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THE GLOW DISCHARGE AS AN ADVANCED PROPULSION DEVICE

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SUMMARY

The generation of thrust directly from electric power through a gaseous discharge is investigated with a one dimensional model. The thrust consists of two parts, the electric pressure and the electric wind. In terms of the ionization constants of gases and of the mobility constants of ions and electrons, both the electric pressure and the electric wind are obtained in simple dimensionless form. At low gas pressures, the thrust contribution from electric pressure is usually much larger than that from electric wind. The latter may be of comparable magnitudes or even much larger at higher (atmospheric) discharge pressures and larger inter-electrode distances. The overall thrust level is proportional to the gas pressure in the discharge squared, p^2. For $p = 1$ mm Hg, the thrust is $\lesssim 1$ dyne/cm^2. The specific thrust per unit power is no greater than 10 gm wt/kw. Thrust is derived from electric wind at a much larger specific power consumption. The low specific thrust alone rules out the possibility of using such a propulsion system as a booster. For application in outer space, it suffers from the limited specific impulse because propellant gas must be carried aboard.

There may, however, be other interesting applications, for example, as a positive control device for hypersonic vehicles at an altitude of 100-200 x 10^3 ft. A detailed feasibility study of this possibility has not yet been made.

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NOMENCLATURE

$a_{\pm}$	coefficient in the experimental drift velocity expresion; see Eqs.(18)(19), p.10
A	saturation ionization constant, cm^{-1} mm Hg^{-1}; see Eq.(3), p.4
B	ionizing potential esu field/(mm Hg); see Eq.(3), p.4
d	distance between electrodes
E	electric field strength in esu, positive in the direction of anode to cathode, <u>i.e.</u>, the positive x-direction
j	current density of each species
J_o	discharge current density esu current/cm^2
J^*	a dimensionless discharge current density defined by Eq.(20), p.11
p	pressure of gas (plasma) in mm Hg
r	charge density esu/cm^3
Th	reactive thrust acting on the system positive in the negative x-direction
T_1	contribution of electric pressure effect to the reactive thrust Th; see Eq.(21), p.11
T_2	contribution of electric wind effect to the reactive thrust Th; see Eq.(22), p.12
u	mean velocity of the neutral plasma cm/sec positive in the positive x-direction
V_d	magnitude of drift velocity of particles relative to the mean motion of plasma
x	axis normal to the discharge electrodes, anode at $x = 0$ and cathode at $x = d > 0$
$\alpha,\beta,\gamma,\delta$	ionization coefficients defined in Eqs.(1)(2)(3)(4)
κ	efficiency factor of momentum transfer from electron to neutral gas; see Eq.(6), p.5 and the following explanation
$\bar{\kappa}$	same mean value of κ over the entire discharge region defined by Eq.(24), p.12
λ	external source current density
ω	coefficient of cathode emission defined in Eq.(4), p.4
ρ	density of gas (in appropriate unit)
$\zeta(z)$	a universal function defined by Eq.(20a), p.11

<u>Subscripts</u>

-	indicates quantities pertaining to electrons
+	indicates quantities pertaining to ions
m	indicates quantities pertaining to metastables
p	indicates quantities pertaining to photo-effects
o	indicates quantities evaluated at anode $x = 0$
d	indicates quantities evaluated at cathode $x = d$

INTRODUCTION

When a dc voltage of the order of tens of kilovolts is applied across a pair of parallel planar electrodes constructed of light wire meshes, the electrodes lift into the air.[1] There is little evidence of thermal expansion of the air between the electrodes. An "electro-kinetic" machine of similar construction[2] operating at >100 kv is reported to turn a wheel with a sustained peripheral speed of the order of 10 ft/sec. The novel feature of these devices is that thrust is derived directly from electric power at room temperature without any moving parts and that the ambient atmosphere is employed as the propellant. Might this serve as an advanced propulsion scheme?

In an electric field a neutral gas with a slight degree of ionization (i.e., a neutral plasma) is known to move in the direction of the field. This is the "electric wind".[3] The structure carrying the electrodes will experience the reaction of the gas motion, that is, the thrust. The electric wind has been qualitatively explained as follows: A massive ion is much more efficient than an electron in transferring to the colliding neutral molecule the momentum and the energy which the ion (or electron) derived from the electric field during the free flight prior to collision. Accordingly, the neutral molecules will gain a net momentum in the direction of motion of the ions although the plasma may be locally neutral. The maximum effect of the electric wind can be estimated by ignoring the presence of electrons.

However, if an electron collides with a great many neutral molecules while drifting back and forth through a distance of the order of its free path, practically all of its momentum derived from the electric field during the drifting motion may be transferred to the neutral molecules. Thus, when the particle density is sufficiently large, depending upon the electric field strength, the rate of momentum transfer to the neutral molecules per unit volume will be $E(\sigma_+ - \sigma_-)$, where E is the local electric field strength and σ_+, σ_- are the magnitudes of the charge densities of ions and electrons, respectively. Thus, any net force experienced by the neutral gas must be due to the local excess of positive or negative electric charges. This is the point of view of the conventional continuum analysis of plasma motion.

Since the ionizing collisions and the drifting motion of the charged particles are the fundamental elements of a gaseous discharge, both mechanisms of thrust generation must be present. Before analyzing the dynamics of a slightly ionized gas under electric discharge conditions, a brief outline of the physical processes and their phenomenological representations appears necessary. For a comprehensive discussion, the reader is referred to standard textbooks, in particular the two volumes of the Handbuch der physik.[4]

PHYSICAL PROCESSES IN GAS DISCHARGE

Gas discharge refers to the passage of electric current through a gaseous conductor by means of mass motion of electrons and ions (supposed to be singly charged positively for the sake of simplicity) toward the anode and the cathode, respectively. Prior to the breakdown of the gas, a steady discharge will have to be maintained by external means, for example, by shining ultraviolet light onto the cathode to cause electron emission. These electrons will be accelerated by the electric field and multiply through ionizing collisions with neutral gas molecules. The current strength will depend upon the intensity of illumination and is usually very small, of the order of $\lesssim 10^{-12}$ amps/cm^2.

When the voltage across the electrodes is sufficiently large, the discharge becomes self-sustaining; i.e., a steady discharge current will be maintained even if the external source of ionization is removed. From here on, a slight increase in voltage will result in a significant increase in current density. This is the Townsend dark discharge , where a current density of $\sim 10^{-6}$ amp/cm^2 may be reached. Further increase of the discharge current density brings about the glow discharge, characterized by the emission of visible light of different patterns. Attempts to increase the discharge current further will bring about an arc discharge at relatively low voltages and high currents with hot cathodes.

When the electric field near the cathode is small, the electrons emitted from the cathode collide with neutral molecules elastically. When the cathode field is large, the electrons may gain a sufficient amount of energy during their free flight. Their collision with neutral molecules may then be inelastic, resulting in many different products. The colliding molecule may be ionized, producing an additional electron and a positive ion. In this manner, the number of electrons in the gas increases with distance from the cathode. The logarithmic increment of the electron concentration along the distance from the cathode due to the direct ionization by electrons is known as the first Townsend coefficient α_-.

