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**THE ROYAL COLLEGE OF SCIENCE AND TECHNOLOGY
GLASGOW**

Department of Mathematics

**Further Studies on the Hodograph
Method in Gas Dynamics**

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AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE

EUROPEAN OFFICE
Contract AF 61(052)-407
TECHNICAL (FINAL) REPORT
June 1961

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FURTHER STUDIES ON THE HODOGRAPH METHOD IN GAS DYNAMICS

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FURTHER STUDIES ON THE HODOGRAPH METHOD
IN GAS DYNAMICS

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TECHNICAL (FINAL) REPORT

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AIR FORCE RESEARCH DIVISION
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This TECHNICAL (FINAL) REPORT is based on the following
SCIENTIFIC PAPERS PUBLISHED ON THE CONTRACT

1. J. B. Helliwell, Unsymmetrical flow patterns past a finite wedge profile in a high subsonic stream. Proc. Camb. Phil. Soc. 57 (1961), 401.
2. J. B. Helliwell, A flow pattern at high subsonic speeds past a wedge at incidence. To be published in J. Maths. Phys.
3. A. G. Mackie, Applications of the theory of the general hodograph equation. I. Kirchhoff-Helmholtz flow past a wedge. To be published in Proc Camb Phil Soc
4. A. G. Mackie, Applications of the theory of the general hodograph equation. II. The non-symmetric flow at sonic speeds past a flat plate. To be published in Proc. Camb. Phil. Soc.

FURTHER STUDIES ON THE HODOGRAPH METHOD IN GAS DYNAMICS

- 1 Work done under contract to the European Office, Air Research and Development Command, United States Air Force, up to 1960, is reported in USAF Report No. TR 60-61 (April 1960), this being the Final Technical Report under Contract No. AF 61(514)-1170. The work discussed below follows directly from this and concludes the study made in the Department.
- 2 Unsymmetrical flow patterns past a finite wedge profile in a high subsonic stream.

The flow past a wedge profile at incidence has been examined. The sides of the wedge were permitted to have unequal lengths so that the stagnation point could be forced to be at the tip of the wedge. First, the Roshko model was taken. In this the flow breaks away with sonic speed from the shoulders of the wedge and turns with this (constant) speed on the bounding streamline until it reaches its final direction; it then decelerates without change of direction until it reaches its final speed. The solution of the boundary-value problem, for the Tricomi equation, for the wedge in a channel was formulated as a pair of dual integral equations. The complete solution was found for the wedge in a free stream and the dimensions of the profile together with the lift coefficient was computed as functions of the transonic similarity parameter. The interesting point emerged from the analysis that, whereas with the Roshko model for a wedge at zero angle of attack in a channel the

basic free stream singularity is simply a dipole, for the unsymmetrical flow the singularity consists of a dipole together with a simple pole, the strength of the latter depending upon the angle of attack in such a manner that they tend to zero together.

The details are given in a paper by Dr. J. B. Helliwell published under the above title in the Proceedings of the Cambridge Philosophical Society, volume 57 (1961) pp. 401-414.

The same problem was attempted with the Weinstein-Cole model, according to which the sonic lines from the shoulders were taken to be orthogonal to the direction of the main stream; the dividing streamline was again forced to be at the tip of the wedge. For a wedge in a free stream the analysis showed that the free stream singularity consists of a dipole and simple pole, just as for the Roshko model. It was found that the relative strength of the singularities is the same for both models, whereas the ratio of dimensions of the wedge faces is independent of the main stream velocity for the Weinstein-Cole model but not for that of Roshko. The lift coefficient for the two models is similar.

For a wedge placed centrally in a channel the model is such that the sonic lines may terminate part-way across the channel. A formal solution was obtained in terms of the solution of a pair of dual integral equations. These have been solved iteratively in the special case of a choked channel in which the sonic lines extend completely across the stream from the profile to the channel walls. The dimensions of the profile and the lift coefficient have been computed for this case and compared with the corresponding model of the flow past a similar profile in a free stream with the same upstream Mach number.

The interference effects of the channel walls were found to be similar to those in incompressible flow, the lift coefficient was increased while the dimensions of the profile were scarcely modified by the presence of the walls.

This work is shortly to be published in the Journal of Mathematics and Physics in a paper entitled "A flow pattern at high subsonic speeds past a wedge at incidence in a free stream and a choked channel" by Dr. J. B. Helliwell.

