

UNCLASSIFIED

AD 263 269

*Reproduced
by the*

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

(10)

263209

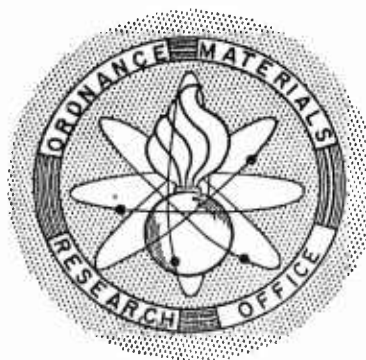
MRL REPORT NO. 103
ASTIA DOCUMENT NO.

HARTREE-FOCK WAVE FUNCTIONS FOR THE $4p$ SHELL ATOMS

REPORT NO. 103
TB-4-001
DA-5-93-32-001

R. E. WATSON
A. J. FREEMAN

JUNE 1961



MATERIALS RESEARCH LABORATORY
ORDNANCE MATERIALS RESEARCH OFFICE
WATERTOWN ARSENAL
WATERTOWN 72, MASSACHUSETTS

XEROX

ASTIA
SEP 21 1961
RECEIVED
TIPDR

Initial distribution of this report has been made in accordance with the distribution list contained herein.

Ordnance Corps installations and other Department of Defense agencies, government agencies outside the Department of Defense, and Ordnance Corps and other Department of Defense agency contractors having approved Field of Interest Registers on file with ASTIA, will make requests for copies direct to Armed Services Technical Information Agency (ASTIA), Document Service Center, Arlington Hall Station, Arlington 12, Virginia.

If no Field of Interest Register is on file with ASTIA, government agencies outside the Department of Defense will make requests for copies to ASTIA through the Commanding Officer, Diamond Ordnance Fuze Laboratories, Washington 25, D. C. Attn: Technical Reference Section.

MEL REPORT NO. 103
ASTIA DOCUMENT NO.

Hartree-Fock Wave Functions for the 4p Shell Atoms

Report No. 103
TB 4-001
DA-5-93-32-001

Authors:

R. E. Watson
R. E. WATSON
Physicist
AVOD, RAD
Wilmington, Mass.

A. J. Freeman

A. J. FREEMAN
Physicist
Materials Research Laboratory

Approved:

R. Beeuwkes, Jr.
R. BEEUWKES, JR.

Chief Scientist
Ordnance Materials Research Office

June 1961

MATERIALS RESEARCH LABORATORY
ORDNANCE MATERIALS RESEARCH OFFICE
WATERTOWN ARSENAL
WATERTOWN 72, MASSACHUSETTS

Signatures of any nonmembers of the organization issuing this report attest not only substantial technical responsibility for contents but compliance with publication policies of their firms or organizations

HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS

R. E. Watson*†

Avco, RAD, Wilmington, Massachusetts

and

A. J. Freeman*

Materials Research Laboratory, Ordnance Materials Research Office
Watertown, Massachusetts

ABSTRACT

Conventional (or restricted) Hartree-Fock wave functions have been obtained for Ga^+ , Ge^{++} , Br^- , Rb^+ and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Analytic Hartree-Fock methods utilizing a version of Nesbet's symmetry and equivalence restrictions, were used to obtain the solutions in analytic form (sums of powers of r times exponentials). Results are given in the form of wave functions, one-electron energies, and total energies. Comparison is made with the earlier results for Zn and Kr, the only atoms in this group for which H-F solutions exist.

*Guests of the Solid State and Molecular Theory Group, M.I.T.

†Part of the work of this author was supported by the Ordnance Materials Research Office.

I. INTRODUCTION

It has become increasingly apparent that Hartree-Fock wave functions for free atoms and ions can play a vital role in providing approximate descriptions of molecular and solid state phenomenon. A striking example of this, and one which was quite unexpected, is the successful use⁽¹⁾ of free ion Hartree-Fock wave functions to describe the recent data on isomeric shifts (i.e., total s electron density at the nucleus) which have been determined by Mössbauer experiments. It goes without saying that basic to any such applications is not only the existence of such functions but their availability in a form suitable for computation. In some of our own studies involving the 4p atoms the complete lack of H-F wave functions, for this part of the periodic table, was only too obvious. In order to overcome this difficulty, we have determined Hartree-Fock solutions for Ga^+ , Ge^{++} , Br^- , Rb^+ and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Of these, Hartree-Fock (H-F) results have been previously available for only $\text{Zn}^{(2)}$ and $\text{Kr}^{(3)}$. These are conventional or restricted Hartree-Fock solutions in that one-electron functions of the same shell are constrained to have the same radial dependence.⁽⁴⁾

As we have said, aside from their own inherent interest as a description of the electronic structure of free atoms, a major purpose of such calculations is to supply a starting point for further investigations. The spherical ion results have been used to obtain the Sternheimer quadrupole antishielding factors⁽⁵⁾ (γ_∞ 's) as these extend our knowledge of theoretical values of γ_∞ to ions for which only estimates could be made previously⁽⁶⁾ (except for Rb^+ for which Hartree calculations exist.) The H-F results have been utilized in several other investigations. Atomic scattering factors have been obtained⁽⁷⁾ and the Ge results have supplied a starting point in an effort to improve on core and valence electron self-consistency in orthogonalized plane wave calculations⁽⁸⁾ for germanium.

