate, as is shown in the typical experimentally obtained response-magnitude and phase-shift curve shown in Figure 3. Furthermore, these viscous and thermal effects vary with the frequency of oscillation; the simplified Helmholtz resonator formulas[4], the steady flow approximations using the Hagen-Poiseuille law[5], and the so-called 'organ-pipe resonance formulas' are not always applicable.

The usual linear analytical treatment of the dynamic response of the pressure system is based on (a) the analogy between the electrical transmission line and the pressure system[6], or (b) on finding an equivalent single-degree-of-freedom system for the pressure system[7]. The usual assumptions are:

(a) Constant pressure over the tube across section

(b) Constant pressure over the gage volume

(c) Laminar flow throughout the system

(d) Small oscillation amplitudes.

For the sinusoidal pressure input, the dynamic response when these assumptions apply is given by

$$\frac{p_g}{p_i} = \frac{1}{\cos kl + \frac{\Lambda_t}{\Lambda_g} \frac{V_g}{V_t} (kl)^2 \frac{\sin kl}{kl}} \tag{1}$$

Mason[6] has given a similar result based on the electrical transmission line analogy. Taback[8], using the propagation velocity calculated from the Rayleigh formula[9] and measured values of the attenuation, has applied the transmission line analogy successfully to pressure systems with tubing lengths, tubing diameters, and instrument volumes usually found in flight applications.

For pressure systems in wind-tunnel-model installations, the tubing lengths, tubing volumes, and instrument volumes are usually such that the approximations

$$\left.\begin{aligned} \cos kl &\cong 1 \\ \frac{\sin kl}{kl} &\cong 1 \end{aligned}\right\} \tag{2}$$

are applicable. The response given in Equation (1) then reduces to

$$\frac{p_g}{p_i} \simeq \frac{1}{1 + \frac{\Lambda_t}{\Lambda_g} \frac{V_g}{V_t} (kl)^2} \tag{3}$$

The lumped constant treatments of the dynamic response of the pressure system frequently found in the literature are asymptotic approximations to Equation (3).

The bulk modulus of elasticity Λ is defined as the ratio of the excess pressure to the condensation[5],

$$\frac{p}{\Delta} = \Lambda \tag{4}$$

The bulk modulus of elasticity is a function of mean density, mean pressure, frequency, geometry, thermal conductivity, and specific heat at constant volume.

Ballantine[10] and Daniels[11, 12] have treated the problem of thermal effects on the bulk modulus of elasticity for simple geometric shapes. For a circular tube,

$$\Lambda_t = P_0\left[1 + (\gamma - 1)\frac{J_2(\beta)}{J_0(\beta)}\right] \tag{5}$$

where the parameter β is defined by

$$\beta = i^{3/2}\left[a\sqrt{\frac{\rho C_v \omega}{\kappa}}\right] \tag{6}$$

The magnitude and phase angle of the ratio of the bulk modulus of elasticity to the mean pressure for air under standard sea level conditions in a circular tube is given in Figure 4. The significant range of the parameter $|\beta|$ is from 0 to 10. Asymptotic values of Equation (5) are

$$\left.\begin{aligned} \lim_{\beta\to 0} \Lambda_t &= P_0 \\ \lim_{\beta\to\infty} \Lambda_t &= \gamma P_0 \end{aligned}\right\} \tag{7}$$

corresponding to isothermal and isentropic conditions, respectively.

For simplicity in calculations, the bulk modulus of elasticity in the circular tube may be taken equal to the bulk modulus of elasticity in the gage volume; the resultant error is probably no greater than the uncertainty in calculating an effective bulk modulus of elasticity. The modulus Λ_g may be modified to take into account diaphragm deflections, if these deflections are important.

The propagation velocity in narrow tubes is a function of the bulk modulus of elasticity and the shear wave number. For a sinusoidal input and a circular tube,

$$k^2 = \frac{-\omega^2}{\left(\frac{\Lambda_t}{\rho} + \frac{\eta i \omega}{\rho}\right)\frac{J_2(\sigma)}{J_0(\sigma)}} \tag{8}$$

The parameter σ is defined by

$$\sigma = i^{3/2}\left[a\sqrt{\frac{\rho\omega}{\eta}}\right] \tag{9}$$

The quantity $a\sqrt{\rho\omega/\eta}$ is the shear wave number; physically, this quantity is a measure of the depth of penetration of the wall shearing effects into the fluid and is related to the boundary-layer thickness[13]. The kinetic theory of gases predicts the relationship[14]

$$\kappa = \eta C_v \tag{10}$$

If Equation (10) holds, the parameter β is identical to the parameter σ. With this assumption, the magnitude and phase angle of quantity $(ka^2\sqrt{P_0\rho})/\eta$ is given as a function of shear wave number in Figure 5.

The shear wave number is an important parameter in the dynamic response of a pressure system for values of the shear wave number less than 10.

These results may be put into functional form for comparing experimental calibrations and for predicting response characteristics from experience. Thus

$$\frac{p_g}{p_i} = F\left[\frac{\omega l}{c}, \frac{V_t}{V_g}, a\sqrt{\frac{\rho\omega}{\eta}}\right] \tag{11}$$

Further insight into the effects of the various parameters is provided by the asymptotic forms of Equation (3). For short lengths of tubing of large diameter, with the ratio of the gage volume to the tubing volume greater than unity, the response approaches that for an undamped Helmholtz resonator, viz.

$$\frac{p_g}{p_i} \simeq \frac{1}{1 - \frac{V_g}{V_t}\frac{l^2\omega^2}{c^2}} \qquad \left[\begin{matrix} |\sigma^2| > 100 \\ \frac{V_g}{V_t} > 1 \end{matrix}\right] \tag{12}$$

For small-diameter tubing, the response approaches that obtained by using the Hagen-Poiseuille law for the pressure drop in the tubing; thus

$$\frac{p_g}{p_i} \cong \frac{1}{1 + i\omega\frac{V_g}{V_t}\frac{8\eta l^2}{P_0 a^2}} \qquad \left[|\sigma^2| < 1\right] \tag{13}$$

The direct combination of Equations (12) and (13) has been used[7] to obtain an equivalent single-degree-of-freedom representation of the pressure system. The result, corresponding to that given in Reference 7, is

$$\frac{p_g}{p_i} \cong \frac{1}{1 - \frac{V_g}{V_t}\frac{l^2\omega^2}{c^2} + i\omega\frac{V_g}{V_t}\frac{8\eta l^2}{P_0 a^2}} \tag{14}$$

Equation (14) may be used as a rough guide, but it must be emphasized that the pressure system is represented by a differential equation with frequency-dependent coefficients. Thurston[13], using D-C 200 silicone fluid, has given a rigorous experimental proof that the equivalent inertia and resistance of a narrow tube are functions of frequency.

A working graphical method for determining the response of a pressure system may be derived from an alternate form of Equation (3), viz:

$$\frac{p_g}{p_i} = \frac{\epsilon}{\epsilon - \sigma^4 \dfrac{J_0(\sigma)}{J_2(\sigma)}} \tag{15}$$

where the parameter ϵ is defined by

$$\epsilon = \frac{A_g}{A_t} \frac{V_t}{V_g} \left(\frac{\rho^2 a^4}{l^2 \eta^2} \right) c^2 \tag{16}$$

The quantity $\sigma^4 J_0(\sigma)/J_2(\sigma)$ has been plotted in polar form in Figure 6 for a range of the parameter $a\sqrt{\rho\omega/\eta}$ from 0 to 3.8. The response of the pressure system may be determined graphically by locating the values of the parameter ϵ on the negative real axis and drawing a vector from this point to the appropriate value of the shear wave number on the curve. The reciprocal of this vector length multiplied by the parameter ϵ is the magnitude of the response; the phase angle is the negative value of the angle formed by the vector and the positive real axis.

From Figure 6 the optimum 'flatness', corresponding to a critical damping ratio near unity, occurs for values of the parameter ϵ between 20 and 40.

A comparison between experimental and calculated values of the response magnitude is shown in Figures 7 and 8. The important effect of the local mean density on the response is demonstrated in Figure 7.

Because of mechanical-drive-system design and other problems, oscillating wind-tunnel models are usually driven in simple harmonic motion, so that the results given above for a sinusoidal pressure input are directly applicable in selecting a pressure-gage-connecting-tubing system. In blast wave and gust entry studies, however, the transient response of the pressure system is important.

