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DATE: February 7, 1958

SUBJECT: Lunar Trajectory Computations

FROM: A. B. Mickelwait

A. Description of fundamental coordinate system:

1. x axis along autumnal equinox
   z axis along earth rotational axis
   y axis completing right handed coordinate system

B. Equations of motion:

(1)
$$\ddot{x} = a_{sx} + x_1 \omega_1^2 + \frac{\partial v}{\partial x}$$
$$\ddot{y} = a_{sy} + y_1 \omega_1^2 + \frac{\partial v}{\partial y}$$
$$\ddot{z} = a_{sz} + z_1 \omega_1^2 + \frac{\partial v}{\partial z}$$

(2) In the above a_{sx}, etc. are accelerations due to free fall about the sun

$$a_{sx} = \rho \omega_2^2 \cos(\omega_2 t + \delta)$$
$$a_{sy} = -\rho \omega_2^2 \sin(\omega_2 t + \delta) \cos i$$
$$a_{sz} = +\rho \omega_2^2 \sin(\omega_2 t + \delta) \sin i$$

ρ = distance of bary center (B. C.) from sun

ω_2 = angular velocity of B. C. about sun

i = inclination of ecliptic to equator (obliquity)

$(\omega_2 t + \delta)$ = angular position of sun in ecliptic measured from autumnal equinox (celestial longitude $- \pi$)

(3) In the above x_1, y_1, z_1 are positions of earth center in B. C. coordinates

$$x_1 = -d_e \left[\cos(\omega_1 t + \mathcal{E}) \sin\phi + \sin(\omega_1 t + \mathcal{E}) \cos\theta \cos\phi\right]$$
$$y_1 = -d_e \left[-\cos(\omega_1 t + \mathcal{E}) \cos\phi + \sin(\omega_1 t + \mathcal{E}) \cos\theta \sin\phi\right]$$
$$z_1 = -d_e \left[-\sin(\omega_1 t + \mathcal{E}) \sin\theta\right]$$

d_e = distance to B. C.

$\theta_1 \phi$ are angular coordinates of moon's rotational axis

ω_1 is angular velocity of moon

$(\omega_1 t + \mathcal{E})$ = angular position of moon in moon's plane measured from ascending equatorial node

(4) $$v = -G\left[\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}\right]$$

G = gravitational constant

m_1 = earth's mass

m_2 = moon's mass

m_3 = sun's mass

r_i = distance to center of m_i from vehicle

$$r_1 = \left[x^2 + y^2 + z^2\right]^{1/2}$$

$$r_2 = \left[(x - D_x)^2 + (y - D_y)^2 + (z - D_z)^2\right]^{1/2}$$

$$r_3 = \left[(x - R_x)^2 + (y - R_y)^2 + (z - R_z)^2\right]^{1/2}$$

where

$$D_x = -D\left[\cos(\omega_1 + \mathcal{E})\sin\phi + \sin(\omega_1 t + \mathcal{E})\cos 0 \cos\phi\right]$$

$$D_y = D\left[\cos(\omega_1 t + \mathcal{E})\cos\phi - \sin(\omega_1 t + \mathcal{E})\cos\theta\sin\phi\right]$$

$$D_z = D\left[\sin(\omega_1 t + \mathcal{E})\sin\theta\right]$$

D = mean separation of earth and moon and

$$R_x = \rho\cos(\omega_2 t + \delta) - d_e\left[\cos(\omega_1 t + \mathcal{E})\sin\phi + \sin(\omega_1 t + \mathcal{E})\cos\theta\cos\phi\right]$$

$$R_y = \rho\sin(\omega_2 t + \delta)\cos i - d_e\left[-\cos(\omega_1 + \mathcal{E})\cos\phi + \sin(\omega_1 t + \mathcal{E})\cos\theta\sin\phi\right]$$

$$R_z = \cdot\ \rho\sin(\omega_2 t + \delta)\sin i + d_e\left[\sin(\omega_1 t + \mathcal{E})\sin\theta\right]$$

C. The equations of motion given to computers were as follows:

1. $$\ddot{x} = a_o\left[a_x\cos\beta - a_1 f_x - G_x\right]$$

$$\ddot{y} = a_o\left[a_y\sin\beta - a_1 f_y \cdot G_y\right]$$

$$\ddot{z} = a_o\left[a_z\sin\beta - a_1 f_z - G_z\right]$$

Note that f_x, f_y, f_z are unit vectors from earth center to B. C. where

$$\beta = (\omega_2 t + \delta)$$

$$f_x = -(A\cos\lambda + B\sin\lambda) \qquad \lambda = (\omega_1 t + \mathcal{E})$$

$$f_y = -(-F\cos\lambda + C\sin\lambda)$$

$$f_z = -E\sin\lambda$$

and

$$G_x = \left[\frac{x}{r_1^3} + \frac{a_2(x - Df_x)}{r_2^3} + \frac{a_3(x - \{C_x\cos\beta + d_e f_x\})}{r_3^3}\right]$$

