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ABSTRACT

The magnetic dipole moment induced in a spinning metallic sphere by a time dependent magnetic field is evaluated. The torque on the sphere is then calculated.

This analysis provides a model for investigating a possible gyroscope malfunction caused by a pulse of radiation. It is found that the angular deflection $\Delta\theta$ of the gyroscope's spin direction, for a pulse of t seconds is $\Delta\theta \cong \frac{1}{2I} (\mathcal{H} t)^2$, where $\mathcal{H}$ is the magnitude of the incident magnetic field, and I is the moment of inertia of the sphere. For $\mathcal{H} \sim 50$ gauss, and $2I \sim 2.5 \times 10^3$ gm - cm^2, $\Delta\theta \simeq t^2$, so that a pulse of $\frac{1}{10}$ sec is required to change the spin direction by 10^{-2} radians.

CONTENTS

I. INTRODUCTION

This report deals with the problem of the interaction of radiation with spinning conductive matter. The specific problem which is analyzed leads to the production of the misalignment of a simple gyroscope, a spinning metallic sphere, induced by a time dependent magnetic field.

When a time dependent magnetic field is applied to a spinning sphere there is a torque $\vec{\tau}$ on the sphere caused by the interaction of the incident magnetic field $\vec{\mathcal{H}}$, with the induced magnetic moment $\vec{M}$, $\vec{\tau} = \vec{M} \times \vec{\mathcal{H}}$. The problem is to calculate the induced magnetic dipole moment. This problem has been solved by Landau and Lifshitz* for the case of a static magnetic field and a sphere of unit permeability. In this paper their method is extended to include the case of sphere with any permeability and a time dependent magnetic field.

It is possible to write the general expression for M in the form

$$\vec{M}_i (t) = \chi_{ij} \vec{\mathcal{H}}_j (t)$$

If it is assumed that the magnetic susceptibility tensor χ_{ij} is independent of the applied field, it has the form

$$\chi_{ij} = \chi_1 \epsilon_{i\ell j} \Omega_\ell + \chi_2 \Omega_i \Omega_j + \chi_3 \delta_{ij}$$

* Reference 1

χ_1, χ_2 and χ_3 are scalar functions and $\vec{\Omega}$ is the sphere's angular velocity.

To evaluate the functions χ_1, χ_2 and χ_3 it is necessary to solve for the magnetic field outside of the sphere. This is most easily done by solving for $\vec{M}$ in a frame which rotates with the sphere and then transforming $\vec{M}$ back to the laboratory frame. The magnetic moment and the torque depend on the magnetic field, the frequency of the magnetic field, and the sphere's permeability, conductivity, radius and angular velocity.

In Section II, the equations for the magnetic field are obtained and then the general forms for the magnetic moment and the torque are deduced. The induced magnetic moment in the rest frame of the sphere is calculated in Section III, which then enables one to determine the functions χ_1, χ_2 and χ_3 (Section IV). The frequency dependence of the torque and magnetic moment is discussed (Section V) and a sample calculation leads to a criteria for when the torque would be significant.

The torque on our gyroscope is found to be appreciable, approximately $\vec{\tau} \simeq \mathcal{H}^2$ dyne-cm. However, for a pulse of $t \approx 10^{-4}$ seconds the deflection of the gyroscope's spin direction is completely negligible. A detectable spin direction deflection of $\Delta\theta = 10^{-2}$ radians is given to our model when

$$\mathcal{H} t \simeq \frac{\sqrt{2I}}{10} \cong 5$$

$\Delta\theta$ is given by

$$\Delta\theta \cong \frac{1}{2I} (\mathcal{H} t)^2 \quad .$$

II. THE SPINNING SPHERE IN A PERIODIC MAGNETIC FIELD

A magnetic field $\vec{H}(t)$ is applied to a spinning sphere

$$H(t) = e^{-i\omega t} H = e^{-i\omega t}\left[H_x \hat{i} + H_z e^{-i\varphi} \hat{k} \right] \quad (1)$$

H_x and H_z are constant amplitudes.* For convenience, it is assumed that $H_y = 0$. The sphere has electrical conductivity σ, permeability μ, dielectric constant $\varepsilon = 1$, radius a, spin $\vec{\Omega}$, moment of inertia I, and angular momentum $J = I\Omega$.

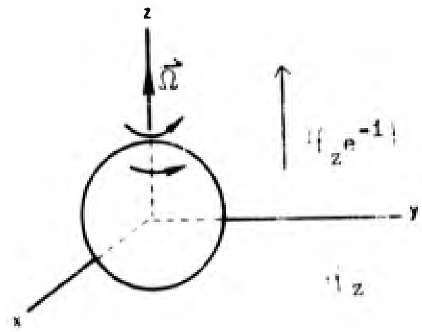

Figure 1

The current density $\vec{j}$ is given by

$$\vec{j} = \sigma(\bar{E} + \bar{v}/c \times \bar{B}) + \rho\bar{v} \quad (2)$$

$$\cong \sigma(E + v/c \times B)$$

with $v = \Omega \times r$, and it is assumed that the convection current $\rho\bar{v}$ is negligible, i.e.

$$\rho v \ll \sigma(\dot{E} + v/c \times B) \quad (3)$$

or

$$\rho \ll \frac{\sigma}{c} |B|$$

*Until Section V, it is usually assumed that $\varphi = 0$. $\varphi \neq 0$ corresponds to a rotating magnetic field.

When the displacement current is neglected,

$$\text{div } \vec{j} \cong 0 \cong \text{div } \vec{E} + \text{div } (\vec{v}/c \times \vec{B})$$

so that

$$\rho \cong \frac{-1}{4\pi} \text{ div } (\vec{v}/c \times \vec{B}) \tag{4}$$

Therefore, the condition for neglecting $\rho\vec{v}$ becomes

$$\frac{v}{L} \ll \sigma \tag{5a}$$

where L is a characteristic length of the problem, either the radius of the sphere a, or the skin depth δ. Since $v \sim \Omega a$ we have

$$\xi \ll \frac{\sigma}{\Omega} \quad \text{or} \quad \Omega \ll \sigma \quad \text{with} \quad \xi \equiv \frac{a}{\delta} \tag{5b}$$

as the condition for neglecting the convection current. That is, the angular velocity of the sphere must be much less than the conductivity. Physically, this means that the conductivity σ must be large enough to prevent a charge build-up generated by the rotation of the sphere.