Analytic H-F methods,⁽⁹⁾ utilizing a version of Nesbet's symmetry and equivalence restrictions,⁽¹⁰⁾ have been used to obtain the wave functions. Some details of the method are discussed in Section II. A fuller discussion has been given previously⁽¹¹⁾ where H-F results for 3p shell atoms were reported. The H-F results appear in Section III along with comparisons of the previously available Zn and Kr functions.

II. DESCRIPTION OF THE CALCULATION

Six of the eleven atoms and ions, for which H-F results will be reported, are closed shell ions. These can be straightforwardly handled⁽¹²⁾ but the remaining five ions with their unfilled 4p and filled 2p and 3p shells raise a problem of maintaining orthogonality between p orbitals. We have discussed this matter at length when reporting⁽¹¹⁾ 3p atom solutions and the reader is referred there for a discussion of this. We have utilized the same version of Nesbet's symmetry and equivalence restrictions⁽¹⁰⁾ as was used in the 3p atom investigation.⁽¹¹⁾ This procedure does not yield the best possible H-F results but it has been our experience,⁽¹³⁾ and that⁽¹⁴⁾ of investigators who have used the numerical H-F method, that the H-F eigenfunctions are insensitive to the way in which the orthogonality "problem" is treated. Because of this, application of the symmetry and equivalence restrictions yields results which are only negligibly inferior to the "best results." The discrepancies introduced are far smaller than the difference between the Hartree-Fock and the "true" many-electron wave functions. It is our opinion that if one requires superior many-electron eigenfunctions or eigenvalues one should be prepared to obtain these by going beyond the conventional Hartree-Fock formalism.

As already noted, we have utilized the analytic Hartree-Fock method to obtain our results. This method uses standard matrix techniques to obtain

orthonormal analytic Hartree-Fock radial orbitals, $U_i(r)$'s, of the form:

$$U_i(r) = \sum_j C_{ij} R_j(r) \quad (1)$$

Their normalization condition is:

$$\int_0^{\infty} |U_i(r)|^2 dr = 1 \quad (2)$$

and the basis functions R_j 's, are of the form:

$$R_j(r) = N_j r^{(\ell + A_j + 1)} e^{-Z_j r} \quad (3)$$

where ℓ is the one-electron angular momentum quantum number appropriate for the one-electron orbital of which $U_i(r)$ is the radial part. The N_j is a normalization constant and is expressible in terms of the other parameters, i.e.:

$$N_j = \left(\frac{(2Z_j)^{2\ell + 2A_j + 3}}{(2\ell + 2A_j + 2)!} \right)^{1/2} \quad (4)$$

$U_i(r)$'s of common ℓ value are constructed from a common set of $R_j(r)$'s. Given the basis sets, i.e. the $R_j(r)$'s, the problem is reduced to solving the Hartree-Fock integro-differential equations for the eigenvectors (the C_{ij} 's) and their eigenvalues. This is done by straightforward matrix diagonalization and manipulation and avoids the problems of numerical accuracy inherent in the integrations of the numerical H-F method.

Recently^(15,16,3) several workers have overcome the problem of numerical integration accuracy. In particular Mayers⁽¹⁶⁾ has not only obtained highly accurate numerical H-F solutions but he has also accurately evaluated (a not easy task!) the one and two-electron integrals making up the total energy. This will allow detailed comparisons to be made for the first time between numerical and analytic results.

In general the two approaches are complementary, sometimes one and sometimes the other provides a) the more suitable method of computation and/or b) results in a more convenient form. We have found the analytic approach to be very useful⁽¹⁷⁾ when going beyond the conventional free atom H-F formalism. We have also found it convenient to have the resultant wave functions in analytic form.

In using the analytic approach we have replaced the problem of accuracy of numerical integration by the problem of choosing adequate basis sets. Let us consider this matter now. First there is the question of the size of the set. A small set is desirable because of economy in computer time and retains the advantages of wave functions of analytic form. These advantages come from the ease, accuracy, and convenience with which matrix elements can be obtained if the functions are in analytic form. Large basis sets allow greater accuracy of solution (providing that we maintain sufficient linear independence among the basis set, otherwise errors accumulate during matrix diagonalization). Having made the choice of the size of the basis set there is then the problem of choosing the individual R_j 's. In the present work we have relied heavily on earlier^(2, 18) H-F investigations for the iron series ions and so we will review these briefly. The first investigation⁽¹⁸⁾ involved obtaining H-F solutions for iron series ions in $3d^n$ (i.e. no 4s electrons) configurations. The basis sets were the largest that could be fitted onto the computer that was used.⁽¹⁹⁾ Their size fixed, they were obtained by an extensive series of computations where the basis functions were varied. This search concentrated on obtaining good basis functions for V through Ni. This investigation was followed by a series of H-F calculations for the neutral $3d^{n-2} 4s^2$ iron series atoms. These were done on a computer of larger capacity and the basis sets were obtained by adding additional s-like R_j 's (for the construction of the outer loop of the 4s orbitals) to the otherwise unmodified basis sets of the first investigation. An extensive basis function search was thus avoided at the

