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PROBLEM OF THE STRENGTH OF ZhRD (LIQUID-FUEL ROCKET ENGINE) BURNER WITH CONNECTED ENVELOPES

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PROBLEM OF THE STRENGTH OF ZhRD (Liquid-Fuel Rocket Engine) BURNER WITH CONNECTED ENVELOPES

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1. Introduction

In the burner cooling systems used at present the pressure of the cooling liquid exceeds the pressure of combustion products. From the viewpoint of heat elimination, it is advantageous to make the burner walls as thin as possible. But then the presence of the cooling liquid's high pressure makes it mandatory to strengthen the interior envelope. A design solution was found in the form of a burner with connected envelopes.

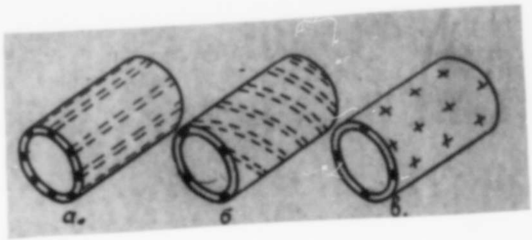
The connection between envelopes in modern burners is realized by means of a contact weld or high-temperature solder. The contact at the weld (solder) points is accomplished either by means of special attached ties or through embossed lugs in the external envelope-jacket.

The systems of connector arrangement can be:

(1) linear (Fig 1a),

(2) spiral (Fig 1b),

(3) point contact (Fig 1в)



1. Systems of connector arrangement.

The arrangement of connectors determines the system of envelope loading.

The choice of the type and system of connecting element arrangement acquires paramount importance in the design of burners with connected envelopes.

Investigations of certain problems associated with the strength and resistance of the cylindrical part of the ZhRD burner with connected envelopes were carried out during 1954-1956 at the MAI (Moscow Aviation Institute). They included both theoretical and experimental research. One of the theoretical investigations is briefly set forth in this article.

2. <u>Loading System</u>

In the examination of the burner it was assumed:

R' is the radius of the median surface of the internal envelope (burner),

R'' is the same of the external envelope (jacket),

h' is the thickness of the burner wall,

h'' is the same of the jacket,

E' is the modulus of elasticity of the first kind of burner material,

E' is the same of the jacket.

Whereupon $v' = v'' = v$, the envelope Poisson factors, were assumed to be equal.

The portion of research described concerns the burner operation in the so-called hydro-pressuring conditions. Characteristic of these conditions are heightened pressure in the inter-jacket space, the absence of counter-pressure of gasses and gas temperature reaction.

During the input of liquid under pressure, reaction forces arise in the inter-jacket space at the envelope contact points. The pressure condition of the burner and jacket will be determined by the value of the liquid pressure (P_{zh} or $P_{zhidkost}$ equals P_l or P_{liquid}) and the values of the reaction forces. The latter values are, in turn, functions of the liquid pressure, and also of the burner envelope rigidity, the arrangement system and number of connecting elements.

For determining the tensions in the connectors, the usual formula for calculating tensions in a stretched rod can be employed:

$$\sigma = \frac{Q}{F}, \qquad (1)$$

where Q is the reaction force in the connector,

F is the connector area (area of envelope contact).

For the linear and spiral systems of connecting the envelopes, the reaction forces must be running (instead of Q - P^k and P^n respectively). Similarly, the contact area must also be taken as a running area.

After excluding from examination the external envelope-jacket, we replace its influence with an equivalent system of forces. Since their magnitude is unknown, the finding of them must precede determination of the burner's pressure state. After making an assumption about the rigidity of connectors, the reaction forces can be calculated, equating the radial shiftings of the envelopes at the points of contact. The following equation will occur:

$$W_{ж} - W'_{p} = -W''_{ж} + W''_{p}, \qquad (2)$$

where W'_{zh} is the radial shifting of the burner from the action of the liquid pressure on the envelope;

W''_{zh} is the same of the jacket;

W'_p is the radial shifting of the burner from the effect of the reaction forces;

W''_p is the same of the jacket.