In principle, the transient response of the pressure system may be obtained from a Fourier integral transformation of Equation (1) or Equation (3). This procedure involves solving a transcendental equation in complex hyperbolic or complex Bessel functions, or both, and in practice is not worth while. Transient pressure changes are often large enough to invalidate the assumptions of small amplitudes; the rate of change of pressure is often such that tubing lags are excessive. For these reasons, flush diaphragm pressure gages are used almost exclusively in blast-wave and shock-tube measurements. However, physical size or other limitations may dictate the use of connecting tubing, in which case the results given above may be indirectly applied.

The pressure-gage-connecting-tubing system is inherently dispersive; that is, the velocity of propagation is a function of frequency, so that a complex pressure wave form is not preserved as it is transmitted down the tubing. This effect may be made small by keeping the tubing as short as possible, the radius as large as possible, and the ratio of the gage volume to the tubing volume small. The resonant frequency is made high and the resonances are removed from the time history recording by means of electrical filters placed in series with the pressure-gage output. This technique should be used with caution; the acoustical resonances may momentarily overload the auxiliary amplifying equipment, and, if carrier-signal excitation is used, the phenomenon of 'frequency fold-over', in which the acoustical resonant frequency modulation of the carrier signal produces spurious outputs with the filter pass-band, may occur.

If the details of the leading edge of the input-pressure waveform are unimportant, as in blowdown tunnel pressure measurements, an alternative approach is to use small-radii tubing and small instrument volumes, so that the tubing system is 'overdamped'. The response is then given approximately by Equation (14), and the methods of Reference 7 may be used to obtain the time lags and the transient response.

3. CALIBRATION EQUIPMENT

In practice, it is difficult to determine the parameters of the pressure system to the degree of precision required, particularly for small-radii, small-instrument-volume systems. The attenuation losses vary as the fourth power of the radius, so that small errors in radius have a disproportionate effect. The end effects on short tubes are open to question. Fittings between the gage and the tubing introduce small, unknown volumes. The fluid constants are never known exactly; for example, from purely theoretical considerations the effective viscosity is a combination of shear viscosity and bulk viscosity, but experiment seems to indicate that the effect of bulk viscosity is negligible. Experimental calibrations remove these uncertainties.

Beranek[15] has given a survey of high-intensity pressure-calibration techniques and equipment used in calibrating high-intensity microphones. Many of these techniques are directly applicable to pressure-gage calibrations.

The pressure calibration equipment should meet the following specifications:

(a) The pressure-oscillation amplitude should be constant over long periods and yet easily varied over the full amplitude range of interest;

(b) The oscillation frequency should be continuously variable over the full frequency range of interest;

(c) The local mean density and local mean temperature should be known and should be variable;

(d) The waveform of the pressure oscillation should be relatively free of harmonics;

(e) The calibrator should have provision for a reference pressure gage 'standard' near the point of measurement;

(f) For added convenience in calibrating, the pressure-oscillation amplitude should remain nearly constant over the full frequency range of interest.

At very low frequencies, these specifications are easily met by a collapsible bellows type of pressure calibrator, where the bellows are operated by an eccentric crank-arm drive. The mean density within the bellows may be controlled by connecting the bellows to a pressure source.

For frequencies between 10 c/sec and 100 c/sec, a conventional piston-type calibrator may be used. Piston leakages cause errors at low frequencies, and at frequencies above 100 c/sec wear of the mechanical drive system causes difficulty.

For frequencies between 20 c/sec and 5,000 c/sec, a unique, cam-type pulsator meeting most of the required specifications is shown in Figure 9. The chamber of the pulsator is rectangular, approximately 1/32 in. by 5/8 in. by 1 in. The pressure-gage-tubing system to be calibrated is connected to the top of the chamber directly opposite a flush-diaphragm reference-pressure gage. The chamber is supplied from a continuously variable high-pressure source through a small orifice; the discharge from the chamber is varied sinusoidally by means of a rotating cam. The space between the cam and the chamber opening is adjusted until the waveform is sinusoidal; the pressure amplitude is varied by means of the high pressure supply. The mean density within the chamber is essentially the same as the density in the region outside the chamber; the mean density may be varied by placing the pulsator within an altitude chamber. The supply pressure is maintained sufficiently high so that the velocity in the orifice is sonic to decouple the supply line from the chamber.

The frequency range of the pulsator is limited by the combination of orifice size, chamber volume, and cam spacing. For the chamber dimensions given, the frequency range is approximately 20 c/sec to 5,000 c/sec; at frequencies below 20 c/sec the waveform distorts. In order to eliminate chamber resonances in the range from 2,000 c/sec to 5,000 c/sec. an irregularly shaped paper liner is placed within the chamber so that no parallel reflecting surfaces exist in the two long dimensions of the chamber.

The pressure oscillation amplitude is continuously variable over a ratio of 70 to 1, with 2500 dynes/cm^2 peak to peak being the lowest practical amplitude.

The cam-type pulsator has proven to be a flexible laboratory calibration apparatus. Air, Freon, and water have been used as calibration media. For transient calibrations, the sine-wave cam has been replaced with a square-wave cam.

A resonant-chamber calibration unit driven by an electromagnetic driver, of the type described by Oberst[16], and shock tubes have been used to calibrate pressure gages at the Langley Aeronautical Laboratory. The theory of the shock tube is too well known to need to be described here; the problems in applying a shock tube to pressure-gage calibrations are primarily practical. For very low shock pressures (0.1 to 1 lb/in.^{2} the principal difficulty lies in obtaining a suitable diaphragm material. Ordinary 0.002 in. thick cellophane, aged by baking at 200° F to 250° F for several hours, has proved satisfactory.

4. INTEGRATED PRESSURE DISTRIBUTIONS

In determinations of unsteady lift and moment by use of pressure gages, the pressure distribution is integrated numerically by adding weight values of the individual instantaneous pressures. This addition can be made by simple, conventional electrical circuits[17].

The question of the minimum number of gages for the maximum accuracy in lift and moment is important from the viewpoint of simplicity and economy. The usual procedure is to locate the pressure gages arbitrarily, with known static-pressure distributions used as a guide. The weights may be determined by assigning areas to each station (rectangular weighting) or by passing a polynomial through three or more stations and solving a set of simultaneous equations. For n pressure stations, an exact integration is possible if the pressure distribution can be represented by a polynomial of the (n-1)th degree.

An alternate procedure is to use the classic Gauss's numbers integration method[18], wherein the pressure orifices are located according to the roots of a Legendre polynomial normalized over the interval 0 to 1. The method has the advantage that for n pressure stations, an exact integration is possible if the pressure distribution can be represented by a polynomial of (2n-1)th degree. For example, if the pressure distribution can be represented by a polynomial of the 13th degree, an exact integration is possible if 14 pressure stations weighted arbitrarily are used, or if 7 pressure stations weighted according to Gauss's method are used. Tables for the pressure stations and weights using Gauss's method may be found in Reference 18.

On the assumption that in practice the pressure distribution can be approximated by a polynomial (this is not true for the theoretical pressure distribution over a thin airfoil, which has a singularity at the leading edge), Gauss's method will always lead to a higher order of accuracy for the same number of pressure stations. If the pressure distribution cannot be represented by a (2n-1)th degree polynomial, Gauss's method will yield a least-squares approximation to the integrated values. If an arbitrarily placed pressure gage is added to n gages weighted according to Gauss's method, the added pressure gage should have zero weight for maximum accuracy (see Appendix).

If the number of pressure gages in the integration interval is increased beyond a certain number (approximately twelve), the probable error in the integration becomes independent of the method of weighting; Gauss's method is advantageous only where space or other considerations limit the number of possible pressure stations. A statistical determination of the probable error in chordwise lift using Gauss's method and twenty static pressure distributions on a 4% thick and a 6% thick airfoil over the transonic speed range is given in Figure 10, as a function of the number of pressure stations. The lift values were compared with an integration obtained by using 100 stations in the integration interval.

5. IN-PHASE AND QUADRATURE LIFT AND MOMENT

The in-phase and quadrature lift and moment may be determined by measuring the magnitude of the integrated pressure distribution and the phase angle between the integrated pressure distributions and the oscillating angle of attack.

The phase angle may be determined from time-history recordings of the pressure signal and the angle of attack by reading cross-over points each half-cycle with respect to a common time base. If many readings are taken and averaged, this procedure tends to eliminate the effect of random errors and of even harmonic distortion; odd harmonic distortion tends to bias the phase-angle readings. Experience at the Langley Aeronautical Laboratory indicates that the best measurement accuracy in determining phase angle from time-history recordings is of the order of 2^0 to 5^0, with most of the error due to random noise. In order to improve the accuracy, band-pass filters may be inserted in series with the pressure signal and the angle-of-attack signal, as described by Rainey[19], or one of the analog procedures for direct determination of the in-phase and quadrature components, such as the contactor cam described by Molyneaux and Ruddleston[2], or the electrical component resolvers[20,21], may be used.