The inelastic collision of an electron and a neutral molecule may lead to results different from ionization. The neutral molecule (or the atom if it is a noble gas) may be left electronically excited and may later emit light by going to a lower state or to the ground state. This is the origin of the visible glow. Some molecules (or atoms) in excited states may possess a relatively long life, i.e., metastable states, which later may collide with another neutral molecule resulting in ionization. The light emitted from the excited atoms may also cause ionization of neutral molecules through the process of photo-ionization. The rates of these processes depend approximately linearly on the electron concentration. Thus, the total logarithmic increment of the electron concentration α may be taken as $\alpha = \alpha_- + \alpha_m + \alpha_p$ where α_m and α_p are the contributions from metastable atoms and photo-ionization, respectively.

The positive ions, produced by the previous ionization processes, may gain a sufficient amount of energy from the electric field to ionize neutral particles upon collisions. The rate of this ionization process will depend linearly on the concentration of the positive ions, or the ion current with logarithmic increment β. This process is, however, very inefficient when compared to ionization by electrons. There are many other possible processes of ionization, including stepwise processes, that depend on the degree of ionization to a higher power. These higher order processes will be neglected for treating discharge problems with a small degree of ionization. Thermal ionization, i.e., ionization due to collision between neutral particles, and photoionization by externally maintained illumination are essentially independent of the local ion density. These may be included in the ion (or electron) material balance as an additional source term. Hence, in terms of the convective currents, j_+ and j_-

$$- \frac{dj_-}{dx} = \alpha j_- + \beta j_+ + \lambda = + \frac{dj_+}{dx} \tag{1}$$

Ions and electrons drift toward the proper electrode where they are neutralized. In order that the discharge may maintain itself, the cathode must emit electrons to maintain the subsequent processes. The electron emission from the cathode may be produced by the incidence of the positive ions or the excited molecules or atoms (metastable states) or of photons from within and/or without the discharge. The electron current density at the cathode will hence depend linearly on the positive ion current density at the cathode with a proportionality constant γ. The effects of metastable atoms and photons are often combined in the steady state into one term proportional to the integral of the electron current density over the discharge, (i.e., proportional to the rate of production of photons and metastables) with proportionality constant δ. The effect of photoemission from the cathode due to external illumination will be neglected. The thermionic emission from the cathode depends on the cathode temperature, and, therefore, the current intensity. We may visualize this thermionic emission as contributing to γ for qualitative purposes. Hence, on the cathode surface ($x = d$)

$$j_-(d) = \gamma j_+(d) + \int_0^d \delta j_-(x)dx \tag{2}$$

All these processes are probably present in any gas discharge to some extent. Whether a particular process may be neglected or not depends on the gaseous pressure and temperature and on the nature of the electrodes. For the Townsend discharge, the gaseous ionization is due primarily to electron avalanche (α_-) and the cathode emission to positive ion incidence (γ). For the glow discharge, the photo-process becomes important, and for arc discharge, the thermionic emission from the hot cathode predominates. Each kind of discharge will be characterized by the predominance of some particular process. The transition from one type of discharge to another is not abrupt, although the transitional states may be unstable. The use of the phenomenological constants is convenient, since the results obtained with these constants are applicable to a wide range of situations so long as the appropriate basic processes are identified with these phenomenological constants.

The ionization constant α_- for electrons are known for many gases. α_- must be proportional to the frequency of collision ($\sim \rho$ or p) and must depend on the energy that the electron acquires during its free flight, i.e., $\sim$ the electric field times the free path, $\sim E/p$. Accordingly, a semi-empirical relation is often adopted.

$$\alpha_- = Ap \cdot \exp(-\frac{Bp}{E}) \simeq \alpha \tag{3}$$

where A and B are characteristic constants of a given gas. Such an empirical correlation gives a good representation of the experimental data over a fairly wide range of p and E/p. The other constants α_m and α_p are less well known. Hence Eq. (3) will be adopted for α instead of α_- with constants A and B slightly adjusted to take α_m and α_p into account.

For the processes on the electrodes, it is convenient to introduce a secondary coefficient ω for the cathode emission with

$$\frac{\omega}{\alpha} \simeq \frac{\gamma + \beta/\alpha + \delta/\alpha}{1 + \gamma + \dfrac{\delta/\alpha}{1 - \beta/\alpha}} \tag{4}$$

a quantity that can be determined experimentally. The magnitude of ω/α does not vary so critically with E/p as α does. The major constituents of ω/α for even the simplest discharge processes are not well understood. A, B and α/ω are the key parameters characterizing the discharge process.

GASDYNAMICS OF THE DISCHARGE

The gas plasma will be assumed slightly ionized and not too much rarified, so that the continuum equation of motion will apply. To illustrate the operating mechanism, we shall consider only the steady state discharge in one spatial dimension x taken to be normal to a pair of parallel planar electrodes with a surface area large compared with the inter-electrode distance d. $x = 0$ is taken as the anode surface and $x = d > 0$ the cathode surface. The negatively charged particles will be taken to be entirely electrons.

Under the assumption of slight ionization, the rate of change of the number of neutral molecules due to ionization, recombination, and diffusion under cataphoresis will be neglected in the mass balance of the neutral gas. Hence

$$\frac{d(\rho u)}{dx} = 0 \quad \text{or} \quad \rho u = \text{const} \tag{5}$$

where ρ is the density of the neutral gas (or of the plasma as a whole) and u is the drift velocity of the neutral gas with respect to the electrodes, positive in the positive x direction (i.e., from anode to cathode).

For the momentum balance of the neutral gas, we write

$$\begin{aligned} \rho u \frac{du}{dx} + \frac{dp}{dx} &= E\sigma_+ - \kappa E\sigma_- \\ &= + E(\sigma_+ - \sigma_-) + (1 - \kappa)E\sigma_- \end{aligned} \tag{6}$$

On the left hand side, the term $\rho u \frac{du}{dx}$ stands for the net out flux of momentum and the term $\frac{dp}{dx}$ the net pressure force. The terms on the right hand side stand for the net rate of momentum transfer from the charged particles to the neutral molecules. Here E is the electric field intensity, positive throughout the region of interest (possible exception in the region of negative glow is ignored). σ is the magnitude of the charge density with subscript + and - designating quantities pertaining to ions and electrons, respectively. The momentum derived from the electric field by the ions is assumed to be transferred in total to the neutral gas molecules. The momentum derived by the electrons is transferred to the neutral gas molecules with an efficiency κ. This efficiency κ will depend very much on the collision processes, elastic or inelastic, on the particle density of the plasma (pressure), and on the electron temperature which may vary with the distance from electrodes. If $\kappa = 1$, Eq.(6) reduces to the conventional continuum momentum equation in which the electric field exerts a force on the neutral gas only through the local net charge density in the plasma. If $\kappa = 0$, i.e., the electrons are considered as incapable of transferring momentum to the neutral gas molecules, the electric field will exert a force on the neutral gas through all the positive ions, but not through electrons, as was conceived in the mechanism of "electric wind".

