GCA TECHNICAL REPORT
60-4-G

(A) CHARGED PARTICLES IN THE UPPER ATMOSPHERE

(B) RECOMBINATION COEFFICIENTS

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JANUARY 1961

JUNE 1961 (REVISED)

GEOPHYSICS CORPORATION OF AMERICA
BEDFORD, MASSACHUSETTS

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A. CHARGED PARTICLES IN THE UPPER ATMOSPHERE

1 ELASTIC COLLISIONS OF SLOW ELECTRONS WITH ATMOSPHERIC PARTICLES

1.1 Molecular Nitrogen

The measurements of Crompton and Sutton (1952) of the Townsend energy coefficient relating to the diffusion of slow electrons in molecular nitrogen taken together with the measurements of the electron drift velocity by Nielsen (1936)* can be used to derive an averaged momentum transfer cross section $\bar{Q}$. Crompton and Sutton deduce that Q decreases as the electron velocity decreases but their observations are restricted to temperatures above about 750°K. The observed decrease is in accord with the microwave conductivity measurements of Phelps, Fundingsland and Brown (1951) which yield a value of $4 \times 10^{-16} \mathrm{cm}^2$ at an electron energy of about 0.04 eV. It is in disagreement with the results of microwave conductivity measurements by Anderson and Goldstein (1956) who deduce that Q increases with decreasing velocity to a value of $17 \times 10^{-16} \mathrm{cm}^2$ at an energy of 0.04 eV.

An explanation of the discrepancy has been suggested by Formato and Gilardini (1960) who believe that in the experiments of Anderson and Goldstein the initial plasma electron temperature is much higher than room temperature.

Further support for a cross section decreasing with electron velocity is provided by recent measurements by Crompton and Hall (1960) of the diffusion coefficient for electrons with energies down to 0.05 eV and by Phelps and Pack (1959) of electron mobilities for electrons with

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*The electron drift velocity measurements of Nielson have been repeated with greater precision by Bowe (1960).

energies in the range from 0.005 eV to 0.05 eV. Crompton and Hall observe that the diffusion coefficient D is a constant. Now for a Lorentzian gas,

$$D = \frac{1}{3n}\left(\frac{\bar{v}}{Q}\right) \qquad \text{(A1)}$$

where n is the gas number density and v is the electron velocity. It follows from the constancy of D that Q is directly proportional to v and Huxley (1959) has derived the result

$$Q = 3.29 \times 10^{-23}\, v \quad \text{cm}^2 \qquad \text{(A2)}$$

Defining a collision frequency according to

$$\nu = v Q n \quad \text{sec}^{-1} \qquad \text{(A3)}$$

(A2) corresponds to

$$\nu = 1.16 \times 10^{-7}\, u\, n \quad \text{sec}^{-1} \qquad \text{(A4)}$$

where u is the electron energy measured in electron volts. From their mobility measurements, Phelps and Pack (1959) have derived the result

$$\nu = \left(2.5 \times 10^{-9}\, u^{1/2} + 1.2 \times 10^{-7}\, u\right) n \quad \text{sec}^{-1} \qquad \text{(A5)}$$

which for electron temperatures between 150°K and 290°K may be replaced by the simpler expression (Phelps 1960)

$$\nu = 1.33 \times 10^{-7}\, u\, n \quad \text{sec}^{-1} \qquad \text{(A6)}$$

Pack and Phelps (1961) have obtained values for the momentum transfer cross section of electrons in N_2 which are about 10% lower than those they reported previously. They now give

$$Q = (5.31 \times 10^{14}\, u^{-1/2} - 3.15 \times 10''\, u^{-3/2})^{-1}\ \mathrm{cm}^2$$

Their drift velocity measurements agree with those made by Nielsen (1936), Klema and Allen (1950) and Colli and Facchini (1952) but are higher than those made by Bortner, Hurst and Stone (1957) and Bowe (1960).

1.2 Molecular Oxygen

Shkarofsky, Bachyuski and Johnston (1959, unpublished) have analysed the measurements of mobility and of the Townsend energy coefficient in molecular oxygen for electrons with average energies above the gas temperature*. According to Phelps (1960), their derived collision cross sections are consistent with a collision frequency

$$\nu = (4 \times 10^{-9} \mu^{1/2} + 5 \times 10^{-8} \mu) n \quad \sec^{-1} \qquad \text{(A7)}$$

for $0.03 < \mu < 0.2$ in eV. Below 0.1 eV, (A7) is represented to an accuracy of ± 10% by

$$\nu = 7 \times 10^{-8} \mu \, n \quad \sec^{-1} . \qquad \text{(A8)}$$

Phelps (1960) has analysed the data of Van Lint (1959, unpublished) on the microwave conductivity of O_2 and shows that if ν is directly proportional to the electron energy μ

$$\nu = 9 \times 10^{-8} \mu \, n \quad \sec^{-1} \qquad \text{(A9)}$$

A compromise between (A8) and (A9) is

$$\nu = 8 \times 10^{-8} \mu \, n \quad \sec^{-1} \qquad \text{(A10)}$$

Van Lint, Hammond and Parez (1961) have reported a value of $1.35 \times 10^{8} p$ $\sec^{-1}$ for the collision frequency of thermal electrons in O_2.