In any case, a measurement time is required, which must be greater than the settling time of the filtering process. The settling time (sampling time, in a statistical sense) in seconds is given approximately by

$$t = \frac{1}{f_2 - f_1} \tag{17}$$

where $f_2 - f_1$ is the band width of the filtering process. For blowdown tunnels, this requirement specifies the minimum running time for a measurement.

The electrical-component resolvers have been used successfully at both the Langley Aeronautical Laboratory and the Ames Aeronautical Laboratory of the N.A.C.A. for obtaining in-phase and quadrature components for several years. An electrical-component resolver is a precision rotary transformer which produces outputs proportional to the product of the primary input voltage and the sine and the cosine of the resolver-shaft position angle. If the signal proportional to lift is applied to the primary, and the resolver shaft is rotated in phase with the oscillating angle of attack, the averaged outputs are analogous to

$$\frac{1}{T}\int_0^T L(t)\cos \omega t \, dt \tag{18}$$

$$\frac{1}{T}\int_0^T L(t)\sin \omega t \, dt \tag{19}$$

which are the Fourier series coefficients for the fundamental-frequency component of the lift signal. For higher harmonic component determinations (as in helicopter blade research), the resolver shaft is rotated as a multiple of the fundamental frequency. Resolvers capable of speeds up to 50 c/sec for extended periods of time are available.

The resolver serves the function of an extremely narrow band-pass filter; the effective band width is inversely proportional to the averaging time. Averaging may be accomplished by means of low-pass electrical filters, overdamped galvanometers, or by numerically averaging the time history output of the resolver.

6. TURBULENCE MEASUREMENTS

Unsteady pressure measurements may be used as a measure of turbulence or tunnel 'roughness' if the results are properly interpreted. At higher frequencies, the unsteady-pressure amplitudes are approximately proportional to the time derivative of the velocity fluctuations; the root-mean-square amplitude of the unsteady pressures in a given frequency band is higher than the root-mean-square amplitude of the unsteady velocity fluctuations. In incompressible potential flow in one dimension, the Bernoulli equation relating velocity and pressure is

$$\frac{\partial}{\partial t}\int U\,dx + \frac{1}{2}U^2 = \frac{H - P_0}{\rho} \qquad (20)$$

Superposing a small-amplitude disturbance in the flow, and discarding the mean values gives for this equation,

$$\frac{\partial}{\partial t}\int v\,dx + Uv = \frac{-p}{\rho} \qquad (21)$$

If the velocity and pressure fluctuations can be represented by a closed Fourier series,

$$\left.\begin{aligned} v &= \sum_{n=-N}^{n=+N} A_n \epsilon^{i\omega nt} \\ p &= \sum_{n=-M}^{n=+M} B_n \epsilon^{i\omega nt} \end{aligned}\right\} \qquad (22)$$

Substitution into Equation (21) yields the relationship between coefficients of the velocity and pressure series expansion,

$$B_n = -\rho U A_n \left(1 + \frac{i\omega n\delta}{U}\right) \qquad (23)$$

where δ is the distance along the path of integration. Asymptotic values are

$$\left.\begin{aligned} B_n &= -\rho U A_n && \left[\frac{\omega n \delta}{U} \ll 1\right] \\ B_n &= -i\rho A_n \omega n\delta && \left[\frac{\omega n \delta}{U} \gg 1\right] \end{aligned}\right\} \qquad (24)$$

An illustrative, hypothetical spectrum of the pressure fluctuations in the wake of a turbine rotor, as compared to the spectrum of the velocity fluctuations, is given in Figure 11. From Figure 11, the overall pressure amplitude is many times the overall velocity amplitude.

These results may be derived by using the general equations for fluid flow and assuming a polytropic expansion for the mean flow and the superposed unsteady flow. An extension of these results to random disturbances may be made by using the concept of the correlation function. The distance δ is a function of the geometry; for a slender conical flow-angle probe pointing into the stream, δ is the distance from the apex of the cone to the pressure orifice.

7. CONCLUDING REMARKS

The solution to the problem of accurately measuring unsteady pressures depends on the proper selection of the pressure-gage-connecting-tubing system, on proper calibration techniques and equipment, and on utilization of simple analog techniques to reduce the labor and expense of data reduction and interpretation. Successful calibration techniques depend on recognition of the fact that the response of the pressure system may change with changes in local mean density and local mean temperature during the wind-tunnel testing; by proper choice of the pressure-system parameters (with subsequent experimental verification) these density and temperature effects may be reduced to a minimum. The application of analog techniques to the data-reduction procedures is based upon the classic Fourier series representations of a complex waveform.