If the sphere has a low conductivity and a high angular velocity so that this condition is not satisfied then it becomes necessary to consider the effects which depend on induced charges. The force density

$$\vec{f} = \rho\vec{E} + \vec{j}/c \times \vec{B} = \rho(\vec{E} + \vec{v}/c \times \vec{B}) + \frac{\sigma}{c}(\vec{E} + \vec{v}/c \times \vec{B}) \times B$$

$$\cong \frac{\sigma}{c}(\vec{E} + \vec{v}/c \times \vec{B}) \times \vec{B} \cong (\vec{\nabla} \times \vec{H}) \times \vec{B}. \tag{6}$$

because we have assumed that $\rho \ll \sigma c B$. That means we can calculate the torque due to the magnetic field and neglect effects which depend directly on ρ.

B. For $\sigma \gg \omega$, the displacement current can be neglected and the equations for the magnetic field inside the sphere are

$$\vec{\nabla} \times \vec{H}_i = \frac{4\pi}{c} \sigma (\vec{E} + \vec{v}/c \times \vec{B}) \quad (7a)$$

$$\vec{\nabla} \cdot \vec{H}_i = 0 \quad (7b)$$

Since $\vec{H}_i = \vec{\nabla} \times \vec{A}_i$ and $\vec{E}_i = -\frac{\mu}{c} \dot{\vec{A}}_i = + \frac{i\mu\omega}{c} \vec{A}_i$ we have

$$(\nabla^2 + k^2) \vec{A}_1 = (- \frac{4\pi\sigma\mu}{c^2}) \vec{v} \times (\vec{\nabla} \times \vec{A}) \quad (8a)$$

and

$$\vec{\nabla} \cdot \vec{A}_i = 0 \quad (8b)$$

with

$$k = \frac{1 + i}{\delta} \quad \text{and} \quad \delta = \frac{c}{\sqrt{2\pi\sigma\mu\omega}} = \text{skin depth.}$$

The source term $\vec{v} \times (\vec{\nabla} \times \vec{A})$ introduces the effect of the sphere's rotation on the eddy currents.

The magnetic field outside the sphere satisfies the equations

$$\vec{\nabla} \times \vec{H}_e = 0 \quad (9a)$$

$$\vec{\nabla} \cdot \vec{H}_e = 0 \quad (9b)$$

It is assumed that retardation effects are negligible so that

$$\omega \ll \frac{c}{a\sqrt{\mu}} \quad (10)$$

Therefore

$$\vec{H}_e = \left[- \vec{\nabla}\phi + \vec{\mathcal{H}} \right] e^{-i\omega t} \quad (11)$$

where $\nabla^2\phi = 0$ and $\phi \to 0$ at ∞.

The solution is then

$$\vec{H}(t) = \vec{\nabla}\left[\vec{M}(t) \cdot \vec{\nabla}\left(1/r\right)\right] + \vec{\mathcal{H}}(t) \qquad (12)$$

$\vec{M}(t)$ must be an axial vector which has the properties that:

a. $\vec{M} \rightarrow 0$ as $\vec{\mathcal{H}} \rightarrow 0$, which assumes that there is no intrinsic moment.

b. $\vec{M}$ is linear in $\vec{\mathcal{H}}$ or the tensor susceptibility is independent of $\vec{\mathcal{H}}$, which assumes that hysteresis effects are negligible.

$\vec{M}$ is then constructed from the vectors $\vec{\mathcal{H}}$ and $\vec{\Omega}$ as

$$\vec{M}(t) = e^{-i\omega t}\left[\chi_1(\vec{\Omega} \times \vec{\mathcal{H}}) + \chi_2(\vec{\mathcal{H}} \cdot \vec{\Omega})\vec{\Omega} + \chi_3 \vec{\mathcal{H}}\right] \qquad (13)$$

where χ_1, χ_2 and χ_3 are complex functions of $\omega, \Omega, \mu, \sigma$, and a.

$$\chi_1 = \chi_1' + i\chi_2'' \quad \text{(etc)}$$

Equation 13 can be written as

$$\vec{M}_i(t) = \chi_{ij}\, \vec{\mathcal{H}}_j(t) \qquad (14a)$$

with

$$\chi_{ij} = \chi_1 \varepsilon_{imj}\, \Omega_m + \chi_2\, \Omega_i\, \Omega_j + \chi_3\, \delta_{ij} \qquad (14b)$$

For $\Omega > 0$, the magnetic susceptibility tensor has off diagonal terms because the rotation of the sphere tends to "drag" the magnetic moment so that it is not parallel to the applied field.

The expression for the torque is

$$\vec{\tau}\,(t) = \vec{M}\,(t) \times \vec{\mathcal{H}}(t)$$

$$= \cos \omega t \Big[(\vec{\Omega} \times \vec{\mathcal{H}}) \times \vec{\mathcal{H}} \,(\chi_1{}' \cos \omega t + \chi_1{}'' \sin \omega t) \qquad (15)$$

$$+ (\vec{\mathcal{H}} \cdot \vec{\Omega})\,(\vec{\Omega} \times \vec{\mathcal{H}})\,(\chi_2{}' \cos \omega t + \chi_2{}'' \sin \omega t) \Big]$$

When this time dependent torque can be averaged over the magnetic field period, the average torque is

$$\vec{\tau}_{av} = \frac{1}{2} \Big[\chi_1{}' \,(\vec{\Omega} \times \vec{\mathcal{H}}) \times \vec{\mathcal{H}} + \chi_2{}' \,(\vec{\mathcal{H}} \cdot \vec{\Omega})\, \vec{\Omega} \times \vec{\mathcal{H}} \Big]$$

$$= \frac{\chi_1{}'}{2} \,(\vec{\Omega} \cdot \vec{\mathcal{H}}) \quad - \frac{\chi_1{}'}{2} \mathcal{H}^2 \vec{\Omega} + \frac{\chi_2{}'}{2} \,(\vec{\mathcal{H}} \cdot \vec{\Omega})\, \vec{\Omega} \times \vec{\mathcal{H}} \qquad (16)$$

This average is meaningful when there are many magnetic field cycles during one gyroscope cycle i.e. $\omega \gg \Omega$. A $(\vec{\Omega} \cdot \vec{\mathcal{H}})^2 \vec{\Omega}$ term in the expression for the torque has been excluded by the conditions on $\vec{M}$.