The shifting (flexure) of the envelope toward the center of its curvature is assumed to be positive.

For the envelope which is under the action of uniformly distributed pressure, the radial flexure is estimated according to the known formula:

$$W = \frac{P \cdot R^2}{E \cdot h}. \tag{3}$$

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For various systems of connecting the burner envelopes, the flexure in axis z from reaction forces will be, of course, variously determined.

For the case of the spiral system of connection, formulas have been derived in one of the studies of B.D.Kartashkin. According to Kartashkin, the flexure of an envelope under the action of running radial forces, is described:

$$W = \frac{P^k \cdot \beta \cdot R^2}{E \cdot h} \cdot \chi_k, \tag{4}$$

where

$$\beta^4 = \frac{3(1-\nu^2)}{R^2 \cdot h^2}, \quad x_k = \frac{\operatorname{sh} \beta \cdot l + \sin \beta \cdot l}{2(\operatorname{ch} \beta l - \cos \beta l)},$$

l is the spacing between radial loads.

The formula for determining the running radial force of reaction was derived on the basis of expressions (2),(3),(4):

$$P^k = \frac{P_m\left(\frac{R'^2}{E'h'} + \frac{R''^2}{E''h''}\right)}{\frac{R'^2}{E'h'} \cdot \beta' x_k' + \frac{R''^2}{E''h''} \cdot \beta'' x_k''}. \quad (5)$$

It is readily noted that the value of the running force of reaction will vary inversely proportional to x_k, i.e. depend on the pitch of the spiral connector (more precisely on the spacing between connectors in the generatrix). As the formula shows, the dependence P^k has an extremum. At a definite spacing between the connectors the value of the running reaction force assumes maximal value.

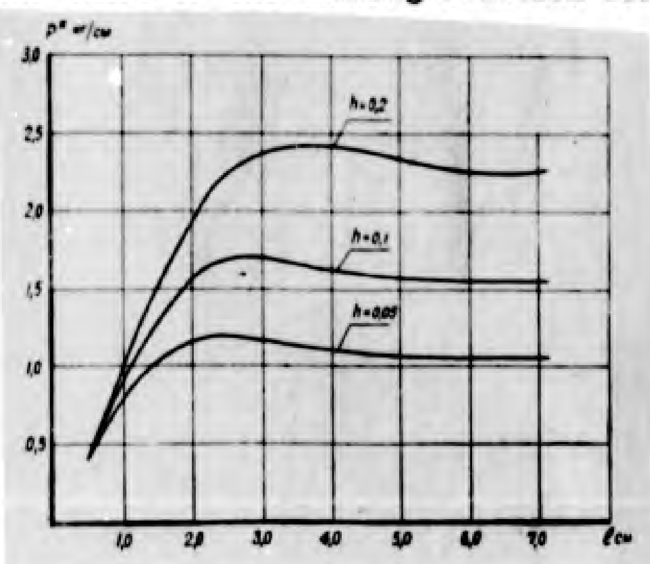

Fig 2. Graphic Dependence $P^k = f(l)$ for burner

$R = R' = R'' = 10$ cm., $h = h' = h'' = v_{ar}$ from material

Ya1-T. $P_{zh} = 1$ kg/cm^2 ; $E = 2 \cdot 10^6$ kg/cm^2.

3. Burner with Linear Connectors

It is essential first of all to know how the flexure (W) of a cylindrical envelope is expressed in case of its loading with n linear loads. For this purpose we use dependencies cited by V.M.Darevsky [1]. Examined in the work indicated is the pressure state of an endlessly long cylindrical envelope 135/136 under the action of an elementary load on it. We remind that under an elementary load is implied a uniform load, distributed in the section of the median surface, limited by two pairs of lines of the envelope curvature.