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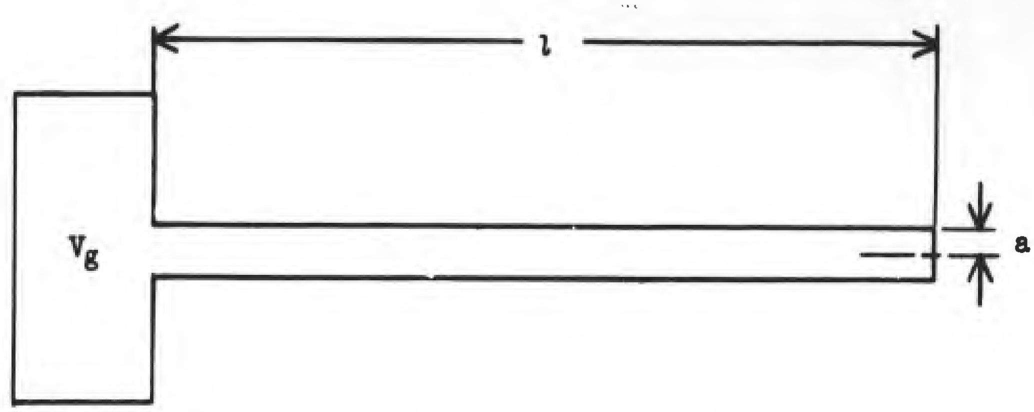
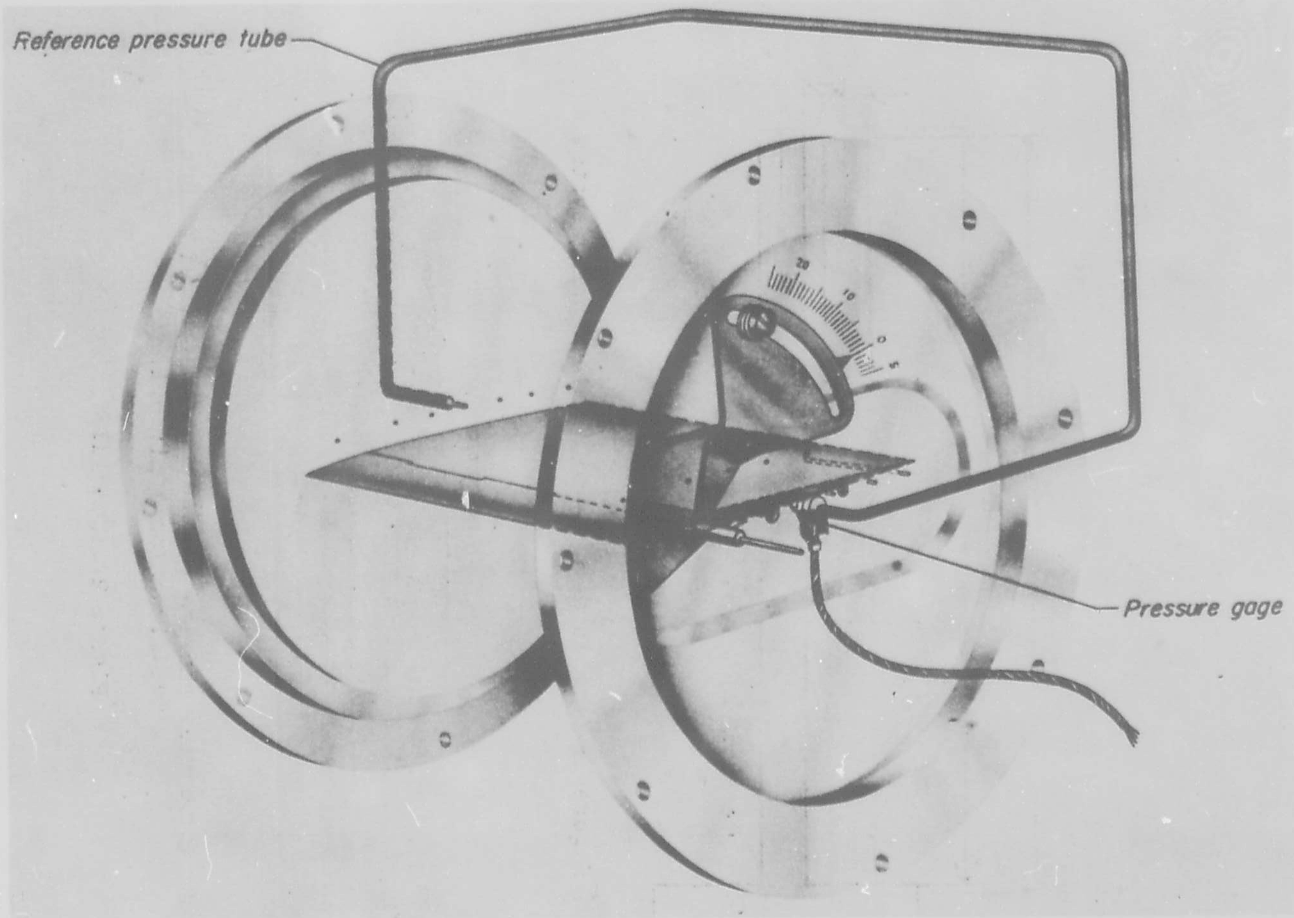
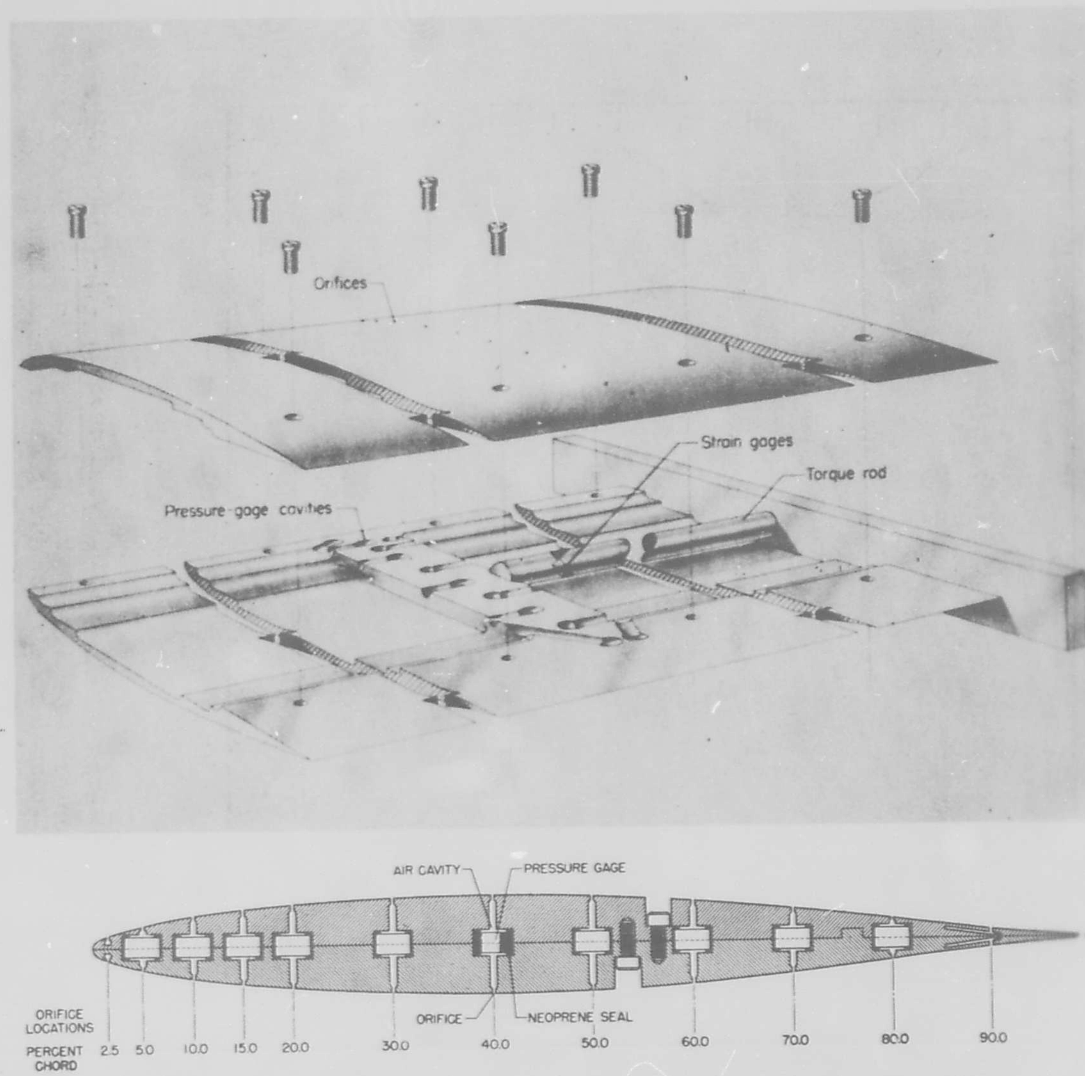


Fig. 1 Pressure gage-connecting-tubing system



(a) Pressure gages mounted outside wind tunnel

Fig.2 Typical wind-tunnel-model pressure gage installations



(b) Exploded view showing pressure gages mounted inside a two-dimensional airfoil

Fig.2 (concluded)

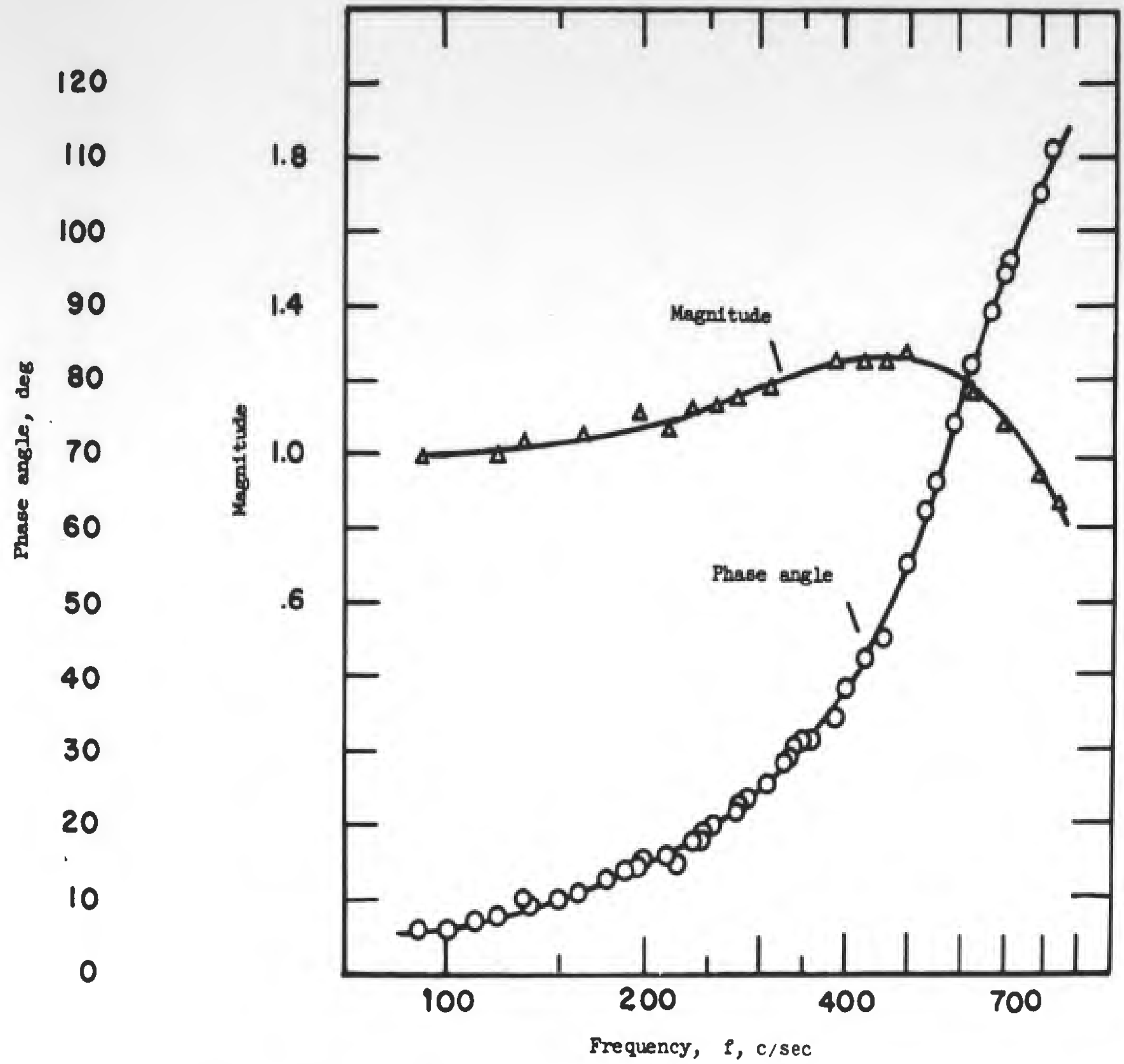


Fig. 3 Experimental response magnitude and phase-angle calibration for 1.38 in. of 0.030 in. nominal-diameter tubing connected to a gage volume of 0.02 cu in. for air at sea-level conditions