1.3 Air

From (A6) and (A10), it follows that for air with nitrogen

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*Further data have been obtained recently by Huxley, Crompton and Bagot (1959).

and oxygen in the ratio of 4:1

$$\nu = 1.22 \times 10^{-7} \mu\, n \qquad \text{sec}^{-1} \tag{A11}$$

where n is here the number density of air molecules. Averaging (A11) over a Maxwellian distribution corresponding to a temperature T gives

$$\bar{\nu} = 1.5 \times 10^{8}\, p \qquad \text{sec}^{-1} \tag{A12}$$

where p is the pressure measured in mm Hg. Expression (A12) may be compared with the value used by Huxley (1959) of

$$\bar{\nu} = 1.2 \times 10^{8}\, p \qquad \text{sec}^{-1} \tag{A13}$$

1.4 Water Vapour

Since a molecule of water vapour has a permanent and large dipole moment, the collision frequency of electrons with water vapour is relatively large and Phelps (1960) has drawn attention to its possible significance in the atmosphere. Altshuler (1957) has derived the formula

$$\nu = 10^{-7} \mu^{-1/2}\, n \qquad \text{sec}^{-1} \tag{A14}$$

which is in fair agreement with experimental measurements (Takeda and Douglal 1960). At 300°K, (A14) is about two hundred times larger than (A11) for equivalent concentrations. Thus for air saturated with water vapour the contributions of (A11) and (A14) to ν will be of the same order.

1.5 Atmospheric Collision Frequencies

Formula (A11) or (A12) should be appropriate up to altitudes of 85 km, Above 85 km, account must be taken of the dissociation of molecular oxygen.

Expressions for the collision frequencies in the atmosphere have been given by Cowling (1945), Nicolet (1953, 1959) and Chapman (1956). These calculations refer to the collision frequency which controls the electrical conductivity of the atmosphere, defined for a weakly ionized gas according to

$$\bar{\nu}_e = \underline{k}T \Big/ m D_e \qquad \text{(A15)}$$

where k is Boltzmann's constant, m is the electron mass and De is the electron diffusion coefficient. This definition is in general not equivalent to the Maxwellian average of (A3). However, using (A1) it may be shown that if Q is directly proportional to v (as seems to be a good approximation) then the two definitions yield identical results.

Experimental measurements of effective collision frequencies in the atmosphere have been made using a variety of techniques. According to Ratcliffe and Weekes (1960), the measurements provide reasonable accuracy for the collision frequency but not always for the height at which the collision frequency is measured. The precise meaning of the derived effective collision frequencies is however obscure and, indeed, may differ from one experiment to another.

The derivations are based upon the Appleton-Hartree magnetoionic theory (cf. Ratcliffe 1959) which incorporates an effective velocity-independent collision frequency. More sophisticated treatments of the propagation of an electromagnetic wave in a weakly ionized gas have been given by Huxley (1937, 1938, 1940), Pfister (1955), Jancel and Kahan(1955), Molmud (1959), Sen and Wyller (1960) and Phelps (1960)*. The study of Phelps is of particular importance and it includes graphs and formulas of direct relevance to radio observations of the D and E layers. The main conclusion is that the concept of an effective collision frequency cannot be employed except under certain limiting conditions. Depending upon the limiting conditions, the derived collision frequencies will normally be within a factor of two of that defined by (A3) and hence that given by (A12).

The safest procedure is to avoid the concept of an effective collision frequency and to compute directly from (A11) the measured quantity. Kane (1959) has recently carried out some radio propagation measurements during polar blackouts. He expresses his data in terms of a function Q defined by

$$Q = \frac{K_x - K_o}{\left(\frac{2\pi}{\lambda} - \mu_x\right) + 1.4\left(\frac{2\pi}{\lambda} - \mu_o\right)} \qquad \text{(A16)}$$

where λ is the wavelength in free space and $\mu - iK$ is the refractive index, the subscript o denoting the ordinary wave and the subscript x the extraordinary wave. Using (A11) (actually using $\nu = 1.25 \times 10^{-7}$ μn sec^{-1}) and also its modification caused by taking the more accurate

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*These more sophisticated treatments are not entirely satisfactory, however, in that they do not take the spatial variations of the wave fully into account.

formula (A5) in place of (A6), Phelps (1960) has computed Q for Kane's experimental conditions as a function of pressure.

Values of Q corresponding to the two pressure distributions given by Nicolet (1960), one of which is consistent with a temperature of 190°K at the mesopause and the other with a temperature of 130°K, are plotted in Figure 1 . Kane's observational data, which refer to two rocket flights, one in November, 1956 and the other in July, 1957, are also shown in the figure. In view of the possible variations in atmospheric pressure which occur in polar regions (Jones, Fischbach and Peterson 1958), the agreement is satisfactory.

1.6 Atomic Oxygen

For altitudes above 85 km the influence of atomic oxygen on the atmospheric collision frequencies must be considered.

There have been several theoretical studies of the scattering of slow electrons by atomic oxygen (Mitra, Ray and Ghosh 1940, Bates and Massey 1947, Yamanouchi 1947, Robinson 1957, Hammerling, Shine and Kivel 1957, Temkin 1957, Klein and Brueckner 1958, Lin and Kivel 1959) and some experimental studies (Lamb and Lin 1957). Both Klein and Brueckner (1958) who obtain a cross section $Q = 9 \times 10^{-16} cm^2$ and Lin and Kivel (1959) who obtain a cross section $Q = 2 \times 10^{-16} cm^2$ claim that their results are in conformity with the measurements of the photodetachment rates of O^- (Branscomb, Burch, Smith and Geltman 1958, Smith 1960). The large uncertainty arises because the scattering of electrons by atomic oxygen occurs in doublet and quartet spin states, the photo-

