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RELATION BETWEEN SURFACE ROUGHNESS AND SPECULAR REFLECTANCE AT NORMAL INCIDENCE

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ABSTRACT. Expressions relating the roughness of a plane surface to its specular reflectance at normal incidence are presented and are verified experimentally. The expressions are valid for the case when the root mean square surface roughness is small compared to the wavelength of light. If light of a sufficiently long wavelength is used, the decrease in measured specular reflectance due to surface roughness is a function only of the root mean square height of the surface irregularities. Long-wavelength specular reflectance measurements thus provide a simple and sensitive method for accurate measurement of surface finish. This method is particularly useful for surface finishes too fine to be measured accurately by conventional tracing instruments. Surface roughness must also be considered in precise optical measurements. For example, a non-negligible systematic error in specular reflectance measurements will be made even if the root mean square surface roughness is less than 0.01 wavelength. The roughness of even optically polished surfaces may thus be important for measurements in the visible and ultraviolet regions of the spectrum.

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FOREWORD

This paper reports a new, high-precision, reflection method of measuring surface roughness. It provides for the replacement of the crude surface measuring devices, such as the profilometers found in machine shops. This capability has been needed to study the mechanism of polishing metal surfaces which has been an art to date. Use of higher temperatures and bearing speeds demands the transfer of metal polishing from an art to a technological basis.

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Relation Between Surface Roughness and Specular Reflectance at Normal Incidence

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Expressions relating the roughness of a plane surface to its specular reflectance at normal incidence are presented and are verified experimentally. The expressions are valid for the case when the root mean square surface roughness is small compared to the wavelength of light. If light of a sufficiently long wavelength is used, the decrease in measured specular reflectance due to surface roughness is a function only of the root mean square height of the surface irregularities. Long-wavelength specular reflectance measurements thus provide a simple and sensitive method for accurate measurement of surface finish. This method is particularly useful for surface finishes too fine to be measured accurately by conventional tracing instruments. Surface roughness must also be considered in precise optical measurements. For example, a non-negligible systematic error in specular reflectance measurements will be made even if the root mean square surface roughness is less than 0.01 wavelength. The roughness of even optically polished surfaces may thus be important for measurements in the visible and ultraviolet regions of the spectrum.

INTRODUCTION

THE reflectance of a surface is a sensitive function of its roughness. However, an adequate and experimentally verified theory relating these properties has been lacking. This paper describes such a theory and its verification for the case of normal incidence.

Several experimental investigations of the relation between the roughness of machined metal surfaces or ground glass surfaces and the specular or diffuse reflectance have been reported.[1-8] Light in the visible region was used and in most cases the reflectance was measured at oblique incidence, since at these wavelengths the surface irregularities are comparable in magnitude to the wavelength and the amount of light which is specularly reflected at normal incidence is quite small. Under these circumstances the reflectance depends not only on the surface roughness but also on other aspects of the surface, e.g., the root mean square slope, so that reflectance measurements have been of only limited usefulness as a method of determining surface roughness.

The present investigation suggests that if somewhat longer wavelengths are used for such surfaces, the characteristics of the surface other than roughness become unimportant and specular reflectance measurements at nearly normal incidence provide a simple and precise method of determining the root mean square roughness of a plane surface. This method can best be applied to surfaces with a root mean square roughness of less than 50 μin. The roughness of surfaces in this range is of considerable practical importance, and current methods of measurement are not completely satisfactory. The reflectance method is easily applied, does not disturb the surface, and is particularly useful for surfaces not amenable to other techniques.