According to [1], the flexure in axis z from the action of the load indicated is expressed so:

$$W = D\Phi. \tag{6}$$

Here D is the linear operator. Its form:

$$D = \frac{(1-\nu^2)^2}{3E^2}\left\{12(1-\nu)\left(\frac{R}{h}\right)^2 \frac{\partial^4}{\partial\xi^4} + \left[24(1-\nu)\left(\frac{R}{h}\right)^2 + 4 - 3\nu - \nu^2\right]\frac{\partial^4}{\partial\xi^2\partial\varphi^2} + (1-\nu)\left[12\left(\frac{R}{h}\right)^2 + 1\right]\frac{\partial^4}{\partial\varphi^4}\right.$$

It is readily shown that without making large assumptions, it can be represented so:

$$D = 4\frac{(1-\nu)(1-\nu^2)^2}{E^3}\left(\frac{R}{h}\right)^3\Delta\Delta.$$

Φ is a special function, expressed by the following series:

$$\Phi = \sum_{p=0,\, n,\, 2n \ldots}^{\infty} f_p(\xi)\cos p\varphi.$$

Used in the expressions for D and Φ are

$\xi = \frac{x}{R}$ the relative axial coordinate,

φ is the angular coordinate.

For our particular case the function Φ can be considerably simplified, if it be taken into account that the loaded linear strip will be extended the entire length of the envelope, and its width can be considered to be approaching zero. It is essential to consider the latter circumstance a definite assumption, for in practice the width of the contact surface has a quite concrete finite value.

For the case when such linear loads n are in the circumference, the flexure in axis Z is represented in the form:

$$W = \frac{P^n \cdot R}{Eh}\left\{\frac{n}{2\pi} + 3(1-\nu^2)\left(\frac{R}{n}\right)^2\left[\frac{\pi}{n} - \zeta + \operatorname{ctg}\frac{\pi}{n} + \right.\right.$$
$$\left.\left. + \frac{\pi}{n}\operatorname{ctg}^2\frac{\pi}{n}\right)\cos\zeta + \left(1 + \zeta\cdot\operatorname{ctg}\frac{\pi}{n}\right)\sin\zeta - \frac{2n}{\pi}\right]\right\}. \qquad (7')$$

In this formula

P^n is the running linear force,

n is the number of linear running forces,

ζ is the current angle of the linear section for which the flexure is determined. For determination of reaction forces, it is necessary to assume the angle equal to zero. Then

$$W = \frac{P^a \cdot R}{Eh}\left[\frac{n}{2\pi} + 3(1-\nu^2)\varkappa_n\left(\frac{R}{h}\right)^2\right]. \qquad (7)$$

Here $\varkappa_n$ is an expression which characterizes the curvature of the span between connectors

$$\varkappa_n = \frac{\pi}{n} + \operatorname{ctg}\frac{\pi}{n} + \frac{\pi}{n}\operatorname{ctg}^2\frac{\pi}{n} - 2\frac{n}{\pi}.$$

The calculation of this function presents certain difficulties, inasmuch as its value is determined as a difference of numbers ascending in value, whereas the difference itself is a monotonic decreasing value.

Employing the expression for calculating the flexure (7), and also (2) and (3), we derive for the running reaction force, the formula:

$$P^a = \frac{P_x\left(\frac{R_1^2}{E'h'} + \frac{R''^2}{E''h''}\right)}{\frac{n}{2\pi}\left(\frac{R'}{E'h'} + \frac{R''}{E''h''}\right) + 3\varkappa_n(1-\nu^2)\left(\frac{R'^3}{E'h'^3} + \frac{R''^3}{E''h''^3}\right)}. \qquad (8)$$

Analysis of the formula shows that increase of the number of linear connecting elements does not always lead to reduction in the reaction forces incoming to one connector (Fig 3).

It is interesting to note that with identical spacings between connectors, in case of linear and spiral systems of connection, the forces arising in connectors from the pressure of liquid in the inter-jacket space, are equal or greater for the radial connectors (Fig 3).