The convective currents j_+ and j_- for ions and electrons in the positive and the negative x directions, respectively, are

$$\begin{aligned} j_+ &= \sigma_+(V_{d+} + u) > 0 \\ j_- &= \sigma_-(V_{d-} - u) > 0 \end{aligned} \tag{7}$$

where V_{d+} and V_{d-} are the magnitudes of the drift velocities of ions and electrons, respectively. V_{d+} is the mean drift velocity of ions toward the cathode relative to the mean gas motion. V_{d-} is that of electrons toward the anode. Both V_{d+} and V_{d-} will be taken as known functions of E/p. The mean gas velocity u is taken to be positive in the positive x direction, i.e., toward the cathode. The field strength E is always positive in the present sign convention. Thus, the space charge condition gives

$$\frac{1}{4\pi} \cdot \frac{dE}{dx} = \sigma_+ - \sigma_- = \frac{j_+}{V_{d+} - u} - \frac{j_-}{V_{d-} + u} \tag{8}$$

All qualities are in electrostatic units. The field strength, $E > 0$, decreases as x increases from zero (proceeding from anode toward cathode) because of the abundance of electrons near the anode. The cathode region is abundant in positive ions, where the field strength increases very rapidly with increasing x when the cathode is approached.

The conservation of ion and electron densities will be written with diffusion neglected. It will be clear later that the neglect of diffusion is inconsequential for the present purpose.

$$\frac{d}{dx}(+j_+) = \alpha j_- + \beta j_+ + \lambda$$
$$\frac{d}{dx}(-j_-) = \alpha j_- + \beta j_+ + \lambda \qquad (9)$$

where ions and electrons are assumed to be produced in pairs with ions all singly charged. The meaning of α, β and λ are explained in the previous sections.

It follows from (9) that

$$\frac{d}{dx}(j_+ + j_-) = 0 \quad \text{or} \quad j_+ + j_- = J_o \qquad (10)$$

where J_o is the total current density, independent of x. The boundary conditions at the anode (x = 0) and cathode (x = d) are

$$\sigma_+(0) = 0 \quad \text{or} \quad j_+(0) = 0$$

and

$$j_-(d) = \gamma j_+(d) + \int_o^d \delta j_-(x)dx \qquad (2)$$

The total voltage drop from the anode to the cathode, ΔV, will be

$$\Delta V = \int_o^d - E\, dx < 0 \qquad (11)$$

The electric quantities can be recognized as governed by the three first order differential equations (8) and (9) for $j_+(x)$, $j_-(x)$ and $E(x)$ subject to the three boundary conditions (2) and (11). If we assume further

$$|u/V_{d+}|, \ |u/V_{d-}| << 1$$

and (12)

$$\Delta p/p << 1$$

to be verified a posteriori, we note that Eqs.(8) and (9) and boundary conditions (2) and (11) are completely independent of the flow variables u and p. Thus, σ_+,σ_- and E may be solved, as a first approximation under conditions (2) and (11), regardless of the flow conditions. The momentum balance of the neutral gas, Eq.(6), can then be solved when the boundary conditions of the flow are specified. Higher order corrections to j_+, j_- and E can be evaluated successively by solving again (8),(9) and so on. These are not necessary for our purposes.

Combining (5),(6) and (8), we can write

$$\begin{aligned} \frac{d}{dx}(p + \rho u^2) &= + \frac{d}{dx}\left(\frac{E^2}{8\pi}\right) + (1 - \kappa)\, E\sigma_- \\ &= + \frac{d}{dx}\left(\frac{E^2}{8\pi}\right) + (1 - \kappa)\, Ej_-/(V_{d-} - u) \end{aligned} \tag{13}$$

The $E^2/8\pi$ term is the energy density of the electric field per unit volume which may be interpreted as the mechanical force per unit area, i.e., the electric pressure. The next term on the right hand side of (13) represents the deficiency of momentum transfer by electrons, i.e., the electric wind effect.

The electric pressure term arises from the presence of net electric charge between the two stations. In the vicinity of the anode, the electric field decreases in the positive x-direction due to the presence of excess electrons. These excess electrons, moving toward the anode (the negative x-direction), transfer their momentum to the neutral gas in the direction against the flow of the neutral gas. The decrease of electric field strength will, therefore, be accompanied by a decrease of the stream thrust of the neutral gas, as indicated in Eq.(13). Across the entire discharge region, there will be a net excess of positive ions, moving in the same direction as the flow of the neutral gas. A net increase of the electric field, accompanied by an increase of the stream thrust of the neutral gas, will, hence, result. The electric wind term, being the defect of the negative momentum transfer by electrons,also acts in the downstream direction to increase the stream thrust of the neutral gas.

Our object is to evaluate the relative importance of the two contributions to the stream thrust of the neutral gas. An increase of the stream thrust of the neutral gas means the presence of a reactive thrust on the system carrying the electrodes per unit sectional area of the discharge. This reactive thrust acts in the negative x-direction, i.e., in the direction opposite to the flow of the gas. By integrating Eq.(13) over $x = 0$ to $x = d$, we obtain the following expression for the magnitude of the reactive thrust

$$Th = \frac{E_d^2 - E_c^2}{8\pi} + \int_0^d [(1 - \kappa) \cdot Ej_-/(V_{d-} - u)]\, dx \tag{14}$$

The field strength E_d at the cathode is always higher than the field E_o at the anode. The efficiency κ is never greater than unity. Hence, both the electric pressure and the electric wind terms are positive, contributing to the reactive thrust on the system of electrodes in the direction from the cathode to the anode. The net force on the plasma exerted by the system is from the anode toward the cathode. In Fig. 1, an approximate sketch of different quantities are given without any attempt of displaying the complicated fine details of the discharge properties.

ELECTRIC PRESSURE VERSUS ELECTRIC WIND

The primary ionization constant α will be taken as given in Eq.(3) with A and B, given by the experimental values in uniform fields, and $E(x)$ by the local field strength. β/α, δ/α, etc., will be taken as some mean values which are usually small. Eqs.(9) can then be readily integrated

$$\begin{aligned} j_+(x) &= (J_o + \frac{\lambda}{\alpha})[1 - e^{f(x)}]/(1 - \beta/\alpha) \\ j_-(x) &= J_o e^{f(x)} - \frac{J_o\beta/\alpha + \lambda/\alpha}{1 - \beta/\alpha}[1 - e^{f(x)}] \end{aligned} \tag{15}$$

where

$$f(x) = -(1 - \beta/\alpha)\int_0^x \alpha(\xi)d\xi \;;\; \alpha = Ap\exp(-\frac{Bp}{E}) \tag{16}$$

The condition at the cathode, $x = d$, given as Eq.(2), requires that

$$J_o[(1 + \gamma + \frac{\delta/\alpha}{1 - \beta/\alpha}) - e^{-f(d)}(\gamma + \frac{\beta}{\alpha} + \frac{\delta}{\alpha})]$$

$$= \frac{\lambda}{\alpha}[-(1 + \gamma + \frac{\delta/\alpha}{1 - \beta/\alpha}) + e^{-f(d)}(1 + \gamma + \delta/\alpha)]$$

Either when thermal ionization or photo-ionization by external sources are neglected ($\lambda/\alpha \cong 0$) or when $\lambda/(\alpha\gamma + \beta + \delta)$ is not >> 1, the coefficient of J_o must be approximately zero since J_o in esu is very large. Hence

$$e^{-f(d)} = \exp[(1 - \frac{\beta}{\alpha}) \int_o^d \alpha(x)dx] = \frac{1 + \gamma + \frac{\delta}{\alpha}/(1 - \beta/\alpha)}{\gamma + \frac{\beta}{\alpha} + \frac{\delta}{\alpha}} = \frac{\alpha}{\omega}$$

i.e., (17)

$$-f(d) = (1 - \frac{\beta}{\alpha}) \int_o^d \alpha(x)dx = \ln(\frac{\alpha}{\omega})$$

When thermionic emission is absent, the experimental values of ω/α are $O(10^{-2} - 10^{-3})$ varying slightly with E/p. For highly abnormal glow discharges with large γ, ω/α may be of the order of 10^{-1}.