Little attention has been given to the effects of surface roughness on optical measurements when the surface irregularities are very small relative to the wavelength and diffraction effects predominate. This situation is rather surprising in view of the extremely large number of measurements of the specular reflectance, transmittance, or polarization of various materials which have been made. Although the reflectance is more sensitive to surface roughness than is the trans-

[1] Lord Rayleigh, Nature 64, 385 (1901).
[2] F. Jentzsch, Z. tech. Physik 7, 310 (1926).
[3] H. Hasunuma and J. Nara, J. Phys. Soc. Japan 11, 69 (1956).
[4] W. E. K. Middleton and G. Wyszecki, J. Opt. Soc. Am. 47, 1020 (1957).
[5] R. S. Hunter, J. Opt. Soc. Am. 36, 178 (1946).
[6] J. Guild, J. Sci. Instr. 17, 178 (1940).
[7] E. A. Ollard, J. Electrodepositor's Tech. Soc. 24, 1 (1949).
[8] J. Halling, J. Sci. Instr. 31, 318 (1954).

mittance,[2,9] a significant systematic error will exist in precise optical measurements of either quantity if the surfaces involved are not sufficiently smooth. The degree of smoothness required is much more critical than has been previously supposed.

THEORY

Expressions for the relation between reflectance and root mean square roughness may be obtained from the statistical treatment of the reflection of electromagnetic radiation from a rough surface derived by Davies.[10] Although this theory was developed in connection with the scattering of radar waves from rough water surfaces, it is equally valid in the optical region. The surface is represented by a statistical model having the following properties: (1) The root mean square roughness σ, defined as the root mean square deviation of the surface from the mean surface level, is small compared with the wavelength λ. (2) The surface is perfectly conducting and hence would have a specular reflectance of unity if it were perfectly smooth. (3) The distribution of heights of the surface irregularities is Gaussian about the mean. (4) The autocovariance[11] function of the surface irregularities is also Gaussian with standard deviation a. The surface has the statistical properties of stationarity and ergodicity with respect to position along the surface.

If a surface is illuminated with a parallel beam of monochromatic light, the reflectance may be divided into two components, one of which arises from specular reflection and the other from diffuse reflection or scattering. Davies' expression for the specular component for a perfect conductor reduces for the case of normal incidence to $\exp[-(4\pi\sigma)^2/\lambda^2]$. Since no material is perfectly conducting, in order to apply Davies' theory to an actual metal surface it is necessary to modify this expression slightly to give for the specular reflectance at normal incidence

$$R_s = R_0 \exp[-(4\pi\sigma)^2/\lambda^2], \tag{1}$$

where R_s is the specular reflectance of the rough surface and R_0 that of a perfectly smooth surface of the same material. The angular dependence of the diffusely reflected light can also be obtained from Davies' theory. If R_0 is included as before, for light at normal incidence the expression reduces to

$$r_d(\theta)d\theta = R_0 2\pi^4 (a/\lambda)^2 (\sigma/\lambda)^2 (\cos\theta+1)^4 \sin\theta \times \exp[-(\pi a \sin\theta)^2/\lambda^2] d\theta. \tag{2}$$

Here $r_d(\theta)d\theta$ refers to the fraction of the reflected light which is scattered into an angle between θ and $\theta+d\theta$ at an angle θ from the normal to the surface. If m is the root mean square slope of the profile of the surface, the autocovariance length a is, as is shown in the Appendix,

$$a = \sqrt{2}\sigma/m. \tag{3}$$

Therefore, if the reflectance on the normal is measured with an instrumental acceptance angle $\Delta\theta$, for light at normal incidence the contribution from diffuse reflectance is

$$\int_0^{\Delta\theta} r_d(\theta)d\theta = R_0 \frac{2^5\pi^4}{m^2} (\sigma/\lambda)^4 (\Delta\theta)^2. \tag{4}$$

Note that this contribution to the measured reflectance decreases very rapidly with increasing wavelength. *For sufficiently long wavelengths the diffuse reflectance may therefore be neglected.* The measured reflectance is then essentially specular and is given by Eq. (1). It depends only on σ and is not affected by the root mean square slope of the surface.

