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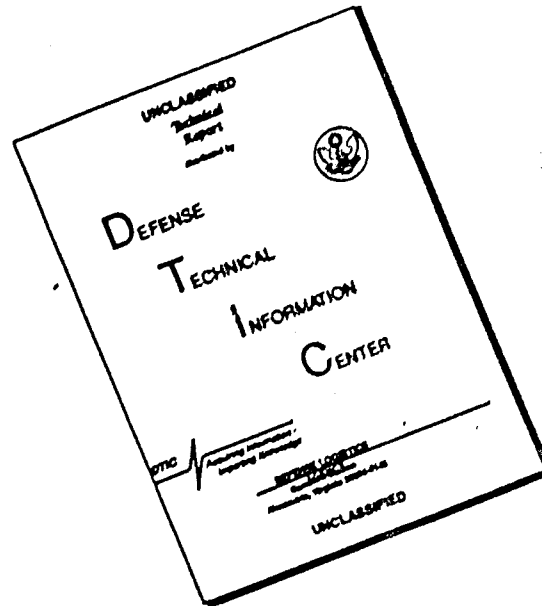
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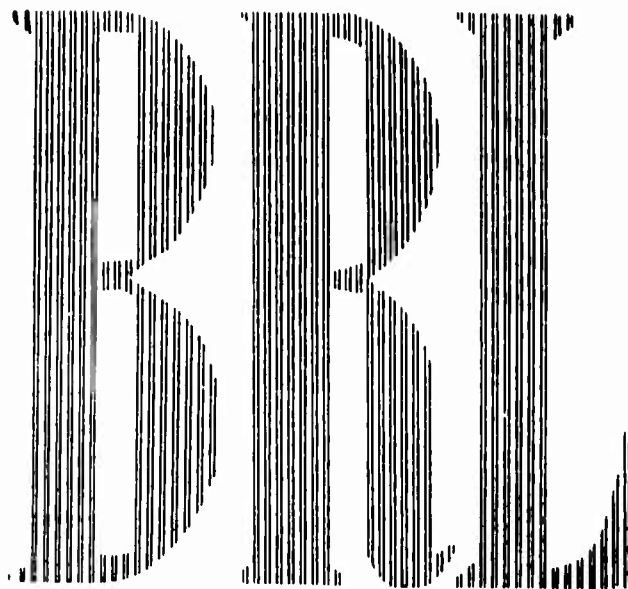
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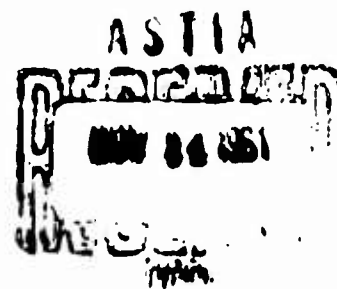
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SEPTEMBER 1961

RELATIONSHIP BETWEEN ADDITIVE AND
NON-ADDITIVE QUANTUM FLUCTUATIONS

L. P. Bolgiano, Jr.

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BALLISTIC RESEARCH LABORATORIES



ABERDEEN PROVING GROUND, MARYLAND

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Ballistic Measurements Laboratory

Department of the Army Project No. 503-05-023
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Aberdeen Proving Ground, Maryland
September 1961

RELATIONSHIP BETWEEN ADDITIVE AND NON-ADDITIVE QUANTUM FLUCTUATIONS

ABSTRACT

The limitation imposed by quantum mechanics on the measurement of electromagnetic signals may be viewed as a non-classical noise. Theory leads both to a minimum noise, which is additive in the sense of being independent of signal strength, and to a non-additive noise dependent on signal strength. The relationship between these two types of noise is discussed. Also a short derivation is given which exhibits their common relationship to the zero-point fluctuation attributed by quantum theory to the electromagnetic field.

