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ARMOUR RESEARCH FOUNDATION OF ILLINOIS INSTITUTE OF TECHNOLOGY

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ARF Project No. 8130

LONG RANGE STUDY PROGRAM
LIGHTWEIGHT ARTILLERY WEAPON

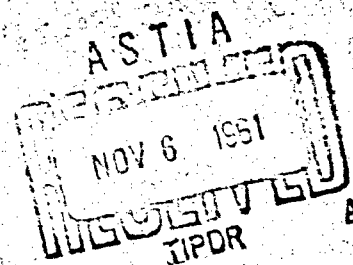
Final Report

R. M. Brach

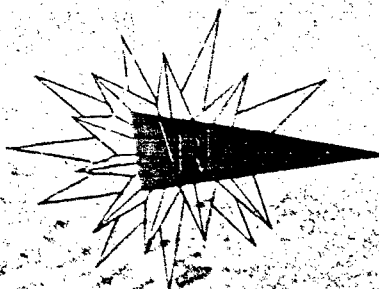
Completed in April, 1961

for

Commanding Officer
Rock Island Arsenal
Rock Island, Illinois



25 years of research



XEROX
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ARMOUR RESEARCH FOUNDATION
of
ILLINOIS INSTITUTE OF TECHNOLOGY
Technology Center
Chicago 16, Illinois

Final Report
LONG-RANGE STUDY PROGRAM
LIGHTWEIGHT ARTILLERY WEAPON

Completed in April, 1961

for

Commanding Officer
Rock Island Arsenal
Rock Island, Illinois

under

Contract No. DA-11-022-ORD-2543
Ordnance Project TW-201
D/A Project No. 5U17-07-021
ILS No. 1-4-1-11
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ARMOUR RESEARCH FOUNDATION OF ILLINOIS INSTITUTE OF TECHNOLOGY

FOREWORD

In June 1957, Armour Research Foundation completed a feasibility study of a lightweight field artillery weapon intended for use by the Marine Corps. The study specifically explored the possibility of development of a helicopter-transportable field weapon with a high-rate-of-fire burst capability. The results of this study indicated that a lightweight field artillery weapon could be developed to use boosted rocket ammunition, be capable of helicopter transport, and fire six rounds in approximately two seconds. A project was then initiated at ARF to design and develop such a launcher.

The first phase of the program consisted of the design and fabrication of the XM70 Launcher, Prototype No. 1. This prototype displayed an exceptionally optimistic performance such that a second phase of the program was initiated. The second phase consisted of the incorporation design improvements and construction of two more prototypes, XM70E1, Prototypes 2 and 3. Subsequently, Prototypes 4 and 5, XM70E2, were designed and built.

Although the early performance of the XM70 Launcher was satisfactory, it was realized that improvements in performance were possible and necessary. In order to determine the best methods of improving performance, specifically the launcher dynamic response and its effect upon accuracy, a long-range study program was initiated. This program consisted of a concentrated effort of both experimental and analytical investigations of the launcher dynamics.

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This long-range study program was begun in October of 1959 and concluded in March, 1961. Progress reports concerning this program were included in the bi-monthly reports of the design and development project. The following staff members contributed significantly to the results of this program: P. Eakins, R. A. Eubanks, L. Gardiner, C. Murray, K. Norikane, N. Pearson, R. Pratt, J. Ross, R. H. Van Beek, and C. Youngdahl.

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
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LONG RANGE STUDY PROGRAM
LIGHTWEIGHT ARTILLERY WEAPON

ABSTRACT

Prototype No. 3 Launcher, XM70E1, 115 mm was instrumented with strain, pressure and displacement gages; these furnished the actual loading and motion of the launcher structure. In addition to certain simple dynamic analyses, a 3-degree-of-freedom, nonlinear mathematical model of the launcher dynamics was derived and programmed for solution on Armour Research Foundation's Univac 1105. The output of the computer program was correlated with experiment and used to study the effect of physical parameter variations.

Regions of instability of the launcher motions were shown to exist for burst firings; relationships between component stiffness and damping were found which optimized the launcher response to firing loads, based upon a simple accuracy criterion. Certain design suggestions were evaluated and shown to benefit the accuracy of the launcher.

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ARMOUR RESEARCH FOUNDATION OF ILLINOIS INSTITUTE OF TECHNOLOGY

LONG-RANGE STUDY PROGRAM
LIGHTWEIGHT ARTILLERY WEAPON

I. INTRODUCTION

A unique automatic rocket launcher in the field artillery weapon class has been designed and developed by Armour Research Foundation under the technical supervision of Rock Island Arsenal. This weapon is a 115mm launcher capable of firing single shots or bursts of up to six rounds. Boost charge variations combined with rocket thrust increments and variable elevation provide an exceptionally wide range capability. The launcher can be traversed $\pm 20^\circ$ without disturbing its original emplacement. Based upon early feasibility studies and also upon the performance of the first prototype, this lightweight, automatic launcher concept showed itself to be unusually capable as an effective weapon. It was realized, however, that production schedule requirements for succeeding prototypes might hinder the development of the concept, because of the tendency to freeze important design features. As a result, this long-range study program was initiated to investigate the dynamic behavior of the launcher during burst firing, relate this behavior to accuracy, and determine the important physical parameter values which will insure good accuracy.

The program was comprised of theoretical and experimental studies of the dynamics of the launcher. These consisted of finding the effects of certain parameter variations upon the launcher motion; accuracy was related to the motion by simple criteria. Some of the important launcher parameters which were considered are: structural rigidity, damping, recoil system characteristics, launcher geometry, and ground restraints.

The majority of the work accomplished, both theoretical and experimental, is presented in the appendices. A description of the work and results as well as the important conclusions are presented in the following sections. Section II is an introductory portion of the report containing a description of the launcher structure and its relationship to its dynamic response. Some of the operational characteristics of the launcher system, as they affect the response, are also discussed.

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Section III presents the results of the experimental work, and is broken into three parts. These respectively describe stiffness measurements of the structural components and two separate firing programs. Stiffness measurements were made, both statically, of all the major structural components, and dynamically of the entire supporting structure assembly (carriages and trails). The second experimental effort was a firing program wherein approximately 75 proof slugs were fired. During these firings the launcher was instrumented with pressure gages, displacement transducers, and strain gages to provide information on the force input, the response of the launcher, and accuracy in the vertical plane. Finally, horizontal dispersion was significantly reduced, as shown by targets, with the use of two fixtures -- one restraining the trails from horizontal motion while the second stiffened the recoiling assembly framework.

In Section IV, the theoretical analyses are described; they consist of the following mathematical models:

1. A planar, nonlinear 3-degree-of-freedom model (called Mathematical Model V in progress reports),
2. A third-order, linear model,
3. A linear, 2-degree-of-freedom model,
4. A three-dimensional, nonlinear, n-degree-of-freedom system,
5. A second-order, linear system.

The first of these was implemented with a computer program and was widely applied to study launcher motion. It contained representations of the hydro-pneumatic recoil system, the indexing cams, and flexibilities and damping of both the supporting structure and elevating system. Information about the response of linear, third-order systems led to a structural optimization of the elevating system. The 2-degree-of-freedom system was an approximation of the launcher used to determine the best location for auxiliary dampers. The fourth model is a relatively complex representation of the launcher; its significant contribution was the inclusion of structural flexibilities of both the recoiling assembly framework and firing tube and also consideration of motion out of the vertical plane. Unfortunately, this model could only be developed to the point where it was programmed for the Univac but not run. The final system was a single-degree-of-freedom system used

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to investigate the important parameter values which can cause the response to decay by a desired amount between rounds of a burst.

The 3-degree-of-freedom model was used to correlate theoretical response curves with the experimental response obtained with various structural changes. The correlation was very good for certain parameter combinations, but the theoretical response curves lacked certain characteristics as some parameters were varied. The reason for this is postulated; allowing conclusions to be drawn which indicate the desirability of certain combinations of structural parameters which correspond to improved accuracy.

A theoretical investigation was made of a specific design recommendation -- delayed recoil rods. Because the 3-degree-of-freedom model had an explicit representation of the recoil system, the behavior resulting from increasing the orifice area during the time the rocket is in the launcher could be studied. This cutback on the recoil rods resulted in near free recoil for a short time.

The final item studied was done as a matter of interest; the terms representing the Coriolis acceleration of the recoiling assembly were neglected during one computer run. Comparison was then made with a normal response to determine the effect.

Although the launcher has a $\pm 20^\circ$ traverse capability, all of the studies considered only zero traverse accuracy because of its order of importance. Ground restraints and soil considerations were included in the n-degree-of-freedom model and could not be studied because of the status of the model. One method of auxiliary ground restraint, horizontal trail ties, was tried and is discussed with horizontal dispersion reduction.

II. DESCRIPTION OF THE LAUNCHER STRUCTURE

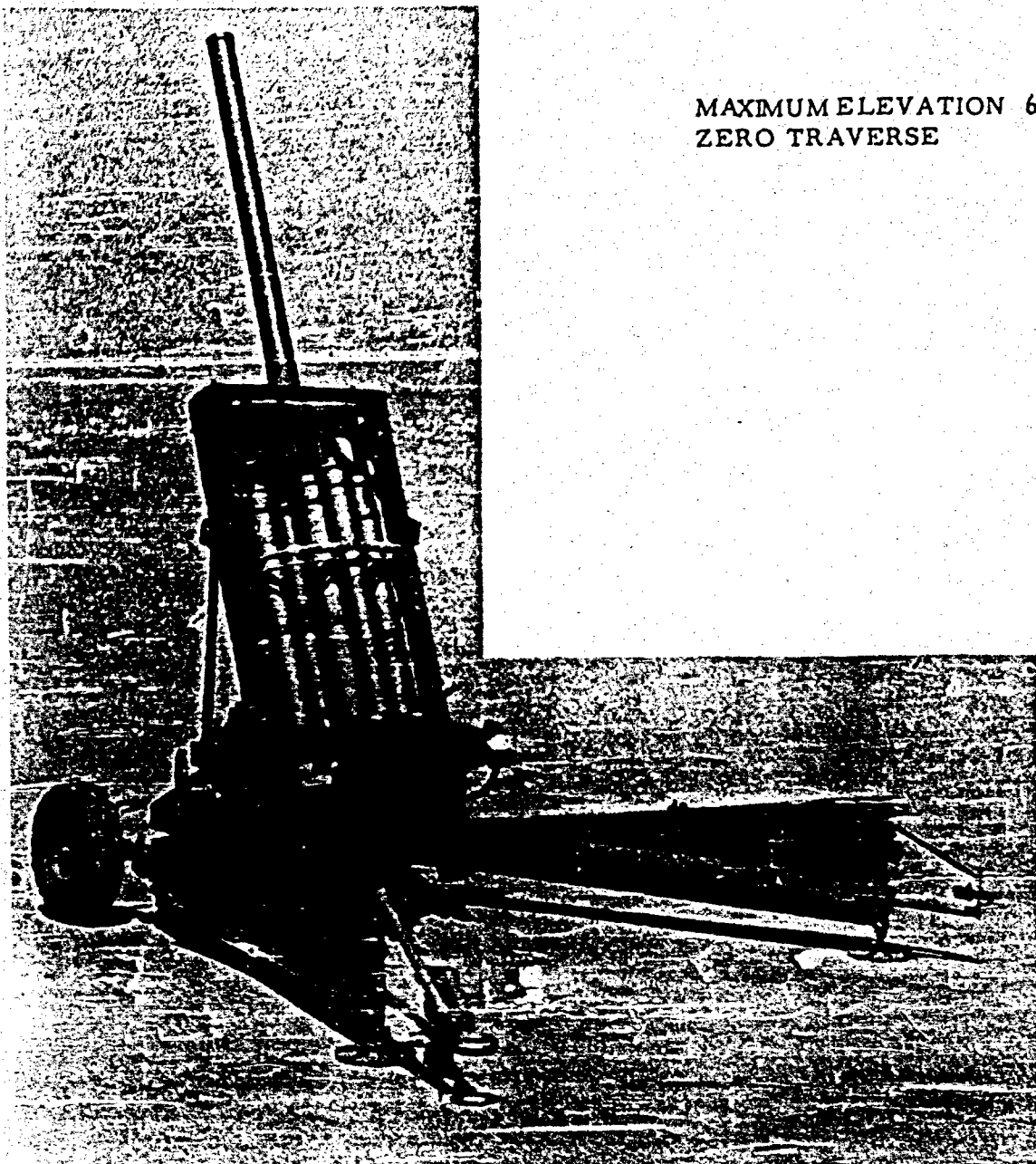
The complete launcher is shown in two firing positions, in Fig. 1 and 2. It consists of two main assemblies: the tipping assembly and the supporting structure. The total weight of the launcher, 3150 lb, is almost equally divided between these two assemblies. The tipping assembly rotates relative to the supporting structure at the trunnion; the elevating system separates them at any fixed position from -6° to 65° .

A. Tipping Assembly

The tipping assembly is composed of a cradle and recoiling assembly; the former is simply the containment for the latter. The firing tube is fixed to the front of the structural framework of the recoiling assembly (see Fig. 3); it also supports two, contrarotating breech clusters of three rounds each and holds the recuperator and recoil system. The framework consists essentially of the front plate, rear plate, two side tie bars, the recuperator cylinder and a lower tie bar; the recoil cylinders are not load bearing members of this framework. Two indexing cams extend into the hubs of the breech clusters and cause them to alternately rotate into firing position during counterrecoil. These cams are fixed to the front of the cradle.

B. Supporting Structure

The supporting structure is the name given to the portion of the launcher below the trunnions; this structure is made up of what are commonly called the upper carriage, lower carriage and trails. An exploded view of the supporting structure in Fig. 4 shows its essential components. This entire structure is composed of hollow, sheet steel construction with various necessary non-structural components accounting for 20% of its total weight. These components are the traverse and elevating mechanisms, wheels, firing base, lunnette and a jack for raising and lowering the firing base. In firing position, the trails are separated by an included angle of 40° which permits $\pm 20^{\circ}$ traverse firing; the upper box, trunnion sides and tipping assembly rotate relative to the remaining structure to attain the desired traverse positions.



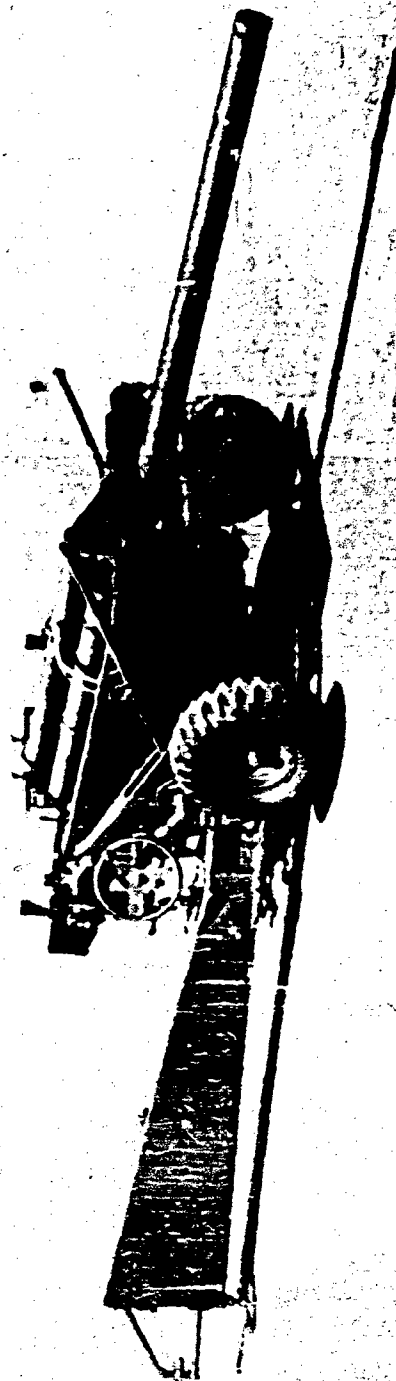
MAXIMUM ELEVATION 65°
ZERO TRAVERSE

11-070-1122-748	ROCK ISLAND ARSENAL ORDNANCE CORPS	May 18, 1960
LAUNCHER, ROCKET, 115MM, TOWED, XM70E1.		
FIRING POSITION, MAXIMUM ELEVATION, (70°) , 0° TRAVERSE, LEFT REAR.		FIG 4
10 OF 27	Project #TW-201	Photographer: E.A. Metzger

Fig. 1 PHOTOGRAPH OF LAUNCHER

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MINIMUM ELEVATION, -6°
ZERO TRAVERSE



11-070-1122-746

ROCK ISLAND ARSENAL ORDNANCE CORPS

May 18, 1960

LAUNCHER, ROCKET, 115MM, TOWED, IN70E1.

FIRING POSITION, MINIMUM ELEVATION, (-6°) , 0° TRAVERSE, RIGHT FRONT.

FIG. 18

16 of 27

Project #TW-201

Photographer: E.A. Metzger

Fig. 2 PHOTOGRAPH OF LAUNCHER

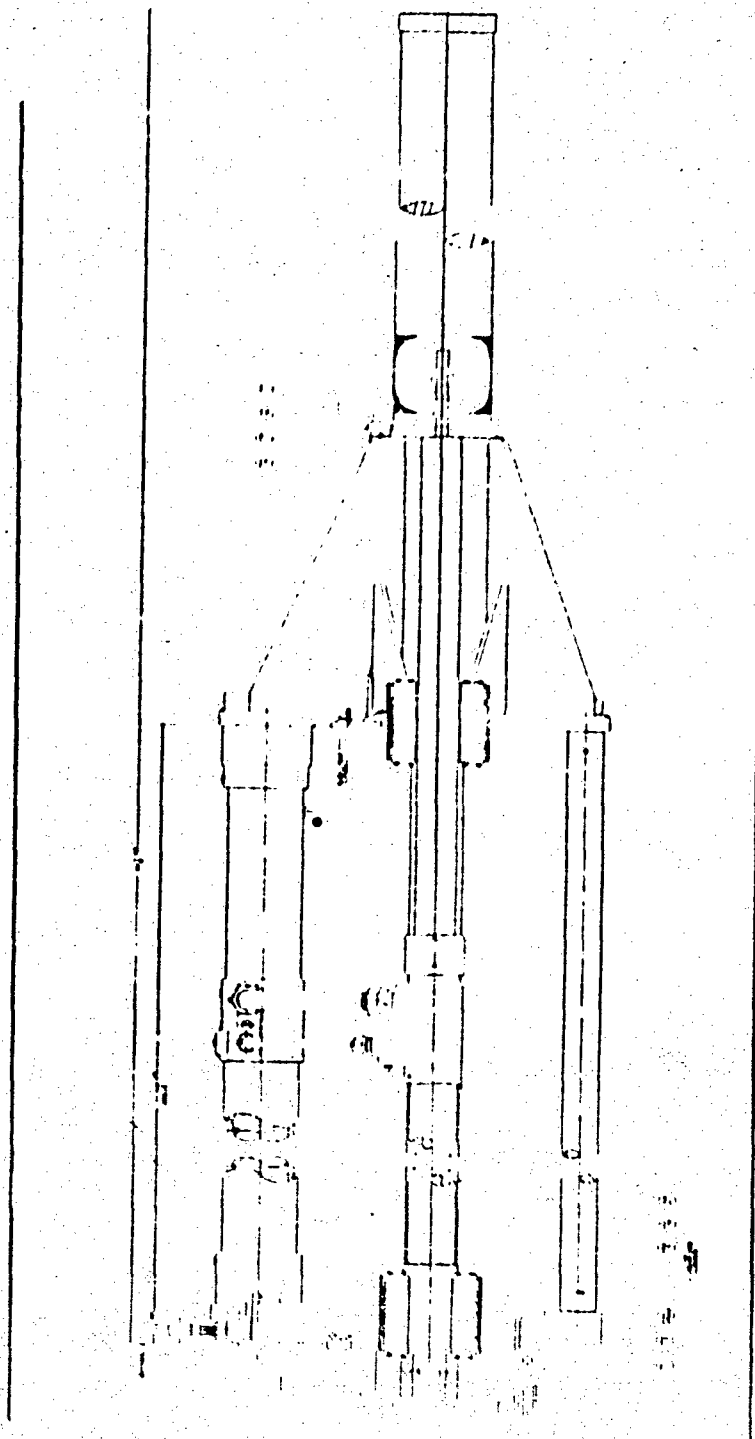


Fig. 3 SKETCH OF RECOILING ASSEMBLY

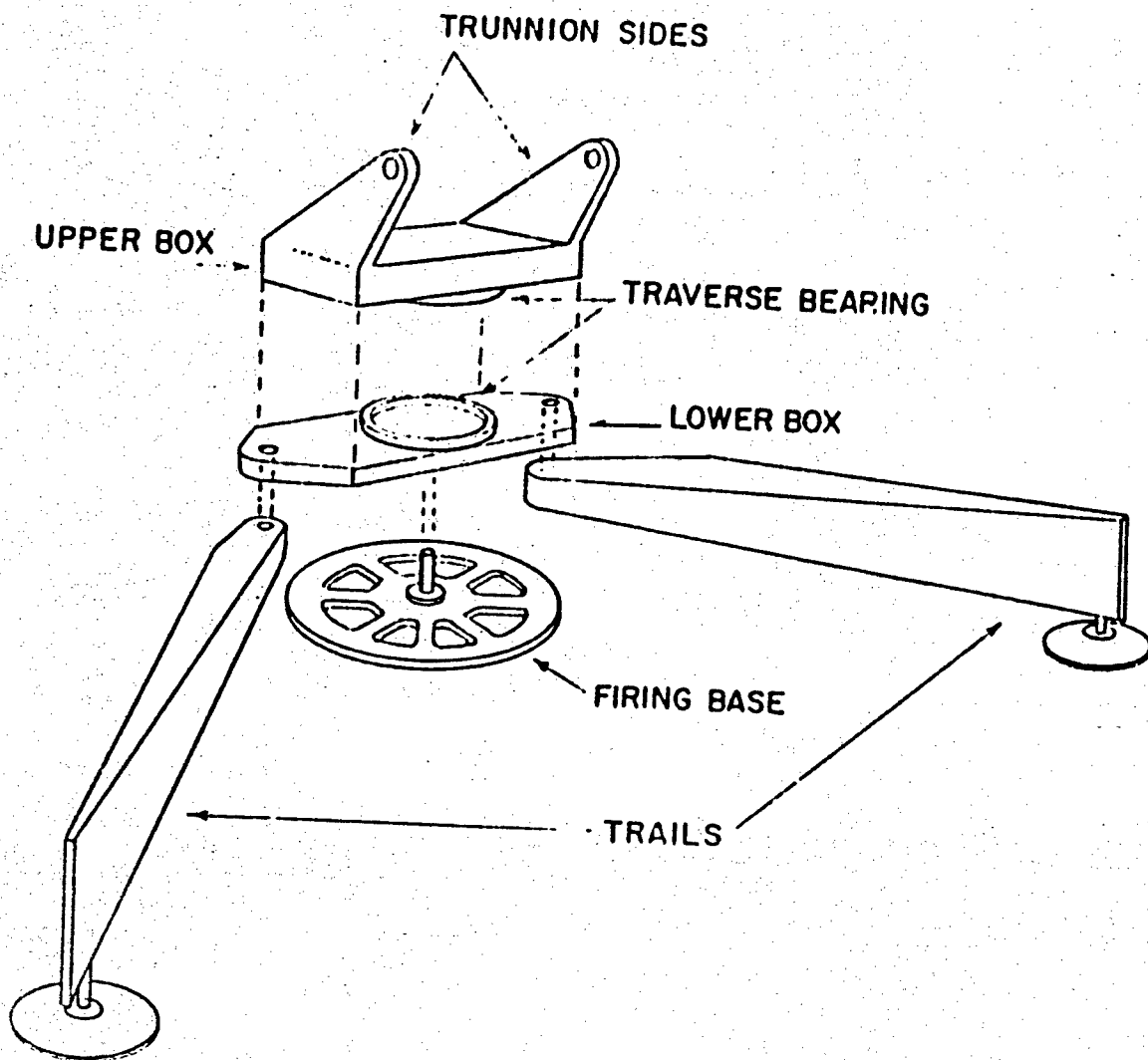


Fig. 4 EXPLODED VIEW OF THE SUPPORTING STRUCTURE

C. Elevating System

The elevating system plays an important role in the dynamic response of the launcher and deserves some discussion. This system is composed of hollow threaded rods passing through ball bearing nut assemblies held in brackets on each side of the cradle. The lower ends of these rods are inserted into the trunnion sides where they are supported by a thrust bearing in a gear box. The rods themselves are relatively stiff in tension and compression; certain other components of this system, however, comprise a significant flexibility. The brackets at the cradle are cantilevered and are inherently flexible; torsion of the cradle sides also contributes to effective relative motion of the tipping assembly and supporting structure. The ball bearings in the screw-nut mechanism, due to unavoidable clearances also add to the system flexibility in the form of nonlinearities. Likewise, the lower connections, through local deflections of the sheet metal box and trunnion sides, are relatively flexible. Thus, significant rotation of the tipping assembly mass center can occur about the trunnions (in the vertical plane) during firing, due to the flexibility of the elevating system.

D. Damping

As designed, damping of the launcher motions can occur from four sources:

1. Structural material damping,
2. Interaction with the soil,
3. The hydraulic recoil system,
4. The support buffers.

The first damping source is negligible in the presence of the others. The second source varies with the type of soil and can be either negligible or significant. The third source of damping is important because relative motion between the cradle and recoiling assembly can cause relatively large amounts of energy to be damped out by forcing oil through the orifice in each recoil cylinder. The support buffers consist of two cylinder and piston assemblies attached to the front of the lower carriage; the cylinders are fixed to the carriage and the piston rods rest against the firing base. When the supporting structure rotates forward, about the ball joint connection to the base, the

pistons force oil through orifices and damp out this forward motion. Small springs in parallel with the pistons restore the position of the supporting structure. These buffers are mainly to prevent the launcher from tipping over following the final shot of a burst when the recoiling assembly returns to battery. These buffers operate during bursts, however, when any forward rotation of the supporting structure occurs.

E. Deflection

Before discussing the dynamic response of the launcher structure to firing loads, it is necessary to first examine the supporting structure to see how it deflects to a load at the trunnions. We will consider zero traverse, a completely symmetric structure and loads in the vertical plane only. Let us assume that the tipping assembly and elevating system are removed; a force at the trunnions (for example, at 45° with respect to the ground) causes the trunnion sides to deflect relative to the upper box. Since these members are essentially short, stubby beams, both bending and shear are of equal significance. We now have a shear force and moment at the sides of the upper box; this causes bending and shear of both sides of this member relative to the bearing. However, this loading also causes torsion of the upper box section, and is the most significant deflection of this member compared to shear and bending deflections. The shear load is further transferred through the bearing down to the ground at the firing base. The moment reaction, however, is transferred through the bearing and causes torsion of the lower box, which eventually places a bending moment on the front of each trail. With respect to trunnion deflection in the vertical plane we then see that the following values of stiffness of the structure components are important:

1. Bending and shear of the trunnion sides,
2. Torsion of the upper and lower box,
3. Separation of the traverse bearing,
4. Bending of the trails.

Significant deflections occur both from the supporting structure and from the elevating system and cause a nonlinear relationship between the various deflections, due to the geometry.

F. Experimental Structure Modifications

During the experimental firings conducted on this program, various modifications to the structure were made. These took essentially two forms, changes in stiffness and changes in damping. Modifications of the structure which essentially affect only the response in the vertical plane are discussed first. The stiffening fixtures are:

1. Variable stiffness elevating rods,
2. Upper and lower box end-clamps,
3. Trunnion-to-trail struts.

The utility of the first fixture is self explanatory. The second and third sets of fixtures were designed to permit a temporary increase in the stiffness of the supporting structure by decreasing the torsion of the upper and lower boxes. The clamps were 1/2 in. thick steel brackets which joined each of the four corners of the boxes and transferred the moment reaction from the base of the trunnion sides directly to the trails. The trunnion-to-trail struts caused most of the trunnion reaction to be directly transferred to the trails. For these struts, the trails had to be rotated inward until they were set directly beneath the trunnions. Having the trails straight back caused an additional stiffness increase in this configuration.

Damping was added to the elevating system in either of two ways. Auxiliary, hydraulic dampers were placed between the cradle front and trunnion sides in parallel to the elevating screws. Figure 5 shows these dampers on the launcher. Provisions were made to change the damping source by the use of frictional ring springs in the variable flexibility elevating rods. A more complete description of these fixtures is presented in Appendix G.

The above fixtures were intended to cause a direct change of the launcher response in the vertical plane (neglecting coupling of vertical and horizontal motions). Another temporary change to the launcher structure was made, this time for the purpose of affecting the horizontal motion of the launcher. This change was the addition of a thin, sheet metal shell which surrounded the recoiling assembly and was attached to the side tie bars. This shell stiffened the framework in a manner such that the front plate and rear plate would be more securely restrained from rotating with respect to

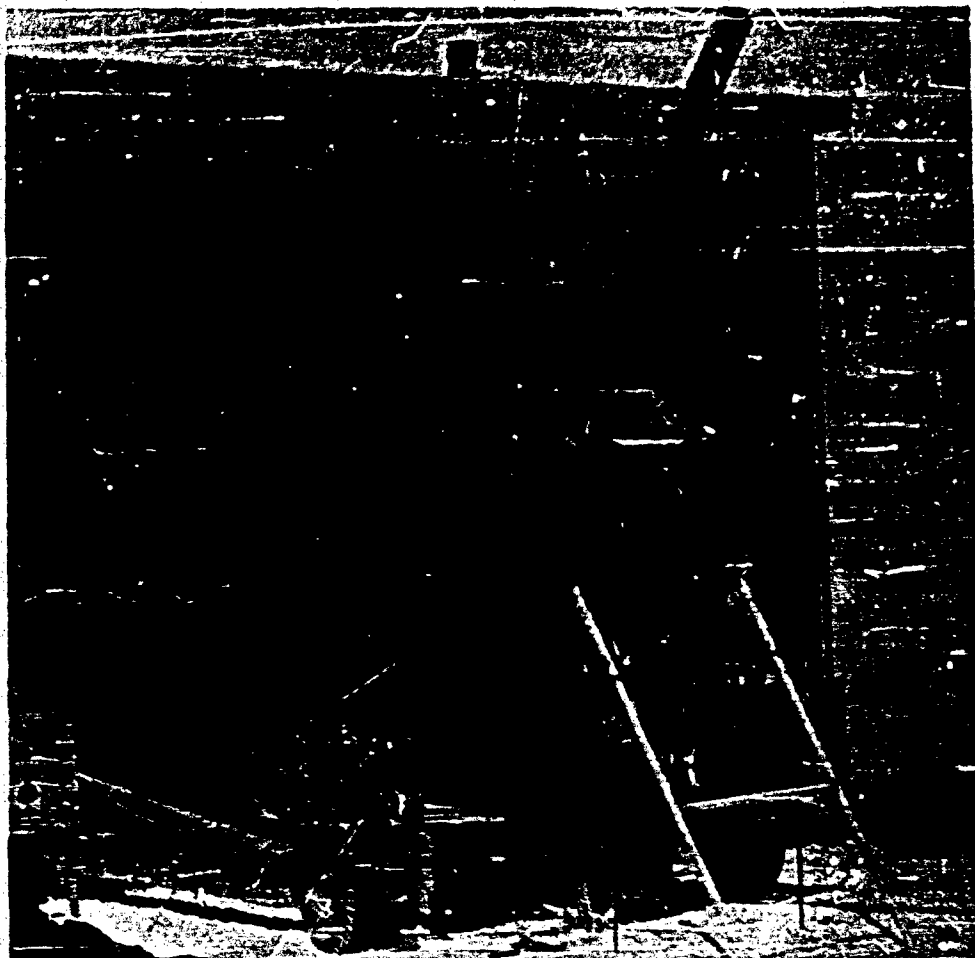


Fig. 5 EXPERIMENTAL LAUNCHER EMPLACEMENT

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each other about a vertical axis. This type of rotation was experienced in the launcher and caused a significant horizontal dispersion. For the same purpose, reduction of horizontal dispersion, the trail ends were fastened by cables to small stakes at the side of each trail. This prevented the trails from rotating (in the plane of the ground) and caused a decrease in horizontal dispersion.

III. EXPERIMENTAL WORK

The experimental effort was composed of three separate ventures:

1. The determination of the structural component flexibilities,
2. Determination of the dynamic response of the launcher in the vertical plane,
3. Testing of fixtures which were to decrease horizontal dispersion.

The values of stiffness of the structural components were needed for two reasons: first, as input to the theoretical studies and second, to determine if the particular structural designs yielded the desired stiffness, as required by earlier analyses. The stiffness was found experimentally because of the complexity of the structure; it was more efficient than corresponding calculations. In cases where calculations were made, the measurements afforded a method of checking.

The dynamic response in the vertical plane was determined for correlation with theory and thereby used to validate the corresponding mathematical model, and/or point out its deficiencies.

A. Physical Parameter Measurements

Static stiffness measurements of the following components or assemblies were made:

1. The entire supporting structure,
2. The entire supporting structure with the upper and lower boxes clamped,
3. The entire supporting structure with the box clamps and with the trunnion-to-trail struts,
4. The entire supporting structure in traverse position,
 - a. 20° right
 - b. 20° left
5. The lower elevating screw support,
6. Trails,
7. Elevating screw - upper bracket combination.

Since rotational motion is the sole significant contributor to dispersion, the moment of the recoil force per unit rotation of a line through the ball joint and the trunnions (in the vertical plane), is used as an indication of supporting structure stiffness. The static stiffness of the

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supporting structure is therefore given in units of in. -lb/radian. For the entire supporting structure at zero traverse, this was measured to be 32×10^6 in. -lb/radian. When the box clamps were added, this stiffness became 38.5×10^6 , approximately 20% greater. A much larger increase occurred when the trunnion-to-trail struts were added; the stiffness rose to 55.5×10^6 , a 73% increase. This was due not only to the direct load transfer from the trunnion to the trails but is also because the trails were straight back; in this position they contributed more rigidity to moments in the vertical plane. During traverse (20° left or right) the stiffness of the supporting structure dropped to 25×10^6 in. -lb/radian for the "as-designed" structure. This decrease is expected since the stiffness of one trail is almost completely lost in this configuration.

The trail stiffness (total, for both trails straight back) for a bending moment applied at the forward end resulted in 145×10^6 in. -lb/radian. In this case, the stiffness was lower than predicted by calculation, and is believed due to local deflections in the area of the trail hinge pins.

Measurement of the stiffness of the lower elevating screw supports illustrated an unanticipated result. The left support was nonlinear up to a load of 800 lb; at this load it assumed a relatively linear, load-deflection curve shape, with a slope of 3.5×10^5 lb/in. The right support had a linear stiffness of 3.5×10^5 lb/in. from zero load. This would indicate that unsymmetrical elevating system deflections would occur. This actually happened and can be seen in the experimental records in Appendix J.

The elevating rods with their upper brackets attached exhibited nonlinear load-deflection curves as assemblies. In this situation, the nonlinearities are due to either or both of the following reasons:

1. Clearances between the ball-nut and the ball bearings.
2. Nonlinear local deflections of the balls and ball-nut.

The clearances can cause nonlinearities because as more balls come into contact at higher loads, the stiffness increases until all the balls are in contact. The resulting stiffness curves are shown in Fig. H-12.

In addition to the static stiffness measurements, dynamic measurements were made of the supporting structure. As opposed to the

static measurements where the deflection is measured under a very slowly varying load, the dynamic stiffness is a measurement of the rms value of a sinusoidally varying force and the rms value of the displacement. (Maximum values also could have been used.) This, of course, varies with frequency.

A standard 600 lb electrodynamic vibration shaker was used to excite the launcher structure for the dynamic stiffness measurements. Figure H-2c shows the physical arrangement. The tipping assembly and elevating rods were removed and a relatively stiff bar was placed across the trunnions. The shaker was coupled to the bar with a dynamic force transducer which yielded the input force. Accelerometers were placed on the trunnions to yield the motion output; the dynamic stiffness, k_D , is simply,

$$k_D = \frac{F\omega^2}{a}$$

where

F = Measured, input force, rms (or peak),

ω = Input force frequency,

a = Measured, linear trunnion acceleration, rms (or peak).

Although the repeatability of the magnitude of the measurements was not good, the shape of the stiffness-frequency curve was the same for each of the two runs. The relative minimum values of this curve (Fig. H-11) correspond to resonance frequencies of the structure. The first mode frequency (the frequency at the first resonant point) of the entire supporting structure is near 70 cps. Since the firing frequency of a burst is slightly less than 3 rounds per second, we can be confident that an undesirable structural resonance will not occur. The recoil force itself could possibly contain variations at a frequency near 70 cps, however, previous experimental and calculated curves show that the magnitude of the recoil force does not have these high frequency variations. The result is that for this launcher, calculation of the first mode response should be sufficient to predict the response accurately.

A rough check of the magnitude of this frequency can be made, combining the measured, static stiffness with the result of a calculation of an equivalent structure inertia. The equivalent structure inertia was found by assuming that the structure and its equivalent both respond harmonically

in the static deflection shape of the actual structure. Imposing the conditions that the maximum deflection and kinetic energy of both systems are identical yields an equivalent inertia; for this structure it is 115 in. -lb-sec². The first mode frequency is found by treating the equivalent system as a 1-degree-of-freedom system. This frequency is:

$$f = \frac{1}{2\pi} \cdot \sqrt{\frac{k_s}{I_e}} = \frac{1}{2\pi} \cdot \sqrt{\frac{32 \times 10^6}{115}}$$

$$f = 84$$

where

k_s = measured static stiffness,

I_e = calculated equivalent inertia about the ball joint.

These two frequencies correspond sufficiently close when experimental errors and simplified calculations are considered. (The methods of determination of the values of stiffness of the launcher structural components are described more fully in Appendix H.)

B. Launcher Response Determination

The dynamic response (both displacement and velocity) of the launcher in the vertical plane to firing loads was obtained directly by instrumenting the launcher during a controlled firing program. The general procedure was to fire a two or three round burst for each structural condition, followed by two single shots. The reason six-round bursts were not necessary for each condition is that the response to rounds 3, 4, 5, and 6 do not differ appreciably from the second. The short test bursts, therefore, illustrated full burst conditions and the single shots provided a reasonable check on repeatability of the type of response. The firing program with the instrumentation and structural configurations listed for each test is shown in Table I-1. A list of the type and use of recorded data is as follows:

1. Strain adjacent to the firing pin (on the rear plate of the recoiling assembly), for a firing signal proportional to the boost charge,
2. Strain of the upper trail flanges near the forward pin connections, to indicate the supporting structure rotation.

3. Cradle displacement, in the vertical plane, at the tip of the cradle. (Combined with similar trunnion motion, this furnishes cradle rotation).
4. Elevating rod strain to furnish the force in the rods,
5. Displacement of the trunnion, parallel and perpendicular to the cradle (some firings only),
6. Damper-support strain to furnish the damper force,
7. Recoil system pressure to indicate the recoil force.

The rotational response of the cradle and the trail strain (structure rotation) comprised the response data of primary significance. The cradle rotation was used as the primary indication of the effectiveness of a particular structural change in reducing dispersion. The trail strain formed an important quantity in correlating theory and experiment. The forcing functions in the form of rear plate strain and recoil system pressure were also necessary quantities. (It should be mentioned that the trail strain measured is linearly proportional to the trail rotation only during positive rotation, the initial direction of deflection. This is because during negative rotation, the supporting structure rotates forward, the trail ends leave the ground and the strain is then a measurement of the trail inertial loads.)

Table 1 summarizes the launcher conditions during the various bursts which are discussed here. Except for the modifications listed, the launcher was as designed.

Table 1

Burst No.	Modifications to Launcher
B-2	None
B-5	Elevating system -- coil springs (total rate of 14,000 lb/in.) and little damping.
B-6	Elevating system -- coil springs and much damping.
B-7	Elevating system -- same as B-6, stiffened supporting structure.
B-8	Elevating system -- stiff rods and much damping, stiffened supporting structure.
B-9	Elevating system -- ring springs (self damping).
B-10	Elevating system -- coil spring (low boost charge, 720 fps muzzle velocity).

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All the burst response curves are shown in Appendix J for each of the significant structural configurations. These response curves indicate that the structure, although theoretically symmetric about the vertical plane, has an unsymmetric response. The difference in stiffness between right and left elevating systems (as determined by measurements) is probably the major cause of this unsymmetrical response. Unsymmetrical supporting structure deflections were noted during the measurements also, but these were much smaller. These differences in response from one side of the launcher to the other increases the difficulty in analyzing these curves especially when comparing them to theoretical, planar response curves. Although these unsymmetrical deflections can increase dispersion, we will discuss the response curves as though they were identical.

In order to judge relative accuracy of these experimental response curves let us use a simple criterion. Since the recoiling assembly is relatively rigid in the plane of motion we are considering, its motion relative to the cradle due to its own flexibilities will be at a higher frequency and should damp out more quickly than the launcher response. Therefore, prior to each subsequent shot of a burst the only residual motion of significant magnitude is the gross launcher motion. The amount of residual gross launcher motion should then indicate the amount of dispersion, since if each round fires with identical initial conditions, the response of the launcher should be the same, neglecting recoil force variations due to weight changes.

Fig. J-5 shows the cradle rotation for burst B-5, fired with a softened and lightly damped elevating system. This burst had the greatest nonsymmetrical response in comparison to all the other firings. This could be due to additional nonsymmetry from the dampers. In discussing the general response, we will still neglect the difference in the curves. The cradle oscillation for this burst is a well damped response; the maximum slope of the response prior to the second shot is near 0.2 radians/sec. The time of shot ejection was not recorded; however, for maximum boost rounds (muzzle velocity of 1000 fps), this time is about 20 msec following ignition. The ignition time is marked, and we see that the slope or velocity is near zero. The trail strain, or supporting structure response in Fig. J-6 indicates some trail hop by the relatively high frequency oscillations about zero, immediately

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following the large initial response. This response is also well damped. Fig. J-7 shows the cradle rotation for burst B-6 which had approximately 15 times more damping in the elevating system. As expected, the cradle response decays more rapidly, but not as much as was expected. The trail strain, Fig. J-8, still shows evidence of rebound of the structure. The maximum cradle velocity of the last oscillation prior to the second shot was reduced to 0.08 radians/sec. At the estimated time of shot ejection the cradle response is very unsymmetrical where one side has a positive velocity near 0.5 radians/sec and the other has a negative velocity near 0.08 radians/sec. Theoretically, the actual rotational velocity in the vertical plane is the average, however, since motion existed out of the plane, this number may be meaningless. The important fact is that the overall response damped out more quickly.

When the supporting structure was stiffened with the box clamps and trunnion to trail struts, burst B-7, the launcher response changed significantly. Fig. J-9 shows the cradle response and Fig. J-10 the corresponding trail strain. The cradle rotation essentially remained positive at all times as did the trail strain. In addition, the magnitude of the cradle rotation was significantly lower; the peak trail strain was slightly lower. The most important change, however, is the shape of the cradle response just prior to the second shot; the slope remains close to zero with only small variations. This indicates small cradle velocity and improved accuracy.

Another modification was made to the launcher by replacing the soft coil springs in the elevating rods by relatively rigid members in burst B-8. The dampers and supporting structure stiffeners were retained. The response of this system (Fig. J-11) was very similar to that of the previous one with possibly a small increase in the residual cradle oscillations. This would seem to indicate that elevating rod flexibility is not a significant parameter. Another possibility exists, however, and is more likely the situation. As indicated in the section describing the launcher, significant elevating system flexibilities occur from sources other than the rods themselves. For this reason, when the rods were stiffened, the system flexibility was not significantly changed. This is due to the fact that for a series system of flexibilities, the overall flexibility is always less than the smallest of the

component flexibilities. An interesting cradle response occurred during this burst (three rounds). The cradle rotation had a larger mean residual value between each of the succeeding rounds; in other words, the response had a tendency to climb. Although the cradle velocity was insignificant prior to each shot, the displacement was increasing. This leads to an undesirable effect upon accuracy.

Burst B-9, Fig. J-13 and J-14, shows the result of using ring springs in the elevating system; ring springs have inherent damping since their deflection is accompanied by sliding upon each other. These were designed to furnish near the maximum possible damping for the stiffness equivalent to the coil springs. Apparently, this amount is insufficient to furnish a good response.

The final firing of a modified launcher, burst B-10, consisted of a two-round burst with a zone 7 boost charge (muzzle velocity of 720 fps). The only change in the launcher structure was the addition of coil springs into the elevating system; a lower zone of boost ammunition was necessary to prevent bottoming of the springs. The cradle response (Fig. J-15) is very similar in shape to the response of burst B-5. The newer response has a generally lower magnitude because of the lower projectile boost. It is very interesting to note that the response is still very well damped even though no damping was added. This is pointed out again and discussed when the response of the unmodified launcher is discussed.

A very important characteristic of this launcher is illustrated in burst B-10, i. e., the variation in rate independent of the response. The second round fired at approximately 0.7 seconds after the first, compared to less than 0.6 seconds in the other bursts. Although the variation in this burst was later discovered to be caused by mechanical interference in the indexing cams, other changes can produce the same effect. For example, changing the bypass valve in the counterrecoil flow can vary the duration of counterrecoil and therefore the firing rate. Also, variations in recuperator preload (which is sensitive to ambient temperature) can also cause rate changes. The point is that a negative cradle velocity prior to ignition of the rounds such as in burst B-5, cannot be depended upon to decrease the value of the initial positive response following ignition. This is because firing time, relative to the cradle

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oscillation, would be very difficult to control. The proposed alternative, sought here, is to reduce the cradle oscillations to a minimum and rely on consistency. This was implicit in the simple criterion previously stated.

One last point to be mentioned about the response of burst B-10 is the very slow initial rise time of the cradle on both shots. This is in comparison to all the other bursts. It is due mainly to the slower rise time of the recoil force for the smaller boost. The type of recoil system used has this characteristic feature.

We will now discuss how these response curves differ from those of the launcher in its as-designed condition. Fig. J-1 shows the cradle response during a 6-round burst, B-2. In comparison to the launcher with a more flexible elevating system, the response is similar in that the negative cradle rotation has appreciable magnitude; the residual oscillation, however, has a very small magnitude as well as negligible variation. (Because of the perfectly flat, residual response, the consideration of instrument error must be made. However, two other 6-round bursts yielded the same result.) It appears that the as-designed structural configuration, by the simple criterion set up, should produce the best burst accuracy. To a limited extent this is true, but there are two important considerations which must be mentioned. The first consideration is the reason why the as-designed structure responded the way it did. If we look at the corresponding trail strain curves (Fig. J-2 and J-3) we can see that because of the high frequency oscillation about zero, the rear of the trails must have lifted from the ground. This is due simply to the elastic rebound, or overshoot, of the structure caused by the recoil force characteristics. This means that the supporting structure rotated forward about the ball joint and compressed the two support buffers between the lower box and the firing base. The amount of trail hop is dependent upon the following quantities:

1. Shape and magnitude of the recoil force,
2. Drop off time of the recoil force relative to the natural period of the system.

Variations in the structural stiffness or the inertia of the launcher can alter the occurrence of hop. This was shown when the supporting structure was stiffened -- the hop was absent.

The second consideration is that while the supporting structure is tipped forward, the launcher can turn about a vertical axis through the ball joint. This allows the launcher to lose its aimed orientation and cause horizontal dispersion. Since a method exists to eliminate this rotation during hop (discussed in the next section), choice can be made to either eliminate the hop, by a stiffer carriage for example, or eliminate the rotation during hop. Either is an acceptable method to insure good horizontal dispersion. Other considerations may enter into a choice of whether or not trail hop should be eliminated, such as the safety of the crew.

C. Horizontal Dispersion

Early tests of the XM70E1 tipping parts indicated that horizontal dispersion was larger than vertical and followed a left-right-left sequence of impact points in burst firings. No analytical attack was made on the dispersion problem when it was first recognized because at that time no logical theory of the behavior could be constructed. In particular, the powder couple did not offer a logical explanation because it does not change direction after each shot.

A test program was planned to investigate the sequential horizontal dispersion. The weapon was prepared and the tests were conducted with the extensive cooperation of the Rock Island Arsenal. It was possible to conduct the tests on the force mount at Rock Island Arsenal because the dispersion evidenced itself with a high degree of repeatability during the proof tests of both Prototype No. 2 and Prototype No. 3.

The first group of tests showed that any round fired from a breech tube in the left cluster would fall 5 in. or more to the left of the vertical boresight line and that any round fired from the right would fall 5 in. or more to the right, at an 85-yd range, for all conditions tried. The boresight spot on the target coincided before and after each firing.

Instrumentation changes were made, and the weapon was prepared for a second series of tests. The tests showed no substantial improvement with slugs more accurately centered in the breech tubes or with disconnected igniter tubes, but some improvement was obtained with the clusters shimmed tight between the front and back plates. The instrumentation indicated that

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the hub of the cluster being fired was not being properly loaded with the inertia force of the cluster.

Instrumentation changes were made, and the weapon was prepared for a third series of tests. These tests showed that a round could be fired accurately from either cluster if that cluster were drawn back tight against the back plate when the round was in the firing position. This was accomplished by preloading the cluster thrust bearing located in the back plate of the recoiling assembly framework. Indexing during firing was then impossible and was prevented by removing the indexing cam followers.

At this time, it was realized that the breech tube being fired was being driven back against the back plate and was carrying the connected cluster rearward with it. This resulted in the application of a large moment to the rear plate since the inertia force of the non-firing cluster was being applied at its thrust bearing.

A final series of tests was conducted with the breech tubes permitted a few thousandths of an inch axial movement in the retaining spiders. This allowed the tube being fired to move rearward without carrying its cluster with it. The horizontal dispersion was very substantially reduced. Several 6-rd bursts at zone 10 (1000 fps muzzle velocity) were fired through targets at 85-yd range. The target pattern was nearly rectangular with overall dimensions of approximately 5 in. vertical by 10 in. horizontal.

The remaining horizontal dispersion is still sequential. Calculations show that because the breech tube being fired is driven rearward primarily by the powder-gas pressure acting on a belled portion at the front of the tube, its mass is lost from the cluster assembly and the loss of that mass then produces the sequential horizontal dispersion. Figure 6 and 7 show schematically how the cluster inertia forces are distributed throughout the recoiling framework.

A series of firing tests ^{1/} was conducted with Prototype No. 3 during September and October, 1960 at Redstone Arsenal to determine the effect of increased rigidity between the front and rear plates (i. e. rigidity

^{1/} Armour Research Foundation Report 8130-20, P. II-4.

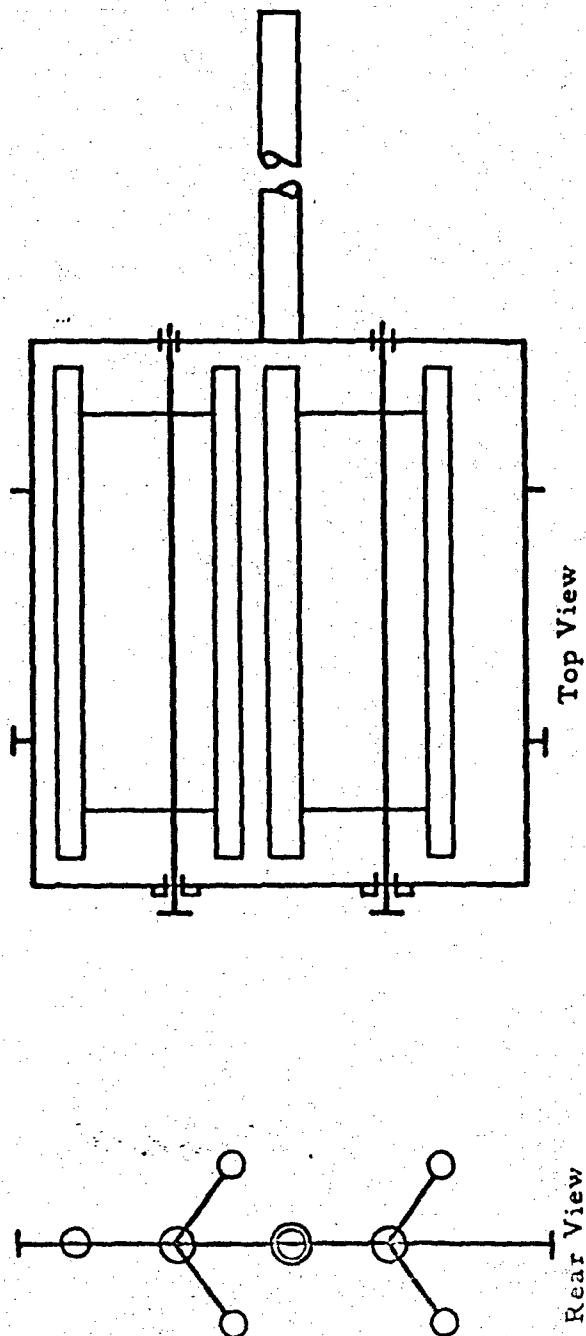


Fig. 3 SCHEMATIC OF RECOILING ASSEMBLY

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of the recoiling assembly framework) on the sequential horizontal dispersion.

A calculation showed that sequential horizontal dispersion would be very small if the recoiling assembly framework was very rigid. The problem then was to find a lightweight structure that would greatly increase the stiffness of the framework. It was decided that the most efficient structure would be a thin metal skin or shell enclosing the entire framework.

The firing program was set up to fire 6-round bursts with and without the shell. To measure shell effectiveness, two quantities were recorded: (1) the bending strain of the firing tube near its base in the horizontal plane, and (2) the target patterns. Six bursts were fired, three with and three without the shell. (The launcher also contained additional stiffening fixtures used in the previous Fort Sheridan firings, consisting of carriage clamps at the front and rear corners of the box sections, trunnion-to-trail struts.)

The records of bending strain in the firing tube without the shell exhibited the alternating direction of initial bending moment. With the shell, the initial direction of the bending moment was consistently in one direction, which can be logically explained by the rifling torque; see Fig. 8. The target patterns, however, did not show any reduction in horizontal dispersion. It was decided that the rigid body rotation of the launcher, due to the trail pads shifting along the ground, was hindering the effectiveness of the shell.

A portion of the test was thus repeated in an attempt to obtain less horizontal dispersion. This time the trails were restrained from rotating about the ball joint in the plane of the ground by placing a pinned connection between a stake and a trail on each side of the launcher. This permitted small rearward and upward trail motions. In this condition, four 6-round bursts were fired, two with and two without the shell. The resulting target patterns are shown in Fig. 9. The two bursts with the shell are rounds No. 93 - 98 and 99 - 104; the two bursts without the shell are No. 106 - 111 and 112 - 117. The firings with the shell had a total horizontal dispersion of approximately 1.5 and 1.9 mils, while the two without the shell had a total horizontal dispersion of 3.7 and 2.9 mils. Because of the single-hole targets, it is impossible to calculate the standard deviation, but it can be inferred

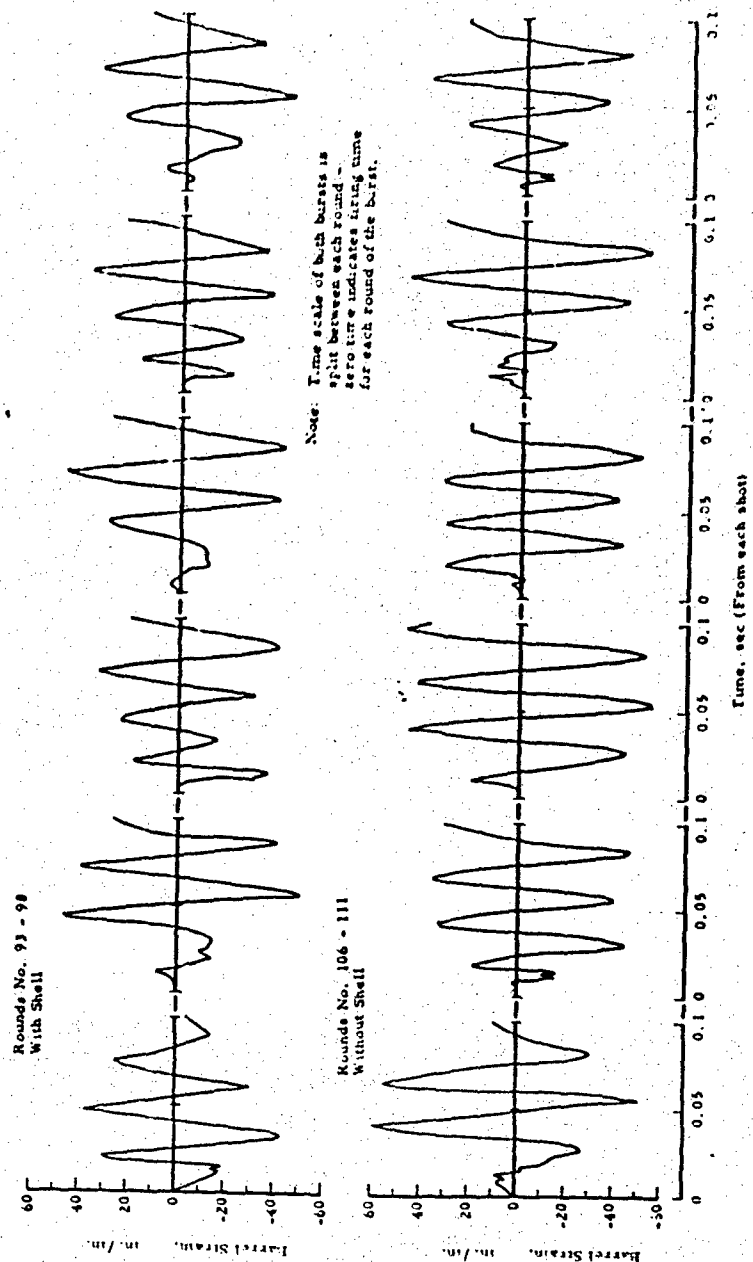


Fig. 8 BARREL STRAIN

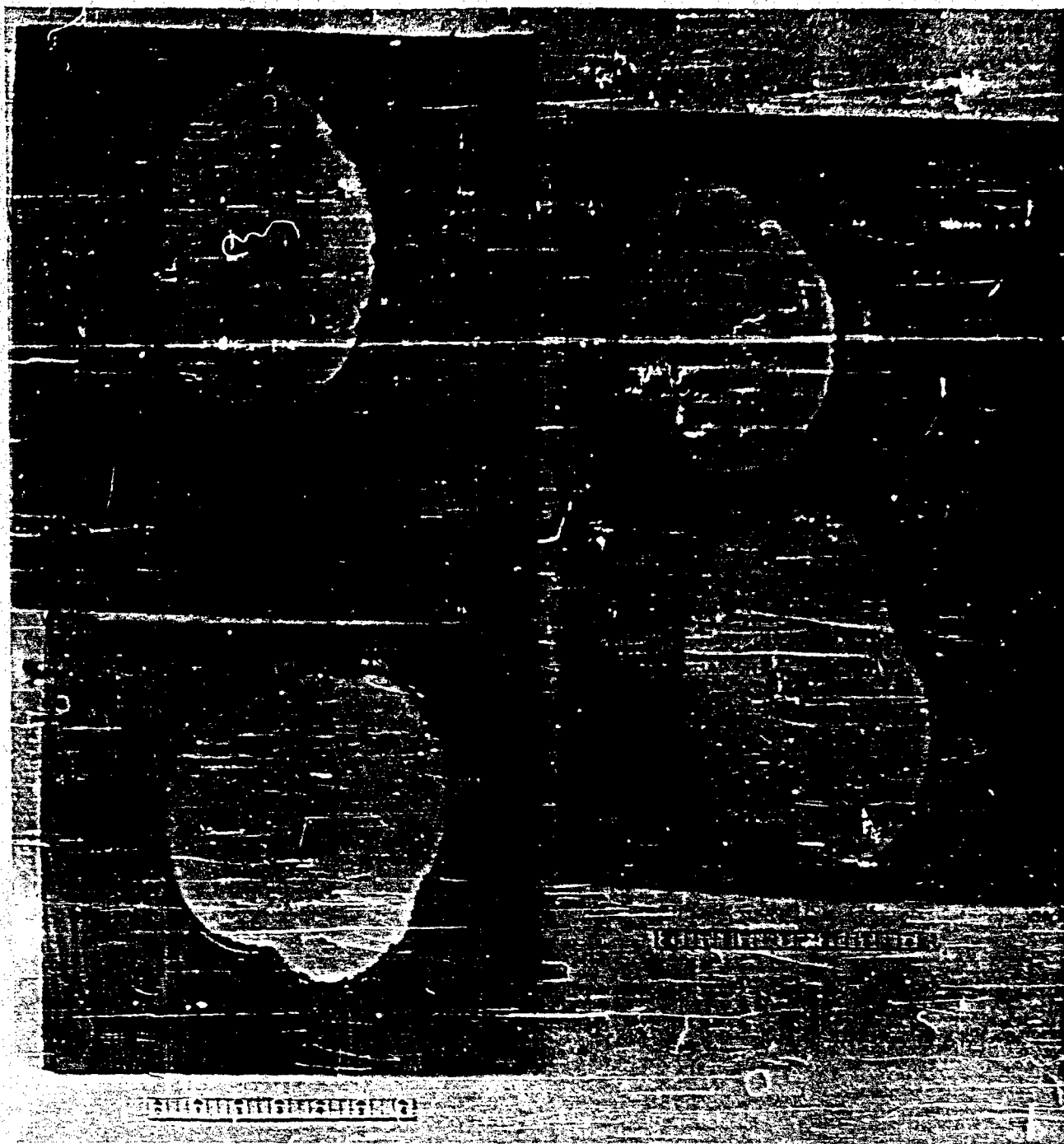


Fig. 9 TARGET PATTERNS

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that, for the two bursts with the shell, the standard deviation is less than 1 mil.

The results of these tests indicate that the shell offers a reduction of horizontal dispersion of nearly 100%. (The trial tests showed a reduction of 200 to 300%.) The reliability or consistency is fairly satisfactory since these targets represent four consecutive bursts fired from Prototype No. 3 under identical conditions. But in order to obtain these results, it must be emphasized that the trails were restrained from rotating about a vertical axis through the ball joint.

IV. THEORETICAL ANALYSES

The theoretical analyses of the launcher dynamics consist of two specific mathematical models to determine launcher motion, and three supplemental studies which used very simple dynamic models in order to directly study the effects of certain parameter variations. The two models which were developed to calculate the launcher motion are:

1. A planar, nonlinear, 3-degree-of-freedom system,
2. A three-dimensional, nonlinear, n-degree-of-freedom system.

The three supplemental studies include:

1. A linear, 2-degree-of-freedom analysis to study the effectiveness of various locations for auxiliary damper locations,
2. A third-order, linear single-degree-of-freedom model to determine optimum elevating system structural parameters.
3. A linear, second-order system to determine the parameters which bring about an optimum response decay.

The derivation, assumptions, and some results of these models and studies are presented in the appendices; a discussion of their development, applications and results are presented in this section.

The derivation and discussion of a mathematical accuracy criterion is also included in this section. This criterion is based upon the sensitivity of the rocket to the different launcher motions studied in the mathematical models.

A. Planar, Nonlinear, 3-Degree-of-Freedom Model

Prior to the initiation of this Long-Range Study Program, some dynamic studies were conducted which aided the formulation of this model. One study, in particular, was a similar, planar, 3-degree-of-freedom model with the following degrees of freedom:

1. Rigid body rotation of the launcher about the trail ends; base hop,
2. Recoil motion,
3. Elastically restrained rotation about the ball joint.

The last type of motion was assumed to the first mode response of the relatively flexible supporting structure, with a rigid tipping mass and rigid elevating system.

Subsequent experience, both from the use of the above model and from actual launcher firings, provided valuable information for making assumptions for the new model. For example, instrumented firings and the output of the above model both showed that rotation about the trail ends was very small. It was therefore neglected in the new model. Recoil motion calculated in the above model agreed fairly well with experiment except during counterrecoil. The difference was traced to seal friction and indexing forces. The former results in longer counterrecoil durations, while the latter brings about two changes. Indexing forces cause small changes in the recoil motion but more important, they significantly affect the cradle response. Consequently, they were both included in the new model. Inability to obtain a good correlation of the experimental response with the output of the prior model led to the conclusion that the elevating system was flexible. Again, this flexibility was therefore included. The assumption of the first mode response of the supporting structure was supported by previous experimental firings, and was reconfirmed with the dynamic analysis described earlier; this assumption was retained. The three degrees of freedom of the model developed on this program correspond to:

1. Elastically restrained rotation of the supporting structure about the ball joint, ϕ_1 .
2. Recoil motion relative to the cradle, u .
3. Rotation of the tipping parts about the trunnion relative to the supporting structure, ϕ_2 .

The last motion is a consequence of the flexible elevating system. Although no auxiliary damping in the elevating system was included in the as-designed launcher, linear damping terms were placed in the equations for dampers in parallel with the elevating rods and also a damper between the lower box and the ground.

The frictional forces were added in the form of a constant frictional force due to the preload plus a term proportional to the recoil pressure. This corresponds to the type of seals employed. The equation of motion of the recoiling mass in the u direction also includes terms representing the constant-force, constant-stopping distance recoil system used in the launcher.

The final equations of motion consisted of three, second-order, nonlinear differential equations. In order to solve these without making any restrictive, simplifying assumptions, they were programmed for ARF's Univac 1105 in the Univac Scientific Exchange (USE) language. This program is described in Appendix D. Solution of the three, second-order differential equations was effected by using the Runge-Kutta numerical integration technique. Launcher parameters such as damping, stiffness, recoil rod shape, initial boost charge force, etc. were all included as input data; a sample output sheet is shown in Fig. D-2.

The purpose of the computer solutions of launcher motion can be grouped into three divisions by considering their purpose. The first group of solutions was aimed at matching the experimental launcher response. The second was to verify the existence of an optimum response and determine the optimum parameters. (This is described later when the optimum system is discussed.) The final group of computer runs was to show the results of various changes, e.g., low zone response, efforts of elevation changes, and delayed recoil.

1. Correlation of Theory with Experiment

The success of correlation between computed and experimental response curves varied and depended upon launcher parameters; in general the agreement was better when less negative carriage rotation occurred. The difficulty was due to the fact that the original model did not represent the nonlinear stiffness and damping displayed by the launcher supporting structure. This was subsequently corrected and closer correlation obtained, though not entirely satisfactory.

Figures 10 through 21 show typical information obtained from the computer program. Figure 18 shows a full burst response. The numerical input corresponded to either measured or calculated values of physical parameters of Prototype No. 3 Launcher. Certain parameters were varied, however, to obtain the correlation. All of the parameters not designated on the curves are listed on the computer program in Appendix D.

The two curves shown superimposed in Fig. 23 are the cradle response from Fig. 15 and the experimental cradle response from firing

R-38, listed in Table I-1. This experimental response was from a single shot with identical launcher conditions to burst B-8, Fig. J-11. The supporting structure stiffness measured for this condition was 5.5×10^7 in. -lb/radian. The numerical value of carriage stiffness used in the computer program was 4.5×10^7 in. -lb/radian and is significantly lower. The fact that this lower value was necessary to match the experimental response is probably due to the flexibility added to the system by the soil in the actual firing. (The structural stiffness measurements were made on concrete.) The measured elevating system stiffness for shot R-38 was approximately 4.5×10^5 compared to 1.4×10^4 used in the computer. The difference, though very large, is probably due to the unmeasured elevating system flexibilities discussed earlier. (Burst B-7 had an elevating system stiffness of 1.4×10^4 lb/in. but had a very similar response to burst B-8, even though the latter supposedly had a much stiffer elevating system. This further supports the conclusion that the high, measured stiffness value is not correct.)

A further comparison can be made between the computed carriage rotation (Fig. 13) and the trail strain from burst B-8, Fig. J-11. These curves agree in less detail, but their main characteristics are the same. The computed curve is generally smoother since it does not contain any higher mode response characteristics.

Figures 26 and 27 show experimentally obtained strain from the elevating screw and damper supports. These curves are tensile and compressive strain in circular, pinned rods and are directly proportional to the forces. They compare reasonably well with the spring force and damper force shown in Fig. 18 and 19 respectively.