cost of building in the shortcomings of the first effort. Subsequently more accurate analytic H-F calculations were done for Zn ($3d^{10}4s^2, ^1S$) which is reported here and for Fe ($3d^6 4s^2, ^5D$), Mn^{++} ($3d^5, ^6S$) and Cu^+ ($3d^{10}, ^1S$) which appear elsewhere.⁽²⁰⁾ The greatest discrepancies between calculations for the same state occur for Cu^+ and Zn which lie outside the group emphasized in the first basis function search.

The present computations were done on the I.B.M. 704 computer at Avco whose capacity (32,000 words) allowed the use of larger basis sets and we have taken advantage of this capability. In fact, the choice of R_j 's is based on the iron series R_j 's and on an extensive basis function search for Kr followed by tests on the resultant sets for other atoms. As a result the H-F functions appearing in this paper are more accurate than the earlier^(2,18) iron series solutions. The parameters defining the R_j 's appear in Table I. We have reported (and used) screening constants (Z_j 's) with four digits after the decimal point. This does not mean that Z_j 's were uniquely established to this many digits. The investigations varying the Z_j 's carried this many digits and since these were kept in the final calculations they are reported here.

III. RESULTS

The eigenvectors (C_{ij} 's) which define the $U_i(R)$'s in terms of the $R_j(r)$'s appear in Table II. Note that the C_{ij} 's are given for normalized $R_j(r)$'s. The C_{ij} 's have not been uniquely established to the number of digits quoted but with these digits they provide well-orthonormalized, well-defined Hartree-Fock orbitals. Total energies, one-electron energies (ϵ_i 's), one-electron kinetic plus nuclear potential energies (K_i 's), and two-electron $F^k(4p,4p)$ integrals⁽²¹⁾ are listed in Table III. The quantities appearing there are accurately evaluated for the functions defined in Table II. The Hartree-Fock one-electron and total energies have not been rigorously

established to the number of digits quoted. More accurate solution of the Hartree-Fock equations (e.g. through the use of larger improved basis sets) will affect the last one or two-digits of these quantities. Note that Hartree atomic units have been used ($1 \text{ au} = 2 \text{ Ry}$).

The K_1 's which do not include interelectronic terms, offer a sensitive test of wave function behavior. For fixed Z , a contraction in a (r) causes an increase in the magnitude of its K_1 . Inspection of the K_1 's of Table III shows in general that the U_1 's of any one element contract when going to a state of higher positive ionization. This is, of course, not surprising. Of greater interest is the exception to this rule which occurs for the 2p and 3p orbitals on going from a state with one or more 4p orbitals (such as Ga or Ge) to a state with none, (such as Ga^+ or Ge^{++}). The requirements of obtaining three simultaneous (hence orthogonal) p-like eigenfunctions for the former state and the partial relaxation of this for the latter has caused a reverse trend in K_{2p} and K_{3p} . It should be noted that the resultant 2p and 3p orbital variations are small.

Let us now compare the present results with those previously available for Kr and Zn. Worsley's numerically obtained⁽⁸⁾ ϵ_1 's for Kr appear in Table IV. K_1 's and the total energy were not obtained by her. We see that there is substantial agreement between the two sets of results. More detailed comparisons can be made with the earlier obtained⁽²⁾ analytic Zn results where K_1 's, ϵ_1 's, and the total energy are available. These appear in Table V. Comparison of ϵ_1 's show larger discrepancies than were seen for Kr. This is due to the previously discussed deficiencies in the basis set of the earlier calculation. If one chose to inspect the ϵ_1 's to learn of orbital variation as has often been done (out of necessity) in the past, one would conclude that the 4s orbitals have changed least (the ϵ_{4s} 's being in best agreement between calculations) and that the 3d orbital of the present calculation is the more contracted (since its ϵ_{3d} is more negative). On the other

hand inspection of the K_1 's shows that neither conclusion is correct. The 4s orbitals show the greatest, not the least, modification and the present calculation's 3d orbital is expanded and not contracted relative to the earlier results. In addition to inspecting individual ϵ_1 's, sums⁽²²⁾ of ϵ_1 's (which also appear in Table V) have in the past been used to estimate the variation in total energy. Such sums suggest that there has been a 1 au change in total energy when in reality a change of one fiftieth that size has occurred. In other words the one-electron energies supply an often misleading "yardstick" to wave function behavior.