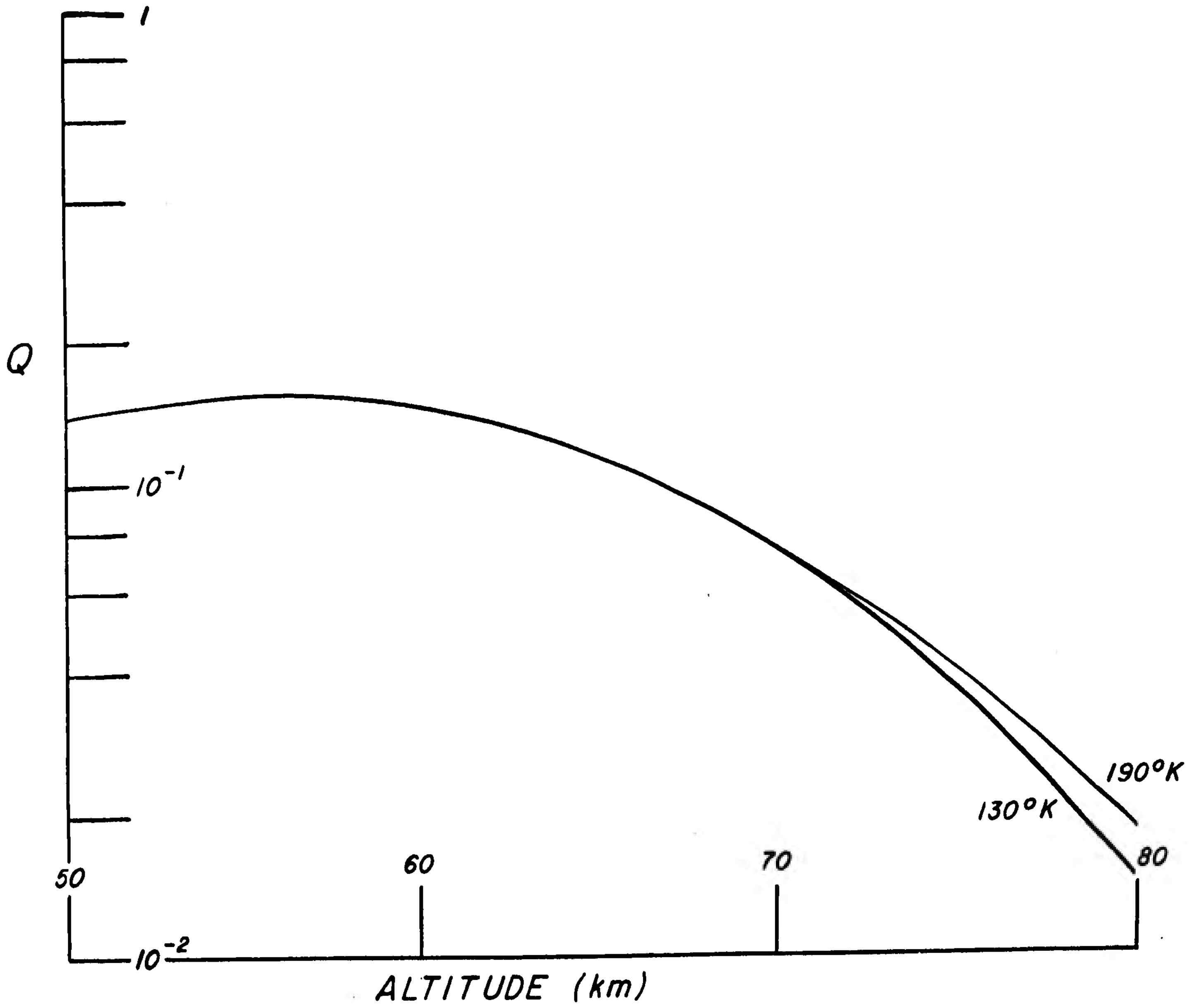

Figure 1. Values of Kane's Function Q

detachment rates providing a check on the doublet contribution only. Because of the uncertainty it is not possible to decide whether the dissociation of O_2 will cause an increase or a decrease in the collision frequency. In the circumstances we shall assume that (A11) and (A12) are valid in the E region.

Measurements of collision frequency in the E-region have been carried out recently by Schlapp (1959) by comparing the change in the reflection coefficient ρ and the changes in group path (P') and phase path (P) resulting from a small change in the frequency of a pulsed ionospheric sounding apparatus. The method is based upon the formula (Appleton 1935)

$$\Delta \ln \rho = \frac{\bar{\nu}}{2c}\left(\Delta P' - \Delta P\right) \qquad \text{(A17)}$$

Although this formula is not uniformly valid, Schlapp's experiments are such that one of the limiting conditions is satisfied which allows the use of an effective collision frequency. From one of the graphs given by Phelps (1960) it appears that (A12) must be multiplied by 5/3 to give

$$\bar{\nu} = 2.5 \times 10^{8}\, p \qquad \mathrm{Sec}^{-1} \qquad \text{(A18)}$$

in order to compare with the values derived by Schlapp from (A17). The pressures in the E region are uncertain. Adopting the values tabulated by Kallman (1959), (A18) yields Figure 2 which is in satisfactory agreement with Schlapp's experimental data. The value of $1.9 \times 10^{-4}\,\mathrm{sec}^{-1}$ at (110 ± 5)km obtained by Whitehead (1959) is also in harmony with the theoretical curve (Figure 2).

The cross section for the elastic scattering of electrons by atomic oxygen has been measured by Neynaber, Marino, Rothe and Trujillo (1961). For energies between 2.3 and 11.6 eV, the cross section is constant and equal to 5 x 10^{-16} cm^2. The behavior below 2 eV is still uncertain but the evidence appears to favor a value of about 2 x 10^{-16} cm^2, as calculated by Bates and Massey (1947).