Because of the similarity of the response of burst B-7 and B-8, we see that the computed response also agrees reasonably well with B-7. In this case the actual launcher had an elevating stiffness of 1.4×10^4 . Burst B-6, Fig. J-7, reveals a large negative dip after the initial positive displacement, which did not occur in burst B-7 or B-8. The only difference is that B-6 did not have a stiffened supporting structure. Reduction of only the supporting structure stiffness in the computer model produced the response in Fig. 24. The expected correspondence does not occur; the cradle response remains essentially positive. The negative supporting structure response, however, doubled compared to the previous theoretical response which indi-

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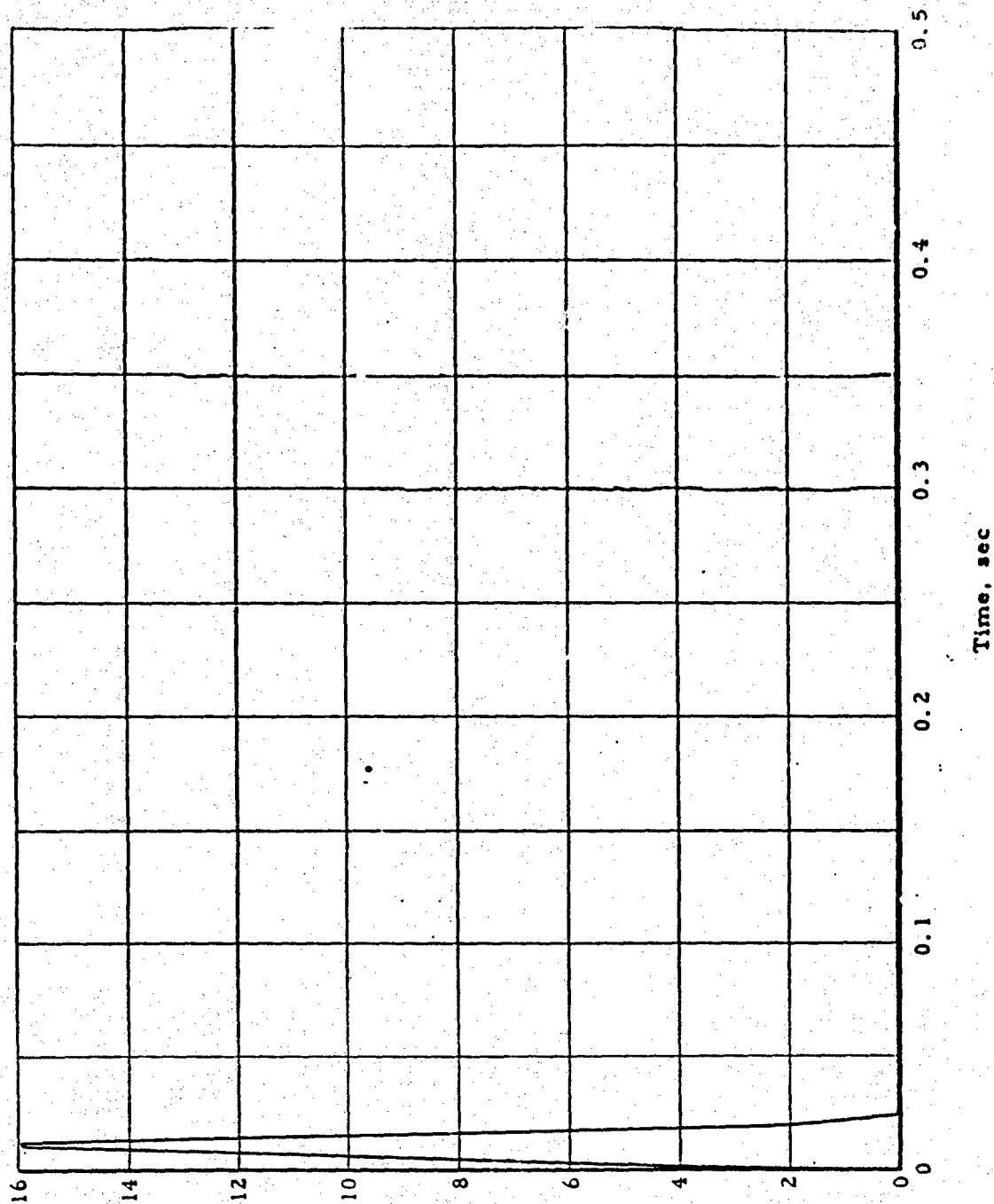


Fig. 10 TYPICAL COMPUTER SOLUTION --- POWDER GAS FORCE

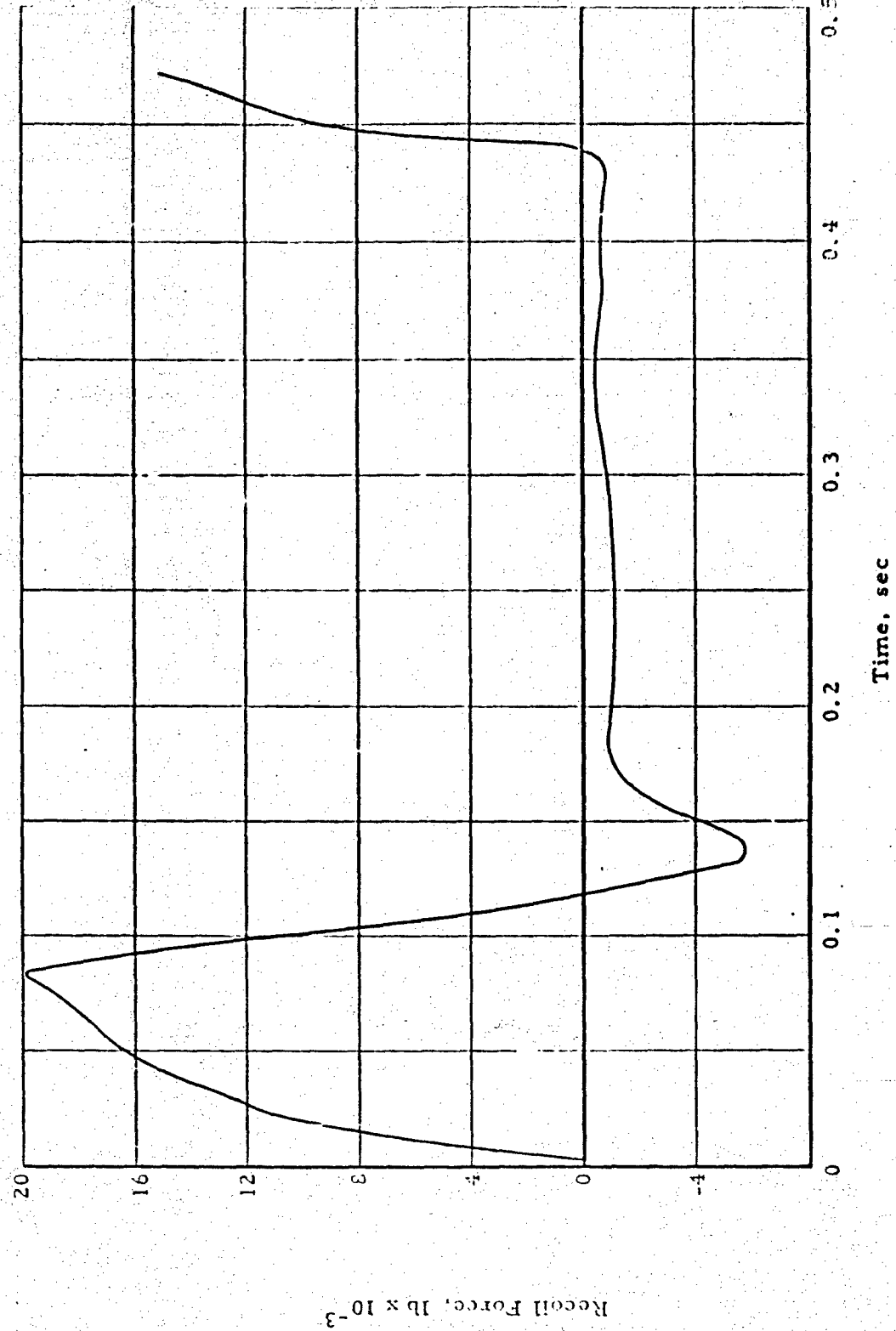


Fig. 11 TYPICAL COMPUTER SOLUTION -- RECOIL FORCE

Recoil Distance, in.

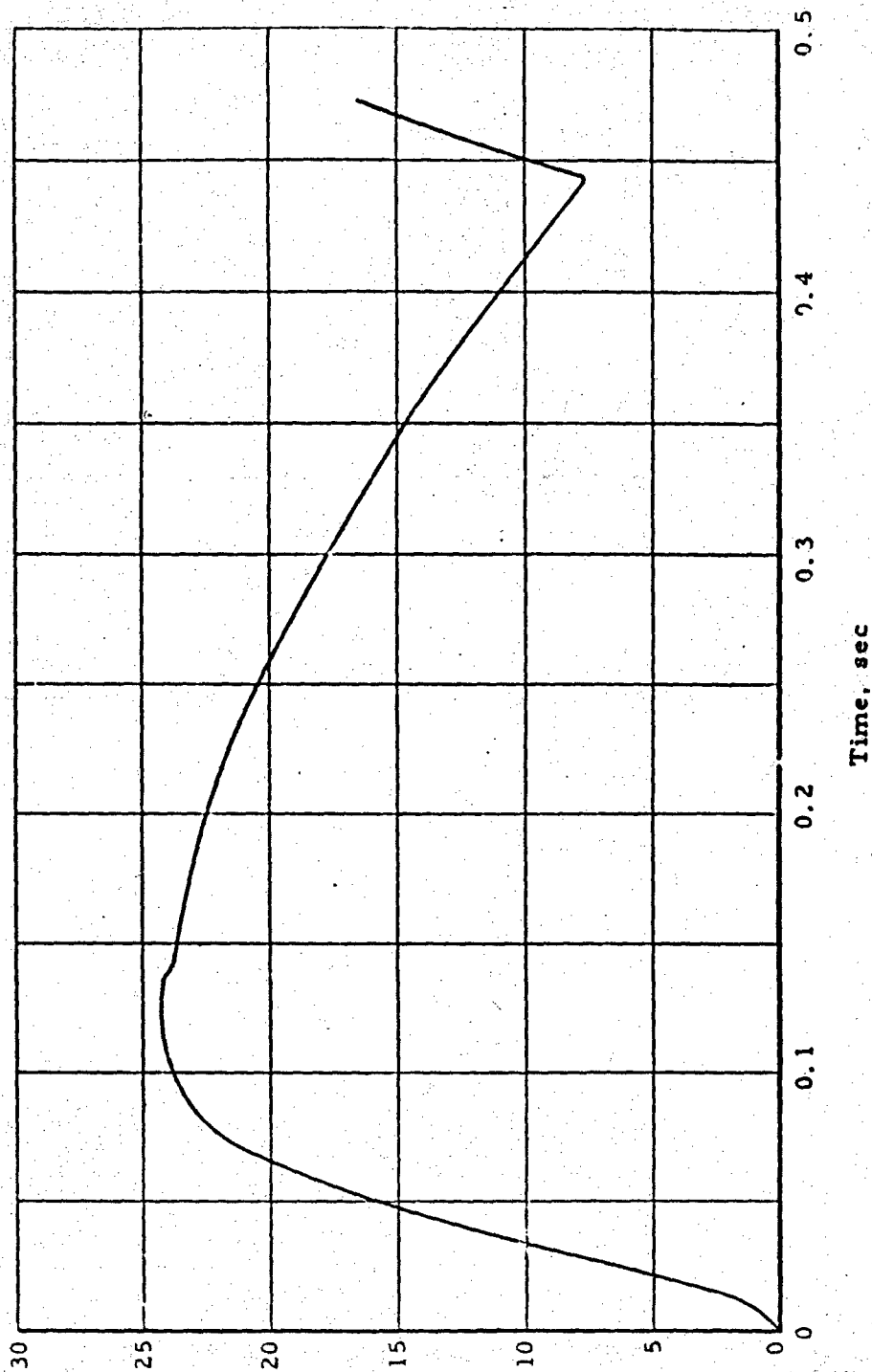


Fig. 12 TYPICAL COMPUTER SOLUTION -- RECOIL DISPLACEMENT

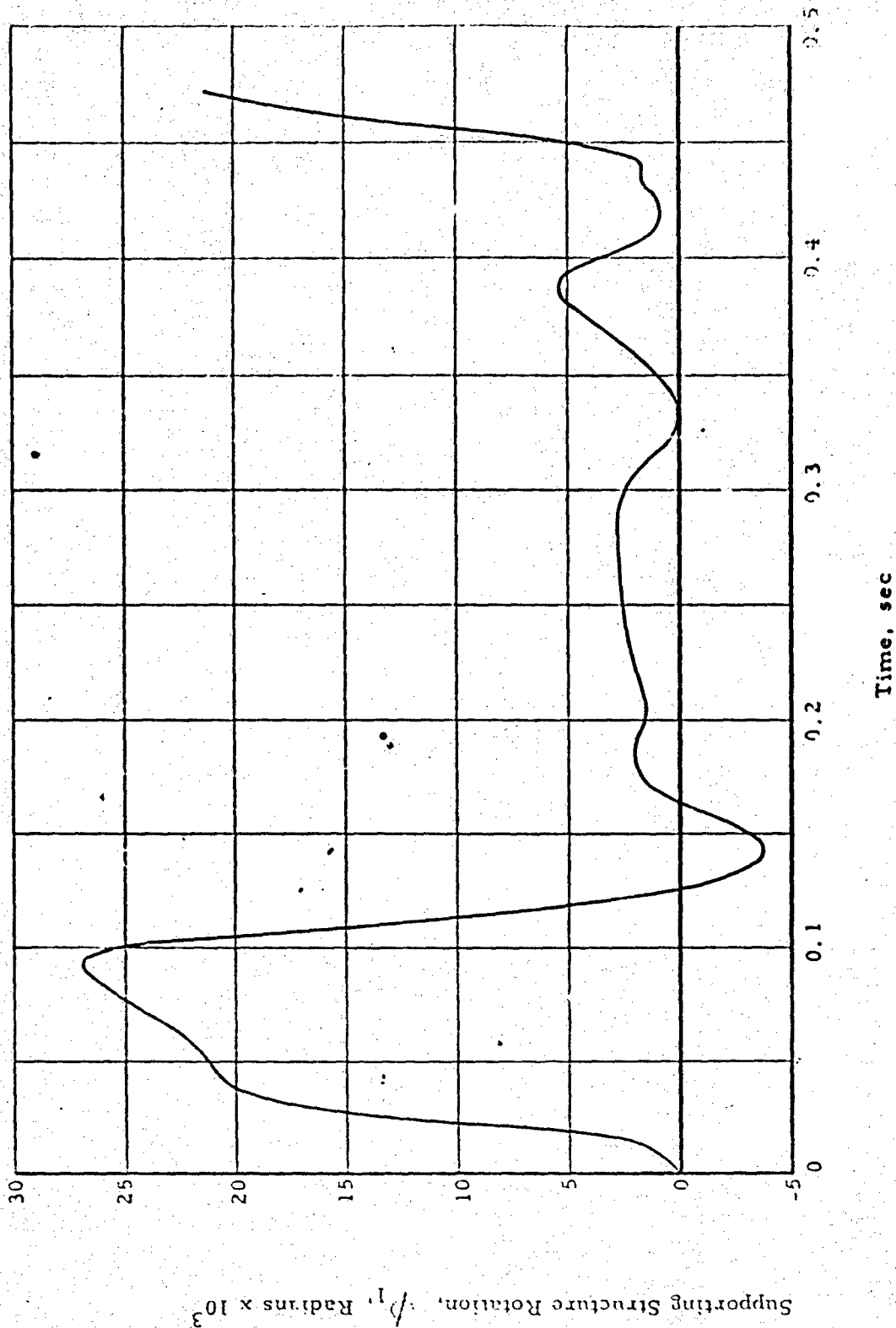


Fig. 13 TYPICAL COMPUTER SOLUTION -- CARRIAGE ROTATION

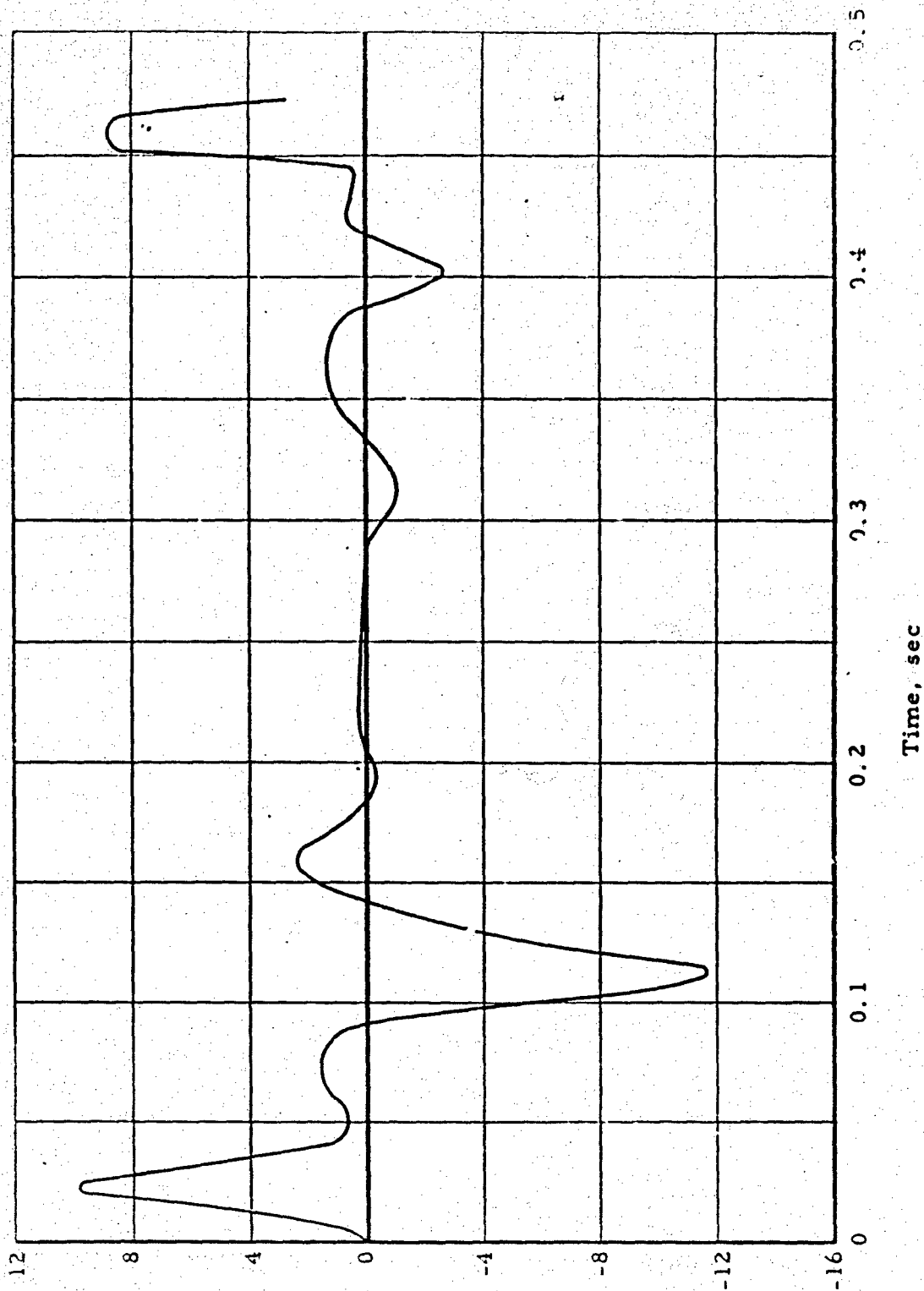


Fig. 14 TYPICAL COMPUTER SOLUTION -- CARRIAGE ROTATIONAL VELOCITY

Cradle Rotation, $(\phi_1 + \phi_2)$, Radians $\times 10^2$

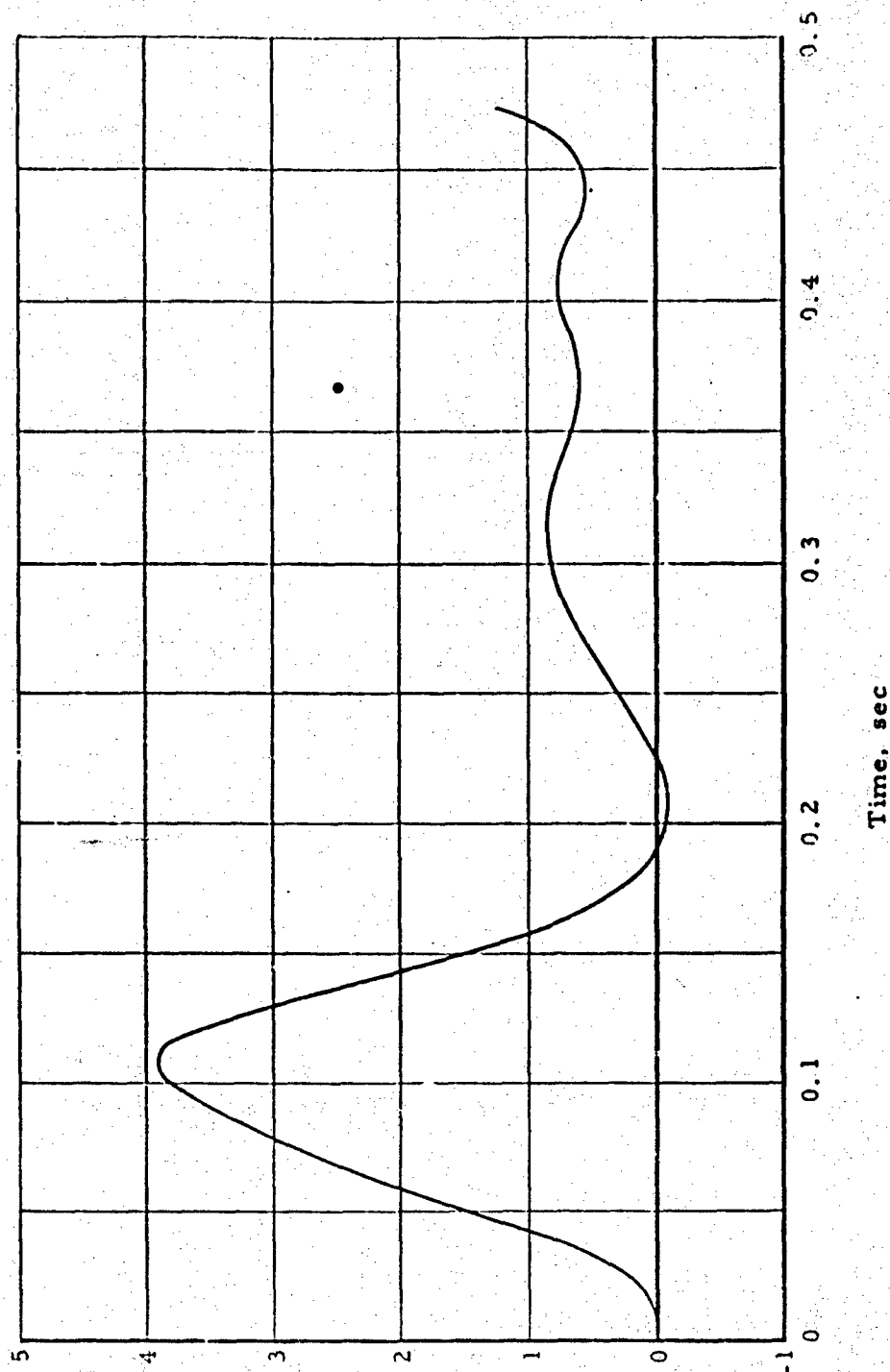


Fig. 15 TYPICAL COMPUTER SOLUTION -- CRADLE ROTATION

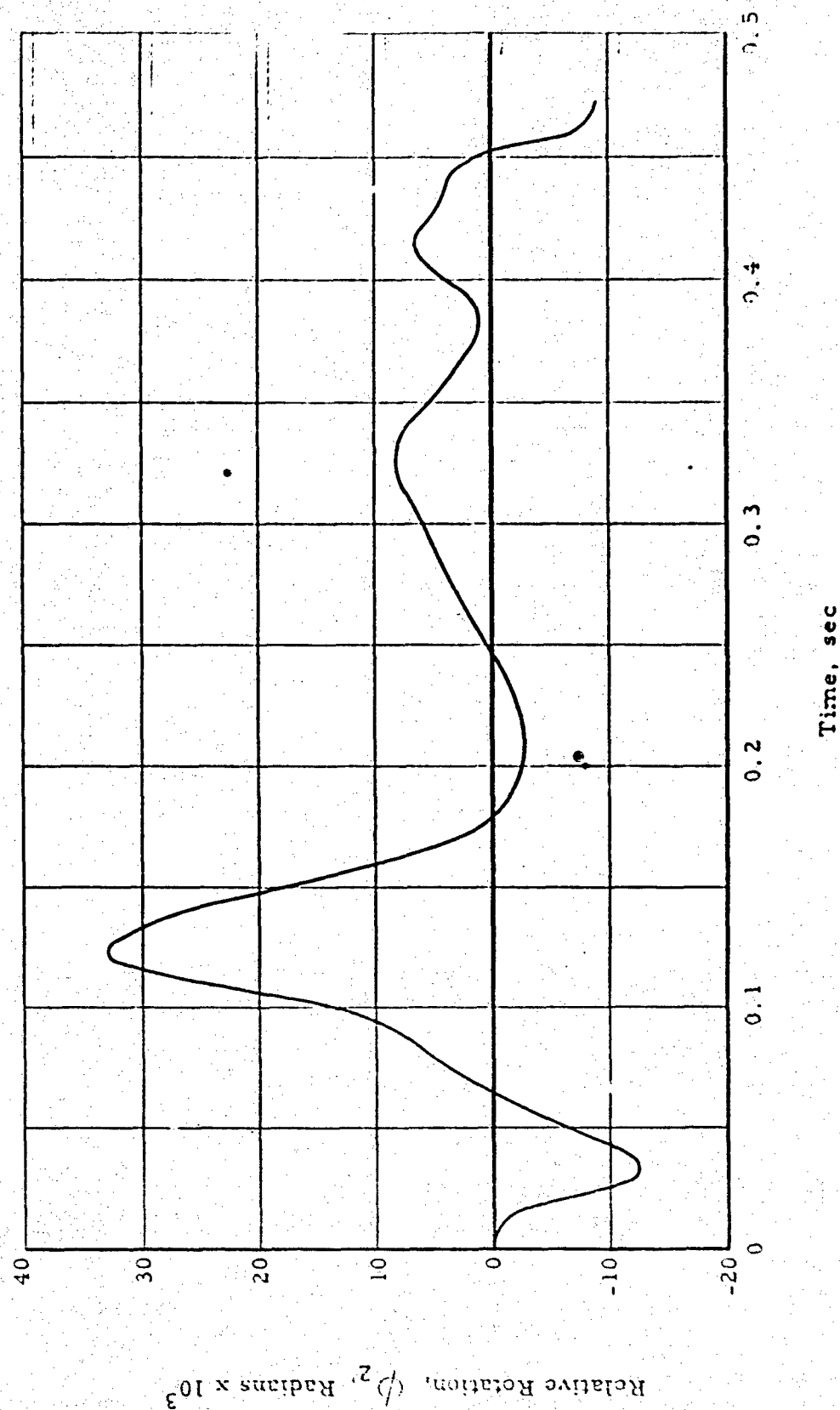


Fig. 16 TYPICAL COMPUTER SOLUTION -- CRADLE-CARRIAGE
RELATIVE ROTATION

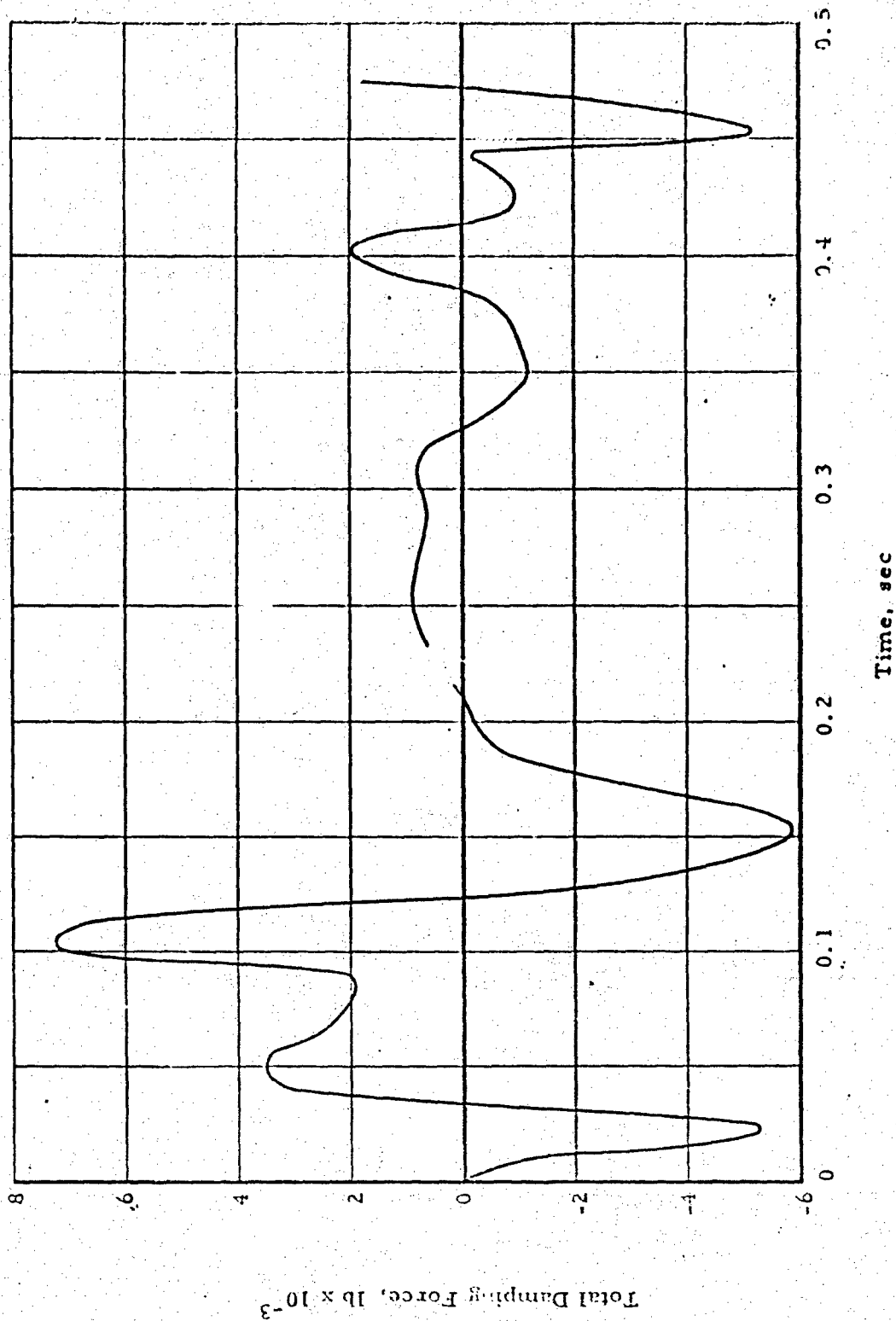


Fig. 17 TYPICAL COMPUTER SOLUTION -- CRADLE-CARRIAGE
RELATIVE ROTATIONAL VELOCITY

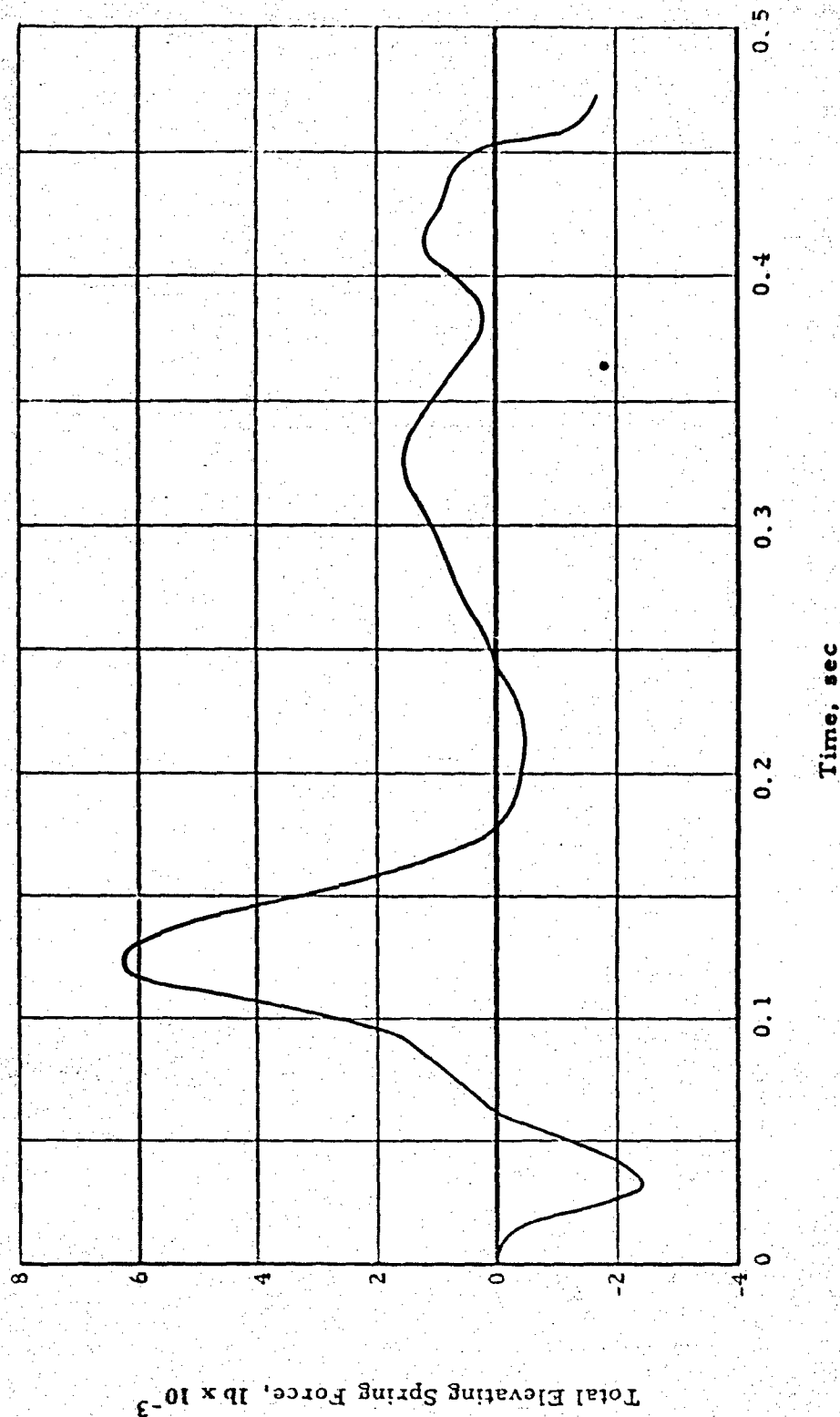


Fig. 18 TYPICAL COMPUTER SOLUTION -- ELEVATING SYSTEM SPRING FORCE

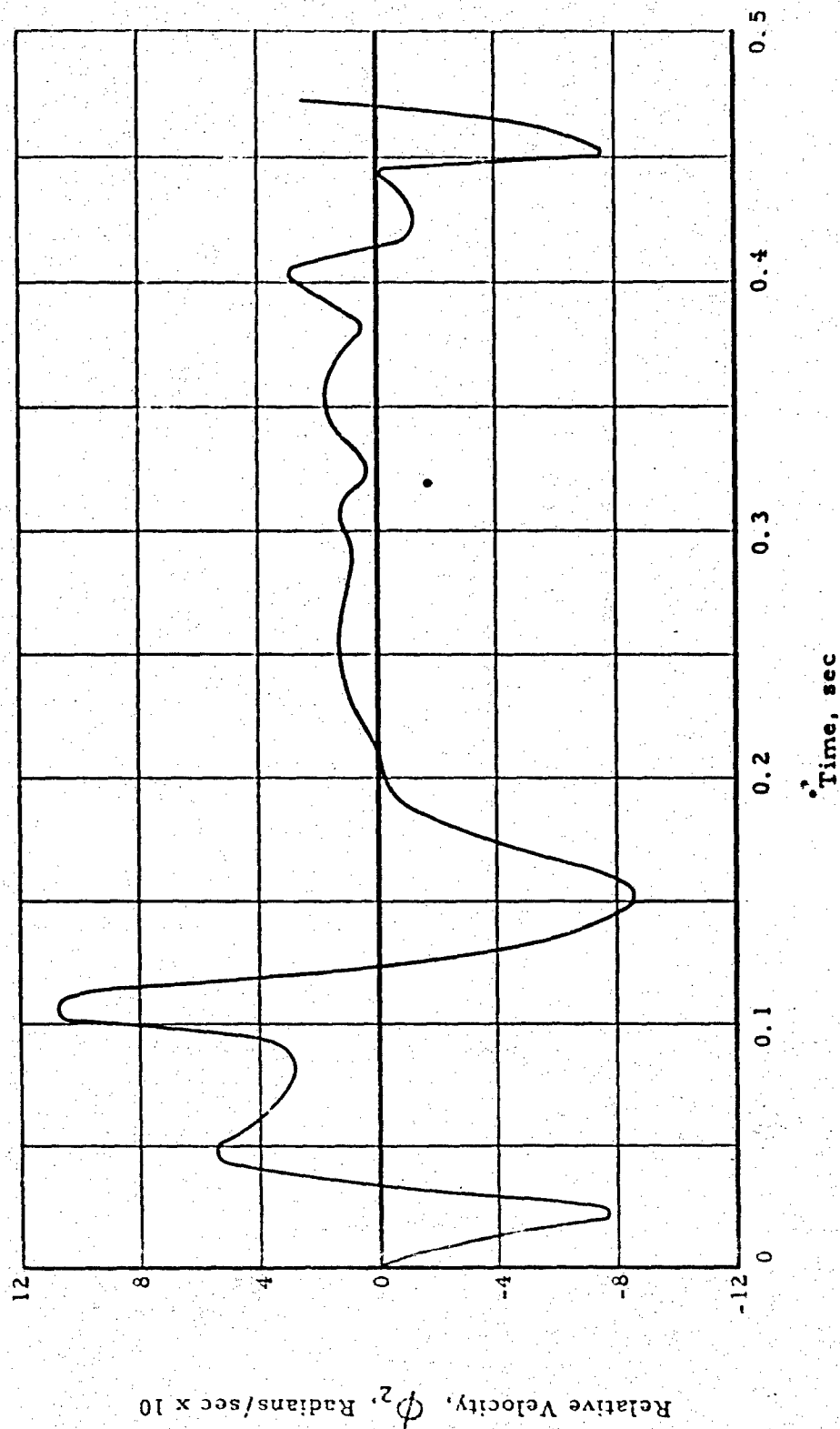


Fig. 19 TYPICAL COMPUTER SOLUTION -- ELEVATING SYSTEM DAMPER FORCE

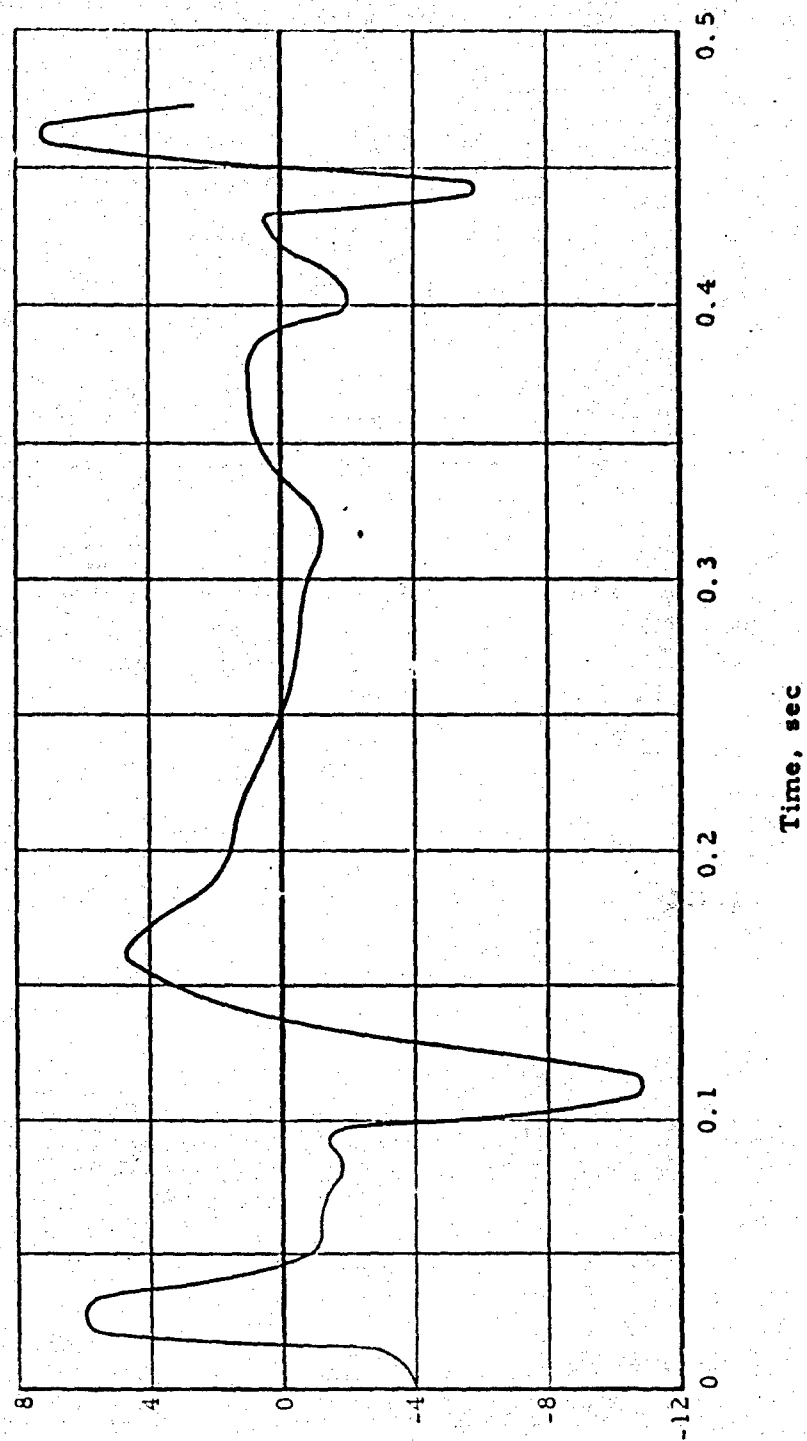


Fig. 20 TYPICAL COMPUTER SOLUTION -- RECOILING ASSEMBLY --
CRADLE MOMENT REACTION

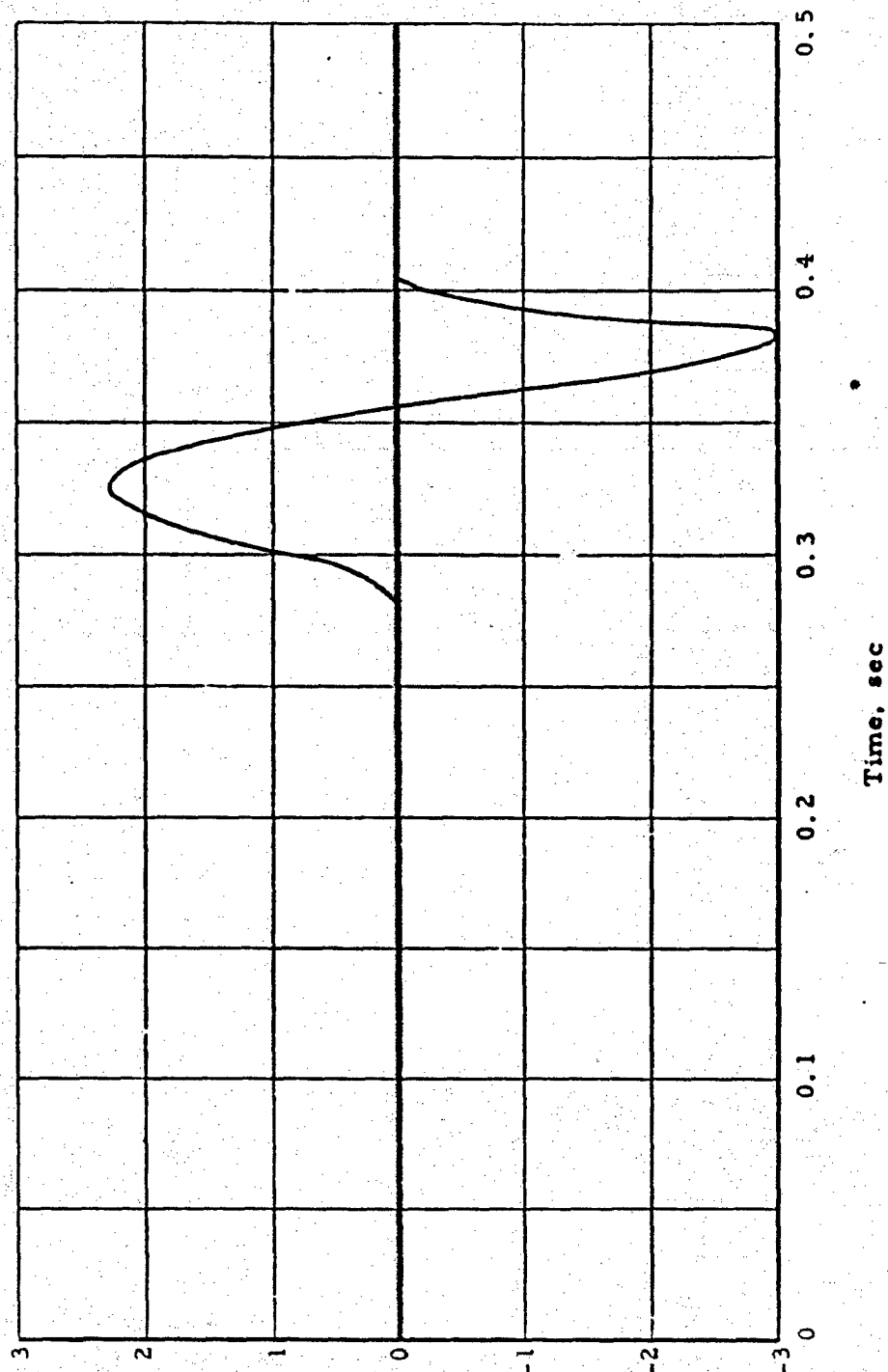


Fig. 21 TYPICAL COMPUTER SOLUTION -- AXIAL CAM FORCE

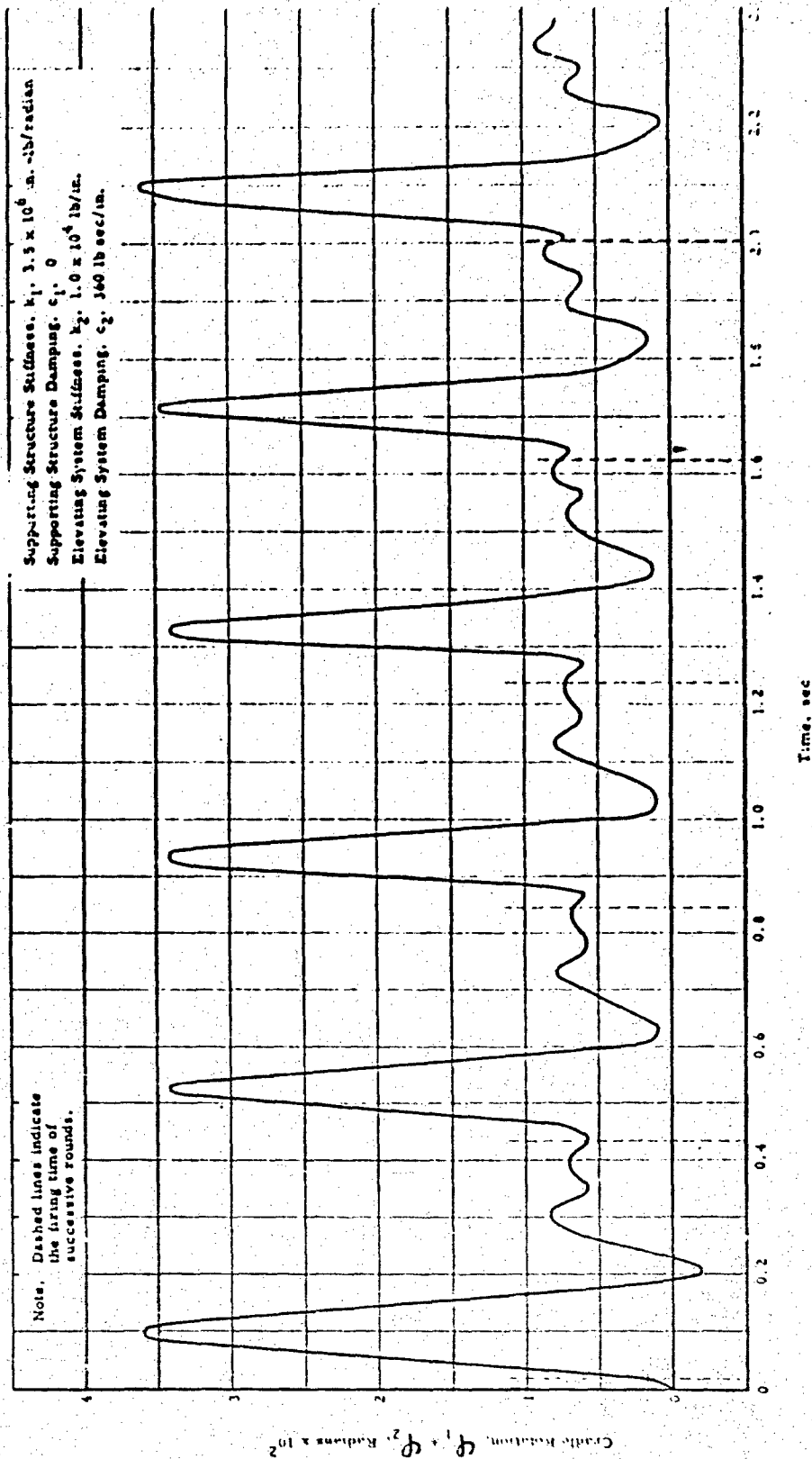


Fig. 22 TYPICAL COMPUTER SOLUTION -- CRADLE ROTATION, 6-RD BURST

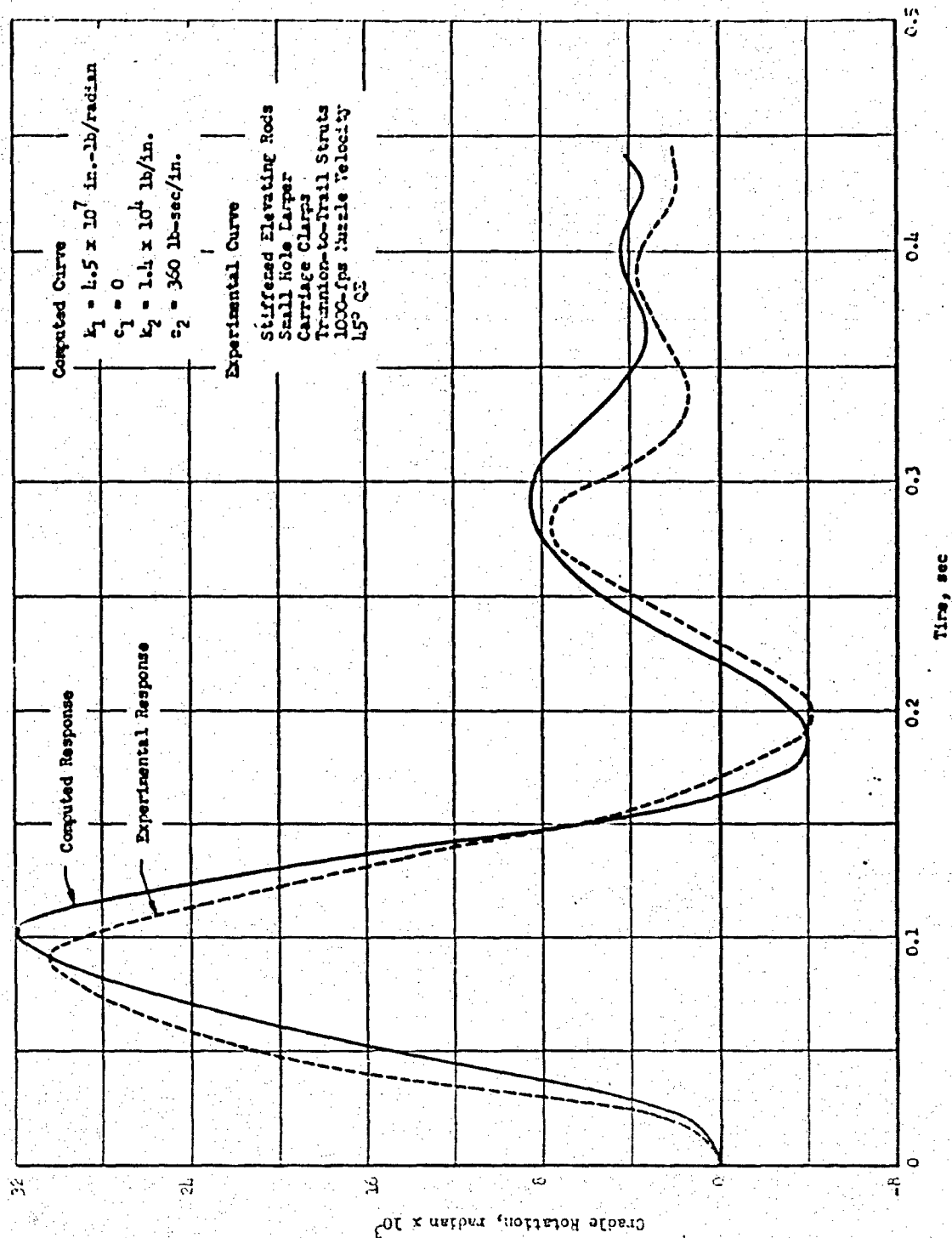


Fig. 23 COMPARISON OF COMPUTED AND EXPERIMENTAL RESPONSE CURVES

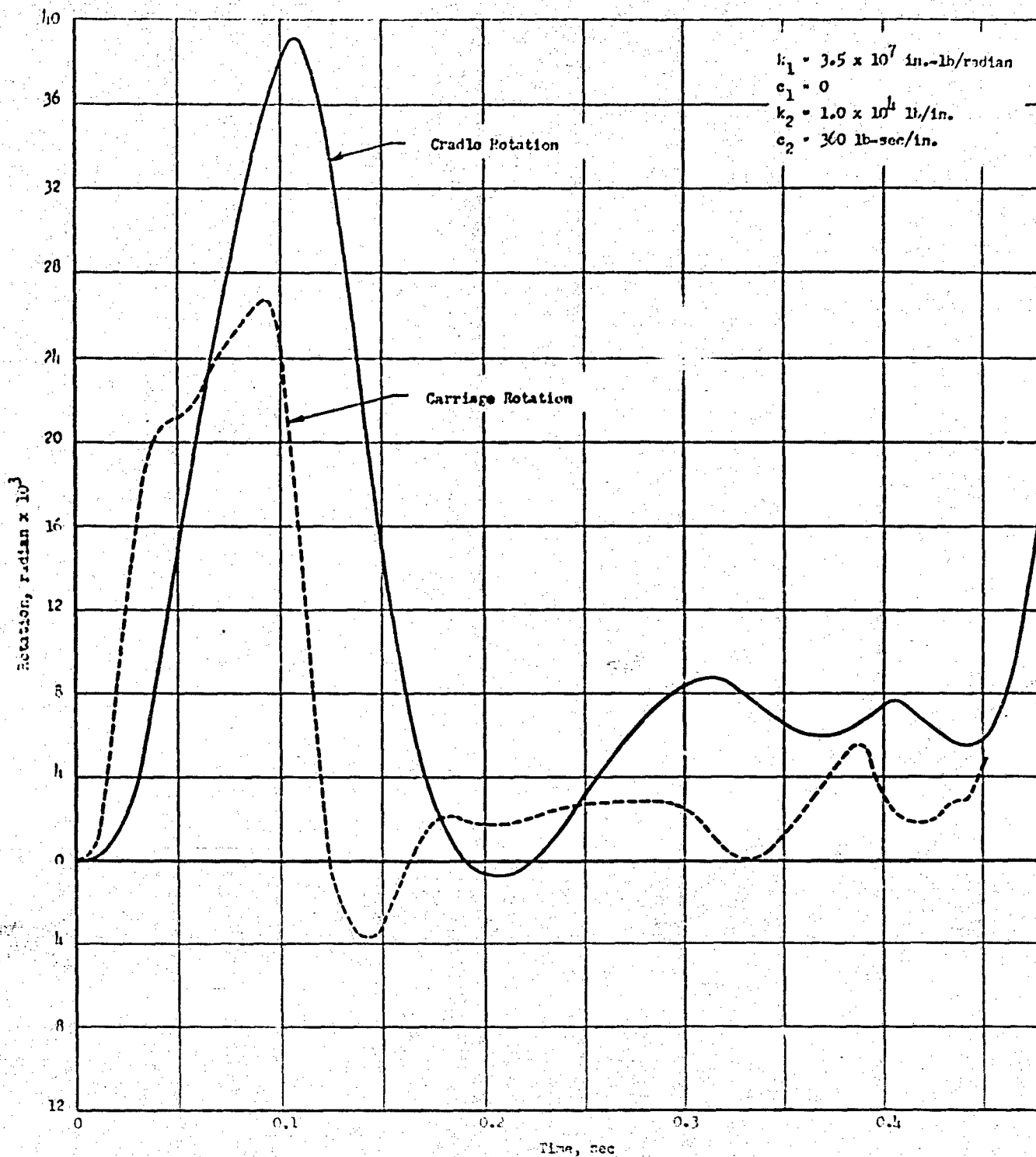


Fig. 24 THEORETICAL RESPONSE WITH REDUCED STRUCTURAL STIFFNESS

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cates that the nonlinear negative supporting structure characteristics are very significant here. To investigate the effects of a bilinear stiffness the computer program was modified. Figure 25 shows the cradle response using a bilinear supporting structure stiffness. Although the agreement is much better, it could probably be improved with slightly different values of stiffness and also the additional simulation of a bilinear damping coefficient. The effect of these nonlinearities has been demonstrated, however, even though perfect correlation was not obtained.

2. Effects of Elevation Changes

It was important to investigate the effects of elevation changes upon the response of the launcher. This was done easily, because the elevation was an input to the computer program. Figures 28 and 29 show the cradle response of the launcher with structural parameters identical to those in the previously computed curves. These responses are at 20° and 60° elevation, respectively. (The previous ones were at 45°). There are some definite changes in the details of the response, especially at the time immediately following the ignition of the rounds. At 20° elevation, the initial rise time (velocity at shot ejection) is significantly higher than at 45° , and at 45° the initial response is higher than that corresponding to 60° . There is a very simple reason for this. The geometry change brings about a change in the inertial moment of the tipping assembly. To demonstrate this, Fig. 30 illustrates that as the supporting structure deflects due to the application of the recoil force, the trunnion moves in a direction perpendicular to a line through the trunnion and ball joint. At 20° elevation, this motion causes the tipping assembly to react with an initial counterclockwise rotation which adds to the normal response. At 60° , however, the initial tendency of the tipping assembly is to rotate clockwise which opposes the normal counterclockwise rotation. The result here is a delay in the cradle response, which occurs at every round of the burst. The response at 45° elevation is similar to 60° , but the delay is smaller in magnitude. We now have a situation where increases in elevation bring about greater accuracy -- an inherent characteristic of this launcher. This characteristic has less importance when delayed recoil is used and is apparent when delayed recoil is discussed in Section IV-E.

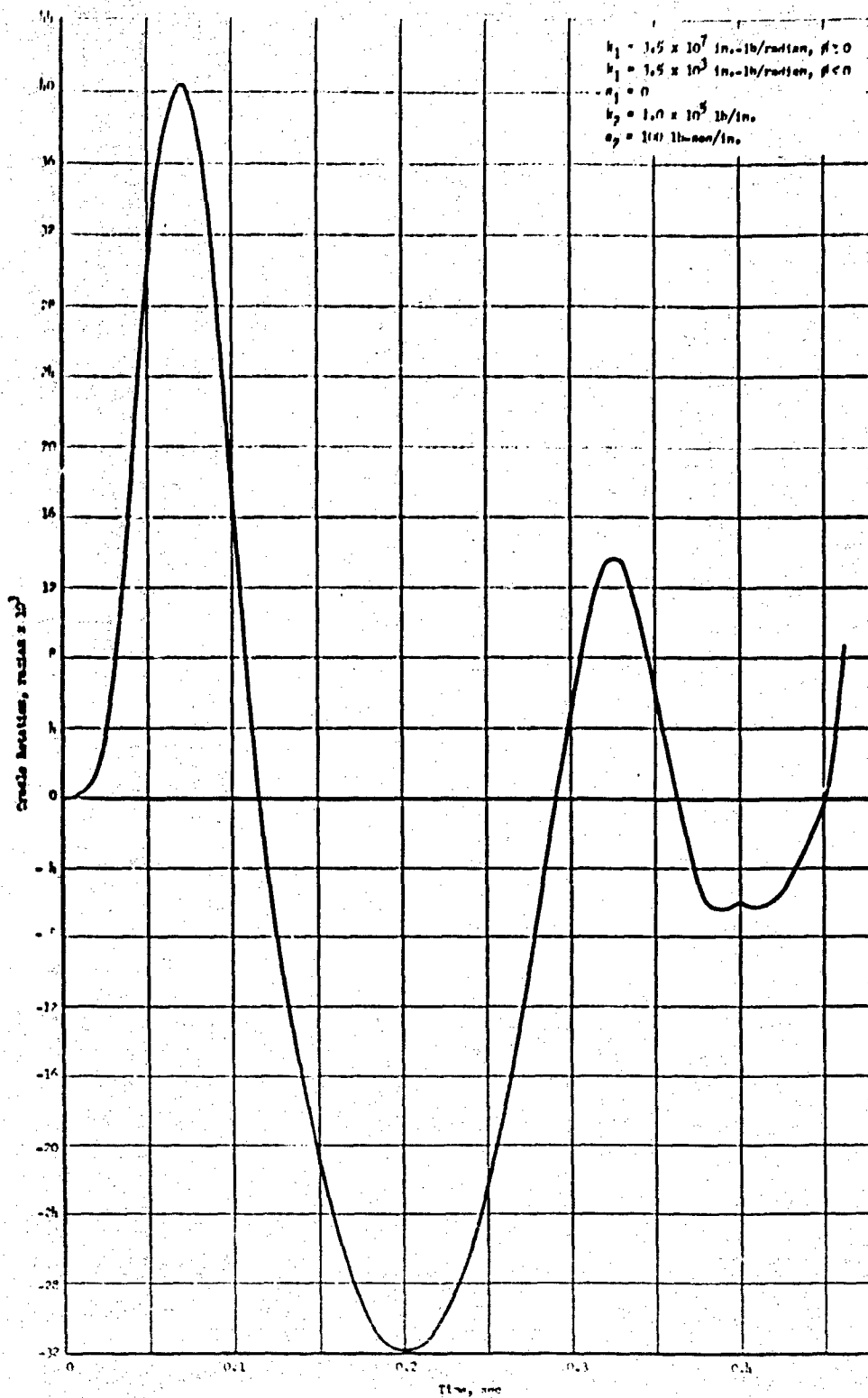


Fig. 25 THEORETICAL RESPONSE WITH A BILINEAR STIFFNESS

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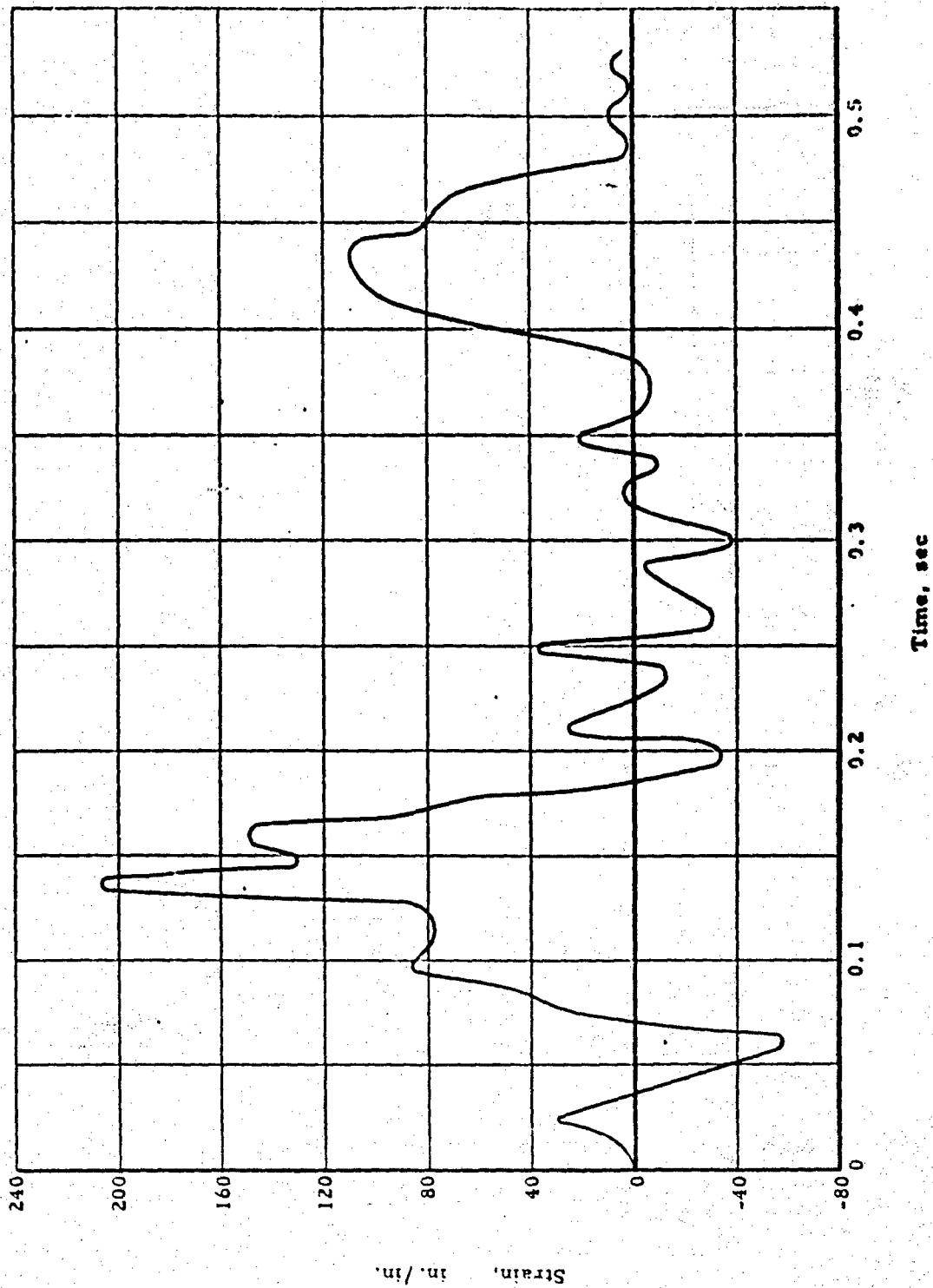


Fig. 26 EXPERIMENTAL ELEVATING ROD STRAIN

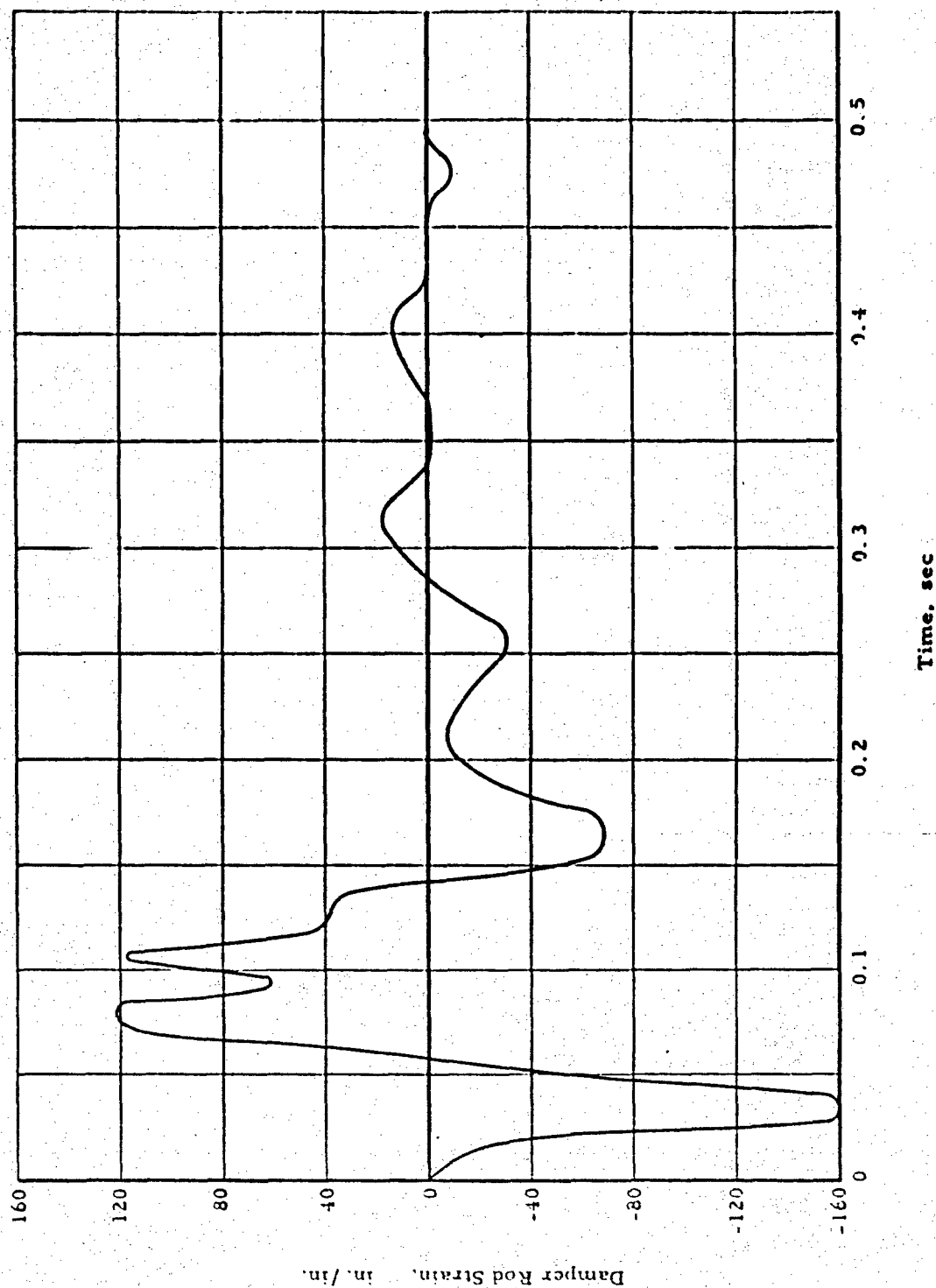


Fig. 27 EXPERIMENTAL ELEVATING SYSTEM DAMPER SUPPORT STRAIN

- Launcher Conditions:
1. Combined carriage-trail stiffness = 3.5×10^7 in.-lb./radian
 2. Elevating system stiffness (total) = 1.0×10^6 lb./in.
 3. Elevating system damping coefficient = 360 lb.-sec./in.
 4. Elevation = 20°
 5. Muzzle velocity = 1000 fps
- The remaining parameters are those of prototype No. 3.