To summarize, the complete expression for the measured reflectance R is

$$R = R_0 \exp[-(4\pi\sigma)^2/\lambda^2] + R_0 \frac{2^5\pi^4}{m^2} (\sigma/\lambda)^4 (\Delta\theta)^2. \tag{5}$$

If reflectance measurements are made at sufficiently long wavelengths, σ can be calculated directly from the measured reflectance since Eq. (5) reduces to Eq. (1). At shorter wavelengths, however, the reflectance near the normal will be a function of both the surface roughness and the root mean square slope of the surface irregularities. By measuring the reflectance at two wavelengths, one of which is long enough so that the effect of the slope may be neglected, it should be possible to determine both the surface roughness and the root mean square slope of the surface irregularities.

When the wavelength is long enough so that the diffuse reflectance may be neglected, Eq. (5) may be written

$$\log_{10} R_0/R = [(4\pi\sigma)^2/2.303](1/\lambda^2). \tag{6}$$

Thus, if R_0/R is plotted on semilog paper vs $1/\lambda^2$, a straight line through the origin with a slope which is directly proportional to σ^2 is obtained. It is convenient to use this equation to calculate the value of the root mean square roughness from the experimental values of the reflectance at normal incidence. Approximate values of the roughness may be obtained in this way even if the contribution from diffuse reflectance is not negligible.

The surface roughness may also be obtained from measurements of the total diffuse reflectance using an integrating sphere. If R/R_0 is near unity, Eq. (5) may

[9] R. W. Wood, *Physical Optics* (The Macmillan Company, New York, 1934), 3rd ed., p. 41.

[10] H. Davies, Proc. Inst. Elec. Engrs. **101**, 209 (1954).

[11] Davies refers to this function as the autocorrelation function. However, we shall use the term autocovariance function as a more appropriate name when the function in question is not normalized. For a discussion of the properties of such functions see J. H. Laning, Jr., and R. H. Battin, *Random Processes in Automatic Control* (McGraw-Hill Book Company, Inc., New York, 1956); and R. B. Blackman and J. W. Tukey, *The Measurement of Power Spectra* (Dover Publications, New York, 1958).

be expanded to give

$$\sigma=\lambda(R_0-R)^{\frac{1}{2}}/4\pi R_0^{\frac{1}{2}}. \qquad (7)$$

Replacing R_0-R in Eq. (7) with R_d, the total diffuse reflectance, yields

$$R_d=R_0(4\pi\sigma)^2/\lambda^2. \qquad (8)$$

If the ratio R_d/R_0 is measured at a fixed wavelength, the surface roughness σ is directly proportional to $(R_d/R_0)^{\frac{1}{2}}$. This proportionality was discovered empirically by Engelhard,[12] who used it to make a correction for the surface roughness of gauge blocks.

The increasingly important part played by the surface roughness as the wavelength becomes longer may be easily understood from a physical point of view. Consider a nominally plane surface made up of many small facets randomly oriented in various directions. If the dimensions of the facets are large compared with the wavelength of light, the reflectance of a surface in a given direction is determined entirely by geometrical optics and is a function only of the inclinations of the facets. As the wavelength becomes longer, diffraction effects become important, and the reflectance is a function of both the inclination and the size of the facets. As the wavelength becomes still longer, so that the dimensions of the facets become very small by comparison, the reflectance of the surface will be determined almost entirely by diffraction effects. The surface roughness will then be the only important parameter.

Although it is assumed in Davies' theory that the surface is perfectly isotropic, this condition is by no means necessary. Frequently in practice, particularly with machined surfaces, there is a preferred direction technically referred to as the lay of the surface. In the case of such an anisotropic surface, the concept of a mean square slope must be extended. Suppose for example that the autocovariance function for such a surface is Gaussian, but with two different autocovariance lengths a and b corresponding to two orthogonal directions x and y along the surface. It can be shown by a derivation similar to that for the isotropic surface that the quantity m^2 in Eq. (5) is replaced by $m_a m_b$, where m_a and m_b are the root mean square slopes measured in the x and y directions, respectively.