1 7 Positive Ions

The possibility that some contribution to $\bar{\nu}$ comes from collisions between electrons and positive ions must be considered, especially since Schlapp (1959) has postulated that $\bar{\nu}$ at a fixed height may increase with increasing sunspot number. The relationship between the effective and the actual collision frequency of electrons and positive ions requires further investigation but no serious error should result in adopting the definition (A15). According to it,

$$\bar{\nu} = \left[34 + 4.18 \log\left(\frac{T^3}{n_e}\right)\right] T^{-3/2} n_i \qquad \text{(A19)}$$

where n_i is the number density of positive ions (Nicolet 1953). It is clear that n_i and n_e must be very nearly equal. Formula (A19) is based upon the Debye cut-off and has received experimental support (Anderson and Goldstein 1955). The formulation of transport phenomena in a fully ionized gas has been improved recently (cf. Ecker and Muller 1959, Liboff 1959, Theimer and Hoffman 1959, Sodha and Varshni 1960) but the modifications are not (yet) significant for upper atmosphere applications.

Tables of (A19) have been prepared by Chapman (1956) and by Nicolet (1959). From these it follows that with T = 200°K and n_e = $10^5 cm^{-3}$, the contribution to $\bar{\nu}$ is 2 x $10^3 sec^{-1}$ and with T = 400°K and n_e = $10^5 cm^{-3}$, the contribution to $\bar{\nu}$ is 6 x $10^2 sec^{-1}$. Collisions between electrons and positive ions are unlikely therefore to be signi-

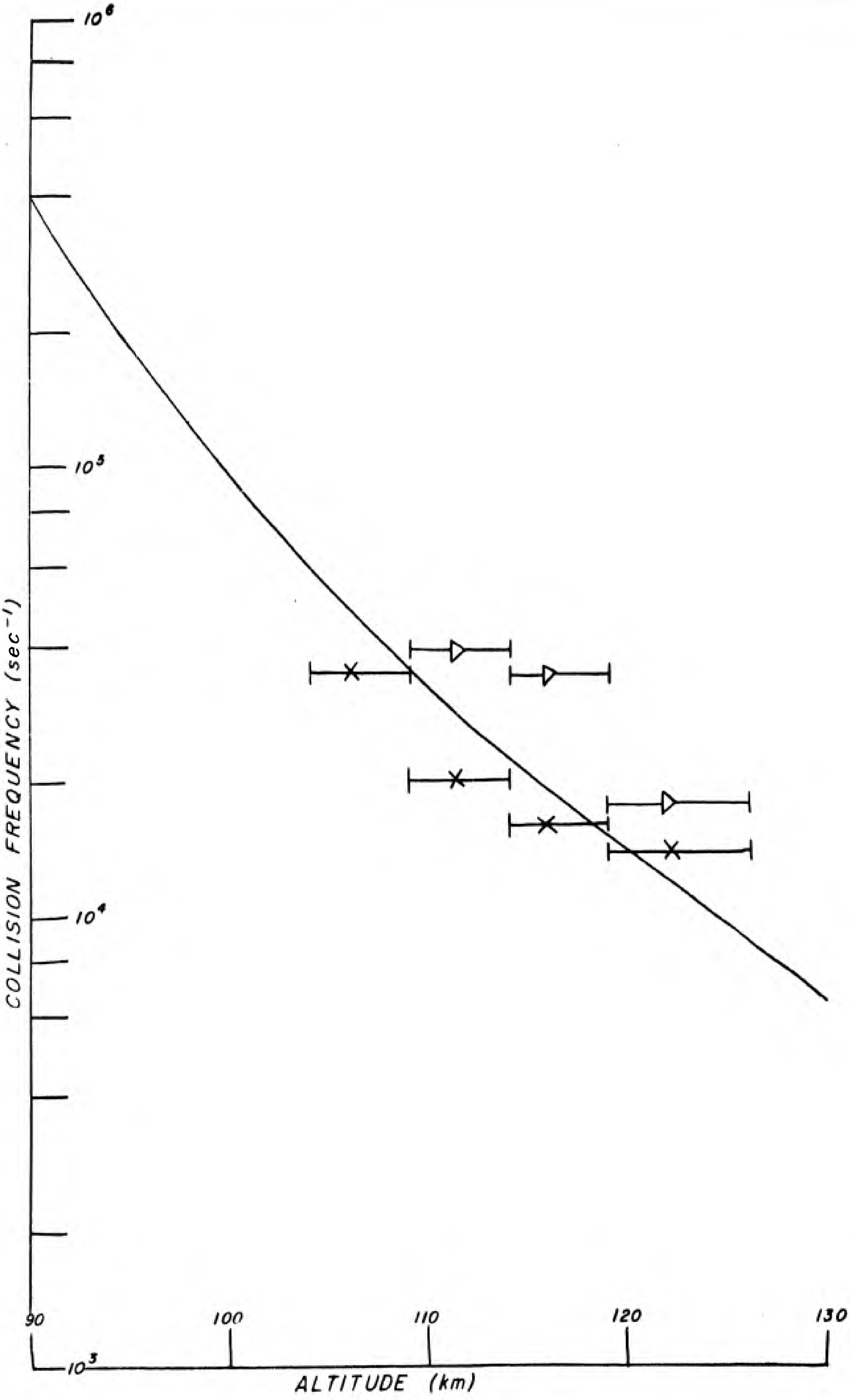

Figure 2. Plot of Collisional Frequency Versus Altitude

ficant in the E region.

The position is different at greater altitudes. As Cowling (1945) and Nicolet (1953) have pointed out, electron-ion collisions are of the same order of importance as electron-neutral particle collisions near the F1 ledge and play a dominant role in the F2 layer.