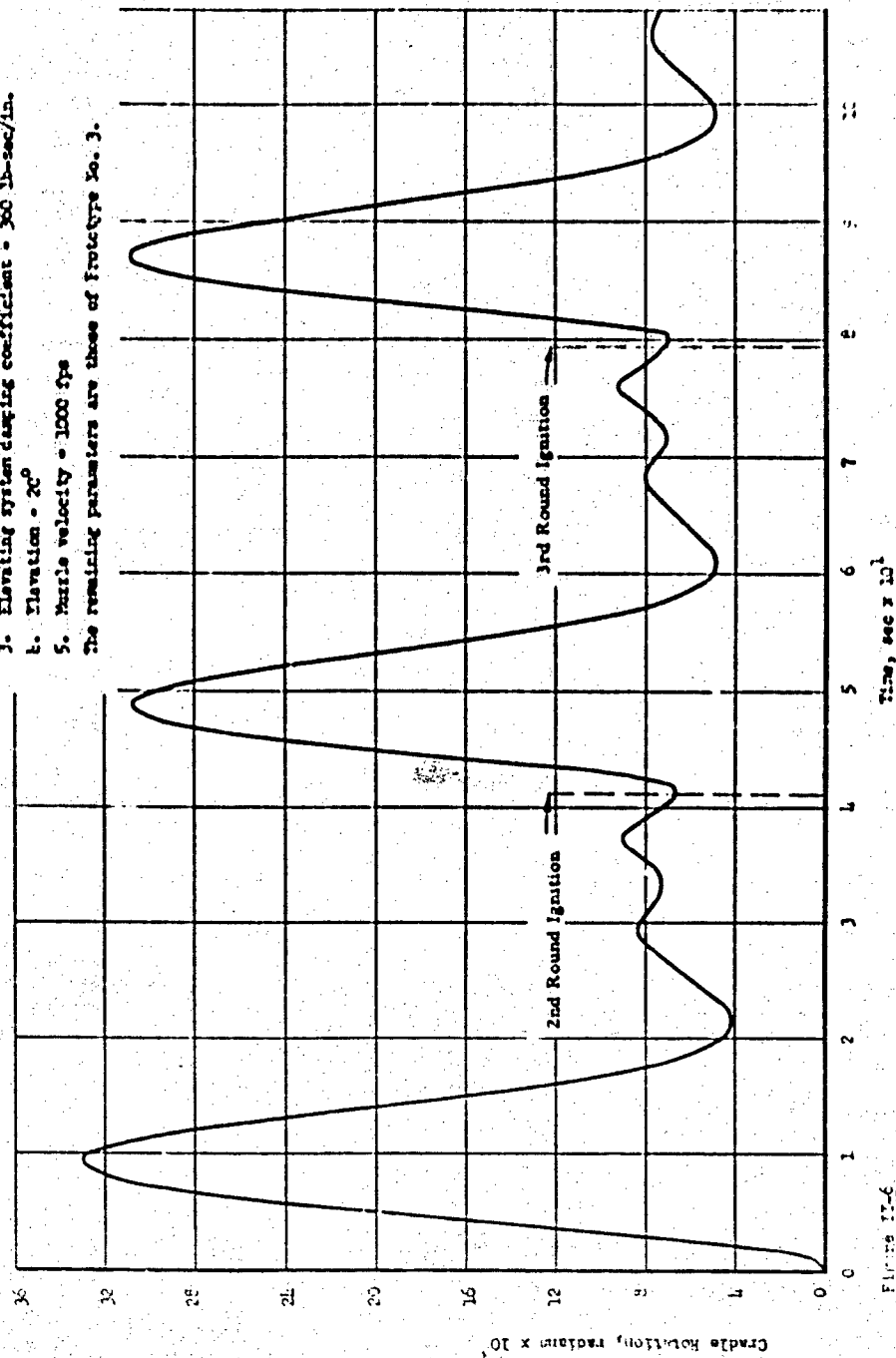


FIGURE 17-6

Fig. 28 CRADLE RESPONSE AT 20° ELEVATION

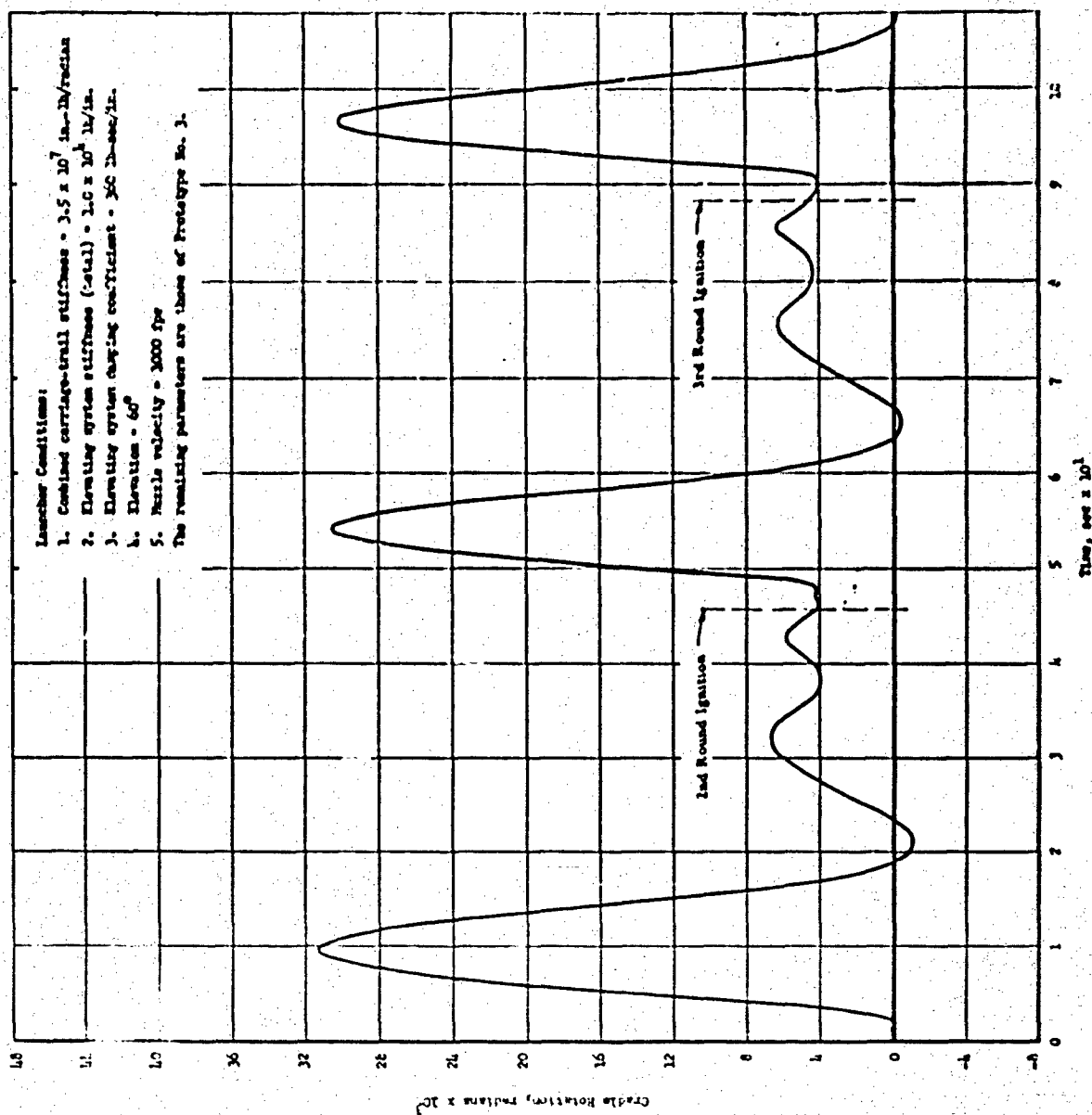


Fig. 29 CRADLE RESPONSE AT 60° ELEVATION

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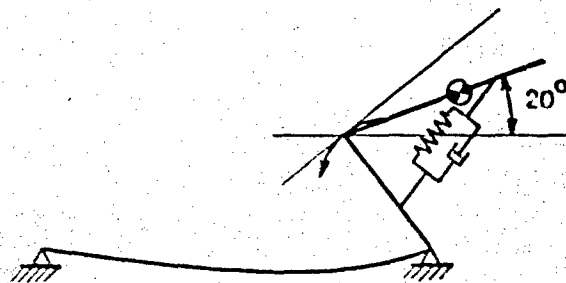
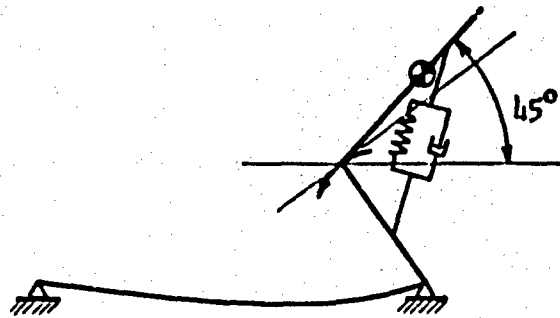
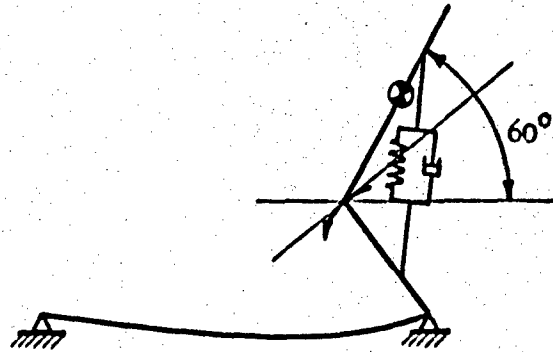


Fig. 30 GEOMETRY OF ELEVATION CHANGES

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3. Low Boost Firings

This launcher has the capability of firing rockets with various muzzle velocities (zones of ammunition) simply by varying the amount of gunpowder. The powder-gas force curve corresponding to a muzzle velocity of 380 fps was used in the computer program. Figure 31 shows the cradle response for 60° elevation at this low boost. (The high value of elevation was necessary to insure a large enough recoil distance to cause automatic indexing.) As would be expected, the rise time of the initial cradle response is much smaller than for the high boost rounds and was due to the slower rise time in the recoil force. The amplitude of the residual motion prior to successive shots is also less, but it is still significant. The variation in the residual motion, however, is very similar to the high zone bursts since the counterrecoil forces developed are a function of the recuperator force and the indexing forces. These are both relatively independent of the boost, because of the constant stopping distance recoil system. The maximum recoil distance for this burst was near 20 inches compared to 24 at higher zones. As a result of the slower rise times of the recoil force, we see that it is much more possible to have a negative cradle velocity at shot ejection. In addition, delayed recoil will have less of an effect in changing the response.

4. Importance of Coriolis Acceleration

Because of the high recoil velocity (400 in./sec) combined with the rotational velocity of the tipping assembly, a question arose concerning the importance of the Coriolis acceleration. This acceleration is proportional to the product of these two velocities. For this mathematical model, because of computer solution, there was no particular reason to neglect this term; in other situations, this might lead to a convenient simplification in the equations of motion. For this reason, Coriolis' effect upon the cradle response was investigated. Because the term is proportional to the product of the velocities $\dot{u}$ and $(\dot{\phi}_1 + \dot{\phi}_2)$, a set of parameters was selected where these velocities were high. The recoil velocity $\dot{u}$ is almost independent of the structural parameters, so the selection was reduced to finding parameters which increased $(\dot{\phi}_1 + \dot{\phi}_2)$. This is done by increasing the stiffness of the elevating system. Figures 32 and 33 show the cradle response with and without the Coriolis terms. The overall response shape has not changed

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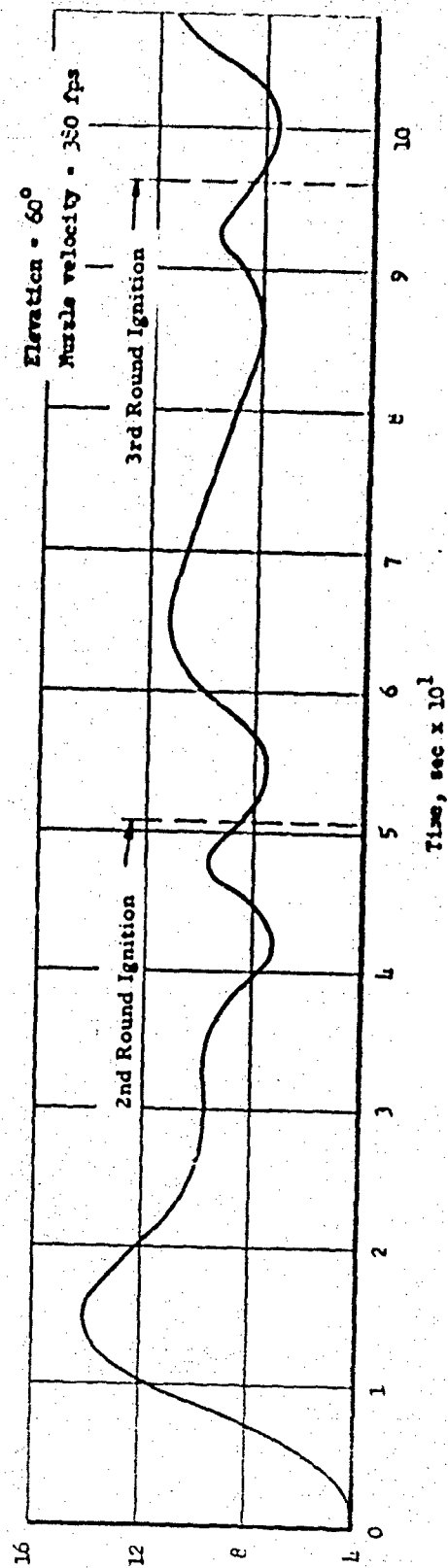


Fig. 31 CRADLE RESPONSE, LOW ZONE BOOST

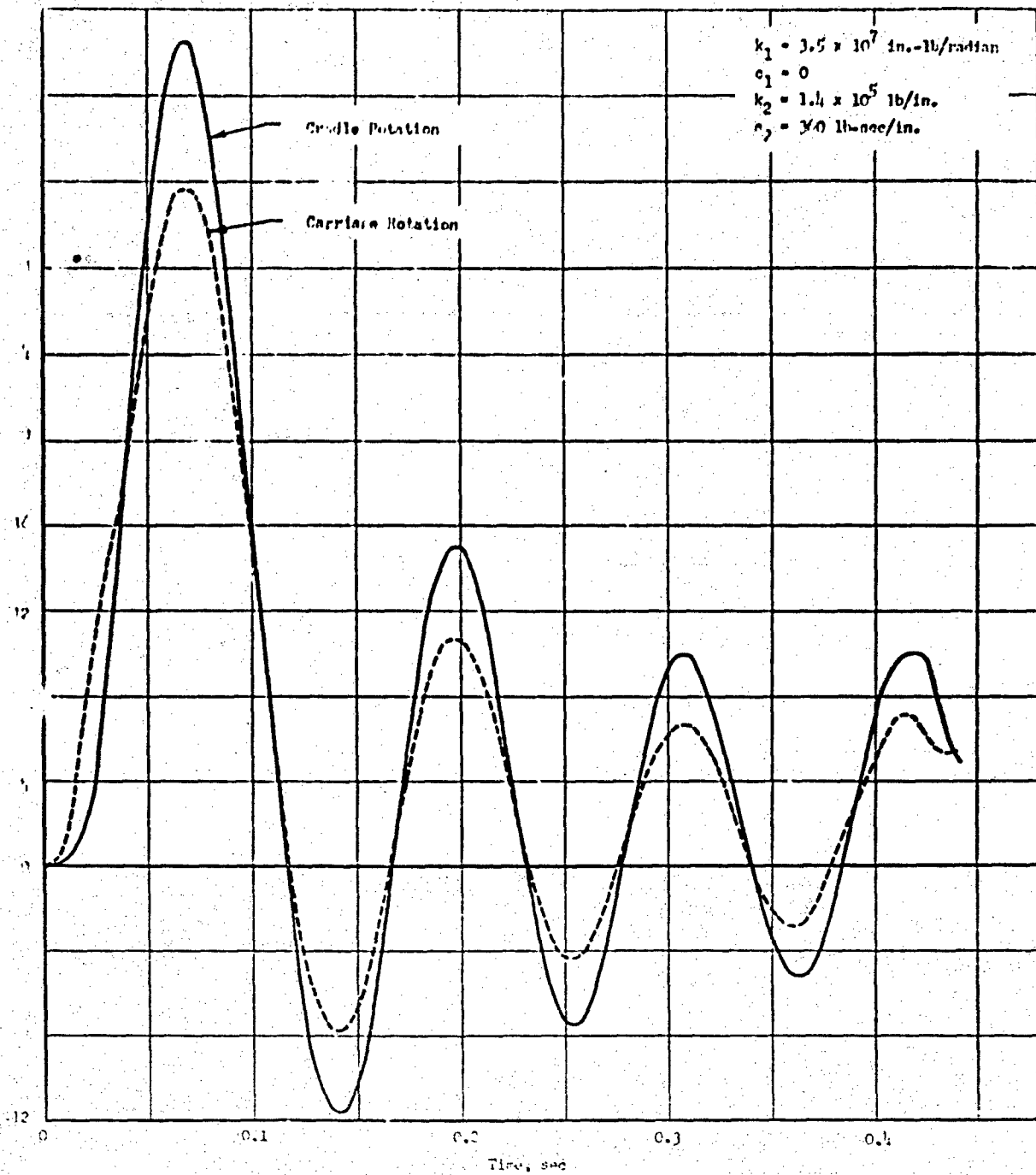


Fig. 32 CRADLE RESPONSE WITH A VERY STIFF ELEVATING SYSTEM

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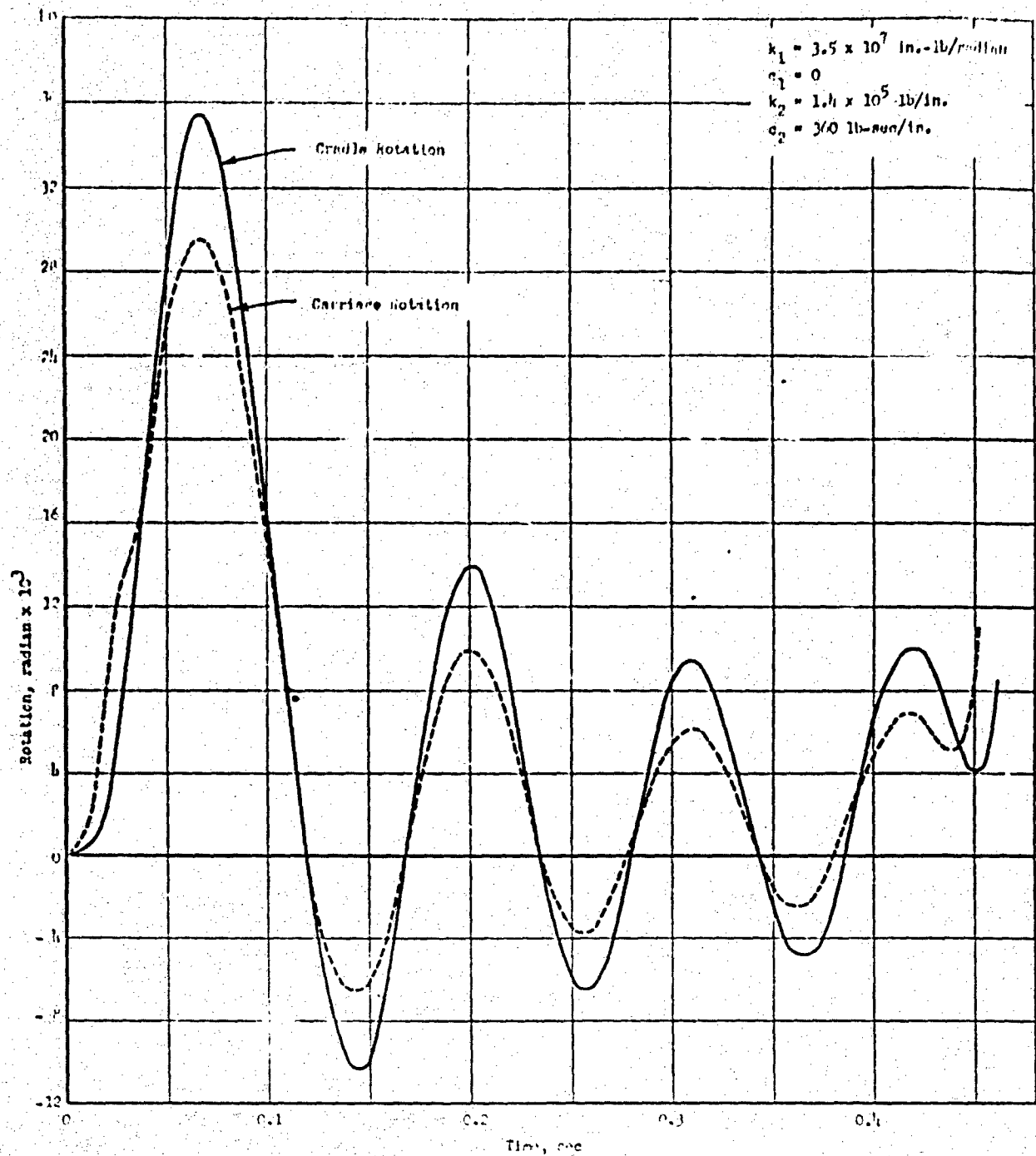


Fig. 33 CRADLE RESPONSE WITH A VERY STIFF ELEVATING SYSTEM NEGLECTING THE CORIOLIS ACCELERATION

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while the peak response has a difference of 7%. This indicates that the Coriolis acceleration is of second order importance with respect to the shape of the cradle response, though the change in magnitude may be significant. This serves to indicate that some of the simplified models used in the supplementary analyses may be a reasonable representation of the launcher for predicting large scale changes in the response.

5. Delayed Recoil

The recoil system on the XM70 Launcher makes use of a variable area orifice which was designed to stop the recoiling assembly in a constant distance with a constant force. The orifice area varies as a function of the recoil distance; this was effected by pulling variable diameter rods through a fixed area orifice plate. A design suggestion was made to modify the recoil system in such a way that the recoil force is zero (or of negligible magnitude) until the rocket has left the firing tube. In this way the carriage deflections will be zero (for the first shot of a burst), the velocity will be small, and the launcher accuracy will be near perfect. Certain problems arise with delayed recoil; the magnitude of the recoil force increases due to the shorter distance over which the recoil force acts, because of the constant stopping distance system. A question also arose about the effect of the delay on shots following the first.

The 3-degree-of-freedom mathematical model was used to study the effect of delayed recoil since the recoil rod shape is entered into the model as a tabular function of the recoil displacement. Inserting a rod shape into this mathematical model corresponds identically to inserting new recoil rods into an actual launcher.

The first rod shape tried was a standard rod with a decreased diameter (increase in orifice area) for the first 11-1/2 in. of recoil motion. The first rocket leaves the launcher at about 3 to 4 in. of recoil; subsequent shots are fired by an adjustable cam, 9 in. out of battery. Because of the residual counterrecoil velocity during all shot ignitions except the first, these rockets leave the launcher at a recoil distance near 11-1/2 in. Figure 34 shows the cradle rotation and the recoil force much larger than obtained for the standard rod. Figure 5 shows a peak force near 20,000 lb compared with the new force

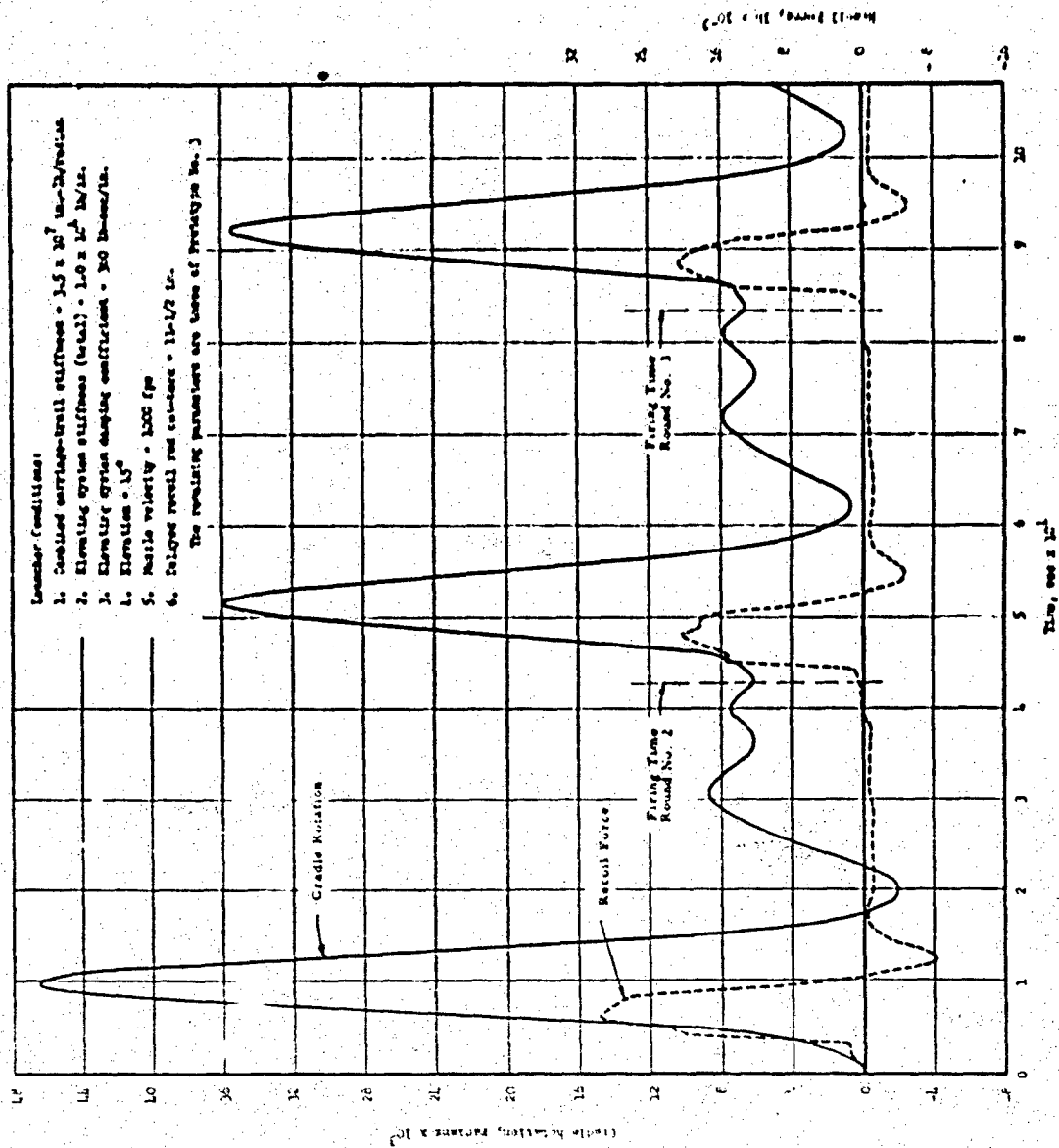


Fig. 34 CRADLE RESPONSE FOR DELAYED RECOIL ROD CUTBACK 11-1/2 IN.

near 30,000 lb. The later shots, however, are near the 20,000 lb peak reflecting the fact that of the 11-1/2 in. cutback, only about 2-1/2 in. of it affect the later shots, compared to the entire 11-1/2 for the first shot. Comparison of the response to that shown in Fig. 20 (optimum elevating system parameters) shows that delayed recoil cut the cradle rotation by about 50% at shot ejection (approximately 20 msec after $t = 0$) and lowered the velocity by approximately 30%. The displacement effect for later shots is almost negligible because it is fairly large to begin with. Close inspection of the curves shows that the velocity reduction is significant for the second shot. The third rocket, however, left the launcher when the cradle had a negative velocity (for the regular control rod). The use of the cutback rods changed the burst characteristics such that the third shot fired prior to a region of positive cradle velocity. We then realize the situation that delayed recoil does not aid the accuracy if the cradle would normally have a negative velocity at the time a round is ignited. The normal launcher response tends to lessen this negative velocity with a desirable result. This leads us to the conclusion that in order to gain the benefits of delayed recoil, the residual cradle motion should be as small as possible. In this situation, delayed recoil is definitely useful.

In order to decrease the high, first-round recoil force a rod was tried with two diameter cutbacks, the first from 0 to 3 in. and the second from 8 to 11-1/2 in. It was expected that the extra raised portion between 3 and 8 in. would help decrease the first force level. Figure 35 shows the result; the peak force is reduced from 30,000 to 26,000 lb. The peak cradle rotation was also reduced. The later rounds were unaffected because the section of the rod over which they operate was unchanged.

We saw earlier that firings at 60° elevation have less initial cradle rotation due to the geometry of the elevating system and tipping assembly mass center. A computed response for a 60° firing using a cutback recoil rods is shown in Fig. 36. In this situation, the cradle velocity is nowhere near that of the cradle for the ordinary launcher. In the same respect, the response for 20° elevation (with a cutback rod) is probably near the ordinary response.

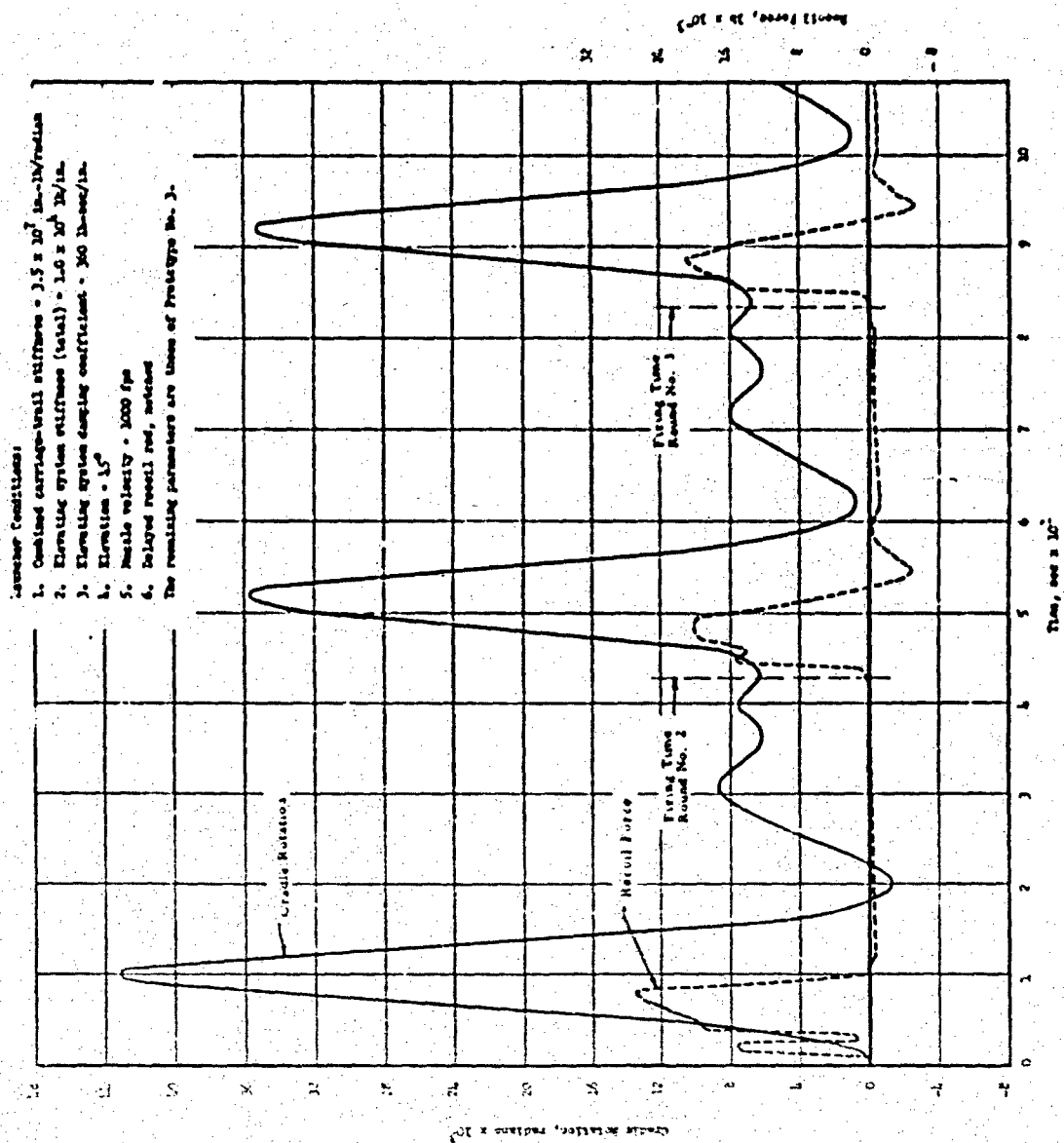


Fig. 35 CRADLE RESPONSE FOR DELAYED RECOIL, NOTCHED ROD

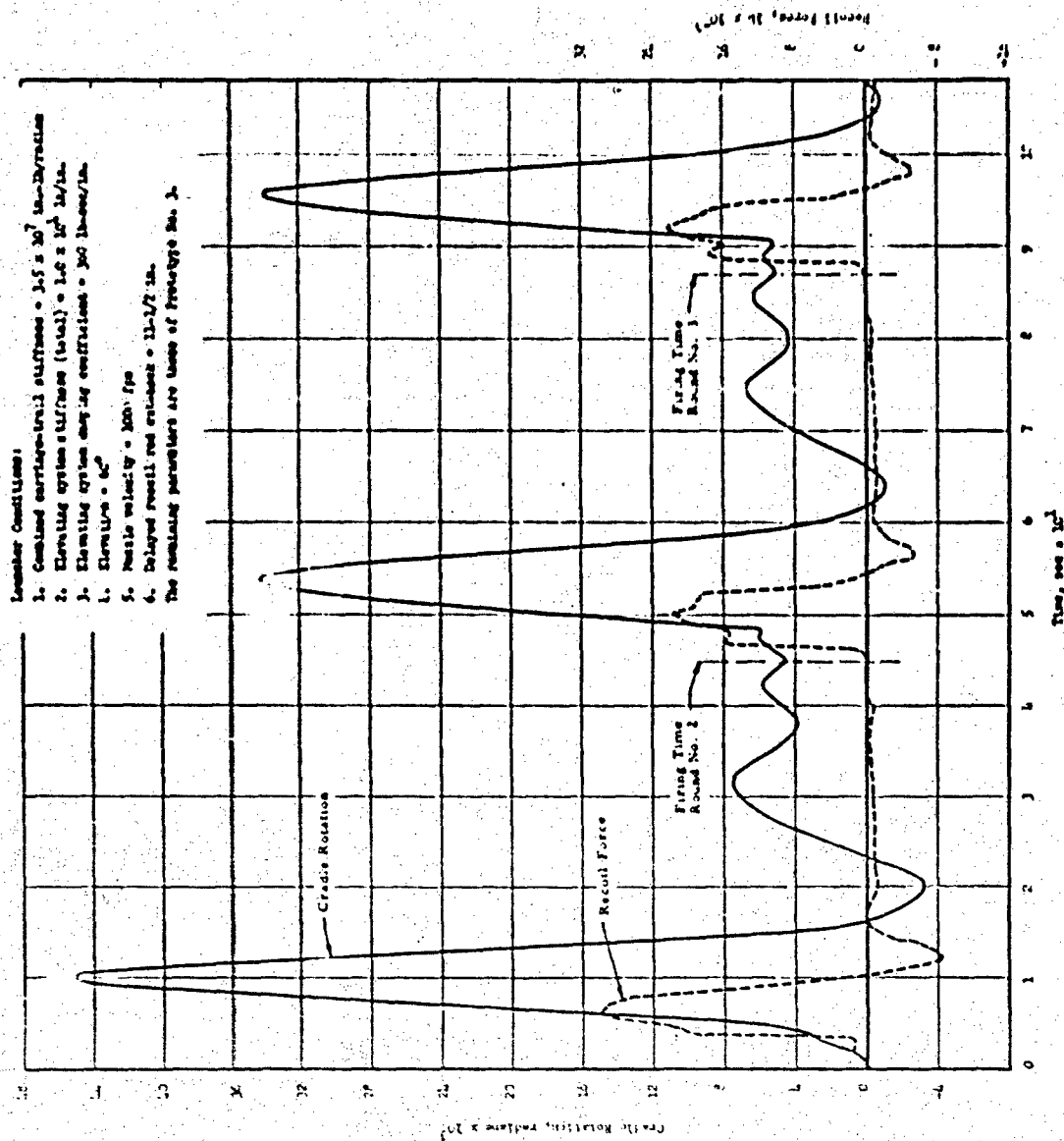


Fig. 36 CRADLE RESPONSE FOR DELAYED RECOIL ROD CUTBACK 11-1/2 IN.

B. Supplementary Analyses

1. Third-Order System

A third-order linear system was developed as a mathematical model and used to find the stiffness and damping which should be incorporated into the elevating system of the launcher. This model was obtained by simplification of the launcher representation shown in Fig. C-1. The simplifications were:

1. The inertia of the carriage and trails, I_0 , was assumed negligible compared to the moment of inertia of the tipping assembly -- a ratio of approximately 115:8000.
2. Nonlinear coupling terms between the motion of tipping assembly and supporting structure were neglected.
3. The recoiling motion was neglected.
4. The recoil force was treated as the primary disturbance and was approximated by an impulse.

These assumptions are considered valid because the resulting solution is not intended to represent the actual launcher motion, but rather to indicate the variation of the response behavior for a wide range of parameter variations. The linear analog of the resulting model is shown in Fig. E-1.

The responses of this third-order system were determined by using the response plots contained in "Dynamic Response Plots and Design Charts for Third-Order Linear Systems" ^{1/}. This report contains the solution of such systems in the form of dimensionless response plots for forty-eight different combinations of parameters. These plots were obtained from evaluation of Equation E-5, as a function of time, on a digital computer.

The design charts in the report ^{1/} show areas of three types of responses.

1. Parameter combinations which produce a normal damped response.
2. Parameter combinations which produce an unstable response.
3. Parameters which result in three real roots to Eq. E-3.

In order to establish the undesirability of the third type of response, a set of

^{1/} Meyfarth, P., Dynamic Response Plots and Design Charts for Third-Order Linear Systems, Research Memo No. R. M. 7401-3, Massachusetts Institute of Technology, 1958.

parameters which produce this was used in the computer solution for the 3-degree-of-freedom system. The resulting cradle response was continuously increasing (non-oscillating), as shown in Fig. E-3, for 150 lb-sec/in. damping and 100 lb/in. stiffness.

Despite the various assumptions, the response curves obtained were a fair representation of the actual launcher motion for a single shot. (Compare Figures E-2 and E-4.) The optimum response for this system was defined as one with both a low first peak displacement and a low ratio of second peak to first peak displacement. Three response plots which satisfy this definition are shown in Figure E-2. The corresponding optimum elevating system parameters are a stiffness of 1.0×10^4 to 1.4×10^4 lb/in. and damping of 360 to 540 lb-sec/in.

2. An Approximate Dynamic Analysis of the Minimum Weapon Weight Required for a Specified Accuracy

A relatively simple mathematical model of the XM70 and a simple dynamic performance requirement for accuracy was formulated in order to evaluate several basic design parameters including the minimum total weight of the weapon. Mathematically, the procedure was one of finding a second order system which had a specified response decay. The details of the analysis are contained in Appendix B of this report.

For the purposes of this analysis, the complete weapon was represented by a linear, forced, damped, single-degree-of-freedom oscillator. The displacement of the oscillator represented angular rotation of the firing tube in a vertical plane. This representation was shown to be useful^{1/} when the stiffness is taken to be the total stiffness of the weapon structure and the mass is a computed dynamic equivalent. Values of the damping coefficient were chosen to be consistent with experimental data of the weapon's response to firing loads. The force applied to the oscillator to simulate firing was a rectangular pulse whose amplitude and duration were chosen to represent the recoil force of the actual weapon.

As a criterion for adequate dynamic performance, it was assumed

^{1/} See ARF Project K130, Bimonthly Report No. 11, Appendix A.

that the amplitude of firing tube oscillations (in a vertical plane) must be less than some small value, which is considered to produce negligible error at the target, prior to firing of each round of a burst.

Under the assumptions of the analysis, a closed form solution can be obtained to the equation of motion of the weapon. The solutions consists of the displacement of the firing tube as a function of time and the stiffness, damping and mass of the weapon but what is ultimately desired from the analysis is the time required for adequate damping of the response as a function of the weapon weight.

The weight of the weapon was explicitly introduced into the solution by writing expressions for the stiffness, damping coefficient, and equivalent mass of the weapon as functions of the weight of the major subassemblies of the XM70. These expressions were obtained by assuming linear relationships and using the XM70 Prototype Model No. 1 weights for reference. Using the solution containing these expressions, it was possible to obtain a time-displacement curve, and hence the time required for adequate damping, for a particular total weapon weight. Several solutions must be obtained in order to obtain a plot of required damping time versus total weight from which the minimum weight can be obtained. The analysis procedure is facilitated by using the envelope of the response curves rather than the curves themselves.

This analysis formed the basis for the computation of the optimum recoiling assembly weight for the XM70 which is reported upon in ARF Project K130 Bimonthly Report No. 8 Appendix B. In addition, this analysis was used to estimate the minimum weight of a weapon having a small recoiling assembly weight (500 lb) and a delayed recoil force. The minimum weight of such a weapon was found to be only 1340 lb., but the resulting ground reaction was too large and the calculated carriage weight was inadequate from a strength standpoint. These calculations led to the development of an auxiliary strength criterion for the carriage and hence to the procedure referred to above for calculating optimum recoiling assembly weight.

3. Linear, 2-Degree-of-Freedom Model

A classical, 2-degree-of-freedom system was another system studied

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to obtain information about the best location for auxiliary damping of the cradle motion. Because only qualitative information was desired, the lack of representation of system nonlinearities is not considered important. Fig. A-1 shows the system which was studied where m_2 represented the tipping assembly, and m_1 , the supporting structure. The three damper locations which were considered correspond to the following:

1. c_1 ; damping between the supporting structure and the ground (the ground was assumed to be capable of furnishing a tensile load).
2. c_2 ; damping between the supporting structure and tipping assembly.
3. c_3 ; damping between the tipping assembly and the ground.

The equations of motion were solved by digital computation where the input disturbance, F_1 , was a tabular representation of an actual recoil force. All three damper locations are physically attainable, though the second is certainly the most convenient location. Equivalent damping between the three locations was determined by estimating the amount of forcible damping for the same weight contribution to the launcher from the respective dampers.

As expected, damping directly between the cradle and ground was most effective, of the three locations. However, damping between the tipping assembly and supporting structure was relatively effective, especially compared to damping between the supporting structure and ground. Combinations of the latter two, c_1 and c_2 , was better, but on the estimated weight basis, the improvement was not significant. Since c_3 -damping is the most difficult to attain, the most promising damper location was concluded to be relative to the supporting structure and tipping assembly.

C. Three-Dimensional, n-Degree-of-Freedom Model

This mathematical model (called Mathematical Model VI in progress reports) was developed with a view toward the incorporation of several of the modes of motion which were not included in earlier models. It contains all of the freedom of the previous models and, in addition, permits the investigation of the following types of behavior.

1. Ground Flexibility

Results of some of the experiments showed that an elastic and plastic deformation of the earth underneath the ball joint and the trail paths can

sizably affect the burst accuracy of the weapon. The new model, consequently, will allow the introduction of an arbitrary force-displacement relationship at the support points.

2. Sidewise Rotation at the Ball Joint

Incorporation of this freedom necessitated that a full three-dimensional representation be utilized. This representation should permit simulation of a portion of the lateral dispersive action associated with the weapon.

3. Left-Right Sequencing

This is also a three-dimensional effect; it arises because the inertial unbalance in the recoiling assembly cannot be eliminated. Left-right sequencing is discussed in detail in another portion of this report. Its incorporation into this model was somewhat simplified through the interpretation of the recoiling assembly framework flexibility in the form of a single torsional spring.

4. Flexible Barrel

The barrel flexibility was considered to be very important to the overall burst dispersion of this weapon. In particular, it would appear that phasing between the barrel vibration and the time of firing of the various rounds is quite significant. This barrel vibration is even more important because it is evidently coupled with the rotation of the weapon about the ball joint to Coriolis force terms and gyroscopic moments which arise due to the moving projectile.

5. Moving Projectile

The flexible barrel is excited by the traveling load induced through a projectile as well as through the gross forces which arise through firing. This operation motivated the inclusion in the model of a representation for the coupled motion of a traveling mass and the barrel proper. This representation is so organized that an experimentally determined time-displacement relationship can be introduced to describe the axial projectile behavior. It is thus possible to determine the instantaneous muzzle position orientation and velocity at the time of projectile exit. It is felt that this

information should be of aid in the development of more complete accuracy criteria for rocket launchers of this type.

This 3-dimensional mathematical model was not completed until near the close of the present project. The equations were programmed for the UNIVAC 1105 digital computer at Armour Research Foundation, but the program was not run on the machine because it was felt that the time and efforts which remained on the project could be more efficiently applied to other facets of launcher behavior. The equations for this model are presented in Appendix F of the present report. These equations are quite complex; their full solution should prove to be time consuming. Conversely, specific items of interest to future rocket launcher programs can certainly be obtained through specification of these very general equations.

D. Accuracy Criterion

The main direction of effort on this program has been to study the response of the XM70 Launcher structure in the vertical plane due to burst firing loads. The intention was to find that combination of launcher parameters, e. g., weight, stiffness, moment of inertia, mass distribution, recoil force, etc., which would optimize or at least improve the response. In order to evaluate the effects of parameter changes, a simple accuracy criterion was used. This criterion was: the most accurate burst has the smallest residual angular cradle displacement and velocity prior to each succeeding round. This was not expressed mathematically, but was applied simply by examining graphs of both computer and experimental responses. A mathematical accuracy criterion is presented here that permits a more accurate basis for judgment. This criterion is a relationship between the launcher response and the points of projectile impact. General application of this criterion is limited only by the fact that it employs the motion characteristics of a short-burning-time, ballistic rocket. Because the development of this criterion came late in this program, its application was not completed. It is presented here, however, because of its generality.

The significant sources of rocket dispersion (of a rocket with a relatively short burning time) are:

1. Initial cross spin, i. e., angular yaw (pitch) velocity when the rocket leaves the launcher,

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2. Launch mal-point, i. e., initial yaw (pitch) when the rocket leaves the launcher,
3. Dynamic unbalance of rocket,
4. Cross winds during burning,
5. Total rocket impulse variations,
6. Rocket spin at burnout,
7. Thrust malalignment.

Only the first three of these are influenced by the launcher. The particular launcher motion that can contribute significantly to dispersion by influencing these three quantities is the cradle motion in combination with the firing tube motion relative to the cradle. The cradle angular displacement and angular velocity in the vertical plane are considered here. At present, we are limited to consider only these because all completed mathematical models assume a rigid firing tube and plane motion only. Because of these two limitations, dynamic unbalance must also be neglected. In addition, during the transition period when the rocket is partially out of the firing tube, the rocket is assumed to have the orientation of the tip of the tube.

The purpose of this criterion is to permit a comparison of launcher responses to burst firings (or a group of single shot firings with the same launcher) and a subsequent selection of the best response; with this in mind, the following characteristics can be imposed upon the criterion:

1. Only relative changes in dispersion or accuracy between burst responses need be indicated by the criterion.
2. In the case of computed responses, an indication of accuracy improvement by the mathematical output of the criterion must correspond to an actual accuracy improvement. (This can be checked experimentally or with a complete, ballistics mathematical model). This correspondence will not be required to be linear, however.
3. The criterion will yield a numerical answer for each burst and will be zero for a perfect burst. (A perfect burst is attained when all rounds impact at the same point.)
4. The criterion must allow for the different contributions to dispersion from unit deviations in initial yaw and initial cross spin.
5. Each round of a burst is to be considered separately, and its sequential position will be neglected.

These are the criterion characteristics that are inferred prior to the development of the criterion simply by considering its purpose and application. The implications of each of these are: Item 1 states that the criterion output for a single burst will not indicate anything about the accuracy of that burst, when compared to the criterion output of another burst; however, it will indicate which of the two is better, i. e., more accurate. Item 2 states that the indication of which of two bursts was more accurate must correspond to the indication of relative accuracy of an actual launcher firing of two bursts under the same conditions: the criterion must agree with reality. In order that the criterion contain the characteristics implied by Item 3, the deviations of both displacement and velocity of the cradle will be measured with respect to their mean. Item 4 states that the criterion must account for the fact that a unit deviation in angular cradle displacement does not necessarily produce the same dispersion as a unit deviation in angular cradle velocity; this is accomplished by using what are known as "unit effects", coefficients determined by the magnitude of dispersion due to a unit deviation in each quantity. The last item, No. 5 merely states that no priority is given to any of the six rounds of a burst.

The combination of the above characteristics narrows the choice of mathematical expressions describing this accuracy criterion to a point where an expression can be deduced:

$$L = U_y \sum_{k=1}^6 \left| \Delta_y \right|_k + U_s \sum_{k=1}^6 \left| \Delta_s \right|_k$$

where

- L = the numerical output from the above expression indicating relative burst accuracy,
- U_y = unit effect for mal-point or initial yaw,
- U_s = unit effect for initial cross spin,
- Δ_y = deviation from the mean in initial yaw,
- Δ_s = deviation from the mean in initial cross spin,
- k = index.

The two deviations are written as:

$$|\Delta_{y|k}| = |\phi_k - \phi_m| \quad |\Delta_s|_k = \dot{\phi}_k - \dot{\phi}_m$$

where ϕ_k and ϕ_m are the initial yaw and cross spin, respectively. In this criterion, the smaller the value of L , the better the burst. Other criteria may be deduced that conform to the given conditions, but for small deviations they will reduce to the above expression.

This criterion was to be incorporated directly into the computer program of the 3-degree-of-freedom mathematical model. A 6-rd burst computation would then automatically give the quantity, L . The unit effects are obtained from solutions of the equations of motion of the rocket; the unit effects for the XM54 rocket were tabulated in previous progress reports of this project. ^{1/}

^{1/} For more information of the unit effects of rocket dispersion, see: Bi-Monthly Report No. 18, ARF Project D124, January 22, 1960.

V. CONCLUSIONS

All the results derived from this study are based upon considerations of cradle motion near the time of shot ejection of each round of a burst. There was no intention of neglecting the effects of firing tube flexibilities, but this effort represents a first step to increase the weapon accuracy. There was no reason to suspect that flexible firing tube motions may be simply additive to cradle motion because of the coupling which exists. However, the correlation obtained between theory and experiment, despite neglecting this coupling theoretically, indicates that the effects upon gross cradle rotation are small. This points to the conclusion that the accuracy criterion based upon cradle motion was valid, i. e., a reduction in the residual cradle motion prior to the successive rounds of a burst will improve accuracy.

The residual cradle oscillations of the as-designed launcher (Prototype No. 3) were seen experimentally to be small. The only disturbing factor was that the as designed structure was subjected to elastic rebound, or trail hop; this permitted serious, gross horizontal dispersion to occur. This horizontal dispersion was eliminated by simple ties restraining the trails from moving in a horizontal plane. An alternative was shown to exist where trail hop was eliminated -- this was shown to be accomplished by stiffening the supporting structure. With this stiffened structure, the ties at the trails are unnecessary.

When trail hop does not occur, an optimum combination of elevating system structural parameters was found. This was based upon a study which showed that the best location (based upon effectiveness per weight) at which auxiliary damping should be added to the launcher is between the tipping assembly and the supporting structure. The optimum elevating system had the following structural parameters:

$$k_2 \text{ (total stiffness) } = 1.4 \times 10^4 \text{ lb/in.}$$

$$c_2 \text{ (Damping coefficient) } = 400 \text{ lb-sec/in.}$$

This combination caused a minimum of residual cradle oscillation between shots of a burst, and is independent of the stiffness of the supporting

structure. The possibility of an unstable response during a burst was demonstrated for very low elevating system stiffness. It should be mentioned that due to the damping of the hydraulic recoil system and possibly soil interaction, the launcher, as designed, displayed much inherent damping.

Elevation changes were shown theoretically to influence the accuracy because of the corresponding geometry changes. The overall effect was to increase accuracy with elevation. Changes in accuracy were also observed due to difference in ammunition boost charge. The difference, however, was much less than might be expected because of the large reduction in the recoil force at the lower zones. This was because the forces developed during counterrecoil, from the recuperator and indexing system, are of the same order of magnitude as during the high boost bursts.

Delayed recoil is effective in reducing the velocity and displacement of the cradle response for single shots. For bursts, the effectiveness is questionable since it will improve accuracy when the ignition of the round occurs simultaneously with positive cradle velocity, but it does not help when ignition occurs during a duration of a negative cradle velocity. Rods cut back to delay the recoil force for the first shot only will probably yield the best overall accuracy improvement. This improves single shot accuracy while it eliminates the possibility of detracting during later shots.

When considering horizontal dispersion, the shell which stiffens the recoiling assembly framework reduces the dispersion by more than 100%. In addition, when trail hop is present, horizontal ties between the trails and the ground are necessary to achieve good accuracy. They may also improve dispersion in the absence of hop if a tendency exists for the trail pads to slide.

Finally, inclusion of Coriolis acceleration was shown to be significant in the calculation of peak response magnitude, but neglecting it did not alter the overall response shape.

In order to completely investigate launcher contributions to dispersion, two additional efforts are recommended. These are a completion of the development of the three-dimensional mathematical model begun in this

project, and a comprehensive experimental program utilizing the stiffening fixtures developed. This experimental program should include firing tube motion instrumentation as well as rocket instrumentation. It is felt that this combined effort can produce a weapon whose accuracy in burst fire exceeds that of present single shot artillery.

APPENDIX A
A STUDY OF THE RELATIVE EFFECTIVENESS
OF VARIOUS DAMPER LOCATIONS

N. P. Pearson, R. M. Brach, and R. H. Van Beek

I. INTRODUCTION

This appendix concerns the theoretical possibility of damping the motions of the supporting structure and tipping parts of an XM70-type launcher between consecutive shots of a burst. Ideal viscous dampers are considered to be located between the ground and supporting structure, and/or between the supporting structure and the tipping parts, and/or between the ground and the tipping parts.

II. THE MATHEMATICAL MODEL

Figure A-1 is a schematic drawing of an XM70-type launcher simplified and idealized to the point where it has only two degrees-of freedom. The general procedure for forming such a discrete system from an elastic system consists of lumping equivalent masses at points where the stiffness is known and/or loads are applied and determining the values of the equivalent masses in such a way that the actual and reduced systems have the same kinetic and potential energy during their first-mode oscillation.^{1/} The accuracy of the technique depends upon the nature of the forcing function and upon the location where the motion is desired. Deflection magnitudes at any given instant are not expected to be accurate because important features, such as the indexing forces, are not considered. However, the relative effectiveness of the three damper locations is expected to be correctly represented. Clearly, in this particular case, if the calculations showed the motions to be thoroughly damped after some relatively long time, say t_0 , and if the actual weapon were equipped with analogous dampers, the motion of the actual carriage or cradle would be thoroughly damped after t_0 also.

1. Jacobsen, L. S., Ayre, R. S., Engineering Vibrations, McGraw Hill Book Co., Inc., New York, 1958.

The equations of motion of the system shown in Fig. A-1 are;

$$I_2 \ddot{\theta}_2 = -k_2 (\theta_2 - \theta_1) - c_2 (\dot{\theta}_2 - \dot{\theta}_1) - c_3 \dot{\theta}_2 \quad A-1$$

and

$$(I_0 + m_r l^2) \ddot{\theta}_1 = k_2 (\theta_2 - \theta_1) + l F(t) - k_1 \theta_1 - c_1 \dot{\theta}_1 \quad A-2$$

or, letting

$$+ c_2 (\dot{\theta}_2 - \dot{\theta}_1)$$

$$I_0 + m_r l^2 = I_1$$

and

$$l F(t) = M(t)$$

then

$$I_1 \ddot{\theta}_1 - k_2 (\theta_2 - \theta_1) + k_1 \theta_1 + c_1 \dot{\theta}_1 + c_2 (\dot{\theta}_2 - \dot{\theta}_1) = M(t) \quad A-3$$

$$I_2 \ddot{\theta}_2 + k_2 (\theta_2 - \theta_1) + c_2 (\dot{\theta}_2 - \dot{\theta}_1) + 3 \dot{\theta}_2 \quad A-4$$

Equations A-3 and A-4 have been solved with a forcing function, $M(t)$, derived from an experimental record of recoil pressure. Figure A-2 shows the forcing function used for all the solutions given in this appendix. The value of the function is just the recoil pressure multiplied by the appropriate area and moment arm. Figures A-3 through A-8 show the calculated response for various choices of the parameters. The stiffness values, k_1 and k_2 , represent the values either measured or calculated for the existing weapons. In addition to the values of damping and stiffness, each of the graphs is labeled with the maximum value of the quantity $c_2 (\dot{\theta}_1 - \dot{\theta}_2)$ which is the maximum force developed in the damper between the supporting structure and the tipping parts.

III. DISCUSSION AND CONCLUSIONS

Figure A-3 describes the undamped response of the system, which is not representative of the actual weapon response, primarily because the actual weapon contains inherent damping. The figure does show, however, that significantly larger amplitudes may be obtained on shots after the first. This phenomenon was observed experimentally for Prototype No. 1. It is worthwhile noting that, in all of these response plots, the negative values of ϕ_1 may be taken as a rough indication of the system's tendency to produce trail hop.

The parameters used in the calculation represented by Fig. A-4, A-5, and A-6 are identical except for the values of damping, with c_1 , c_2 , and c_3 alternately taking on the value 1.25×10^5 lb-sec²/in., the other two being zero in each case. Note from Fig. A-6 that damping between the ground and the tipping parts, c_3 -damping, gives the best results. The relative effectiveness of c_2 -damping, as shown by Fig. A-5, and the relative ineffectiveness of c_1 -damping, as shown by Fig. A-4, constitute the most important result of this study.

Damping between the tipping parts and the ground, c_3 -damping, is the most difficult type to obtain practically. Figure A-7 shows an attempt to obtain the effectiveness of c_3 -damping using only c_1 - and c_2 -damping. The results are seen to be not as good as c_3 -damping alone. This is particularly discouraging when the weight added by the two dampers is considered. Figure A-8 shows the results of attempts to compensate for the weight of the two dampers by reducing the supporting structure stiffness and, hence, its weight. This performance is seen to be slightly less satisfactory, but further conclusions are unwarranted because of the simplicity of the model which does not include indexing forces and elevating system geometry. Because these factors affect the optimum stiffness-damping relationship, this relationship will be studied by means of a more elaborate mathematical model.