3 Applications of the theory of the general hodograph equation

The techniques for solving the generalized hodograph equation

$$\frac{\partial^2 \psi}{\partial \beta^2} + k(\beta) \frac{\partial^2 \psi}{\partial \theta^2} = 0$$

described by Dr. Mackie in an earlier paper (Proc. Camb. Phil. Soc. 54 (1958), 538) were applied to determine the Kirchhoff-Helmholtz flow past a wedge in which the free stream Mach number had any value from 0 to 1 inclusive. The flow was supposed to leave the shoulder of the wedge in two "free" streamlines on which the velocity was that at infinity. By means of the earlier techniques a closed expression in the form of a contour integral was found for the drag coefficient C_D . This agreed with the result given by Lamb (Hydrodynamics) for liquids, that is, for $M_1 = 0$ and also with the results obtained by Mackie (loc. cit.) for $M_1 = 1$. The extended formula enabled the intermediate values to be surmised. Previously results were available only at these two limits. Thus an expansion for small α revealed that, for any given fixed value of $M_1 < 1$, the leading term in the drag coefficient is $(1 - M_1^2)^{-\frac{1}{2}}$ times that value for which $M_1 = 0$. In other words it confirmed that the Prandtl-Glauert

law is valid for this type of problem. Near $M_1 = 1$ it was possible to find the shape of the C_D, M curve by determining $\frac{1}{\alpha} \left(\frac{d C_D}{d M_1} \right)_{M_1=1}$ for small α , where α is the semi-wedge angle. This value turned out to be 1.18, thereby refuting a hypothesis of Tulin who argued on physical grounds that it was probably zero.

In conclusion asymptotic forms were found for the free streamlines far downstream. As anticipated it turned out that $y \sim x^{\frac{1}{2}}$ for $M_1 < 1$ and $y \sim x^{\frac{2}{5}}$ for $M_1 \approx 1$.

This work is to be reported in a paper entitled "Applications of the theory of the general hodograph equation. Part I. Kirchhoff-Helmholtz flow past a wedge", by Dr. A. G. Mackie, to be submitted for publication in the Proceedings of the Cambridge Philosophical Society.

In a second paper under the same heading, sub-titled "Part II. The non-symmetric flow at sonic speeds past a flat plate", to be submitted to the same Proceedings, Dr. Mackie determined the stream function for the flow of a general gas past a flat plate held at an angle $\frac{1}{2}\pi - \epsilon$ to the stream. At infinity the Mach number M_1 was taken to be unity and free streamlines on which the Mach number was unity were supposed to extend from the edges of the plate to infinity downstream. The method of solution required the determination of the correct singularity corresponding to the free stream and the subsequent satisfying of the boundary conditions. The drag coefficient was found by considering the drag from the stagnation point to each edge of the plate separately and the results were then compounded. The formula was checked with the known results for the cases $M_1 = 1, \epsilon = 0$ and $M_1 = 0, \epsilon \neq 0$. The form

of C_D was given by

$$C_D^{-1} = \frac{1}{2} + \frac{\cos \epsilon}{2\pi i} \int_C \frac{v^2 \cos v\epsilon + v \tan \epsilon \sin v\epsilon}{(v^2 - 1) \sin \frac{v\pi}{2}} L'(0, v) dv$$

in the notation of Mackie (loc. cit.). It was not found possible to obtain detailed numerical results with the facilities available. If computations were to be undertaken it would seem that the best approach would be from tabulating the values of $L'(0, v)$ for purely imaginary values of v .

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Final Technical Report

FURTHER STUDIES ON THE HODOGRAPH METHOD IN GAS DYNAMICS

Department of Mathematics, The Royal College of Science
and Technology, Glasgow, Scotland

June 1961 5 p.

ABSTRACT: The flow of a free stream past a wedge profile at incidence has been examined for the Roshko and Weinstein-Cole models, with the stagnation point forced to be at the tip of the wedge. The free stream singularity consisted of a dipole and a simple pole. The solution of the boundary value problem for a wedge placed centrally in a channel was formulated in terms of dual integral equations. Lift coefficients and the

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dimensions of the profile have been computed as functions of the transonic similarity parameter.

The generalized hodograph equation has been solved to determine the Kirchhoff-Helmholtz flow past a wedge with free stream Mach number between zero and unity. The variation of drag coefficient with Mach number has been studied and asymptotic forms found for the free streamlines downstream. The stream function has also been found for the flow of a general gas past a flat plate held at an angle of incidence in a sonic stream, and the drag coefficient expressed in analytical form.

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