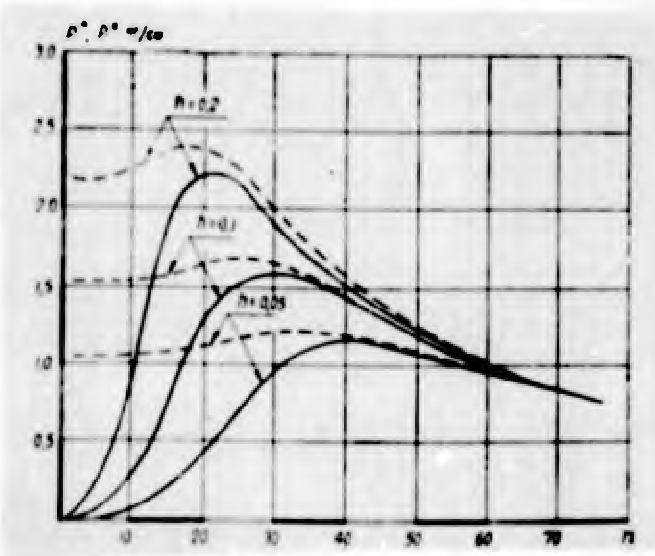

Fig 3. Graphic Dependence $P^k = f(m_k)$ and $P^n = f(n)$ for burner $R = R' = R'' = 10$ cm, $h = h' = h'' = v_{ar}$ from material Ya1-T. $P_{zh} = 1$ kg/cm^2; $E = 2 \cdot 10^6$ kg/cm^2.

Conventional symbols: ——— linear connectors,

------- radial connectors.

4. Burner with Point Contact Connectors

The method proposed in his time by B.D.Kartashkin can be used in approaching the determination of the reaction forces in the connectors. At first we determine the running reactions derived in analysis of the burner's pressure state according to the formulas cited in sections 2 or 3. For this purpose we seemingly unite adjacent points either in continuous rings or in continuous generatrix. After that, taking into account the spacing between points, we replace the running force of reaction by the point force:

$$Q_k = \frac{2\pi R}{n} \cdot p^k, \tag{9}$$

where Q_k is the point force of reaction.

And similarly

$$Q_n = p^n \cdot l. \tag{10}$$

where Q_n is the point force of reaction determined by means of formula (8).

In the first case as well as in the second, the value of the point force of reaction depends on n and l, the basic parameters of the point contact system of connecting envelopes.

The indicated dependencies are graphically presented in Fig 4.

Stricter determination of the point contact forces of reaction is associated with considerable difficulties.

After disconnection of envelopes at the weld point, the influence of the provisionally discarded external envelope is replaced by equally distributed pressure (q) applied at the contact points, so that the sum of this pressure in the contact area is equal to the full force of reaction (Q).

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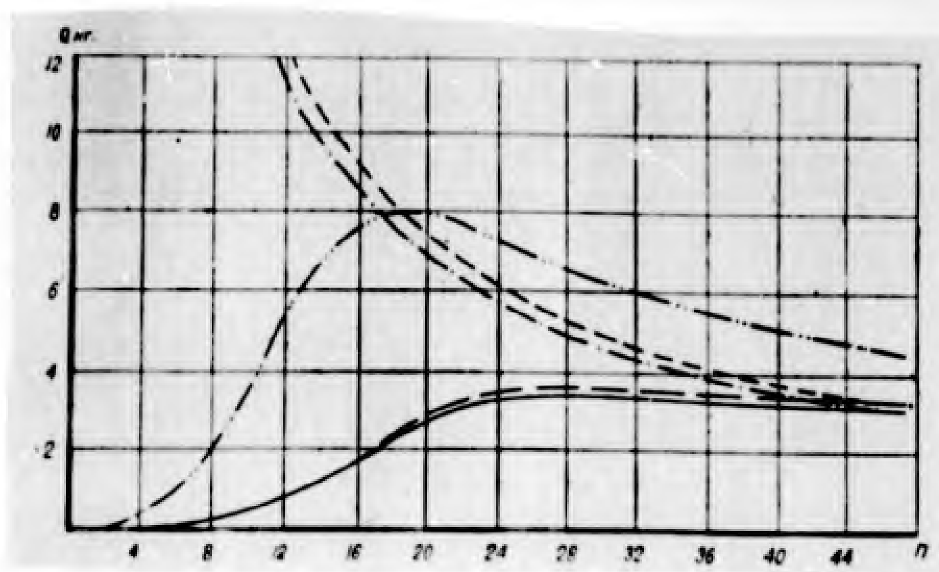