EXPERIMENTAL

To test the theory presented above, a series of flat $1\frac{1}{2}$-in.-diam disks of various roughnesses were prepared. The disks were overcoated with an opaque, evaporated aluminum film, and the reflectance was measured as a function of wavelength. Both steel and plate-glass disks were used. The steel disks were made of AISI type 01 tool steel hardened to Rockwell 58–60, and a fine feed surface grinder was used to obtain the finish. Roughnesses of the samples were $2\frac{1}{2}$, 8, and 32 μin. root mean square as measured with a profilometer. Some lapping was necessary for the $2\frac{1}{2}$-μin. sample. The plate-glass disks were ground using grinding particles of 5–22 μ average particle size. The ground disks and a plane plate-glass reference disk were aluminized in one evaporation, and the reflectance at essentially normal incidence was measured as a function of wavelength in the 2–22-μ infrared region using the reflectometer reported previously.[13] For highly reflecting samples an accuracy in measured reflectance values of about 0.1% can be obtained with this instrument. Since a solid angle of only 0.03 sr about the direction of specular reflectance is accepted by the instrument on each of the two reflections from the sample, only light reflected almost specularly is recorded.

RESULTS

The theoretical wavelength dependence of the decrease in reflectance caused by surface roughness is in good agreement with experiment. A typical relative reflectance vs wavelength curve is shown in Fig. 1. The circles represent experimental values. The solid line was computed on the assumption that only the specularly reflected light need be considered. The reflectance is then only a function of σ and not of m. The solid line fits the experimental points quite well above 0.90 reflectance, but begins to decrease too rapidly for lower reflectances indicating that here there is an appreciable contribution from diffuse reflectance near the normal. The dotted line, which was computed assuming a contribution from both specular and diffuse reflectance, fits the experimental data to about 0.75 reflectance. At lower reflectances the requirement that $\sigma \lessapprox \lambda$ is violated, and the theory would not be expected to hold. These curves demonstrate that if σ is to be determined from Eq. (1) without

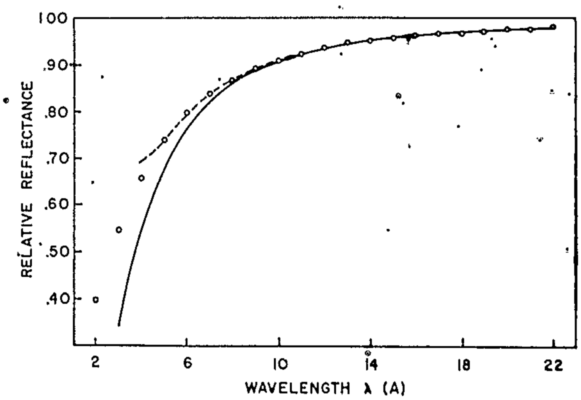

FIG. 1. Relative reflectance of a finely ground glass surface as a function of wavelength. Circles indicate experimental points. The dotted curve was calculated from Eq. (5) and the solid curve from Eq. (1). Both curves coincide above $R/R_0=0.90$.

[12] E. Engelhard, Natl. Bur. Standards Circ. No. 581, 1 (1957).

[13] H. E. Bennett and W. F. Koehler, J. Opt. Soc. Am. 50, 1 (1960).

danger of a significant systematic error, the wavelength must be sufficiently long that the diffuse reflectance near the normal is negligible. It is interesting to observe, however, that even at a relative reflectance of 0.40 the value of σ computed from the experimental values of the reflectance and neglecting the contribution from the diffuse reflectance is in error by less than a factor of 2.