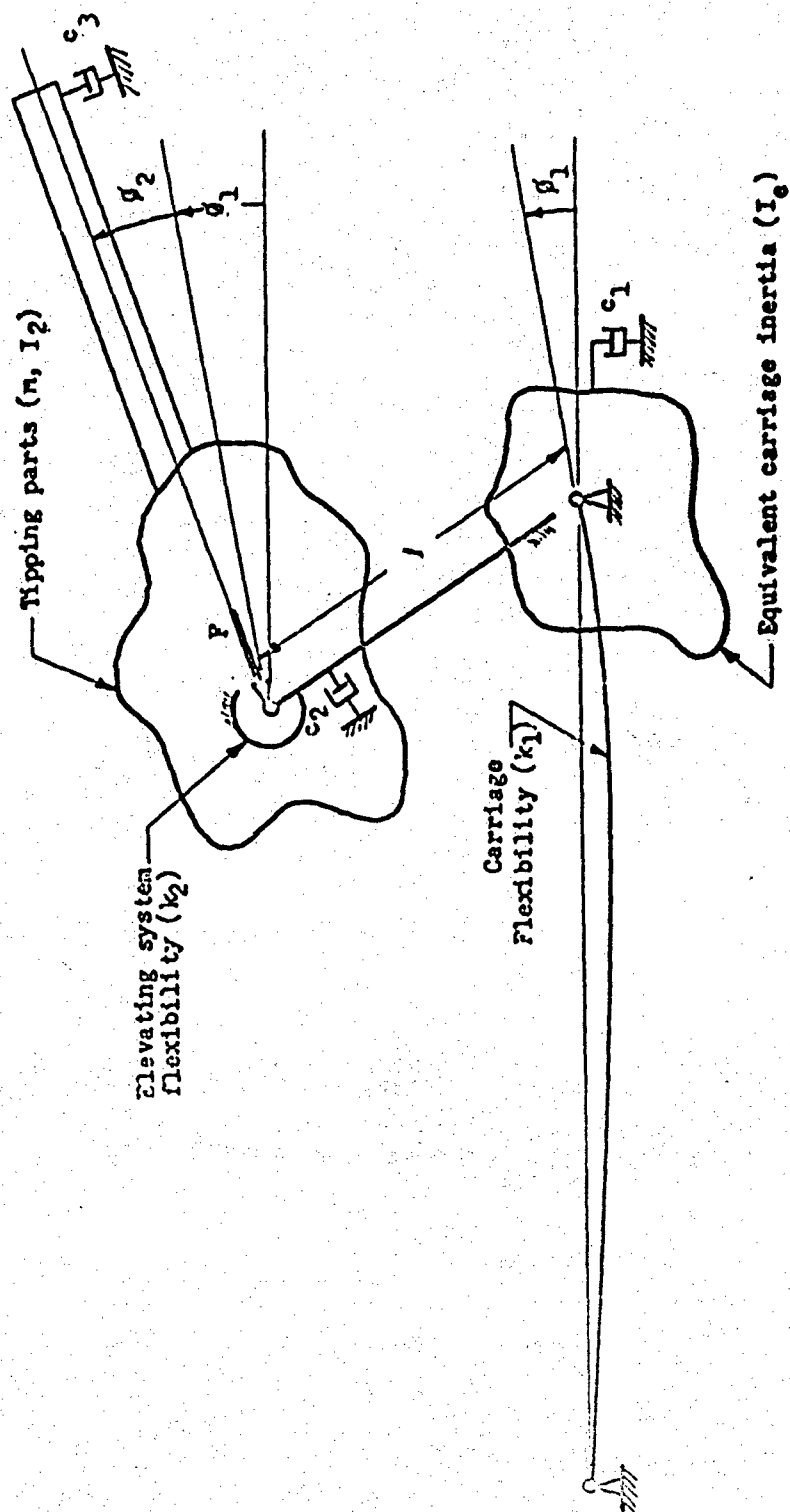


Fig. A-1 TWO-DEGREE-OF-FREEDOM REPRESENTATION OF A XM70 LAUNCHER

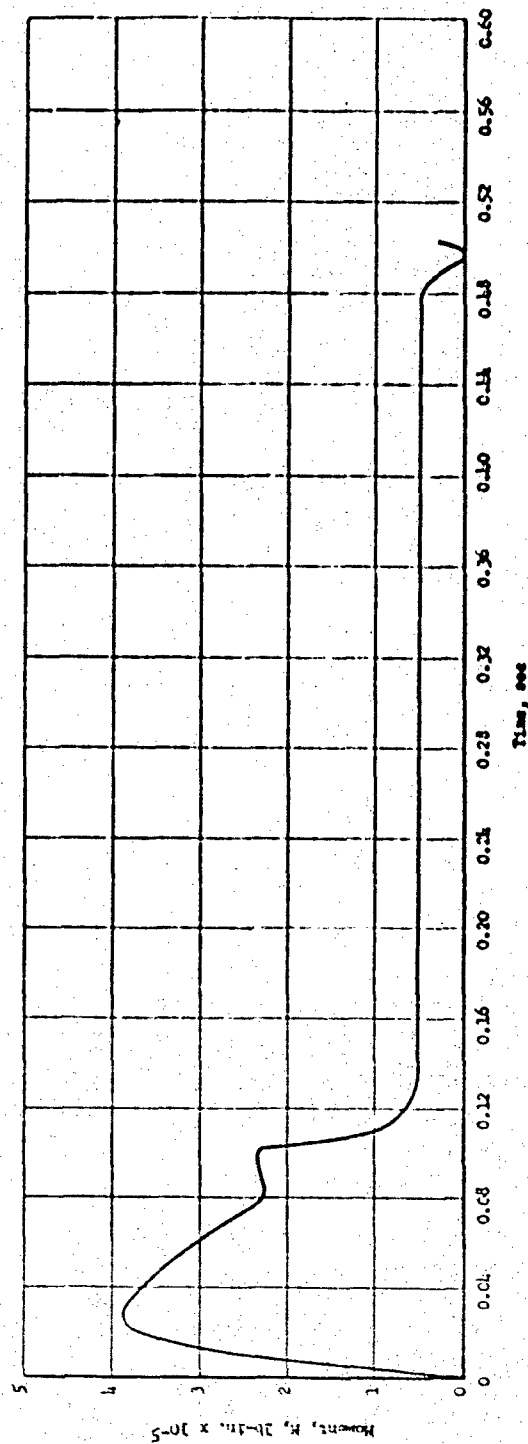


Fig. A-2 MOMENT PRODUCED BY THE RECOIL FORCE

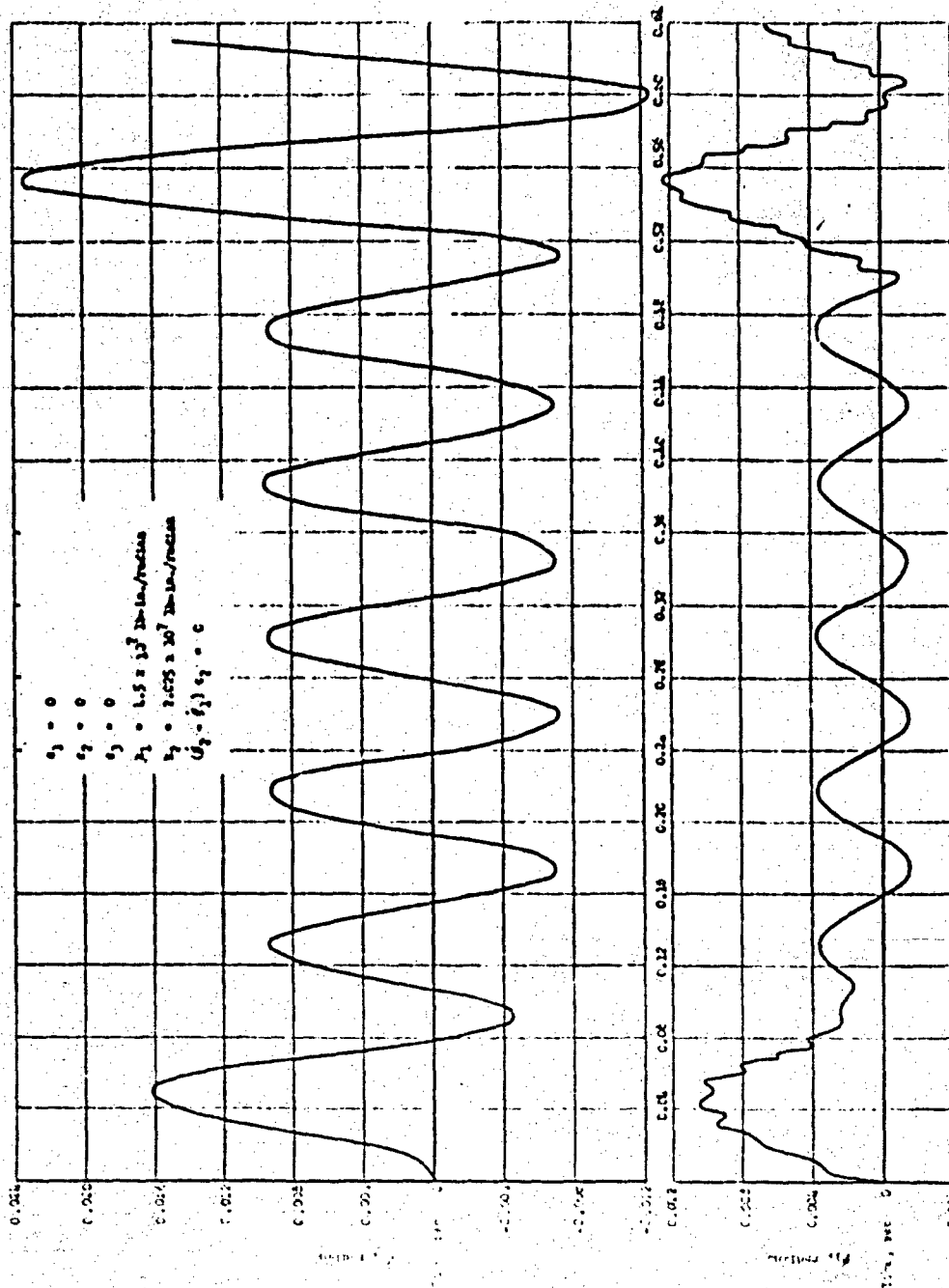


Fig. A-3 CALCULATED UNDAMPED RESPONSE OF AN XM70-TYPE LAUNCHER

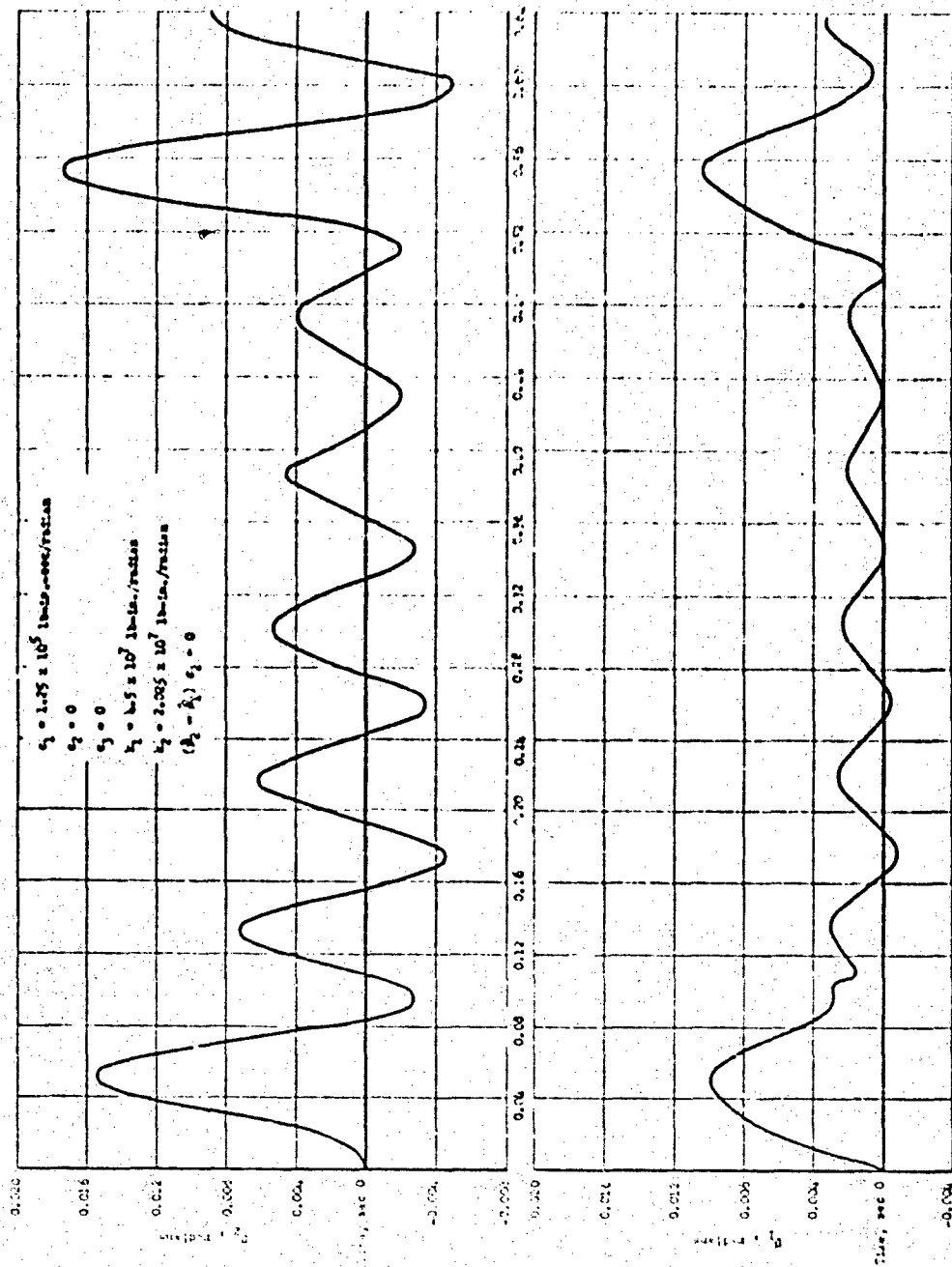


Fig. A-4 CALCULATED RESPONSE OF AN XM70-TYPE LAUNCHER WITH c_1 -DAMPING ONLY

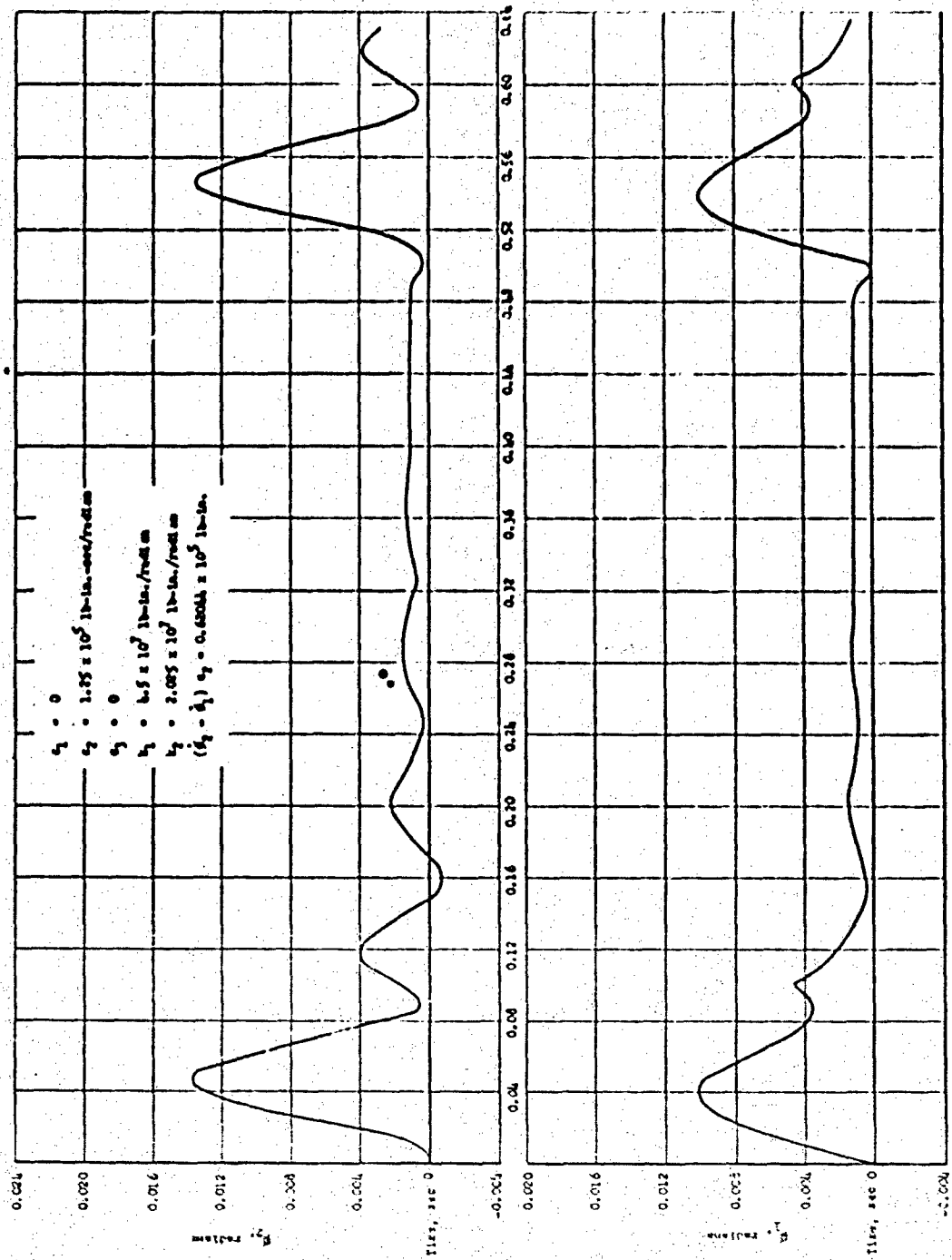


Fig. A-5 CALCULATED RESPONSE OF AN XM70-TYPE LAUNCHER WITH c_2 -DAMPING ONLY

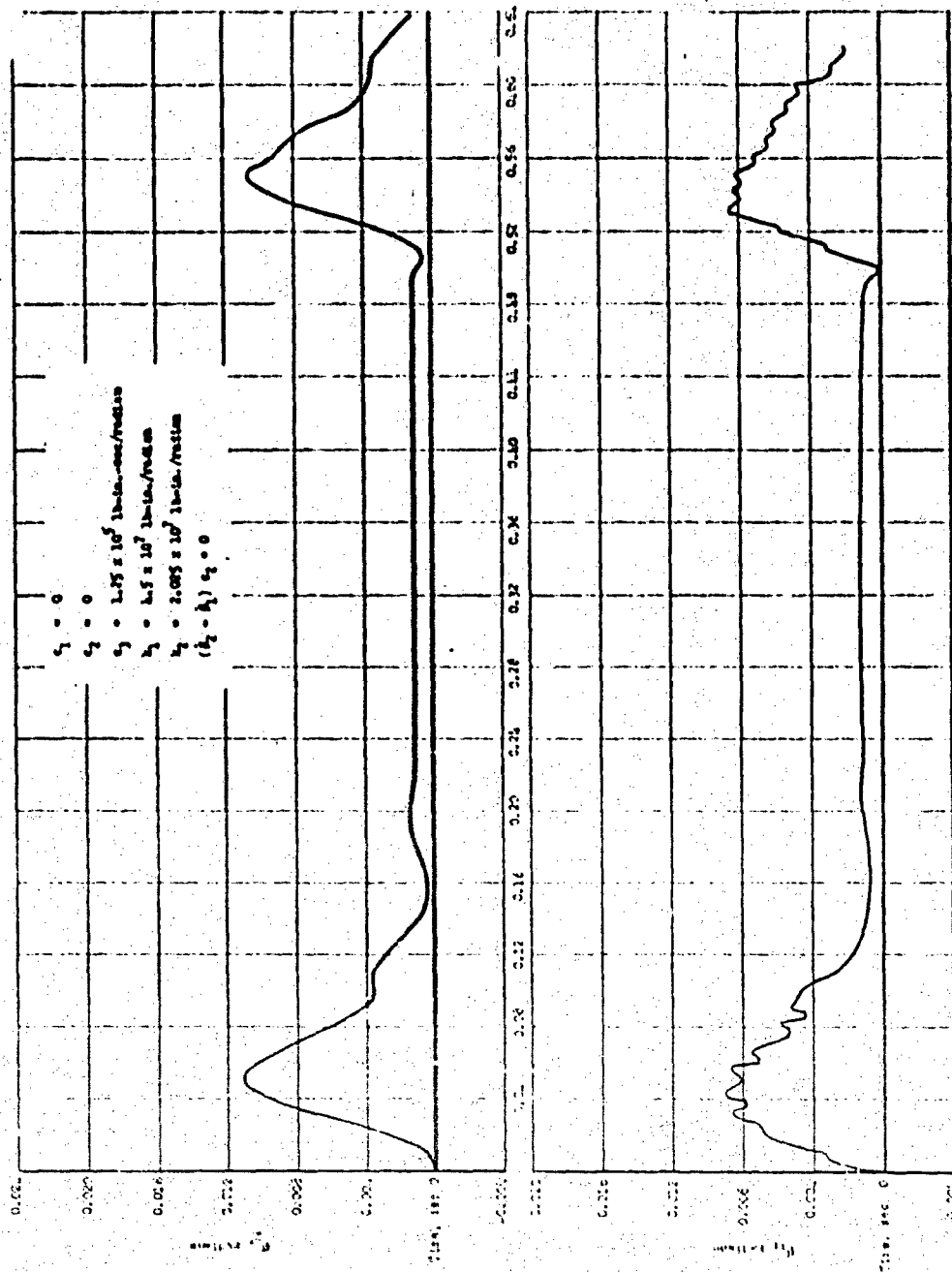
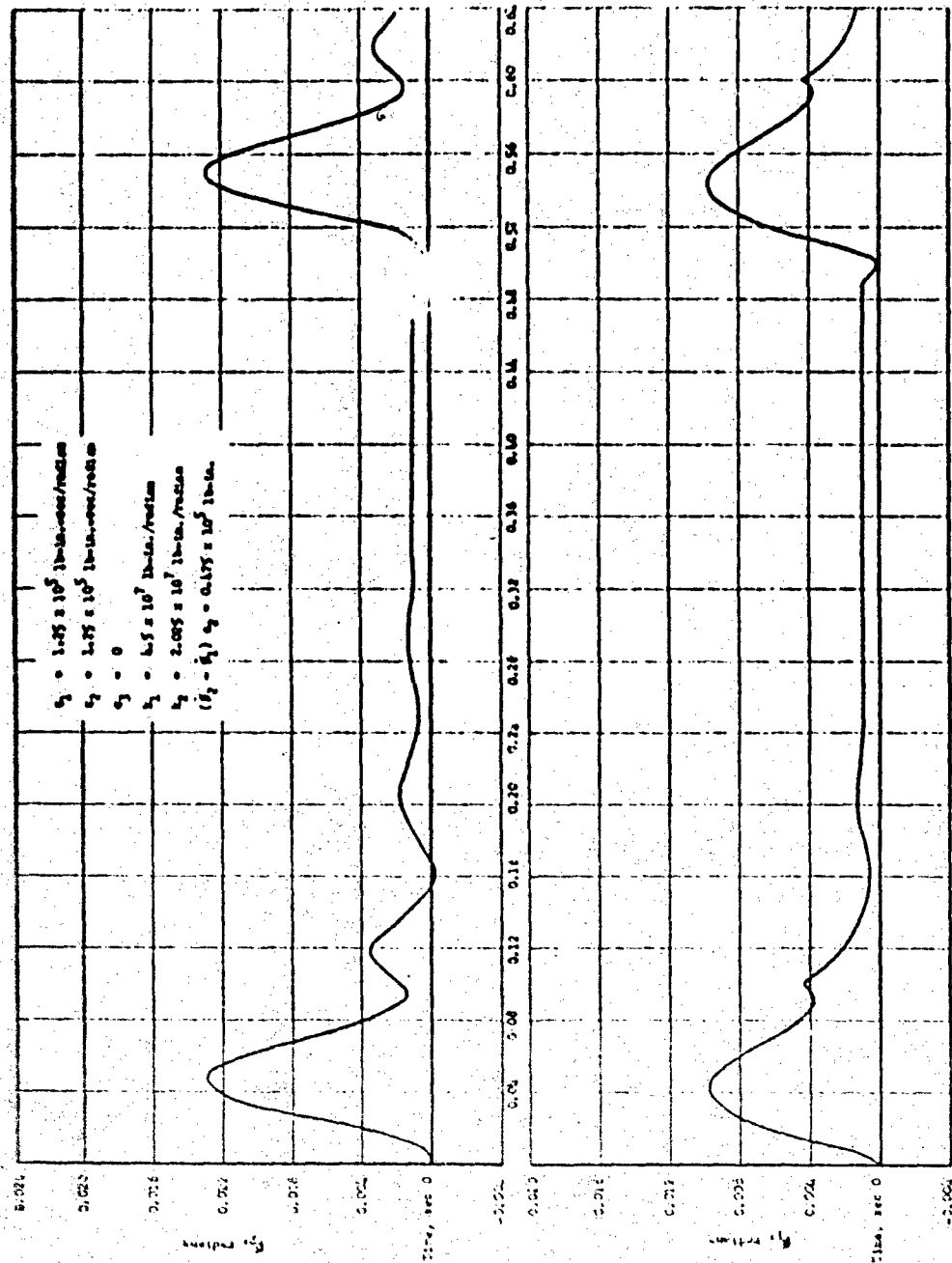
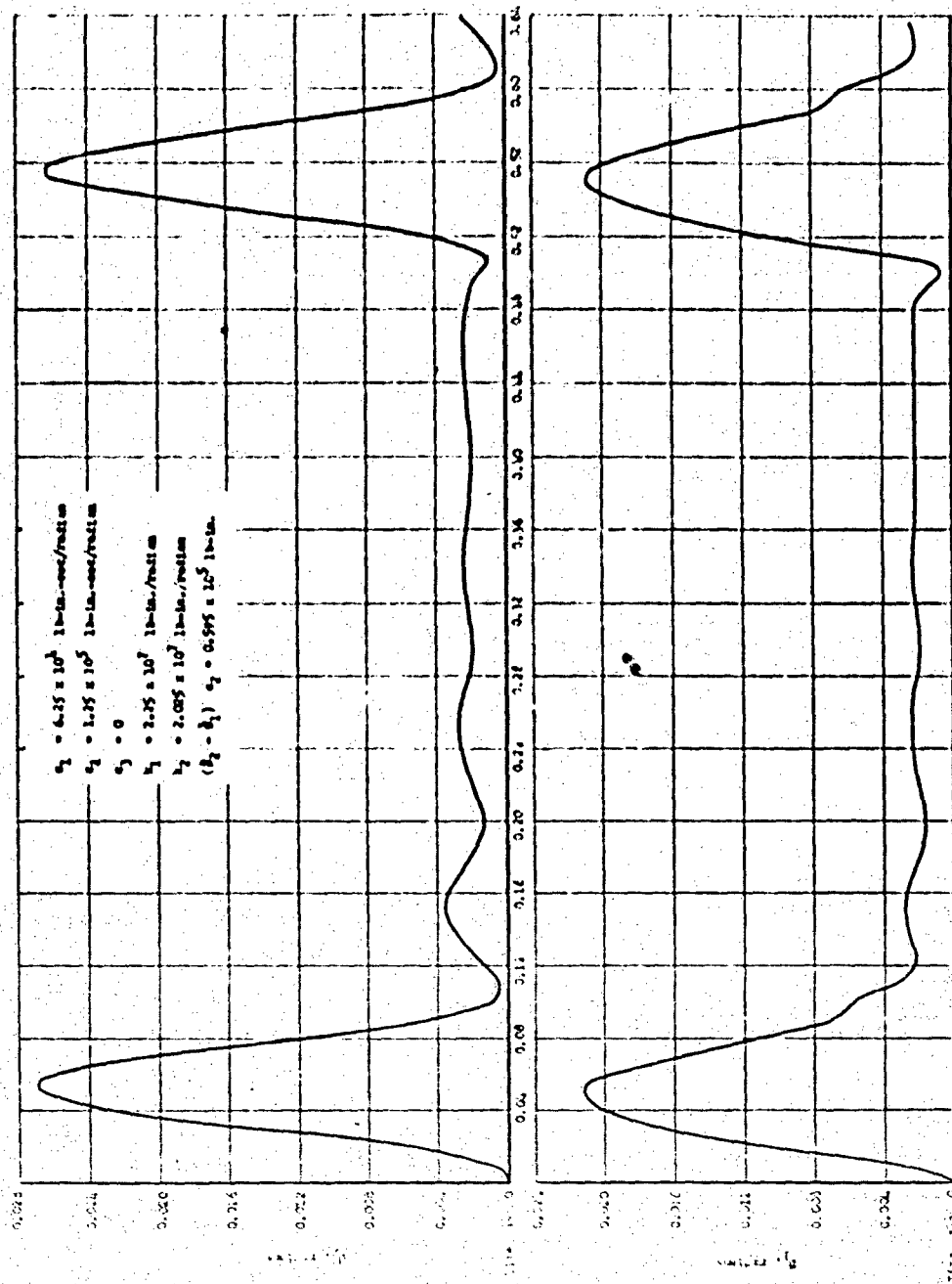


Fig. A-6 CALCULATED RESPONSE OF AN XM70-TYPE LAUNCHER WITH c_3 - DAMPING ONLY



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Fig. A-7 CALCULATED RESPONSE OF AN XM70-TYPE LAUNCHER WITH c_1 - AND c_2 - DAMPING COMBINED



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Fig. A-8 CALCULATED RESPONSE OF AN XM70-TYPE LAUNCHER WITH c_1 - AND c_2 -DAMPING COMBINED AND WITH REDUCED CARRIAGE STIFFNESS

APPENDIX B

A DYNAMIC ANALYSIS OF SOME BASIC DESIGN PARAMETERS

By R. H. Van Beek

I. INTRODUCTION

This appendix contains a dynamic analysis which was intended to aid in determining some of the basic design parameters, such as weight and stiffnesses, of weapons of the XM70 type. This analysis uses a relatively simple mathematical model of a weapon together with semiempirical data from the XM70 Prototype Model No. 1. A nomenclature list is at the end of this Appendix.

II. ANALYSIS

It has been shown that the weapon can be represented by a single-degree-of-freedom dynamic model with the recoil force (rod pull) as a forcing function. The assumed model and forcing function are shown in figures B-1 and B-2.

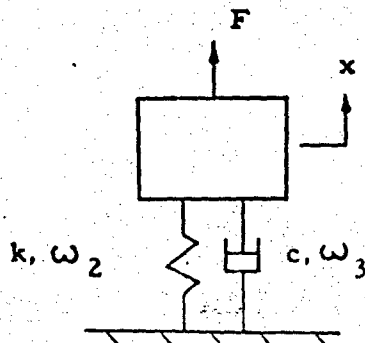


Fig. B-1 MATHEMATICAL MODEL

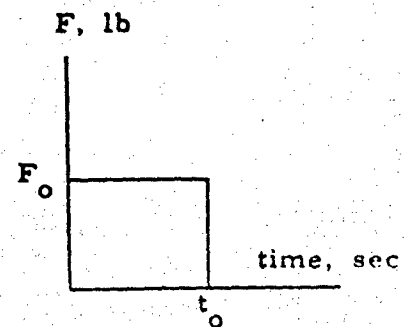


Fig. B-2 FORCING FUNCTION

The differential equation for the system of figure B-1 is

$$M \ddot{x} + c \dot{x} + kx = F \quad \text{Eq. (B-1)}$$

$$\text{Let } \eta \equiv \sqrt{\frac{K}{M}} t_0, \quad \mu \equiv \frac{c}{2\sqrt{KM}}, \quad \tau \equiv \frac{t}{t_0}, \quad \chi \equiv \frac{XM}{F_0 t_0^2} \quad \text{Eq. (B-2)}$$

The solution to differential equation (B-1) in terms of these quantities is

$$x(\tau) = \frac{1}{\eta^2} \left[1 - e^{-\eta\mu\tau} \left(\frac{\mu}{\sqrt{1-\mu^2}} \sin \eta \sqrt{1-\mu^2} \tau + \cos \eta \sqrt{1-\mu^2} \tau \right) \right] \quad \text{for } 0 \leq \tau \leq 1 \quad \text{Eq. (B-3)}$$

$$x(\tau) = e^{-\eta\mu\tau} \left[L_1 \sin \eta \sqrt{1-\mu^2} \tau + L_2 \cos \eta \sqrt{1-\mu^2} \tau \right] \quad \text{for } \tau \geq 1 \quad \text{Eq. (B-4)}$$

$$\text{where } L_1 = \frac{1}{\eta^2} \left[\frac{\mu}{\sqrt{1-\mu^2}} + e^{\mu\eta} \left(\frac{\mu}{\sqrt{1-\mu^2}} \cos \eta \sqrt{1-\mu^2} + \sin \eta \sqrt{1-\mu^2} \right) \right] \quad \text{Eq. (B-5)}$$

$$\text{and } L_2 = -\frac{1}{\eta^2} \left[1 + e^{\mu\eta} \left(\frac{\mu}{\sqrt{1-\mu^2}} \sin \eta \sqrt{1-\mu^2} - \cos \eta \sqrt{1-\mu^2} \right) \right] \quad \text{Eq. (B-6)}$$

For a single degree-of-freedom torsional system, the same solution holds with the dimensionless variables given by

$$x = \frac{\theta I}{T_o t_o^2} \cdot \eta = \sqrt{\frac{k}{I}} t_o, \quad \mu = \frac{c}{2\sqrt{kI}}, \quad \tau = \frac{t}{t_o} \quad \text{Eq. (B-7)}$$

A typical displacement time plot is shown in figure B-3.

Adequate dynamic performance, i.e. acceptable accuracy, results when t_c is less than the time to a subsequent firing and θ_c is a displacement which produces negligible error at the target. Several primary design parameters can be logically determined by finding the minimum weight weapon which will give the required performance.

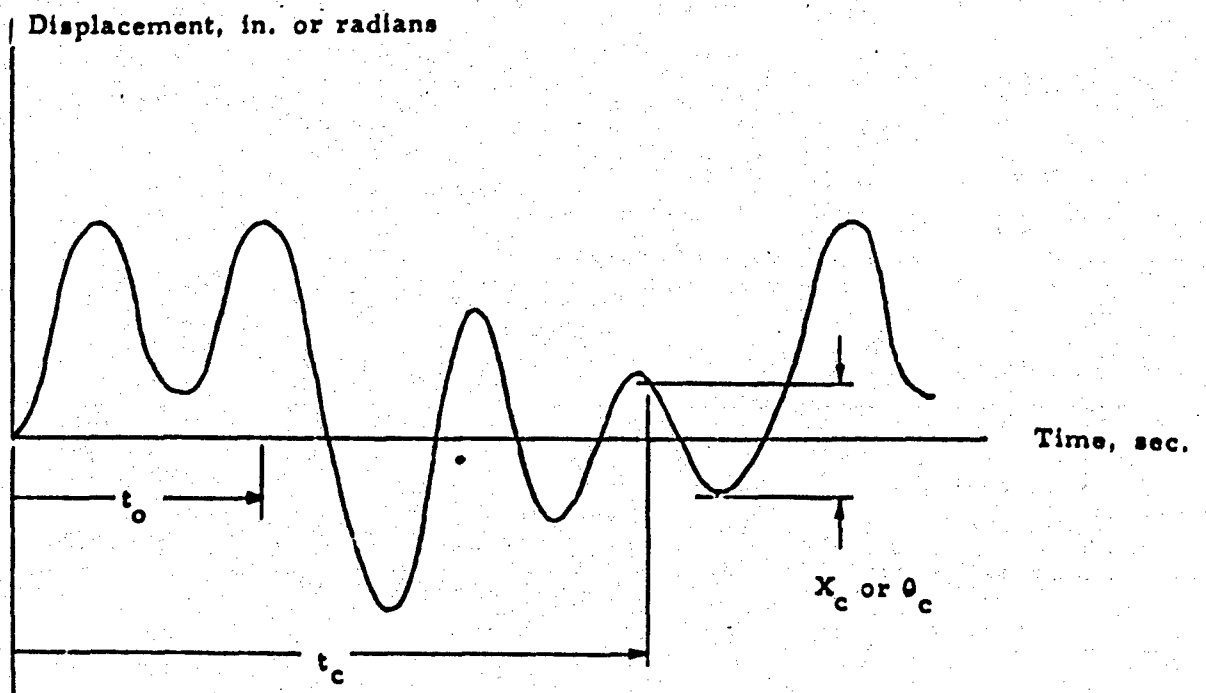


FIG. B-3 TYPICAL TIME DISPLACEMENT CURVE
OF THE XM70 LAUNCHER

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For given μ and η the graph of τ versus θ can be drawn. The last intersection of the curve with $\theta = \pm \theta_c$ determines τ_c . Repeating this procedure for a number of combinations of μ and η gives data for a plot of τ_c versus η for various values of μ . For a given τ_c , a plot of η versus μ can then be drawn. All points on this curve are such that the corresponding η and μ produce a displacement of θ_c at $\tau = \tau_c$ and $\theta < \theta_c$ for $\tau > \tau_c$. All points give values of μ and η for which τ_c is less than the value of τ for which the curve is drawn. Figure B-4 shows a typical graph of τ versus η .

As an approximation, the envelope of equation (B-4) can be used. It is given by

$$\theta_c = L_1^2 + L_2^2 e^{-\eta\mu\tau} \quad \text{Eq. (B-8)}$$

$$L_1^2 + L_2^2 = \frac{\sqrt{1 - 2e\mu\eta} \cos \eta \sqrt{1 - \mu^2} + e^{2\mu\eta}}{\eta^2 \sqrt{1 - \mu^2}} \quad \text{Eq. (B-9)}$$

Then

$$(\theta_c)_c = L_1^2 + L_2^2 e^{-\eta\mu\tau_c} \quad \text{Eq. (B-10)}$$

This eliminates the necessity of drawing the first two of the above mentioned graphs, since, given τ_c and θ_c , for each μ the corresponding η can be found numerically by trial and error. In all cases where the graphs of μ versus η were found by both methods the difference between the graphs was negligible.

Let the total weight of the weapon be given by

$$W_t = W_1 + W_2 + W_3 + W_{NS} \quad \text{Eq. (B-11)}$$

Assume that the weight of the carriage is related to its rotational stiffness by the linear relation

$$W = \alpha k \quad \text{Eq. (B-12)}$$

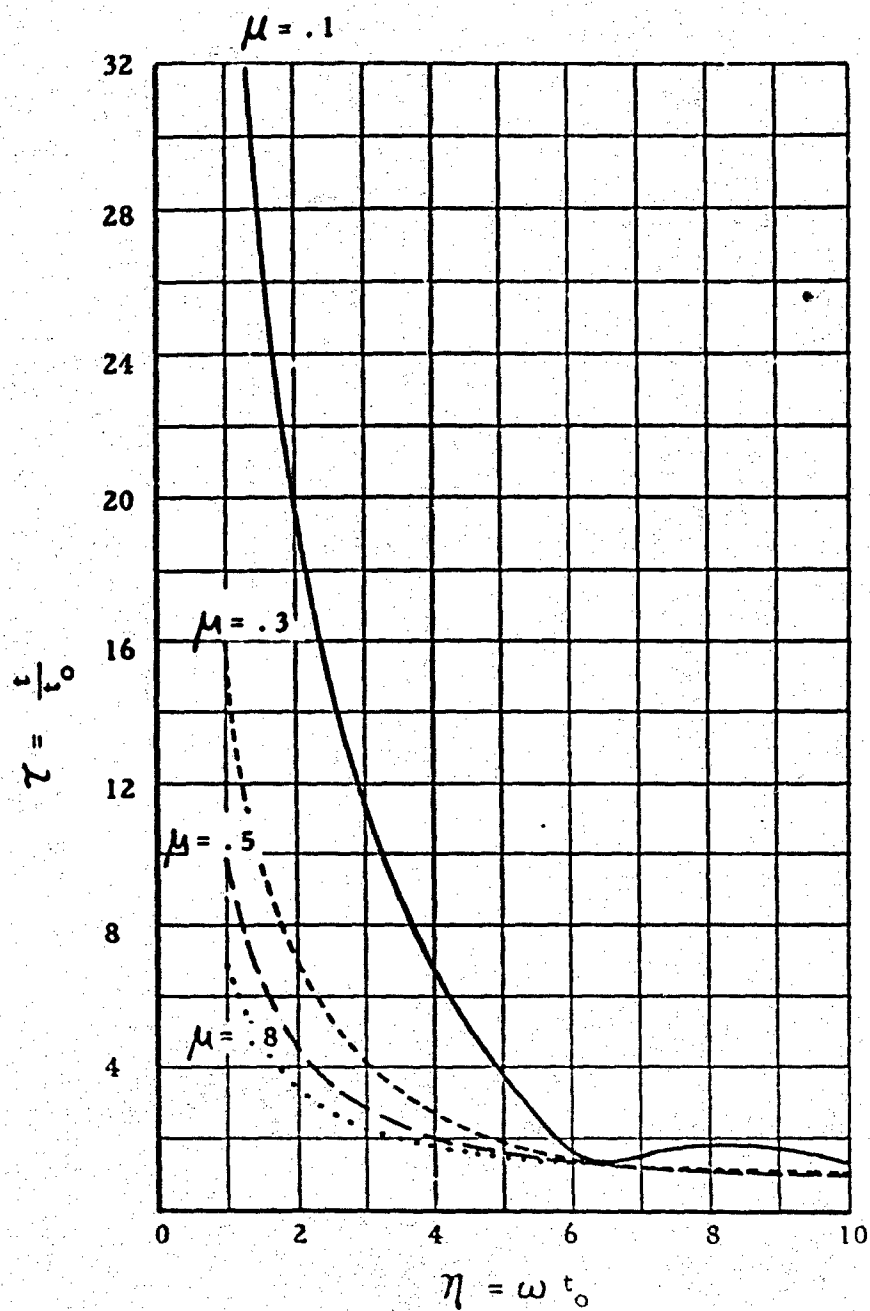


FIG. B-4 TYPICAL GRAPH OF DIMENSIONLESS TIME VERSUS
FREQUENCY FOR VARIOUS DAMPING RATIOS

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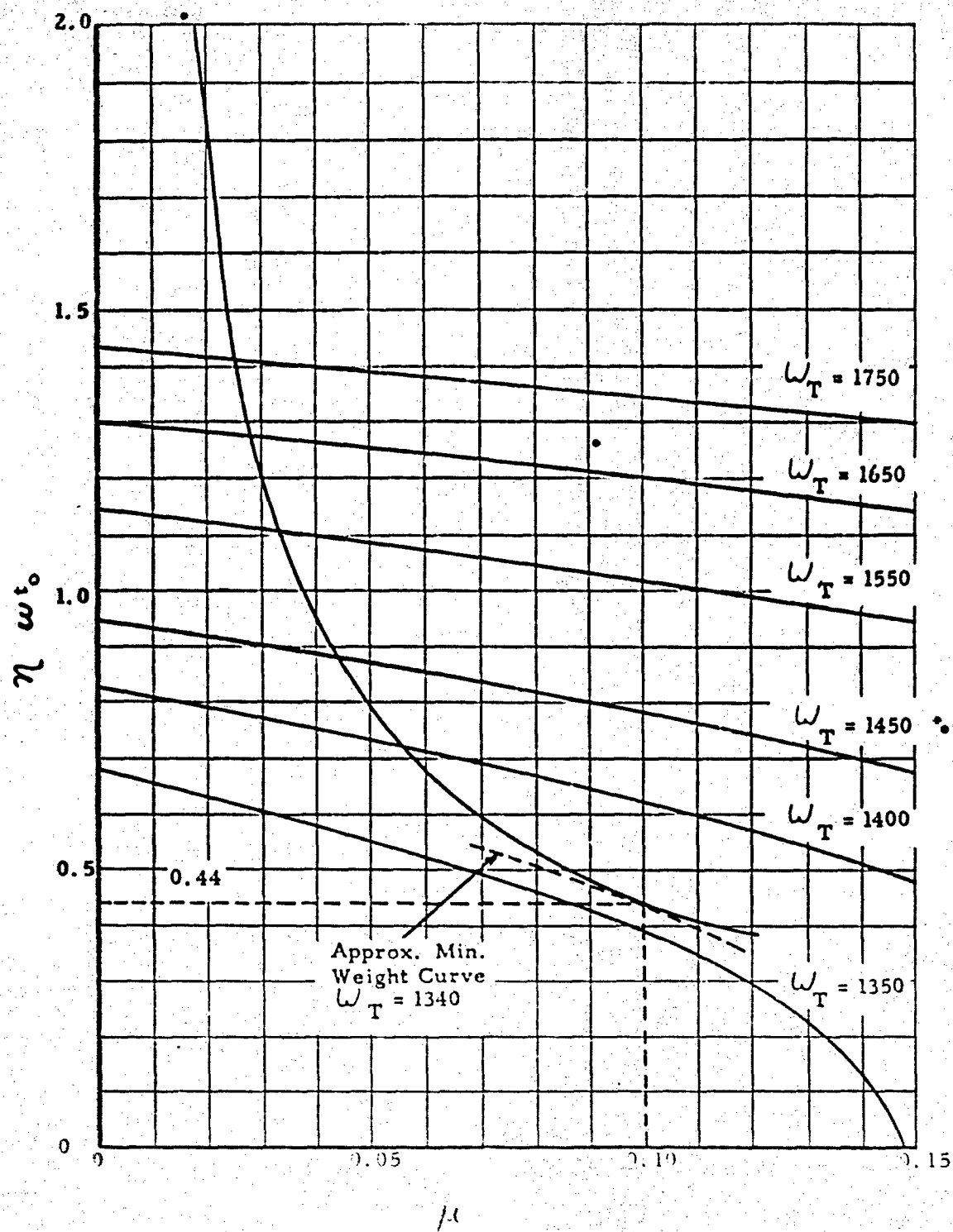


FIG. B-5 MINIMUM WEIGHT REQUIRED TO
SATISFY ACCURACY CRITERION

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In addition, assume that the weight of the damper is related to the damping ratio by the linear relation

$$W_3 = \beta \mu \quad \text{Eq. (B-13)}$$

By (B-1), (B-2), and (B-3)

$$W_T - W_H - W_1 = \alpha k + \beta \mu \quad \text{Eq. (B-14)}$$

By definition

$$\eta = \omega t_0 = \frac{k}{I} t_0 \quad \text{Eq. (B-15)}$$

Assuming that one-third of the moment of inertia of the carriage should be added to the moment of inertia of the recording assembly given

$$\begin{aligned} I &= I_1 + 1/3 I_2 \\ I &= \frac{W_1}{g} r_1^2 + \frac{W_2}{3g} r_2^2 \\ I &= \frac{W_1}{g} r_1^2 + \frac{2k}{3g} r_2^2 \end{aligned} \quad \text{Eq. (B-16)}$$

By eliminating k and I between Eq. (B-4), (B-5), and (B-6), the relation between η and μ for constant W_T is obtained, assuming W_1 and W_H are constants.

$$\eta^2 = \frac{3 g t_0^2}{2 r_2^2} \left[\frac{\gamma}{3 \frac{r_1^2}{r_2^2} + \gamma} \right] \quad \text{Eq. (B-17)}$$

where $\gamma \equiv \frac{W_T - W_H - W_1 - \beta\mu}{W_1}$ Eq. (B-18)

III. RESULTS

From this relationship, curves of η versus μ for various values of W_T can be plotted. If a curve of η versus μ for the desired value of τ is plotted on the same graph, the lowest possible weight giving acceptable accuracy can be determined. The situation is illustrated in Fig. 5. The constant γ curve can be thought of as representing acceptable performance and the other curves are the constant weight curves. Then the lowest weight curve which just intersects the performance curve gives the minimum total weight and the point of intersection determines the desired damping ratio and η . Then by the use of equations (B-11), (B-12), and (B-13), the carriage stiffness and weight and the damper weight can be calculated. Fig. 5 also shows the results of applying this analysis with the following input data:

Projectile weight*	50 lb.
Muzzle velocity*	1000 ft/sec.
Firing rate	20 rnds/min.
Maximum Amplitude of Vibration at triggering (θ_c)	0.0005 rad.
Maximum Recoil Displacement (Free Recoil until shot ejection is assumed and requires 12 in. of recoil)	24 in.
Recoiling Assembly Weight	500 lb.

The inertia-weight relationships and the carriage stiffness - weight relationship, (α), of the XM 70 were assumed. It will be noted that the hypothetical weapon has a very low firing rate and small recoiling assembly weight and hence has a small minimum weight. From the graph, the minimum total weight which gives the desired accuracy is 1340 lb. Included in this is a total accessory or non-structural weight which is assumed to be 750 lb. This is the same as the XM70 and includes everything except the recoiling assembly and the carriage structure. Using the values of μ and η from the graph, the damper weight is found to be 67.5 lb. and the carriage weight

* these quantities determine the forcing function

22.5 lb. The carriage stiffness required is 1.12×10^6 lb. in./rad.

This example design, although it theoretically satisfies the accuracy criterion, is impossible because a carriage weighing 22.5 lb. could not carry the rod pull. This result led to the development of strength criterion and a method of optimizing recoiling assembly weight. This latter method was used for the XM70 Prototype Model No. 2 and is presented in ARF Project K130 Bi-monthly Report No. 8, Appendix B.

IV. NOMENCLATURE

W_T total weight of gun

W_{NS} weight of gun hardware

W_1 weight of recoiling assembly

W_2 weight of carriage

W_3 weight of damper

k torsional stiffness of carriage

K spring stiffness for linear vibration system

c damping stiffness for linear vibration system

M mass stiffness for linear vibration system

$$b = \frac{c}{2M}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{k}{I}}, \text{ natural frequency of undamped system}$$

I effective moment of inertia of torsional vibration system

μ ratio of damping to critical damping

$$\eta = \omega t_0, \text{ non-dimensional frequency}$$

x displacement for linear vibration system

θ displacement for torsional vibration system

t time

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$$\chi = \frac{Mx}{F_0 t_0^2} = \frac{I_0}{T_0 t_0^2} \quad \text{non-dimensional displacement}$$

t_0 length of pulse

F_0 amplitude of pulse-linear vibration system

T_0 amplitude of pulse-torsional vibration system

$\tau = t/t_0$ non-dimensional time

I_1 moment of inertia of recoiling assembly about the base

I_2 moment of inertia of carriage about the base

APPENDIX C

EQUATIONS OF MOTION OF THE LAUNCHER WITH THREE DEGREES OF FREEDOM

R. M. Brach

I. INTRODUCTION

The following equations apply to launchers with a structural geometry corresponding to that of the XM70 automatic rocket launcher. It is a three-degree-of-freedom analysis of gross launcher motion in the vertical plane. These equations are to be used to investigate the stability of the system and the influence on stability of changes in: (1) elevating system stiffness and damping, (2) carriage and trail stiffness, (3) mass distribution, (4) recoil force, and (5) indexing cam length. They can also furnish an indication of the effects due to variations in the muzzle momentum of the projectile. However, because the analysis considers a rigid firing tube, it cannot predict the precise projectile motion at the time of muzzle ejection.

The three degrees of freedom (see Fig. C-1) are the following: (1) the linear displacement of the recoiling mass relative to the cradle, u , (2) the angular motion of the tipping parts relative to the carriage, ϕ_2 , and (3) the absolute angular motion of the supporting structure, ϕ_1 . The supporting structure in this analysis is taken to be the entire structure that supports the tipping parts; i. e., the supporting structure includes a rigid, massless, trunnion side, an inertia, and a massless, flexible trail. The inertia is an equivalent inertia found by letting the above support contain kinetic and potential energy equal to that of the actual launcher vibrating in its static deflection shape. The supporting structure is assumed to rotate about the ball joint, point 0 in Fig. C-1; a provision for damping the rotation is included. The elevating system is assumed to be a massless spring and damper. The cradle and recoiling mass are both assumed to be rigid bodies.

The equations of motion of the recoiling mass include those describing the constant-force, constant-stopping-distance recoil system.

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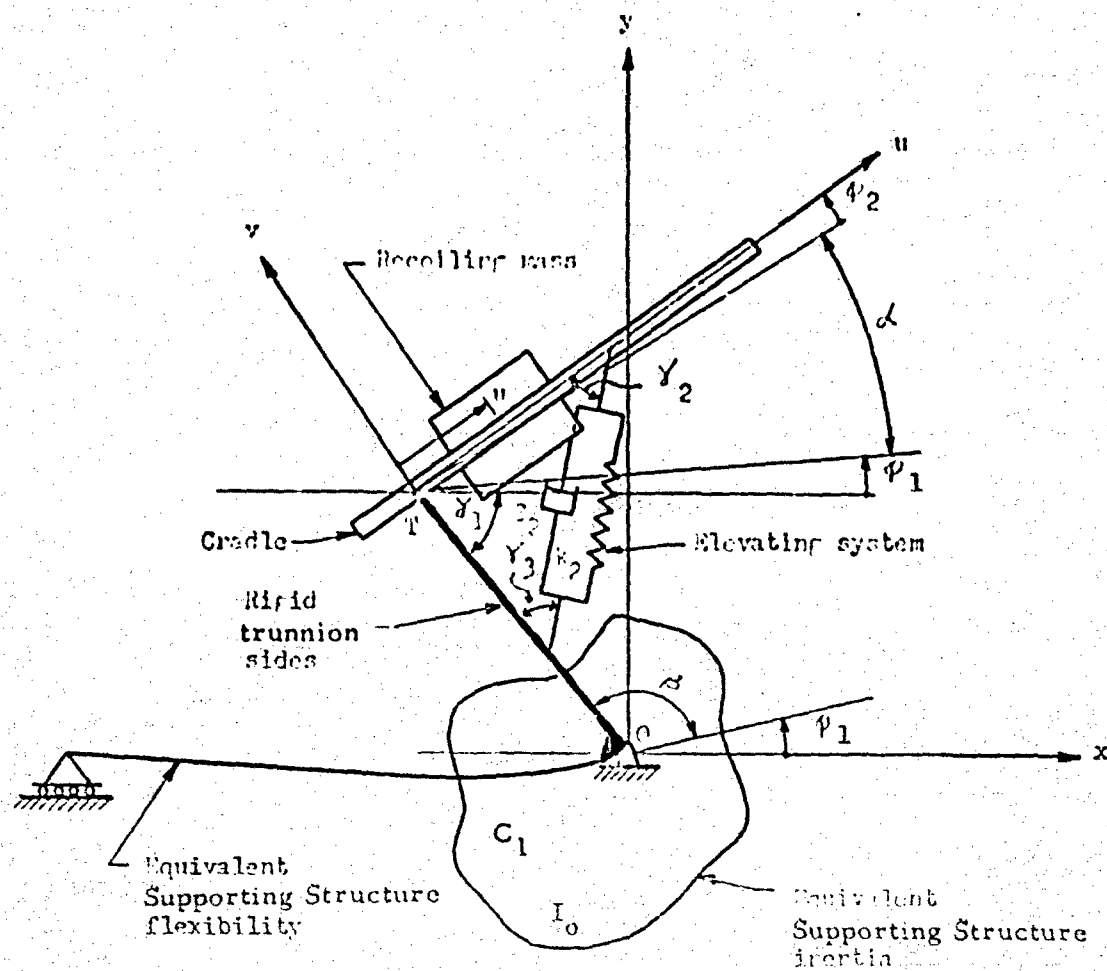


Fig. C-1 SCHEMATIC REPRESENTATION OF THE LAUNCHER

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used in the XM70, and also the axial reactions of the indexing cam since these have been found to be significant from past experience.

II. ANALYSIS

A. Kinematics

If the motion of the recoiling mass is described by x_r and y_r with respect to the ground and by u with respect to the cradle, we see from Fig. C-2 that

$$\begin{aligned} x_r &= l \cos(\beta + \phi_1) + u \cos(\alpha + \phi_1 + \phi_2) \\ \text{and} \quad y_r &= l \sin(\beta + \phi_1) + u \sin(\alpha + \phi_1 + \phi_2) \end{aligned} \quad \text{C-1}$$

In these equations, l is the distance from the point 0 to the trunnion, point T, β is the fixed obtuse angle made with ground and the trunnion sides, α is the angle of elevation, ϕ_1 is the angular motion of the supporting structure and ϕ_2 is the angular motion of the tipping parts with respect to the supporting structure. The velocities, $\dot{x}_r$ and $\dot{y}_r$, and the accelerations, $\ddot{x}_r$ and $\ddot{y}_r$, are found by differentiation to be

$$\begin{aligned} \dot{x}_r &= -l\dot{\phi}_1 \sin(\beta + \phi_1) + \dot{u} \cos(\alpha + \phi_1 + \phi_2) \\ &\quad - u(\dot{\phi}_1 + \dot{\phi}_2) \sin(\alpha + \phi_1 + \phi_2) \\ \dot{y}_r &= l\dot{\phi}_1 \cos(\beta + \phi_1) + \dot{u} \sin(\alpha + \phi_1 + \phi_2) \\ &\quad + u(\dot{\phi}_1 + \dot{\phi}_2) \cos(\alpha + \phi_1 + \phi_2) \end{aligned} \quad \text{C-2}$$

and

$$\begin{aligned} \ddot{x}_r &= -l\ddot{\phi}_1 \sin(\beta + \phi_1) - l\dot{\phi}_1^2 \cos(\beta + \phi_1) \\ &\quad + \ddot{u} \cos(\alpha + \phi_1 + \phi_2) - \dot{u}(\dot{\phi}_1 + \dot{\phi}_2) \sin(\alpha + \phi_1 + \phi_2) \\ &\quad - u(\ddot{\phi}_1 + \ddot{\phi}_2) \sin(\alpha + \phi_1 + \phi_2) - \dot{u}(\dot{\phi}_1 + \dot{\phi}_2) \sin \\ &\quad (\alpha + \phi_1 + \phi_2) - u(\dot{\phi}_1 + \dot{\phi}_2)^2 \cos(\alpha + \phi_1 + \phi_2) \end{aligned} \quad \text{C-3}$$

$$\ddot{y}_r = l\ddot{\phi}_1 \cos(\beta + \phi_1) - l\dot{\phi}_1^2 \sin(\beta + \phi_1) + \ddot{u} \sin(\alpha + \phi_1 + \phi_2) + \dot{u}(\dot{\phi}_1 + \dot{\phi}_2) \cos(\alpha + \phi_1 + \phi_2) + u(\ddot{\phi}_1 + \ddot{\phi}_2) \cos(\alpha + \phi_1 + \phi_2) + \dot{u}(\dot{\phi}_1 + \dot{\phi}_2) \cos(\alpha + \phi_1 + \phi_2) - u(\dot{\phi}_1 + \dot{\phi}_2)^2 \sin(\alpha + \phi_1 + \phi_2) \quad C-4$$

The vector sum of the component $\ddot{x}_r$ and $\ddot{y}_r$ may be resolved into components in the u- and v- directions. These components are denoted as $\ddot{\eta}$ and $\ddot{\xi}$, respectively. From Fig. C-3, these are:

$$\ddot{\eta} = \ddot{x}_r \cos \omega_1 + \ddot{y}_r \sin \omega_1 \quad C-5$$

$$\ddot{\xi} = \ddot{y}_r \cos \omega_1 - \ddot{x}_r \sin \omega_1 \quad C-6$$

where

$$\omega_1 = \alpha + \phi_1 + \phi_2 \quad C-7$$

Substituting Eq. C-3 and 4 into Eq. C-5 and 6 gives the following expression,

$$\ddot{\eta} = -l\ddot{\phi}_1 \sin \omega_2 - l\dot{\phi}_1^2 \cos \omega_2 + \ddot{u} - u(\dot{\phi}_1 + \dot{\phi}_2)^2 \quad C-8$$

$$\ddot{\xi} = l\ddot{\phi}_1 \cos \omega_2 - l\dot{\phi}_1^2 \sin \omega_2 + 2\dot{u}(\dot{\phi}_1 + \dot{\phi}_2) + u(\ddot{\phi}_1 + \ddot{\phi}_2) \quad C-9$$

where

$$\omega_2 = \beta - \alpha - \phi_2 \quad C-10$$

B. Equations of Motion

Equations C-8 and C-9 are the components of the absolute acceleration of the recoiling mass. The free body diagram of the recoiling mass is shown in Fig. C-4. Summing forces acting through the mass center, we obtain

$$m_r \ddot{\eta} = -F(t) + \Phi(t) + e\mu |N| - m_r g \sin \omega_1 \quad C-11$$

$$m_r \ddot{\xi} = N - m_r g \cos \omega_1 \quad C-12$$

In these equations the following nomenclature is used:

$F(t)$ applied force

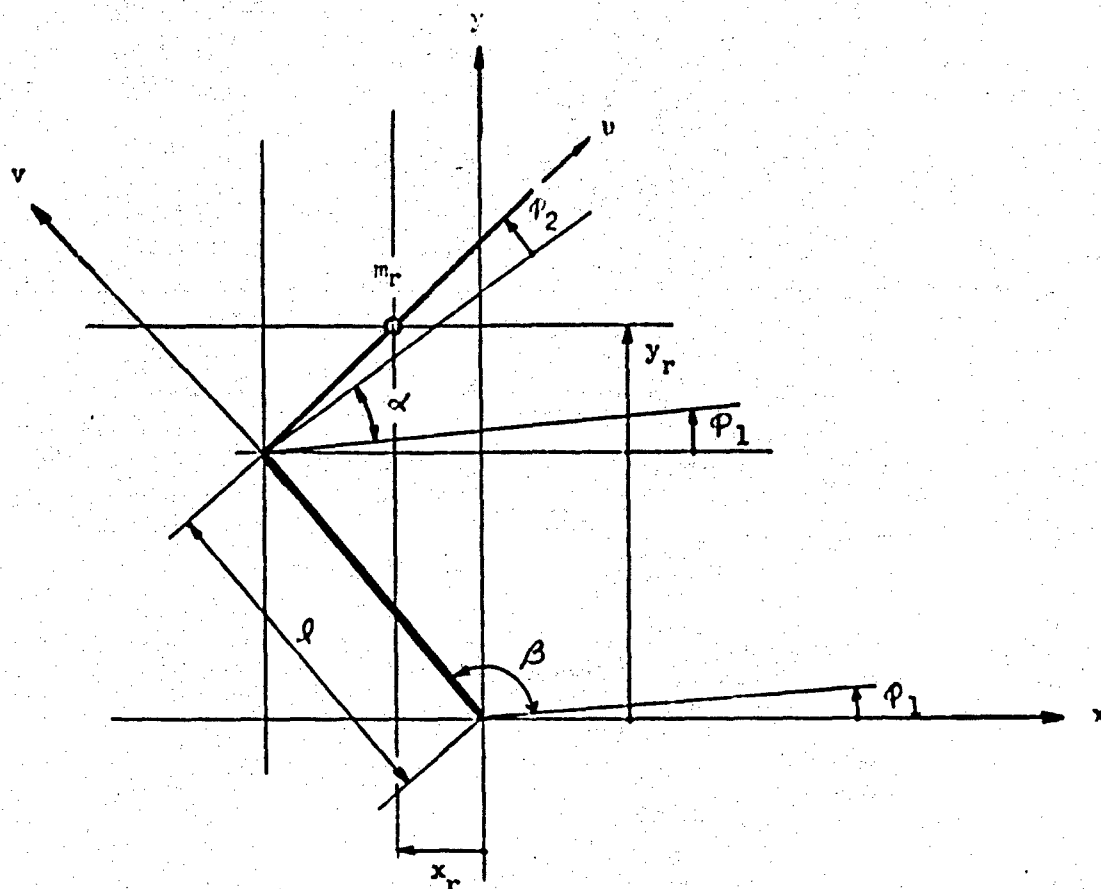


Fig. C-2 GEOMETRIC REPRESENTATION OF AN XM70-TYPE LAUNCHER

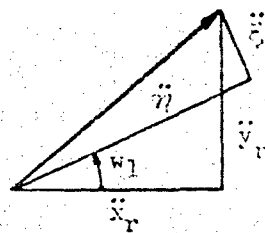


Fig. C-3 VECTOR COMPONENT TRANSFORMATION

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$T'(t)$	applied moment (powder couple equal to $\epsilon F(t)$ where ϵ is the moment arm from the bore center to the recoiling mass cg.
$\ddot{\Phi}(t)$	reactions of the recoil system and cam
N	normal reaction force between the cradle and recoiling mass
T	moment reaction between the cradle and recoiling mass
μ	coefficient of Coulomb friction between the cradle and recoiling mass
e	$e = 1, \dot{u} < 0$; $e = -1, \dot{u} > 0$; $e = 0, \dot{u} = 0$

The XM70 launchers have very low friction ball bearings between the cradle and recoiling parts; the force $\mu|N|$ is therefore negligible compared to the seal friction. Because the absolute value sign, the term becomes unwieldy in later expressions and is neglected at this point.

If I_r is the moment of inertia of the recoiling mass about an axis through the mass center, we may write

$$I_r (\ddot{\Phi}_1 + \ddot{\Phi}_2) = T + T' \quad C-13$$

In order to evaluate the retarding force, $\Phi(t)$, we must know what type of resistance is furnished by the recoil system. The system used in the XM70 launchers contains a preloaded recuperator or air spring to hold the mass in battery position at angles of elevation. Sufficient accuracy will be obtained if a linear spring rate is used to represent the recuperator. The recoil force is furnished by forcing oil through a variable-area orifice. The total retarding force is then made up of the following terms:*

$$K, k(u_0 - u), R(\dot{u}^2, u), H_1, H_3(\dot{u}^2, u) \quad C-14$$

The nomenclature used is

K	recuperator preload
k	recuperator spring rate
R	hydraulic resistance
H_1	constant friction force
H_3	hydraulic system seal frictional force
u_0	in-battery position of the recoiling mass

* For a more complete description of the recoil system, see Bimonthly Report No. 7 of this Project (8130)

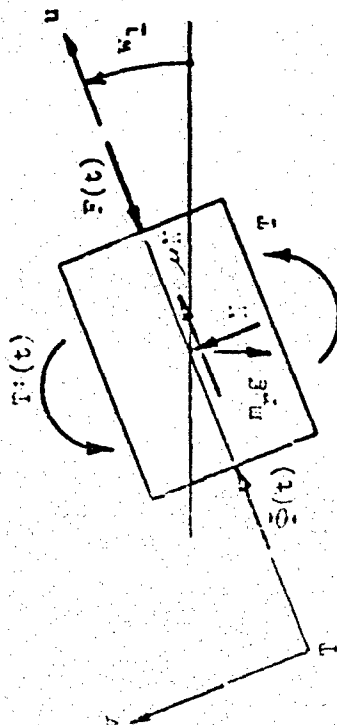


Fig. C-4 FREE BODY DIAGRAM OF THE RECOILING MASS

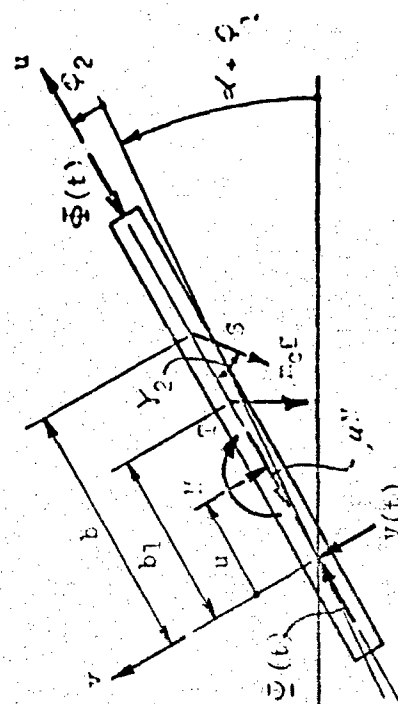


Fig. C-5 FREE BODY DIAGRAM OF THE CRADLE

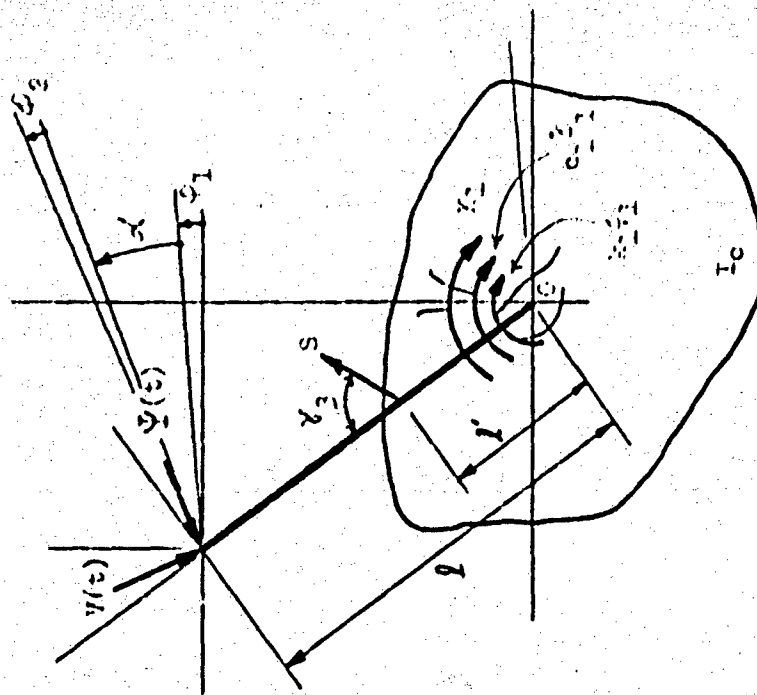


Fig. C-6 FREE BODY DIAGRAM OF THE EQUIVALENT FIRING SUPPORT

The cam reaction force in the direction of recoil may be represented by $C(\ddot{u}, \dot{u}, u)$; the actual functions will be introduced later, in order to keep the lengths of the expressions to a minimum. The total retarding force is, then,

$$\ddot{\Phi}(t) = K + k(u_0 - u) + eR(\dot{u}^2, u) + eH_1 + eH_3(\dot{u}^2, u) + e f C(\ddot{u}, \dot{u}, u). \quad C-15$$

In this expression, $f = 0$, except when $u_1 < u < u_2$ and $u < 0$, that is when the recoiling mass is counterrecoiling through the cam, beginning at u_1 and ending at u_2 , otherwise $f = +1$.

We will now consider a rigid cradle with a free body diagram as shown in Fig. C-5. It is assumed that there is no relative angular motion between the cradle and recoiling mass; because of this, the coordinates of the mass center of the cradle may be immediately written from Eq. C-1. This is done by substituting the constant length b_1 for u . These equations are

$$\begin{aligned} x_c &= l \cos(\beta + \phi_1) + b_1 \cos \omega_1 \\ \text{and} \\ y_c &= l \sin(\beta + \phi_1) + b_1 \sin \omega_1 \end{aligned} \quad C-16$$

The length b_1 is the distance along the u -axis from the trunnion to the cradle mass center, and x_c and y_c are the absolute coordinates of the mass center of the cradle. The corresponding velocity and acceleration components may be found by differentiation of Eq. C-16 or by similar substitution of b_1 for u in Eq. C-2, 3, and 4. The components of acceleration of the cradle mass center are:

$$\begin{aligned} \ddot{x}_c &= -l \ddot{\phi}_1 \sin(\beta + \phi_1) - l \dot{\phi}_1^2 \cos(\beta + \phi_1) \\ &\quad - b_1 (\ddot{\phi}_1 + \ddot{\phi}_2) \sin \omega_1 - b_1 (\dot{\phi}_1 + \dot{\phi}_2)^2 \cos \omega_1 \end{aligned} \quad C-17$$

$$\begin{aligned} \ddot{y}_c &= l \ddot{\phi}_1 \cos(\beta + \phi_1) - l \dot{\phi}_1^2 \sin(\beta + \phi_1) \\ &\quad + b_1 (\ddot{\phi}_1 + \ddot{\phi}_2) \cos \omega_1 - b_1 (\dot{\phi}_1 + \dot{\phi}_2)^2 \sin \omega_1 \end{aligned} \quad C-18$$

If we let the vector sum of the components of x_c and y_c be resolved into components ζ and λ , respectively, in the u -, v -coordinate system using Fig. C-3, we have

$$\ddot{\zeta} = -l \ddot{\phi}_1 \sin \omega_2 - l \dot{\phi}_1^2 \cos \omega_2 - b_1 (\ddot{\phi}_1 + \ddot{\phi}_2) \quad C-19$$

and

$$\ddot{\lambda} = l\ddot{\phi}_1 \cos \omega_2 - l\dot{\phi}_1^2 \sin \omega_2 - b_1 (\ddot{\phi}_1 + \ddot{\phi}_2) \quad C-20$$

The equations of motion of the cradle mass center may now be found by summing forces in the u- and v-directions. These are:

$$m_c \ddot{\xi} = \Psi(t) - \Phi(t) - S \cos \gamma_2 - m_c g \sin \omega_1 \quad C-21$$

$$m_c \ddot{\lambda} = V(t) - N - m_c g \cos \omega_1 - S \sin \gamma_2, \quad C-22$$

where the following nomenclature is used:

- $\Psi(t)$ trunnion reaction in the u-direction
- $V(t)$ trunnion reaction in the v-direction
- S force in the elevating system.

The displacement of one end of the elevating system relative to the other is given by $b\phi_2 \sin \gamma_2$. The force developed, assuming a linear spring and damping proportional to the first derivative of the displacement, is

$$S = K_2 + k_2 b (\sin \gamma_2) \phi_2 + c_2 b (\sin \gamma_2) \dot{\phi}_2 \quad C-23$$

The preload is given by K_2 , the rate by k , and the damping coefficient by c_2 . In this expression, the angle γ_2 is assumed constant. The angular equation of motion of the cradle is found by summing moments about the mass center. From this we get

$$I_c (\ddot{\phi}_1 + \ddot{\phi}_2) = -b_1 V(t) - T + N(b_1 - u) - S(b - b_1) \sin \gamma_2 \quad C-24$$

In Eq. C-24, I_c is the moment of inertia of the cradle about the mass center. The preload K_2 is the preload necessary to balance the component of force due to the weight of the recoiling mass and the cradle itself.

The final equation of motion may now be found by summing moments on the firing support about the fixed point 0 (see Fig. C-6):

$$I_o \ddot{\phi}_1 = \Psi(t) l \sin \omega_2 - V(t) l \cos \omega_2 - S l' \sin \gamma_3 - K_1 - k_1 \phi_1 - c_1 \dot{\phi}_1 \quad C-25$$

For any angle of elevation, it will be assumed that γ_3 is a constant; also the following relationship will be used:

$$\gamma_3 \approx \gamma_3 \Big|_{\alpha=0} - \alpha \equiv \gamma_0 - \alpha \quad \text{C-26}$$

We must now express the recoil and cam forces in terms of the variables of this analysis. The recoil force is given as

$$eR(\dot{u}^2, u) = \frac{\gamma A_p^3}{2 g C_o^2 \alpha_o^2} \dot{u}^2, \quad \text{C-27}$$

where the following nomenclature is used:

γ	density of the oil
A_p	piston area
C_o	orifice coefficient
α_o	orifice area (function of u)

For this analysis, it is assumed that the seal friction is directly proportional to the recoil pressure (or force) and may be expressed as

$$H_3 = \delta R, \quad \text{C-28}$$

where δ is a constant.

A cycloidal indexing cam shape is used in this analysis (as on the XM70 launcher). The axial component of force is

$$C = \frac{I_B}{R_1^2} \left[\frac{h^2}{(u_2 - u_1)^2} \right] \left[\frac{2\pi}{u_2 - u_1} \sin 2\pi \frac{u - u_1}{u_2 - u_1} \right. \\ \left. (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1}) \dot{u}^2 + (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1})^2 \ddot{u} \right] \quad \text{C-29}$$

The launcher has two breech clusters containing three rounds each. For this analysis, it is assumed that only one cluster exists containing six rounds; however, the proper moment of inertia, I_B , corresponding to the combined, double cluster will be used. In Eq. C-29, h is the height of the cam path development and R_1 is the pitch radius.

Substituting the last three relationships into Eq. C-11, 12, 13, 21, 22, 24, and 25, and eliminating the four unknown reactions, T , V , N , and Ψ , leaves three equations. These three equations are second order, non-linear, differential equations, with the variables ϕ_1 , ϕ_2 , and u .

They are:

$$\begin{aligned}
 & m_r (l \sin \omega_2 + u) \ddot{\phi}_1 + \left[ef \frac{I_B}{R_1^2} \frac{h^2}{(u_2 - u_1)^2} (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1})^2 - m_r \right] \ddot{u} \\
 & = -m_r u (\dot{\phi}_1 + \dot{\phi}_2)^2 + F(t) - K - k(u_0 - u) + m_r l \cos \omega_2 \dot{\phi}_1^2 \\
 & - e \left[(1 + \delta) \left(\frac{\gamma_A^3}{2g C_o^2 \alpha_o^2} \right) + f \frac{I_B}{R_1^2} \left(\frac{2\pi h^2}{(u_2 - u_1)^3} \sin 2\pi \frac{u - u_1}{u_2 - u_1} \right) \right. \\
 & \left. \cdot (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1}) \right] \dot{u}^2 - eH_1 + m_r g \sin \omega_1 \quad C-30
 \end{aligned}$$

and

$$\begin{aligned}
 & \ddot{\phi}_1 \left[I_o + m_c l^2 + (l \cos \omega_2 + u) m_r l \cos \omega_2 + m_c b_1 l \cos \omega_2 \right] + \\
 & + \ddot{\phi}_2 \left[m_r u l \cos \omega_2 + m_c b_1 l \cos \omega_2 \right] + \ddot{u} \left[-ef l \sin \omega_2 \cdot \right. \\
 & \left. \cdot \frac{I_B h^2}{R_1^2 (u_2 - u_1)^2} (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1})^2 \right] = l \sin \omega_2 \left[-b_1 m_c \cdot \right. \\
 & \left. \cdot (\dot{\phi}_1 + \dot{\phi}_2)^2 + m_r l \cos \omega_2 + K_o + k(u_0 - u) + m_c g \sin \omega_1 \right] + \\
 & l \cos \omega_2 \left[-2 m_r \dot{u} (\dot{\phi}_1 + \dot{\phi}_2) - g \cos \omega_1 (m_r + m_c) \right] + \\
 & + e l \sin \omega_2 \left[(1 + \delta) \frac{\gamma_A^2}{2g C_o^2 \alpha_o^2} \dot{u}^2 + H_1 + f \frac{I_B 2\pi h^2}{R_1^2 (u_2 - u_1)^3} \right. \\
 & \left. \sin 2\pi \frac{u - u_1}{u_2 - u_1} (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1}) \dot{u}^2 \right] - C_1 \dot{\phi}_1 + \\
 & - k_1 \phi_1 - K_1 + \left[K_2 + k_2 b(\sin \gamma_2) \phi_2 + C_2 b(\sin \gamma_2) \dot{\phi}_2 \right] \\
 & \cdot \left[l \sin (\omega_2 - \gamma_2) - l' \sin (\gamma_o - \alpha) \right] \quad C-31
 \end{aligned}$$

and

$$\begin{aligned}
 & \ddot{\phi}_1 \left[I_c + I_r + m_r u (l \cos \omega_2 + u) + b_1 m_c (l \cos \omega_2 + b_1) \right] \\
 & + \ddot{\phi}_2 \left[I_c + I_r + m_r u^2 + m_c b_1^2 \right] = \dot{\phi}_1^2 l \sin \omega_2 (m_r u + m_c b_1) +
 \end{aligned}$$

$$- 2 m_r u \dot{u} (\dot{\phi}_1 + \dot{\phi}_2) - g \cos \omega_1 (m_r u + m_c b_1) + \epsilon F(t)$$

$$- b \sin \gamma_2 \left[K_2 + b \sin \gamma_2 (k_2 \phi_2 + c_2 \dot{\phi}_2) \right] . \quad C-32$$

III. DISCUSSION

Based upon the assumption made, the solution to the three differential equations of motion just derived will furnish the first mode response of the launcher in the vertical plane. The reaction forces and moments may also be found when the equations of motion are solved. Because of the complexity of these expressions, their solution is obtained on the Technology Center Univac 1105 digital computer. The programming of these equations and the results obtained is presented in other appendices.

APPENDIX D
DIGITAL COMPUTER PROGRAM,
THREE-DEGREE-OF-FREEDOM MATHEMATICAL MODEL

R. M. Brach

I. INTRODUCTION

The equations of motion of a three-degree-of-freedom system representing an XM70-type launcher are presented in Appendix C. Because of the complexity of these equations, they are being solved numerically on Armour Research Foundation's Univac 1105 digital computer. In this appendix, the computer program is described and some results are compared with previous work.

II. COMPUTER PROGRAM

Basically, the computer program constitutes the numerical solution of a set of three, second-order, non-linear differential equations. The solution was programmed in a non-algebraic, symbolic language called USE (Univac Scientific Exchange). A side-by-side listing of the program, as written, and its octal representation are shown in Fig. D-3. Program output format is illustrated in Fig. D-4, which is the first page of output of a typical run.

A subroutine was written for general use by Robert Floyd of Armour Research Foundation based on the numerical technique of Runge and Kutta for the solution of up to 50, first-order, simultaneous differential equations. It is applied here to solve the three, second-order equations after a reduction of their order by three trivial substitutions, such as $\dot{u} = s$. The subroutine is shown listed after the main program with the call address, RWF999. One other special subroutine was written, RWF998, to allow the use of two tabular functions: in this case, an experimental powder-gas forcing function and a table of recoil-system orifice areas. This subroutine employs linear interpolation between points. It is also listed after the main program. All other subroutines used are from the standard ARF library.

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Fig. D-1 shows a general program flow diagram. A detailed flow diagram showing how the time of application of the powder gas force is derived from the position and velocity of the recoiling parts is shown in Fig. D-2.

III. RESULTS

Two comparisons of the output of this program will be made in this appendix: (1) a comparison with a previous computer program of the parent project and (2) a comparison with experimental curves obtained from firings of the Prototype No. 1 launcher.

The physical launcher parameters used for the first comparison are taken from Prototype No. 1. The response curve from an early computer program is shown in Fig. D-5; the corresponding response curve from the present computer program is shown in Fig. D-6. Table D-1 points out the significant similarities of and differences between the two sets of equations of motion and launcher parameters.

The two curves are seen to differ slightly in amplitude and frequency. The amplitude differences may be accounted for by the difference in recoil forces between the two: the recoil force in the current program peaks significantly near the end of recoil as does the recoil force of the actual launcher, whereas in the early program, the recoil force remains nearly constant. The frequency differences probably are due to the inclusion of the elevating system flexibility in the current program, even though that system is very stiff.

The second comparison utilizes the experimental record of the first shot of Burst No. L68-L73, which was fired at Redstone Arsenal on February 22, 1959 (presented in Appendix II-A of Report No. 11 of the parent project). Figure D-7 shows the trail strain, cradle motion, and recoil force measured during that burst. The response as given by the current computer program is shown in Fig. D-8. The input data to the computer program to be compared are shown in Table D-2. The chosen elevating system stiffness value is seen to be relatively low; it may be recalled that the elevating screw brackets on the Prototype No. 1 were found to be relatively flexible.

Table D-1

COMPARISON OF MODELS AND LAUNCHER PARAMETERS

Item	Old Program	New Program
Degrees of freedom	Recoil motion, carriage rotation, hop of firing base	Recoil motion, carriage rotation, cradle rotation relative to the carriage
Launcher Geometry	Pilot No. 1 at 15° elevation	Pilot No. 1 at 15° elevation
Values of Mass	Pilot No. 1	Pilot No. 1
Elevating System Stiffness	Rigid	1×10^6 lb/in.
Equivalent Carriage Stiffness	100×10^6 in. -lb/radian	100×10^6 in. -lb/radian
Recoil System	Computed from equations of a theoretical recoil system	Computed from equations of a theoretical recoil system using actual orifice areas from Pilot No. 3 rods.
Auxiliary damping	None	None

Table D-2

INPUT DATA TO COMPUTER PROGRAM

Item	Value in Program
Launcher Geometry	Pilot No. 3 at 45° elevation
Values of Mass	Pilot No. 3 (See parameter list in Program, Fig. D-3)
Elevating System Stiffness	1.405×10^4 lb/in.
Elevating System Damping Coefficient	90 lb-sec/in.
Equivalent Carriage Stiffness	45×10^6 in. -lb/radian

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The similarity between the experimental and computed curves is striking, even down to details such as the influence of indexing forces between times 0.26 and 0.4. Especially important is the reproduction of the computed curve of cradle motion in the experimental curve just prior to the time of firing of the second round. This cradle motion, the most undesirable part of the gross response of the Prototype No. 1 launcher, was subsequently reduced by designing a stiffer elevating system. By means of an accurate computer program, however, the optimum combination of parameters can easily be found for a desired response.

IV. CONCLUSIONS

The examples presented in this appendix indicate: (1) that the current computer program corresponds closely to previous theoretical work, which means that errors do not exist either in the equations that were programmed or in the program itself; and (2) that the data derived from the current computer program represent gross experimental launcher response very accurately, which means that the program may be used to improve that response.