Fig 4. Graphic Dependencies $Q = f(n)$ for burner $R = 10.0$ cm, $h = h' = h'' = 0.2$ cm from material Yal-T. $P_{zh} = 1$ kg/cm^2; $E = 2 \cdot 10^6$ kg/cm^2. Conventional symbols: ——— one ring of "n" lugs; — • — • — one continuous ring, ----- ring of "n" lugs with $l = 3.5$ cm, – – – – – continuous ring with $l = 3.5$ cm, ••— •• — •• — generatrix $l = 3.5$ cm.

For determining the reaction forces that interest us, we again employ the derivations of V.M. Darevsky's work [1] and [2] about the envelope experiencing the action of an elementary load.

In the case under examination, the envelope is loaded with a system of elementary loads (q•a•b = Q). The arrangement of them in the envelope, corresponding to the layout of embossed lugs, can be "rectangular" and "checkered". Just as before,

l is the spacing between elementary loads in generatrix, 138/139

n is the number of embossed lugs (elementary loads) in circumference,

m is the number of embossed lugs (elementary loads) in generatrix; the number is infinitely large, since an infinitely long envelope is being examined.

Determination of the flexure (W) from the system of elementary loads must precede calculation of the reaction. It is evidently formed from the sum of flexures:

$$W = W_0 + 2W_l + 2W_{2l} + \ldots + 2W_{ml} = W_0 + 2\sum_{m=1}^{m} W_{ml}. \qquad (11)$$

This is for the "rectangular" system of lug arrangement. While for the "checkered", it is:

$$W = W_0 + 2\sum_{m=1}^{m} W_{ml} + 2\sum_{m=1}^{m} W_{m\frac{l}{2}}. \qquad (12)$$

In the expressions (11) and (12)

W_0 is the flexure from the ring of "n" elementary loads; this ring lying in the coordinate plane "xy",

W_l is the flexure from one ring of "n" elementary loads but one that is spaced at distance l from the coordinate beginning, etc.

For the case with the "checkered" arrangement, the nearest ring of n embossed lugs is spaced at distance $l/2$ away and shifted at angle $\varphi_0/2$.

$\varphi_0 = 2\pi Q/n$ is the angle between the lugs in circumference.

Determination of the components of a full flexure embraces a series of operations.

Just as before, in calculation of the flexure from linear forces

$$W = D\Phi,$$

where D is the same dual Laplacian operator:

$$D = 4\frac{(1-\nu)(1-\nu^2)^2}{E^3}\left(\frac{R}{h}\right)^2\Delta\Delta\ldots,$$

and Φ is a special function in the form

$$\Phi = \sum_{p=0}^{\infty} f_p(\xi)\cos p\varphi$$

for the "rectangular" system.

When calculating the flexure from the elementary loads arranged in the envelope according to the "chekered" system, the function Φ will be:

$$\Phi = \sum_{p=0}^{\infty} f_p(\xi)\sin p\varphi.$$

We will not dwell on derivation of an equation for determining the function $f_p(\xi)$ in the case when one elementary load acts on the envelope; it has been described in adequate detail in [2]. We note only that use can be made of them, if the characteristic equation of the following form is solved:

$$\eta^8 + 4P^2\cdot\eta^6 + (6P^4 + 4\varkappa^4)\cdot\eta^4 + 4P^2(P^2-1)^2\cdot\eta^2 + P^4(P^2-1)^2 = 0.$$