The root mean square surface roughness values for ground glass surfaces obtained by this reflectance method are recorded in Table I and are in good agreement with results obtained using other methods of measurement. In cases where the relative specular reflectance was less than 0.90 at the longest wavelength measured, a small correction was applied to the observed reflectance for the diffuse reflectance near the normal in accordance with Eq. (5). In making this correction the value of m^2 for the rougher ground glass surfaces was assumed to be equal to that measured for the smoother surfaces. Although the irregularities on a ground glass surface are so closely spaced and so irregular in shape that the usual techniques of roughness measurement fail, Preston[14] has reported a value of about 1 wavelength of visible light for the average peak to valley depth of a finely ground glass surface. This figure is in good agreement with the results reported in the last column of Table I.

The root mean square surface roughness values for the ground steel surfaces obtained by this reflectance method are recorded in Table II along with the corresponding profilometer values and the ratio σ(optical)/σ(profilometer). Although the surface roughness of the samples changes by an order of magnitude, the roughness values obtained by these two methods are generally compatible. However, as the surface becomes smoother, the discrepancy between the two methods increases until for the smoothest surface the roughness obtained from the optical measurements is $2\frac{1}{2}$ times that obtained using the profilometer. This result would be expected if the tip of the profilometer stylus did not bottom in the grooves on the smoother surfaces. Since the diameter of the tracing stylus is 1000 μin., or about two orders of magnitude larger than the roughness to be measured, failure of the tip to bottom is easily understood.

TABLE I. Roughnesses of ground glass surfaces.

Grade	Composition	Average particle size (microns)	σ (microns)
302	emery powder	22	1.0
W6	garnet powder	12	0.7
$303\frac{1}{2}$	emery powder	11	0.6
W10	garnet powder	5	0.2
305	emery powder	5	0.15

[14] F. W. Preston, Trans. Opt. Soc. (London) 23, 141 (1922).

TABLE II. Roughnesses of ground steel surfaces.

σ Profilometer (microinches)	σ Optical (microinches)	σ Optical (microns)	σ Optical/ σ profilometer
32	51.0	1.30	1.6
8	15.8	0.40	2.0
2.5	6.2	0.16	2.5

An important advantage of the reflectance method when smoother surfaces are to be measured is most easily explained by comparison with the profilometer. The profilometer stylus is of a fixed size, and hence the percent uncertainty in the measurements increases as the surface becomes smoother. However in the reflectance method, a shorter wavelength is used for smoother surfaces so that the percent uncertainty remains the same. Thus, when using the reflectance method, the accuracy with which the roughness of a rather smooth surface can be measured is the same as that for a much rougher one.

The precision of measurement of surface roughness which may be obtained using the reflectance method is good. Since the square of σ appears in the exponent in Eq. (5), the reflectance is sensitive to a small change in σ. Even if a crude reflectometer is used and the uncertainty in the measured reflectance is $\pm 1\%$, the uncertainty in the value of σ is $\pm 5\%$ at a relative reflectance of 0.90. For a 2-μin. root mean square roughness surface, this uncertainty is ± 0.1 μin. With the best reflectometer in our laboratory, this uncertainty can be reduced by an order of magnitude.

The reflectance method is free from other disadvantages of previously used methods of measuring surface roughness. For example, most conventional methods employ a diamond tracing stylus which leaves a deep scratch on metal surfaces. Such instruments are insensitive to roughnesses of less than a few microinches, even if the surface is composed of very broad, shallow grooves. If the surface is irregular with deep microscratches in which the diamond point cannot bottom, these tracing instruments cannot be used at all. Under restricted conditions an interferometric method can be used,[15] but determining root mean square roughness in this way is difficult and time consuming compared to the reflectance method.

If a small and relatively inexpensive spectrometer were fitted with sodium chloride and cesium bromide prisms, it would be possible to cover the 1–40-μ wavelength range with sufficient resolution for surface roughness measurement. If a smooth, flat sample of the same material were used as a standard, the data necessary for calculating the root mean square roughness from Eq. (1) could be obtained by simply turning the wavelength control until the reflectance of the rough surface is 90% of that of the standard. If the wave-

[15] W. F. Koehler and W. C. White, J. Opt. Soc. Am. 45, 1011 (1955).

length drum were calibrated in values of the root mean square roughness, no calculation would be necessary, and the roughness could be read directly from the drum setting. Although the surfaces to be measured would have to be carefully cleaned, this measuring procedure would be rapid, versatile, and nondestructive. It could be performed by unskilled personnel and would not be subject to operator error.