It is now necessary to solve equation 8a for $\vec{A}_i$ in order to evaluate the functions χ_1, χ_2 and χ_3. It would be possible to assume that

$$\vec{A}_i = \vec{\nabla} \times (a\, f_1 \vec{\Omega} + b\, f_2 \vec{\mathcal{H}} + c\, f_3 \vec{\Omega} \times \vec{\mathcal{H}})$$

and determine f_1, f_2 and f_3 so that equation 8a is satisfied. The conditions that $\vec{B}_{normal}$ and $\vec{H}_{tangential}$ be continuous would then lead to an evaluation of χ_1, χ_2 and χ_3. However, it is easier to use the trick of solving for $\vec{M}$ in a rotating frame in which $\vec{\Omega} = 0$ and then transform $\vec{M}$ back to the original frame.

III. THE MAGNETIC MOMENT IN THE ROTATING FRAME

A. It is assumed that

$$\frac{v}{c} \sim \frac{\Omega a}{c} \ll 1 \tag{17}$$

so that we can transform to a frame $\xi \eta z$ which rotates with the sphere. Fields such as $E' \sim \vec{v}/c \times \vec{B}$ and inertial forces are neglected in this transformation.

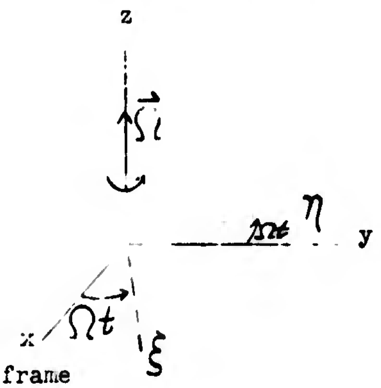

Figure 2

In the rotating frame

$$\vec{\mathcal{H}}(t) = \hat{\xi}\,\mathcal{H}_x \cos \omega t \cos \Omega t - \hat{\eta}\,\mathcal{H}_x \cos \omega t \sin \Omega t + \hat{k}\,\mathcal{H}_z \cos(\omega t + \varphi) \tag{18a}$$

Therefore, in $\xi \eta Z$, there is a static sphere with an applied field,

$$\vec{\mathcal{H}}(t) = \hat{\xi}\,\frac{\mathcal{H}_x}{2}\left(e^{-i\gamma_+ t} + e^{-i\gamma_- t}\right) + \hat{\eta}\,\frac{\mathcal{H}_x}{2i}\left(e^{-i\gamma_+ t} + e^{-i\gamma_- t}\right) + \hat{k}\,\mathcal{H}_z\, e^{-i\varphi}\, e^{-i\omega t} \tag{18b}$$

with $\gamma_\pm = \Omega \pm \omega$

The magnetic dipole moment induced in a static sphere by a periodic field and then by the applied field (equation 18b) is now required. The dipole moment of the rotating sphere is then obtained by transforming back to the original (xyz) frame.

B. A periodic magnetic field of frequency γ is applied to a static conducting sphere.

$$\vec{\mathcal{H}}(t) = \vec{\mathcal{H}} e^{-i\gamma t} \tag{19}$$

$\vec{\mathcal{H}}$ is a constant amplitude.

For $v/c \ll 1$, Maxwell's equations are valid in the rotating frame, therefore, the equations for the vector potential A_i in $\xi \eta \zeta$ are

$$(\vec{\nabla}^2 + k^2)\vec{A}_i = 0 \tag{20a}$$

$$\vec{\nabla} \cdot \vec{A}_i = 0 \tag{20b}$$

Now

$$\delta = \frac{c}{\sqrt{2\pi\sigma\mu|\gamma|}}$$

with

$$k = \frac{1+i}{\delta} \quad \text{for } \gamma > 0$$

$$k = -\frac{1-i}{\delta} \quad \text{for } \gamma < 0$$

The solution is then

$$\vec{A}_i = \beta\,(\vec{\nabla} \times f\vec{\mathcal{H}})\, e^{-i\gamma t} \tag{21a}$$

where β is a constant and $(\nabla^2 + k^2)f = 0$. For the lowest multipole, that is the magnetic dipole moment, the boundary conditions will be satisfied with $f = \frac{\sin kr}{r}$. $\vec{A}_i$ also satisfies the condition that the rotation properties of the solution should depend on $\vec{\mathcal{H}}$ (the only specified direction in this problem) and that $\vec{A}_i$ is a polar vector.

The inside magnetic field is

$$\vec{H}_i = \beta \vec{\nabla} x (\vec{\nabla} x f \vec{\mathcal{H}}) e^{-i\delta t} \quad (21b)$$

$$= \beta e^{-i\delta t} \left[(\frac{f'}{r} + k^2 f)\vec{\mathcal{H}} - (\frac{3f'}{r} + k^2 f)(\vec{\mathcal{H}}\cdot\hat{n})\hat{n} \right]$$

$\hat{n} = \frac{\vec{r}}{r}$

Outside the sphere the equations are

$$\vec{H}_e = \left[-\vec{\nabla}\phi + \vec{\mathcal{H}}\right] e^{-i\delta t} \quad (22)$$

with $\nabla^2 \phi = 0$ and $\phi \rightarrow 0$ at $r \rightarrow \infty$.

The pseudo-scalar solution which depends on $\vec{\mathcal{H}}$ and is the lowest multipole solution is $\phi = -V\alpha \left(\vec{\mathcal{H}}\cdot\vec{\nabla}\right)(1/r)$ (23a)

$V = \frac{4\pi}{3}a^3$; $\nabla^2(1/r) = 0$ for $r \geq a$.