IV. ACKNOWLEDGEMENT

We are pleased to thank R. K. Nesbet for advice and assistance with the computer programs. The computations were performed on the IBM 704 at Avco and we thank the staff of that facility for their cooperation.

REFERENCES

- ¹L. R. Walker, G. K. Wertheim, and V. Jaccarino, Phys. Rev. Letters, 6, 98 (1961) and S. DeBenadetti, G. Lang, and R. Ingalls, Phys. Rev. Letters 6, 60 (1961).
- ²R. E. Watson, Phys. Rev. 118, 1036 (1960).
- ³B. H. Worsley, Proc. Roy. Soc. A247, 390 (1958).
- ⁴Among recent discussions see: R. E. Watson and A. J. Freeman, Phys. Rev. 120, 1125 (1960) and R. K. Nesbet, Revs. Modern Phys. 33, 28 (1961).
- ⁵R. M. Sternheimer, Phys. Rev. 80, 102 (1950); 84, 244 (1951); 86, 316 (1953) and 95, 736 (1954).
- ⁶R. M. Sternheimer and H. M. Foley, Phys. Rev. 102, 731 (1956) and E. G. Wikner and T. P. Das, Phys. Rev. 109, 360 (1958).
- ⁷A. J. Freeman and R. E. Watson (to be published).
- ⁸F. Quelle (to be published).
- ⁹C. A. Coulson [Proc. Cambridge Phil. Soc. 34, 204 (1938)] appears to have been the first to have used an expansion technique in a molecular problem, while C. C. J. Roothaan [Revs. Modern Phys. 23, 69 (1951)] presented the approach in a particularly desirable form for closed-shell molecules. Nesbet, with his symmetry and equivalence restrictions, extended the method to nonclosed shells and emphasized its use for atomic cases [see reference 10 and also Quarterly Progress Reports No. 15, January, 1955, p. 10; No. 16, April, 1955, p. 38 and p. 41; No. 18, October, 1955, p. 4, Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (unpublished)]. Recently C. C. J. Roothaan [Revs. Modern Phys. 32, 179 (1960)] has extended his formalism to cover the nonclosed shell case for the conventional restricted Hartree-Fock method where nonzero off-diagonal Lagrange multipliers occur. S. Huzinaga (Phys. Rev. 120, 886 (1960); *ibid* 122, 131 (1961)) has extended Roothaan's formalism further.
- ¹⁰R. K. Nesbet, Proc. Roy. Soc. A230, 312 (1955).
- ¹¹R. E. Watson and A. J. Freeman, Phys. Rev. (to be published); Section II and the appendix of this paper supply sufficient information for the reader to construct the Hartree-Fock equations solved in the present paper.
- ¹²See D. R. Hartree, The Calculation of Atomic Structures, (John Wiley & Sons Inc., New York, 1957) for details concerning the conventional Hartree-Fock formalism and for the derivation of Hartree-Fock equations.

- 13 This observation is based on the work reported in Ref. 11 and in R. K. Nesbet and R. E. Watson, *Ann. Phys. (New York)* 9, 260 (1960).
- 14 In two cases (D. R. Hartree and W. Hartree, *Proc. Roy. Soc. A* 193, 299 (1948); W. Hartree, D. R. Hartree and M. Manning, *Phys. Rev.* 60, 857 (1948), small "off-diagonal" Lagrange multipliers were included for orthogonality, otherwise [D. R. Hartree and W. Hartree, *Proc. Roy. Soc. A* 164, 167 (1938); D. A. Goodings (to be published); and unpublished work of D. F. Mayers (1958) the problem was ignored, i.e., the multipliers were set equal to zero and orthogonality was reasonably maintained.
- 15 D. F. Mayers (to be published) has obtained, among others, numerical Hartree-Fock results for Zn but at the time of writing these results were in a preliminary stage and thus were not available for comparison.
- 16 D. A. Goodings (to be published) has obtained accurate numerical "unrestricted" Hartree-Fock solutions for a number of low Z atoms.
- 17 For example, the analytic method was used to obtain Hartree-Fock solutions (1) for ions in external "crystalline" fields [R. E. Watson, *Phys. Rev.* 117, 742 (1960) and R. E. Watson and A. J. Freeman *Phys. Rev.* 120, 1134 (1960).]; (2) for a non-magnetic ion in the "exchange field" of neighboring magnetic ions [A. J. Freeman and R. E. Watson, *Bull. Am. Phys. Soc.* 6, 234 (1961)]; and (3) to obtain one-electron orbitals which are not separable into a product of a radial and an angular function C. Sonnenschein, (to be published). In (2) and (3) the analytic method has several advantages in making the computations possible.
- 18 R. E. Watson, *Phys. Rev.* 119, 1934 (1960).
- 19 M.I.T.'s Whirlwind I was used.
- 20 For definitions see E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra*, (Cambridge University Press, Cambridge, 1953), p. 177.
- 21 The Mn^{++} results appear in an investigation [R. E. Watson and A. J. Freeman (to be published)] of hyperfine effects, those for Cu^+ in an investigation of quadrupole polarizabilities [A. J. Freeman and R. E. Watson, *Bull. Am. Phys. Soc.* 6, 166 (1961)], and the Fe results are unpublished.