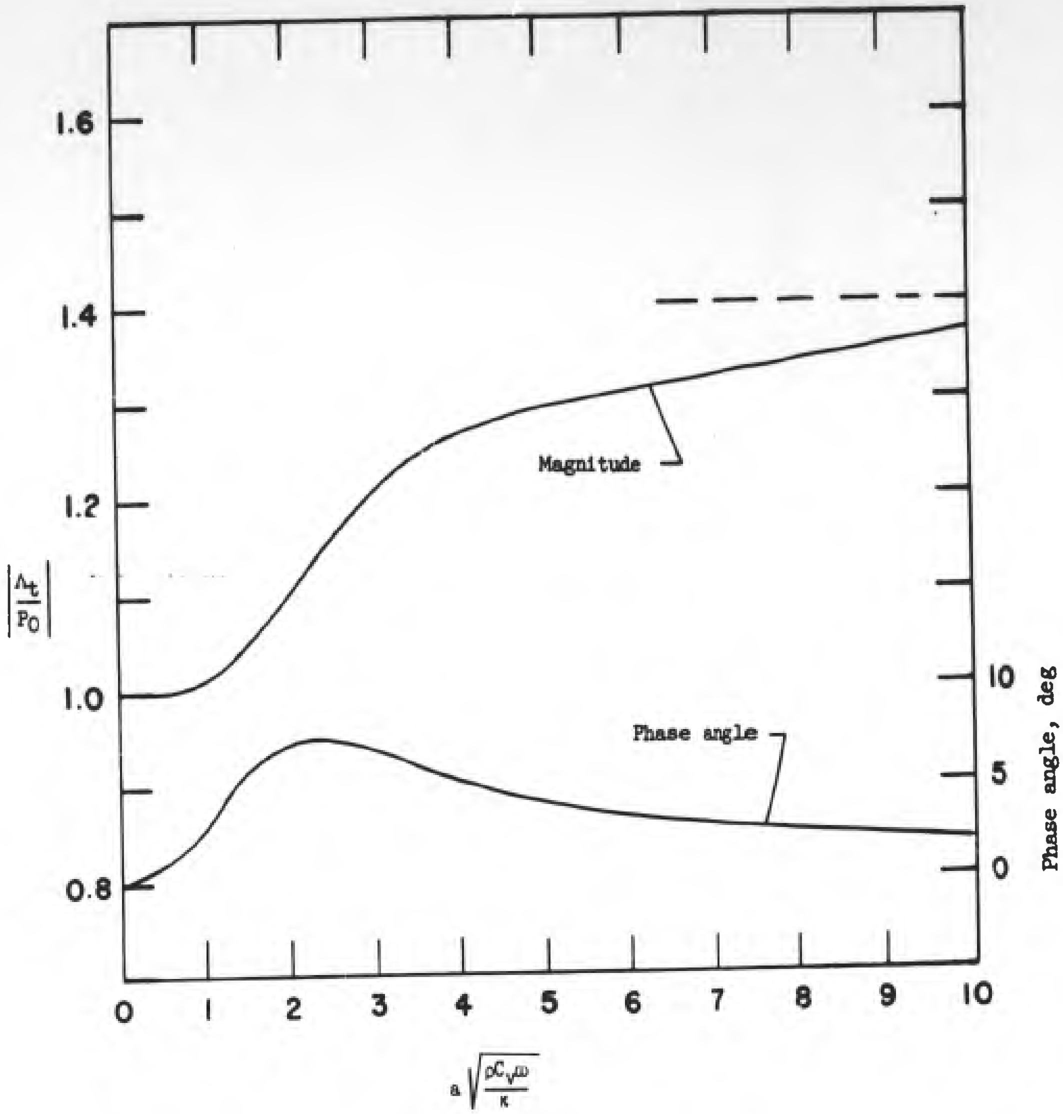


Fig.4 Magnitude and phase angle of the ratio of the bulk modulus of elasticity to the mean pressure as a function of the termal wave number for air in a cylindrical tube at standard sea-level conditions

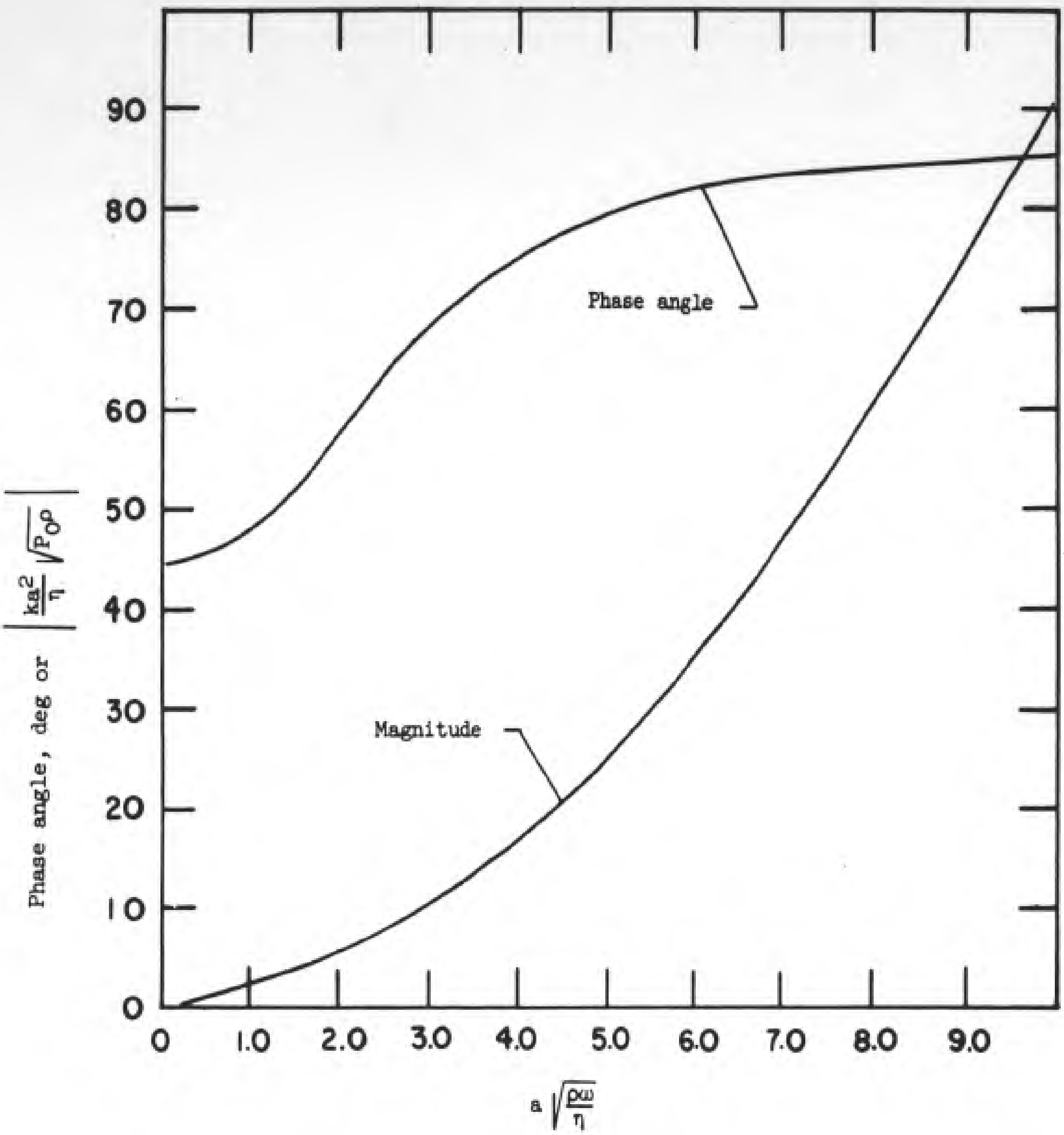


Fig. 5 Plot of the magnitude and phase angle of the parameter $(ka^2\sqrt{P_0\rho})/\eta$ as a function of the shear wave number for air in a cylindrical tube at standard sea-level conditions