Several measurements of collision frequency in the F region have been reported but, it seems that the variability in amplitude produced by focussing effects causes the results to be of little value (Ratcliffe and Weekes 1960). According to Schlapp (1960), the only valid assertion is that $\bar{\nu}$ is less than $5 \times 10^{3}\,\mathrm{sec}^{-1}$. This bound is too large to give a useful temperature. The observation by Whitehead (1959) that the collision frequency is substantially constant from 150 km to 200 km is at least in qualitative agreement with an increasing electron density and an increasing ambient temperature.

1.8 The Electrical Conductivity of the Atmosphere

Chapman (1956) has reviewed the state of knowledge about the electrical conductivity of the atmosphere. His numerical data can be improved by using the electron collision frequencies given by (A12) and (A19), and the ion collision frequencies given in section 3.

2. ENERGY LOSSES OF SLOW ELECTRONS

The rate at which slow electrons of mean energy $\bar{E}$ lose energy when moving in a gas of mean thermal energy Eg is an important parameter in theories of radio wave interaction (cf. Huxley 1952). The rate may be written as the sum of a part due to elastic collisions and a part due to inelastic collisions. The average rate for the elastic energy loss of an electron of energy E is given by

$$\left(\frac{dE}{dt}\right)_{el} = \frac{2m}{M}\, h\, Y_e \left(E - \frac{4}{3} E_g\right) \qquad \text{(A20)}$$

where M is the mass of a molecule of the gas. Provided E is not so large that vibrational excitation is possible, the average rate for the inelastic energy loss is given by

$$\left(\frac{dE}{dt}\right)_{in} = V \sum_j n_j\, \sigma(j, j')\, \Delta \varepsilon_{jj'} \qquad \text{(A21)}$$

where n_j is the number density of the molecules in the jth rotational level, σ (j, j') is the cross section for the j-j' transition and $\Delta \varepsilon$ jj' is the associated energy change.

Theories of radio wave interaction involve the quantity G which relates the mean rate of energy loss $G\nu$ to the mean electron energy according to

$$\overline{\frac{dE}{dt}} = G\nu\left(\bar{E} - E_g\right) \qquad \text{(A22)}$$

When only elastic collisions can occur,

$$G = 8m / 3M \tag{A23}$$

When inelastic collisions can also occur, it is often assumed that G is still a constant but this is not correct, in general

Much of the experimental data refer to the mean fractional energy loss for collision λ which is given by

$$\lambda = \frac{1}{h\nu E} \frac{dE}{dt} \tag{A24}$$

If the gas is atomic and $E \gg E_g$, then λ is $2\,m/M$.

Measurements of λ for N_2 have been reported by Crompton and Sutton (1952) and by Anderson and Goldstein (1956), and their results are shown in Figure 3

A theoretical analysis of rotational excitation of N_2 by Gerjuoy and Stein (1955) suggests that in the low energy limit the cross sections $\sigma(j, j')$ are proportional to the square of the quadrupole moment Q of the molecule. Adopting a value of 0.27×10^{-16} cm^2 for $|Q|$ and a constant momentum transfer cross section of 4.2×10^{-16} cm^2, Gerjuoy and Stein have computed λ as a function of electron energy. We have repeated their calculations using the momentum transfer cross section corresponding to (A16) and the resulting values of λ are compared in Figure 3 with the experimental data, which include vibrational excitation, the threshold of which is 0.29 eV. Part of the discrepancy may be attributed to the uncertainty in the value of λ, but further