$$G_y = \left[\frac{y}{r_1^3} + \frac{a_2(y - Df_y)}{r_2^3} + \frac{a_3(y - \{C_y\sin\beta + d_e f_y\})}{r_3^3}\right]$$

$$G_z = \left[\frac{z}{r_1^3} + \frac{a_2(z - Df_z)}{r_2^3} + \frac{a_3(z - \{C_z\sin\beta + d_e f_z\})}{r_3^3}\right]$$

where

$$r_1 = \left[x^2 + y^2 + z^2\right]^{1/2}$$

$$r_2 = \left[(x - Df_x)^2 + (y - Df_y)^2 + (z - Df_z)^2\right]^{1/2}$$

$$r_3 = \left[(x - \{C_x\cos\beta + d_e f_x\})^2 + (y - \{C_y\sin\beta + d_e f_y\})^2 + (z - \{C_z\sin\beta + d_e f_z\})^2\right]^{1/2}$$

2. The input data and constants that must be specified are as follows. (The values are preliminary):

1. $a_o = GM_1$, where G = gravitational constant, m_1 = earth's mass ($1.40766.10^{16}$ ft^3/sec^2)

2. $a_x = -\rho\omega_2^2/a_o = -1.39338.10^{-18}$ ft^{-2}

3. $a_y = -a_x \cos i = -1.2783.10^{-18}$ ft^{-2}

4. $a_z = a_x \sin i = 5.5450.10^{-19}$ ft^{-2}

5. $a_1 = d_e \omega_1^2 / a_0 = 7.79287.10^{-21}$ ft^{-2}

6. $a_2 = m_2/m_1 = (81.45)^{-1}$ (moon's mass to earth's)

7. $a_3 = m_3/m_1 = 3.33432.10^5$ (sun's mass to earth's)

8. ρ = mean distance from B. C. to sun ($4.9035.10^{11}$ ft)

9. $\omega_1 = 2.6616995.10^{-6}$ rad/sec

10. $\omega_2 = 2.10^{-7}$ rad/sec

11. δ = position of sun in ecliptic at t = 0, measured from autumnal equinox

12. ϵ = position of moon in moon's plane at t = 0, measured from ascending equatorial node

13. d_e = mean distance from center of earth to B. C. ($1.54838.10^7$ ft)

14. D = mean earth moon separation ($1.26116.10^9$ ft)

15. i = sun's obliquity with respect to the equator ($23^\circ 27'$)

16. $A = \sin\phi$

17. $B = \cos\theta \cos\phi$

18. $C = \cos\theta \sin\phi$

19. $E = -\sin\theta$

20. $F = \cos\phi$

21. θ = obliquity of earth-moon plane with respect to the equator

22. $\phi = (\sigma - \pi/2)$, where σ is angular position in equator of moon's ascending equatorial node

23. $C_x = \rho$

24. $C_y = \rho \cos i$

25. $C_z = -\rho \sin i$

26. t_f = shut off time

27. Δt_1 = time interval for integration close to earth

28. Δt_2 = time interval for integration in cis lunar space

29. Δt_3 = time interval for integration near moon

30. r_e = distance criteria measured from earth for switch-over from fine mesh at Δt_1 to Δt_2 ($2.81264 \cdot 10^7$ ft)

31. r_m = distance criteria measured from moon for switch-over from Δt_2 to Δt_1 ($3 \cdot 10^6$ ft)

32. η = longitude of launch at t_o with respect to the autumnal equinox, east positive

33. θ_L = launch latitude

34. ϕ_L = launch longitude

35. α_L = azimuth at launch from north in clockwise direction

36. R = down range distance from launch

37. h = altitude at burnout

38. v_o = burnout velocity on non rotating earth

39. β = burnout flight path angle from vertical

40. a = radius of earth = $2.09264 \cdot 10^7$ ft

41. ω_s = rotational spin of earth

D. The input data transformations are as follows:

1. The input data is in the following form:
   - (a) Launch coordinates θ_o, $\phi_o = 0$
   - (b) Launch azimuth, α_o
   - (c) Down range distance during powered flight, R
   - (d) Altitude at burnout, h
   - (e) Non rotating earth burnout velocity, v_o
   - (f) Burnout flight path angle, β

2. The burnout position and velocity are computed as follows:
   - (a) Let θ_b be latitude from equator, ϕ_b be the longitude from launch (positive, eastward), α_b the burnout azimuth, and v_b the inertial velocity at burnout, then:

1. $\theta_b = \sin^{-1}\left\{\sin\theta_o \cos\left(\frac{R}{a}\right) + \cos\theta_o \sin\left(\frac{R}{a}\right)\cos\alpha_o\right\}$

2. $\alpha_b = \sin^{-1}\left\{\frac{\cos\theta_o}{\cos\theta_b}\sin\alpha_o\right\}$

3. $\phi_b = \sin^{-1}\left\{\frac{\sin\left(\frac{R}{a}\right)\sin\alpha_o}{\cos\theta_b}\right\}$

4. $v_b = v_o + \omega_s (a + h)\cos\theta_b \sin\alpha_b \sin\beta$

(b) Note: The vehicle is assumed to fly a great circle (relative to the earth) from launch; the input β is assumed to be inertial; the burnout azimuth is not corrected for earth's rotation.