OPTICALLY POLISHED SURFACES

Since very small surface roughnesses may be of importance in optical measurements, it is important to consider whether the theory fits the experimental data for very small values of σ/λ. The derivation is not valid when $\sigma/\lambda \ll 1$, and indeed under these circumstances Huygens' principle, on which the theory is based, would be expected to fail. However, the change in relative reflectance with wavelength calculated from Eq. (5) fits the experimental data for the smoothest ground-glass and steel surfaces at the longest wavelengths which could be measured with the present equipment. Typical results for a ground-glass surface are shown in Fig. 2. The solid line represents the calculated decrease in reflectance caused by a surface roughness so small that the ratio of σ/λ was only about 0.01. Note that the calculated and measured reflectances show no tendency to diverge even for the smallest value of σ/λ, where the specular reflectance of the ground surface was only $\frac{3}{4}\%$ less than that of a plane surface. Since the theoretical expression approaches the correct limit as the surface becomes perfectly smooth and the error in measured reflectance caused by surface roughness becomes zero, it seems probable that the calculated curve holds for even smaller values of σ/λ than were obtained experimentally.

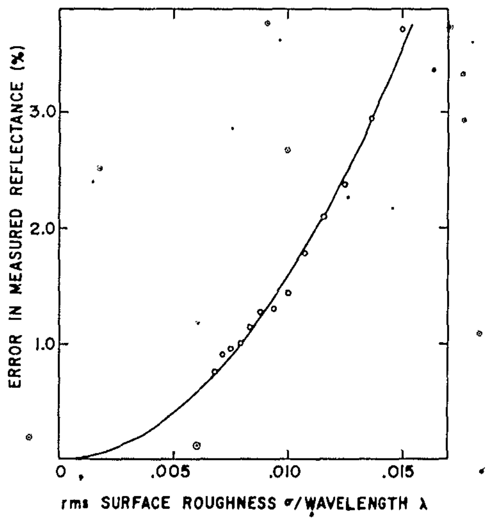

FIG. 2. Error made in reflectance measurements when surface roughness is neglected. Circles represent experimental points. The solid line was calculated from Eq. (1).

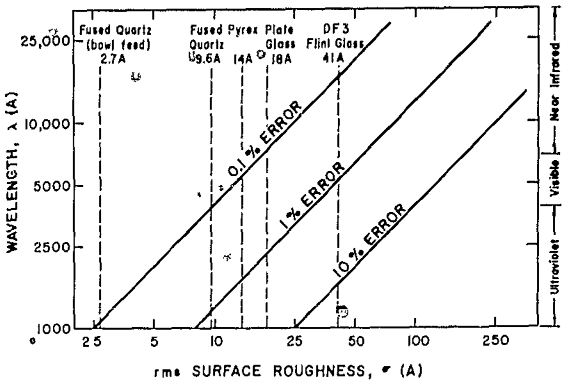

FIG. 3. Error caused by surface roughness in reflectance measurements as a function of wavelength. Dotted lines indicate the surface roughnesses of typical optically polished glass surfaces calculated from the experimental data of Koehler and White.