Table 1 Parameters (A_i and Z_i), which define the basis orbitals (R_i 's)

z_j is for											
j	A_j	$Zn(4s^2, ^1S)$	$Ga(4s^2, ^1S)$	$Ge(4s^2, ^1S)$	$As(4s^2 4p^3, ^4S)$	$Se(4s^2 4p^4, ^3P)$	$Br(4s^2 4p^5, ^2P)$	$Kr(4s^2 4p^6, ^1S)$	$Rb(4s^2 4p^6, ^1S)$	$Ga(4s^2 4p, ^2P)$	$Ge(4s^2 4p^2, ^3P)$
s orbitals ($\ell = 0$)											
1	0	31.6557	32.7951	33.9345	35.0739	36.2133	37.3527	37.4921	38.6315	32.7951	33.9345
2	1	27.9415	28.9818	30.0221	31.0624	32.1027	33.1430	33.1833	34.2236	28.9818	30.0221
3	1	14.4013	14.9772	15.5531	16.1290	16.7049	17.2808	17.3208	18.4326	14.9772	15.5531
4	2	13.5516	14.1895	14.7273	15.2651	15.8029	16.3407	16.3807	17.4163	14.1895	14.7273
5	2	6.8235	7.1430	7.4622	7.7814	8.1006	8.4198	8.4198	9.0582	7.1430	7.4622
6	2	5.1970	5.4823	5.7676	6.0529	6.3382	6.6235	6.6235	6.9088	5.4823	5.7676
7	3	5.4626	5.7224	5.9822	6.2420	6.5018	6.7616	6.7616	7.0214	5.7224	5.9822
8	3	2.3494	2.6961	3.0428	3.3895	3.7362	4.0829	4.0829	4.3479	2.6961	2.9504
9	3	1.3648	1.6592	2.0145	2.3698	2.7251	3.0804	3.0804	3.3454	1.6592	1.9107
10	3	0.8829	1.0954	1.3508	1.6062	1.8615	2.1168	2.1168	2.3718	1.0954	1.3465
p orbitals ($\ell = 1$)											
1	0	19.6537	20.3930	21.1223	21.8516	22.5809	23.3102	23.3102	24.0396	20.3930	21.1223
2	0	12.3337	12.9976	13.6615	14.3254	14.9893	15.6532	15.6532	16.3171	12.9976	13.6615
3	1	11.7281	12.3476	12.9671	13.5866	14.2061	14.8256	14.8256	15.4451	12.3476	12.9671
4	1	7.4809	7.9029	8.3249	8.7469	9.1689	9.5909	9.5909	10.2129	7.9029	8.3249
5	1	4.6219	4.9255	5.2291	5.5327	5.8363	6.1399	6.1399	6.7471	4.9255	5.2291
6	2	4.3975	4.6067	4.7359	4.9651	5.1943	5.4235	5.4235	5.8819	4.6067	4.7359
7	2	1.9142	2.0473	2.2704	2.4935	2.7166	2.9397	2.9397	3.3859	2.0473	2.2704
8	2	1.0254	1.0911	1.2458	1.4005	1.5552	1.7099	1.7099	1.8646	1.0911	1.2458
9	2	0.6758	0.7335	0.8512	0.9689	1.0866	1.2043	1.2043	1.3220	0.7335	0.8512
d orbitals ($\ell = 2$)											
1	0	1.8017	2.0478	2.2939	2.5400	2.7861	3.0322	2.9679	3.2783	2.0478	2.2939
2	0	2.9851	3.3364	3.6877	4.0390	4.3903	4.7416	4.7416	5.0929	3.3364	3.6877
3	0	2.3914	2.7709	3.1504	3.5299	3.9094	4.2889	4.2889	4.6684	2.7709	3.1504
4	0	7.9320	8.3627	8.7934	9.2241	9.6548	10.0855	10.0855	10.5162	8.3627	8.7934
5	0	13.5738	14.0706	14.5674	15.0642	15.5610	16.0578	16.0578	16.5486	14.0706	14.5674

[illegible]

1s

2s(4s², 6s)

3s(4s², 5s)

6s(4s², 1s)

As(4s², 3s, 5s)

Se(4s², 4p, 3p)

Br(4s², 4p⁵, 2p)

Kr(4s², 4p⁵, 1s)

Rb(4s², 4p⁵, 1s)

Ga(4s², 4p⁵, 2p)