When α is taken from Eq.(3) as a known function of E, Eq.(15) and (16) express j_+ and j_- in terms of E. When $|u/V_{d+}|$, $|u/V_{d-}|$ << 1 (or if u is known from previous iteration) and when V_{d+} and V_{d-} are taken from experimentally known functions of E/p, the space charge equation (8) stands as a first order differential equation for E(x). Thus E(x) can be solved in principle. The condition (11) determines E(x) completely when the total voltage drop across the electrodes is specified. Solution of equations in this manner, although straightforward, is tedious and does not appear necessary for our purpose.

The drift velocity for ions may be written according to experimental data[5] (Figure 59-61) under ordinary vacuum conditions

$$\begin{aligned} V_{d+} &= b_+ \cdot 10^5(E/p) \quad \text{for} \quad (E/p) \ll 1 \\ &= a_+\ 10^5(E/p)^{\frac{1}{2}} \quad \text{for} \quad (E/p) \gtrsim O(1) \end{aligned} \tag{18}$$

where V_d is expressed in cm/sec and E/p in esu field/mm Hg pressure, the coefficients b_+, a_+ are of order of unity for most gases. Similarly, the drift velocity for electrons in gases can be expressed as

$$\begin{aligned} V_{d-} &= b_-\ 10^9(E/p)^{\frac{1}{2}} \quad \text{for} \quad (E/p) \ll 10^{-1} \\ &= a_- \cdot 10^8(E/p) \quad \text{for} \quad (E/p) \gtrsim O(10)^{-1} \end{aligned} \tag{19}$$

Note the difference in the qualitative dependence of V_{d+} and V_{d-} on E/p. While the a's and b's are of the same order of magnitude, the ratio V_{d+}/V_{d-} becomes increasingly smaller than 10^{-3} with larger E/p (in esu/mm Hg). Thus, multiply the space charge equation (9) by $(V_{d+} + u) \cdot \alpha = Ap \exp[-Bp/E] \cdot a_+ \cdot 10^5 (E/p)^{\frac{1}{2}}$ and integrate from x = 0 to x = d. With $V_{d+}/V_{d-} \ll 1$ and $\lambda/\alpha \ll 1 \ll J_o$ (in esu current), we obtain

$$\zeta(\frac{E_d}{Bp}) - \zeta(\frac{E_o}{Bp}) = J^* = \frac{4\pi}{a_+ \; AB^{3/2}} [\ln(\frac{\alpha}{\omega}) - 1] \frac{J_o}{p^2} \cdot 10^{-5} \qquad (20)^\dagger$$

where

$$\zeta(z) = - \int_{\infty}^{\tau = \frac{1}{z}} \tau^{-5/2} e^{-\tau} d\tau \; ; \; z = \frac{E}{Bp} \qquad (20a)$$

$\zeta(z)$ is a universal function which may be expressed in terms of error function. If $V_{d+} \sim (E/p)^s$ should be adopted with $s \neq 1/2$, Eq.(20) would be modified as follows: $B^{3/2} \rightarrow B^{1+s}$ and $\zeta(z)$ would be the integral of $\tau^{-(2+s)} e^{-\tau}$ which can be expressed in terms of an incomplete Γ function of order $-(2+s)$. Eq.(20a) therefore defines a universal function $\zeta(z)$ once $\alpha(E/p)$ and $V_{d+}(E/p)$ of a discharge system are selected. The function $\zeta(z)$ is illustrated in Fig. 2 for s = 1/2. The right hand side of Eq.(20) is a dimensionless parameter J^* for the reduced current density, J_o/p^2, involving the characteristic constants, a_+, A, B and α/ω, of the gas in the discharge system.

When the dimensionless field at the anode $\frac{E_o}{pB}$ and the dimensionless current density J^* are fixed, Eq.(20) will give the dimensionless field at the cathode, E_d/pB. Hence, the dimensionless electric pressure term in (14)

$$\frac{T_1}{B^2 p^2} = \frac{1}{8\pi} \left[(\frac{E_d}{pB})^2 - (\frac{E_o}{pB})^2 \right] \qquad (21)$$

can be calculated.

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†This relation is, in many respects, similar to, but different from, the universal relation of Von Engle and Steenbeck[6] for the cathode region; i.e., V_e/p versus J_o/pL.

The electric wind term in (14) may be written as

$$T_2 = \int_0^d (1 - \kappa) \cdot \frac{Ej_-}{V_{d-} - u} \, dx = \frac{p}{a_- 10^8} \int_0^d (1 - \kappa) j_-(x) dx \tag{22}$$

Let us write $j_+(x) = J_o - j_-(x)$ in the space charge equation (8), multiply through by $V_{d+} + u = a_+ 10^5 (E/p)^{\frac{1}{2}}$ and integrate from $x = 0$ to $x = d$. We obtain $\int_0^d (1 - \kappa) j_-(x) dx$ in terms of $J_o/p^2, (E_d/p)^{3/2}$ and $(E_o/p)^{3/2}$. Eq.(20) may be used to put J_o/p^2 in terms of $\zeta(\frac{E_d}{Bp}) - \zeta(\frac{E_o}{Bp})$. Thus, the dimensionless electric wind term in Eq.(14) becomes

$$\frac{T_2}{B^2 p^2} = \frac{J_o}{p^2} \frac{p \int_0^d (1 - \kappa) dx}{a_- \cdot 10^8} - \frac{a_+}{a_-} \frac{10^{-3}}{B^{1/2}} \int_0^d \frac{(1 - \kappa)}{6\pi} \frac{d}{dx} \left(\frac{E}{Bp}\right)^{3/2} \cdot dx$$

with (23)

$$\frac{J_o}{p^2} \frac{1}{a_-} \cdot 10^{-8} = \frac{a_+}{a_-} \frac{A \cdot 10^{-3}}{4\pi \, B^{1/2}} \frac{1}{\ln \frac{\alpha}{\omega} - 1} \left[\zeta\left(\frac{E_d}{Bp}\right) - \zeta\left(\frac{E_o}{Bp}\right)\right]$$

The second part of T_2/B^2p^2 is negligible compared with T_1/B^2p^2. This is because the integral in the last term is at most of the same order as T_1/B^2p^2 while the coefficient is $O(10^{-3}) \ll 1$. The electric wind term T_2/B^2p^2, that may be of any significance in producing thrust, is the first term in (23), which is linearly proportional to the reduced discharge current density, J_o/p^2.