An example of the importance which the surface roughness of optically polished surfaces may have when measuring the intrinsic specular reflectance of materials in the visible and ultraviolet regions of the spectrum is shown in Fig. 3. In this figure the error in the measured specular reflectance caused by surface roughness is plotted vs wavelength. Since a log-log plot is used, the calculated values of σ which give a particular error in reflectance lie on straight lines when plotted as a function of wavelength. Since optically polished glass surfaces are among the smoothest surfaces available, the roughnesses of some typical optically polished glasses are shown for comparison purposes. Data on the roughness of polished glass were taken from the work of Koehler and White.[15,16] Since their data indicated that the irregularities on a polished glass surface were not symmetrical about an average plane, they used an unsymmetrical distribution function. Also, they report only the distribution of the maxima of the irregularities rather than that of all points on the surface. In order to compare their data with the theory presented here, however, it is necessary to use a Gaussian distribution function. A triangular shape was assumed for the irregularities. The numbers presented here for the roughnesses of various polished glass surfaces are thus only approximate, but do indicate the range of errors which one might expect to find in reflectance measurements at shorter wavelengths using even optically polished surfaces. For example, if a material having the surface roughness of DF 3 flint glass were used, an error of about 1% would be expected in reflectance measurements made in the visible. This error is an order of magnitude greater than the accuracy with which the reflectance can be measured.[13] It would be expected to be nearly two orders of magnitude or 100 times greater than the accuracy of measurement in the ultraviolet.

[16] W. F. Koehler, J. Opt. Soc Am. 43, 743 (1953).

CONCLUSION

The expressions relating the reflectance at normal incidence to the surface roughness give a wavelength dependence which is verified by experiment. In addition, from measurement of the reflectance a value of the root mean square surface roughness can be obtained which is in good agreement with the results using other techniques. Since the surface roughness can be determined with precision regardless of the root mean square slope of the surface irregularities if the measurements are made at sufficiently long wavelengths, a possible application to measurement of surface finish is suggested. The theory may also be applied to optically polished surfaces, since experimental results show that it holds even for surfaces which have a root mean square roughness of less than one hundredth of the wavelength employed. A non-negligible systematic error in reflectance measurements made in the visible and ultraviolet may result from surface roughness even if good optically polished surfaces are employed.

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APPENDIX

In this appendix we develop the relationship between m, the root mean square slope, and a, the autocovariance length, of the surface. Let $z(x,y)$ represent the surface height as a function of position along the surface. Following Davies,[10] we approximate the actual surface by a surface of infinite extent with the same statistical character throughout. The mean square slope m^2 measured in an arbitrary direction, which we take as the x direction, is then defined by

$$m^2=\lim_{X,Y\to\infty}(1/X)(1/Y)\int_{-X/2}^{X/2}\int_{-Y/2}^{Y/2}(\partial z/\partial x)^2dxdy,\quad(9)$$

where we assume that z^2 and $(\partial z/\partial x)^2$ are noninfinite at all points on the surface. The relationship between m^2 and parameter a^2 of the autocovariance function involves the Fourier transform of the autocovariance function. In order to establish this relationship, one may apply the two-dimensional Parseval relation[17] to Eq. (9). However in order to do this the equation must first be expressed in an appropriate form.

To convert Eq. (9) to the proper form for application of the Parseval relation, we introduce the new function $z_{XY}(x,y)=z(x,y)r_X(x)r_Y(y)$. Here r_X and r_Y are functions which may be used to control the behavior of $z_{XY}(x,y)$ in the region $X/2<|x|<\infty$, $Y/2<|y|<\infty$.

[17] S. Bochner and K. Chandrasekharan, *Fourier Transforms* (Princeton University Press, Princeton, New Jersey, 1949), p. 67.