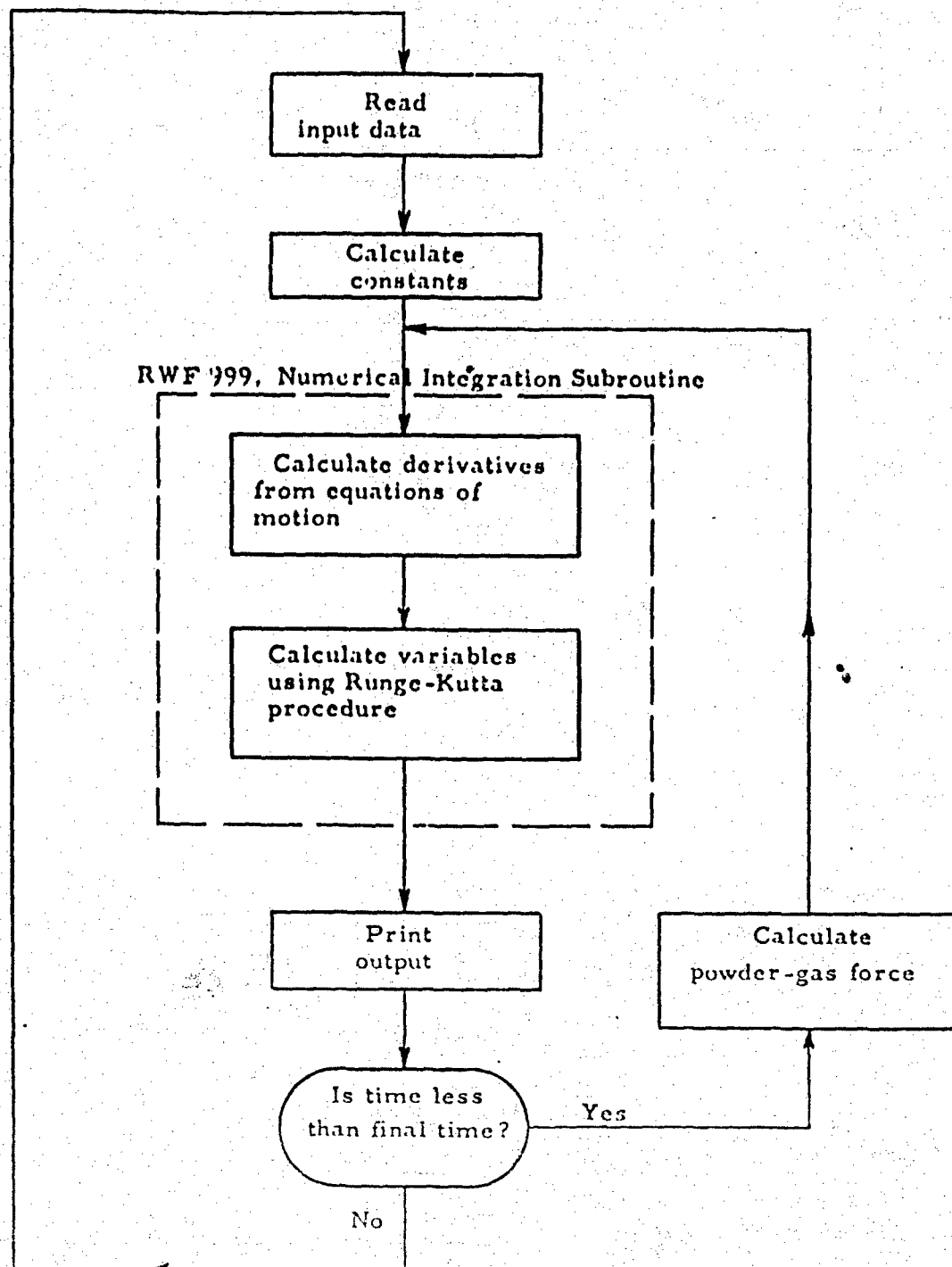
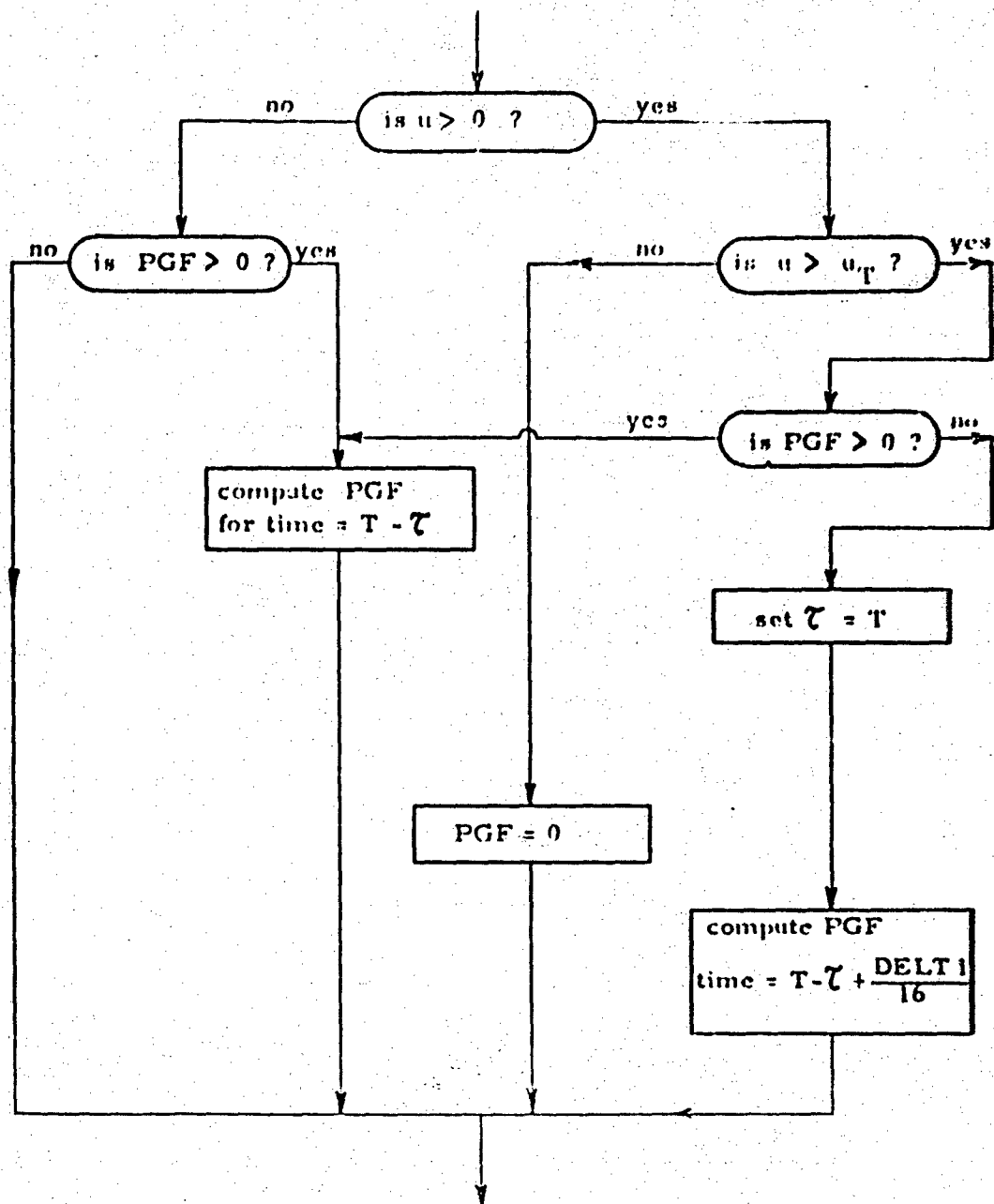


Fig. D-1 COMPUTER PROGRAM FLOW CHART

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PGF Powder-gas force

T Time

UT Trigger Position

τ Time between shots of a burst

DELTA 1 Increment of tabular powder-gas force function

Fig. D-2 POWDER-GAS FORCE SELECTION FLOW DIAGRAM

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FIG. D-3 COMPUTER PROGRAM (Continued)

D-8

FIG. D-3 COMPUTER PROGRAM (Continued)

D-10

01231	01231	70	01231	01231	2420	.	70	.	FL	01231	01
01232	01232	70	01232	01232	2430	.	70	.	FL	01232	01
01233	01233	70	01233	01233	2440	.	70	.	FL	01233	01
01234	01234	70	01234	01234	2450	.	70	.	FL	01234	01
01235	01235	70	01235	01235	2460	.	70	.	FL	01235	01
01236	01236	70	01236	01236	2470	.	70	.	FL	01236	01
01237	01237	70	01237	01237	2480	.	70	.	FL	01237	01
01238	01238	70	01238	01238	2490	.	70	.	FL	01238	01
01239	01239	70	01239	01239	2500	.	70	.	FL	01239	01
01240	01240	70	01240	01240	2510	.	70	.	FL	01240	01
01241	01241	70	01241	01241	2520	.	70	.	FL	01241	01
01242	01242	70	01242	01242	2530	.	70	.	FL	01242	01
01243	01243	70	01243	01243	2540	.	70	.	FL	01243	01
01244	01244	70	01244	01244	2550	.	70	.	FL	01244	01
01245	01245	70	01245	01245	2560	.	70	.	FL	01245	01
01246	01246	70	01246	01246	2570	.	70	.	FL	01246	01
01247	01247	70	01247	01247	2580	.	70	.	FL	01247	01
01248	01248	70	01248	01248	2590	.	70	.	FL	01248	01
01249	01249	70	01249	01249	2600	.	70	.	FL	01249	01
01250	01250	70	01250	01250	2610	.	70	.	FL	01250	01
01251	01251	70	01251	01251	2620	.	70	.	FL	01251	01
01252	01252	70	01252	01252	2630	.	70	.	FL	01252	01
01253	01253	70	01253	01253	2640	.	70	.	FL	01253	01
01254	01254	70	01254	01254	2650	.	70	.	FL	01254	01
01255	01255	70	01255	01255	2660	.	70	.	FL	01255	01
01256	01256	70	01256	01256	2670	.	70	.	FL	01256	01
01257	01257	70	01257	01257	2680	.	70	.	FL	01257	01
01258	01258	70	01258	01258	2690	.	70	.	FL	01258	01
01259	01259	70	01259	01259	2700	.	70	.	FL	01259	01
01260	01260	70	01260	01260	2710	.	70	.	FL	01260	01
01261	01261	70	01261	01261	2720	.	70	.	FL	01261	01
01262	01262	70	01262	01262	2730	.	70	.	FL	01262	01
01263	01263	70	01263	01263	2740	.	70	.	FL	01263	01
01264	01264	70	01264	01264	2750	.	70	.	FL	01264	01
01265	01265	70	01265	01265	2760	.	70	.	FL	01265	01
01266	01266	70	01266	01266	2770	.	70	.	FL	01266	01
01267	01267	70	01267	01267	2780	.	70	.	FL	01267	01
01268	01268	70	01268	01268	2790	.	70	.	FL	01268	01
01269	01269	70	01269	01269	2800	.	70	.	FL	01269	01
01270	01270	70	01270	01270	2810	.	70	.	FL	01270	01
01271	01271	70	01271	01271	2820	.	70	.	FL	01271	01
01272	01272	70	01272	01272	2830	.	70	.	FL	01272	01
01273	01273	70	01273	01273	2840	.	70	.	FL	01273	01
01274	01274	70	01274	01274	2850	.	70	.	FL	01274	01
01275	01275	70	01275	01275	2860	.	70	.	FL	01275	01
01276	01276	70	01276	01276	2870	.	70	.	FL	01276	01
01277	01277	70	01277	01277	2880	.	70	.	FL	01277	01
01278	01278	70	01278	01278	2890	.	70	.	FL	01278	01
01279	01279	70	01279	01279	2900	.	70	.	FL	01279	01
01280	01280	70	01280	01280	2910	.	70	.	FL	01280	01
01281	01281	70	01281	01281	2920	.	70	.	FL	01281	01
01282	01282	70	01282	01282	2930	.	70	.	FL	01282	01
01283	01283	70	01283	01283	2940	.	70	.	FL	01283	01
01284	01284	70	01284	01284	2950	.	70	.	FL	01284	01
01285	01285	70	01285	01285	2960	.	70	.	FL	01285	01
01286	01286	70	01286	01286	2970	.	70	.	FL	01286	01
01287	01287	70	01287	01287	2980	.	70	.	FL	01287	01
01288	01288	70	01288	01288	2990	.	70	.	FL	01288	01
01289	01289	70	01289	01289	3000	.	70	.	FL	01289	01
01290	01290	70	01290	01290	3010	.	70	.	FL	01290	01
01291	01291	70	01291	01291	3020	.	70	.	FL	01291	01
01292	01292	70	01292	01292	3030	.	70	.	FL	01292	01
01293	01293	70	01293	01293	3040	.	70	.	FL	01293	01
01294	01294	70	01294	01294	3050	.	70	.	FL	01294	01
01295	01295	70	01295	01295	3060	.	70	.	FL	01295	01
01296	01296	70	01296	01296	3070	.	70	.	FL	01296	01
01297	01297	70	01297	01297	3080	.	70	.	FL	01297	01
01298	01298	70	01298	01298	3090	.	70	.	FL	01298	01
01299	01299	70	01299	01299	3100	.	70	.	FL	01299	01

FIG. D-3 COMPUTER PROGRAM (Continued)

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01311	01111	30	00000	10713	2910	.	00	.	AL	05
01312	01112	30	00000	00000	2920	.	00	.	.	05
01313	01113	30	00000	00000	2930	.	00	.	.	05
01314	01114	55	00000	00000	2940	.	00	.	TIME	05
01315	01115	32	00000	20000	2950	.	00	.	TIME	05
01316	01116	32	00000	20000	2960	.	00	.	TIME	05
01317	01117	47	00000	10000	2970	.	00	.	TIME	05
01318	01118	32	00000	20000	2980	.	00	.	TIME	05
01319	01119	32	00000	20000	2990	.	00	.	TIME	05
01320	01120	32	00000	20000	3000	.	00	.	TIME	05
01321	01121	32	00000	20000	3010	.	00	.	TIME	05
01322	01122	32	00000	20000	3020	.	00	.	TIME	05
01323	01123	32	00000	20000	3030	.	00	.	TIME	05
01324	01124	32	00000	20000	3040	.	00	.	TIME	05
01325	01125	32	00000	20000	3050	.	00	.	TIME	05
01326	01126	32	00000	20000	3060	.	00	.	TIME	05
01327	01127	32	00000	20000	3070	.	00	.	TIME	05
01328	01128	32	00000	20000	3080	.	00	.	TIME	05
01329	01129	32	00000	20000	3090	.	00	.	TIME	05
01330	01130	32	00000	20000	3100	.	00	.	TIME	05
01331	01131	32	00000	20000	3110	.	00	.	TIME	05
01332	01132	32	00000	20000	3120	.	00	.	TIME	05
01333	01133	32	00000	20000	3130	.	00	.	TIME	05
01334	01134	32	00000	20000	3140	.	00	.	TIME	05
01335	01135	32	00000	20000	3150	.	00	.	TIME	05
01336	01136	32	00000	20000	3160	.	00	.	TIME	05
01337	01137	32	00000	20000	3170	.	00	.	TIME	05
01338	01138	32	00000	20000	3180	.	00	.	TIME	05
01339	01139	32	00000	20000	3190	.	00	.	TIME	05
01340	01140	32	00000	20000	3200	.	00	.	TIME	05
01341	01141	32	00000	20000	3210	.	00	.	TIME	05
01342	01142	32	00000	20000	3220	.	00	.	TIME	05
01343	01143	32	00000	20000	3230	.	00	.	TIME	05
01344	01144	32	00000	20000	3240	.	00	.	TIME	05
01345	01145	32	00000	20000	3250	.	00	.	TIME	05
01346	01146	32	00000	20000	3260	.	00	.	TIME	05
01347	01147	32	00000	20000	3270	.	00	.	TIME	05
01348	01148	32	00000	20000	3280	.	00	.	TIME	05
01349	01149	32	00000	20000	3290	.	00	.	TIME	05
01350	01150	32	00000	20000	3300	.	00	.	TIME	05
01351	01151	32	00000	20000	3310	.	00	.	TIME	05
01352	01152	32	00000	20000	3320	.	00	.	TIME	05
01353	01153	32	00000	20000	3330	.	00	.	TIME	05
01354	01154	32	00000	20000	3340	.	00	.	TIME	05
01355	01155	32	00000	20000	3350	.	00	.	TIME	05
01356	01156	32	00000	20000	3360	.	00	.	TIME	05
01357	01157	32	00000	20000	3370	.	00	.	TIME	05
01358	01158	32	00000	20000	3380	.	00	.	TIME	05
01359	01159	32	00000	20000	3390	.	00	.	TIME	05
01360	01160	32	00000	20000	3400	.	00	.	TIME	05
01361	01161	32	00000	20000	3410	.	00	.	TIME	05
01362	01162	32	00000	20000	3420	.	00	.	TIME	05
01363	01163	32	00000	20000	3430	.	00	.	TIME	05
01364	01164	32	00000	20000	3440	.	00	.	TIME	05
01365	01165	32	00000	20000	3450	.	00	.	TIME	05
01366	01166	32	00000	20000	3460	.	00	.	TIME	05
01367	01167	32	00000	20000	3470	.	00	.	TIME	05
01368	01168	32	00000	20000	3480	.	00	.	TIME	05
01369	01169	32	00000	20000	3490	.	00	.	TIME	05
01370	01170	32	00000	20000	3500	.	00	.	TIME	05

FIG. D-3 COMPUTER PROGRAM (Continued)

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FIG. D-3 COMPUTER PROGRAM (Continued)

D-13

FIG. D-3 COMPUTER PROGRAM (Continued)

D-14

FIG. D-3 COMPUTER PROGRAM (Continued)

D-16

01671	01671	56	13700	13700	4930	.	.	FM	03	.	05	
01672	01672	56	13700	13710	4940	.	.	FM	03	.	05	
01673	01673	56	13700	13715	4950	.	.	FM	03	.	05	
01674	01674	56	13700	13720	4960	.	.	FM	03	.	05	
01675	01675	56	13700	13725	4970	.	.	FM	03	.	05	
01676	01676	56	13700	13730	4980	.	.	FM	03	.	05	
01677	01677	56	13700	13735	4990	.	.	FM	03	.	05	
01678	01678	56	13700	13740	5000	.	.	FM	03	.	05	
01679	01679	56	13700	13745	5010	.	.	FM	03	.	05	
01680	01680	56	13700	13750	5020	.	.	FM	03	.	05	
01681	01681	56	13700	13755	5030	.	.	FM	03	.	05	
01682	01682	56	13700	13760	5040	.	.	FM	03	.	05	
01683	01683	56	13700	13765	5050	.	.	FM	03	.	05	
01684	01684	56	13700	13770	5060	.	.	FM	03	.	05	
01685	01685	56	13700	13775	5070	.	.	FM	03	.	05	
01686	01686	56	13700	13780	5080	.	.	FM	03	.	05	
01687	01687	56	13700	13785	5090	.	.	FM	03	.	05	
01688	01688	56	13700	13790	5100	.	.	FM	03	.	05	
01689	01689	56	13700	13795	5110	.	.	FM	03	.	05	
01690	01690	56	13700	13800	5120	.	.	FM	03	.	05	
01691	01691	56	13700	13805	5130	.	.	FM	03	.	05	
01692	01692	56	13700	13810	5140	.	.	FM	03	.	05	
01693	01693	56	13700	13815	5150	.	.	FM	03	.	05	
01694	01694	56	13700	13820	5160	.	.	FM	03	.	05	
01695	01695	56	13700	13825	5170	.	.	FM	03	.	05	
01696	01696	56	13700	13830	5180	.	.	FM	03	.	05	
01697	01697	56	13700	13835	5190	.	.	FM	03	.	05	
01698	01698	56	13700	13840	5200	.	.	FM	03	.	05	
01699	01699	56	13700	13845	5210	.	.	FM	03	.	05	
01700	01700	56	13700	13850	5220	.	.	FM	03	.	05	
01701	01701	56	13700	13855	5230	.	.	FM	03	.	05	
01702	01702	56	13700	13860	5240	.	.	FM	03	.	05	
01703	01703	56	13700	13865	5250	.	.	FM	03	.	05	
01704	01704	56	13700	13870	5260	.	.	FM	03	.	05	
01705	01705	56	13700	13875	5270	.	.	FM	03	.	05	
01706	01706	56	13700	13880	5280	.	.	FM	03	.	05	
01707	01707	56	13700	13885	5290	.	.	FM	03	.	05	
01708	01708	56	13700	13890	5300	.	.	FM	03	.	05	
01709	01709	56	13700	13895	5310	.	.	FM	03	.	05	
01710	01710	56	13700	13900	5320	.	.	FM	03	.	05	
01711	01711	56	13700	13905	5330	.	.	FM	03	.	05	
01712	01712	56	13700	13910	5340	.	.	FM	03	.	05	
01713	01713	56	13700	13915	5350	.	.	FM	03	.	05	
01714	01714	56	13700	13920	5360	.	.	FM	03	.	05	
01715	01715	56	13700	13925	5370	.	.	FM	03	.	05	
01716	01716	56	13700	13930	5380	.	.	FM	03	.	05	
01717	01717	56	13700	13935	5390	.	.	FM	03	.	05	
01718	01718	56	13700	13940	5400	.	.	FM	03	.	05	
01719	01719	56	13700	13945	5410	.	.	FM	03	.	05	
01720	01720	56	13700	13950	5420	.	.	FM	03	.	05	
01721	01721	56	13700	13955	5430	.	.	FM	03	.	05	
01722	01722	56	13700	13960	5440	.	.	FM	03	.	05	
01723	01723	56	13700	13965	5450	.	.	FM	03	.	05	
01724	01724	56	13700	13970	5460	.	.	FM	03	.	05	
01725	01725	56	13700	13975	5470	.	.	FM	03	.	05	
01726	01726	56	13700	13980	5480	.	.	FM	03	.	05	
01727	01727	56	13700	13985	5490	.	.	FM	03	.	05	
01728	01728	56	13700	13990	5500	.	.	FM	03	.	05	
01729	01729	56	13700	13995	5510	.	.	FM	03	.	05	
01730	01730	56	13700	14000	5520	.	.	FM	03	.	05	
01731	01731	56	13700	14005	5530	.	.	FM	03	.	05	
01732	01732	56	13700	14010	5540	.	.	FM	03	.	05	
01733	01733	56	13700	14015	5550	.	.	FM	03	.	05	
01734	01734	56	13700	14020	5560	.	.	FM	03	.	05	
01735	01735	56	13700	14025	5570	.	.	FM	03	.	05	
01736	01736	56	13700	14030	5580	.	.	FM	03	.	05	
01737	01737	56	13700	14035	5590	.	.	FM	03	.	05	
01738	01738	56	13700	14040	5600	.	.	FM	03	.	05	
01739	01739	56	13700	14045	5610	.	.	FM	03	.	05	
01740	01740	56	13700	14050	5620	.	.	FM	03	.	05	
01741	01741	56	13700	14055	5630	.	.	FM	03	.	05	
01742	01742	56	13700	14060	5640	.	.	FM	03	.	05	
01743	01743	56	13700	14065	5650	.	.	FM	03	.	05	
01744	01744	56	13700	14070	5660	.	.	FM	03	.	05	
01745	01745	56	13700	14075	5670	.	.	FM	03	.	05	
01746	01746	56	13700	14080	5680	.	.	FM	03	.	05	
01747	01747	56	13700	14085	5690	.	.	FM	03	.	05	
01748	01748	56	13700	14090	5700	.	.	FM	03	.	05	
01749	01749	56	13700	14095	5710	.	.	FM	03	.	05	
01750	01750	56	13700	14100	5720	.	.	FM	03	.	05	

FIG. D-3 COMPUTER PROGRAM (Continued)

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01751	01751	11	01750	01750	5300	.	.	TP	.	Q	.	TP	.	5300
01752	01752	55	01750	01750	5400	.	.	FS	.	Q	.	TP	.	5400
01753	01753	11	01750	01750	5410	.	.	TP	.	Q	.	TP	.	5410
01754	01754	11	01750	01750	5420	.	.	TP	.	Q	.	TP	.	5420
01755	01755	15	01750	01750	5430	.	.	TP	.	Q	.	TP	.	5430
01756	01756	11	01750	01750	5440	.	.	TP	.	Q	.	TP	.	5440
01757	01757	54	01750	01750	5450	.	.	TP	.	Q	.	TP	.	5450
01758	01758	11	01750	01750	5460	.	.	TP	.	Q	.	TP	.	5460
01759	01759	15	01750	01750	5470	.	.	TP	.	Q	.	TP	.	5470
01760	01760	11	01750	01750	5480	.	.	TP	.	Q	.	TP	.	5480
01761	01761	15	01750	01750	5490	.	.	TP	.	Q	.	TP	.	5490
01762	01762	11	01750	01750	5500	.	.	TP	.	Q	.	TP	.	5500
01763	01763	15	01750	01750	5510	.	.	TP	.	Q	.	TP	.	5510
01764	01764	15	01750	01750	5520	.	.	TP	.	Q	.	TP	.	5520
01765	01765	55	01750	01750	5530	.	.	TP	.	Q	.	TP	.	5530
01766	01766	11	01750	01750	5540	.	.	TP	.	Q	.	TP	.	5540
01767	01767	15	01750	01750	5550	.	.	TP	.	Q	.	TP	.	5550
01768	01768	11	01750	01750	5560	.	.	TP	.	Q	.	TP	.	5560
01769	01769	15	01750	01750	5570	.	.	TP	.	Q	.	TP	.	5570
01770	01770	11	01750	01750	5580	.	.	TP	.	Q	.	TP	.	5580
01771	01771	15	01750	01750	5590	.	.	TP	.	Q	.	TP	.	5590
01772	01772	11	01750	01750	5600	.	.	TP	.	Q	.	TP	.	5600
01773	01773	15	01750	01750	5610	.	.	TP	.	Q	.	TP	.	5610
01774	01774	15	01750	01750	5620	.	.	TP	.	Q	.	TP	.	5620
01775	01775	55	01750	01750	5630	.	.	TP	.	Q	.	TP	.	5630
01776	01776	11	01750	01750	5640	.	.	TP	.	Q	.	TP	.	5640
01777	01777	15	01750	01750	5650	.	.	TP	.	Q	.	TP	.	5650
01778	01778	11	01750	01750	5660	.	.	TP	.	Q	.	TP	.	5660
01779	01779	15	01750	01750	5670	.	.	TP	.	Q	.	TP	.	5670
01780	01780	11	01750	01750	5680	.	.	TP	.	Q	.	TP	.	5680
01781	01781	15	01750	01750	5690	.	.	TP	.	Q	.	TP	.	5690
01782	01782	11	01750	01750	5700	.	.	TP	.	Q	.	TP	.	5700
01783	01783	15	01750	01750	5710	.	.	TP	.	Q	.	TP	.	5710
01784	01784	11	01750	01750	5720	.	.	TP	.	Q	.	TP	.	5720
01785	01785	15	01750	01750	5730	.	.	TP	.	Q	.	TP	.	5730
01786	01786	11	01750	01750	5740	.	.	TP	.	Q	.	TP	.	5740
01787	01787	15	01750	01750	5750	.	.	TP	.	Q	.	TP	.	5750
01788	01788	11	01750	01750	5760	.	.	TP	.	Q	.	TP	.	5760
01789	01789	15	01750	01750	5770	.	.	TP	.	Q	.	TP	.	5770
01790	01790	11	01750	01750	5780	.	.	TP	.	Q	.	TP	.	5780
01791	01791	15	01750	01750	5790	.	.	TP	.	Q	.	TP	.	5790
01792	01792	11	01750	01750	5800	.	.	TP	.	Q	.	TP	.	5800
01793	01793	15	01750	01750	5810	.	.	TP	.	Q	.	TP	.	5810
01794	01794	11	01750	01750	5820	.	.	TP	.	Q	.	TP	.	5820
01795	01795	15	01750	01750	5830	.	.	TP	.	Q	.	TP	.	5830
01796	01796	11	01750	01750	5840	.	.	TP	.	Q	.	TP	.	5840
01797	01797	15	01750	01750	5850	.	.	TP	.	Q	.	TP	.	5850
01798	01798	11	01750	01750	5860	.	.	TP	.	Q	.	TP	.	5860
01799	01799	15	01750	01750	5870	.	.	TP	.	Q	.	TP	.	5870
01800	01800	11	01750	01750	5880	.	.	TP	.	Q	.	TP	.	5880
01801	01801	15	01750	01750	5890	.	.	TP	.	Q	.	TP	.	5890
01802	01802	11	01750	01750	5900	.	.	TP	.	Q	.	TP	.	5900
01803	01803	15	01750	01750	5910	.	.	TP	.	Q	.	TP	.	5910
01804	01804	11	01750	01750	5920	.	.	TP	.	Q	.	TP	.	5920
01805	01805	15	01750	01750	5930	.	.	TP	.	Q	.	TP	.	5930
01806	01806	11	01750	01750	5940	.	.	TP	.	Q	.	TP	.	5940
01807	01807	15	01750	01750	5950	.	.	TP	.	Q	.	TP	.	5950
01808	01808	11	01750	01750	5960	.	.	TP	.	Q	.	TP	.	5960
01809	01809	15	01750	01750	5970	.	.	TP	.	Q	.	TP	.	5970
01810	01810	11	01750	01750	5980	.	.	TP	.	Q	.	TP	.	5980
01811	01811	15	01750	01750	5990	.	.	TP	.	Q	.	TP	.	5990
01812	01812	11	01750	01750	6000	.	.	TP	.	Q	.	TP	.	6000
01813	01813	15	01750	01750	6010	.	.	TP	.	Q	.	TP	.	6010
01814	01814	11	01750	01750	6020	.	.	TP	.	Q	.	TP	.	6020
01815	01815	15	01750	01750	6030	.	.	TP	.	Q	.	TP	.	6030
01816	01816	11	01750	01750	6040	.	.	TP	.	Q	.	TP	.	6040
01817	01817	15	01750	01750	6050	.	.	TP	.	Q	.	TP	.	6050
01818	01818	11	01750	01750	6060	.	.	TP	.	Q	.	TP	.	6060
01819	01819	15	01750	01750	6070	.	.	TP	.	Q	.	TP	.	6070
01820	01820	11	01750	01750	6080	.	.	TP	.	Q	.	TP	.	6080
01821	01821	15	01750	01750	6090	.	.	TP	.	Q	.	TP	.	6090
01822	01822	11	01750	01750	6100	.	.	TP	.	Q	.	TP	.	6100
01823	01823	15	01750	01750	6110	.	.	TP	.	Q	.	TP	.	6110
01824	01824	11	01750	01750	6120	.	.	TP	.	Q	.	TP	.	6120
01825	01825	15	01750	01750	6130	.	.	TP	.	Q	.	TP	.	6130
01826	01826	11	01750	01750	6140	.	.	TP	.	Q	.	TP	.	6140
01827	01827	15	01750	01750	6150	.	.	TP	.	Q	.	TP	.	6150
01828	01828	11	01750	01750	6160	.	.	TP	.	Q	.	TP	.	6160
01829	01829	15	01750	01750	6170	.	.	TP	.	Q	.	TP	.	6170
01830	01830	11	01750	01750	6180	.	.	TP	.	Q	.	TP	.	6180
01831	01831	15	01750	01750	6190	.	.	TP	.	Q	.	TP	.	6190
01832	01832	11	01750	01750	6200	.	.	TP	.	Q	.	TP	.	6200
01833	01833	15	01750	01750	6210	.	.	TP	.	Q	.	TP	.	6210
01834	01834	11	01750	01750	6220	.	.	TP	.	Q	.	TP	.	6220
01835	01835	15	01750	01750	6230	.	.	TP	.	Q	.	TP	.	6230
01836	01836	11	01750	01750	6240	.	.	TP	.	Q	.	TP	.	6240
01837	01837	15	01750	01750	6250	.	.	TP	.	Q	.	TP	.	6250
01838	01838	11	01750	01750	6260	.	.	TP	.	Q	.	TP	.	6260
01839	01839	15	01750	01750	6270	.	.	TP	.	Q	.	TP	.	6270
01840	01840	11	01750	01750	6280	.	.	TP	.	Q	.	TP	.	6280
01841	01841	15	01750	01750	6290	.	.	TP	.	Q	.	TP	.	6290
01842	01842	11	01750	01750	6300	.	.	TP	.	Q	.	TP	.	6300
01843	01843	15	01750	01750	6310	.	.	TP	.	Q	.	TP	.	6310
01844	01844	11	01750	01750	6320	.	.	TP	.	Q	.	TP	.	6320
01845	01845	15	01750	01750	6330	.	.	TP	.	Q	.	TP	.	6330
01846	01846	11	01750	01750	6340	.	.	TP	.	Q	.	TP	.	6340
01847	01847	15	01750	01750	6350	.	.	TP	.	Q	.	TP	.	6350
01848	01848	11	01750	01750	6360	.	.	TP	.	Q	.	TP	.	6360
01849	01849	15	01750	01750	6370	.	.	TP	.	Q	.			

FIG. D-3 COMPUTER PROGRAM (Continued)

D-19

FIG. D-3 COMPUTER PROGRAM (Continued)

D-20

FIG. D-3 COMPUTER PROGRAM (Continued)

D-21

FIG. D-3 COMPUTER PROGRAM (Continued)

D-22

02331	02331	46	11000	01015	7700	.	FM	03	00	00
02332	02332	57	11000	01063	7700	.	FD	03	00	00
02333	02333	56	11000	01014	7700	.	FA	03	00	00
02334	02334	44	11000	01015	7700	.	FM	03	00	00
02335	02335	47	11000	01043	7800	.	FD	03	00	00
02336	02336	11	11000	12735	7910	.	FM	03	00	00
02337	02337	54	15724	12732	7920	.	FA	03	00	00
02338	02338	54	11000	13731	7930	.	FA	03	00	00
02339	02339	54	11000	13047	7930	.	FA	03	00	00
02340	02340	54	11000	13046	7930	.	FA	03	00	00
02341	02341	11	11000	12731	7930	.	FM	03	00	00
02342	02342	57	15735	12735	7930	.	FD	03	00	00
02343	02343	57	15735	12735	7930	.	FD	03	00	00
02344	02344	57	15735	12735	7930	.	FD	03	00	00
02345	02345	11	11000	13045	7930	.	FM	03	00	00
02346	02346	57	15735	12735	7930	.	FD	03	00	00
02347	02347	57	15735	12735	7930	.	FD	03	00	00
02348	02348	57	15735	12735	7930	.	FD	03	00	00
02349	02349	57	15735	12735	7930	.	FD	03	00	00
02350	02350	11	11000	13045	7930	.	FM	03	00	00
02351	02351	57	15735	12735	7930	.	FD	03	00	00
02352	02352	11	11000	13045	7930	.	FM	03	00	00
02353	02353	57	15735	12735	7930	.	FD	03	00	00
02354	02354	55	11000	13045	7930	.	FM	03	00	00
02355	02355	13	11000	13045	7930	.	FM	03	00	00
02356	02356	57	15735	12735	7930	.	FD	03	00	00
02357	02357	11	11000	13045	7930	.	FM	03	00	00
02358	02358	57	15735	12735	7930	.	FD	03	00	00
02359	02359	57	15735	12735	7930	.	FD	03	00	00
02360	02360	57	15735	12735	7930	.	FD	03	00	00
02361	02361	45	11000	13045	8010	.	FM	03	00	00
02362	02362	13	11000	13045	8020	.	FM	03	00	00
02363	02363	57	15735	12735	8030	.	FD	03	00	00
02364	02364	11	11000	13045	8040	.	FM	03	00	00
02365	02365	57	15735	12735	8050	.	FD	03	00	00
02366	02366	55	11000	13045	8060	.	FM	03	00	00
02367	02367	13	11000	13045	8070	.	FM	03	00	00
02368	02368	57	15735	12735	8080	.	FD	03	00	00
02369	02369	11	11000	13045	8090	.	FM	03	00	00
02370	02370	57	15735	12735	8100	.	FD	03	00	00
02371	02371	11	11000	13045	8110	.	FM	03	00	00
02372	02372	57	15735	12735	8120	.	FD	03	00	00
02373	02373	55	11000	13045	8130	.	FM	03	00	00
02374	02374	13	11000	13045	8140	.	FM	03	00	00
02375	02375	57	15735	12735	8150	.	FD	03	00	00
02376	02376	11	11000	13045	8160	.	FM	03	00	00
02377	02377	57	15735	12735	8170	.	FD	03	00	00
02378	02378	45	11000	13045	8180	.	FM	03	00	00
02379	02379	13	11000	13045	8190	.	FM	03	00	00
02380	02380	57	15735	12735	8200	.	FD	03	00	00
02381	02381	11	11000	13045	8210	.	FM	03	00	00
02382	02382	57	15735	12735	8220	.	FD	03	00	00
02383	02383	55	11000	13045	8230	.	FM	03	00	00
02384	02384	13	11000	13045	8240	.	FM	03	00	00
02385	02385	57	15735	12735	8250	.	FD	03	00	00
02386	02386	11	11000	13045	8260	.	FM	03	00	00
02387	02387	57	15735	12735	8270	.	FD	03	00	00
02388	02388	57	15735	12735	8280	.	FD	03	00	00
02389	02389	57	15735	12735	8290	.	FD	03	00	00
02390	02390	57	15735	12735	8300	.	FD	03	00	00
02391	02391	57	15735	12735	8310	.	FD	03	00	00
02392	02392	57	15735	12735	8320	.	FD	03	00	00
02393	02393	57	15735	12735	8330	.	FD	03	00	00
02394	02394	57	15735	12735	8340	.	FD	03	00	00
02395	02395	57	15735	12735	8350	.	FD	03	00	00
02396	02396	57	15735	12735	8360	.	FD	03	00	00
02397	02397	57	15735	12735	8370	.	FD	03	00	00
02398	02398	57	15735	12735	8380	.	FD	03	00	00
02399	02399	57	15735	12735	8390	.	FD	03	00	00
02400	02400	57	15735	12735	8400	.	FD	03	00	00
02401	02401	57	15735	12735	8410	.	FD	03	00	00
02402	02402	57	15735	12735	8420	.	FD	03	00	00
02403	02403	57	15735	12735	8430	.	FD	03	00	00
02404	02404	57	15735	12735	8440	.	FD	03	00	00
02405	02405	57	15735	12735	8450	.	FD	03	00	00
02406	02406	57	15735	12735	8460	.	FD	03	00	00
02407	02407	57	15735	12735	8470	.	FD	03	00	00
02408	02408	57	15735	12735	8480	.	FD	03	00	00
02409	02409	57	15735	12735	8490	.	FD	03	00	00
02410	02410	57	15735	12735	8500	.	FD	03	00	00
02411	02411	57	15735	12735	8510	.	FD	03	00	00
02412	02412	57	15735	12735	8520	.	FD	03	00	00
02413	02413	57	15735	12735	8530	.	FD	03	00	00
02414	02414	57	15735	12735	8540	.	FD	03	00	00
02415	02415	57	15735	12735	8550	.	FD	03	00	00
02416	02416	57	15735	12735	8560	.	FD	03	00	00
02417	02417	57	15735	12735	8570	.	FD	03	00	00
02418	02418	57	15735	12735	8580	.	FD	03	00	00
02419	02419	57	15735	12735	8590	.	FD	03	00	00
02420	02420	57	15735	12735	8600	.	FD	03	00	00
02421	02421	57	15735	12735	8610	.	FD	03	00	00
02422	02422	57	15735	12735	8620	.	FD	03	00	00
02423	02423	57	15735	12735	8630	.	FD	03	00	00
02424	02424	57	15735	12735	8640	.	FD	03	00	00
02425	02425	57	15735	12735	8650	.	FD	03	00	00
02426	02426	57	15735	12735	8660	.	FD	03	00	00
02427	02427	57	15735	12735	8670	.	FD	03	00	00
02428	02428	57	15735	12735	8680	.	FD	03	00	00
02429	02429	57	15735	12735	8690	.	FD	03	00	00
02430	02430	57	15735	12735	8700	.	FD	03	00	00
02431	02431	57	15735	12735	8710	.	FD	03	00	00
02432	02432	57	15735	12735	8720	.	FD	03	00	00
02433	02433	57	15735	12735	8730	.	FD	03	00	00
02434	02434	57	15735	12735	8740	.	FD	03	00	00
02435	02435	57	15735	12735	8750	.	FD	03	00	00
02436	02436	57	15735	12735	8760	.	FD	03	00	00
02437	02437	57	15735	12735	8770	.	FD	03	00	00
02438	02438	57	15735	12735	8780	.	FD	03	00	00
02439	02439	57	15735	12735	8790	.	FD	03	00	00
02440	02440	57	15735	12735	8800	.	FD	03	00	00
02441	02441	57	15735	12735	8810	.	FD	03	00	00
02442	02442	57	15735	12735	8820	.	FD	03	00	00
02443	02443	57	15735	12735	8830	.	FD	03	00	00
02444	02444	57	15735	12735	8840	.	FD	03	00	00
02445	02445	57	15735	12735	8850	.	FD	03	00	00
02446	02446	57	15735	12735	8860	.	FD	03	00	00
02447	02447	57	15735	12735	8870	.	FD	03	00	00
02448	02448	57	15735	12735	8880	.	FD	03	00	00
02449	02449	57	15735	12735	8890	.	FD	03	00	00
02450	02450	57	15735	12735	8900	.	FD	03	00	00
02451	02451	57	15735	12735	8910	.	FD	03	00	00
02452	02452	57	15735	12735	8920	.	FD	03	00	00
02453	02453	57	15735	12735	8930	.	FD	03	00	00
02454	02454	57	15735	12735	8940	.	FD	03	00	00
02455	02455	57	15735	12735	8950	.	FD	03	00	00
02456	02456	57	15735	12735	8960	.	FD	03	00	00
02457	02457	57	15735	12735	8970	.	FD	03	00	00
02458	02458	57	15735	12735	8980	.	FD	03	00	00
02459	02459	57	15735	12735	8990	.	FD	03	00	00
02460	02460	57	15735	12735	9000	.	FD	03	00	00
02461	02461	57	15735	12735	9010	.	FD	03	00	00
02462	02462	57	15735	12735	9020	.	FD	03	00	00
02463	02463	57	15735	12735	9030	.	FD	03		

FIG. D-3. COMPUTER PROGRAM (Continued)

D-27

D-29

17361	17361	11	30000	17374	X	362.	.	AMP	.	TP	.	FILL	FA	IS
17362	17362	16	31000	17351	X	363.	.	.	.	ST	.	13	L-1	IS
17363	17363	55	17375	17374	X	364.	.	.	.	FS	.	FB	FA	IS
17364	17364	01	15437	17374	X	365.	.	.	.	FP	.	FRACT	FA	IS
17365	17365	11	31000	17336	X	366.	.	.	.	TP	.	13	ANS	IS
17366	17366	45	00000	17335	X	367.	.	.	.	HJ	.	.	EXIT	IS
17367	17367	00	00000	00001	X	368.	.	.	.	X	.	11	.	IS
17370	17370	23	56000	00000	X	369.	.	.	.	323	.	34000	00000	IS
17371	17371	00	00000	00000	X	370.	.	.	.	300	.	.	.	IS
17372	17372	00	00436	00000	X	371.	.	.	.	INTGR	.	THL1	.	IS
17373	17373	00	00532	00000	X	372.	.	.	.	THL2	.	THL2	.	IS
17374	17374	00	00000	00000	X	373.	.	.	.	FA	.	.	.	IS
17375	17375	00	00000	00000	X	374.	.	.	.	00	.	.	.	IS
					X	375.	.	.	.	ENDSUB	.	.	.	IS
					X	376.	.	.	.	SUB	.	ARF074	74 WORDS	
					X	377.	.	.	.	SUB	.	ARF051	334 WORDS	
					X	378.	.	.	.	SUB	.	ARF052	51 WORDS	
					X	379.	.	.	.	SUB	.	ARF034	200 WORDS	
					X	380.	.	.	.	SUB	.	ARF055	24 WORDS	

FIG. D-3 COMPUTER PROGRAM (CONTINUED)

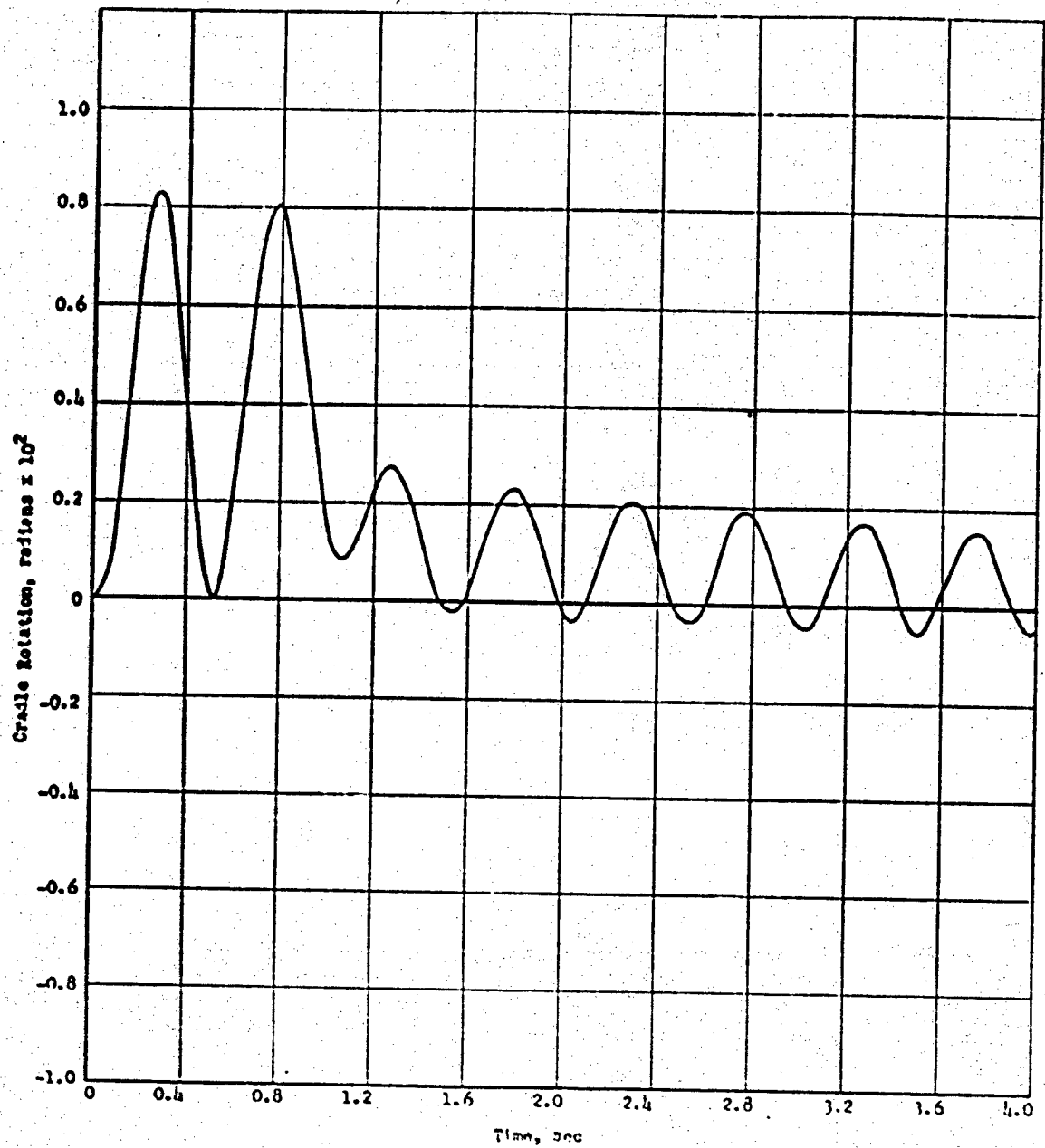


Fig. D-5 LAUNCHER RESPONSE FROM EARLY COMPUTER PROGRAM

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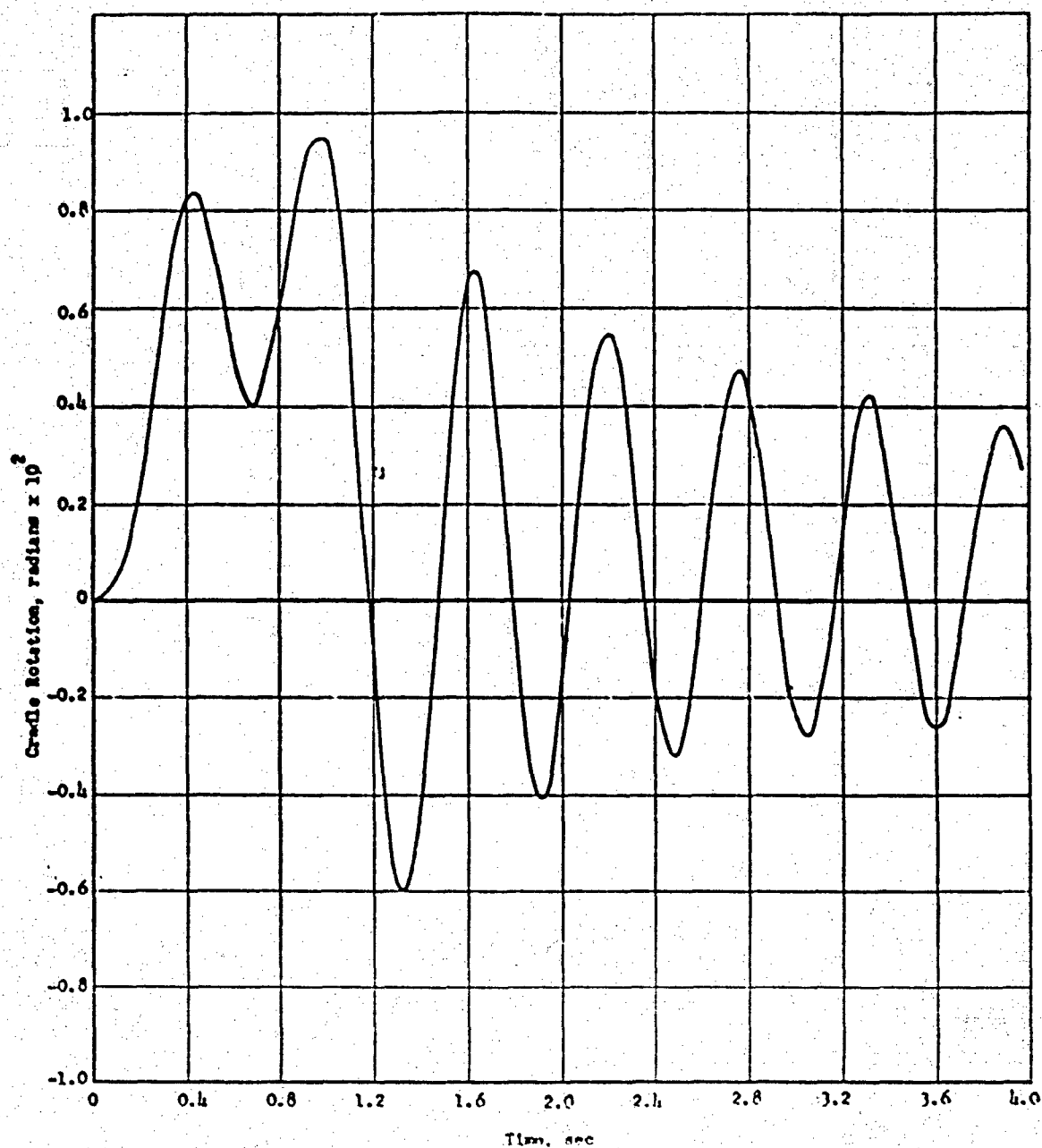


Fig. D-6 LAUNCHER RESPONSE FROM PRESENT COMPUTER PROGRAM

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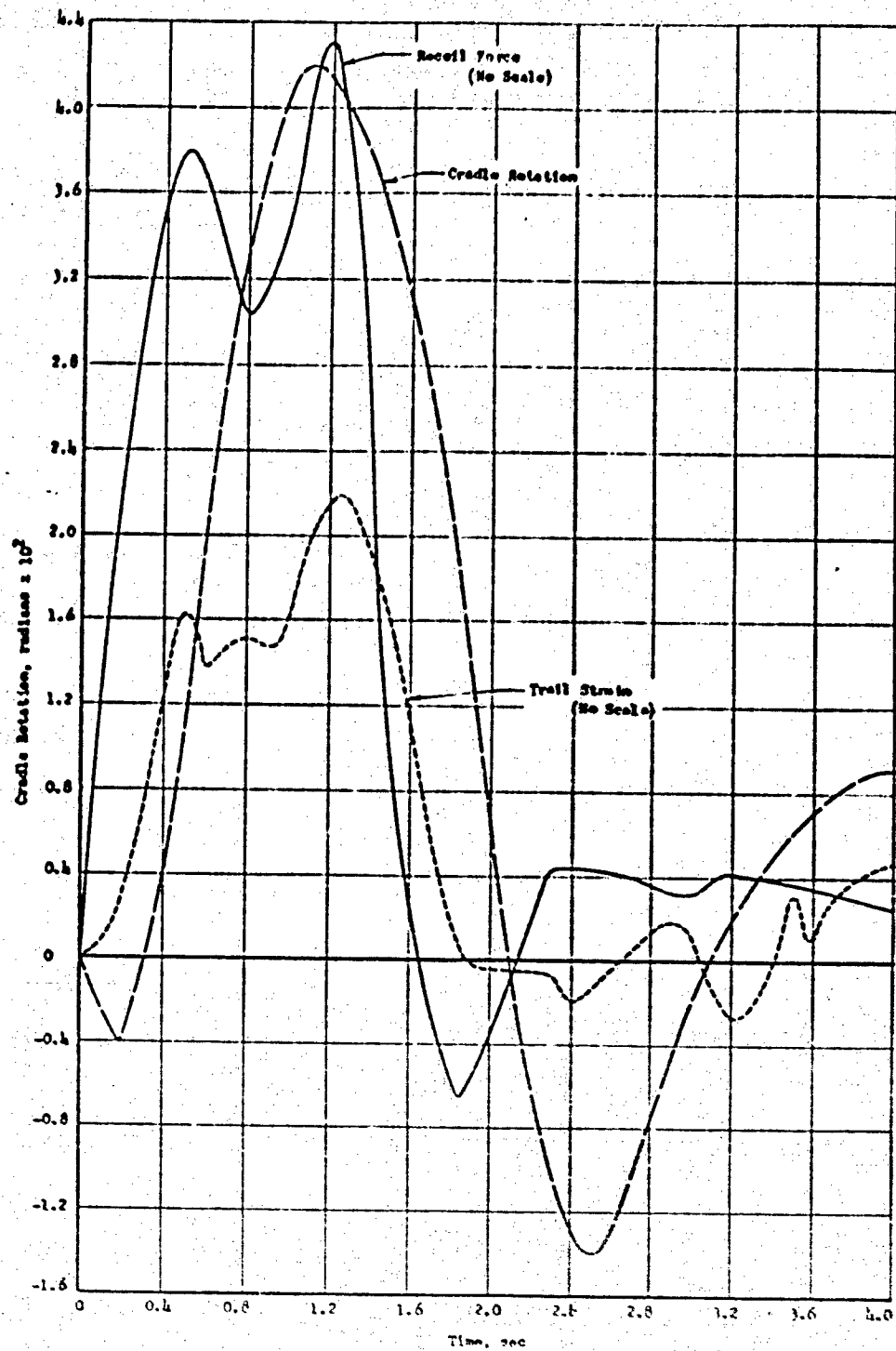


Fig. D-7 EXPERIMENTAL LAUNCHER RESPONSE

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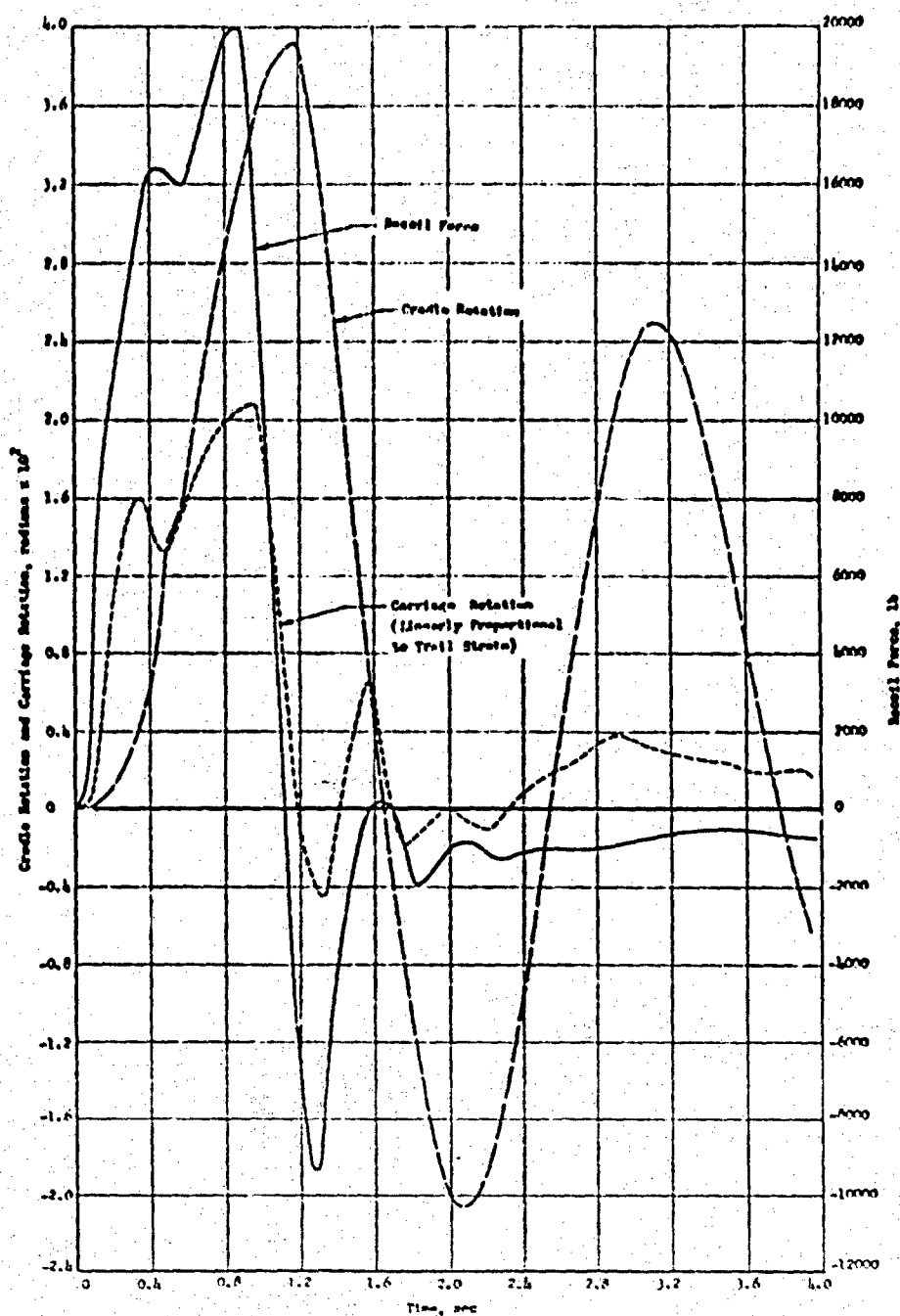


Fig. D-3 LAUNCHER RESPONSE FROM PRESENT COMPUTER PROGRAM
CORRESPONDING TO EXPERIMENTAL RESPONSE

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APPENDIX E
OPTIMUM ELEVATING SYSTEM PARAMETERS

Coleen Murray

I. INTRODUCTION

The purpose of this study was to determine the effects of various combinations of elevating system stiffness and damping upon gross launcher motion and to find the optimum combination of such stiffness and damping. The optimum combination was defined as that combination which yields a consistently low value of cradle displacement prior to the firing of each of the successive rounds of a burst and which concurrently yields a minimum corresponding cradle velocity. The work was based on the assumption that cradle oscillations sustained during a burst firing contribute directly to launching errors that cause dispersion of the shots of a burst. This study is concerned primarily with cradle motion, although barrel motion relative to the cradle is also a major cause of launching errors.

This study utilizes the work of Philip Meyfarth ^{1/} in order to define the general area of optimum damping for a given stiffness. Optimization of Meyfarth's third-order system yielded an optimum damping coefficient for any given elevating system stiffness. The damping determined by Meyfarth's work was used to indicate a value about which the damping should be varied in the Model V solutions in order to obtain the actual optimum. The ranges of optimum damping thus obtained for the several values of stiffness were then compared to each other, and a range of optimum stiffness, with corresponding optimum damping, was chosen. The third-order system was used because of the availability of the response data in the reference and the resulting simplification of the optimization procedure for the three-degree-of-freedom computer solution.

A few variations of equivalent carriage stiffness for the optimum elevating system were also investigated to determine the effect of this stiffness

^{1/} Meyfarth, P., Dynamic Response Plots and Design Charts for Third-Order Linear Systems, Research Memo No. R. M. 7401-3, Massachusetts Institute of Technology, 1958.

on cradle motion and trail hop, which is the upward deflection of the trails observed during firing of the XM70E1 launcher when the recoiling assembly is in its extreme recoil position. The values of launcher parameters maintained constant in the study are those of the XM70E1 Prototype No. 3 launcher at 0° traverse and 45° elevation.

II. DISCUSSION

A. Third-Order Analysis

The angular motion of the launcher was approximated by the motion of the following linear system with the proper conversion of units:

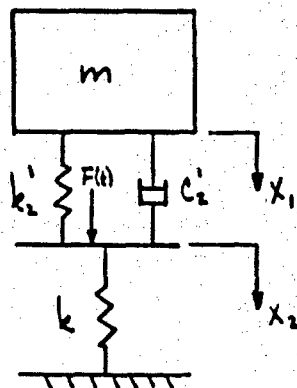


Fig. E-1 MODEL OF THIRD-ORDER SYSTEM

where

- m is an equivalent mass moment of inertia of the tipping parts about the trunnion, 7500 in. -lb-sec²
- k_2 is the equivalent angular stiffness of the elevating system
- c_2 is the equivalent angular damping coefficient of the elevating system
- k is the combined angular stiffness of the carriages and trails, 3.5×10^7 in. -lb/radian
- $F(t)$ is a forcing function applied to the trunnion.

The equation of motion for m is:

$$m \ddot{x}_1 = -k'_2 (x_1 - x_2) - c'_2 (\dot{x}_1 - \dot{x}_2) \quad \text{E-1}$$

From the statics at x_2 , we have:

$$kx_2 = k'_2 (x_1 - x_2) + c'_2 (\dot{x}_1 - \dot{x}_2) + F(t) \quad \text{E-2}$$

By eliminating x_2 from E-1 and rewriting in terms of differential operators, one obtains:

$$\begin{aligned} D^3 x_1 (c'_2) + D^2 x_1 (k + k'_2) + D x_1 (c'_2 k/m) + x_1 (k \cdot k'_2 / m) \\ = (c'_2 / m) D [F(t)] + (k'_2 / m) [F(t)] \end{aligned} \quad \text{E-3}$$

The left-hand side of Eq. E-3 may be factored into three factors, each representing a root of the system*. When two of these roots are complex conjugates, the parameters τ , ω , and ξ may be defined by the following equations:

$$\frac{c'_2 m}{k k'_2} = \frac{\tau}{\omega^2} \quad \text{E-4a}$$

$$\frac{(k + k'_2) m}{k k'_2} = \frac{(2\xi\tau\omega + 1)}{\omega^2} \quad \text{E-4b}$$

$$\frac{c'_2}{k'_2} = \frac{(\tau\omega + 2\xi)}{\omega} \quad \text{E-4c}$$

where

- τ is the constant of the factor representing the real root
- ξ is the damping ratio associated with the complex-conjugate pair
- ω is the undamped natural frequency associated with the complex-conjugate pair.

Let the right-hand side of Eq. E-3 be an impulse function of strength 1; then, the solution of Eq. E-3 is:

* For a complete discussion of the following technique consult Meyfarth, Reference 1.

$$\frac{k' k'_2}{m \omega^2} y(\omega t) = \frac{1}{\tau \omega - 2\xi + \tau \omega} \left\{ e^{-t/\tau} + e^{-\xi \omega t} \right. \\ \left. \cdot \left[\frac{\left[\frac{1}{\tau \omega} - 2\xi \right] + \xi}{\sqrt{1 - \xi^2}} \sin \sqrt{1 - \xi^2} \omega t - \cos \sqrt{1 - \xi^2} \omega t \right] \right\} \quad \text{E-5}$$

The first of the two terms in the right-hand side of Eq. E-5 is an inverse exponential which decreases with increasing time; the second is the product of an inverse exponential and a sinusoidal function. The sinusoidal function will decay in the shortest time when the values of the coefficients of t in the exponents are minima. The values of τ , ω , and ξ which appeared to yield the best damping were chosen from the graphs in Reference 1 and the corresponding value of damping was then evaluated.

Figure E-2 shows the response for several values of k'_2 with the corresponding optimum damping. The angular stiffness and angular damping coefficient, k'_2 and c'_2 , were converted to values corresponding to linear restraints by multiplying by 10^{-3} (the inverse of the square of the distance from the trunnion to the elevating screw bracket).

The following comments are made on the statement preceding Equation E-4a, concerning the pair of complex roots. Large values of stiffness always result in two of the roots being complex conjugates. However, below a certain stiffness it is possible to get three real roots for some ranges of damping. Furthermore, as the stiffness decreases, there is an increase in the range of damping which causes this. When these damping values were used as input to the computer solutions, they resulted in a continuously increasing (non-oscillatory) displacement, as shown in Figure E-3.

B. Computer Solutions

Initially, the computer solutions were investigated for the combined carriage and trail stiffness of 45×10^6 in.-lb/radian, which is about 30% higher than the measured stiffness. A range of very good response was found for stiffness of 1.0×10^4 lb/in. and damping of 360 to

540 lb-sec/in. Several graphs of cradle rotations resulting from paired values of stiffness and damping in this range are shown in Fig. E-4. The choice of an optimum combination here would be arbitrary because, at the time of each succeeding firing, the cradle displacement is uniform (Fig. E-5) and the velocity is small for all of the responses.

To illustrate that this optimum system is an improvement, its response is compared in Fig. E-6 with a response similar to that of the present weapon (with little damping). Also shown is an over-damped response, to demonstrate that an optimum damping does exist.

It was realized that there were limits on the obtainable values of stiffness and damping in the elevating system. Very high stiffness was shown to be difficult to achieve in previously built launchers, and extremely low stiffness presents a problem of stability during sighting and also is subject to excessive preload deflections. Comparison of experimental and analytical results indicates that damping corresponding to at least 90 lb-sec/in. is inherent in the launcher system; thus, no smaller damping is feasible.

Next, a much lower stiffness with increased damping was investigated to discover if cradle rotation would thus be decreased. Fig. E-3 shows that for damping between the non-oscillating range up to the maximum damping used, increased damping gives better response. Nevertheless, these responses are not as good as those with the higher spring rate (see Fig. E-7 for comparison). Also, the study of a 4-rd burst (Fig. E-8) showed that cradle rotation increased with each round.

Finally, the computer solutions were investigated for the combined carriage and trail stiffness of 35×10^6 in.-lb/radian; this is very close to the experimental value for Prototype No. 3. Although the initial maximum cradle rotation was somewhat greater than for the stiffer structure, the range of good response was similar. The optimum response still occurred for a stiffness of 1.0×10^4 to 1.4×10^4 lb/in. and damping of 360 to 540 lb-sec/in.

Trail hop was significantly affected by the carriage stiffness, k_1 , as shown in Fig. E-9, which is a plot of the cradle rotation for three systems with otherwise identical parameters. The maximum negative

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rotation at about 0.2 sec of the supporting structure is an indication of the relative magnitude of the hop, illustrating that trail hop greatly decreases with increasing carriage stiffness.

During the course of the study, it was noted that the optimum damping determined from the computer solutions agreed with the optimum predicted by the third-order analysis, where linear stiffnesses from 1.0×10^4 to 1.4×10^4 lb/in. corresponded to optimum damping from 460 to 510 lb-sec/in. Such close agreement was interesting and unexpected, in that the Model V equations include the actual input force, cam path, and many nonlinear launcher effects not considered in the third-order system.

III. CONCLUSIONS

On the basis of the investigations described above, it is concluded that improved firing accuracy can be achieved for the system studied by adding damping to the elevating system. Specifically, the stiffness of the elevating system should be about 1.0×10^4 to 1.4×10^4 lb/in. and the damping of the elevating system should be about 360 to 540 lb-sec/in., for the maximum improvement.