Therefore

$$\vec{H}_e = \left[V\alpha \; \vec{\nabla}(\vec{\mathcal{H}}\cdot\vec{\nabla}1/r) + \vec{\mathcal{H}}\right] e^{-i\delta t} \quad (23b)$$

$$= \left\{\frac{V\alpha}{r^3}\left[3\hat{n} \; (\vec{\mathcal{H}}\cdot\hat{n}) - \vec{\mathcal{H}}\right] + \vec{\mathcal{H}}\right\} e^{-i\delta t}$$

which is the field of a magnetic dipole

$$\vec{M} = V\alpha \; \vec{\mathcal{H}} \; e^{-i\delta t} \quad . \quad (24)$$

The boundary conditions that B_{normal} and $H_{tangential}$ be continuous*

imply

$$\frac{-2f'(a)\mu\beta}{a} = \frac{2V\alpha}{a^3} + 1 \quad (25a)$$

*For $\vec{v} = \vec{\Omega} x \vec{r}$ the boundary conditions are unaffected by the transformation to $\xi \eta z$.

and

$$\left(\frac{f'(a)}{a} + k^2 f(a)\right)\beta = -\frac{V\alpha}{a^3} + 1 \quad ** \tag{25b}$$

The constant β is eliminated from equations 25a, and 25b and it is then found that the magnetic dipole polarizability α is

$$\alpha = \alpha' + i\alpha'' = -\frac{3}{8\pi}\left\{\frac{1 + (2\mu + 1)(R + iI)}{1 + (1 - \mu)(R + iI)}\right\} \tag{26a}$$

with

$$R = \frac{1}{2\xi}\,\frac{\sin 2\xi - \sinh 2\xi}{\cosh 2\xi - \cos 2\xi} \tag{26b}$$

$$I = [\operatorname{sgn}\gamma]\ \frac{1}{2\xi}\left(\frac{1}{\xi} - \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}\right) \quad ***$$

$$\xi = \frac{a}{\delta} = \frac{a}{c}\sqrt{2\pi\sigma\mu|\gamma|}$$

The expressions for α' and α" are

$$\alpha' = -\frac{3}{8\pi}\left\{1 + \frac{3\mu\left[R + (1 - \mu)(R^2 + I^2)\right]}{\left[1 + (1 - \mu)R\right]^2 + (1 - \mu)^2 I^2}\right\} \tag{26c}$$

and

$$\alpha'' = -\frac{9\mu}{8\pi}\left\{\frac{I}{\left[1 + (1 - \mu) R\right]^2 + (1 - \mu)^2 I^2}\right\} \tag{26d}$$

In the limit of small skin depth $\xi \gg 1$

$$R \rightarrow -\frac{1}{2\xi} \qquad I \rightarrow -[\operatorname{sgn}\gamma]\left(\frac{1}{2\xi}\right) \tag{26e}$$

** $f(a) = \dfrac{\sin k a}{a}$ and $f'(a) = \dfrac{k \cos k a}{a} - \dfrac{\sin k a}{a^2}$

*** $\operatorname{sgn}\gamma$ is + for $\gamma > 0$
- for $\gamma < 0$

and then

$$\alpha' = -\frac{3}{8\pi}\left\{1 - \frac{6\mu\left[\xi + \mu - 1\right]}{\left[2\xi + (\mu - 1)\right]^2 + (\mu - 1)^2}\right\} \tag{26f}$$

$$\alpha'' = +\left[\operatorname{sgn}\gamma\right]\left(\frac{9\mu}{8\pi}\right)\left\{\frac{2\xi}{\left[2\xi + (\mu - 1)\right]^2 + (\mu - 1)^2}\right\} \tag{26g}$$

The dipole moment $\vec{M}$ of the static sphere, induced by a field $\mathcal{H}e^{-i\gamma t}$ is

$$\vec{M} = V\alpha\vec{\mathcal{H}}e^{-i\gamma t} = V\vec{\mathcal{H}}(\alpha' \cos\gamma t + \alpha'' \sin\gamma t) \tag{27}$$

where α' and α'' are evaluated from equations 26b, 26c and 26d, if $\xi \gg 1$ equations 26f, 26g are used. This expression for $\vec{M}$ in a periodic field is now used to find the magnetic moment induced by our applied field (equation 18b).

C. In Section IIB, the magnetic moment induced in a static sphere by a periodic field has been found (equation 27). The problem now is to find the magnetic moment induced in the static sphere by the applied field of equation 18b which involves the frequencies $\Omega + \omega$, $\Omega - \omega$, and ω. Equation 24 is used for each frequency in the applied field so that the magnetic dipole moment in $\xi\eta\zeta$ is by superposition

$$\begin{aligned} \vec{M}(t) = {} & \hat{\xi}\frac{V\mathcal{H}_x}{2}\left[\alpha_+ e^{-i\gamma_+ t} + \alpha_- e^{-i\gamma_- t}\right] \\ & + \hat{\eta}\frac{V\mathcal{H}_x}{2i}\left[\alpha_- e^{-i\gamma_+ t} + \alpha_- e^{-i\gamma_- t}\right] + \\ & + \hat{k}\, V\mathcal{H}_z\, \alpha_\omega e^{-i\varphi} e^{-i\omega t} \end{aligned} \tag{28}$$

where $\alpha_\pm$ and α_ω are evaluated for $\gamma_\pm = \Omega \pm \omega$ and ω respectively. It is then found that

$$M_\xi = M_x \cos \Omega t + M_y \sin \Omega t$$
$$M_\eta = M_y \cos \Omega t - M_x \sin \Omega t \tag{29}$$

where

$$M_x = \frac{V\mathcal{H}_x}{2}\left[(a_+' + a_-') \cos \omega t + (a_+'' - a_-'') \sin \omega t\right],$$
$$M_y = \frac{V\mathcal{H}_x}{2}\left[(a_+'' + a_-'') \cos \omega t + (-a_+' + a_-') \sin \omega t\right], \tag{30}$$

and

$$M_z = V\mathcal{H}_z \left[a_\omega' \cos (\omega t + \varphi) + a_\omega'' \sin (\omega t + \varphi)\right]$$

are the components of the moment in the laboratory frame.