Fig. 2 shows that $\zeta/\frac{E}{Bp}$ is almost always less than unity. The values of $A/B^{\frac{1}{2}}$ for molecular and rare gases are around 10 with A in $(\text{cm-mm-Hg})^{-1}$ and B in esu field/mm Hg. Except for arc discharges, α/ω is substantially larger than 1 and $\ln \frac{\alpha}{\omega} - 1$ is not small. Consequently, the electric wind effect T_2 will contribute to the stream thrust comparable to the electric pressure effect T_1 only when

$$p \cdot d \cdot (1 - \bar{\kappa}) = p \int_0^d (1 - \kappa) dx > O(10^2) \text{ cm - mm Hg} \tag{24}$$

When condition (24) is fulfilled, we have

$$\frac{Th}{B^2p^2} = \frac{T_1}{B^2p^2} + \frac{T_2}{B^2p^2} = \frac{1}{8\pi}\left[\left(\frac{E_d}{Bp}\right)^2 - \left(\frac{E_o}{Bp}\right)^2\right] + \frac{J_o}{p^2} \cdot \frac{p \cdot d(1 - \bar{\kappa})}{a^- \cdot 10^8} \tag{25}$$

The electric wind effect becomes more significant when the discharge pressure p is higher and when the inter-electrode distance d is larger with the reduced current density J_o/p^2 kept constant. If $\bar{\kappa} \ll 1$, the electric wind effect becomes appreciable in discharges at 1 mm Hg when the electrodes are several meters apart. At higher pressures and larger inter-electrode distances, the electric wind effect may be orders of magnitude larger than electric pressure when $pd \cdot (1 - \bar{\kappa}) \cdot 10^{-2} \gg O(1)$.

The magnitude of κ, i.e., the efficiency of momentum transfer by electrons upon colliding with neutral molecules, varies considerably, especially when inelastic collisions are of importance. It is considerably larger than what is indicated by the ratio of electron mass to molecular mass. In a reduced electric field, as small as 5 - 10 V/cm mm Hg, the value of the coefficient of energy transfer is $O(10^{-1})$ for electrons in molecular gases like N_2 because of the tightly (or closely) packed vibrational levels.[7] The corresponding value of the coefficient of momentum transfer is expected to be of the same order of magnitude. Hence, κ cannot be taken as zero throughout the region of discharge for the quantitative evaluation of the electric wind effect if the electric wind effect should contribute appreciably to the total thrust, as indicated by Eq.(25), either for discharges at low pressures between widely separated electrodes or for discharges at higher pressures (even sub-atmospheric) for moderate inter-electrode distances of several centimeters apart.

Eq.(20) is a universal relation between the cathode field $\frac{E_d}{pB}$ and the dimensionless current density J^*. The anode field E_o/pB may be recognized as a parameter characterizing the discharge in place of ΔV, the overall voltage drop across the electrodes. Thus, the thrust due to electric pressure (T_1/B^2p^2) can be given as a set of universal curves for different values of J^* and E_o/Bp, (Fig. 3). Note that the magnitude of E_o/Bp is of secondary importance and that when $J^* \gtrsim 1$

$$\frac{T_1}{B^2p^2} \cong 0.2\ J^* \tag{26}$$

The cathode field (E_d/Bp) is also given as a set of universal curves (Fig. 4). For estimating the orders of magnitude, $(1 - \bar{\kappa})$ may be taken as of the order of unity so that

$$\frac{T_2}{B^2p^2} \cong \left(\frac{J_o}{p^2}\right) \frac{(p \cdot d)}{a^- \cdot 10^8} \tag{27}$$

All quantities are expressed in esu and mm Hg pressure. These rough estimates will guide the following discussion.

The characteristic constants of the gas, like A, B, etc., appear only in the dimensionless parameters. The results of the analysis are applicable in general for any gas. We shall, however, take air as a typical example to facilitate further discussion. For discharges in air:

$$\begin{aligned} a_- &= 1.4 \text{ cm sec}^{-1}(\text{esu field/mm Hg}) \\ a_+ &= 1.7 \text{ cm sec}^{-1}(\text{esu field/mm Hg})^{1/2} \\ A &= 15 \text{ cm}^{-1} \text{ mm Hg}^{-1} \\ B &= 1.22 \text{ esu field (mm Hg)}^{-1} \end{aligned}$$

α/ω may vary considerably even for a glow discharge, ranging from $O(10^{-1})$ to $O(10^{-3})$, depending on the cathode material and cathode surface conditions. The abnormal glow discharge, in which thermionic emission from the cathode becomes appreciable, may be considered as one with γ large, i.e., $1 > \frac{\alpha}{\omega} \simeq O(10^{-1})$. If the thermionic emission is predominant at a hot cathode, as in arc discharge, there will be severe variations of gas temperature and pressure which are not accounted for in the previous analysis. (p should be replaced by the gas density ρ in the dimensionless variables.) We shall not, however, seriously consider the arc discharge in the following because its application to propulsive devices is more likely associated with its thermal effects.

The following values, obtained from Fig. 3, illustrate the orders of magnitudes of the quantities at 1 mm Hg pressure when the electric wind effect is neglected, i.e., Th $\cong T_1$.

Thrust dynes/cm²	J^*	T_1/B^2p^2	J_o µA/cm²	E_d V/cm	$-\Delta V$ volts	Th/power dynes/watt or gm wt/kw
0.38	1.25	0.25	90 ~ 20	1000	O(kv)	1-10
1.0	4.0	0.8	300 ~ 60	1650	O(kv)	1-10

For a discharge between electrodes of a given distance d, the total voltage drop is essentially the sum of the cathode fall (~300 v for normal glow discharge in air) and the voltage drop in the positive column. The anode fall is small. The reduced cathode dark space pd_n for air is ~0.2 - 0.5 cm-mm-Hg for most electrode materials. Hence, the length of the positive column is essentially the same as the inter-electrode distance d, i.e., $\frac{d_n}{d} \ll 1$. The field in the positive column is smaller than the anode field, but not much different. A reasonable estimate of the voltage drop in the positive column may be $E_o \cdot d$. Thus, we obtain a rough estimate of the overall voltage drop and hence the power consumption. The calculated results agree with the experimental data of Ref. 8, both in the orders of magnitudes of numerical quantities and in the qualitative trends.

DISCUSSION

Let us first consider the situation when the electric wind effect, T_2, is negligibly small. The proportionality of the thrust T_1 to the dimensionless parameter J^* in Eq.(26), and the definition of J^* in Eq.(20), imply that:

1. A smaller discharge current, J_o, will be required to produce a given thrust, T_1, if the ratio ω/α is smaller.
2. A smaller ion mobility constant a_+ (~larger ion mass, etc.) and a smaller saturation ionization constant A tend to reduce the discharge current for a given thrust T_1.
3. Both T_1 and J_o increase with the square of the pressure of the gas in a given discharge system, i.e., ~p^2.

The first inference shows that abnormal glow discharge with larger ω/α, due to thermionic emission at the cathode, tends to require more discharge current to produce unit thrust. The abnormal cathode fall also increases with the discharge current. Hence, an abnormal glow discharge will imply higher specific power consumption, although a larger thrust may be achieved per unit discharge area. The optimum operating condition will be the normal glow discharge covering the entire electrode. Larger thrust can be achieved more economically by increasing the electrode area than by operating in the abnormal glow region.

The cathode conditions (the field E_d and the current density J_o) of the glow discharge are not affected by shortening the inter-electrode distance within certain limits. The entire column of positive glow may be eliminated with the overall voltage across the electrodes reduced accordingly. The power consumption in the positive column does not produce useful thrust when the electric wind effect is negligibly small.