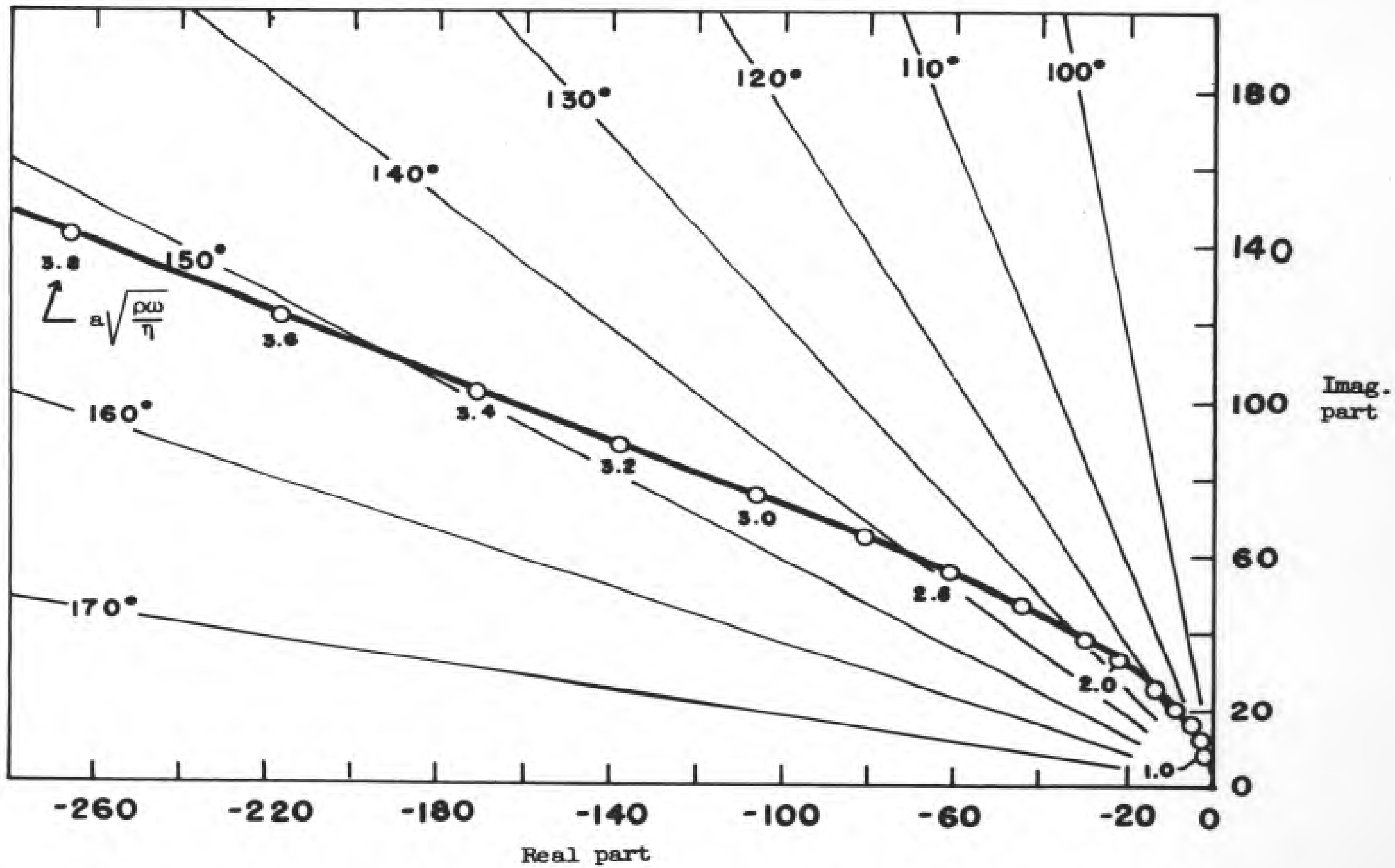


Fig. 6 Polar plot of the quantity $\sigma^4 J_0(\sigma)/J_2(\sigma)$ as a function of the shear wave number $a\sqrt{\rho\omega/\eta}$

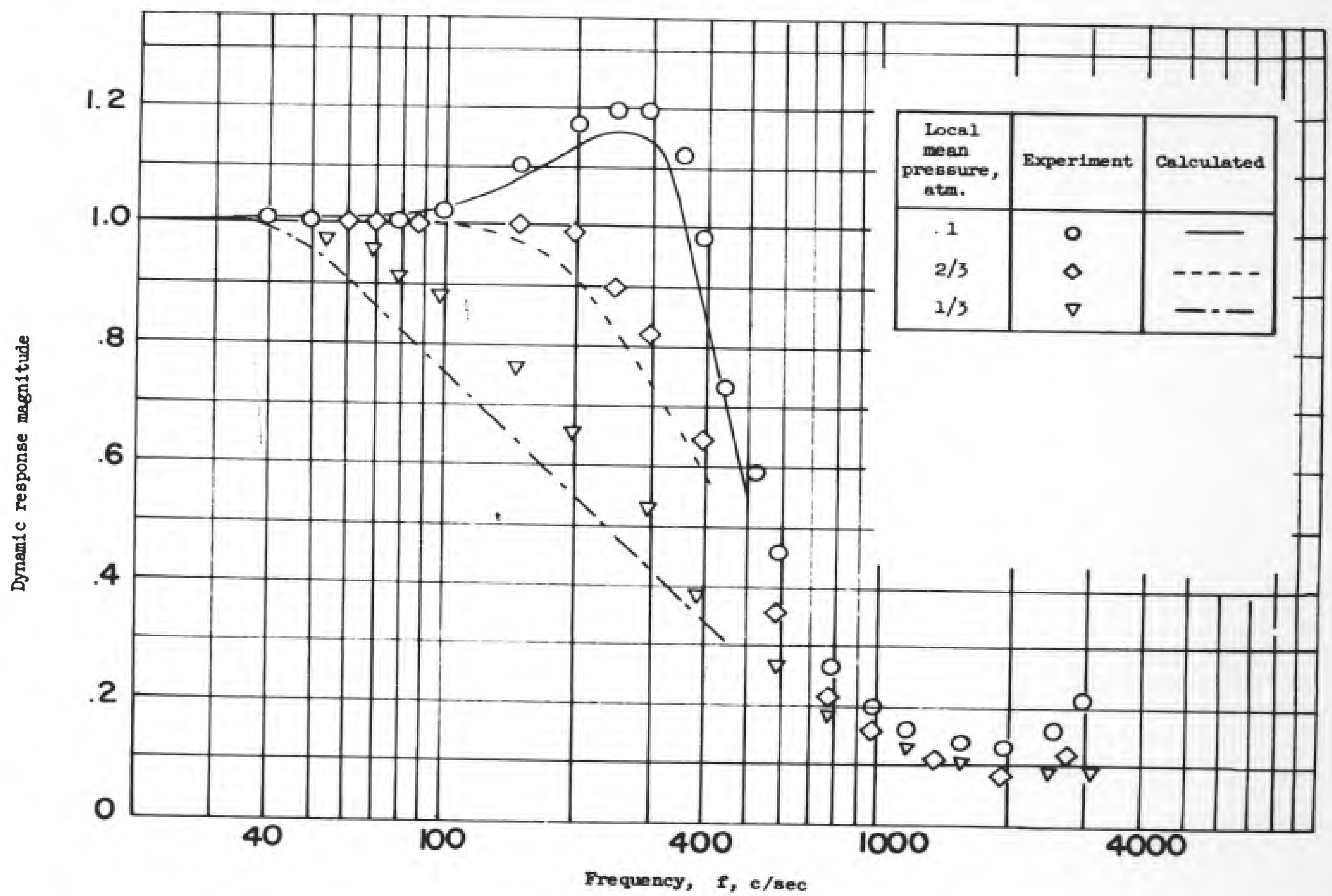


Fig. 7 The dynamic-response magnitude for a nominal 0.020 in. dia. tube 2 in. in length connected to a pressure gage with a volume of 0.015 cu in. at three values of local mean density