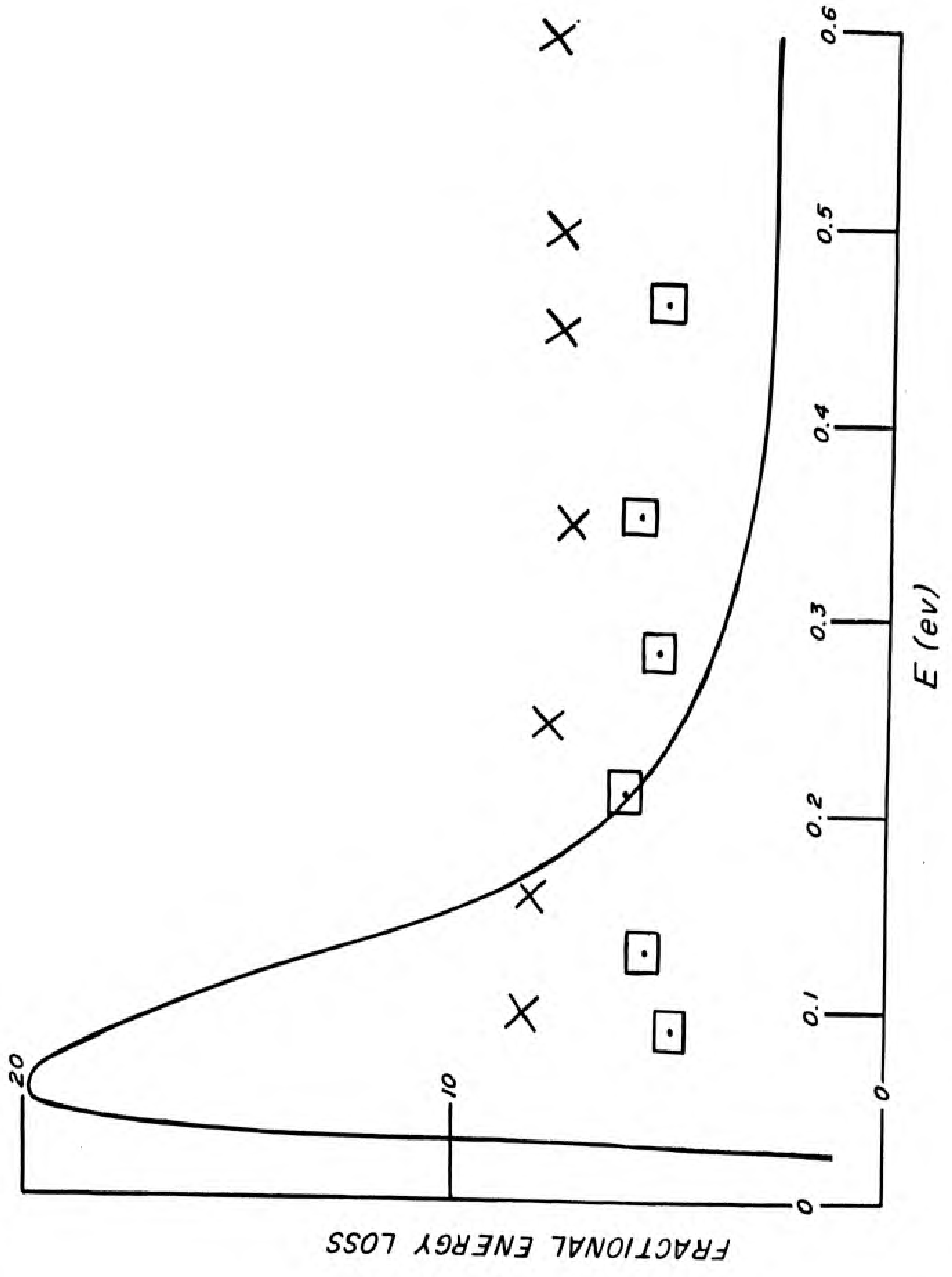

Figure 3. Mean Fractional Energy Loss in N_2

experimental and theoretical work at low energies is desirable.

Measurements of λ have been made by Crompton, Huxley and Sutton (1953) for air and by Crompton and Sutton (cf. Huxley 1956) for molecular oxygen at electron temperatures in excess of 1000°K. The data have been discussed by Huxley (1959) who remarks that they are consistent with the assumption that the rate of energy loss in O_2 is equal to that in N_2 at <u>all</u> electron velocities.

This assumption is not consistent with theoretical predictions. Although the theory of Gerjuoy and Stein (1955) is not strictly applicable to rotational excitation of O_2, it should suffice for a rough estimate. According to Smith and Howard (1950), the value of $\mathcal{Q}$ for O_2 is less than $0.09 \times 10^{-16} \text{cm}^2$ so that rotational loss in O_2 is probably less than one tenth as efficient as in N_2.

From the data on N_2, Huxley (1959) has derived the formula

$$G\nu = n\left[\frac{1.08 \times 10^{-7}}{T^{3/2}} \exp\left(-213/T\right) + 7.82 \times 10^{-16}\, T\right] \qquad \text{(A25)}$$

and has applied it to radio wave interaction assuming that n is the number density of air molecules. It may be more accurate to take n as the number density of N_2 molecules. Formula (A25) is then appropriate at all altitudes below the level at which the collision frequency due to the scattering of electrons by positive ions becomes dominant compared to that for scattering by neutral atoms. Values of $G\nu/n$ are given for various ambient temperatures in table A1; values at 288°K and at 196°K have been given previously (Huxley 1959).

TABLE A1

Values of $(G\gamma/n)$ and G for Various Temperatures

T°K	130	150	170	190	210	240	270	300
$\frac{G\gamma}{n} \times 10^{11}$	1.42	1.42	1.39	1.35	1.29	1.20	1.11	1.02
$G \times 10^{3}$	9.9	8.6	7.4	6.5	5.6	4.5	3.7	3.1

Measurements of radio wave interaction lead to values of $G\nu$ between 1×10^3 and 3×10^3 and the heights of the seat of cross-modulation lie between 82 km and 90 km (Huxley 1952, 1959). From table A1, these measurements imply a number density of N_2 molecules near 10^{14} cm^{-3} in agreement with rocket density data.

For some applications, a knowledge of the quantity G may be useful and some values are therefore included in the table. The measurements of Fejev and Vice (1959) on the D region suggest that $G = 0.01$.

3. ELASTIC COLLISIONS OF POSITIVE IONS WITH ATMOSPHERIC PARTICLES

Most of the data on cross sections describing the elastic collisions of positive ions with gas particles are provided by measurements of the ion mobilities, a subject which has been reviewed recently by Dalgarno, McDowell and Williams (1958). The mobility K of an ion in a gas is related to the diffusion coefficient D_i according to

$$K = \frac{e D_i}{k T} \tag{A26}$$

and the collision frequency of ions with neutral atoms is given by (Cowling 1945)

$$\nu_i = \frac{kT}{M_i D_i} = \frac{e}{M_i K} \tag{A27}$$

where M_i is the mass of the ion and it is assumed that the number density of ions is small compared to that of neutral particles. The mobility K is inversely proportional to the gas number density and the reported experimental values are usually referred to standard temperature and pressure in units of cm^2 volt^{-1}sec^{-1}. Denoting these values by K_s and measuring mass in units of the proton mass, values of D_i and ν_i appropriate to a number density n cm^{-3} are given by

$$D_i = \frac{2.3 \times 10^{15}}{n} K_s T \quad \text{cm}^2 \ \text{sec}^{-1} \tag{A28}$$

and

$$\nu_i = \frac{3.5 \times 10^{-8}\, n}{M_i K_s} \quad \text{sec}^{-1} \tag{A29}$$