Essentially the same thrust can be derived from much smaller electric power consumption by shortening the inter-electrode distance to the nearly constricted condition. The optimum specific thrust, dynes/watt, is then achieved when the overall voltage drop is taken as the normal cathode fall, ~300 v for air. Thus, without the electric wind effect, the thrust to power ratio cannot be expected to improve by orders of magnitude from 10 dynes/watt, or 10 gm wt/kw.

The second conclusion indicates that for the same thrust, the discharge current density, and hence the power consumption, may be reduced by using gases with larger ion mass and smaller saturation ionization constant A. However, the range of values of the mobility constant a_+ and the ionization constants A and B for all gases (molecular or rare gases) are quite limited. Changes in orders of magnitude of the specific power consumption cannot be expected by using different gases. By proper choice of electrode material, ω/α may be reduced by several orders of magnitude. However, the effect is felt only through the logarithm. Therefore, reduction in discharge current and power consumption by an order of magnitude can neither be expected by selecting different electrode materials.

The third conclusion, that increasing the discharge pressure increases both the current density, J_o, and the thrust, Th, in proportion to p^2, means that the specific power consumption is not affected by increasing p.

Consider now the effect of electric wind under the condition corresponding to an electric pressure of 1 dyne/cm² at 1 mm Hg and a specific thrust of 10 dynes/watt. If the electric wind term T_2/B^2p^2 is to contribute 1 dyne/cm², we would have (Eq.25), $p \cdot d \cong 10^3$ cm-mm Hg. Corresponding to $d \sim 10$ m, the potential drop across the positive column in volts would then be

$$\Delta V_+ = 300\ B \cdot \left(\frac{E_+}{pB}\right) \cdot p \cdot d = 365 \cdot 10^3 \cdot \left(\frac{E_+}{pB}\right)$$

where (E_+/pB) is the dimensionless field parameter in the positive column, which is somewhat smaller than E_o/pB at the anode, but is definitely of the same order. Let us take a conservative value of $E_+/pB \cong 0.03$ (12 v/cm in air). The voltage drop across the positive column will be 10 kv, i.e., 30 times the normal cathode fall of 300 v. Thus, the additional 1 dyne/cm² thrust is obtained through an electric wind at 30 times the electric power consumption than the 1 dyne/cm² from electric pressure. It is thus clear that the electric wind effect cannot improve the specific power consumption of the thrust device.

The thrust level that may be derived from discharges at ambient condition ($p \cong 760$ mm Hg) could be of the order of 0.5 kg/cm^2 if the 1 dyne/cm^2 at 1 mm Hg could be extrapolated according to p^2 law. (The thrust achievable from electric wind effect is neglected.) A thrust of the order of 5000 kg or 10,000 lbs might be obtained over an electrode area of one square meter. If the electric power is supplied from a separate source, it is conceivable that under the condition of subnormal glow discharge (with little visible glow) corresponding to current densities of the order of microamperes/cm^2, a structure, weighing less than 5 gm/cm^2 electrode area, may lift itself against the earth's gravity. The demonstration referred to in the introduction is clearly feasible. This demonstration, however, does not imply the feasibility of such a device as a self-contained propulsion system. If the system is to carry its own power source and lift itself from the ground, the thrust per unit power must be larger than the specific weight of the power plant. It does not appear possible in the near future, even with excessive extrapolation of the development of present technology, that there will be any power plant with specific weight of the order of 10 gm/kw.

When the discharge takes place in an ambient atmosphere to provide thrust, no propellant need be carried aboard the vehicle. The specific impulse is then of no concern. The gas discharge system may therefore find some interesting applications on vehicles for operation in the lower atmosphere with stand-by electric power supply. For example, for vehicles operating at about 10^5 ft altitude (~30 km) where $p \sim 10$ mm Hg, a thrust of 1 kg wt can be obtained over 1 square meter area of control surface. This may serve as a simple positive control of hypersonic vehicles. Another example may be the propulsion of high Mach number ramjets. Instead of adding heat to the hot air in the diffuser, just to produce dissociation of the air rather than thrust, the gas discharge scheme will produce thrust without adding much heat to the air. For such an application in ramjets, the inter-electrode distance can be made very large (if liberal supply of electric power is available) to take advantage of the electric wind effect. There may be other special circumstances where the gas discharge as a thrust device may find interesting applications.

At altitudes of $2 \cdot 10^5$ ft (~60 km) and above, the atmospheric pressure becomes a small fraction of a mm Hg. Eq.(26), even if applicable, would predict a thrust much too small to be of practical significance. When the atmospheric pressure reaches the range of 10^{-3} mm Hg (one micron) at about 100 km, the free paths of the ions and electrons become so large that they move in an electric field as electron or ion beams, rather than drift as swarms of random particles. They may also attain energy levels high enough to cause many new phenomena. Therefore, discharges through the highly rarefied "atmosphere" at such altitudes cannot be discussed within the present context. If the gas discharge in the ordinary sense is to be applied at such high altitudes or in the environment of outer space as a propulsion device, the gas must be stored aboard the vehicle under a pressurized chamber and allowed to flow through the region of discharge into outer space. The electric pressure is likely to be much smaller than the gas storage pressure. The electric wind effect can neither be counted on to increase greatly the thrust level, because the loss of neutral gas through diffusion into outer space will be excessive. The low specific impulse will put the gas discharge system in a rather unfavorable competitive position for space applications.

In conclusion, the glow discharge in gases (without resorting to thermal effects like the arc discharge) is limited in its potentiality as an advanced propulsion device. Within a reasonable expectation of the specific power-weight ratio of electric power plants, the gas discharge system cannot serve the purpose of a primary propulsion system. However, there may be other interesting applications, like the auxiliary thrust device for vehicle control, and the propulsive device for high Mach number ramjets. Detailed study for the specific function has yet to be made.