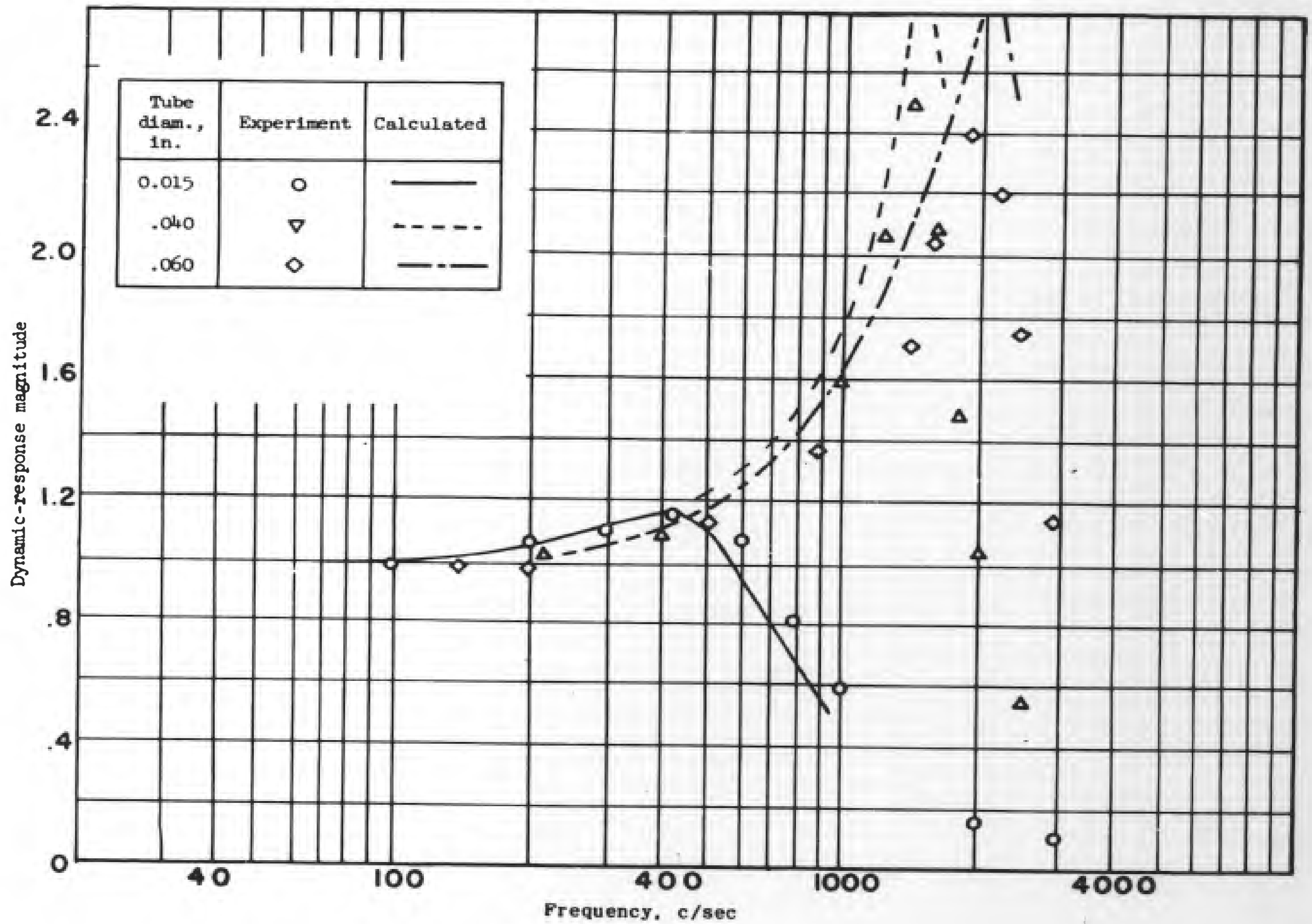


Fig. 8 The dynamic-response magnitude for 0.5 in. of a 0.015 in. nominal-diameter tubing, 0.5 in. of a 0.040 in. nominal-diameter tubing, and 0.5 in. of a 0.060 in. nominal-diameter tubing, connected to a gage volume of 0.015 cu in. at standard sea-level conditions