At low temperature the mobility of an ion moving in an atomic gas is given by

$$K_s = \frac{35.9}{(\alpha\mu)^{1/2}} \quad \mathrm{cm^2\ volt^{-1}\ sec^{-1}} \tag{A30}$$

where α is the polarizability of the gas in atomic units a_o^3 (=1.48 x 10^{-25}cm^3) and μ is the reduced mass in units of the proton mass:

$$\mu = \left(\frac{M_i M_j}{M_i + M_j}\right) \tag{A31}$$

M_j being the mass of the gas atom. Except at very low temperatures, (A30) is valid for a molecular gas with α taken as the spherical average of the parallel and perpendicular polarizabilities $\alpha_{\parallel}$ and $\alpha_{\perp}$, respectively:

$$\alpha = \frac{1}{3}\left(\alpha_{\parallel} + 2\alpha_{\perp}\right) \tag{A32}$$

For molecular nitrogen, $\alpha_{\parallel} = 16.1\ a_o^3$ and $\alpha_{\perp} = 9.8\ a_o^3$ and hence

$$K_s(N_2) = \frac{10.4}{\mu^{1/2}} \quad \mathrm{cm^2\ volt^{-1}\ sec^{-1}} \tag{A33}$$

A comparison of (A33) with the values measured at 291°K (Mitchell and Ridler 1934, Tyndall 1938), reduced to standard temperature and pressure, is given in table A2.

For molecular oxygen, $\alpha_{\parallel} = 15.9\ a_o^3$ and $\alpha_{\perp} = 12.1\ a_o^3$ and hence

TABLE A2

Mobilities K_s in N_2 at 291°K

Ion	Li	Na	K	Rb	Cs	Al	Ga	Kr	In	Xe	Ba	H	Tl
Measured	3.95	2.85	2.53	2.24	2.20	2.72	2.28	2.26	2.15	2.09	2.09	2.03	2.04
Equation (27)	4.41	2.92	2.57	2.26	2.21	2.81	2.33	2.27	2.19	2.17	2.16	2.10	2.10

$$K_s(O_2) = \frac{10.9}{\mu^{1/2}} \quad \mathrm{cm^2\ Volt^{-1}\ sec^{-1}} \tag{A34}$$

while for atomic oxygen, $\alpha = 5.2\,a_0^3$ (Alpher and White 1959) and hence

$$K_s(O) = \frac{15.8}{\mu^{1/2}} \quad \mathrm{cm^2\ Volt^{-1}\ sec^{-1}} \tag{A35}$$

The mobility of an ion in a mixture of gases is given in terms of the individual mobilities by Blanc's law

$$K_s = \left(\sum_i f_i \big/ K_s(i) \right)^{-1} \tag{A36}$$

where f_i is the fraction of the ith component. To a good approximation we may write for an air mixture consisting of N_2 and O_2 in the ratio 4:1

$$D_i = \frac{2.4 \times 10^{16}}{n} \left(\frac{1 + M_i/29}{M_i} \right)^{1/2} T \quad \mathrm{cm^2\ Sec^{-1}} \tag{A37}$$

$$\nu_i = \frac{3.3 \times 10^{-9}}{\left[M_i \left(1 + M_i/29 \right) \right]^{1/2}}\, n \quad \mathrm{sec^{-1}} \tag{A38}$$

Values of (A37) and (A38) for H^+, He^+, N^+, O^+ and NO^+ are given in table A3. The cases of O_2^+ and N_2^+ will be considered in the following section.

For an air mixture consisting of atomic oxygen only, the formulae

TABLE A3

Diffusion Coefficients and Collision Frequencies of Positive Ions in Air at T < 300°K

Ion	$(D_i\ n/T)$	(ν_i/n)
H^+	2.5×10^{16}	3.2×10^{-9}
He^+	1.3×10^{16}	1.5×10^{-9}
N^+	7.8×10^{15}	7.2×10^{-10}
O^+	7.8×10^{15}	6.6×10^{-10}
NO^+	5.3×10^{15}	4.2×10^{-10}

analogous to (A37) and (A38) are

$$D_i = \frac{3.6 \times 10^{16}}{n} \left(\frac{1 + M_i/16}{M_i} \right)^{1/2} T \quad \text{cm}^2 \ \text{sec}^{-1} \tag{A39}$$

and

$$\gamma_i = \frac{2.7 \times 10^{-4}}{\left[M_i \left(1 + M_i/16 \right) \right]^{1/2}} \tag{A40}$$

Values of (A39) and (A40) for H^+, He^+, N^+, N_2^+, NO^+ and O_2^+ are given in table A4. The case of O^+ will be considered in the following section.

It is apparent from tables A3 and A4 that the dissociation of molecular oxygen will cause an increase in D_i and a decrease in γ_i but the changes are not large. The ion O^+ is an important exception to this statement.

The results are accurate for ambient temperatures up to about 250°K or 300°K. For higher temperatures, the variation of Ks $\mu^{1/2}$ with T depends upon the specific nature of the ion and of the air constituents. In the absence of experimental data appropriate to a particular case only a rough guide can be given and a typical variation can be illustrated, the variations of D_i and γ_i being roughly as $T^{\frac{1}{2}}$ for large T. See Figure 4.