IV. THE MAGNETIC MOMENT OF THE SPINNING SPHERE

The magnetic dipole moment induced in the rotating sphere (equation 30) can be expressed in the form of equation 13.

$$\vec{M}(t) = e^{-i\omega t}\left[\chi_1 (\vec{\Omega} \times \vec{H}) + \chi_2 (\vec{H} \cdot \vec{\Omega})\vec{\Omega} + \chi_3 \vec{H}\right] \qquad (31)$$

where

$$\begin{aligned} \chi_1 &= \chi_1' + i\chi_1'' = \left(\frac{V}{\Omega}\right)\left[\frac{a_+ - a_-^*}{2i}\right] \\ \chi_2 &= \chi_2' + i\chi_2'' = \left(\frac{V}{\Omega}\right)\left[a_\omega - \frac{a_+ + a_-^*}{2}\right] \\ \chi_3 &= \chi_3' + i\chi_3'' = V\left[\frac{a_+ + a_-^*}{2}\right] \end{aligned} \qquad (32)$$

These functions indicate how effective the magnetic field is in inducing a magnetic moment in the sphere. They are now readily computed using the known expressions for $a_\pm$ and a_ω.

As was shown in Section IIB, the magnetic susceptibility tensor (equation 14b), the torque (equation 15), and the average torque (equation 16), are now known for any ω, Ω, ψ, σ, and a. The restrictions are that

$$\begin{aligned} &\omega \ll \sigma \\ &\Omega \ll \sigma \\ &\Omega \ll \frac{c}{a} \\ &\omega \text{ and } |\Omega \pm \omega| \ll \frac{c}{a}\frac{1}{\sqrt{\mu}} \\ &\xi \ll \frac{\sigma}{\Omega} \end{aligned} \qquad (33)$$

and that the permeability μ is constant.

The exact expressions for χ_1, χ_2, χ_3 are obtained from equations 32 and 26 .

For example,

$$\chi_1' = -\frac{9\mu V}{16\pi\Omega}\left\{\frac{I_+}{\left[1+(1-\mu)R_+\right]^2+(1-\mu)^2 I_+^{\,2}} + \frac{I_-}{\left[1+(1-\mu)R_-\right]^2+(1-\mu)^2 I_-^{\,2}}\right\} \tag{34}$$

and

$$\chi_2' = -\frac{9\mu V}{8\pi\Omega^2}\left\{\frac{R_\omega+(1-\mu)\left(R_\omega^{\,2}+I_\omega^{\,2}\right)}{\left[1+(1-\mu)R_\omega\right]^2+(1-\mu)^2 I_\omega^{\,2}} - \frac{1}{2}\left[\frac{R_+ +(1-\mu)(R_+^{\,2}+I_+^{\,2})}{\left[1+(1-\mu)R_+\right]^2+(1-\mu)^2 I_+^{\,2}} + \frac{R_- +(1-\mu)\left(R_-^{\,2}+I_-^{\,2}\right)}{\left[1+(1-\mu)R_-\right]^2+(1-\mu)^2 I_-^{\,2}}\right]\right\} \tag{35}$$

where the R and I's are calculated from Equation 26 using the corresponding ξ

i.e.

$$\begin{aligned} \xi_\omega &= \frac{a}{c}\sqrt{2\pi\sigma\mu}\,\sqrt{\omega} \\ \xi_+ &= \xi_\omega\sqrt{\frac{\Omega}{\omega}+1} \\ \xi_- &= \xi_\omega\sqrt{\left|\frac{\Omega}{\omega}-1\right|} \end{aligned} \tag{36}$$

V. THE FREQUENCY DEPENDENCE

A. The frequency dependence of the magnetic moment and of the torque can now be examined. In the limit of a static magnetic field $\omega \ll \Omega$ we have,

$$a_+ \longrightarrow a_- \longrightarrow a_\Omega \quad \text{and}$$

$$\begin{aligned} X_1 &\longrightarrow \left(\frac{V}{\Omega}\right)\left[\frac{a_\Omega - a_\Omega^{*}}{2i}\right] = \left(\frac{V}{\Omega}\right) a_\Omega'' \\ X_2 &\longrightarrow \left(\frac{V}{\Omega}\right)\left(a_{\omega\to 0} - a_\Omega'\right)^{\dagger} \\ X_3 &\longrightarrow V\, a_\Omega' \end{aligned} \tag{37}$$

so that

$$\vec{M} \longrightarrow V\left[a_\Omega'\, \mathcal{H}_x\, \hat{i} + a_\Omega''\, \mathcal{H}_x\, \hat{j} + (a'_{(\omega\to 0)})\, \mathcal{H}_z\, \hat{k}\right] \tag{38}$$

and the torque is

$$\begin{aligned} \vec{\tau} \longrightarrow V\Big[&a_\Omega''\, \mathcal{H}_x \mathcal{H}_z\, \hat{i} + (a'_{\omega\to 0} - a_\Omega')\mathcal{H}_x\mathcal{H}_z\, \hat{j} \\ &+ (-a_\Omega''\, \mathcal{H}_x^2)\, \hat{k}\Big] \end{aligned} \tag{39}$$

Therefore, a static magnetic field exerts a torque on a rotating sphere. For $\mu = 1$, equation 39 is the solution given by Landau and Lifshitz for the torque exerted on a rotating sphere by a static field.**

For a high frequency magnetic field $\omega \gg \Omega$ we have

$$a_+ \longrightarrow a_-^{*} \longrightarrow a_\omega$$

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† For $\mu = 1$ $\quad a_{(\omega\to 0)} \longrightarrow 0$

**Reference 1, page 193

and

$$\chi_1 \rightarrow 0, \quad \chi_2 \rightarrow 0$$

$$\chi_3 \rightarrow Va_\omega \quad \text{and} \quad \vec{M}(t) \rightarrow Ve^{-i\omega t} a_\omega \vec{\mathcal{H}} \tag{40}$$

Therefore, for $\omega \gg \Omega$ and $\varphi = 0$

$$\vec{\tau} \longrightarrow V\,(a_\omega' \cos\omega t + a_\omega'' \sin\omega t)\,\vec{\mathcal{H}} \times (\vec{\mathcal{H}} \cos\omega t) \longrightarrow 0 \tag{41}$$

The magnetic moment becomes parallel to the magnetic field and the torque is zero in the high frequency limit.