3. The transformation from burnout data to machine integration coordinates is as follows:

(a) $x(0) = (a + h)\cos\theta_b \cos(\phi_b + \eta)$

(b) $y(0) = (a + h)\cos\theta_b \sin(\phi_b + \eta)$

(c) $z(0) = (a + h)\sin\theta_b$

(d) $\dot{x}(0) = v_b\left[\left\{\cos\beta\cos\theta_b - \sin\beta\sin\theta_b\cos\alpha_b\right\}\cos(\phi_b + \eta) - \sin\beta\sin\alpha_b\sin(\phi_b - \eta)\right]$

(e) $\dot{y}(0) = v_b\left[\left\{\cos\theta_b\cos\beta - \sin\beta\cos\alpha_b\sin\theta_b\right\}\sin(\phi_b + \eta) + \sin\beta\sin\alpha_b\cos(\phi_b + \eta)\right]$

(f) $\dot{z}(0) = v_b\left[\cos\beta\sin\theta_b + \cos\theta_b\sin\beta\cos\alpha_b\right]$

where $(\phi_b + \eta)$ is the longitude of burnout from autumnal equinox measured eastward, η is itself the longitude of launch measured from the autumnal equinox.

E. The output data transformations are as follows:

1. Earth centered, inertial, earth-moon plane rectangular, (x axis along moon's ascending equatorial node)

$$x_1 = -x\sin\phi + y\cos\phi = \lambda_N$$
$$y_1 = \cos\theta\left[-x\cos\phi - y\sin\phi\right] + z\sin\theta = -\cos\theta\,\lambda_\phi + z\sin\theta$$
$$z_1 = \sin\theta\left[x\cos\phi + y\sin\phi\right] + z\cos\theta = \sin\theta\,\lambda_\phi + z\cos\theta$$

2. **Earth centered, rotating, earth-moon plane, rectangular, (moon at + x_2)**

$$x_2 = x_1 \cos(\omega_1 t + \mathcal{E}) + y_1 \sin(\omega_1 t + \mathcal{E})$$
$$y_2 = -x_1 \sin(\omega_1 t + \mathcal{E}) + y_1 \cos(\omega_1 t + \mathcal{E})$$
$$z_2 = z_1$$

3. **Earth centered, inertial, earth-moon plane, spherical polar**

$$r_e = \left[x_1^2 + y_1^2 + z_1^2\right]^{1/2}$$

$$\theta_e = \cot^{-1}\left[\frac{z_1}{(x_1^2 + y_1^2)^{1/2}}\right]$$

$$\phi_e = \tan^{-1}\left[\frac{y_1}{x_1}\right]$$ = angle from moon's ascending node measured in E-M plane

4. **Centered at B. C.** rotating in earth-moon plane, **moon at + x_3**

$$x_3 = \lambda_N \cos(\omega_1 t + \mathcal{E}) - \sin(\omega_1 t + \mathcal{E}) \left[\lambda_\phi \cos\theta - z \sin\theta\right] - d_e$$

$$y_3 = -\lambda_N \sin(\omega_1 t + \mathcal{E}) - \cos(\omega_1 t + \mathcal{E}) \left[\lambda_\phi \cos\theta - z \sin\theta\right]$$

$$z_3 = \lambda_\phi \sin\theta + z \cos\theta$$

5. Moon centered, rotating in **earth-moon plane rectangular, (earth at $-x_4$)**

$$x_4 = x_3 + d_e - D$$

$$y_4 = y_3$$

$$z_4 = z_3$$

6. Moon centered, rotating in earth-moon plane, **spherical polar**

$$r_m = \left[x_4^2 + y_4^2 + z_4^2\right]^{1/2}$$

$$\theta_m = \cot^{-1}\left[\frac{z_4}{(x_4^2 + y_4^2)^{1/2}}\right]$$

$$\phi_m = \tan^{-1}\left(\frac{y_4}{x_4}\right)$$

7. Velocity with respect to the moon

$\dot{r}_m$, $r_m \dot{\theta}_m$, $r_m \sin\theta_m \dot{\phi}_m$

8. Jacobi integral:

$$J = \left[\dot{x}_3^2 + \dot{y}_3^2 + \dot{z}_3^2\right] - \omega_1^2 (x_3^2 + y_3^2) - 2GM_1 \left[\frac{1}{r_1} + \frac{a_2}{r_2}\right]$$

$$r_1 = \left[(x_3 + d_e)^2 + y_3^2 + z_3^2\right]^{1/2}$$

$$r_2 = \left[(x_3 + d_e - D)^2 + y_3^2 + z_3^2\right]^{1/2}$$

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ABM:jb

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