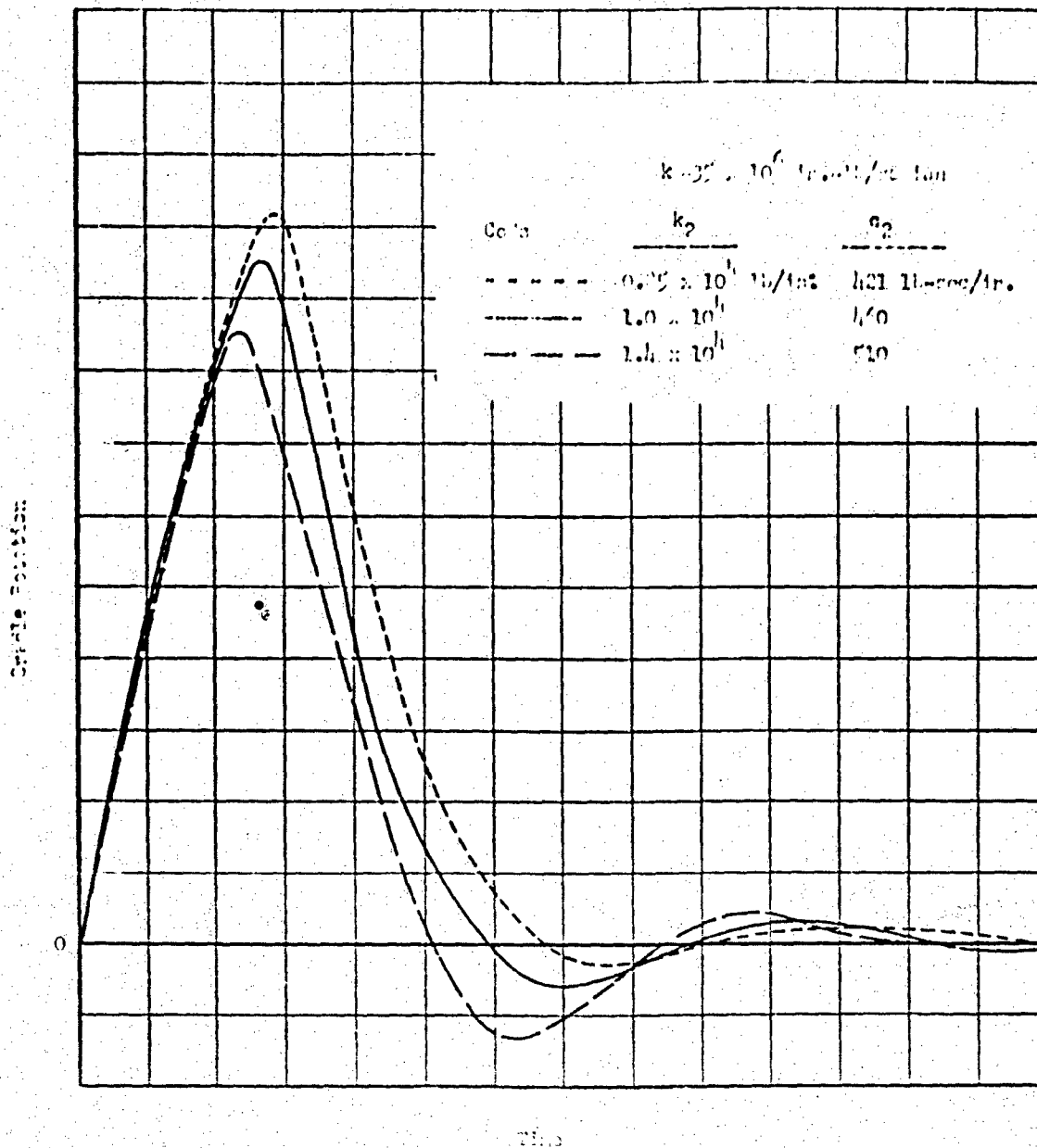


Fig. E-2 CRADLE ROTATION FROM THIRD-ORDER SYSTEM ANALYSIS

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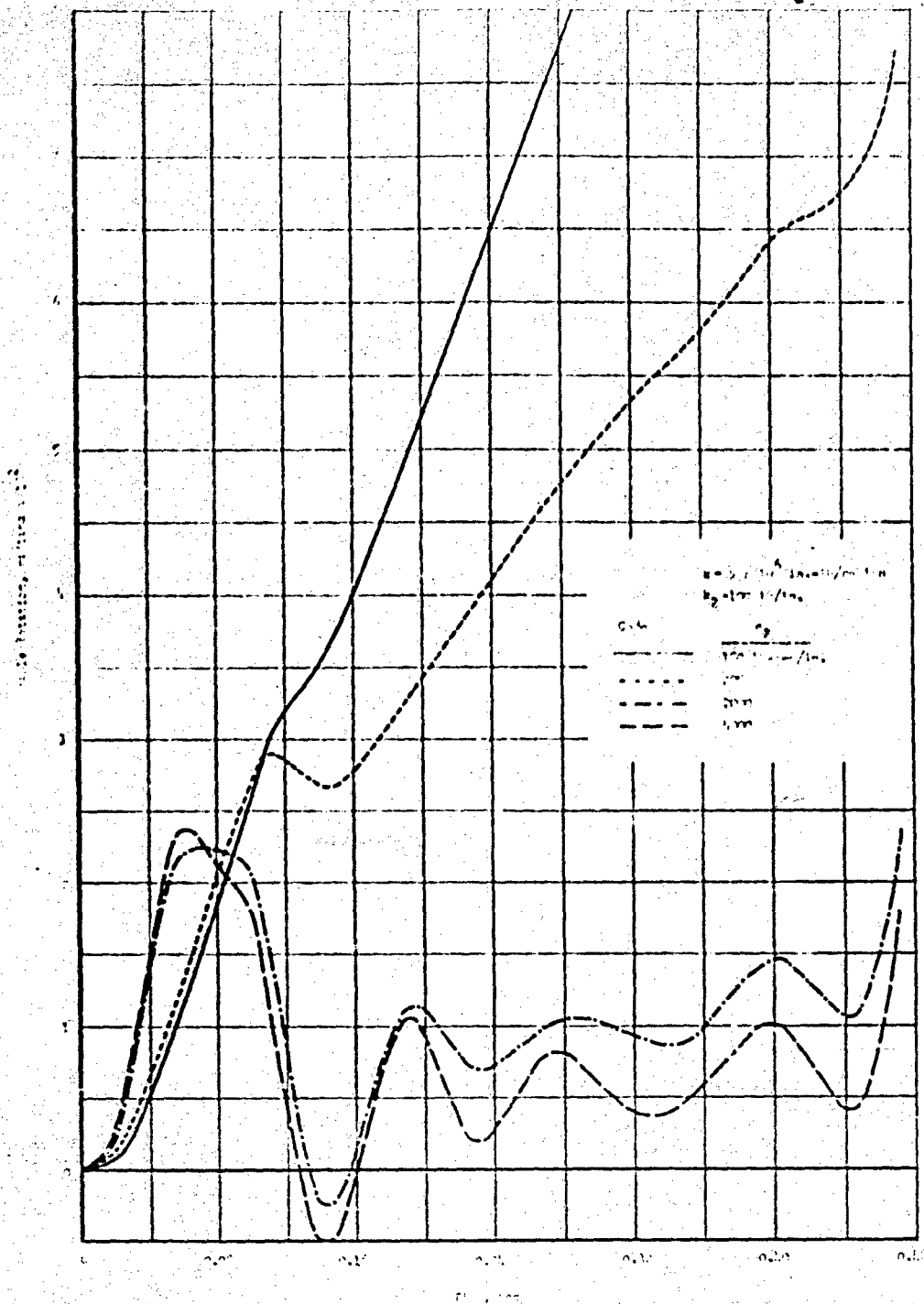


Fig. E-3 EFFECTS OF DAMPING ON CRADLE ROTATION
WITH LOW ELEVATING SYSTEM STIFFNESS

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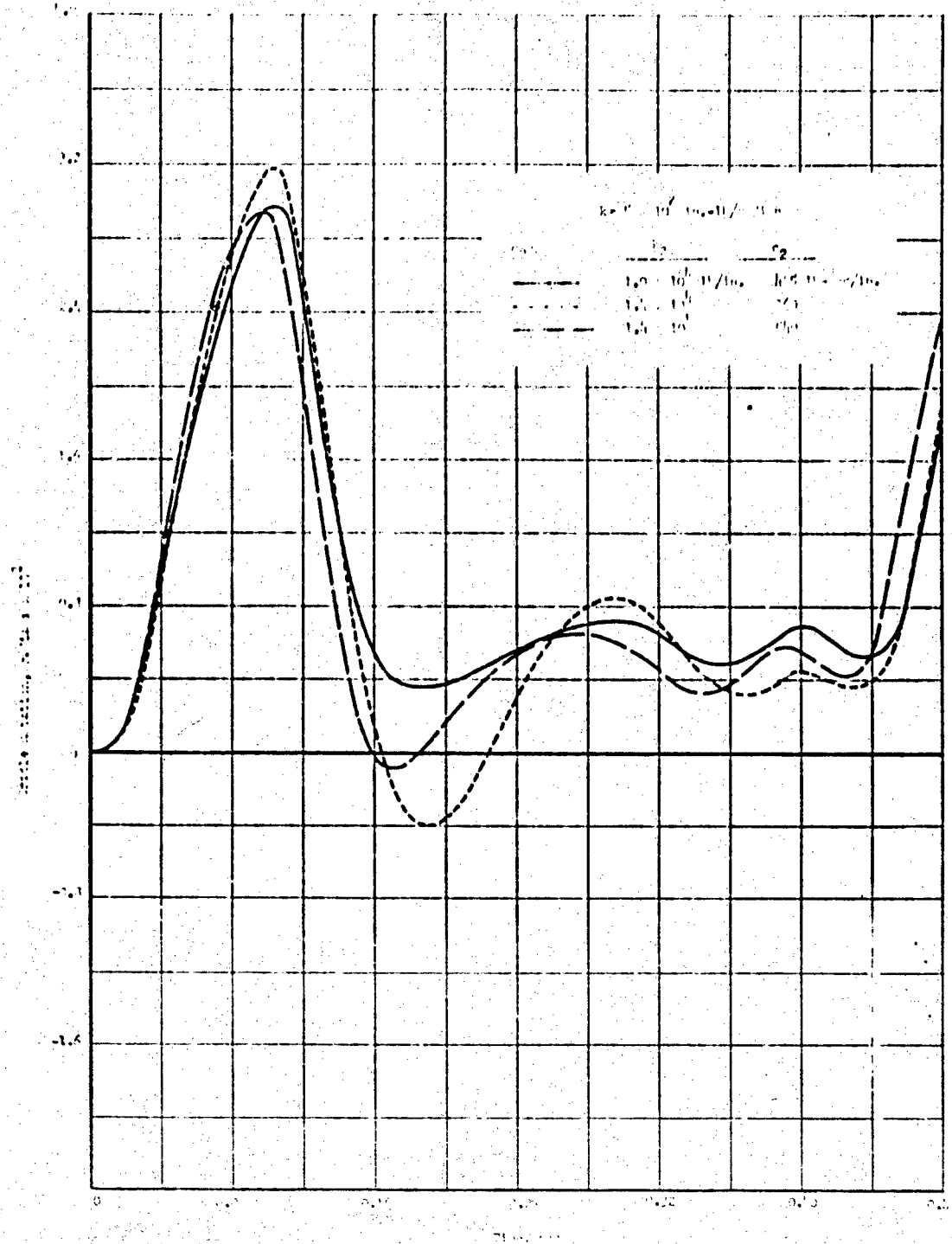


Fig. E-4 OPTIMUM RESPONSES

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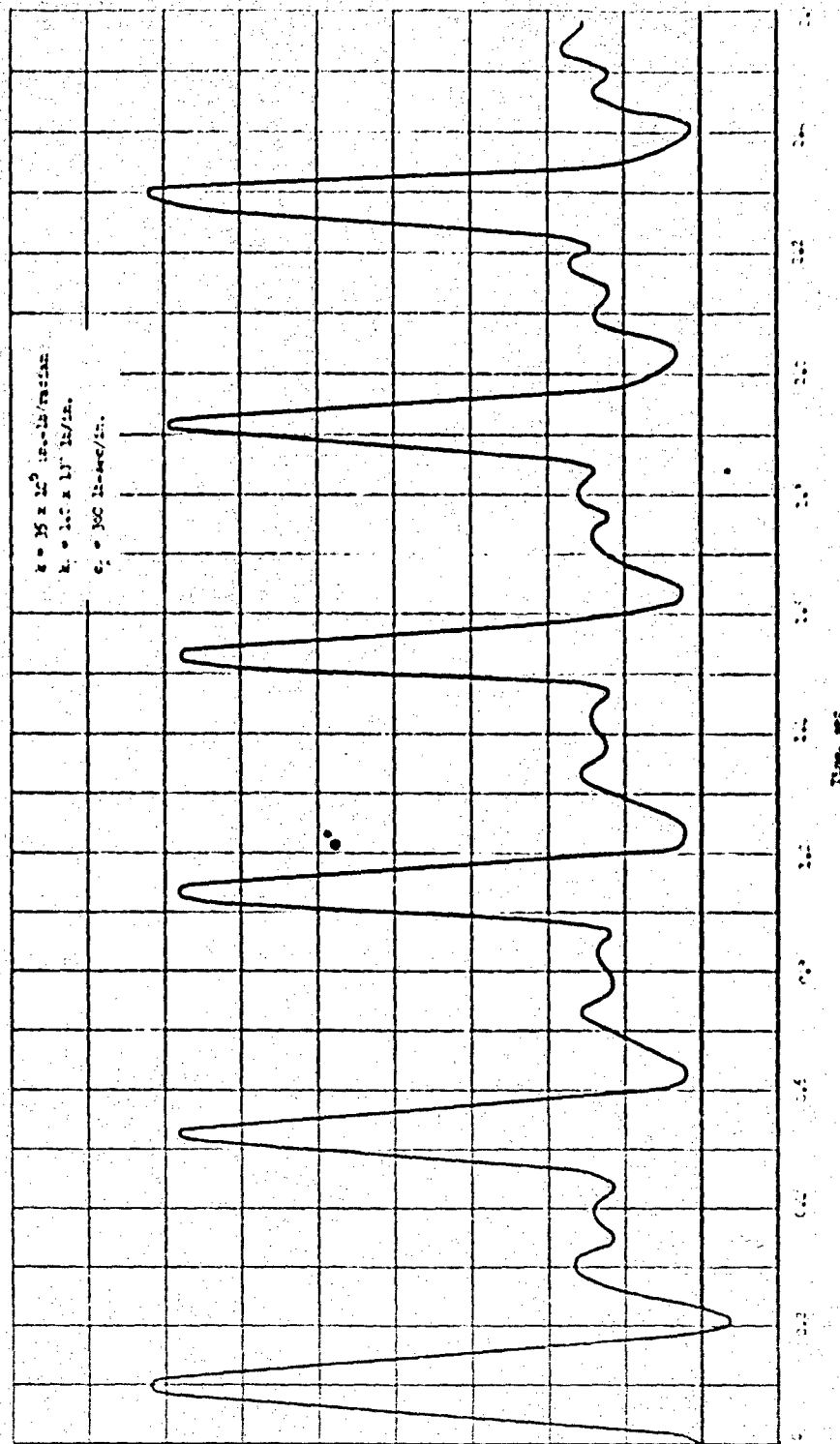


Fig. E-5 CRADLE ROTATION FOR A 6-RD BURST WITH OPTIMUM ELEVATING SYSTEM

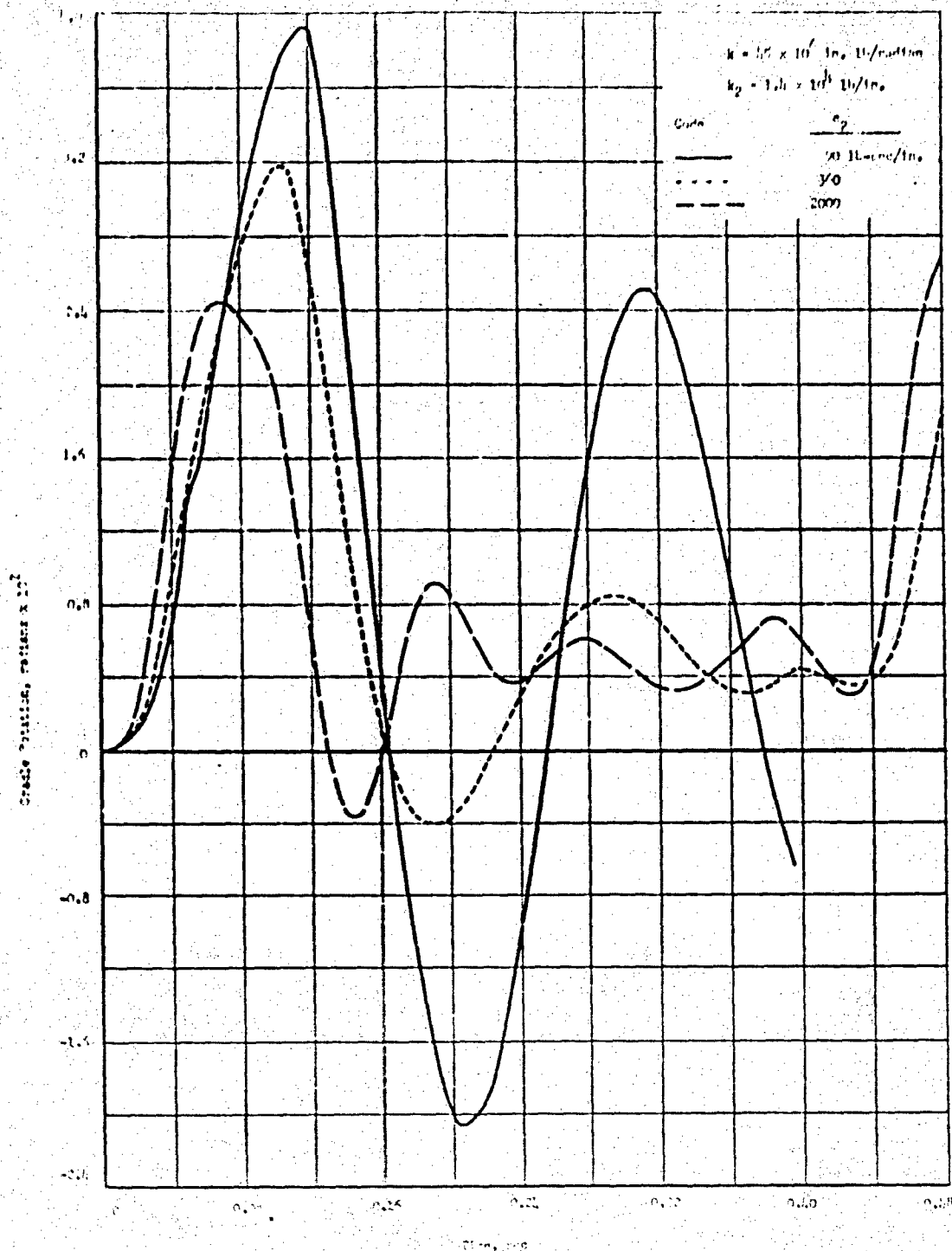


Fig. E-6 RESPONSES FOR VARIOUS ELEVATING
SYSTEM DAMPING COEFFICIENTS

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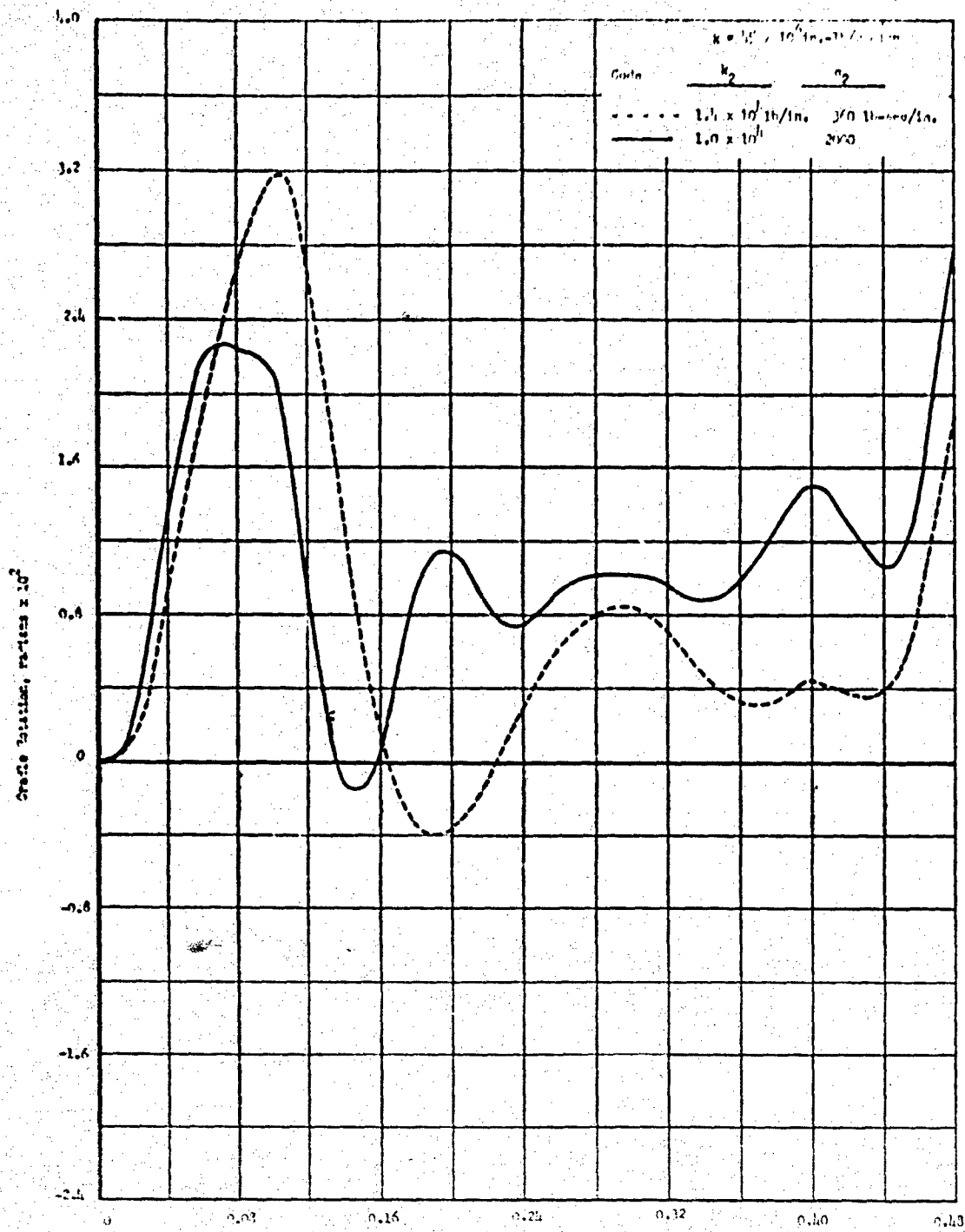


Fig. E-7 COMPARISON OF CRADLE ROTATION FOR A LOW ELEVATING STIFFNESS TO AN OPTIMUM SYSTEM

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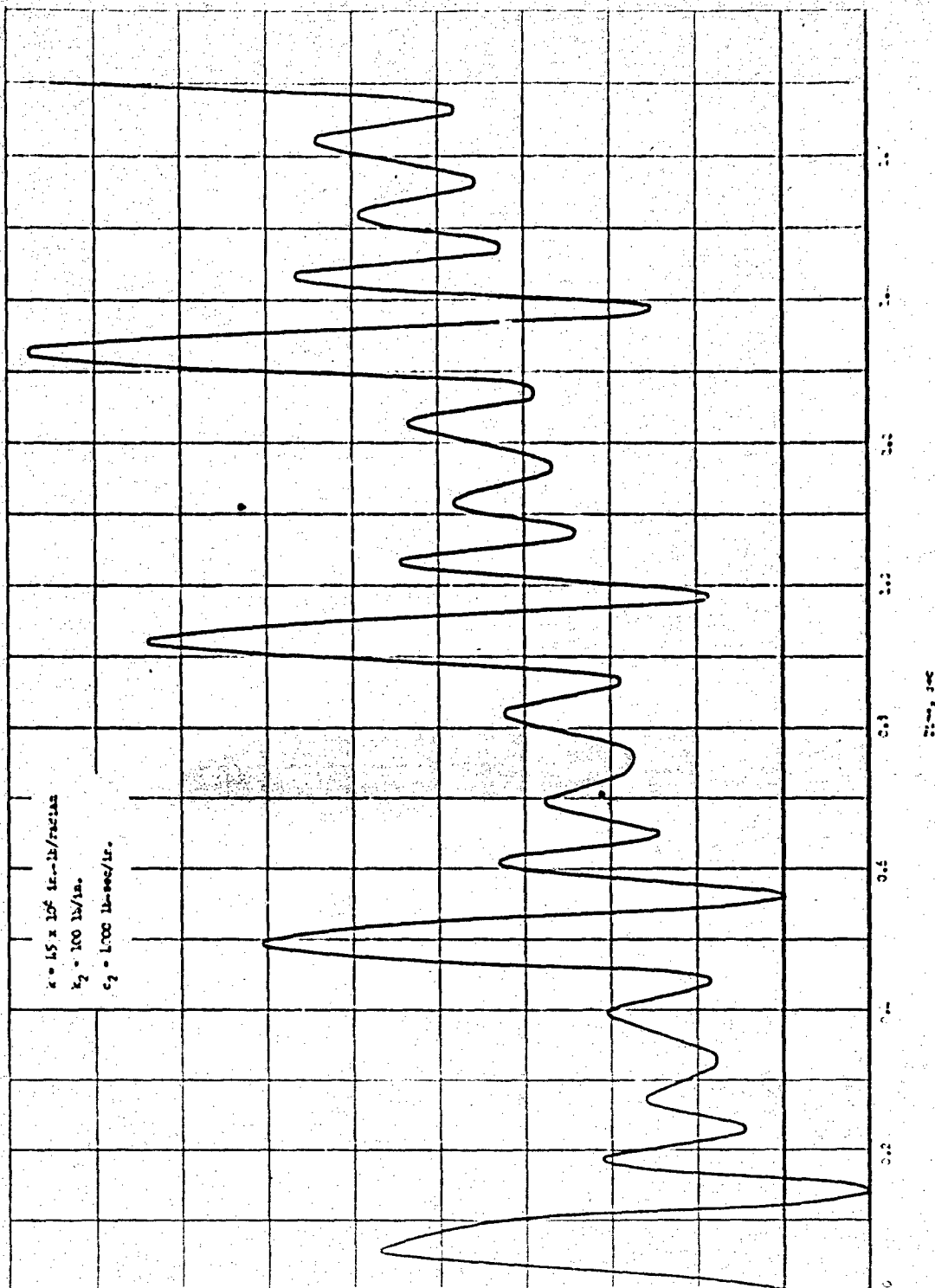


Fig. E-8 CRADLE ROTATION FOR A 4-RD BURST
WITH LOW ELEVATING SYSTEM STIFFNESS

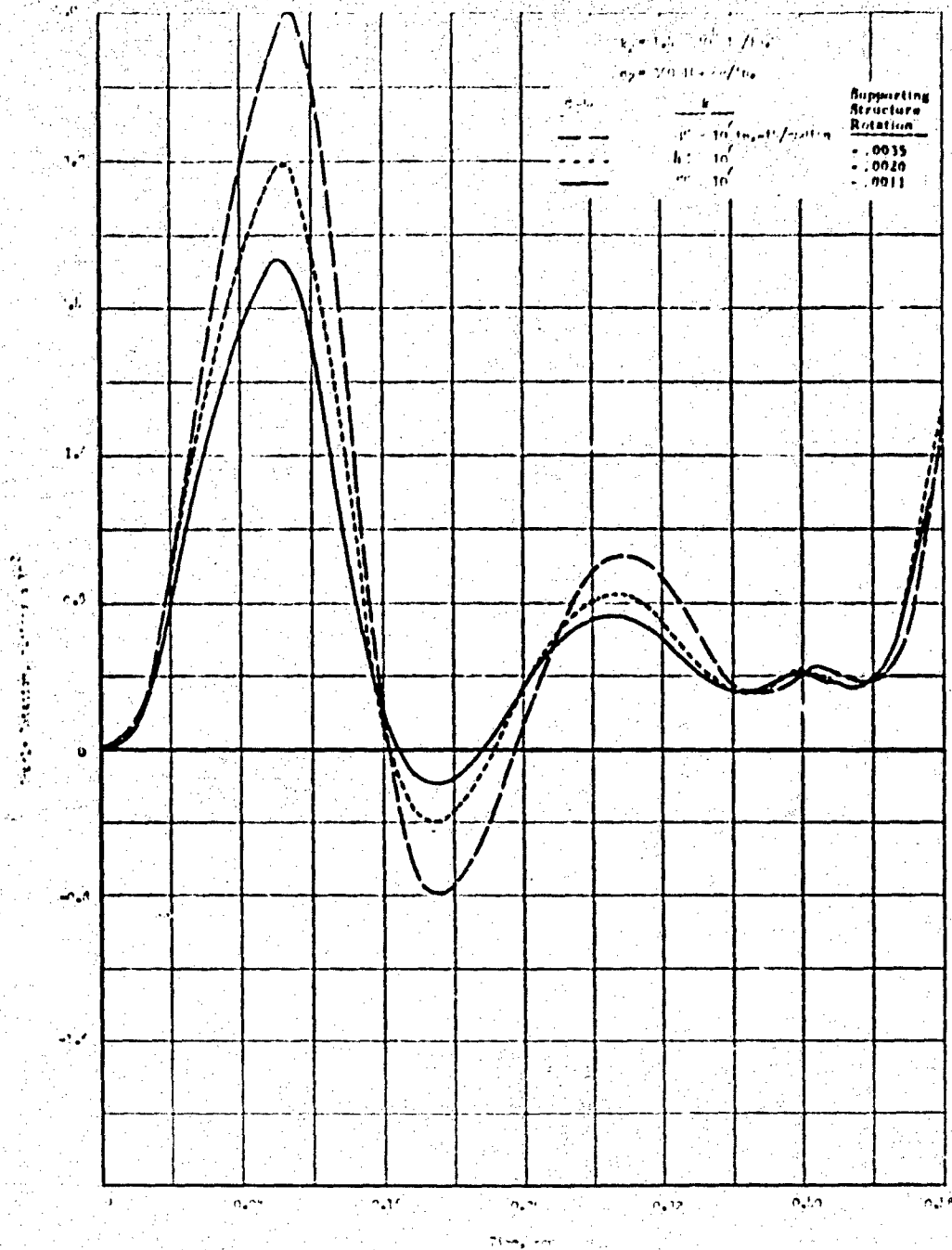


Fig. E-9 EFFECT OF CARRIAGE STIFFNESS ON CRADLE ROTATION

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APPENDIX F

GENERAL EQUATIONS OF MOTION OF LAUNCHER DYNAMICS

I. INTRODUCTION

The 3-degree-of-freedom model, presented in Appendix C, was programmed for a digital computer and has been used extensively. As a 3-degree-of-freedom model, it has the advantage of relative simplicity and also reproduces much of the primary behavior of the XM70-type launcher. On the other hand, the very simplicity precludes its application to many of the presumed secondary features of the launcher. In order to study these features, a more complete model with up to n-degrees of freedom was constructed.

The following are some of the salient features of this more complete model that are present in addition to the features of the 3-degree-of-freedom model:

1. The base of the launcher is not assumed to be fixed to a rigid foundation; instead, the trail pads and base plate are placed on visco-elastic supports which can be assigned parameter values corresponding to various terrain compositions (clay, sand, concrete, etc.). The model takes cognizance of the fact that this may be a one-way support condition so that trail hop and base hop are directly reproduced. Should the need arise, the model can be easily altered to reproduce progressive trail dig-in and base settlement.
2. It is recognized that the launcher does not recoil straight back, maintaining its position in its initial vertical plane. The tendency for three-dimensional rotation about the ball joint is allowed for. Reaction forces and moments at the ball joint, the trunnion supports, the cradle, and the recoiling parts are admitted as three-dimensional quantities.
3. The flexibility of the recoiling assembly is provided for, as are the lateral and vertical variations of center of gravity and inertial force location from shot to shot. The additional features allow the reproduction and study of the "left-right sequencing" phenomenon observed in tests of the XM70 prototype launchers.
4. The transmission of rifling torque to the launcher structure is included, as are the force parameters and geometrical freedoms necessary for interpretation of its effect.

5. The rifled firing tube is represented as a flexible piece with many degrees of freedom in both the vertical and horizontal planes. This permits the investigation of tube whip and vibrations and their effect on both the supporting structure and the projectile.
6. The projectile itself is a part of the model. The resultant effects include the variation of mass during firing and the influence of traveling forces on launcher tube motion.
7. A result of the inclusion of the increased geometrical freedom and applied forces is the introduction of Coriolis forces (in three dimensions) and gyroscopic moments not previously considered. An additional result is the inclusion of more of the coupled effects of the motion of the various launcher components.

This appendix presents a summary of the derivation of the basic equations of this model, as well as graphical and verbal definitions of the known and unknown parameters involved with the basic equations that govern the model. The motion of the launcher can be best expressed in terms of a set of interdependent right-hand three-dimensional coordinate systems, as shown in Fig. F-1. Coordinate system 1 ($[S_1]$) is fixed in the earth at the trail pads with y_1 vertical. Then, x_1 is defined by a plane through y_1 and the initial tube centerline. Systems $[S_2]$ and $[S_3]$ have origins at the ball joint and arise from a translation and a rotation through angle $(\theta_1 + \theta_2)$ and the x_1, y_1 plane. Angle θ_1 is a measure of trail flexibility, while angle θ_2 is a ball-joint rotation. System $[S_4]$ arises through a rotation through angle θ_3 about y_3 . Thus, y_4 coincides with y_3 . System $[S_5]$ arises through a rotation through angle θ_4 about x_4 ; x_5 coincides with x_4 . It is apparent that $\theta_3, \theta_4, \theta_5$ represent freedoms at the ball joint. We arrive at $[S_6]$ by means of a translation through distance l_2 and a rotation through angle $\alpha + \theta_5$; the origin of $[S_6]$ is at the trunnions. The angle α is the angle of elevation and θ_5 is the freedom in elevation permitted by the flex system. The tube base is located a distance l_7 along the x_6 axis. Left-right sequencing forces the tube to swing through an angle θ_6 about an axis parallel to y_6 . In addition to coordinate systems, Fig. F-1 illustrates many of the quantities which are defined on the following pages and which appear in the equations.

II. SPECIFIED QUANTITIES

l_1	trail length
l_2	carriage side length
l_3	trunnion-to-cradle center of gravity
l_4	ball joint to flex attachment
l_5	y_6 -position of breech center of gravity
l_6	z_6 -position of breech center of gravity
l_7	distance from breech center of gravity to tube base
L	tube length
u_0	initial x_6 -position of breech center of gravity
α	elevation angle
ξ_0	initial projectile position with reference to tube base
l_8	distance from trunnion to flex attachment point
l_9	horizontal (z_6) distance of cluster axle from tube centerline
β_1	carriage side orientation
β_2	flex system orientation
I	tube moment of inertia = $\frac{\pi}{4} (r_2^4 - r_1^4)$
ρ	tube mass density per unit length
m_p	projectile mass
J_x $J_y = J_z$	mass polar inertial moments of projectile
$\varphi(t), \varphi = \varphi^*$ $\varphi = \varphi_9$	
m_B	breech mass

J_{Bx}	}	mass polar inertial moments of breech
J_{By}		
J_{Bz}		
m_A		cradle mass
I_{Ax}	}	mass polar inertial moments of cradle
I_{Ay}		
I_{Az}		
m_1		concentrated mass at rear of trails (one-half trail mass)
I_{xc}	}	mass polar inertial moments of carriage about ball joint
I_{yc}		
I_{zc}		
m_B		one-half mass of trails plus mass of carriage and trunnion sides concentrated at ball joint
m_r		mass of right (+ z_0) cluster and projectiles
m_l		mass of left (- z_0) cluster and projectiles
k_1, γ_1		spring and damping constants at trail rear
k_2, γ_2		spring and damping constants at trail front
k_3		angular trail and carriage stiffness (z-axis)
k_4		carriage stiffness for torque about y-axis
k_5		carriage stiffness for torque about x-axis
k_6, γ_6		spring and damping constants for flex system
E		Young's modulus (3×10^7)
μ		coefficient of friction between breech and cradle
$F(t)$		resultant "powder-gas force"
k_8		breech "stiffness" for right-left sequencing
V_n		eigen values

b_n	wave numbers	$b_n^4 = \frac{\rho}{EI} v_n^2$
	$L b_1 = 1.875$	
	$L b_2 = 4.694$	
	$L b_3 = 7.855$	
	$L b_4 = 10.996$	
	$L b_5 = 14.137$	
	$L b_6 = 17.279$	
K	recuperator preload	
k_7	recuperator spring rate	
H_1	constant recuperator friction force	
γ	oil density	
A_p	piston area	
C_o	orifice coefficient	
$\alpha_o(u)$	orifice area	
g	acceleration of gravity	
δ	proportionality constant; seal friction to recoil force	
I_B	cluster inertia moment	
R_p	cam pitch radius	
h	height of cam path development	
u_1	initial cam engagement point	} $u_1 \leq u \leq u_2$
u_2	final cam engagement point	

III. LOGICAL SYMBOLS

$$H(\xi) = \begin{cases} 0 & \text{for } \xi \leq 0 \\ 1 & \text{for } \xi > 0 \end{cases} \quad (\xi \text{ generic})$$

$$\epsilon = \begin{cases} 1 & \text{for } \dot{u} < 0 \\ -1 & \text{for } \dot{u} > 0 \\ 0 & \text{for } \dot{u} = 0 \end{cases}$$

$|A|$ = absolute value of A (A generic)

IV. PHYSICAL VARIABLES (Unknowns)

y_T	trail end deflection
y_B	ball joint deflection
u	breech displacement along cradle measured from trunnion
x_f	displacement of tube base: $x_f = u + l_7$
$\{$	position of projectile measured from tube base; positive in x_6 direction
v	y_6 -displacement of tube at point x
w	z_6 -displacement of tube at point x
θ_1	angle due to "rigid" base movement $\theta_1 = \arcsin \frac{y_B - y_T}{l_1}$
θ_2	flexural rotation of combined carriage flexibility
θ_3	transverse angle of rotation about y_3
θ_4	angle of rotation about x_4
θ_5	elevating system flexibility
θ_6	right-left sequencing flexibility of breech
$q_n(t)$	time history for n th vibrational mode of tube in y_6 -direction
$h_n(t)$	time history for n th vibrational mode of tube in z_5 -direction
$\theta_7 = \psi_y$	rotation about z_8 . . . $\psi_y = - \frac{\partial w}{\partial x}$
$\theta_8 = \psi_z$	rotation about y_8 . . . $\psi_z = \frac{\partial v}{\partial x}$
$\left. \begin{matrix} R_{6x} \\ R_{6y} \\ R_{6z} \end{matrix} \right\}$	ball joint force reactions at trunnion; positive acting on carriage; components in system 6
$\left. \begin{matrix} E_{6x} \\ E_{6y} \end{matrix} \right\}$	flex system force components; positive acting on carriage; components in system 5

T_{6x}	} trunnion moment reactions; components in system 6; positive acting on carriage
T_{6y}	
P_{cx}	} force reaction between cradle and breech; components in system 6; positive acting on cradle
P_{cy}	
P_{cz}	
N_{cx}	} moment reaction between cradle and breech; components in system 6; positive acting on cradle
N_{cy}	
N_{cz}	
$S_x(0, t)$	} shear force between breech and tube
$S_y(0, t)$	
$M_y(0, t)$	} moments between breech and tube
$M_z(0, t)$	
N_{py}	} force reaction between projectile and tube; components in system 6; positive when acting on projectile
N_{pz}	
M_{px}	} moment reaction between projectile and breech, when projectile is in breech; components in system 6; positive when acting on projectile
M_{py}	
M_{pz}	
C_{px}	moment reaction between projectile and tube when projectile is in tube -- transferred rigidly to tube-breech connection; component in system 6; positive when acting on projectile
C_5	$= \cos(\alpha + \theta_5)$
S_5	$= \sin(\alpha + \theta_5)$
C_9	$= \cos \varphi$
S_9	$= \sin \varphi$
R_1	ground reaction and gravity force at y_T
R_2	ground reaction and gravity force at y_B

$$\begin{aligned}
 z_x &= l_2 \cos \beta_1 - l_4 \cos \beta_2 + l_8 C_5 \\
 z_y &= l_2 \sin \beta_1 - l_4 \sin \beta_2 + l_8 S_5 \\
 z &= \sqrt{z_x^2 + z_y^2} \\
 &= \left[l_2^2 + l_4^2 + l_8^2 - 2 l_2 l_4 \cos (\beta_1 - \beta_2) \right. \\
 &\quad \left. + 2 l_2 l_8 \cos (\alpha + \theta_5 - \beta_1) - 2 l_4 l_8 \cos \right. \\
 &\quad \left. (\alpha + \theta_5 - \beta_2) \right]^{1/2}
 \end{aligned}$$

V. DERIVATION OF EQUATIONS OF MOTION

The equations of motion were derived with the use of standard matrix techniques. These procedures are fully covered in "Elementary Matrices and Some Applications to Dynamics and Differential Equations" by R. A. Frazer, W. J. Duncan, and A. R. Collar, Cambridge University Press, 1957.

It is well known that the motion of a body can be represented by equations which characterize the motion of the center of gravity of the body together with those which characterize the rotation of the body about its center of gravity. Figure F-2 illustrates the general case. The coordinates X, Y, Z are fixed in space, while x, y, z are fixed in the body. The motion of point P is defined through the rate of change of $\bar{R}$ or, equivalently, the rates of change of $\bar{p}$ and $\bar{r}$. If the point P is fixed in the body, G , the rate of change of $\bar{r}$ is completely defined by the rotation of the body.

$\bar{R}$ is normally referred to X, Y, Z , as is $\bar{p}$. On the other hand, $\bar{r}$ is normally expressed in terms of x, y, z . If $\bar{A}$ is the matrix for transformation from X, Y, Z to x, y, z we then have

$$\bar{r} = \bar{A} (\bar{R} - \bar{p})$$

with respect to the moving coordinates the velocity of P is

$$\bar{A} \dot{\bar{R}} = \bar{A} \dot{\bar{p}} + \dot{\bar{r}} + \bar{\omega} \bar{r},$$

where $\bar{\omega}$ is the rotational velocity matrix, defined through

$$\bar{\omega} = - \dot{\bar{A}} \bar{A}^{-1}$$

where $\bar{A}^{-1}$ is the inverse of $\bar{A}$.

It should be noted that

$$\dot{\bar{R}} = \frac{d\bar{R}}{dt}$$

is the velocity of P with respect to the fixed coordinates and that $\bar{A} \dot{\bar{R}}$ merely is the result of changing the component bases.

Differentiation leads to the following expression for the acceleration referred to the moving axes

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$$\bar{A} \ddot{\bar{R}} = \bar{A} \dot{\bar{p}} + 2 \bar{\omega} \dot{\bar{r}} + \dot{\bar{\omega}} \bar{r} + \bar{\omega}^2 \bar{r} + \ddot{\bar{r}}$$

The kinematic relations can be used directly to obtain the equations of motion for a point mass or the center of gravity of a rigid body. In the moving coordinate system

$$M \bar{A} \ddot{\bar{R}} = \bar{F}$$

where M is the mass and $\bar{F}$ is the force vector. For the rotational equations of motion we observe that the angular velocity can be represented as the matrix, $\bar{\omega}$ or alternatively as the vector $\bar{p}$.

$$\bar{\omega} = \begin{vmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{vmatrix}$$

$$\bar{p} = (w_x, w_y, w_z)$$

Let $\bar{J}$ be the polar moment of inertia matrix of the body represented by the body axes. Then the angular momentum vector in the moving system is

$$\bar{H} = \bar{J} \bar{p}$$

and the time rate of change of angular momentum is

$$\frac{d}{dt} \bar{H} = \bar{M} = \dot{\bar{J}} \bar{p} + \bar{J} \dot{\bar{p}} + \bar{\omega} \bar{J} \bar{p}$$

where $\bar{M}$ is the vector of applied moments.

Coordinate Transformations

The applicable coordinate systems are shown in Fig. F-1. These systems are related through the following transformations.

$$\bar{R}_2 = \bar{R}_1 - \bar{T}_{21}$$

$$\bar{T}_{21} = (l_1, y_B, 0)$$

$$\bar{R}_3 = \bar{A}_{32} \bar{R}_2$$

$$\bar{A}_{32} = \begin{vmatrix} \cos(\theta_1 + \theta_2) & \sin(\theta_1 + \theta_2) & 0 \\ -\sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$R_4 = \bar{A}_{43} R_3$$

$$\bar{A}_{43} = \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 \\ 0 & 1 & 0 \\ \sin \theta_3 & 0 & \cos \theta_3 \end{bmatrix}$$

$$R_5 = \bar{A}_{54} R_4$$

$$\bar{A}_{54} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_4 & \sin \theta_4 \\ 0 & -\sin \theta_4 & \cos \theta_4 \end{bmatrix}$$

$$R_6 = \bar{A}_{65} (R_5 - T_{65})$$

$$T_{65} = [l_2 \cos \beta_1, l_2 \sin \beta_1, 0]$$

$$\bar{A}_{65} = \begin{bmatrix} \cos(\alpha + \theta_5) & \sin(\alpha + \theta_5) & 0 \\ -\sin(\alpha + \theta_5) & \cos(\alpha + \theta_5) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_7 = R_6 - T_{76}$$

$$T_{76} = (x_f, 0, 0)$$

$$R_8 = R_7 - T_{87}$$

$$T_{87} = (x, 0, 0)$$

$$R_9 = \bar{A}_{98} R_8$$

$$\bar{A}_{98} = \begin{bmatrix} \cos \psi_Z & \sin \psi_Z & 0 \\ -\sin \psi_Z & \cos \psi_Z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{A}_{98} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi^* & \sin \varphi^* \\ 0 & -\sin \varphi^* & \cos \varphi^* \end{bmatrix} \cdot \begin{bmatrix} \cos \psi_y & 0 & -\sin \psi_y \\ 0 & 1 & 0 \\ \sin \psi_y & 0 & \cos \psi_y \end{bmatrix}$$

$$\bar{A}_{98} = \begin{bmatrix} \sin \varphi^* \sin \psi_y \sin \psi_z & \cos \varphi^* \sin \psi_z \\ + \cos \psi_y \cos \psi_z & \cos \varphi^* \cos \psi_z \\ \sin \varphi^* \sin \psi_y \cos \psi_z & -\sin \varphi^* \\ - \cos \psi_y \sin \psi_z & \\ \cos \varphi^* \sin \psi_y & \end{bmatrix}$$

$$\begin{bmatrix} \sin \varphi^* \cos \psi_y \sin \psi_z \\ - \sin \psi_y \cos \psi_z \\ \sin \varphi^* \cos \psi_y \cos \psi_z \\ + \sin \psi_y \sin \psi_z \\ \cos \varphi^* \cos \psi_y \end{bmatrix}$$

Utilization of these transformations leads to the following equations of motion.

VI. AUXILIARY QUANTITIES AND EXPRESSIONS

$$A_n = V_n''(0) = 2 b_n^2 / \sqrt{\rho L}$$

$$B_n = V_n'''(0) = - \frac{2 b_n^3}{\sqrt{\rho L}} \left(\frac{\cosh b_n L + \cos b_n L}{\sinh b_n L + \sin b_n L} \right)$$

$$V_n(\xi) = \frac{1}{\sqrt{\rho L}} \left[\cosh b_n \xi - \cos b_n \xi + \left(\frac{\cosh b_n L + \cos b_n L}{\sinh b_n L + \sin b_n L} \right) (-\sinh b_n \xi - \sin b_n \xi) \right]$$

$$V_n'(\xi) = \frac{b_n}{\sqrt{\rho L}} \left[\sinh b_n \xi + \sin b_n \xi + \left(\frac{\cosh b_n L + \cos b_n L}{\sinh b_n L + \sin b_n L} \right) (\cos b_n \xi - \cosh b_n \xi) \right]$$

$$v_0(\xi) = - \frac{\rho g \cos \alpha}{24 E I} (\xi^4 - 4 \xi^3 L + 6 \xi^2 L^2)$$

$$v_0'(\xi) = - \frac{\rho g \cos \alpha}{6 E I} (\xi^3 - 3 \xi^2 L + 3 \xi L^2)$$

$$v(\xi, t) = v_0(\xi) + \sum_{n=1}^{\infty} V_n(\xi) q_n(t)$$

$$v(\xi, t) = \sum_{n=1}^{\infty} V_n(\xi) h_n(t)$$

$$\dot{v}(\xi, t) = \sum_{n=1}^{\infty} V_n(\xi) \dot{q}_n(t) + \xi v_0' + \xi \sum_{n=1}^{\infty} V_n'(\xi) q_n(t)$$

$$\dot{v}(\xi, t) = \sum_{n=1}^{\infty} V_n(\xi) \dot{h}_n(t) + \xi \sum_{n=1}^{\infty} V_n'(\xi) h_n(t)$$

$$\theta_7(\xi, t) \equiv \psi_7(\xi, t) = -\sum_{n=1}^{\infty} V_n'(\xi) h_n(t)$$

$$\theta_8(\xi, t) \equiv \psi_8(\xi, t) = \sigma_0'(\xi) + \sum_{n=1}^{\infty} V_n'(\xi) q_n(t)$$

$$\ddot{U}(\xi, t) = \ddot{\xi} \sigma_0' + \xi^2 \sigma_0'' + \sum_{n=1}^{\infty} V_n(\xi) \ddot{q}_n + 2 \dot{\xi} \sum_{n=1}^{\infty} V_n' \dot{q}_n + \\ + \xi \sum_{n=1}^{\infty} V_n' \ddot{q}_n(t) + \dot{\xi}^2 \sum_{n=1}^{\infty} V_n'' q_n(t)$$

$$\ddot{u}(\xi, t) = \sum_{n=1}^{\infty} V_n(\xi) \ddot{h}_n + 2 \dot{\xi} \sum_{n=1}^{\infty} V_n' \dot{h}_n + \xi \sum_{n=1}^{\infty} V_n' \ddot{h}_n + \\ + \dot{\xi}^2 \sum_{n=1}^{\infty} V_n'' h_n$$

$$V_n''(\xi) = \frac{b_n^2}{\sqrt{\rho L}} \left[\cosh b_n \xi + \cos b_n \xi + \right. \\ \left. + \left(\frac{\cosh b_n L + \cos b_n L}{\sinh b_n L + \sin b_n L} \right) (-\sinh b_n \xi - \sin b_n \xi) \right]$$

$$V_n'''(\xi) = \frac{b_n^3}{\sqrt{\rho L}} \left[\sinh b_n \xi - \sin b_n \xi + \right. \\ \left. + \left(\frac{\cosh b_n L + \cos b_n L}{\sinh b_n L + \sin b_n L} \right) (-\cosh b_n \xi - \cos b_n \xi) \right]$$

$$\dot{\theta}_7 = -\sum_{n=1}^{\infty} V_n' \dot{h}_n - \xi \sum_{n=1}^{\infty} V_n'' \dot{h}_n$$

$$\dot{\theta}_8 = \dot{\xi} \sigma_0''(\xi) + \sum_{n=1}^{\infty} V_n' \dot{q}_n + \dot{\xi} \sum_{n=1}^{\infty} V_n'' q_n(t)$$

$$\ddot{A}_1 = -\sum_1 \ddot{V}_n' \dot{h}_n - 2\dot{\xi} \sum_1 \ddot{V}_n'' \dot{h}_n - \dot{\xi}^2 \sum_1 \ddot{V}_n''' h_n - \dot{\xi} \sum_1 \ddot{V}_n'' h_n$$

$$\ddot{\Theta}_9 = \ddot{\xi} v_0''(\xi) + \dot{\xi}^2 v_0'''(\xi) + \sum_1 \ddot{V}_n' \ddot{q}_n + 2\dot{\xi} \sum_1 \ddot{V}_n'' \dot{q}_n + \dot{\xi}^2 \sum_1 \ddot{V}_n''' q_n(t) + \dot{\xi} \sum_1 \ddot{V}_n'' q_n(t)$$

$$v_0''(\xi) = -\frac{pq \cos \alpha}{2EI} (\xi^2 - 2\xi L + L^2)$$

$$v_0'''(\xi) = -\frac{pq \cos \alpha}{EI} (\xi - L)$$

$$\dots \text{ LET } \Theta_9 = \varphi(\xi(t))$$

$\varphi(\xi)$ IS THE RIFLING EQUATION

$$\varphi'(\xi) = \frac{d}{d\xi} \varphi(\xi)$$

$$\varphi''(\xi) = \frac{d^2}{d\xi^2} \varphi(\xi)$$

$$\text{THEN } \dot{\Theta}_9 = \dot{\xi} \varphi'(\xi)$$

$$\ddot{\Theta}_9 = \ddot{\xi} \varphi'(\xi) + (\dot{\xi})^2 \varphi''(\xi)$$

$$\Delta_1 = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_5)^2 + (\dot{\theta}_3 S_5 + \dot{\theta}_4 C_5)^2$$

$$\Delta_2 = 2[(\dot{\theta}_1 + \dot{\theta}_2)(\theta_4 S_5 - \theta_3 C_5) + \dot{\theta}_3 S_5 + \dot{\theta}_4 C_5]$$

$$\Delta_3 = (\ddot{\theta}_1 + \ddot{\theta}_2)(\theta_4 S_5 - \theta_3 C_5) - (\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_5)(\theta_3 C_5 - \theta_4 S_5) + \ddot{\theta}_3 S_5 + \ddot{\theta}_4 C_5$$

$$\Delta_4 = (\dot{\theta}_3 S_5 + \dot{\theta}_4 C_5)(\dot{\theta}_3 C_5 - \dot{\theta}_4 S_5)$$

$$\Delta_5 = 2(\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_3 \theta_4 + \dot{\theta}_5)$$

$$\Delta_6 = C_5 - (\theta_1 + \theta_2) S_5$$

$$\Delta_7 = \{[\dot{\theta}_3^2 + (\dot{\theta}_1 + \dot{\theta}_2)^2] S_5 + (\ddot{\theta}_1 + \ddot{\theta}_2 - \theta_4 \ddot{\theta}_3) C_5\} L_2 \cos \beta_1 + \{(\ddot{\theta}_1 + \ddot{\theta}_2 - \theta_4 \ddot{\theta}_3 - 2\dot{\theta}_4 \dot{\theta}_3) S_5 - [(\dot{\theta}_1 + \dot{\theta}_2)^2 + \dot{\theta}_4^2] C_5\} L_2 \sin \beta_1$$

$$\Delta_8 = \dot{\theta}_3^2 + \dot{\theta}_4^2$$

$$\Delta_9 = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_5)(\dot{\theta}_3 C_5 - \dot{\theta}_4 S_5)$$

$$\Delta_{10} = (\ddot{\theta}_1 + \ddot{\theta}_2)(\theta_4 C_5 + \theta_3 S_5) + \ddot{\theta}_3 C_5 - \ddot{\theta}_4 S_5 + (\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_5)(\dot{\theta}_3 S_5 + \dot{\theta}_4 C_5)$$

$$\Delta_{11} = 2[(\dot{\theta}_1 + \dot{\theta}_2)(\theta_4 C_5 + \theta_3 S_5) + \dot{\theta}_3 C_5 - \dot{\theta}_4 S_5]$$

$$\Delta_{12} = [\ddot{\theta}_4 - (\ddot{\theta}_1 + \ddot{\theta}_2) \theta_3] L_2 \sin \beta_1 - [\ddot{\theta}_3 + (\ddot{\theta}_1 + \ddot{\theta}_2) \theta_4] L_2 \cos \beta_1$$

$$\Delta_{13} = (\theta_1 + \theta_2) C_5 + S_5$$

$$\Delta_{14} = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_5)^2 + (\dot{\theta}_3 C_5 - \dot{\theta}_4 S_5)^2$$

$$\Delta_{12} = \ddot{\theta}_1 + \ddot{\theta}_2 - \ddot{\theta}_3 \theta_4 + \ddot{\theta}_5 - \ddot{\theta}_3 \dot{\theta}_4$$

$$\Delta_{16} = (\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_5)(\dot{\theta}_3 \zeta_5 + \dot{\theta}_4 C_5)$$

$$\Delta_{17} = \{(\ddot{\theta}_1 + \ddot{\theta}_2 - \ddot{\theta}_3 \theta_4) S_5 - [\dot{\theta}_3^2 + (\dot{\theta}_1 + \dot{\theta}_2)^2 C_5]\} l_2 \cos \beta_1 + \\ - \{(\ddot{\theta}_1 + \ddot{\theta}_2 - \ddot{\theta}_3 \theta_4 - 2 \dot{\theta}_3 \dot{\theta}_4) C_5 + [\dot{\theta}_4^2 + (\dot{\theta}_1 + \dot{\theta}_2)^2 \zeta_5]\} l_2 \sin \beta_1$$

A. PROJECTILE EQUATIONS

$$F(t) = m_p \{ \ddot{\xi} + \ddot{u} + \Delta_{13} \ddot{y}_B - (\Delta_5 + \theta_6 \Delta_2) \dot{v} + (\Delta_{11} + 2 \dot{\theta}_6) \dot{\omega} + \\ - (\Delta_{14} + 2 \theta_6 \Delta_{16} + \dot{\theta}_6 \Delta_{11}) (\xi + u + x + l_1) + \\ + [\Delta_4 - \Delta_{15} - \theta_6 (\Delta_9 + \Delta_3)] v + \Delta_{17} - \theta_6 \Delta_{12} + \\ + [\Delta_{10} + \Delta_{16} + \theta_6 (\Delta_8 - \Delta_{14}) + \dot{\theta}_6] \omega \} \quad (F-1)$$

$$N_{py} = m_p \{ \dot{v} + \Delta_6 (\ddot{y}_B + g) + (\Delta_5 + \theta_6 \Delta_2) (\ddot{\xi} + \ddot{u}) - (\Delta_2 - \theta_6 \Delta_5) \dot{\omega} + \\ - \Delta_1 v + [\Delta_4 + \Delta_{15} - \theta_6 (\Delta_9 - \Delta_3) + \dot{\theta}_6 \Delta_2] (\xi + u + x + l_1) + \\ + [\Delta_9 - \Delta_3 + \theta_6 (\Delta_4 + \Delta_{15}) + \dot{\theta}_6 \Delta_5] \omega + \Delta_7 \} \quad (F-2)$$

$$\begin{aligned}
 N_{pz} = m_p \{ & \ddot{w} + (\theta_6 \Delta_{13} - \theta_4)(\ddot{y}_B + g) - (\Delta_{11} + 2\dot{\theta}_6)(\dot{\xi} + \dot{u}) + \\
 & + (\Delta_2 - \theta_6 \Delta_5) \dot{v} - [\Delta_8 - 2\theta_6 \Delta_{16} + \dot{\theta}_6 \Delta_{11}] \omega + \\
 & + [\Delta_{16} - \Delta_{10} + \theta_6(\Delta_8 - \Delta_{14}) - \ddot{\theta}_6](\xi + u + x + l_1) + \\
 & + [\Delta_9 + \Delta_3 + \theta_6(\Delta_4 - \Delta_{15})] \sigma + \Delta_{12} + \theta_6 \Delta_{17} \}
 \end{aligned} \quad (F-3)$$

$$\begin{aligned}
 C_{px} = J_x \{ & \ddot{\theta}_9 + \Delta_3 + \theta_6 C_9 \Delta_{10} + \frac{1}{2} [(\dot{\theta}_8 S_9 - \dot{\theta}_6 - \dot{\theta}_7) \Delta_5 + \dot{\theta}_8 C_9 \Delta_{11} + \\
 & + (\theta_8 S_9 - \theta_7 - \theta_6) \Delta_{15}] \}
 \end{aligned} \quad (F-4)$$

$$\begin{aligned}
 C_y = -J_y \{ & C_9 (\ddot{\theta}_6 + \ddot{\theta}_7) - \ddot{\theta}_9 \theta_8 - \dot{\theta}_9 \dot{\theta}_8 - \dot{\theta}_9 \dot{\theta}_6 S_9 - \dot{\theta}_9 \dot{\theta}_7 S_9 + \\
 & + (\theta_6 S_9 + \theta_7 S_9 - \theta_8) \Delta_3 + C_9 \Delta_{10} + S_9 \Delta_{15} + \\
 & + \frac{1}{2} [(\dot{\theta}_6 S_9 + \dot{\theta}_7 S_9 - \dot{\theta}_8) \Delta_2 - \dot{\theta}_9 S_9 \Delta_{11} + \dot{\theta}_9 C_9 \Delta_5] \} + \\
 & - (J_x - J_y) [\dot{\theta}_8 \dot{\theta}_9 - S_9 \dot{\theta}_9 (\dot{\theta}_7 + \dot{\theta}_6) + C_9 \Delta_{16} - S_9 \Delta_4]
 \end{aligned} \quad (F-5)$$

$$\begin{aligned}
 C_z = -J_z \{ & \ddot{\theta}_8 - (\ddot{\theta}_6 + \ddot{\theta}_7) S_9 - (\dot{\theta}_6 + \dot{\theta}_7) \dot{\theta}_9 C_9 - S_9 \Delta_{10} + \\
 & + (\theta_6 + \theta_7) C_9 \Delta_3 + C_9 \Delta_{15} + \frac{1}{2} [(\dot{\theta}_6 + \dot{\theta}_7) C_9 \Delta_2 - (S_9 + C_9) \dot{\theta}_9 \Delta_5] \} + \\
 & + (J_x - J_y) [C_9 \dot{\theta}_9 (\dot{\theta}_7 + \dot{\theta}_6) + C_9 \Delta_4 + S_9 \Delta_{16}]
 \end{aligned} \quad (F-6)$$

B. DETERMINATION OF 6

$$k_{\theta} \theta_6 = (m_L - m_P)(\ddot{u} + \Delta_{13} \ddot{y}_B - \Delta_{14} \dot{u} + \Delta_{17}) + \quad (F-7)$$

$$- (m_L + m_P) l_B (\Delta_{10} + \Delta_{16})$$

C. TUBE EQUATIONS

$$\ddot{q}_n + [V_n^2 - \Delta_1] q_n = -[N_{PY} V_n(\xi) + C_Y V_n'(\xi)] H(\xi) H(L-\xi) +$$

$$- \frac{B_n E I q}{V_n^4} \cos \alpha (\Delta_1 - V_n^2) - (\Delta_2 - \theta_6 \Delta_5) \dot{h}_n +$$

$$+ [\Delta_9 - \Delta_3 + \theta_6 (\Delta_4 + \Delta_{16}) + \dot{\theta}_6 \Delta_5] \dot{h}_n +$$

$$+ [\Delta_{15} + \Delta_4 - \theta_6 (\Delta_9 - \Delta_3) + \dot{\theta}_6 \Delta_2] \frac{\rho \Delta_n}{L_n^4} + \quad (F-8)$$

$$- \frac{\rho B_n}{b_n^4} \left\{ [\Delta_{15} + \Delta_4 - \theta_6 (\Delta_9 - \Delta_3) + \dot{\theta}_6 \Delta_2] (u + l_7) + \right.$$

$$\left. + (\Delta_5 + \theta_6 \Delta_2) \dot{u} + \Delta_6 (\ddot{y}_B + g) + \Delta_7 \right\}$$

$$\ddot{h}_n + [V_n^2 - (\Delta_8 - 2\theta_6 \Delta_{16} + \dot{\theta}_6 \Delta_{11})] \dot{h}_n =$$

$$- [N_{PY} V_n(\xi) - C_Y V_n'(\xi)] H(\xi) H(L-\xi) +$$

$$+ (\Delta_2 - \theta_6 \Delta_5) \left[\dot{q}_n - \frac{B_n E I q}{V_n^4} \cos \alpha \right] + \quad (F-9)$$

$$+ [\Delta_{16} - \Delta_{10} + \theta_6 (\Delta_3 - \Delta_{14}) - \dot{\theta}_6] \left[\frac{\rho \Delta_n}{b_n^4} - \frac{\rho B_n}{b_n^4} (u + l_7) \right] +$$

$$- \frac{\rho B_n}{b_n^4} \left[\Delta_{12} + \theta_6 \Delta_{17} + (\theta_6 \Delta_{13} - \theta_4) (\ddot{y}_B + g) + \right.$$

$$\left. - (\Delta_{11} + 2\theta_6) \dot{u} \right]$$

D. END SHEARS AND MOMENTS

$$\left. \begin{aligned} S_y(0,t) &= -\rho g L \cos \alpha + EI \sum_1^{\infty} B_n q_n \\ M_z(0,t) &= -\frac{1}{2} \rho g L^2 \cos \alpha + EI \sum_1^{\infty} A_n q_n \\ S_z(0,t) &= EI \sum_1^{\infty} B_n h_n \\ M_y(0,t) &= -EI \sum_1^{\infty} A_n q_n \end{aligned} \right\} \quad (F-10)$$

E. BREECH EQUATIONS

$$\begin{aligned} (\rho L + m_B) [\ddot{u} + \Delta_{13} (\ddot{y}_B + q) - \Delta_{14} u + (\Delta_4 - \Delta_{15}) l_3 + \\ + (\Delta_{10} + \Delta_{16}) l_6 + \Delta_{17}] = \end{aligned} \quad (F-11)$$

$$= \rho L (l_1 + \frac{1}{2} L) \Delta_{14} - P_{ex} - F(t)$$

$$\begin{aligned} m_E [\Delta_6 (\ddot{y}_E + q) + \Delta_5 \ddot{u} - \Delta_1 (l_2 + (\Delta_4 + \Delta_{15}) u + \\ + (\Delta_9 - \Delta_3) l_6 + \Delta_7] = \end{aligned} \quad (F-12)$$

$$= -P_{ey} - S_y(0,t) - [1 - H(\xi)] N_{Fy}$$

$$\begin{aligned} m_E [-\Delta_4 (\ddot{y}_B + q) - \Delta_{11} \ddot{u} - \Delta_8 l_6 + \Delta_{12} + (\Delta_{16} - \Delta_{10}) l_7 + \\ + (\Delta_9 + \Delta_3) l_5] = \end{aligned} \quad (F-13)$$

$$= -P_{ez} - S_z(0,t) - [1 - H(\xi)] N_{Pz}$$

$$\begin{aligned}
 (J_{Ex} + 2\rho L I) \Delta_3 + (J_{Ex} - J_{By}) \Delta_9 &= \\
 = -N_{cx} - P_{cy} l_6 + P_{cz} l_5 - C_{px} H(\xi) H(L - \xi) + & (F-14) \\
 + [H(\xi) - 1] J_x (\ddot{\theta}_9 + \Delta_3)
 \end{aligned}$$

$$\begin{aligned}
 J_{By} \Delta_{10} + (J_{Bx} - J_{Bz}) \Delta_{16} &= \\
 = -N_{cy} + M_y(0, t) + l_7 S_z(0, t) + & (F-15) \\
 + [1 - H(\xi)] [(l_7 + \xi) N_{py} - J_y (C_9 \Delta_{10} + S_9 \Delta_{15}) + \\
 - (J_x - J_y) (C_9 \Delta_{16} - S_9 \Delta_4)]
 \end{aligned}$$

$$\begin{aligned}
 J_{Bz} \Delta_{16} + (J_{By} - J_{Ex}) \Delta_4 &= \\
 = -N_{cz} - l_7 S_y(0, t) + M_z(0, t) - F(t) l_5 + & \\
 + [H(\xi) - 1] [(l_7 + \xi) N_{py} + J_y (C_9 \Delta_{15} - S_9 \Delta_{10}) + & (F-16) \\
 - (J_x - J_y) (C_9 \Delta_4 + S_9 \Delta_{16})] + \\
 - \rho L l_5 [\ddot{u} + \Delta_{13} \ddot{y}_B - \Delta_{14} (u + l_7 + \frac{1}{2}L) + \Delta_{17}]
 \end{aligned}$$

F. COUNTERRECOIL MECHANISM

$$\begin{aligned}
 P_{cx} &= -K - eH_1 - k_7 (u_0 - u) - \frac{e(\Delta_p^3 (1 + \epsilon) \dot{u}^2}{2\eta C_0^2 \alpha_0^2} + & (F-17) \\
 &- H(u - u_1) H(u_2 - u) \frac{I_B h^2}{R_p^2 (u_2 - u_1)} \left[2\pi \sin 2\pi \frac{u - u_1}{u_2 - u_1} (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1}) \dot{u}^2 \right. \\
 &\left. + (1 - \cos 2\pi \frac{u - u_1}{u_2 - u_1})^2 \right] \frac{1 - \epsilon}{2}
 \end{aligned}$$

G. TRAILS AND GROUND REACTIONS

DEFINITION:

$$R_1 = \begin{cases} -k_1 y_T - \delta_1 \dot{y}_T & \text{FOR } y_T \leq 0 \\ -m_1 (\ddot{y}_T + g) & \text{FOR } y_T > 0 \end{cases} \quad (F-18)$$

$$R_2 = \begin{cases} -m_2 \ddot{y}_B - k_2 y_B - \delta_2 \dot{y}_B & \text{FOR } \dot{y}_B \leq 0 \\ -m_2 (\ddot{y}_B + g) & \text{FOR } \dot{y}_B > 0 \end{cases} \quad (F-19)$$

$$\theta_1 = \frac{y_B - y_T}{L_1} \quad (F-20)$$

$$L_1 R_1 = k_3 \theta_2 \quad (F-21)$$

H. VERTICAL EQUILIBRIUM AT BALL JOINT

$$R_1 + R_2 + \Delta_{13} R_{6x} + \Delta_{6y} R_{6y} - \theta_4 R_{6z} + \\ + (\theta_1 + \theta_2) E_{6x} + E_{6y} = 0 \quad (F-22)$$

I. ANGULAR MOTION OF THE BALL JOINT

$$L_1 R_1 \theta_3 - k_5 \theta_4 + R_{6z} L_2 \sin \beta_1 + T_{6x} C_5 - T_{6y} S_5 + \\ - I_{cz} (\ddot{\theta}_1 + \ddot{\theta}_2) \dot{\theta}_3 - I_{xc} [\ddot{\theta}_4 - (\ddot{\theta}_1 + \ddot{\theta}_2) \theta_3 - (\dot{\theta}_1 + \dot{\theta}_2) \dot{\theta}_3] = 0 \quad (F-23)$$

$$-L_1 R_1 \theta_4 - k_4 \theta_3 - R_{6z} L_2 \cos \beta_1 + T_{6x} S_5 - T_{6y} C_5 + \\ - (I_{xc} - I_{zc}) (\dot{\theta}_1 + \dot{\theta}_2) \dot{\theta}_4 = 0 \quad (F-24)$$

$$\begin{aligned}
 & -l_1 R_1 + k_4 \theta_3 \theta_4 - E_{6x} l_4 \sin \beta_2 + R_{6x} \sin(\alpha - \beta_1 + \theta_3) + \\
 & + E_{6y} l_4 \cos \beta_2 + R_{6y} \cos(\alpha - \beta_1 + \theta_3) + I_{x6} \ddot{\theta}_3 \dot{\theta}_4 + \quad (F-25) \\
 & - I_{z6} (\ddot{\theta}_1 + \ddot{\theta}_2 - \ddot{\theta}_3 \theta_4 \cdot \dot{\theta}_3 \dot{\theta}_4) = 0
 \end{aligned}$$

J. ANGULAR MOTION OF CRADLE

$$\Delta_3 I_{xA} + (I_{2A} - I_{7A}) \Delta_9 = N_{cy} - l_6 P_{cy} - T_{6x} \quad (F-26)$$

$$\begin{aligned}
 \Delta_{10} I_{7A} + (I_{7A} - I_{1A}) \Delta_{16} = N_{cy} + l_6 P_{cx} - (u - l_3) P_{cz} + \\
 - T_{6y} - l_3 R_{6z} \quad (F-27)
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{15} I_{1A} + (I_{7A} - I_{xA}) \Delta_4 = N_{cz} + (u - l_3) P_{cy} + l_3 R_{6y} + \\
 + (l_5 - l_3) S_B E_{6x} - (l_5 - l_3) C_S E_{6y} \quad (F-28)
 \end{aligned}$$

K. LINEAR MOTION OF CRADLE CENTER OF GRAVITY

$$\begin{aligned}
 m_A [\Delta_{13} (\ddot{y}_B + g) - \Delta_{14} l_3 + \Delta_{17}] = \\
 = P_{cx} - R_{6x} - E_{6x} C_S - E_{6y} S_S \quad (F-29)
 \end{aligned}$$

$$\begin{aligned}
 m_A [\Delta_6 (\ddot{y}_B + g) + (\Delta_4 + \Delta_{15}) l_3 + \Delta_7] = \\
 = P_{cy} - R_{6y} + E_{6x} S_S - E_{6y} C_S \quad (F-30)
 \end{aligned}$$

$$m_A [-\theta_4 (\ddot{y}_B + g) - (\Delta_{10} - \Delta_{16}) l_3 + \Delta_{12}] = P_{C2} - R_{C2} \quad (F-31)$$

L. FLEXIBLE SYSTEM FORCE

$$E_{6x} = k_6 \frac{\ddot{z}^x}{\ddot{y}} l_8 (C_5 - \cos \alpha) + \quad (F-32)$$

$$+ k_6 \frac{\ddot{z}^x}{\ddot{y}} l_8 \dot{\theta}_5 [l_4 \sin(\alpha + \theta_5 - \beta_2) - l_2 \sin(\alpha + \theta_5 - \beta_1)]$$

$$E_{6y} = k_6 \frac{\ddot{z}^y}{\ddot{y}} l_8 (C_5 - \cos \alpha) + \quad (F-33)$$

$$+ k_6 \frac{\ddot{z}^y}{\ddot{y}} l_8 \dot{\theta}_5 [l_4 \sin(\alpha + \theta_5 - \beta_2) - l_2 \sin(\alpha + \theta_5 - \beta_1)]$$

VII. COMPILATION OF UNKNOWNNS

Unknown	Equation	Unknown	Equation	Unknown	Equation
E_{6x}	631	y_T	619	y_B	621
3	623	5	627	T_{6y}	626
R_{6x}	628	R_{6z}	630	E_{6y}	632
P_{cy}	611	N_{cx}	613	N_{cz}	615
q_n	607	6	606	C_y	604
	600	N_{pz}	602	2	620
1	624	4	622	T_{6x}	625
P_{cx}	616	R_{6y}	629	u	610
P_{cz}	612	N_{cy}	614	S_y, S_z	609
				M_z, M_y	
h_n	608	C_{px}	603	C_z	605
N_{py}	601				

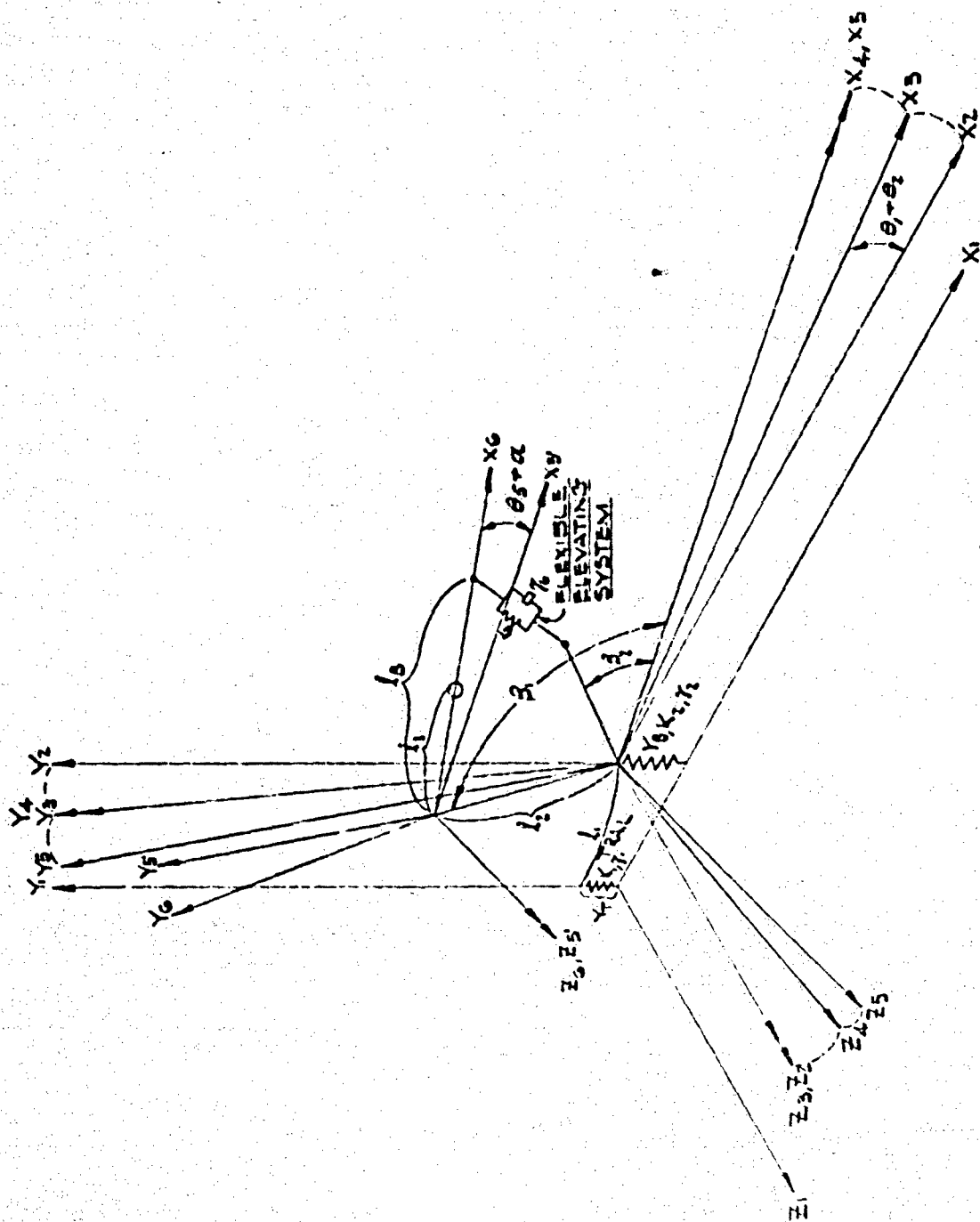


Fig. F-1 DIAGRAM OF COORDINATE SYSTEMS DESCRIBING LAUNCHER MOTION
FOR MATHEMATICAL MODEL VI

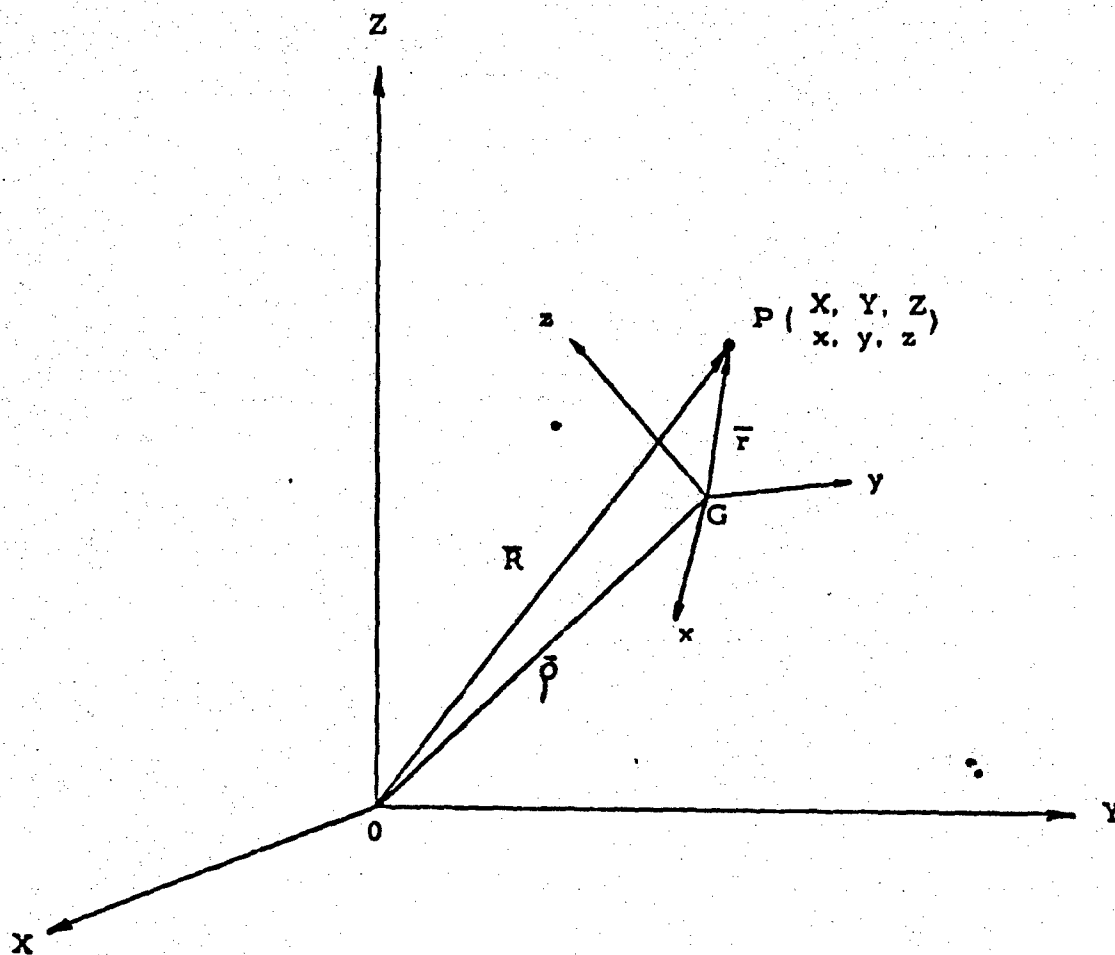


Fig. F-2 COORDINATE SYSTEM

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APPENDIX G

FIXTURE DESIGNS

I. INTRODUCTION

The purpose and some of the operations of the experimental structure modifications are described in the body of the report. This appendix shows the details of the construction of the more important fixtures, the dampers, and the elevating system flexibilities. Not only the springs and dampers used on the program are shown, but also a design showing a combination spring and damper which could be used in the actual launcher. (The fixture used in the experimental program prohibited changes in elevation.)