9

2s

1 27981497

2 16563476

3 67545885

4 45234459

5 14904477

6 95822943

7 07862839

8 00063612

9 00035545

10 000013818

3s

1 10598677

2 05959651

3 24825317

4 34366733

5 73912139

6 06520175

7 53561260

8 03048772

9 01676536

10 00578473

4s

1 02116275

2 01228575

3 05101164

4 06936675

5 06769632

6 02688527

7 04751508

8 40669291

9 59138808

10 12181579

5s

1 02657713

2 01503131

3 06143794

4 06936675

5 15760614

6 06964691

7 42371276

8 46785939

9 59138808

10 02950113

6s

1 03099578

2 01748191

3 07020964

4 11036951

5 20611348

6 06553725

7 13267654

8 23269557

9 07560098

10 02523804

7s

1 03230841

2 01831023

3 07120779

4 12016399

5 22339614

6 20283058

7 04524901

8 13877892

9 44939749

10 57503311

8s

1 03330359

2 01946686

3 07462571

4 12924824

5 16206355

6 04750869

7 08919578

8 49367388

9 54483003

10 11299232

9s

1 03589367

2 02058588

3 07775195

4 13592181

5 23182777

6 06523353

7 20277922

8 50286503

9 4957107

10 09000001

10s

1 03840633

2 02210015

3 08238557

4 14692594

5 23657743

6 08366033

7 22089864

8 44938380

9 44936095

10 07764163

11s

1 02487901

2 01409084

3 05759839

4 08700354

5 17852820

6 07461854

7 10279938

8 173308

9 59289441

10 06878169

THE ORIGINAL DOCUMENT WAS OF POOR QUALITY. BEST POSSIBLE REPRODUCTION FROM COPY FURNISHED ASTIA.

THE ORIGINAL DOCUMENT WAS OF POOR QUALITY. BEST POSSIBLE REPRODUCTION FROM COPY FURNISHED ASTIA.

Table III Hartree-Fock one-electron energies (ϵ_i^1), one-electron nuclear potential + kinetic energies (ϵ_i^1), total energies and two-electron $F^2(3p, 3p)$ integrals. All energies are in atomic units ($1 \text{ au} = 2 \text{ Ry}$).

	$Zr(4s^2, 1S)$	$Ga(4s^2, 1S)$	$As(4s^2 4p^3, 4S)$	$Se(4s^2 4p^4, 3P)$	$Br(4s^2 4p^5, 2P)$	$Rb(4s^2 4p^6, 1S)$	$Ga(4s^2 4p^6, 1S)$	$Ga(4s^2 4p^2, 3P)$
ϵ_{1s}	-379.120	-379.120	-432.578	-460.862	-490.052	-520.157	-551.658	-405.235
ϵ_{2s}	-48.469	-48.469	-56.306	-60.666	-65.194	-69.904	-75.250	-52.145
ϵ_{3s}	-6.635	-6.635	-8.029	-8.931	-9.889	-10.850	-12.333	-7.189
ϵ_{4s}	-0.294	-0.294	-0.688	-0.837	-0.952	-1.153	-1.721	-0.424
ϵ_{2p}	-38.921	-38.921	-50.150	-54.256	-58.546	-63.011	-68.107	-42.490
ϵ_{3p}	-3.837	-3.837	-5.880	-6.661	-7.475	-8.332	-9.688	-4.482
ϵ_{4p}	-1.494	-1.494	-0.370	-0.403	-0.457	-0.524	-0.609	-0.287
ϵ_{3d}	-0.780	-0.780	-2.112	-2.649	-3.218	-3.826	-4.933	-1.192
ϵ_{1s}	-480.346	-480.346	-544.343	-577.841	-621.339	-647.838	-684.336	-480.346
ϵ_{2s}	-118.164	-118.164	-134.098	-142.441	-151.035	-159.878	-168.973	-118.164
ϵ_{3s}	-46.384	-46.384	-53.241	-56.860	-60.602	-64.457	-68.438	-46.384
ϵ_{4s}	-15.140	-15.140	-18.540	-20.586	-22.631	-24.685	-27.247	-14.470
ϵ_{2p}	-117.508	-117.508	-133.449	-141.795	-150.391	-159.238	-168.336	-117.508
ϵ_{3p}	-44.206	-44.206	-51.123	-54.760	-58.518	-62.366	-66.384	-44.206
ϵ_{4p}	-38.541	-38.541	-45.140	-48.068	-51.035	-54.044	-57.233	-38.541
ϵ_{3d}	-34.722	-34.722	-42.390	-45.198	-48.068	-51.042	-54.044	-34.722
$F^2(4p, 4p)$	-3605	-3605	-3942	-3942	-4293	-4643	-5229	-2650
$F^2(4p, 4p)$	-1667	-1667	-2038	-2038	-2221	-2406	-2750	-1340
Total Energy	-1777.841	-1923.059	-2234.239	-2399.867	-2572.443	-2752.085	-2938.220	-2075.354

Table IV Kr one-electron energies for the present calculation
and as obtained by Worsley^a

	Worsley's results ^a	present calculation
ϵ_{1s}	-520. au	-520.167 au
ϵ_{2s}	- 69.9	- 69.904
ϵ_{3s}	- 10.8 ₅	- 10.850
ϵ_{4s}	- 1.151 ₅	- 1.153
ϵ_{2p}	- 63.1	- 63.011
ϵ_{3p}	- 8.33 ₅	- 8.332
ϵ_{4p}	- 0.53	- 0.524
ϵ_{3d}	- 3.80	- 3.826

^aSee ref. 3

Table V Zn one-electron energies (ϵ_i 's), one-electron kinetic + nuclear potential energy integrals (K_i 's), sums of one-electron energies, and total energies for the present and earlier^a analytic Hartree-Fock calculations.