Now, in the case that $\omega \gg \Omega$ and $\varphi \neq 0$, which corresponds to a high frequency, rotating magnetic field, we have

$$\vec{\tau} \longrightarrow \text{Re}\,(Va_\omega \vec{\mathcal{H}}(t)) \times \text{Re}\,\vec{\mathcal{H}}(t) \tag{42}$$

with $\vec{\mathcal{H}}(t) = e^{-i\omega t}\,(\mathcal{H}_x \hat{i} + \hat{k}\,\mathcal{H}_z e^{-i\varphi})$

then

$$\vec{\tau} \longrightarrow \hat{j}\,\mathcal{H}_x \mathcal{H}_z V \, a_\omega'' \sin\varphi \tag{43}$$

Since $a''_{\omega} \xrightarrow[(\omega\gg\Omega)]{} 0$, $\vec{\tau} \xrightarrow[\omega\gg\Omega]{} 0$ also.

Equation 43 indicates that a rotating magnetic field tends to spin the sphere. This torque is a consequence of the "drag" on the magnetic moment, as the magnetic field rotates the induced magnetic moment lags behind. The effect given by equation 43 is not to be confused with a similar hysteresis effect, for our case it has been assumed that μ is a constant.

B. The known functions $\chi_1(\omega)$, $\chi_2(\omega)$, and $\chi_3(\omega)$ give the frequency dependence of the magnetic moment and of the torque for assumed values of Ω, σ, a and μ. For example, the torque averaged over a magnetic field cycle is given by equation 16 or

$$\frac{\vec{\tau}_{av} \cdot \hat{i}}{\mathcal{H}_x \mathcal{H}_z} = - \frac{\vec{\tau}_{av} \cdot \hat{k}}{\mathcal{H}_x^2} = \frac{\Omega}{2} \chi_1'(\omega) \qquad (44)$$

$$\frac{\vec{\tau}_{av} \cdot \hat{j}}{\mathcal{H}_x \mathcal{H}_z} = \frac{\Omega^2}{2} \chi_2'(\omega)$$

As a special case, the sphere is assumed to have a:

radius: $a = 2.5$ cm

conductivity: $\sigma = \frac{1}{7} \times 10^7 \frac{\text{mho}}{\text{m}} = \frac{9}{7} \times 10^{16}$ esu units*

angular velocity: $\Omega = 1.2 \times 10^3$ rad/sec.

For a steel sphere, this corresponds to a moment of inertia $I = 1.25 \times 10^3$ gm - cm^2, and an angular momentum $J = 1.5 \times 10^6$ gm - cm^2/sec. These parameters are chosen to approximate actual gyroscope values.

The frequency dependence of equation 44 is given in figure 3 for $\mu = 1$ and figures 4,5 for $|\mu - 1| = 10, 10^2, 10^3$. For $\mu = 1$, the magnetic moment is determined by the induced eddy currents. These eddy currents are complicated by the rotation of the sphere so that the net magnetic moment is not parallel to $\vec{\mathcal{H}}$. That is, the sphere tends to drag the magnetic moment.

---

* cgs units are used, the conversion factor is $1 \frac{\text{mho}}{\text{m}} = 9 \times 10^9$ esu units of conductivity.

As the frequency ω increases, the drag effect and the skin depth change* in such a way that the net magnetic moment in the $\vec{\Omega} \times \vec{\mathcal{H}}$ direction changes sign. That means that there is a change from paramagnetism to diamagnetism in the $\vec{\Omega} \times \vec{\mathcal{H}}$ direction as ω increases, which results in a change in the average torque $\left(\text{ie } \frac{\vec{\tau}_{av} \cdot \hat{i}}{\mathcal{H}_x \mathcal{H}_z} \right)$ as shown in figure 3 .

The effect of increasing permeability is illustrated in figures 4, 5. Now the mechanism causing the magnetic moment becomes even more complicated. The skin depth is decreased, which changes the eddy current contribution to $\vec{M}$ and the $\vec{v}/c \times \vec{H}$ term becomes more important. In addition to the eddy currents, the magnetic susceptibility $\frac{\mu - 1}{4\pi}$ also contributes to the magnetic moment and to the torque. The magnetic fields inside the sphere cause the magnetic moment associated with $\frac{\mu - 1}{4\pi}$ to have components which are not parallel to $\vec{\mathcal{H}}$.

* The "drag" effect is contained in the term $\frac{-4\pi\sigma\mu}{c^2}\left|\vec{v} \times H\right|$ of equation 8a, as the frequency ω increases, this term becomes less important (compared to $\frac{\omega^2}{c^2} \vec{A}_i$). The skin depth decreases with ω as $\frac{\text{const}}{\sqrt{\omega}}$, and therefore, the eddy current contributions to the magnetic moment change because of the variation of eddy current with depth of penetration into surface. For example, for a plane surface, the eddy current varies as $e^{-z/\delta} \left[\cos(\omega t - \frac{z}{\delta})\right]$ (z is penetration depth), for a sphere the eddy current variation with depth is more complex.

VI. CONCLUSION

The functions X_1, X_2, and X_3 can be calculated for any assumed values of ω, Ω, μ, a, and σ, subject to equation 33 . Therefore, the induced moment and the torque are known.

It is possible to calculate the net torque on the sphere due to a magnetic field pulse $\vec{\mathcal{H}}(t)$, taking equation 15 as the torque exerted on the sphere by each fourier component $\vec{\mathcal{H}}_\omega$ of $\vec{\mathcal{H}}(t)$.

Figure 4 indicates a possible torque of magnitude $|\vec{\tau}| \sim \mathcal{H}^2$ dyne-cm, which would tend to change the spin direction by $\Delta\theta$ with

$$\Delta\theta = \frac{1}{2I}(\mathcal{H}t)^2 \tag{45}$$

Assuming that a $\Delta\theta = 10^{-2}$ radians would be detectable this means that if $\mathcal{H}t \sim \frac{\sqrt{2I}}{10}$, it would be necessary to consider a possible gyroscope malfunction.

For our model

$$t \sim \frac{5}{\mathcal{H}} \tag{46}$$

which means that for a field of 50 gauss a pulse of $t \sim .1$ sec could lead to a spin direction charge of 10^{-2} radians.

It is possible to reduce the torque on the sphere by picking μ, σ, Ω, and a so that we are near the zeros of the X_1, X_2, and X_3 functions for some central frequency of the pulse.