II. FIXTURE DESCRIPTION

Figure G-1 shows the assembly of the auxiliary dampers used for the firing program. It is simply a piston submerged in a fluid inside a cylinder; the piston has orifices so that its motion gives a force approximately proportional to the square of the velocity. A detail drawing of the piston is shown in Figure G-2.

Figure G-3 shows the assembly of the flexibility added to the elevating rod; Figure G-4 gives the spring characteristics. This assembly was inserted into the present elevating system simply by moving the existing elevating screw up, and placing the spring assembly between the support in the upper carriage side and the screw. The flexibility could be removed, if desired, by replacing the spring by a rigid spacer. An alternate method was used, however, that of replacing the entire elevating screw by more rigid rods.

Figure G-5 shows a possible configuration of a spring-damper combination. The entire spring is submerged in hydraulic fluid; in this case the spring end retainers have orifice holes and act as the damper pistons. The lower, smaller spring and piston assembly is a device which acts as a fluid reserve which compensates for changes in ambient temperature. The entire assembly would be keyed so that it could transmit the torques for elevating the launcher.

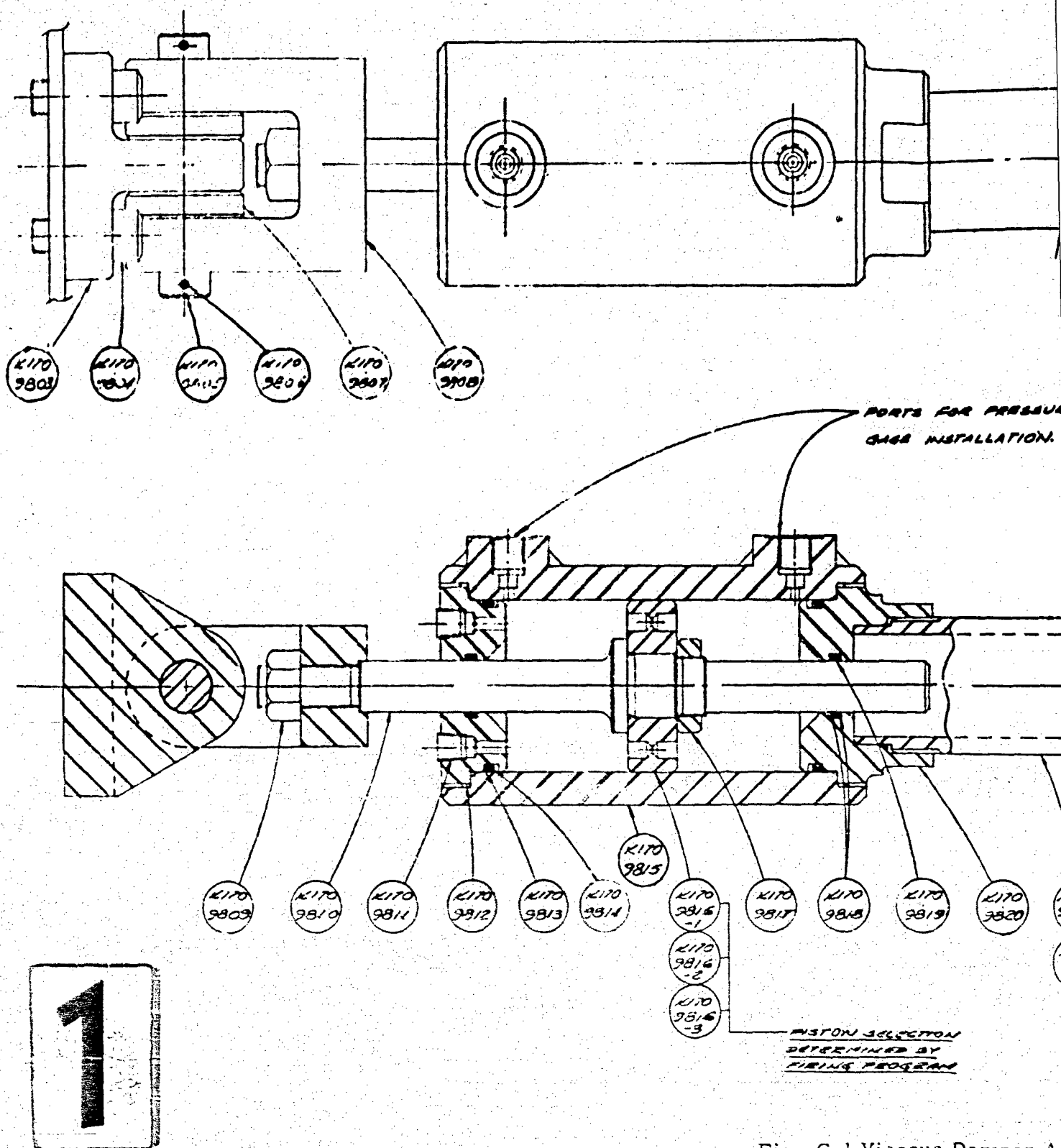
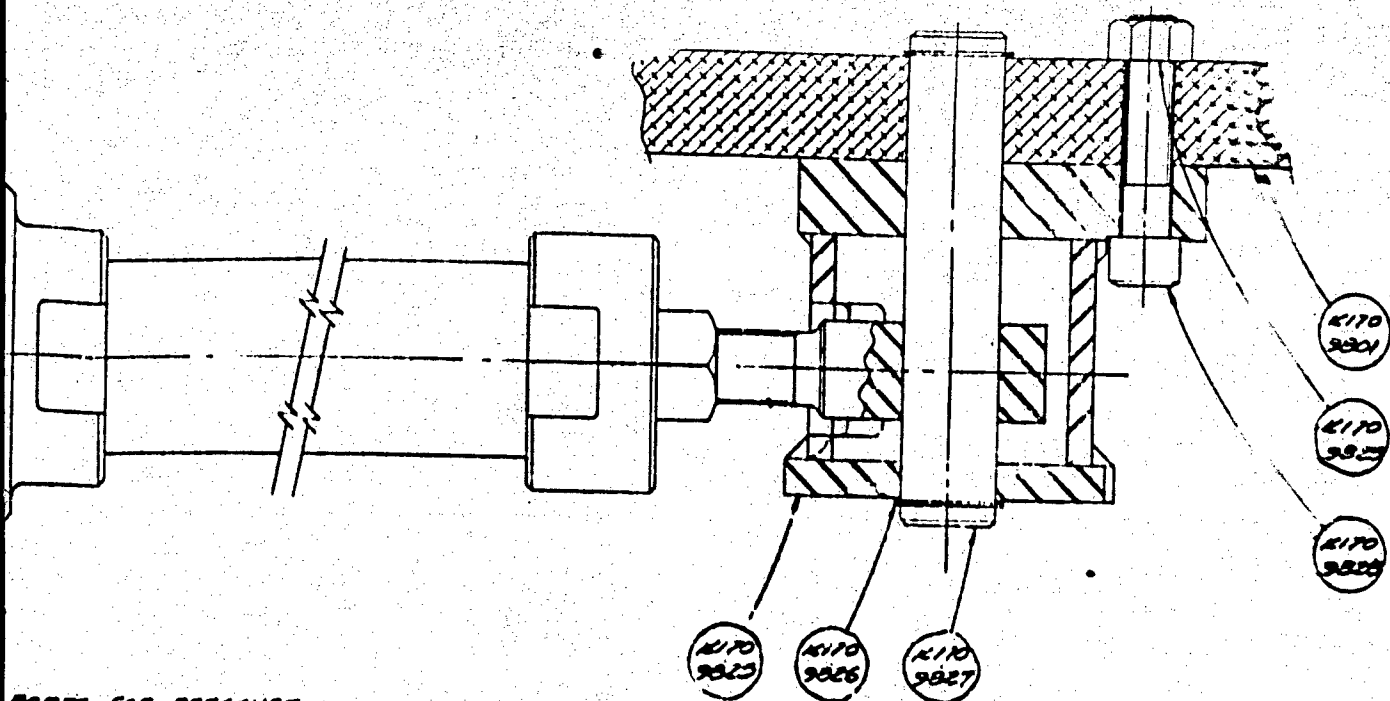
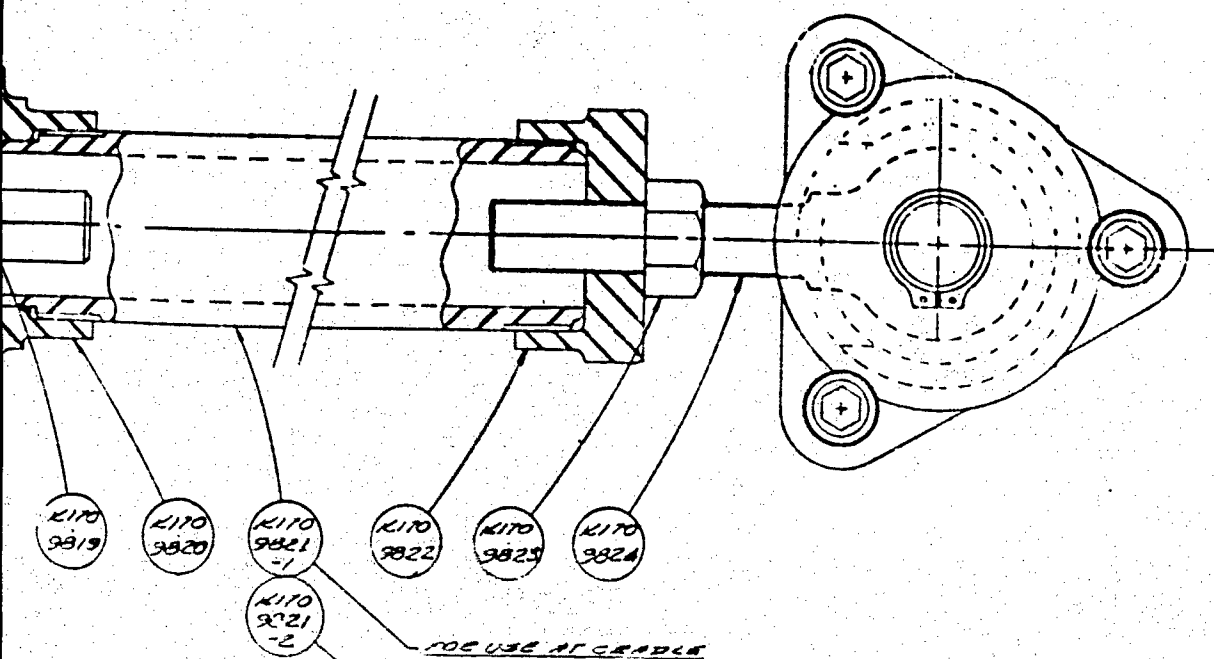


Fig. G-1 Viscous Damper Ass



PORTS FOR PRESSURE
GAGE INSTALLATION.



ELECTION
VED BY
ROSEAN

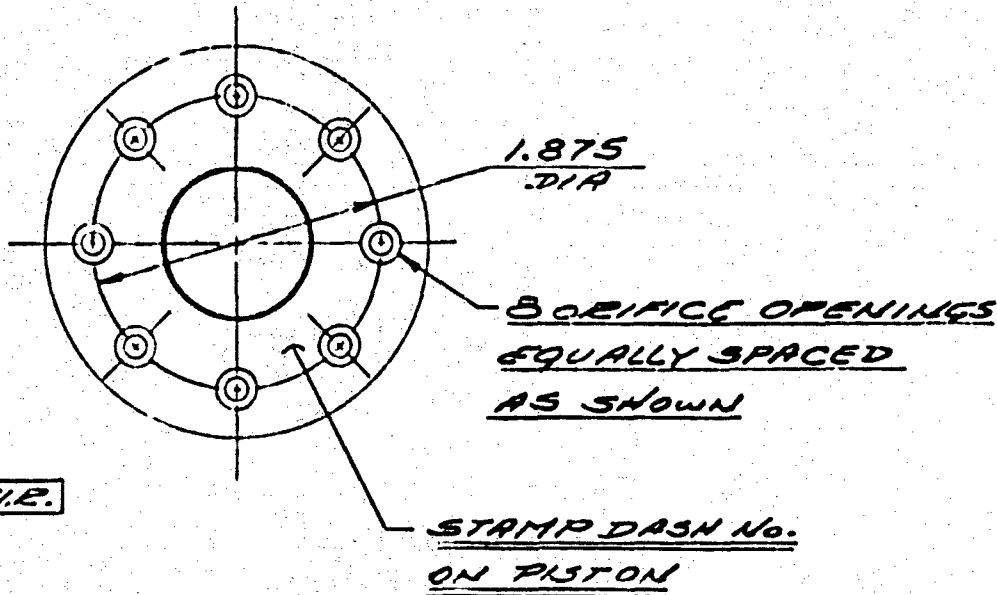
FOR USE AT GRADE
ELEVATION OF 45°

FOR USE AT GRADE
ELEVATION OF 22 1/2°

2

Viscous Damper Assembly

PART No.	DIM "A"	No REQ.
K1709816-1	.045	2
K1709816-2	.089	2
K1709816-3		



MAX.
ICE
09815

-.001 T.I.E.

FINISH
EXCEPT AS NOTED
ARCO-LUBRIZE

2

G-2 Damper-Piston

G-3

1

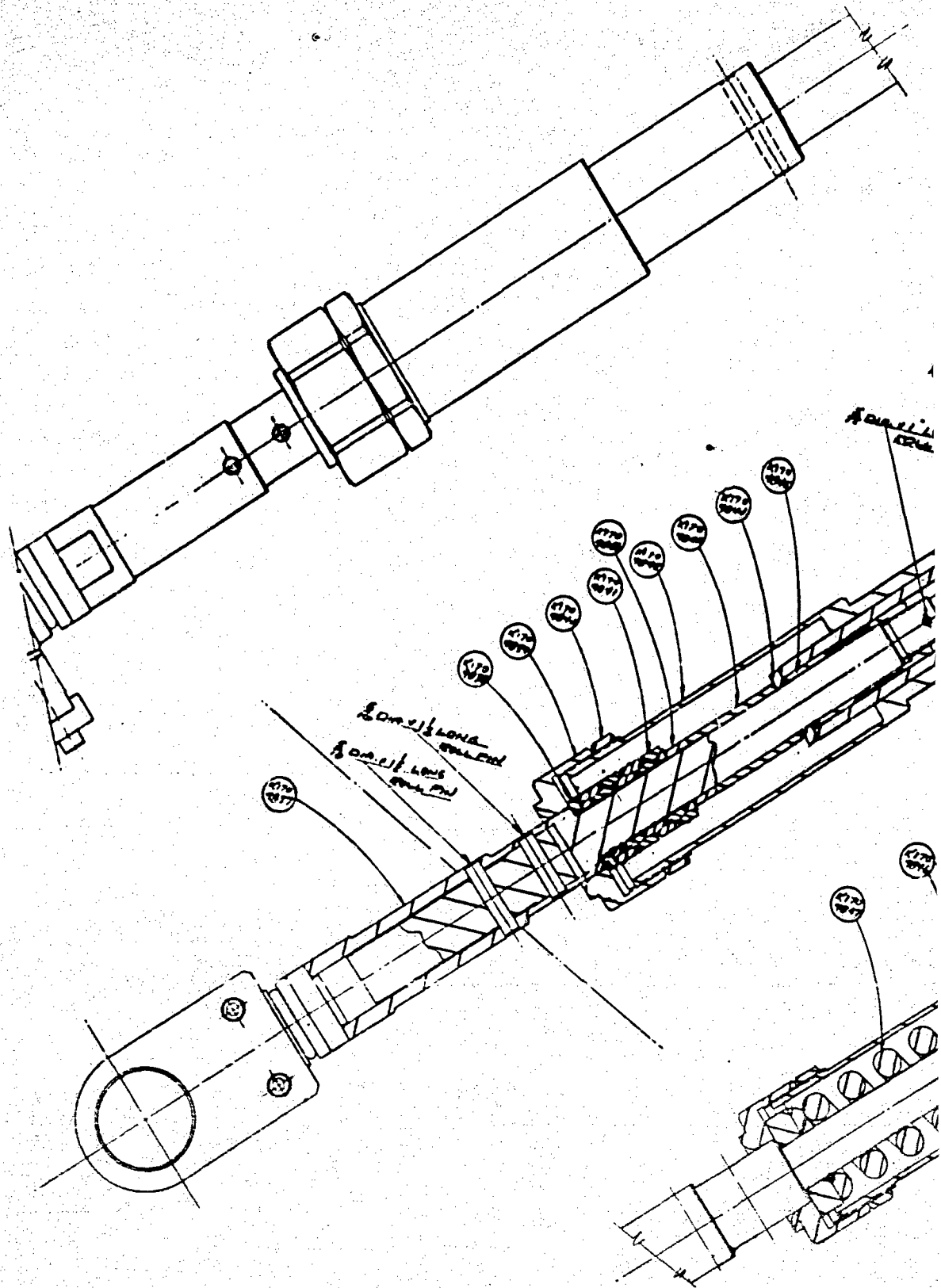


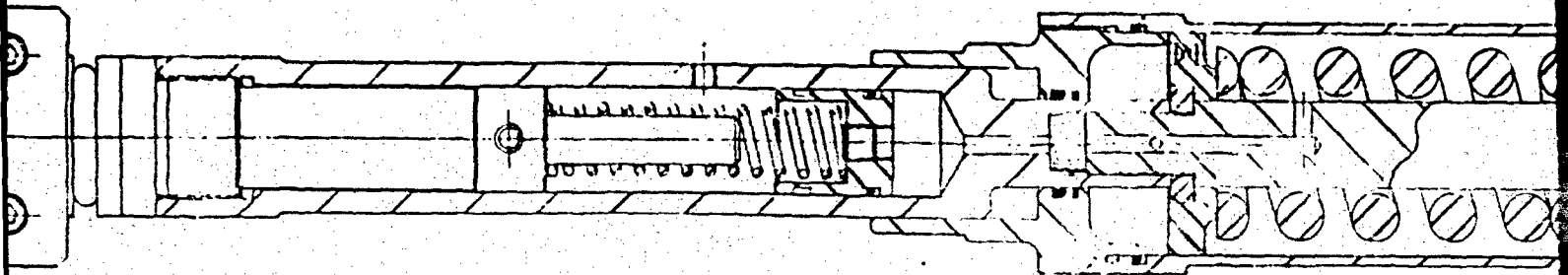
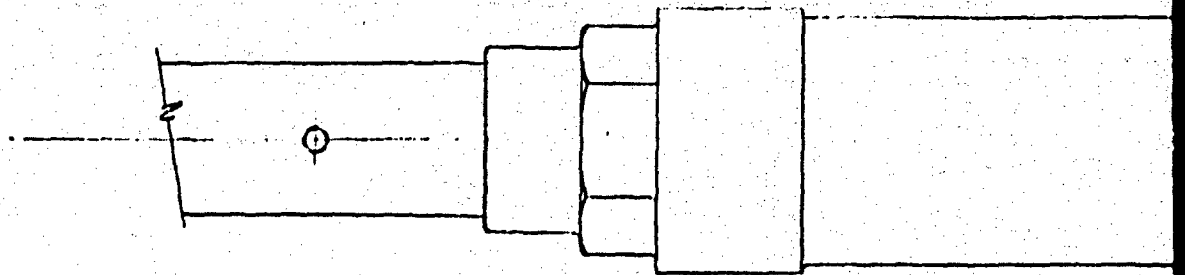
Fig. G-3 Spring Damper Combination

SPRING DATA

MATL. AISI 1095 SPRING STL.
WIRE DIA. 1/2 "
MEAN DIA. 1-7/16"
ACTIVE COILS 4-1/3
TOTAL COILS 6-1/3
RATE 7000 lb/in.
FREE HEIGHT 3-13/16
SOLID HEIGHT 3-3/16
INSTALLED HEIGHT 3-5/8
PRE-LOAD 1200 lb
DEFLECTION 572"
MAX. LOAD 4000 lb
STRESS 115,000 p. s. i.
WOUND R. H. - L. H.
ENDS SQUARED & GROUND

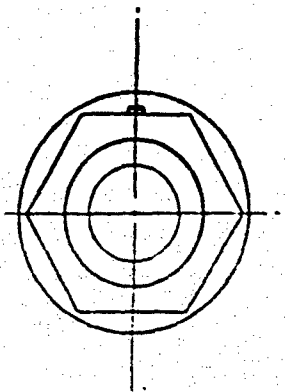
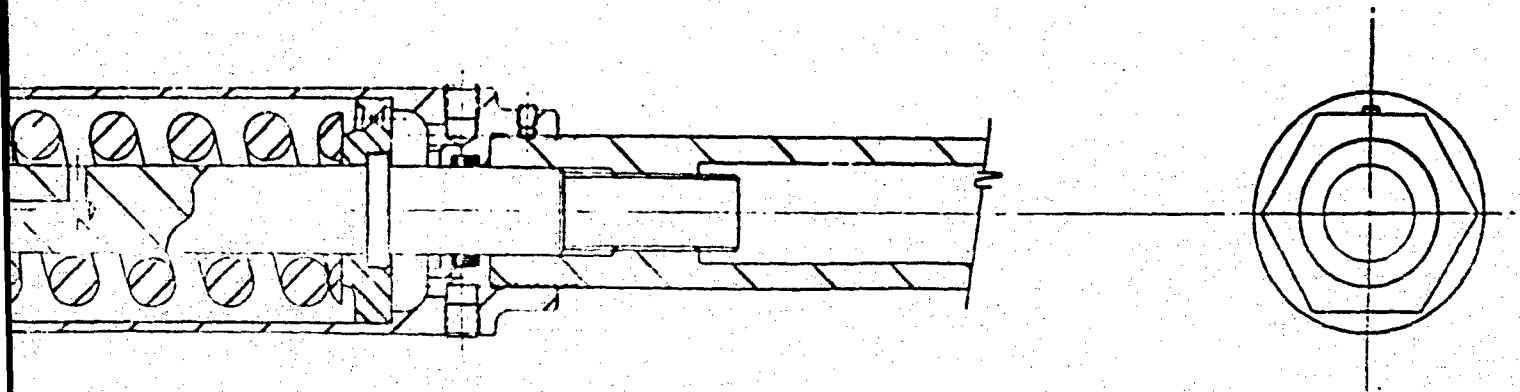
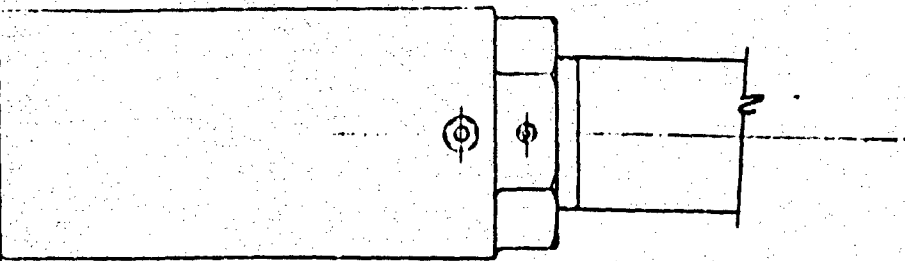
Fig. G-4 SPRING DATA

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1

Fig. G-5 Spring Damper Comb



Spring Damper Combination

G-6

APPENDIX H
STRUCTURAL STIFFNESS MEASUREMENTS

R. M. Brach and R. H. Van Beek

I. INTRODUCTION

In order to improve launcher accuracy by choosing optimum launcher parameters, the individual effect of each significant parameter must be determined. Studies with mathematical models are being used to determine these effects, followed by firing programs to substantiate the theoretical conclusions. The success of using mathematical models for this purpose depends upon at least two major points: (1) the degree to which the models qualitatively represent the dynamics of the launcher and (2) the quantitative agreement between the mathematical model output values and actual measurements of launcher motion. In order to optimize the quantitative agreement, the numerical input values to the mathematical models should correspond closely to measured values for the actual launcher. For this reason, static stiffness measurements were made of a number of launcher structural components using Prototype No. 3 as well as the trails of Prototype No. 1. Another equally important reason for making these measurements is to enable comparison of the actual stiffness values of the manufactured parts with the intended design stiffness.

The overall, or combined, stiffness of the launcher structure below the trunnions is desired. It would also be very useful to determine which components, such as the carriage box-sections, trail hinge-pins, etc., contribute the most to the total flexibility. As it would have been time consuming and costly to disassemble the entire structure and measure the stiffness of each part, the stiffness of the entire structure was first measured, and then, wherever disassembly was simple, as for example, the removal of the trails, separate measurements were taken. An attempt was made to determine the stiffness of certain components by eliminating their contributions to flexibility by means of auxiliary stiffeners. In general, no special consideration was given to the measurement procedures to develop

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extreme accuracy since stiffness values accurate within 5% to 10% were regarded as sufficient with respect to the mathematical models. Standard laboratory test machines were used wherever possible.

II. MEASUREMENTS

Static stiffness measurements were made for the following components and assemblies:

1. The entire structure below the trunnions(supporting structure).
2. The entire structure below the trunnions with the upper and lower carriage box-sections clamped together.
3. The entire structure below the trunnions with the carriage box-sections clamped together and struts inserted between the trunnions and trails. (Trails were parallel to the firing tube.)
4. The entire structure without stiffeners-but traversed:
 - a. 20° right
 - b. 20° left
5. Elevating screw support
6. Trails:
 - a. Prototype No. 3
 - b. Prototype No. 1

The stiffness of the entire supporting structure was measured by applying a known load perpendicular to a bar supported through the trunnions, and measuring the deflection of the trunnions with dial gages. A side elevation of this system is shown in Fig. H-1. The load was measured with calibrated strain gages on the loading bar.

The trail stiffness was measured with the two trails clamped together by a relatively stiff fixture at their hinge-pin ends. This enabled the measurement and loading of the system to be performed as a simply supported beam (see Fig. H-2b). Again, the deflections were measured with dial gages.

The load-deflection characteristics of the jack screw supports in the upper carriage sides were measured by the procedure shown in Fig. H-2a. The compressive load applied to the screw was determined, as was the displacement of the screw relative to the upper carriage.

One parameter required by the mathematical models is a dynamic equivalent for the mass moment of inertia of the supporting structure, about a line through the ball joint perpendicular to the plane of elevation. This equivalent inertia, which permits the elastic carriage to be represented by a single flexibility and single inertia, can be measured indirectly by measuring the first mode natural frequency of the elastic carriage. In order to do this, an electro-dynamic shaker was attached to the bar through the trunnions for the purpose of applying to the launcher a harmonic force of variable frequency. The dynamic stiffness was measured as

$$k_D = \frac{Fw^2}{a},$$

where F is the applied force, w is the frequency, and a is the acceleration. Zero or very small values of the dynamic stiffness indicate resonance, and the smallest of the resonant frequencies is the first normal mode frequency.

III. DISCUSSION OF RESULTS AND CONCLUSIONS

Figure H-3 shows the load-deflection characteristics for the carriage. When converted to angular rotation about the base pivot, the average of the two sides yields a stiffness for the carriage of 32×10^6 lb-in./radian. Figure H-4 shows the stiffness of the same assembly, but with wedges between the upper and lower carriages and links of ties bolting the two together. The stiffness is 38.5×10^6 lb-in./radian. Figure H-5 shows the stiffness of the same assembly as Figure H-4, but with additional links connecting the trunnions to the trails. The stiffness is 55.5×10^6 lb-in./radian.

Figures H-6 and H-7 show the load-deflection characteristics of the carriage in full traverse, right and left, respectively. Traverse to the right stiffens the right trunnion and adds flexibility to the left trunnion, and vice versa; but there is a net decrease in the average stiffness. For both right and left traverse, the average stiffness is 25×10^6 lb-in./radian.

Figure H-8 shows the load-deflection characteristics of the lower elevating screw supports loaded in compression. This deflection occurs in the complicated structure consisting of the elevating screw gear box and its mounting.

Figure H-9 shows the load-deflection characteristics of the Prototype No. 3 trails joined at the hinge-pins and loaded as a single, simply supported beam. Converted to angular rotation, which would be produced about the base pivot, the combined stiffness of the two trails is 145×10^6 lb-in./radian. Figure H-10 gives the same data for the Prototype No. 1 trails, for which the combined stiffness is 109×10^6 . The trails on the two prototypes are of different lengths, and the stiffness varies as the length squared. For purposes of comparison, the stiffness of the Prototype No. 3 trails was extrapolated to the value they would have if their length were the same as that of the Prototype No. 1 trails. This value is 223×10^6 lb-in./radian. The weight of the Prototype No. 3 trails is 385 lb; of the Prototype No. 1 trails, 351 lb. On the basis of these stiffnesses and weights, the stiffness-to-weight ratio of the Prototype No. 3 trails is computed to be 87% greater than that of the Prototype No. 1 trails.

Figure H-11 shows the results of the dynamic stiffness measurements on the carriage. The data are not conclusive, but it is probable that resonance in the first normal mode is indicated at 70 cps. The inertia of a torsional system having a stiffness of 32×10^6 lb-in./radian and a natural frequency of 70 cps agrees reasonably well with the calculated equivalent inertia of the carriage.

Each elevating screw and upper bracket assembly was loaded in compression in a universal testing machine. Fig. H-12 shows the resulting load-deflection curve for each screw-bracket assembly. The nonlinearities are due primarily to nonlinear local deflections under the balls in the ball nuts and also to clearances between the balls and the nut.

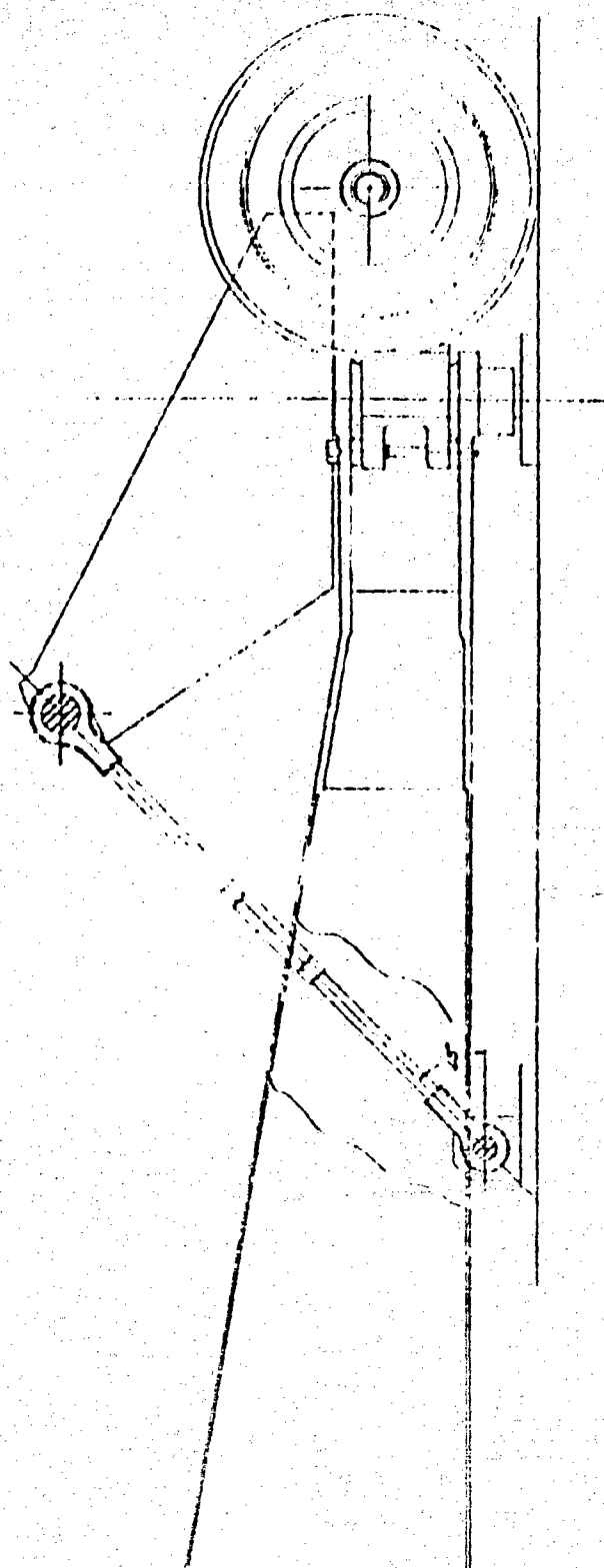


Fig. H-1 TEST SETUP FOR MEASURING CARRIAGE STIFFNESS AT TRUNNIONS

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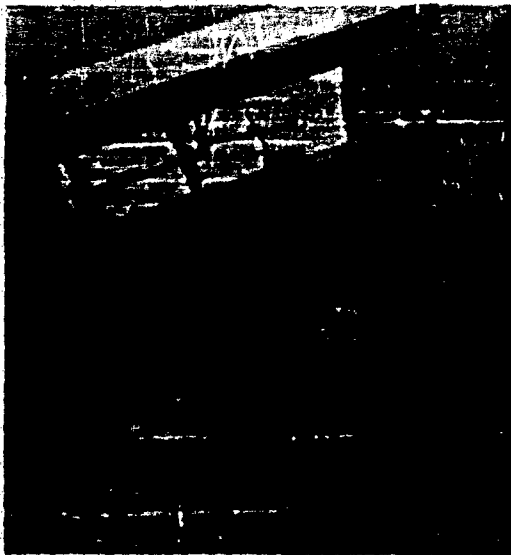


Fig. H-2a Stiffness of
Elevating Screw Support

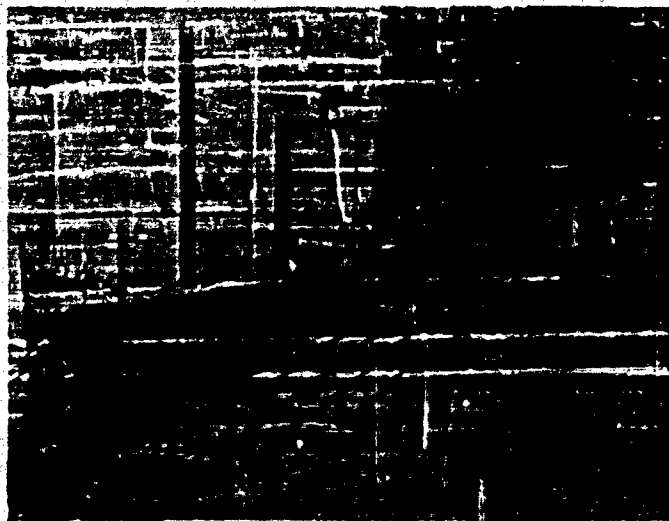


Fig. H-2b Stiffness of Trails



Fig. H-2c Natural Frequency of Carriage

Fig. H-2 LAUNCHER COMPONENTS UNDERGOING TESTS

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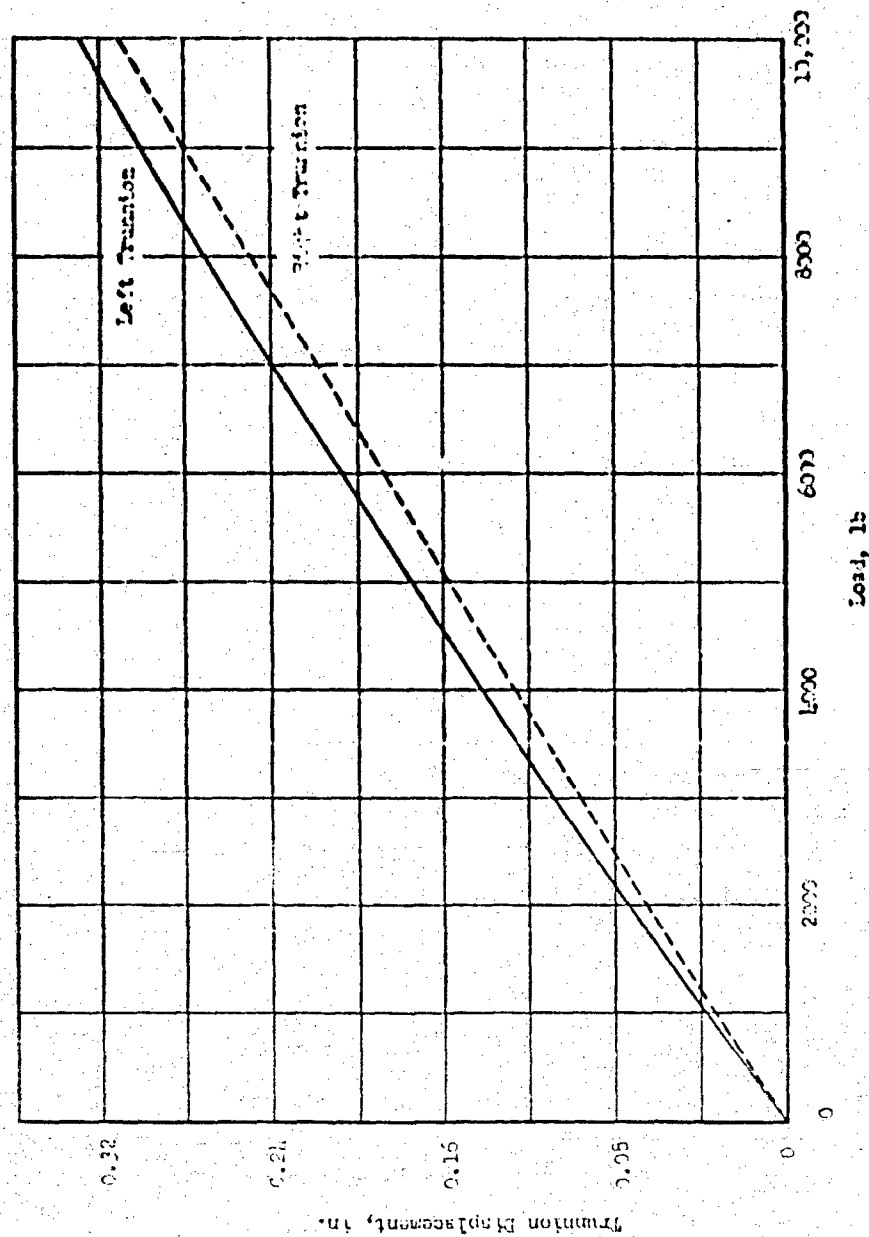


Fig. H-3 STIFFNESS OF CARRIAGE AT TRUNNIONS WITHOUT STIFFENERS

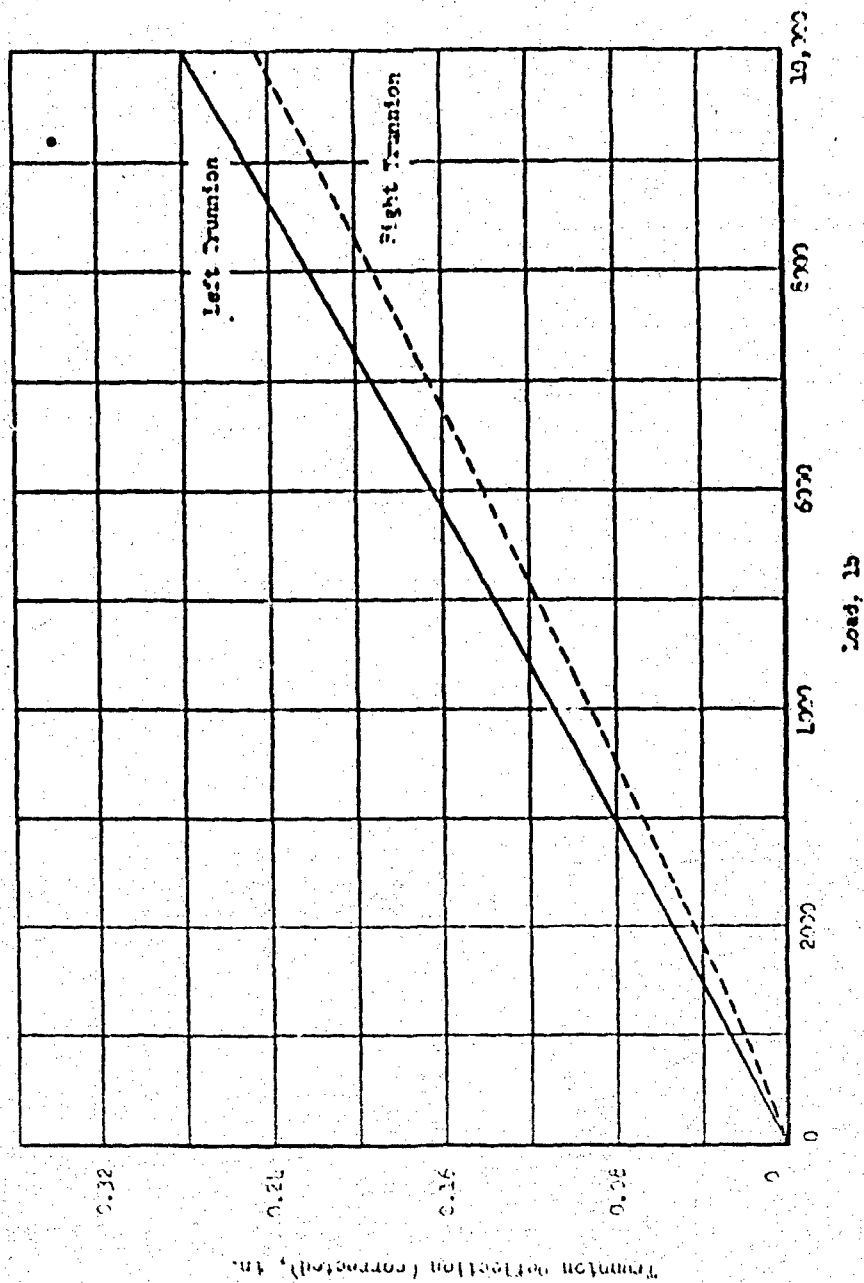


Fig. H-4 STIFFNESS OF CARRIAGE AT TRUNNIONS WITH WEDGES AND TIES

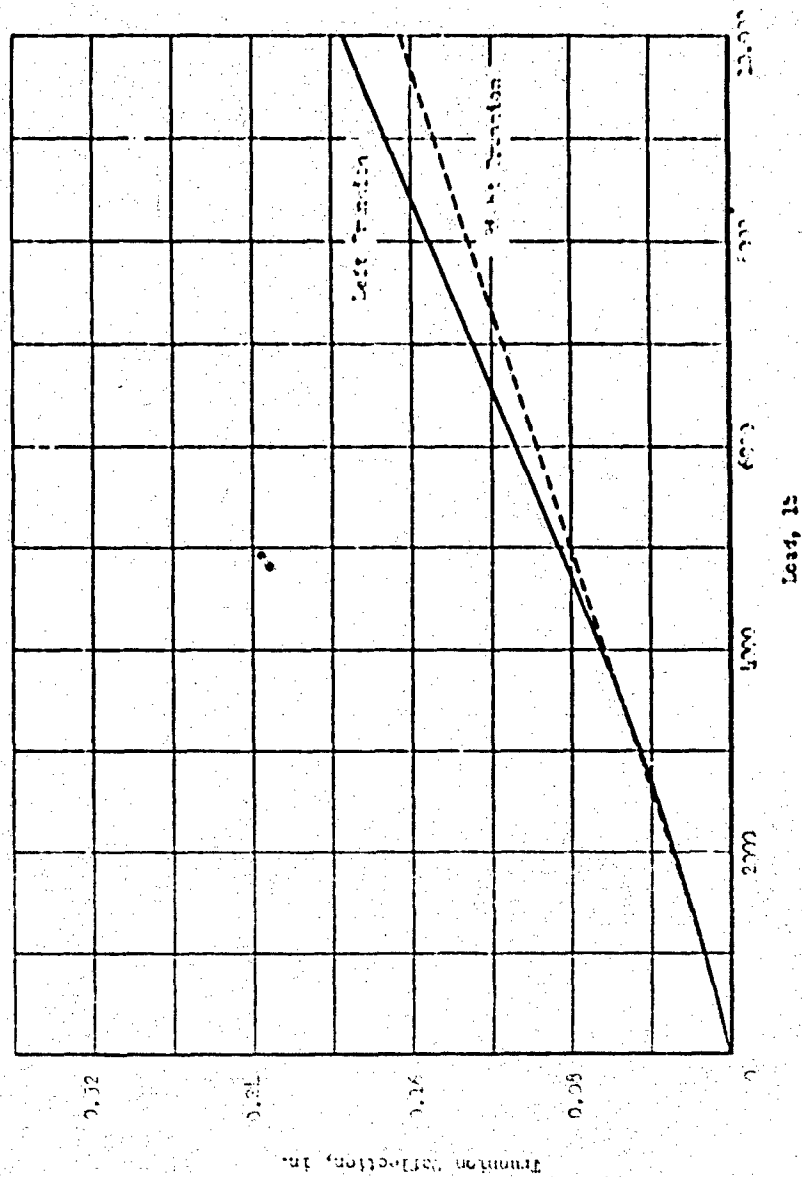


Fig. H-5 STIFFNESS OF CARRIAGE AT TRUNNIONS WITH WEDGES, TIES,
AND LINKS BETWEEN TRUNNIONS AND TRAILS

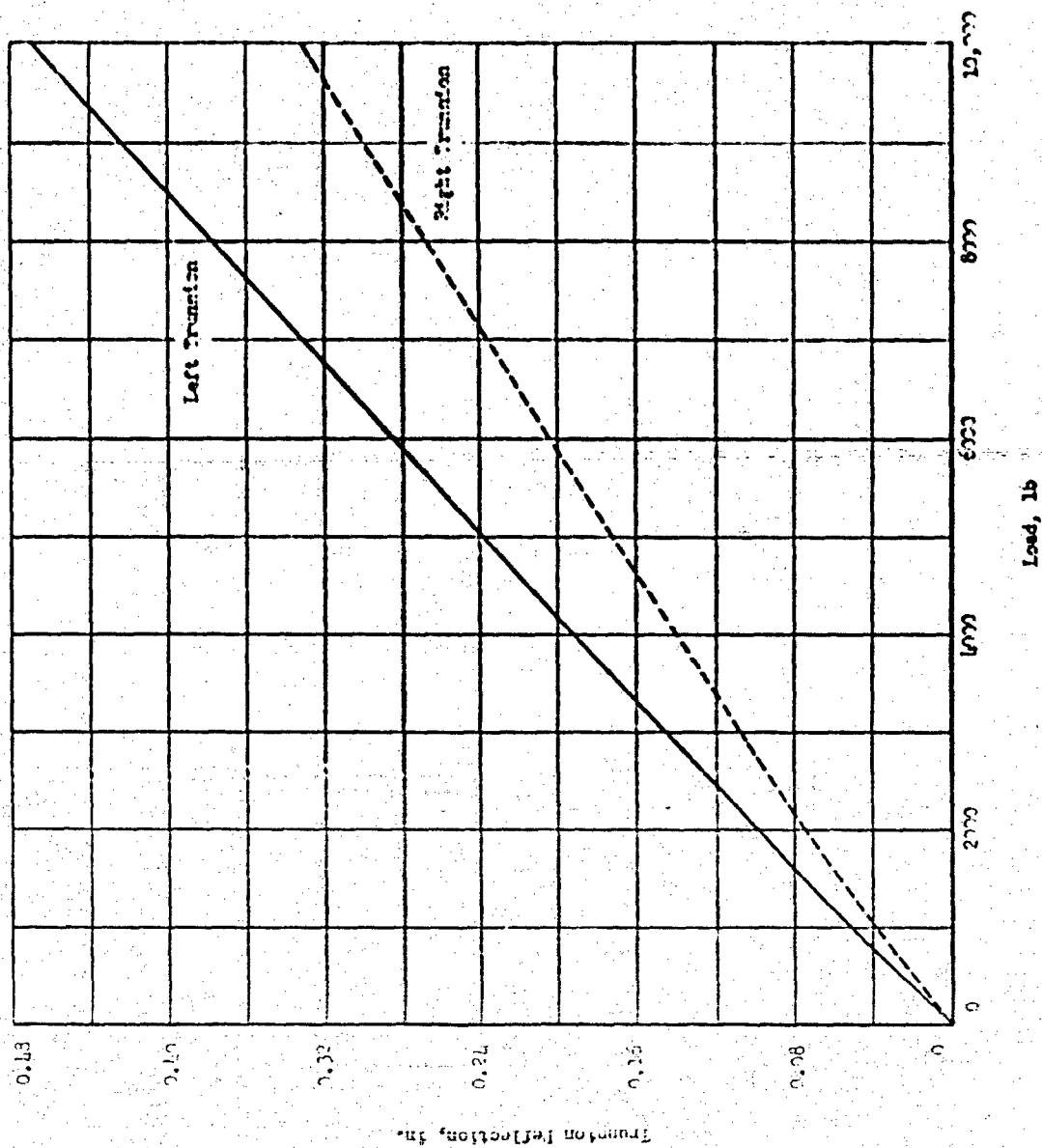


Fig. H-6 STIFFNESS OF CARRIAGE AT FULL TRAVERSE, 20° RIGHT

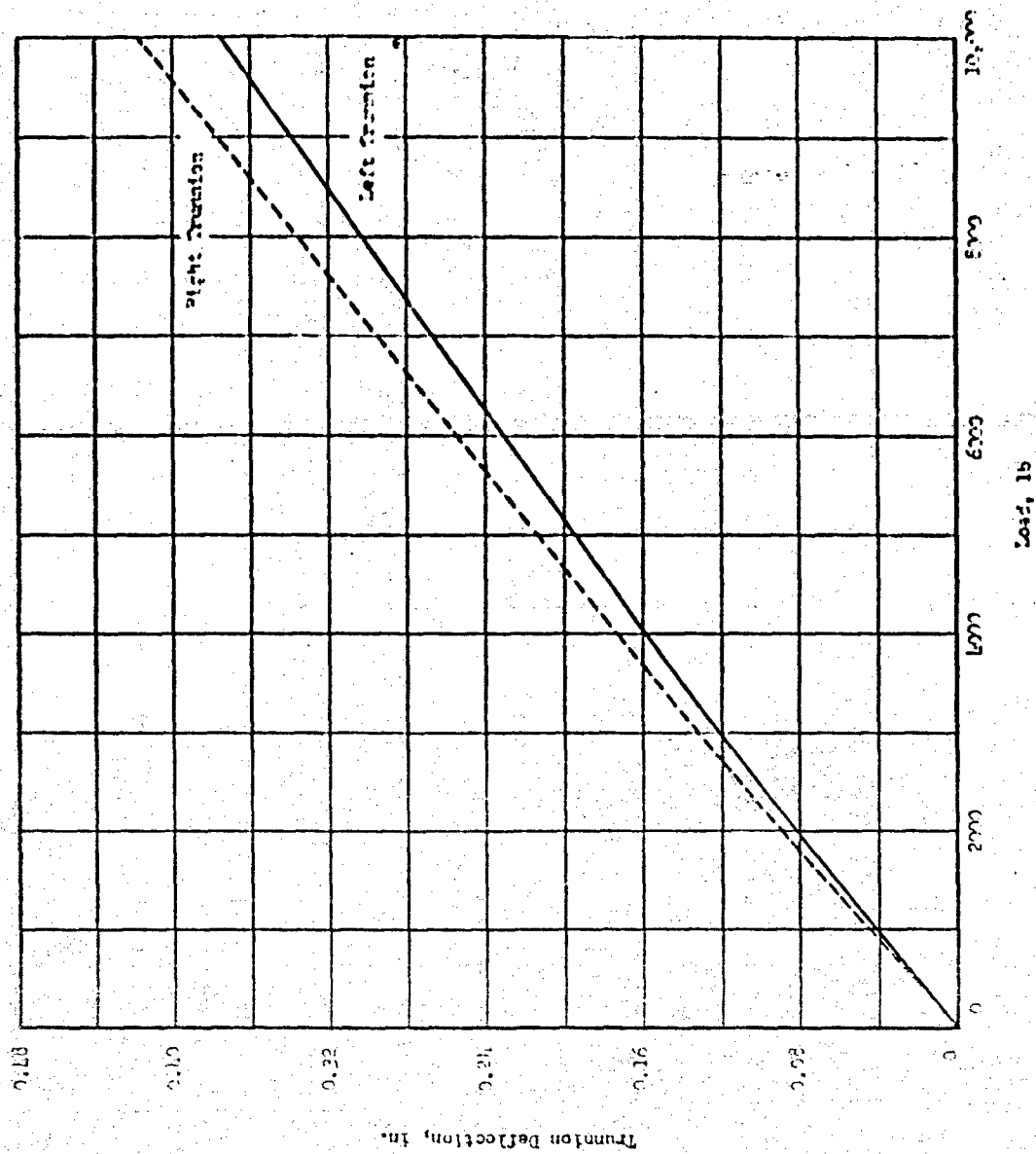


Fig. H-7 STIFFNESS OF CARRIAGE AT FULL TRAVERSE, 20° LEFT

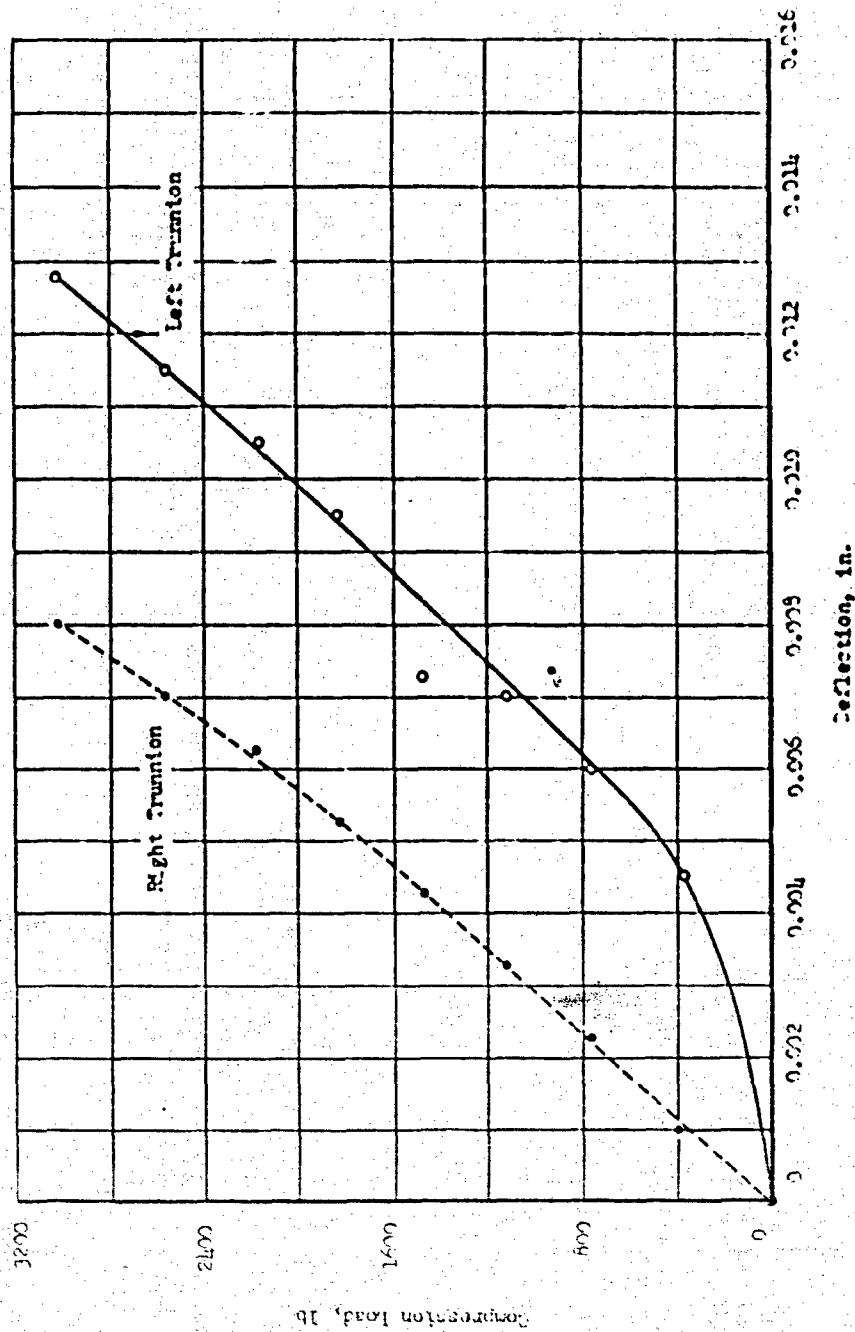


Fig. H-8 STIFFNESS OF ELEVATING SCREW SUPPORTS

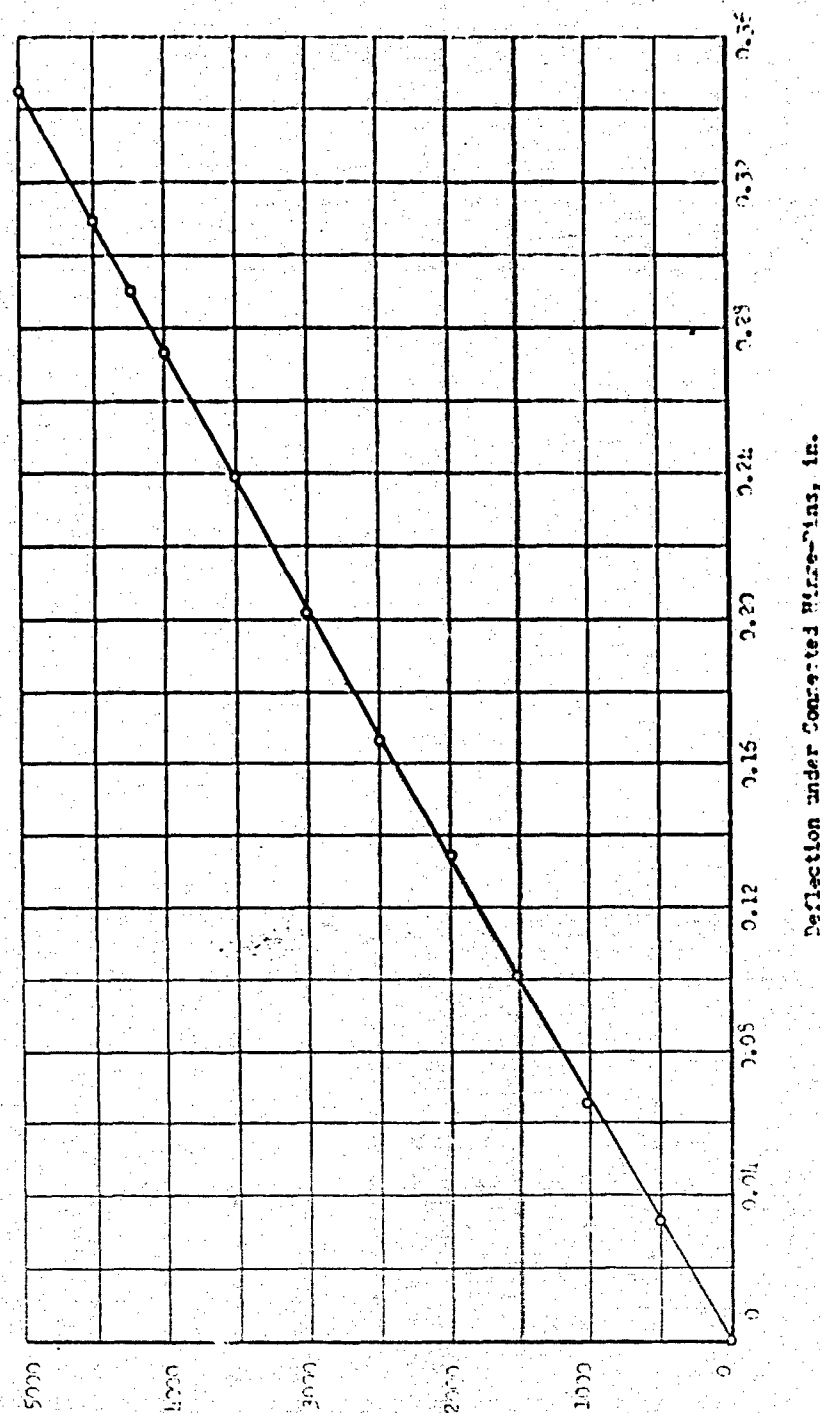


Fig. H-9 STIFFNESS OF PROTOTYPE NO. 3 TRAILS

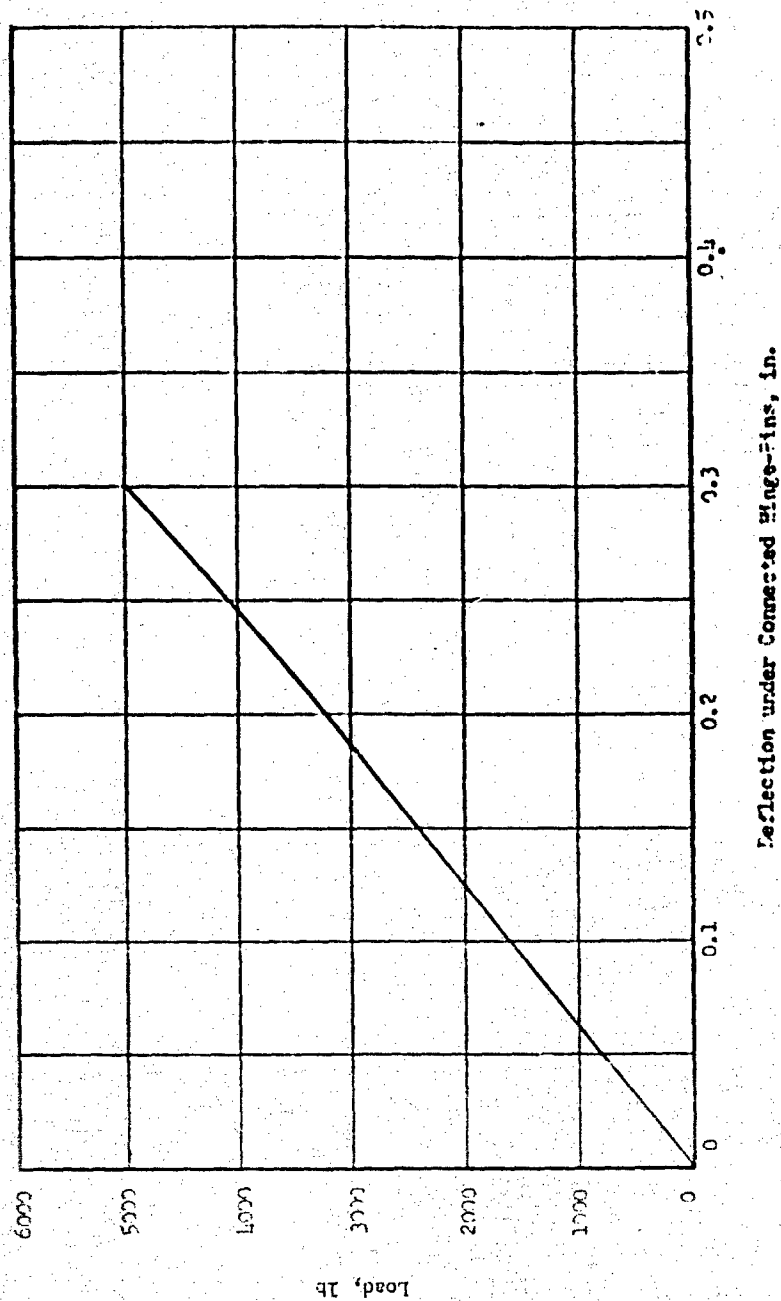
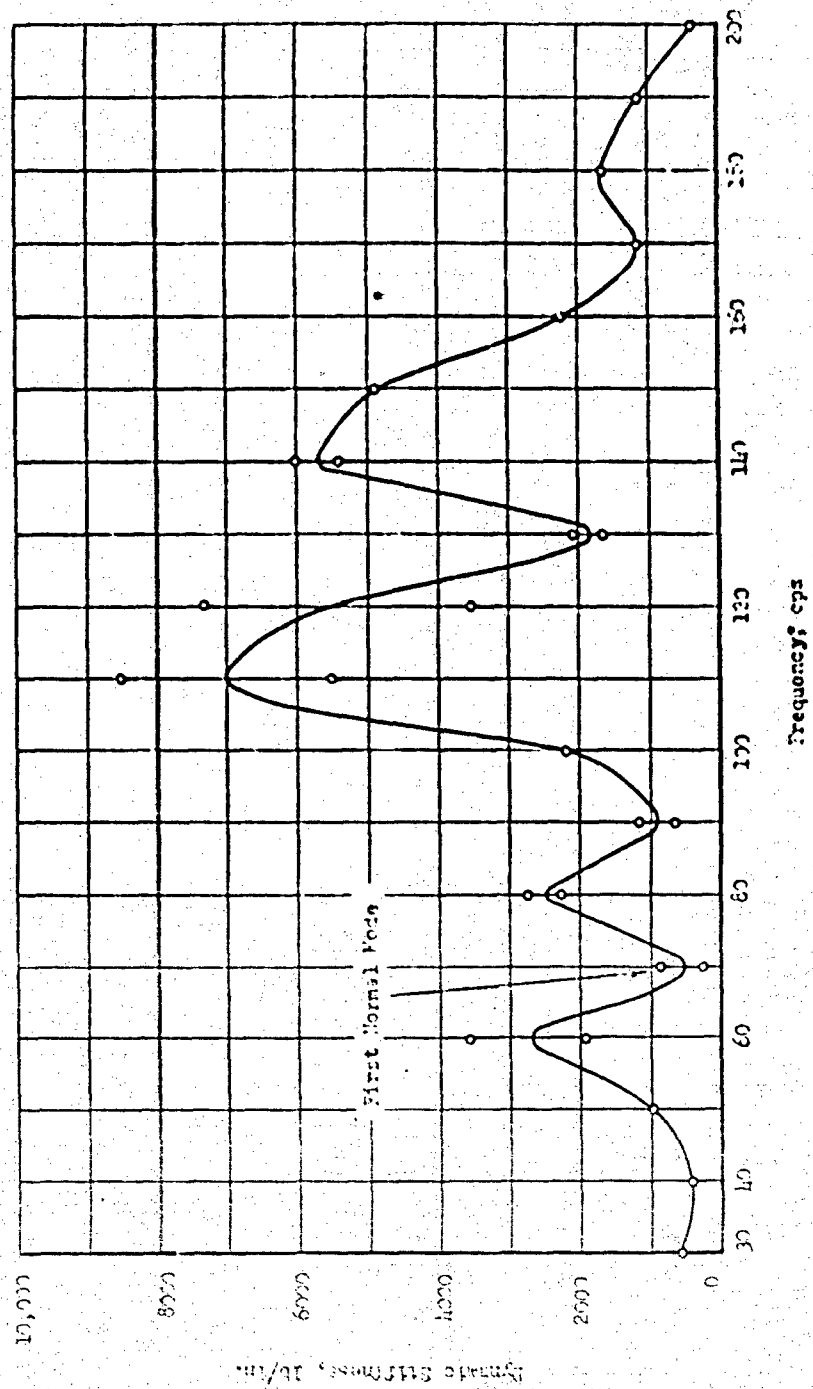


Fig. H-10 STIFFNESS OF PROTOTYPE NO. 1 TRAILS



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Fig. H-11 NATURAL FREQUENCY OF LAUNCHER CARRIAGE

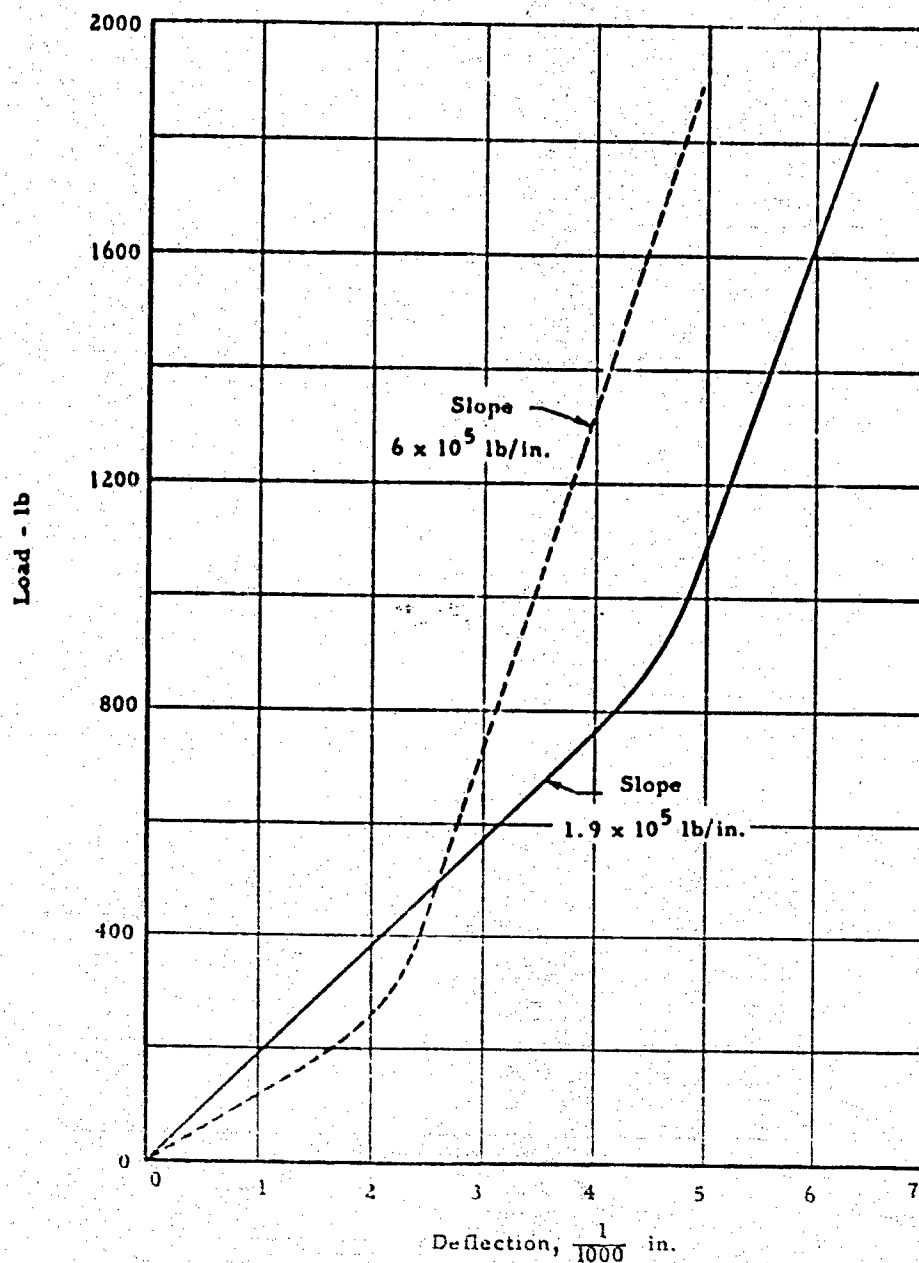


Fig. H-12 STIFFNESS OF ELEVATING SCREWS
AND UPPER BRACKETS

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APPENDIX I
FIRING PROGRAM

This Appendix presents a list of the firings conducted in order to obtain the launcher response in the vertical plane (see Appendix J). The instrumentation for each firing is listed as well as the structural configuration. The bursts are coded with B followed with a number; each round is individually marked by R with a corresponding number.