In discussions of quantum noise limitations in masers^(1,2,3,4), there appear at first sight to be two separate types of quantum noise predicted. Spontaneous emission noise, or equivalently zero-point fluctuation of the field, leads to a minimum noise when no signal is present. On the other hand, the statistical nature of the amplification process which does not correlate specific output photons to specific input photons gives rise to an uncertainty in the precision with which the mean number of incident quanta, N , can be measured equal to the $\sqrt{N}$ (for large N).

That some relationship should exist between these fluctuations is suggested by Heisenberg's⁽⁵⁾ derivation of the Einstein formula⁽⁶⁾ for fluctuations in black-body radiation. In this calculation the mean square "particle" fluctuation, equal to the number of quanta involved, results directly from the zero-point energy of the field oscillators. Serber and Townes⁽⁷⁾, in fact, point out that a $\sqrt{N}$ fluctuation in signal energy can be obtained by assuming that the signal amplitude is augmented by a random fluctuation of amount corresponding to the in-phase component of a noise signal whose energy corresponds to the zero-point fluctuation.

We wish here to point out that, using a similar argument, it is possible to obtain a single formula which gives both the $\sqrt{N}$ fluctuation expected for large N and also the minimum fluctuation $1/2 hf$ per degree of freedom expected for $N = 0$.

We begin by considering the description given by classical wave theory for a sine wave plus thermal noise. That is we assume a waveform representable by an equation of the form

$$I = A_s \cos \omega t + A_N(t) \cos (\omega t + \phi(t)) \quad (1)$$

where the signal amplitude A_s is constant and the random amplitude $A_N(t)$ and phase $\phi(t)$ of the noise vary slowly relative to ω . If this expression is squared to compute power, there result three terms corresponding to signal power, noise power, and beats between the two. It may seem strange to speak of beats when only one frequency appears

explicitly in equation (1). However, the slow variation of $A_N(t)$ and $\phi(t)$ could be equivalently represented by writing the noise as a Fourier expansion over a frequency band of width equal to the highest frequency contained in $A_N(t)$ and $\phi(t)$.

If we consider only time variations slow with respect to ω , the signal power term is steady and the noise power term fluctuates with a mean square fluctuation equal to the mean square noise power itself. The interaction term, which averages zero, alternately adds and subtracts from the total power contributing a mean square fluctuation proportional to the product of signal and noise power. The total mean square fluctuation in power which results may be expressed⁽⁸⁾ by

$$\overline{\Delta P^2} = \overline{P_N^2} + 2P_S \overline{P_N} \quad (2)$$

where P_S denotes signal power and $\overline{P_N}$ mean noise power.

In order to obtain an estimate of quantum fluctuations, we shall interpret this power fluctuation as an energy fluctuation per unit time at frequency f . We express the signal energy per unit time as Nhf , and take for mean noise power and mean square noise power $1/2 hf$ and $(1/2 hf)^2$ respectively, to correspond to the zero-point fluctuation. This gives for the mean square energy fluctuation

$$\begin{aligned} \overline{\Delta W^2} &= (1/2 hf)^2 + 2(Nhf) (1/2 hf) \\ &= (N + 1/4) (hf)^2, \end{aligned} \quad (3)$$

and for the root mean square fluctuation in energy

$$\sqrt{\overline{\Delta W^2}} = hf \sqrt{N + 1/4} \quad (4)$$

For large N this formula gives the non-additive $\sqrt{N}$ fluctuation normally associated with Poisson's "law of rare events". For $N = 0$, it also gives the minimum zero-point fluctuation $1/2 hf$.

The significance of the non-additive $\sqrt{N}$ fluctuation for microwave radiometry was discussed in a previous paper⁽⁹⁾. Since only the non-additive quantum fluctuation was considered, it was necessary to qualify the results as applying only if N is not too small. The derivation given here suggests that this restriction might be removed by replacing N by $N + 1/4$ in the formula found for the mean square energy fluctuation.

L. P. Bolgiano Jr.
L. P. BOLGIANO, JR.

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