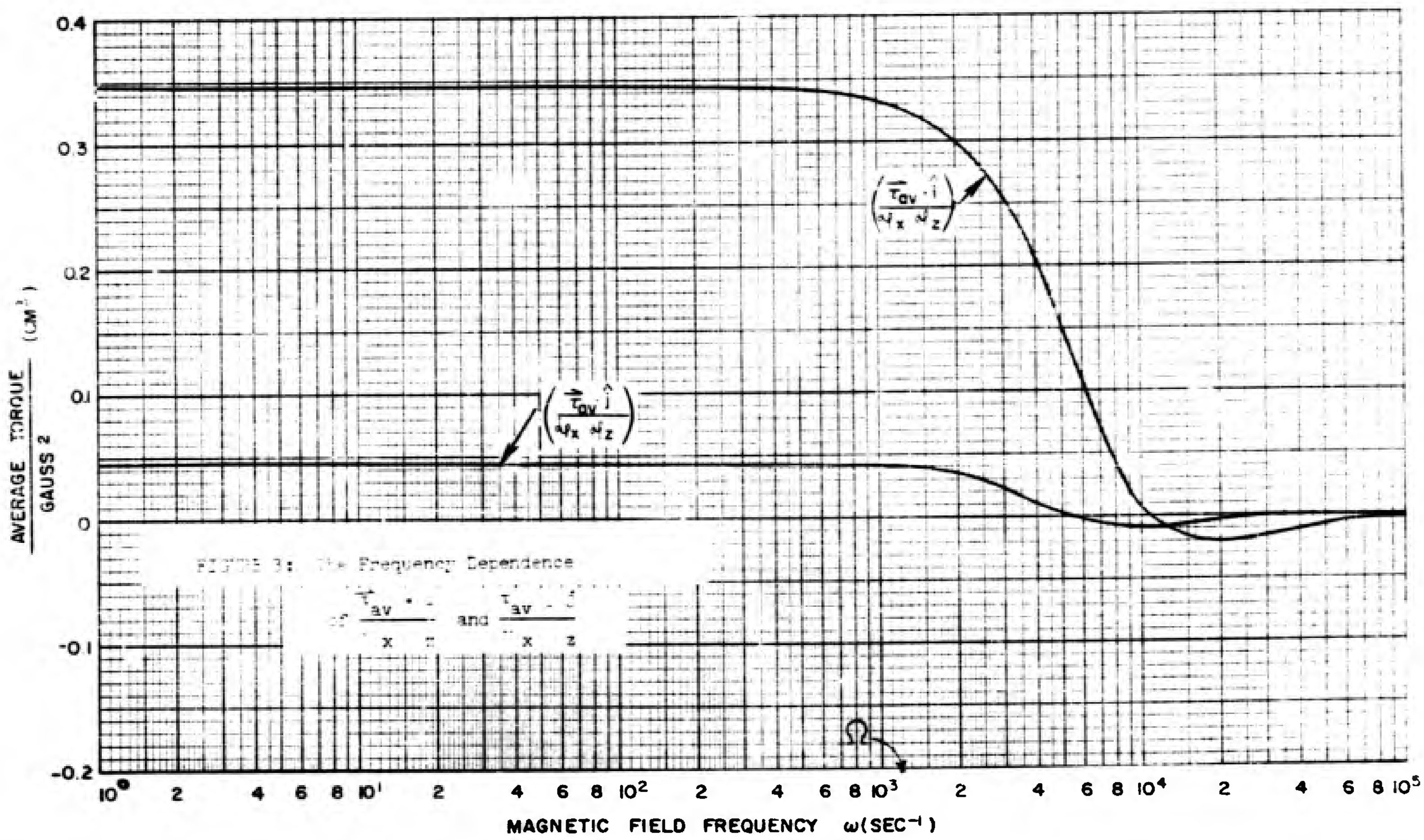


FIGURE 3: The Frequency Dependence of $\frac{\tau_{av} \cdot \hat{i}}{x\ z}$ and $\frac{\tau_{av} \cdot \hat{j}}{x\ z}$