	Earlier calculation ^a	Present calculation
ϵ_{1s}	-353.261 au	-353.299 au
ϵ_{2s}	-44.319	-44.358
ϵ_{3s}	-5.600	-5.635
ϵ_{4s}	-0.286	-0.291
ϵ_{2p}	-38.882	-38.921
ϵ_{3p}	-3.804	-3.837
ϵ_{3d}	-0.751	-0.780
K_{1s}	-449.849	-449.848
K_{2s}	-110.569	-110.571
K_{3s}	-43.136	-43.155
K_{4s}	-12.060	-12.218
K_{2p}	-109.912	-109.913
K_{3p}	-40.996	-40.981
K_{3d}	-34.842	-34.722
Total Energy	-1777.823	-1777.843
$\sum_{i=1}^{30} \epsilon_i$	= -1070.558	-1071.514

^aSee ref. 2

ORDNANCE MATERIALS RESEARCH OFFICE

TECHNICAL REPORT DISTRIBUTION

REPORT NO. 103 TITLE: Hartree-Back Wave Functions for the 4p
Shell Atoms

TO:	NO. OF COPIES	TO:	NO. OF COPIES
OMRO - ORDBE-Z		OTHER:	
Materials Research Laboratory	5		
Author	20	ASTIA	10
RPD	2	OTS	2
		WADD	1
OFFICE, CHIEF OF ORDNANCE		OSR	1
ORDIM-Ammunition		ONR	1
ORDIR-Artillery		ARO	1
ORDIS-Small Arms		AOSWAC	1
ORDIT-Automotive		ERO (Germany)	1
ORDTA-Art. Ammunition		U. S. Standardization Grp. (London)	1
ORDTB-Res. & Materials	1	Dr. Lucas	1
ORDTQ-Ammunition Dev.		AEC	1
ORDTR-Artillery Dev.			
ORDTS-Small Arms Dev.			
ORDTT-Tank Automotive			
ORDTU-Rocket Dev.			
ORDTX-Executive			
ORDTX-AR-Executive Library	1		
ORDNANCE CORPS AGENCIES			
ORDHA-Frankford Arsenal			
ORDBB-Picatinny Arsenal			
ORDBC-Rock Island			
ORDBD-Springfield Armory			
ORDBE-Watertown Arsenal	1		
ORDBF-Waterlyet Arsenal			
ORDBG-Aberdeen Proving Ground			
ORDDW-Redstone Arsenal			
ORDJR-Raritan Arsenal			
ORDMX-Detroit Arsenal			
ORDNANCE WEAPONS COMMAND	1		
OFFICE OF ORDNANCE RESEARCH	1		
JET PROPULSION LABORATORY	1		
DIAMOND ORDNANCE FUZZ LABS.	1		

AD Accession No. 103, June 1961, 16 pp - tables -
Materials Research Laboratory OMRO, Watertown 72, Mass.
HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS -
R. E. Watson and A. J. Freeman
MRL Report No. 103, June 1961, 16 pp - tables -
DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified
Report

Conventional (or restricted) Hartree-Fock wave-
functions have been obtained for Ga, Ge, Br, Rb⁺
and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr.
Analytic Hartree-Fock methods utilizing a version of
Nesbet's symmetry and equivalence restrictions, were
used to obtain the solutions in analytic form (sums of
powers of r times exponentials). Results are given
in the form of wave functions, one-electron energies,
and total energies. Comparison is made with the
earlier results for Zn and Kr, the only atoms in this
group for which H-F solutions exist.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED
1. Wave
functions
for
4p atoms

I. Watson,
R. E.
II. Freeman,
A. J.

III. DA Proj
5-93-32-001

IV. OO Proj
TB 4-001

AD Accession No. 103, June 1961, 16 pp - tables -
Materials Research Laboratory OMRO, Watertown 72, Mass.
HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS -
R. E. Watson and A. J. Freeman
MRL Report No. 103, June 1961, 16 pp - tables -
DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified
Report