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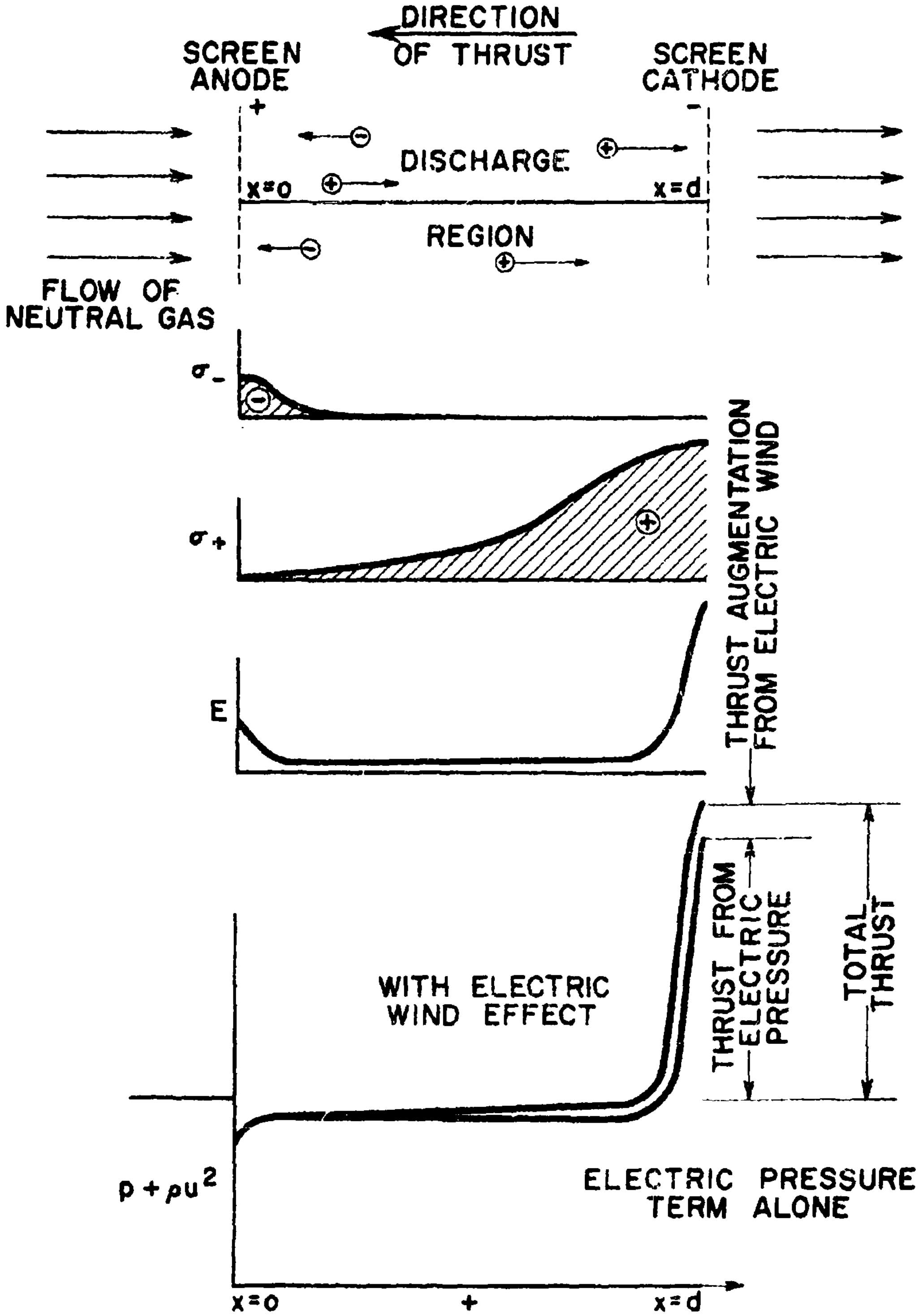