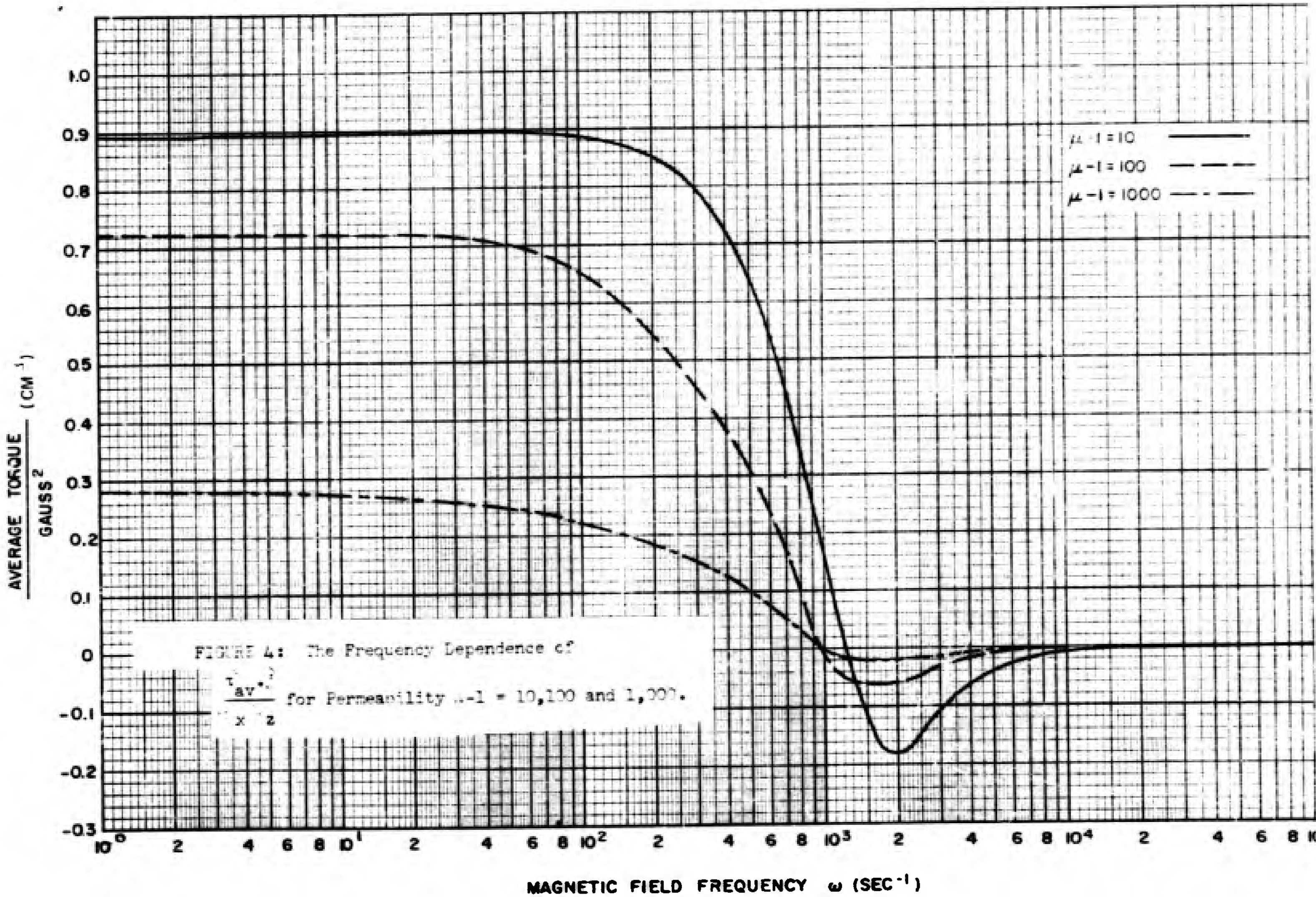
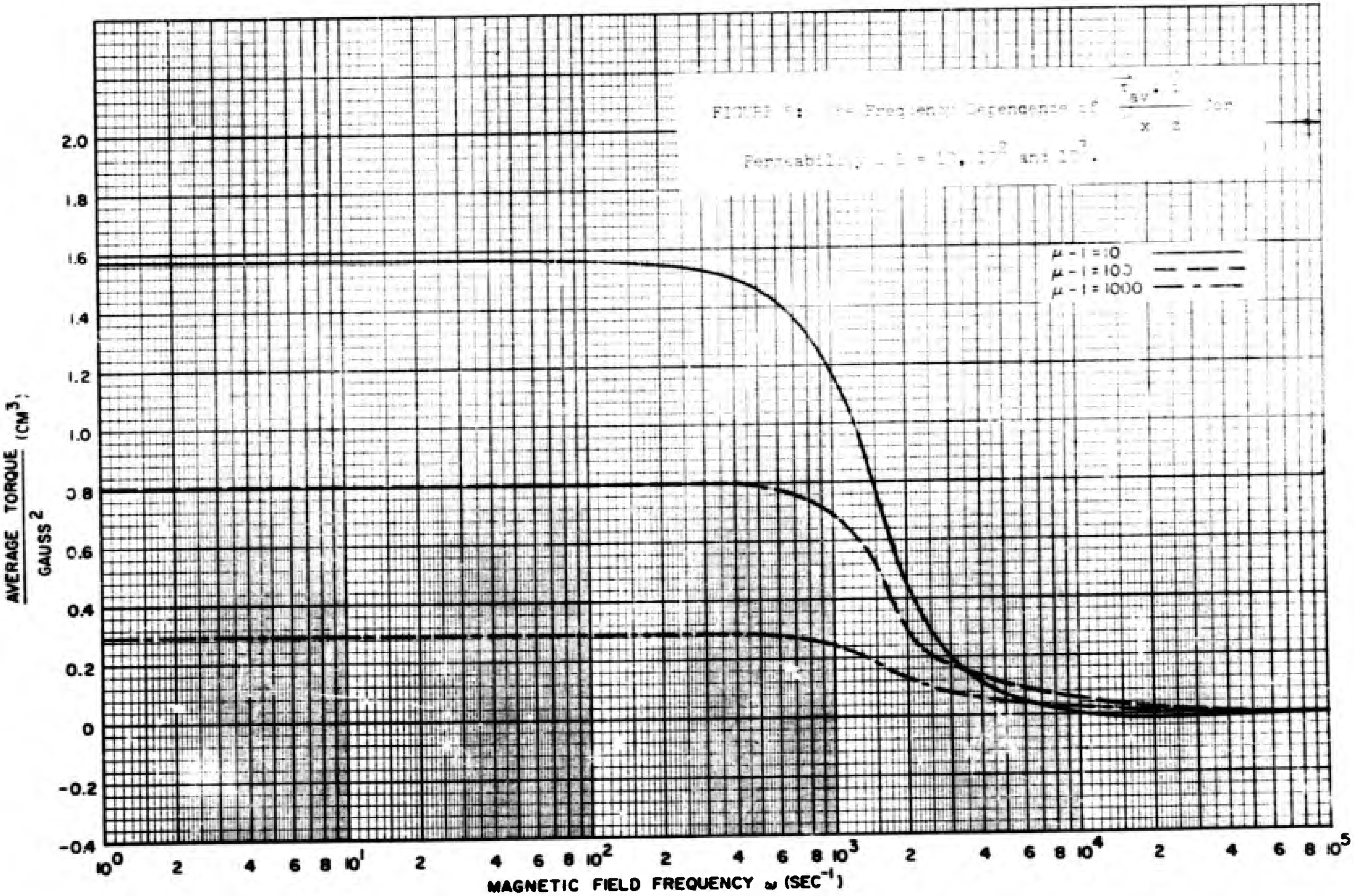


FIGURE 4: The Frequency Dependence of $\frac{\tau_{av}}{H_x H_z}$ for Permeability μ-1 = 10,100 and 1,000.


AVERAGE TORQUE (CM³) / GAUSS²
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
10^0 2 4 6 8 10^1 2 4 6 8 10^2 2 4 6 8 10^3 2 4 6 8 10^4 2 4 6 8 10^5
MAGNETIC FIELD FREQUENCY ω (SEC⁻¹)
μ - 1 = 10
μ - 1 = 100
μ - 1 = 1000

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694

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