Conventional (or restricted) Hartree-Fock wave-
functions have been obtained for Ga, Ge, Br, Rb⁺
and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr.
Analytic Hartree-Fock methods utilizing a version of
Nesbet's symmetry and equivalence restrictions, were
used to obtain the solutions in analytic form (sums of
powers of r times exponentials). Results are given
in the form of wave functions, one-electron energies,
and total energies. Comparison is made with the
earlier results for Zn and Kr, the only atoms in this
group for which H-F solutions exist.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED
1. Wave
functions
for
4p atoms

I. Watson,
R. E.
II. Freeman,
A. J.

III. DA Proj
5-93-32-001

IV. OO Proj
TB 4-001

AD Accession No. 103, June 1961, 16 pp - tables -
Materials Research Laboratory OMRO, Watertown 72, Mass.
HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS -
R. E. Watson and A. J. Freeman
MRL Report No. 103, June 1961, 16 pp - tables -
DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified
Report

Conventional (or restricted) Hartree-Fock wave-
functions have been obtained for Ga, Ge, Br, Rb⁺
and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr.
Analytic Hartree-Fock methods utilizing a version of
Nesbet's symmetry and equivalence restrictions, were
used to obtain the solutions in analytic form (sums of
powers of r times exponentials). Results are given
in the form of wave functions, one-electron energies,
and total energies. Comparison is made with the
earlier results for Zn and Kr, the only atoms in this
group for which H-F solutions exist.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED
1. Wave
functions
for
4p atoms

I. Watson,
R. E.
II. Freeman,
A. J.

III. DA Proj
5-93-32-001

IV. OO Proj
TB 4-001

AD Accession No. 103, June 1961, 16 pp - tables -
Materials Research Laboratory OMRO, Watertown 72, Mass.
HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS -
R. E. Watson and A. J. Freeman
MRL Report No. 103, June 1961, 16 pp - tables -
DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified
Report

Conventional (or restricted) Hartree-Fock wave-
functions have been obtained for Ga, Ge, Br, Rb⁺
and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr.
Analytic Hartree-Fock methods utilizing a version of
Nesbet's symmetry and equivalence restrictions, were
used to obtain the solutions in analytic form (sums of
powers of r times exponentials). Results are given
in the form of wave functions, one-electron energies,
and total energies. Comparison is made with the
earlier results for Zn and Kr, the only atoms in this
group for which H-F solutions exist.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED
1. Wave
functions
for
4p atoms

I. Watson,
R. E.
II. Freeman,
A. J.

III. DA Proj
5-93-32-001

IV. OO Proj
TB 4-001

AD Accession No. <u>Materials Research Laboratory OMRO, Watertown 72, Mass.</u> <u>HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS</u> - R. E. Watson and A. J. Freeman MRL Report No. 103, June 1961, 16 pp - tables - DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified Report Conventional (or restricted) Hartree-Fock wave functions have been obtained for Ga, Ge, Br, Rb⁺ and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Analytic Hartree-Fock methods utilizing a version of Nesbet's symmetry and equivalence restrictions, were used to obtain the solutions in analytic form (sums of powers of r times exponentials). Results are given in the form of wave functions, one-electron energies, and total energies. Comparison is made with the earlier results for Zn and Kr, the only atoms in this group for which H-F solutions exist. NO DISTRIBUTION LIMITATIONS	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001
AD Accession No. <u>Materials Research Laboratory OMRO, Watertown 72, Mass.</u> <u>HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS</u> - R. E. Watson and A. J. Freeman MRL Report No. 103, June 1961, 16 pp - tables - DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified Report Conventional (or restricted) Hartree-Fock wave functions have been obtained for Ga, Ge, Br, Rb⁺ and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Analytic Hartree-Fock methods utilizing a version of Nesbet's symmetry and equivalence restrictions, were used to obtain the solutions in analytic form (sums of powers of r times exponentials). Results are given in the form of wave functions, one-electron energies, and total energies. Comparison is made with the earlier results for Zn and Kr, the only atoms in this group for which H-F solutions exist. NO DISTRIBUTION LIMITATIONS	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001
AD Accession No. <u>Materials Research Laboratory OMRO, Watertown 72, Mass.</u> <u>HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS</u> - R. E. Watson and A. J. Freeman MRL Report No. 103, June 1961, 16 pp - tables - DA Proj 5-93-32-001, OO Proj TB 4-001, Unclassified Report Conventional (or restricted) Hartree-Fock wave functions have been obtained for Ga, Ge, Br, Rb⁺ and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Analytic Hartree-Fock methods utilizing a version of Nesbet's symmetry and equivalence restrictions, were used to obtain the solutions in analytic form (sums of powers of r times exponentials). Results are given in the form of wave functions, one-electron energies, and total energies. Comparison is made with the earlier results for Zn and Kr, the only atoms in this group for which H-F solutions exist. NO DISTRIBUTION LIMITATIONS	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001	UNCLASSIFIED I. Wave functions for 4p atoms I. Watson, R. E. II. Freeman, A. J. III. DA Proj 5-93-32-001 IV. OO Proj TB 4-001

UNCLASSIFIED

UNCLASSIFIED