In the example considered, P is the multiple of n the number of embossed lugs in circumference, and in any case larger than 6 to 8. The indicated characteristic equation can, therefore,be simplified. In the new writing, it assumes the form:

$$\eta^8 + 4P^2\cdot\eta^6 + 6P^4\cdot\eta^4 + 4P^6\cdot\eta^2 + P^8 + 4\varkappa^4\cdot\eta^4 = 0$$

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or

$$(\eta^2 + P^2)^4 + 4\varkappa^4\cdot\eta^4 = 0.$$

If we denote

$$\frac{\eta^2}{P^2} = x, \quad \frac{\varkappa^2}{P^2} = k,$$

then

$$(x+1)^4 + 4k^2x^2 = 0.$$

This equation is easily solved.

Omitting all essential transformations, the expression can be imagined for the full flexure with "rectangular" system of load arrangement:

$$W = \frac{Q}{4\pi E}\left[k_1 n(\mu_0 - \mu_l) + k_2(\mu_n + \mu_{nl})\right].$$

$$\mu_0 = 1 - e^{-\varkappa p}\cdot\cos\varkappa p; \qquad (13)$$

In this formula

$$\mu_l = \sum_{m=1}^{m} (e^{-x(ml-\rho)} \cdot \cos x|ml-\rho| + e^{-x(ml+\rho)} \cdot \cos x|ml+\rho|);$$

$$\mu_n = \sum_{p=n;\,2n}^{\infty} K_{p\zeta}[1 + (Cp_1 \cdot \sin pAp_1)\rho| + Cp_2 \cos pAp_2\rho)e^{-pBp_1}|\rho) +$$
$$+ (Cp_3 \sin p \cdot Ap_2|\rho| - Cp_4 \cdot \cos p \cdot Ap_2\rho)e^{-pBp_2}|\rho||;$$

$$\mu_{nl} = \sum_{m=1;}^{m} \sum_{p=n;\,2n}^{\infty} K_{p\zeta}|Cp_1 \sin pAp_1|ml+\rho| - Cp_2 \cos pAp_1|ml+$$
$$+\rho|e^{-pBp_1|ml+\rho|} - (Cp_1 \sin pAp_1|ml-\rho| -$$
$$- Cp_2 \cos pAp_1|ml-\rho|)e^{-pBp_1|ml-\rho|} + (Cp_3 \sin pAp_2|ml+\rho| -$$
$$- Cp_4 \cos pAp_2|ml-\rho|)e^{-pBp_2|ml+\rho|} - (Cp_3 \cdot \sin pAp_2)(ml-\rho| -$$
$$- Cp_4 \cos pAp_2|ml-\rho|)e^{-pBp|ml-\rho|}].$$

Calculation of flexure from the action of the system of elementary loads with "checkered" load arrangement also does not cause special difficulties. It can be shown that sufficient for this purpose is merely to calculate additionally the series

$$\mu_{\frac{l}{2}} = \sum_{m=1}^{m} (e^{-x\left(m\frac{l}{2}-\rho\right)} \cdot \cos x\left|m\frac{l}{2}-\rho\right| - e^{-x\left(m\frac{l}{2}-\rho\right)} \cdot \cos x\left(m\frac{l}{2}-\rho\right).$$

Then the formula for the flexure assumes the form:

$$W = \frac{Q}{4\pi E}[K_1 n(\mu_0 - \mu_{\frac{l}{2}} - \mu_l) + K_2(\mu_n + \mu_{nl}). \qquad (14)$$

In the formulas cited above the coefficients Ap_1, Bp_1, Ap_2, Cp_1 and so forth, and also $K_1, K_2, Kp\zeta$ are used. They can be calculated by the formulas:

$$Ap_1 = \sqrt{\frac{\sqrt{j^2+i^2}-j}{2}}, \quad Bp_1 = \sqrt{\frac{\sqrt{j^2+i^2}+j}{2}}, \qquad \frac{140}{141}$$

$$Ap_2 = \sqrt{\frac{\sqrt{\theta^2 + i^2} + \theta}{2}}, \quad Bp_2 = \sqrt{\frac{\sqrt{\theta^2 + i^2} - \theta}{2}},$$