FIG. 1 APPROXIMATE CHARACTERISTICS (SCHEMATIC) OF GLOW DISCHARGE AS A THRUST DEVICE

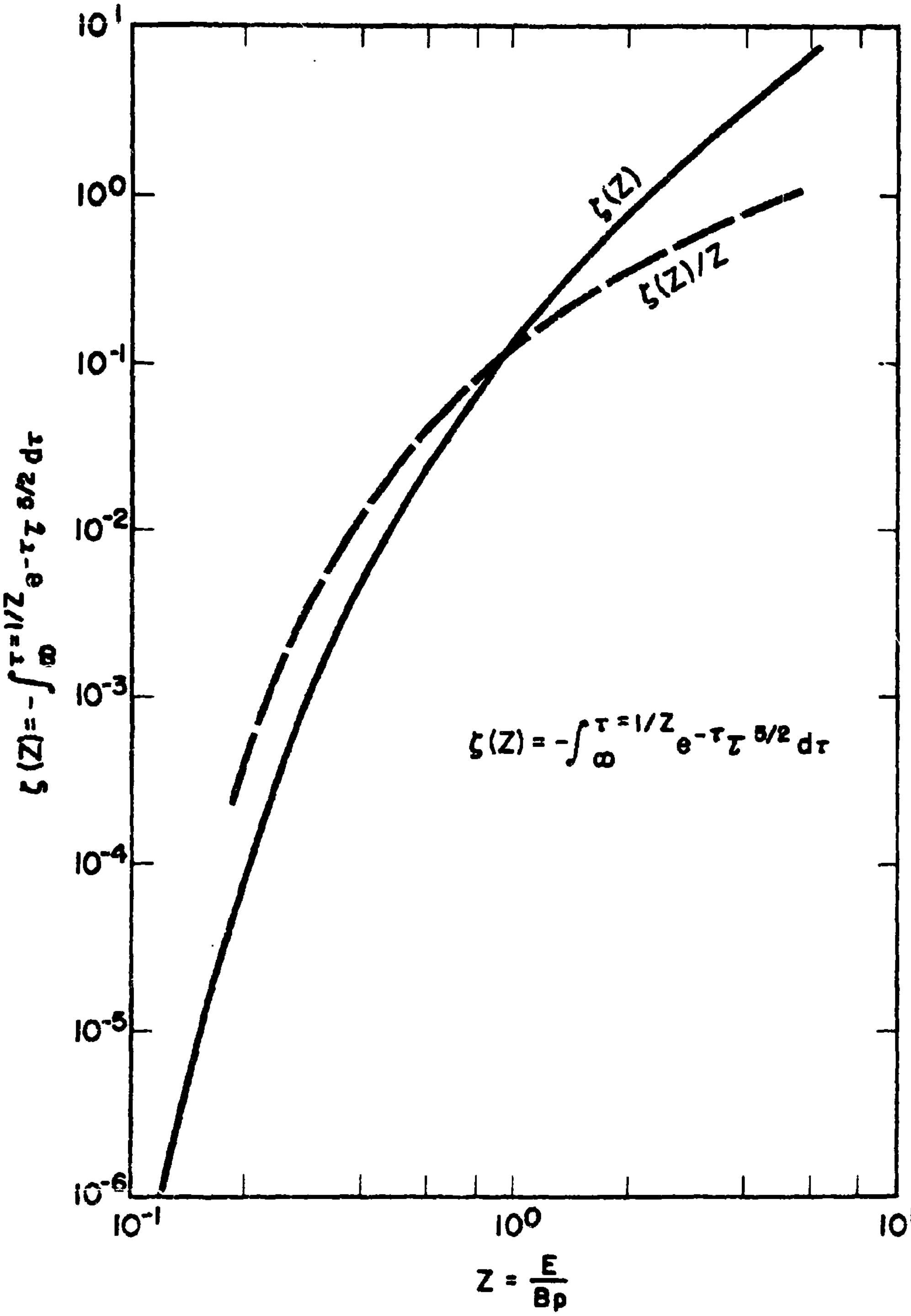

FIG. 2 UNIVERSAL FUNCTION $\zeta(\frac{E}{Bp})$

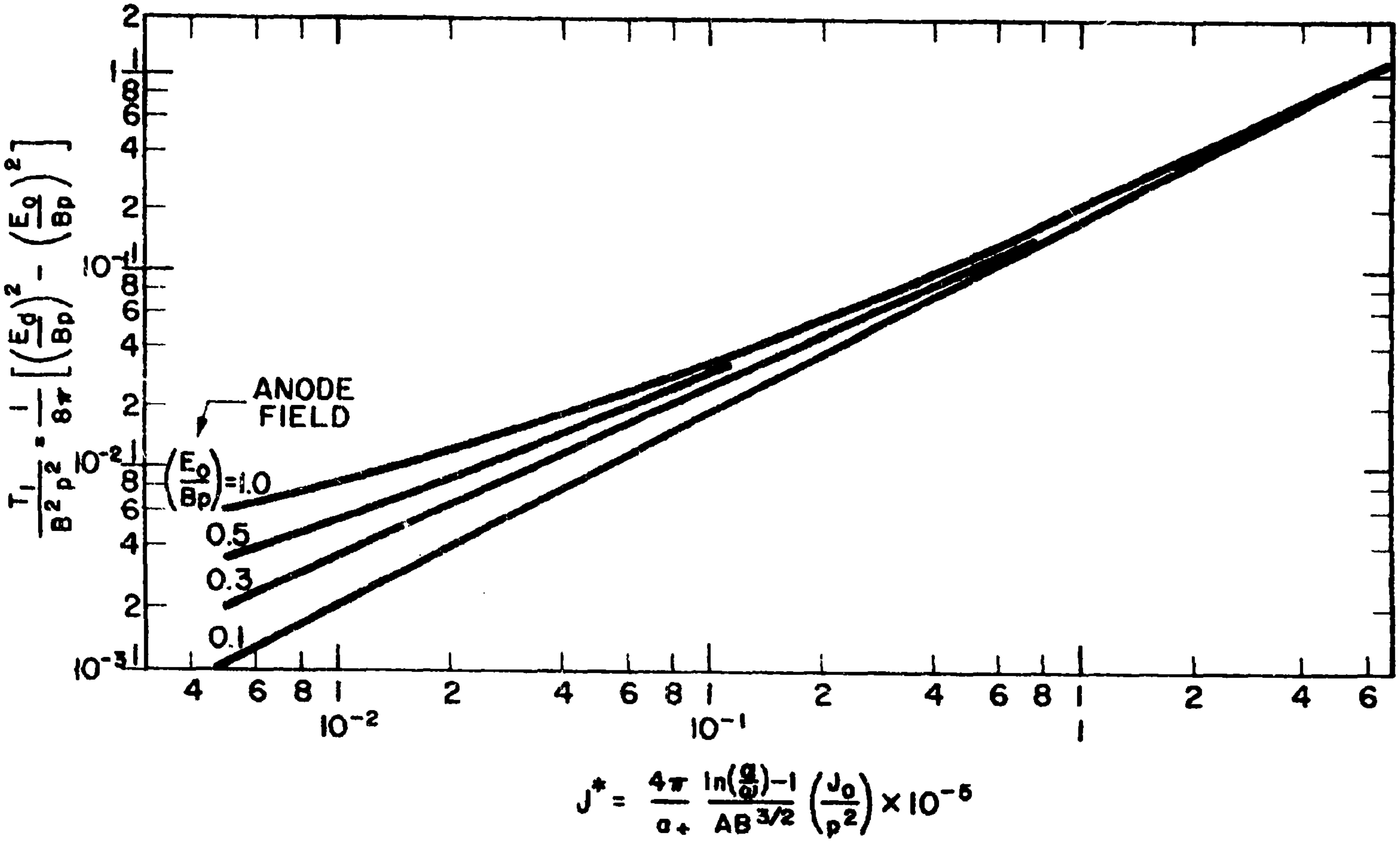

FIG. 3 DIMENSIONLESS THRUST VS. DISCHARGE CURRENT IN A GLOW DISCHARGE

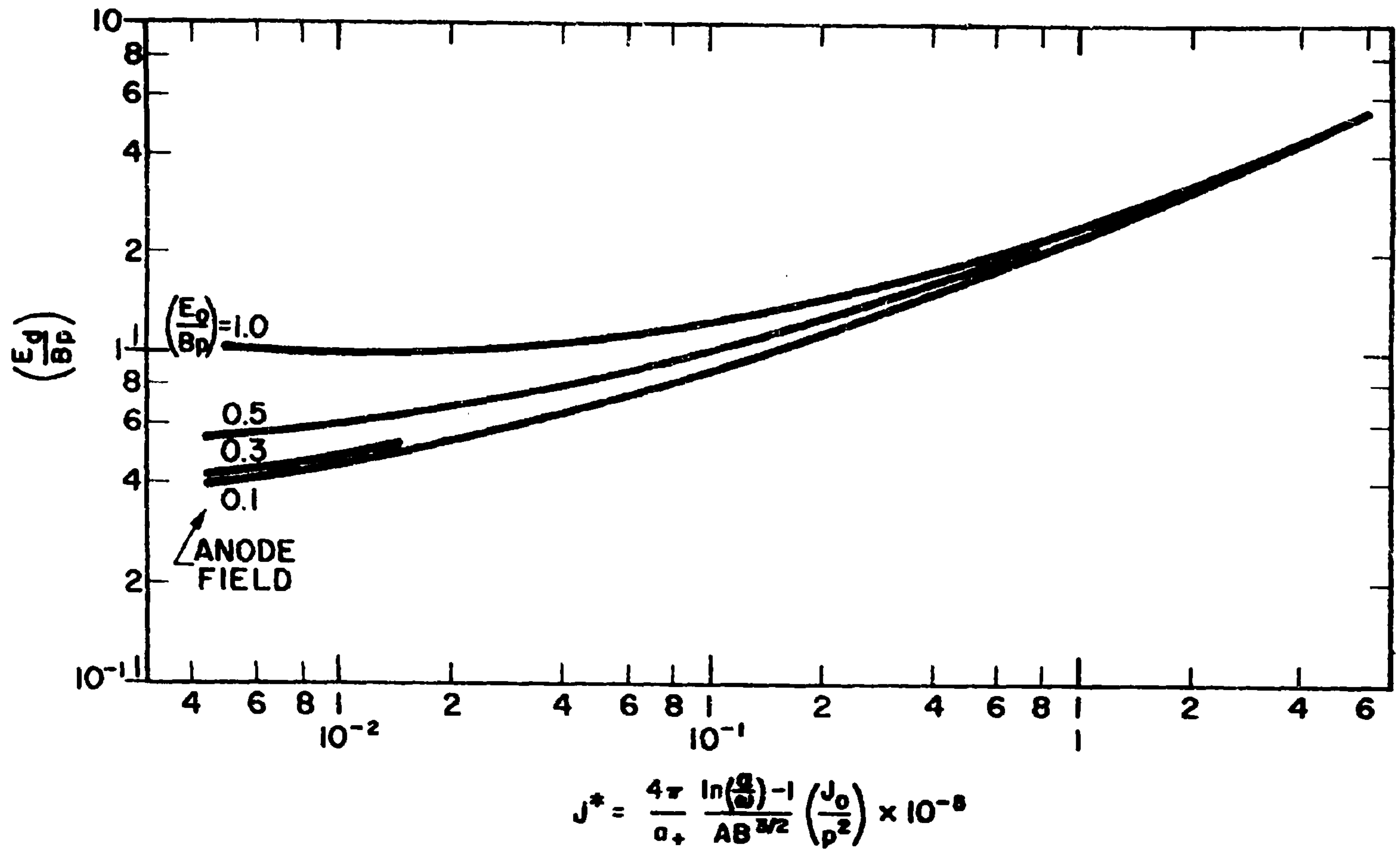

FIG. 4 DIMENSIONLESS CATHODE FIELD VS. DISCHARGE CURRENT IN A GLOW DISCHARGE