Because of instrumentation difficulties, some bursts were repeated. These are not listed because they were under identical conditions. Also some additional firings were made such as those to study the horizontal dispersion problem; these are described in the report body and not listed here.

TABLE 1-1
FIRING SCHEDULE

Burst No.	Round No.	Zone		Target	Instrumentation	Fixtures	Movies	Comments
		Elevation	Traverse					
S. S.	R-1	8/60°/0°		No	4, 5, 6, 8, 10	None	None	To seat firing base and stakes
S. S.	R-2	8/60°/0°		No	4, 5, 6, 8, 10	None	None	and also to adjust C' recoil buffer. Checkout instrumentation.
S. S.	R-3	8/60°/0°		No	4, 5, 6, 8, 10	None	None	
B-1	R-4	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On	This will be a burst to be used
	R-5	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	Target	for comparison purposes but
	R-6	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	(See	also to check out the left-right
	R-7	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	Comments)	sequencing. Have a movie camera
	R-8	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	Black &	era on the target to record sequence. Regular 16MM camera
B-2	R-9	8/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	White	at high speed.
	R-10	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	Same as previous burst
	R-11	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
	R-12	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
	R-13	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
B-3	R-14	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
	R-15	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
	R-16	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	Repeat of previous burst to
	R-17	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	check similarity of results.
	R-18	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	(Mainly to check the similarity
	R-19	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	of target patterns.)
	R-20	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	
	R-21	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	On target	

Burst No.	Round No.	Zone		Target	Instrumentation	Fixtures	Movies	Comments
		Elevation	Traverse					
B-4 {	R-22	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	None	To complete a set of three records at this condition for comparison.
	R-23	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	None	None	Record round sequence through target.
B-5 {	R-24	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	1a, 3	None	Large damper holes. Record round sequence through target.
	R-25	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	1a, 3	None	
S. S.	R-26	9/45°/0°		No	3, 4, 5, 6, 8, 10	1a, 3	None	Single shot repeat of preceding burst.
S. S.	R-27	9/45°/0°		No	3, 4, 5, 6, 8, 10	1a, 3	None	Repeat of preceding test.
B-6 {	R-28	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	1a, 3	None	Change to small diameter holes in hydraulic damper. Record round sequence through target.
	R-29	9/45°/0°		Yes	3, 4, 5, 6, 8, 10	1a, 3	None	Single shot repeat of preceding test.
S. S.	R-30	9/45°/0°		No	3, 4, 5, 6, 8, 10	1a, 3	None	Repeat of preceding test.
S. S.	R-31	9/45°/0°		No	3, 4, 5, 6, 8, 10	1a, 3	None	Repeat of preceding test.
B-7 {	R-32	9/45°/0°		Yes	3, 4, 5, 6, 7	1a, 3, 4, 5, 6	None	Small dia. holes in damper coil spring in elevating system.
	R-33	9/45°/0°		Yes	3, 4, 5, 6, 7	1a, 3, 4, 5, 6	None	
S. S.	R-34	9/45°/0°		No	3, 4, 5, 6, 7	1a, 3, 4, 5, 6	None	Single shot repeat of preceding test.
S. S.	R-35	9/45°/0°		No	3, 4, 5, 6, 7	1a, 3, 4, 5, 6	None	Repeat of preceding test.

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Burst No.	Round No.	Zone		Target	Instrumentation	Fixtures	Movies	Comments
		Elevation	Traverse					
B-8 {	R-36	9/45°/0°		Yes	3, 4, 5, 6, 7, 8	2, 3, 4, 5, 6	None	Small dia. damper holes.
	R-37	9/45°/0°		Yes	3, 4, 5, 6, 7, 8	2, 3, 4, 5, 6	None	record target sequence.
S. S.	R-38	9/45°/0°		No	3, 4, 5, 6, 7, 8	2, 3, 4, 5, 6	None	Single shot repeat of preceding test.
	R-39	9/45°/0°		No	3, 4, 5, 6, 7, 8	2, 3, 4, 5, 6	None	Repeat of preceding test.
B-9 {	R-40	9/45°/0°		No	3, 4, 5, 6	1b	None	Install ring springs into elevating system.
	R-41	9/45°/0°		No	3, 4, 5, 6	1b	None	
S. S.	R-42	9/45°/0°		No	3, 4, 5, 6	1b	None	Single-shot repeat of preceding test.
	R-43	9/45°/0°		No	3, 4, 5, 6, 7	1b, 3	None	Repeat of previous test with damper added. Small dia. holes.
B-10 {	R-44	9/45°/0°		No	3, 4, 5, 6, 7	1b, 3	None	
	R-45	7/45°/0°		No	3	1a	None	No recoil pressure
B-11 {	R-46	7/45°/0°		No	3	1a	None	

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The following fixtures were required for the experimental firings of Pilot No. 3 at Ft. Sheridan. The corresponding numbers in the following list will be used in the firing schedule.

1. Variable flexibility elevating rods with: a.) coil springs and b.) friction damped ring springs,
2. Rigid elevating rods,
3. Hydraulic dampers between carriage sides and cradle with interchangeable pistons,
4. Shorter pad placement on the trails,
5. Trunnion-to-trail stiffening struts,
6. Brackets and wedges for stiffening the carriage box-sections.

The following instrumentation was required for the experimental firings of Pilot No. 3 at Ft. Sheridan. The combination of instruments to use for each particular firing will be found on the firing schedule. Each of the following list will be referred to by number.

1. One strain gage on the rear plate of the recoiling parts for a firing signal.
2. One strain gage on each trail.
3. Two linear pots between cradle and ground, one on each side of the cradle, perpendicular to it at 45° elevation, a few inches behind the cradle front. Maximum expected motion, ± 3 in.
4. One strain gage on each elevating rod including the fixtures. Maximum expected strain, 300×10^{-6} in./in.
5. Two linear pots, one on each side of the launcher, trunnion to ground, parallel to the cradle at 45° elevation. Expected maximum motion, $\pm 3/4$ in.
6. Two linear pots, one on each side, trunnion to ground, perpendicular to the cradle at 45° elevation. Maximum expected motion, $\pm 1/2$ in.
7. Two strain gages, one on each damper rod, to measure tension and compression. Maximum expected strain, 200×10^{-6} in./in.
8. One linear pot on each trail end, perpendicular to the ground. Maximum expected vertical displacement ($+6$, $-1/4$) in.
(The firing base will be staked to prevent rearward displacements.)
9. Strain gages on the indexing system shafts.
10. Recoil system pressure (one gage. right or left side)

INSTRUMENTATION NOTE: Gages 1, 2 and 11 (rear plate, trail, & recoil pressure) will be used for all firings unless otherwise noted.

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APPENDIX J

EXPERIMENTAL LAUNCHER RESPONSE CURVES

I. INTRODUCTION

The list of the firings and the instrumentation for each burst in Appendix I shows that a very large amount of data was recorded. The most significant portion of this data is that which actually illustrates the response of the launcher. In order to limit the amount of data to a reasonable level, only the cradle response and trail strain are shown. (Exceptions to this are Fig. 22 and 23 in the body of the report.) These curves are presented for each of the structural configurations.

II. DISCUSSION

The cradle response is shown in inches of displacement. This is the motion of the cradle front perpendicular to the plane containing the trunnions and firing tube. If the trunnions remain motionless, this cradle motion, by dividing by the proper distance, gives cradle rotation. For this purpose, the trunnion motion in the vertical plane was found experimentally to be negligible; dividing the cradle displacement by 60 in. is a good approximation of cradle rotation.

If the trails respond primarily in their first mode shape, the trail strain is approximately proportional to the angular deflection of the trail at its connection with the lower box member. Again, experiments show that the use of the trail strain to indicate rotation is a good assumption. This strain and the cradle displacement serve to show the launcher response.

The following figures show the cradle response and trail strains for the first conditions described in Appendix I.

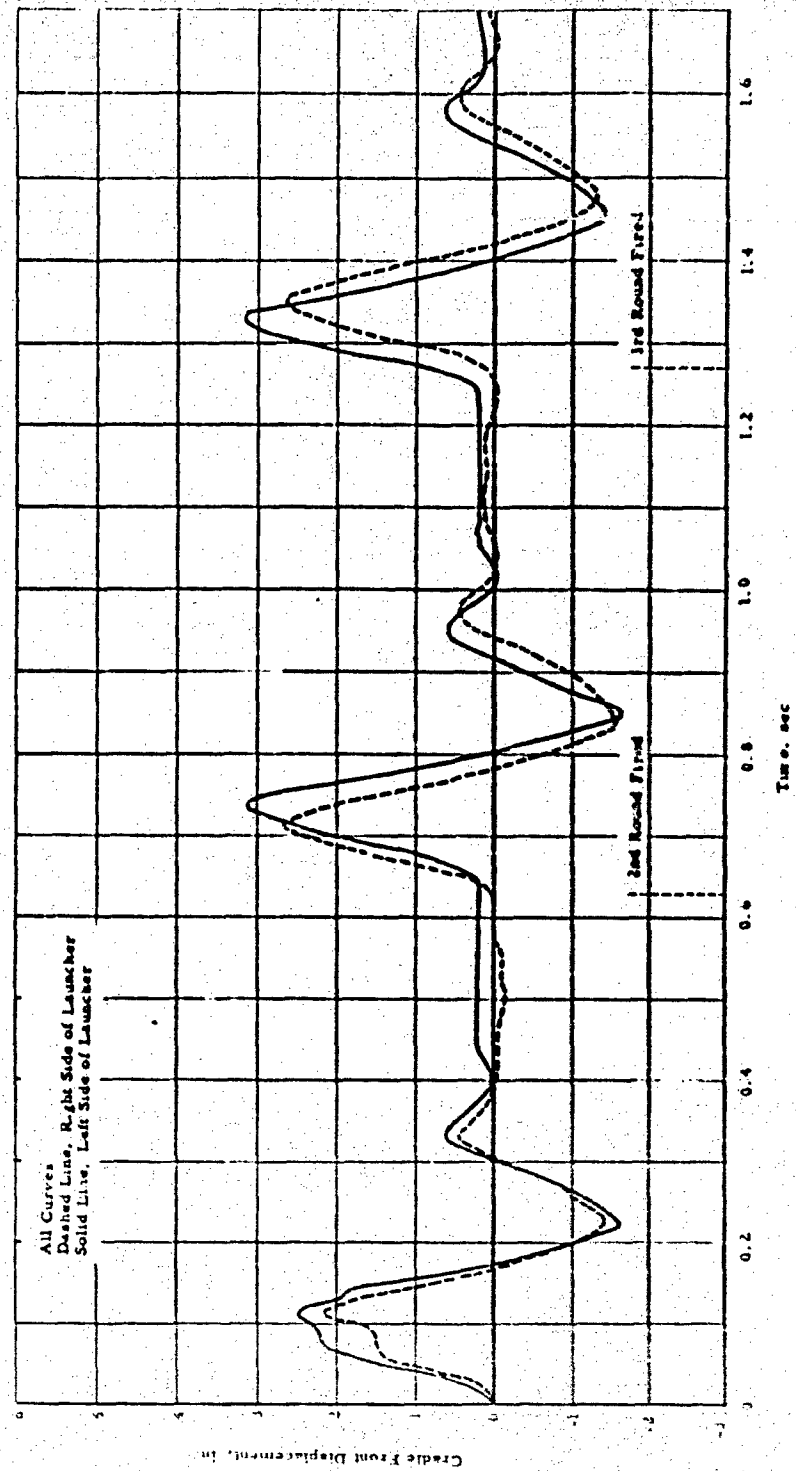


Fig. J-1 CRADLE MOTION, BURST B-2

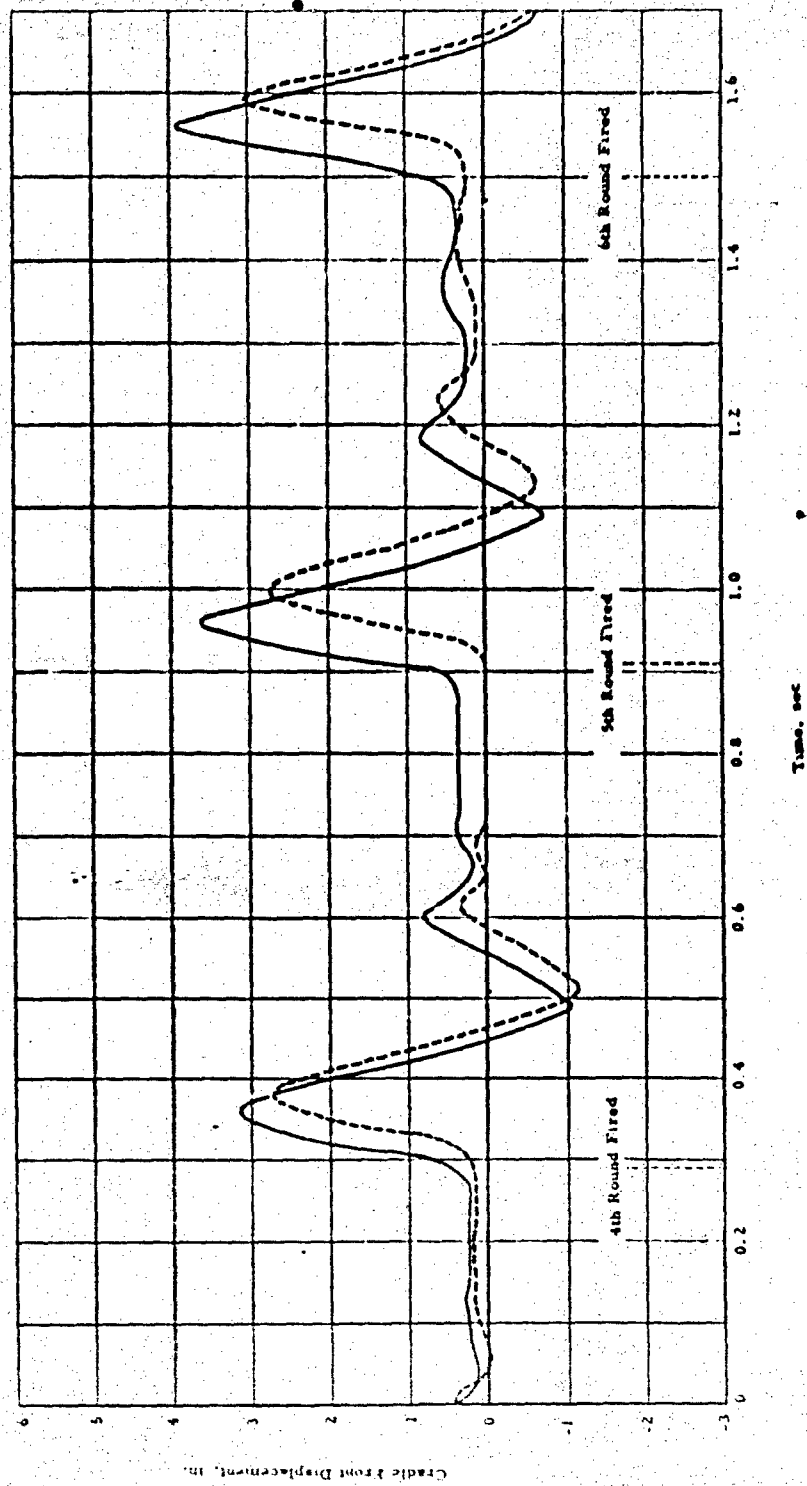


Fig. J-2 CRADLE MOTION, BURST B-2

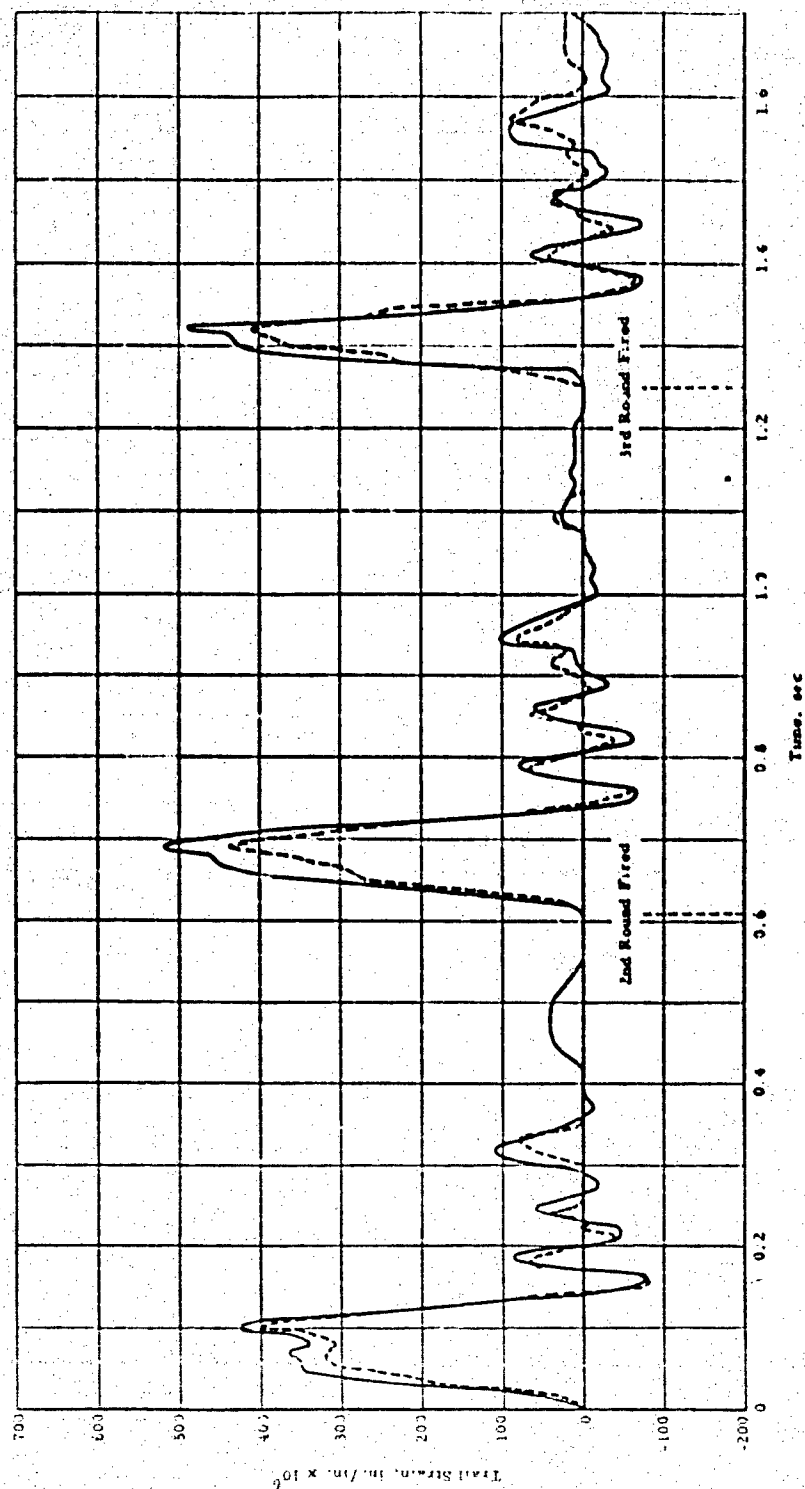


Fig. J-3 TRAIL STRAIN, BURST B-2

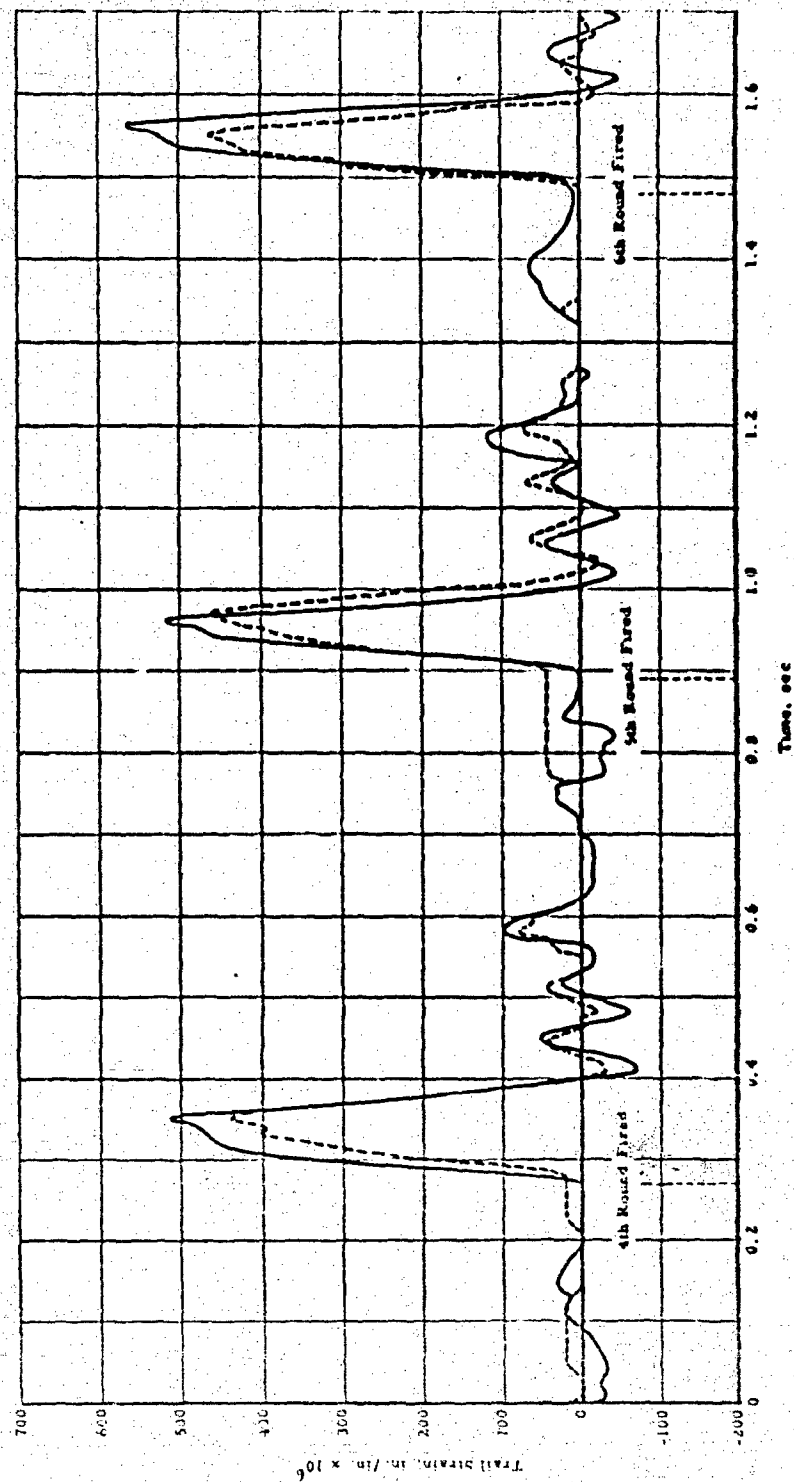


Fig. J-4 TRAIL STRAIN, BURST B-2

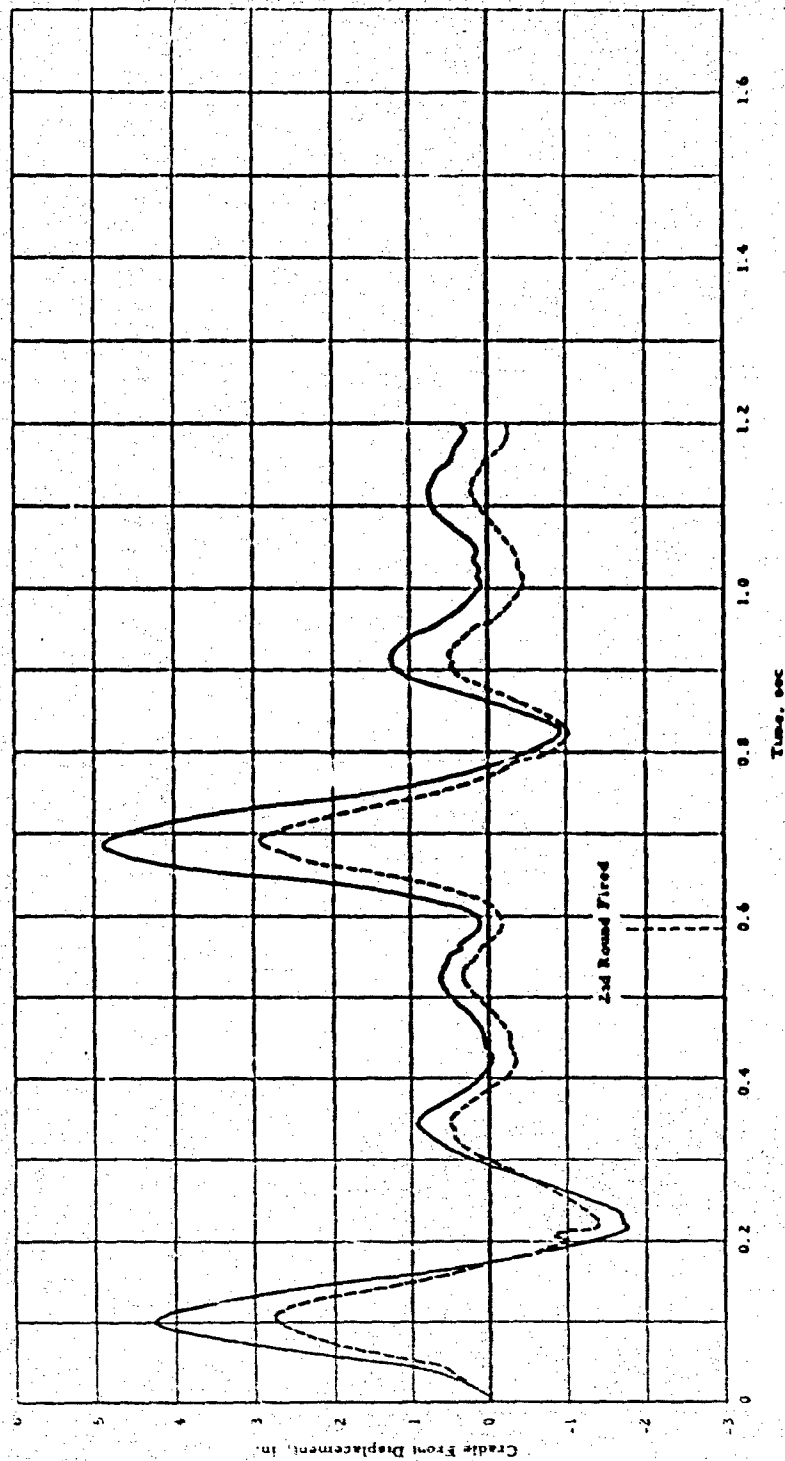


Fig. J-5 CRADLE MOTION, BURST B-5

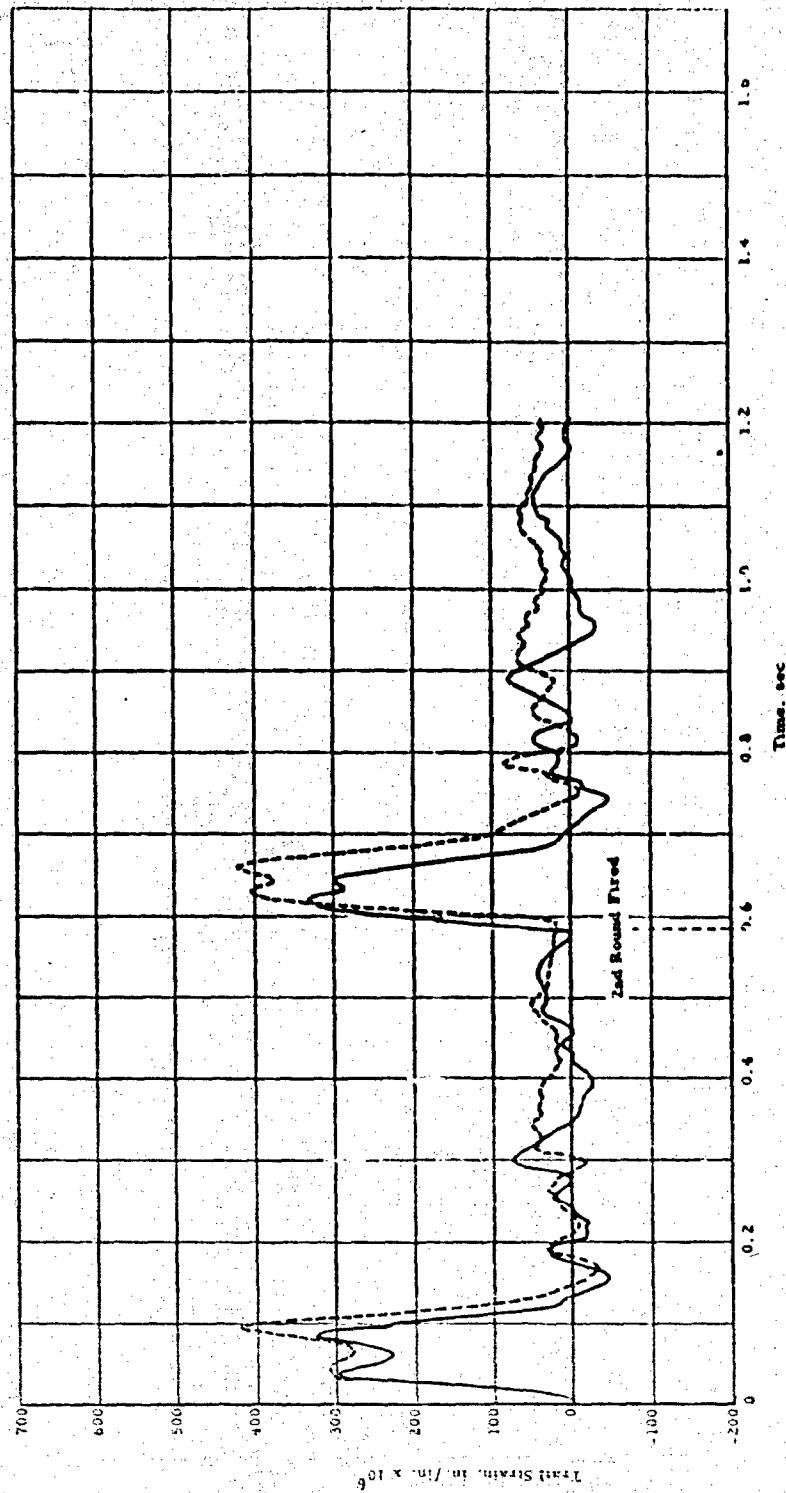


Fig. J-6 TRAIL STRAIN, BURST B-5

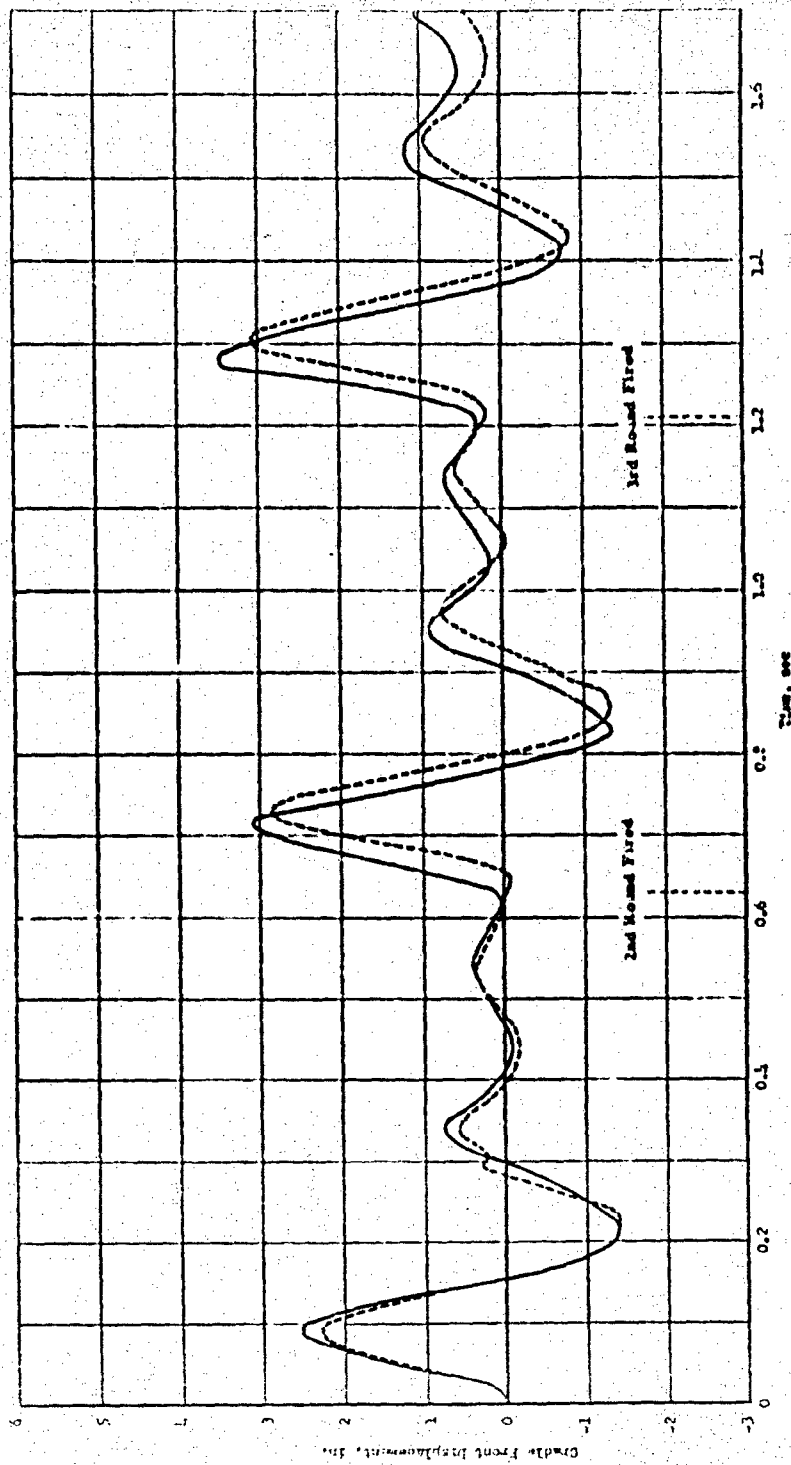


Fig J-7 CRADLE MOTION, BURST B-6

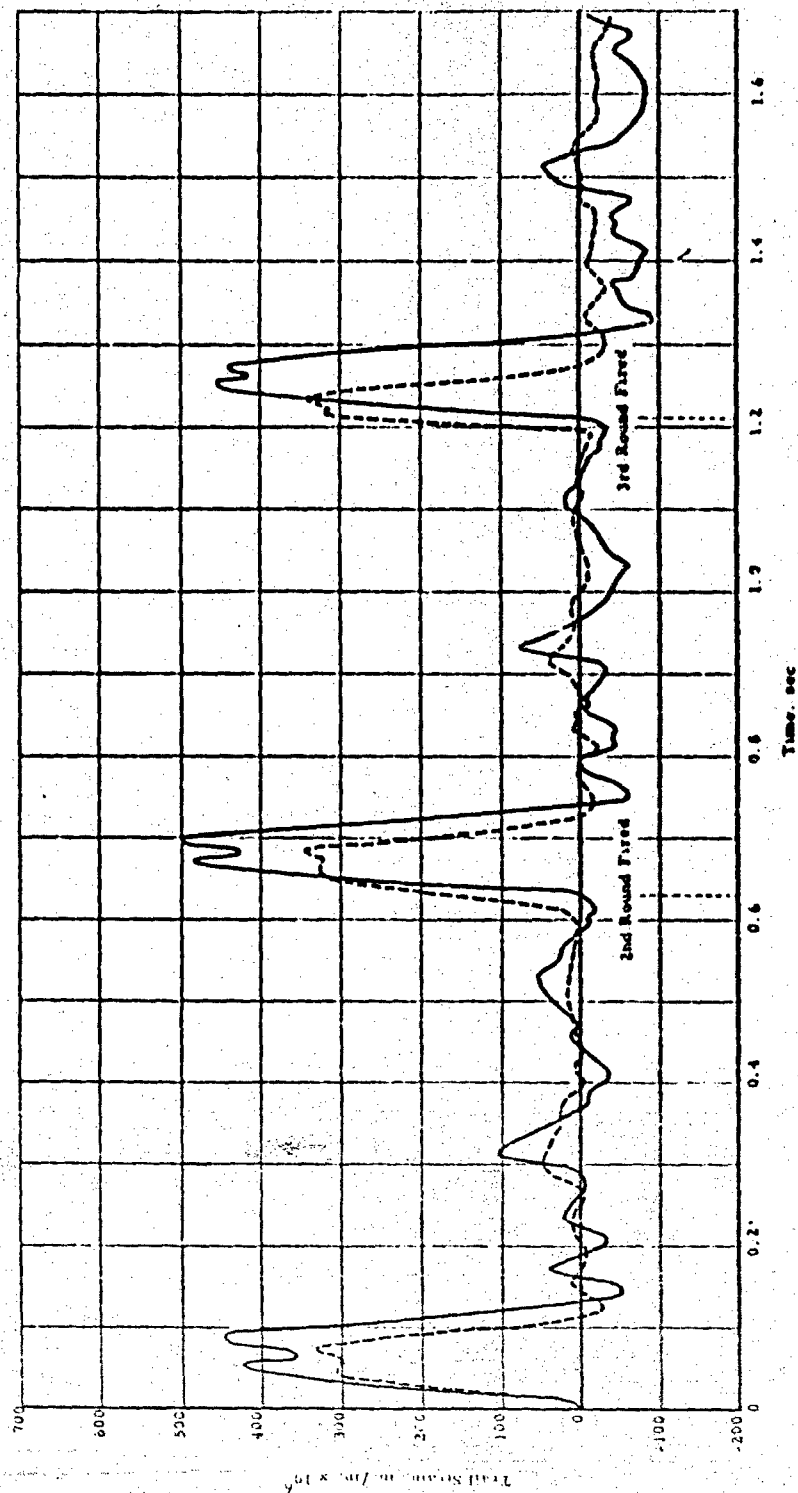


Fig. J-8 TRAIL STRAIN, BURST B-6

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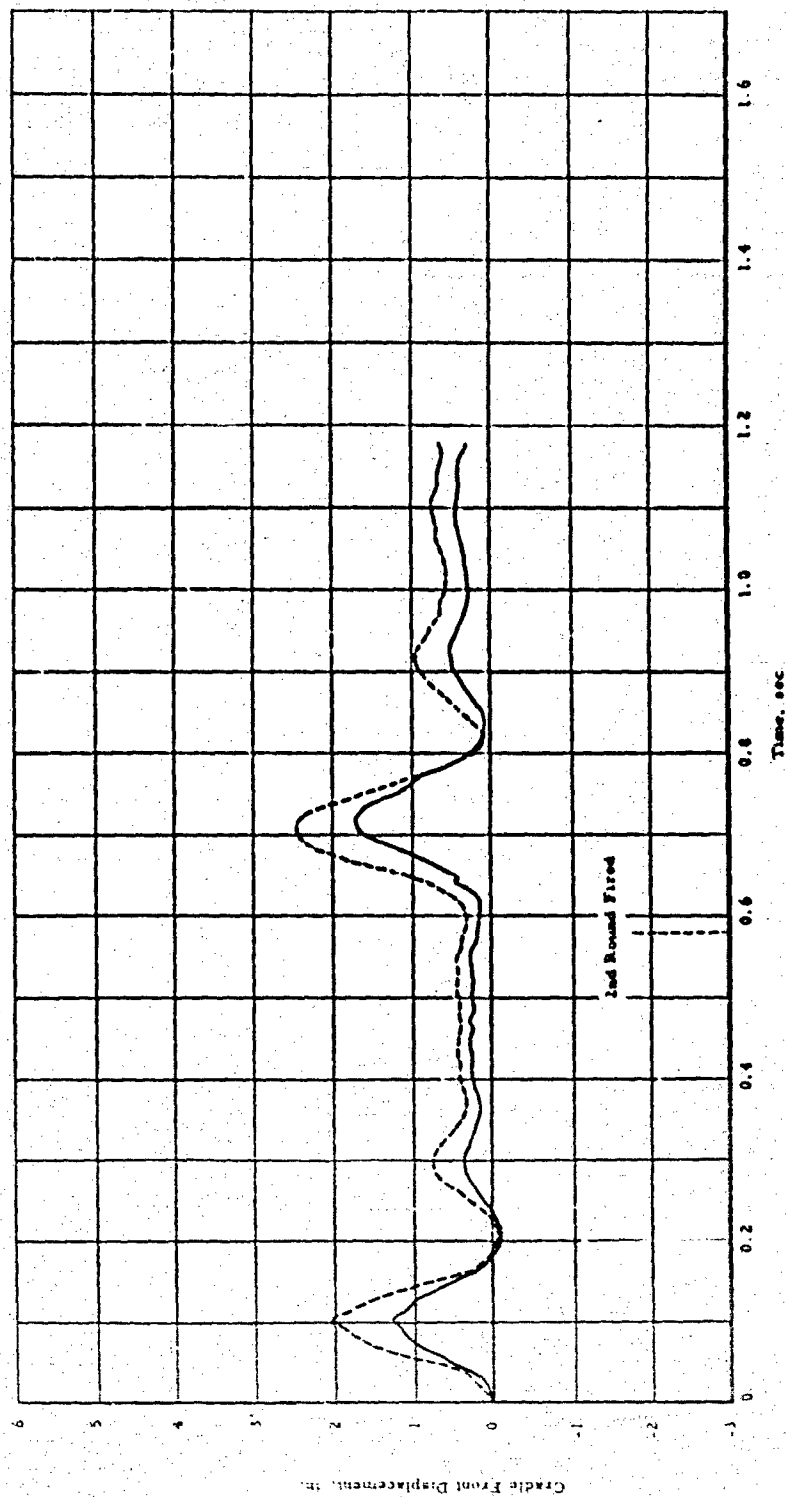


Fig. J-9 CRADLE MOTION, BURST B-7

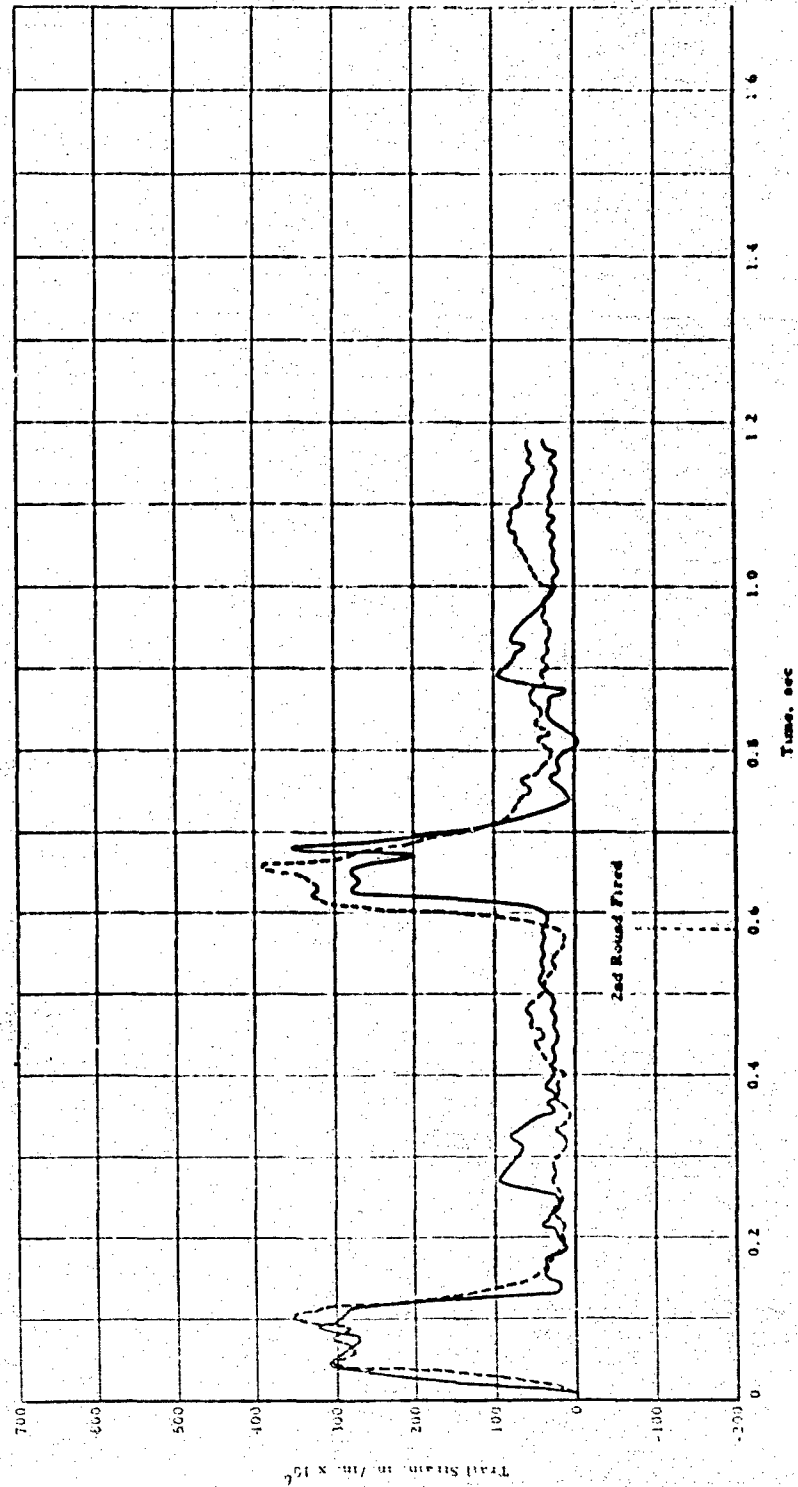


Fig. J-10 TRAIL STRAIN, BURST B-7

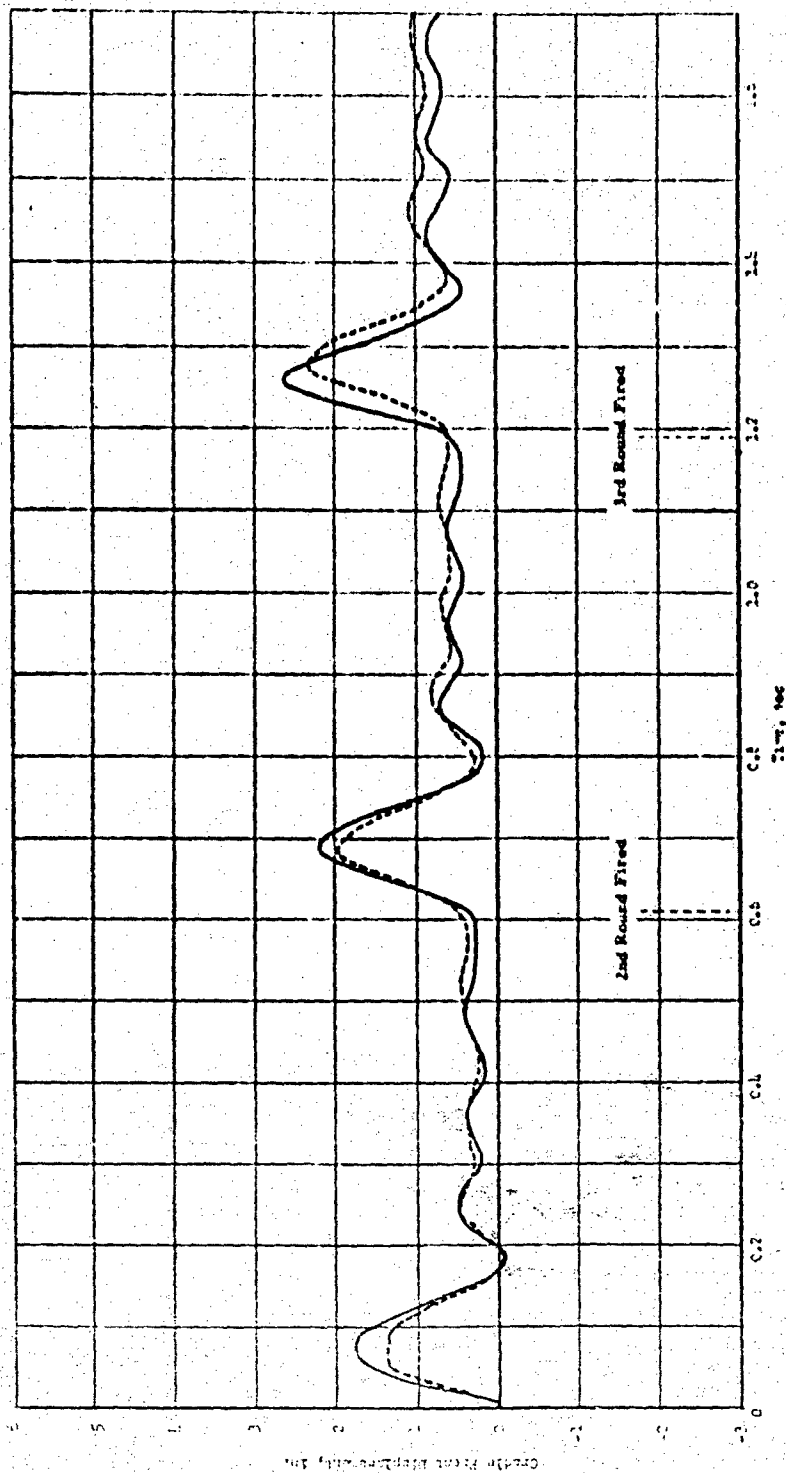


Fig. J-11 CRADLE MOTION, BURST B-8

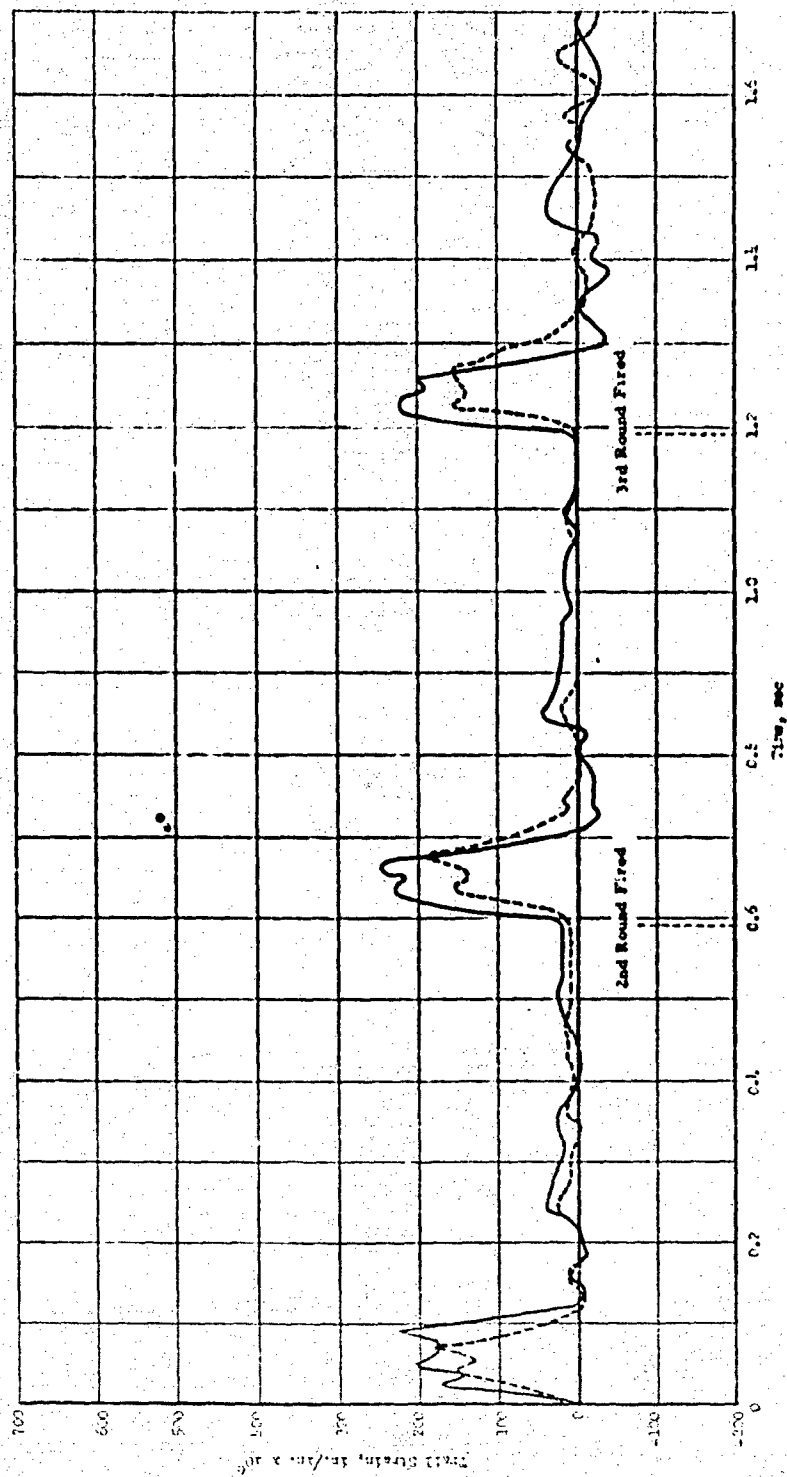


Fig. J-12 TRAIL STRAIN, BURST B-8

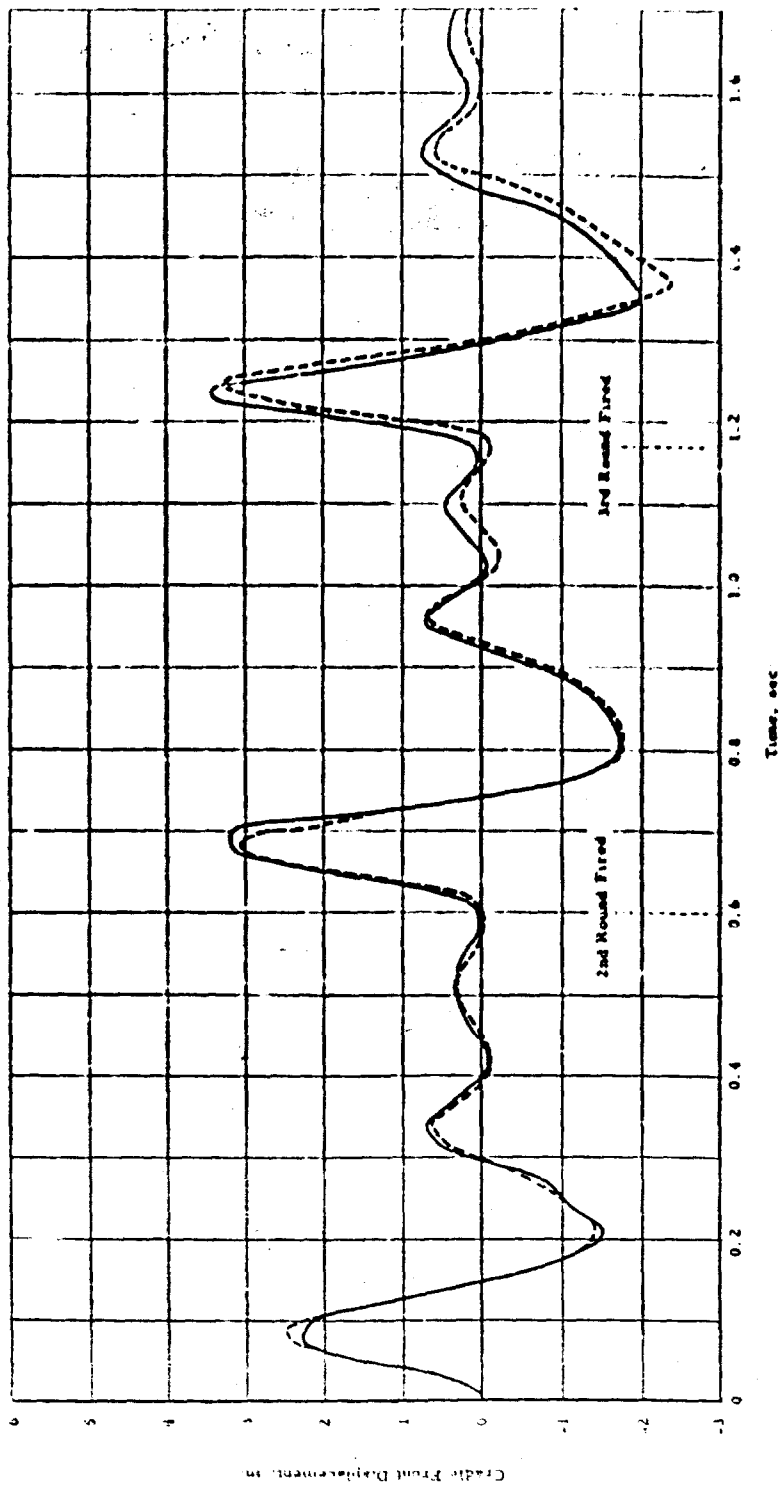


Fig. J-13 CRADLE ROTATION, BURST B-9

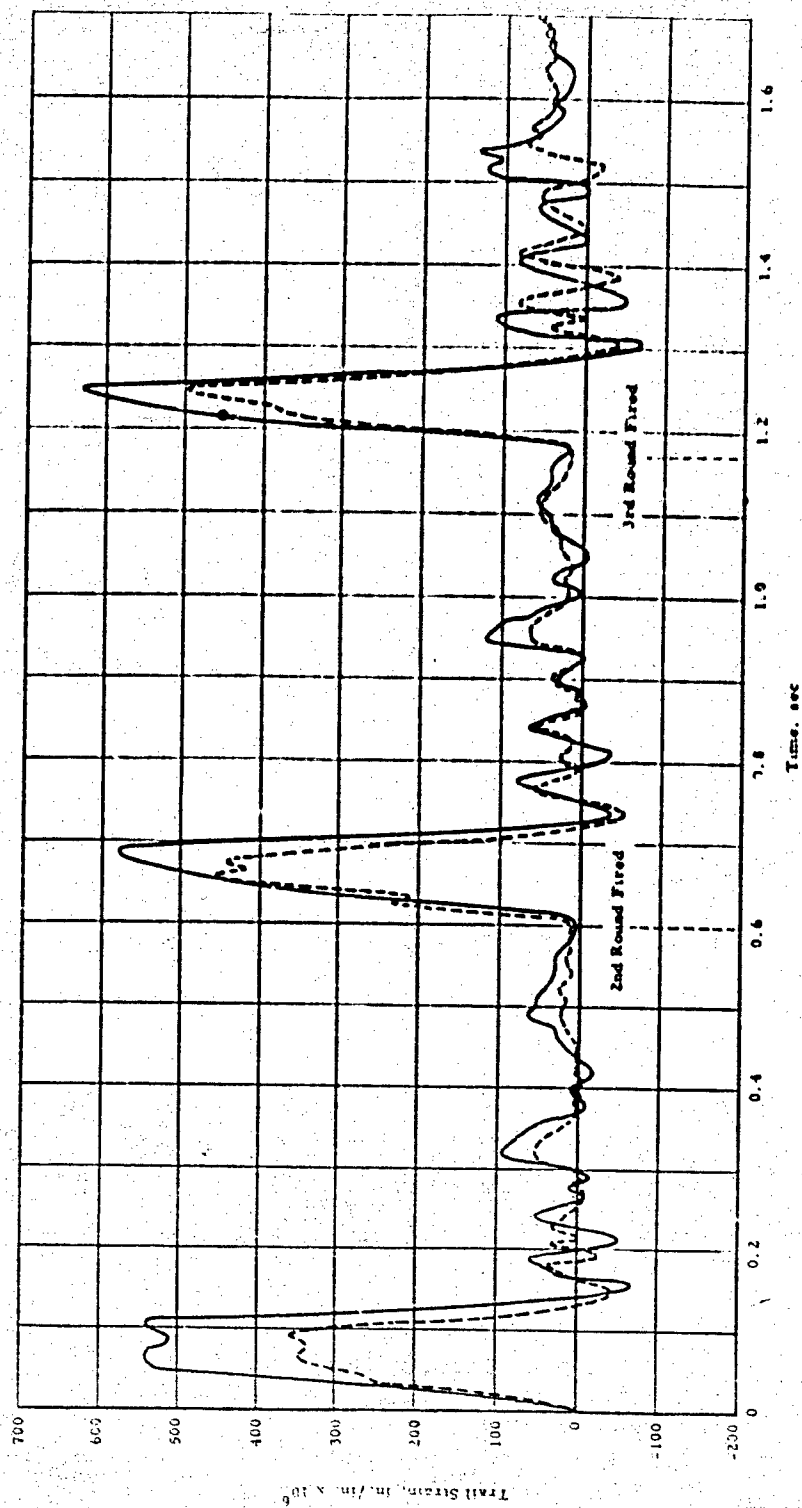


Fig. J-14 TRAIL STRAIN, BURST B-9

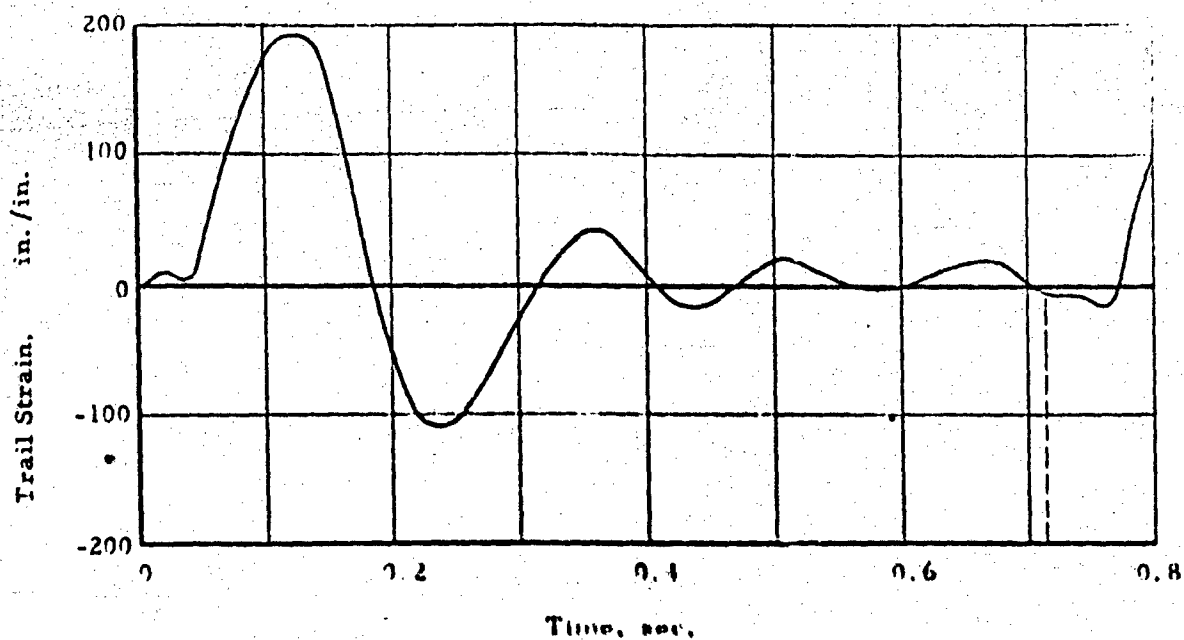


Fig. J-15 TRAIL STRAIN, BURST B-10

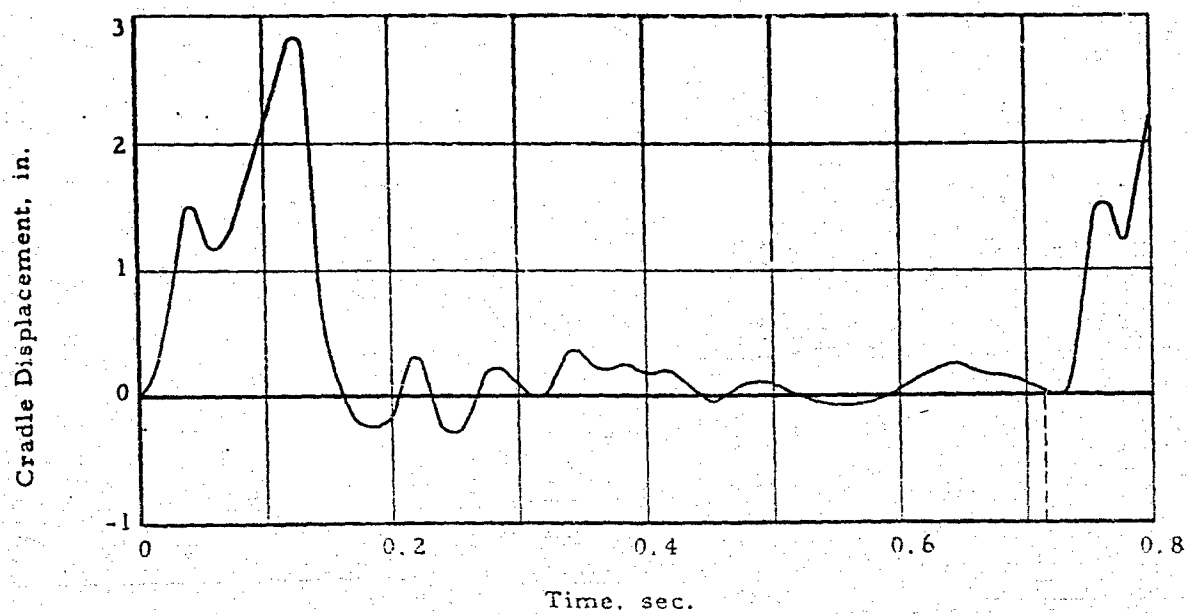


Fig. J-16 CRADLE MOTION, BURST B-10

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