$$j = a + 1, \; i = b + k, \; \theta = a - 1, \; f = b - k.$$

$$a = \sqrt{\frac{\sqrt{k^4 + 4k^2} - k^2}{2}}, \quad b = \sqrt{\frac{\sqrt{k^4 + 4k^2} + k^2}{2}},$$

$$Cp_1 = 2(b + k)(a \cdot Rn_1 - k \cdot Rn),$$

$$Cp_2 = -2(b + k)(kRn_1 - aRn),$$

$$Cp_3 = 2(b - k)[kRn_1 + a(1 - Rn)],$$

$$Cp_4 = -2(b - k)[aRn_2 - k(1 - Rn)],$$

$$Rn = \frac{\theta^2 + f^2}{4(k + b^4)}[2ki + j(k^2 + a^2)],$$

$$Rn_2 = \frac{f^2}{4j(k^2 + b^4)}[j(a^2 - bk) + i^2 a],$$

$$Rn_1 = \frac{\theta^2 + f^2}{4(k^2 + b^4)}(2a^2 + a - bk),$$

$$K_1 = \frac{1}{n\rho}, \quad \rho = \frac{d}{2R},$$

$$x = \sqrt[4]{3(1 - \nu^2)}\sqrt{\frac{R}{h}}.$$

d is the diameter of the welded point.

$$K_2 = 24(1 - \nu^2)\frac{R^3}{h^3 \cdot \rho^2};$$

$$K_{p\varphi} = \frac{\sin p\varphi \cdot \cos p\varphi}{(p^2 - 1)^2}.$$

For reaction determination angle $\Phi = 0$.

It should also be indicated that in practical calculations when summarizing in P, it is sufficient to confine one's self to two (with very small numbers n, to three) terms of the μ_n, μ_n series. Moreover, for the majority of engineering calculations of burners, encountered in practice, it is sufficient to limit the formula for flexures to terms μ_c and μ_n. This is equivalent to determination of the flexure from the action of one ring of elementary loads on the envelope. Then, under any system of loading ("rectangular" or "checkered") the expression for the flexure assumes the form:

$$W_Q = \frac{Q}{4\pi E} \cdot (K_1 n \mu_0 + K_2 \mu_n). \tag{15}$$

Knowing the flexure under one of the elementary loads, when a system of such forces is acting on the cylindrical envelope, one can turn to determination of the point reaction forces arising in the point contact connectors of the burner. For this purpose it is necessary to equate one to the other the shiftings of the external and internal envelopes at the contact points. That is, after using (2),(3) and (13) or (14) or even approximate (15), we derive a formula for estimating the reaction forces in case of the point contact system of connecting the burner envelopes:

$$Q = \frac{4\pi P_{\text{н}} \left(\frac{R'^2}{E' n'} + \frac{R''^2}{E'' n''} \right)}{[K_1 n (\mu_0 - \mu_l) + K_2 (\mu_n + \mu_{nl})]' E'^{-1} + [\dots]'' E''^{-1}} \cdot \tag{16}$$

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That is for the "rectangular" system.

But for the "checkered" system is

$$Q = \frac{4\pi P_{\text{н}} \left(\frac{R'^2}{E' n'} + \frac{R''^2}{E'' n''} \right)}{E'^{-1} [K_1 n (\mu_0 - \mu_{\frac{l}{2}} - \mu_l) + K_2 (\mu_n + \mu_{nl})]' + [\dots]'' E''^{-1}} \cdot \tag{17}$$

If the value of the reaction forces is sufficiently close

(which is the usual result), then

$$Q = \frac{4\pi P_m\left(\frac{R'^3}{E'n'} + \frac{R''^3}{E''n''}\right)}{(K_1 m_0 + K_2 n_n)' E'^{-1} + (\ldots)' E'^{-1}} . \tag{18}$$

Design of real envelops shows that the methods previously known (for example, the method of Kartashkin, Feodos'yev) give results much too high (Fig 4).

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