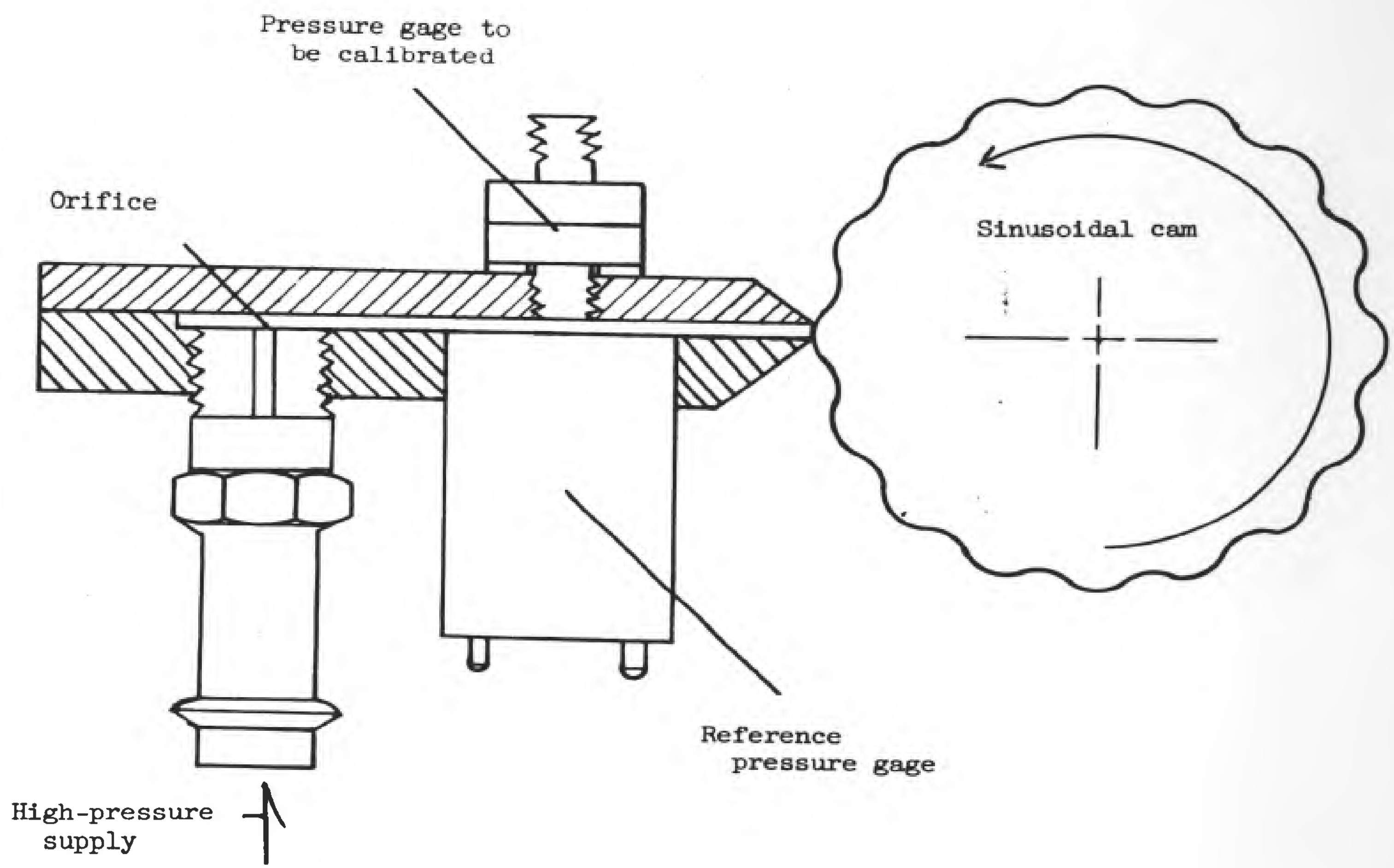


Fig.9 The cam-type pulsator calibrator

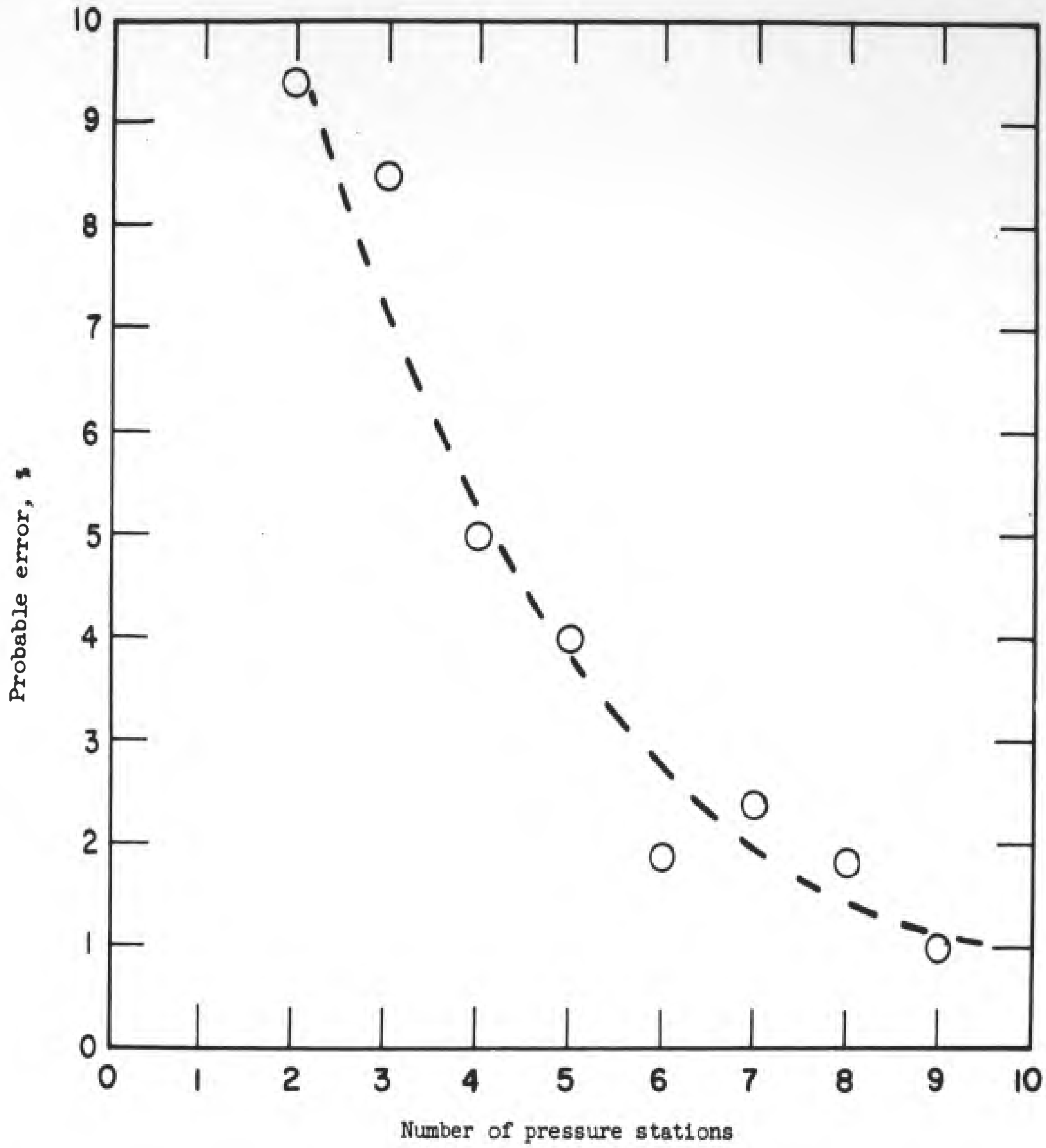


Fig. 10 The probable error in chordwise lift as a function of the number of chordwise pressure stations, using Gauss's method of weighting

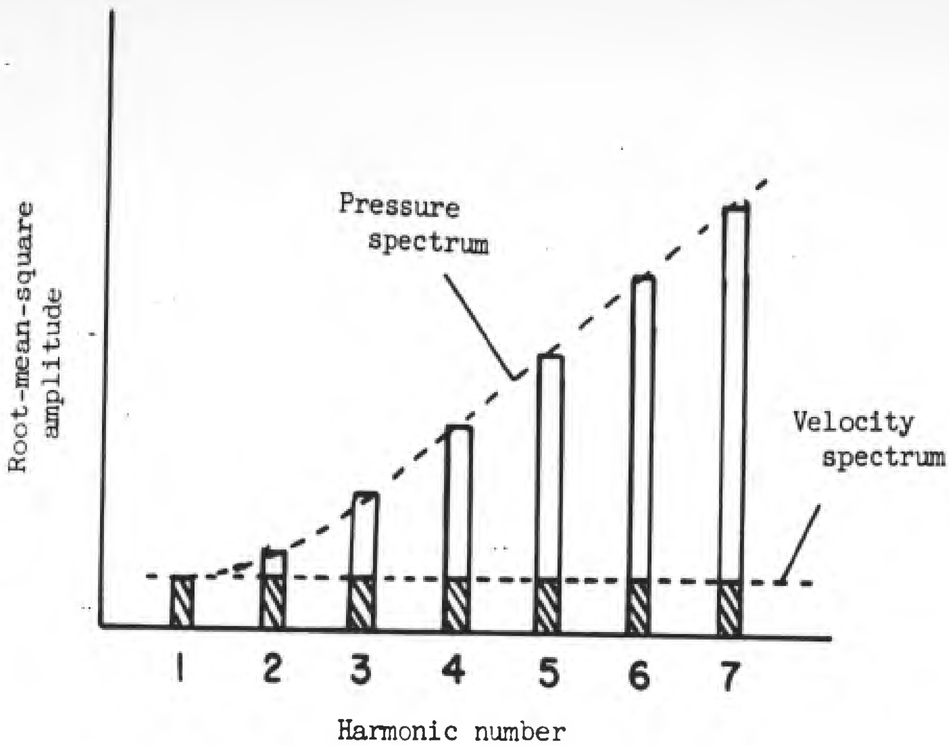


Fig. 11 Illustration of relationship between the unsteady-pressure spectrum and unsteady-velocity spectrum

APPENDIX

THE WEIGHTING OF AN ARBITRARILY PLACED PRESSURE GAGE ADDED TO n PRESSURE GAGES WEIGHTED ACCORDING TO GAUSS'S METHOD

APPENDIX

The Weighting of an Arbitrarily placed Pressure Gage added to n Pressure Gages Weighted according to Gauss's Method

Let n pressure stations be located at $x_1, x_2, \ldots x_k \ldots x_n$. Assume that the pressure distribution can be represented exactly by a polynomial of degree (2n - 1). The pressure at any point on the line connecting the stations is, by Lagrange's interpolation formula,

$$p(x) = \sum_{k=1}^{n} I_k^{(n)}(x)p(x_k) \qquad (A.1)$$

where

$$I_k^{(n)}(x) = \frac{\prod_{j=1}^{n}{}'(x - x_j)}{\prod_{j=1}^{n}{}'(x_k - x_j)} \qquad (A.2)$$

where the prime indicates the omission of the k^{th} term from the product.

The lift on a strip of unit width connecting the stations is given by the integral of Equation (A.1):

$$L = \sum_{k=1}^{n} W(x_k)p(x_k) \qquad (A.3)$$

where the weighting of the k^{th} station, $W(x_k)$, is given by

$$W(x_k) = \int_0^1 I_k^{(n)}(x)dx \qquad (A.4)$$

If the locations $x_1, x_2, \ldots x_k \ldots x_n$ are the roots of a Legendre polynomial normalized over the interval 0 to 1, then, by Equation (A.2),

$$W(x_k) = \frac{1}{\prod_{j=1}^{n}{}'(x_k - x_j)} \int_0^1 \frac{P_n(x)}{x - x_k}\,dx \qquad (A.5)$$

where $P_n(x)$ is a Legendre polynomial of order n normalized over the interval 0 to 1.

If an additional pressure gage is located at an arbitrary point $x = x_m$, the weighting of this pressure gage is

$$W(x_m) = \frac{1}{\prod_{j=1}^{n}{}' (x_m - x_j)} \int_0^1 P_n(x)dx \qquad (A.6)$$

However, by definition,

$$\int_0^1 P_n(x)dx = 0$$

Hence, the weighting of the additional gage is zero. Using the identity

$$\int_0^1 x^m P_n(x)dx = 0 \qquad (m \leqq n-1) \qquad (A.7)$$

It can be shown that the best weighting for m arbitrarily placed gages added to n gages weighted according to Gauss's method is zero, until the number m is equal to or greater than the number n.

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Eugene L. Davis, Jr.
1958
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