We then consider

$$\lim_{X,Y\to\infty}(1/X)(1/Y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\partial z_{XY}/\partial x)^2dxdy$$

$$=\lim_{X,Y\to\infty}(1/X)(1/Y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}r_Y^2(y)[(\partial z/\partial x)^2r_X^2(x)+2z(\partial z/\partial x)r_X(x)(dr_X/dx)+z^2(dr_X/dx)^2]dxdy.\quad(10)$$

We now let

$$\begin{aligned} r_X(x)&=(X/X+\xi)[(x/\xi)+(X/2\xi)+1];\quad -(X/2)-\xi<x<-(X/2)\\ &=(X/X+\xi);\quad |x|<(X/2)\\ &=(X/X+\xi)[-(x/\xi)+(X/2\xi)+1];\quad (X/2)<x<(X/2)+\xi \end{aligned}\quad(11)$$

where ξ is an arbitrarily small constant, and

$$\begin{aligned} r_Y(y)&=1;\quad |y|<(Y/2)\\ &=0;\quad |y|>(Y/2). \end{aligned}\quad(12)$$

This causes all terms except the first in the square bracket of Eq. (10) to vanish outside of a finite region, and hence give a vanishing contribution to the integral in the limit. Furthermore, the right-hand side of Eq. (10) becomes equivalent to the right-hand side of Eq. (9). Combining these results yields

$$m^2=\lim_{X,Y\to\infty}(1/X)(1/Y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\partial z_{XY}/\partial x)^2dxdy.\quad(13)$$

Application of the two-dimensional Parseval relation to Eq. (13), which is now in proper form, yields

$$m^2=\lim_{X,Y\to\infty}(1/X)(1/Y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|\left[\frac{\partial z_{XY}}{\partial x}\right]_f(u,v)\right|^2dudv\quad(14)$$

where $[\partial z_{XY}/\partial x]_f(u,v)$ denotes the two-dimensional Fourier transform of $(\partial z_{XY}/\partial x)$. To obtain the connection with the autocovariance function of z, we write the integrand of Eq. (14) in terms of $[z_{XY}]_f$ directly. This may be done by partial differentiation of the equation defining the inverse Fourier transform:

$$z_{XY}(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[z_{XY}]_f(u,v)e^{2\pi i(xu+yv)}dudv.\quad(15)$$

We thus infer that

$$\left[\frac{\partial z_{XY}}{\partial x}\right]_f(u,v)=2\pi iu[z_{XY}]_f(u,v)\quad(16)$$

and finally that

$$m^2 = \lim_{X,Y\to\infty} (1/X)(1/Y) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 4\pi^2 u^2 |[z_{XY}]_f(u,v)|^2 du dv$$
$$= 4\pi^2 \mu_{20}\{A_f(u,v)\} \quad (17)$$

where $A_f(u,v)$ denotes the Fourier transform of the autocovariance function of z, and μ_{20} denotes the second moment with respect to u and the zeroth moment with respect to v of $A_f(u,v)$.

To show that the expression

$$A_f(u,v) = \lim_{X,Y\to\infty} (1/X)(1/Y)|[z_{XY}]_f(u,v)|^2$$

is in accordance with the customary definition of the autocovariance function is perfectly straightforward. Taking the inverse Fourier transform of both sides yields

$$A(s,t) = \lim_{X,Y\to\infty} (1/X)(1/Y) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [z(x,y) r_X(x) r_Y(y)]$$
$$\times [z(s-x,\, t-y) r_X(s-x) r_Y(t-y)] dx dy. \quad (18)$$

Using the above definition of r_X and r_Y causes this expression to reduce to

$$A(s,t) = \lim_{X,Y\to\infty} (1/X)(1/Y)$$
$$\times \int_{-X/2}^{X/2}\int_{-Y/2}^{Y/2} z(x,y) z(s-x,\, t-y) dx dy, \quad (19)$$

which is the usual definition of the autocovariance function.

All that remains to obtain the relationship between m^2 and a^2 is to substitute the Fourier transform of the Gaussian autocovariance function

$$A(s,t) = \sigma^2 \exp[-(s^2+t^2)/a^2]$$

into Eq. (17). One thus obtains

$$m^2 = 4\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u^2 [\sigma^2 \pi a^2 \exp -\pi^2 a^2 (u^2+v^2)] du dv \quad (20)$$

or

$$m^2 = 2(\sigma/a)^2. \quad (21)$$

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