Some information on diffusion in the upper atmosphere is available from meteor studies (Greennow and Neufeld 1955, Weiss 1955, Murray 1959). The diffusion observed is ambipolar throughout most of the height range. The corresponding coefficient Da is related to Di by

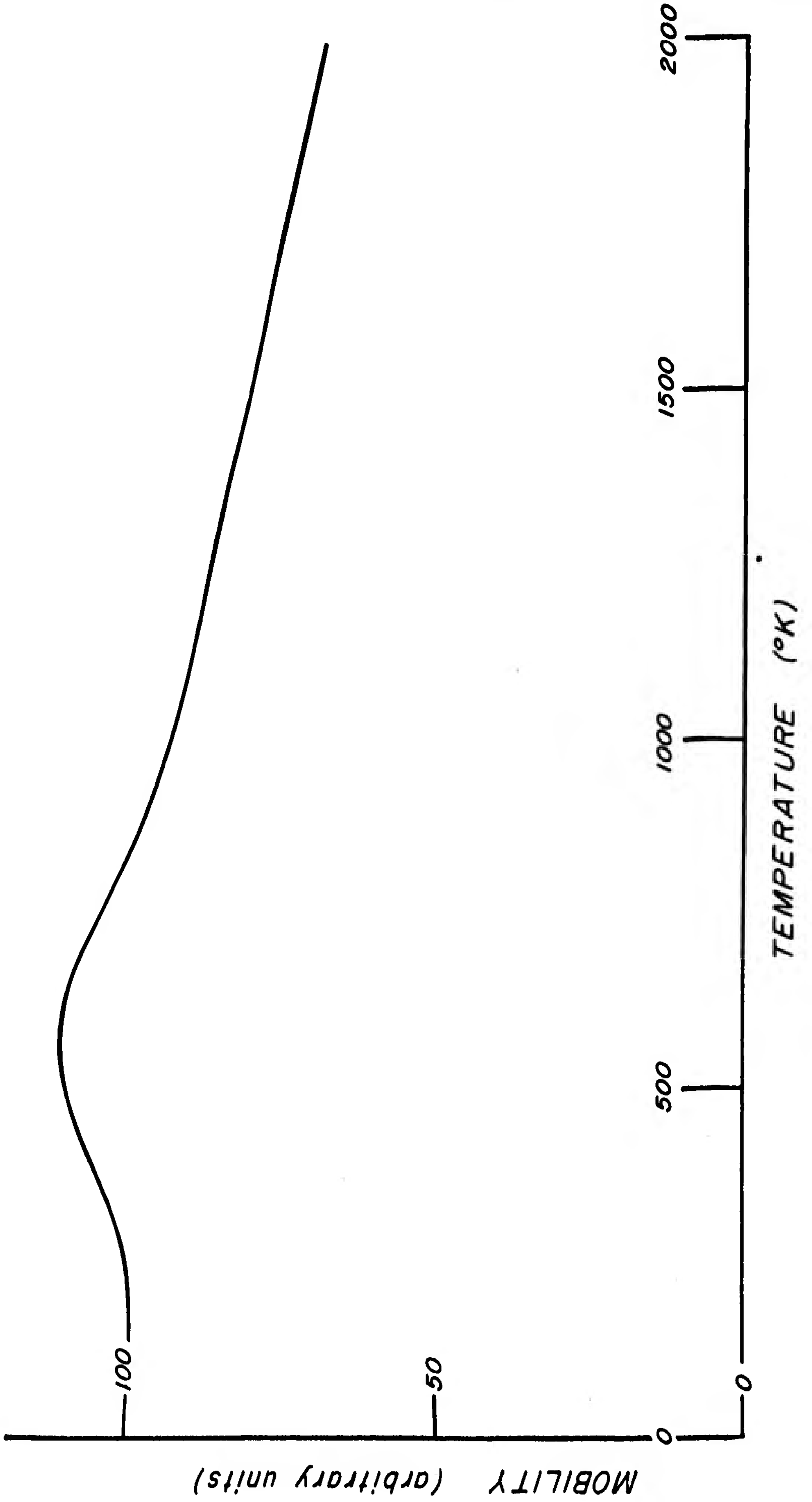

Figure 4. Mobility as a Function of Temperature

TABLE A4

Diffusion Coefficients and Collision Frequencies of Positive Ions in Atomic Oxygen at $T < 250^{\circ}K$

Ion	$(D_i\ n/T)$	(ν_i/n)
H^+	3.7×10^{16}	2.1×10^{-9}
He^+	2.0×10^{16}	9.8×10^{-10}
N^+	1.3×10^{16}	4.3×10^{-10}
N_2^+	1.1×10^{16}	2.5×10^{-10}
NO^+	1.1×10^{16}	2.4×10^{-10}
O_2^+	1.1×10^{16}	2.3×10^{-10}

$$D_a \sim D_i \left(1 + T_e / T_i \right) \tag{A41}$$

where Te and Ti are respectively the electron and ion temperatures.

If $T_e = T_i$,

$$D_a = 2 D_i \tag{A42}$$

It is clear from table A2 that for meteor ions, the variation with mass is small and we take as appropriate to the meteor observations

$$D_a = 1.04 \times 10^{16} \; T / n \tag{A43}$$

using $K_s = 2.25$ from Table A2 and substituting A42 into A28.

Values of (A43) corresponding to the two pressure and temperature distributions given by Nicolet (1960) are compared with the data (Figure 5) of Greenhow and Neufeld (1955), of Weiss (1955) and of Murray (1959). The agreement is not unsatisfactory in view of the uncertainties attending the derivation of Da from the observational data, which show considerable scatter (Murray 1959)*. No conclusions can be drawn about the atmospheric content of atomic oxygen, which has been ignored in deriving (A43).

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*Greenhow and Hall (1960) have recently reported further measurements of Da. Their results suggest the presence of a large diurnal variation in Da at a fixed height which they interpret as being due primarily to a diurnal variation of pressure, the fractional variation increasing with altitude.

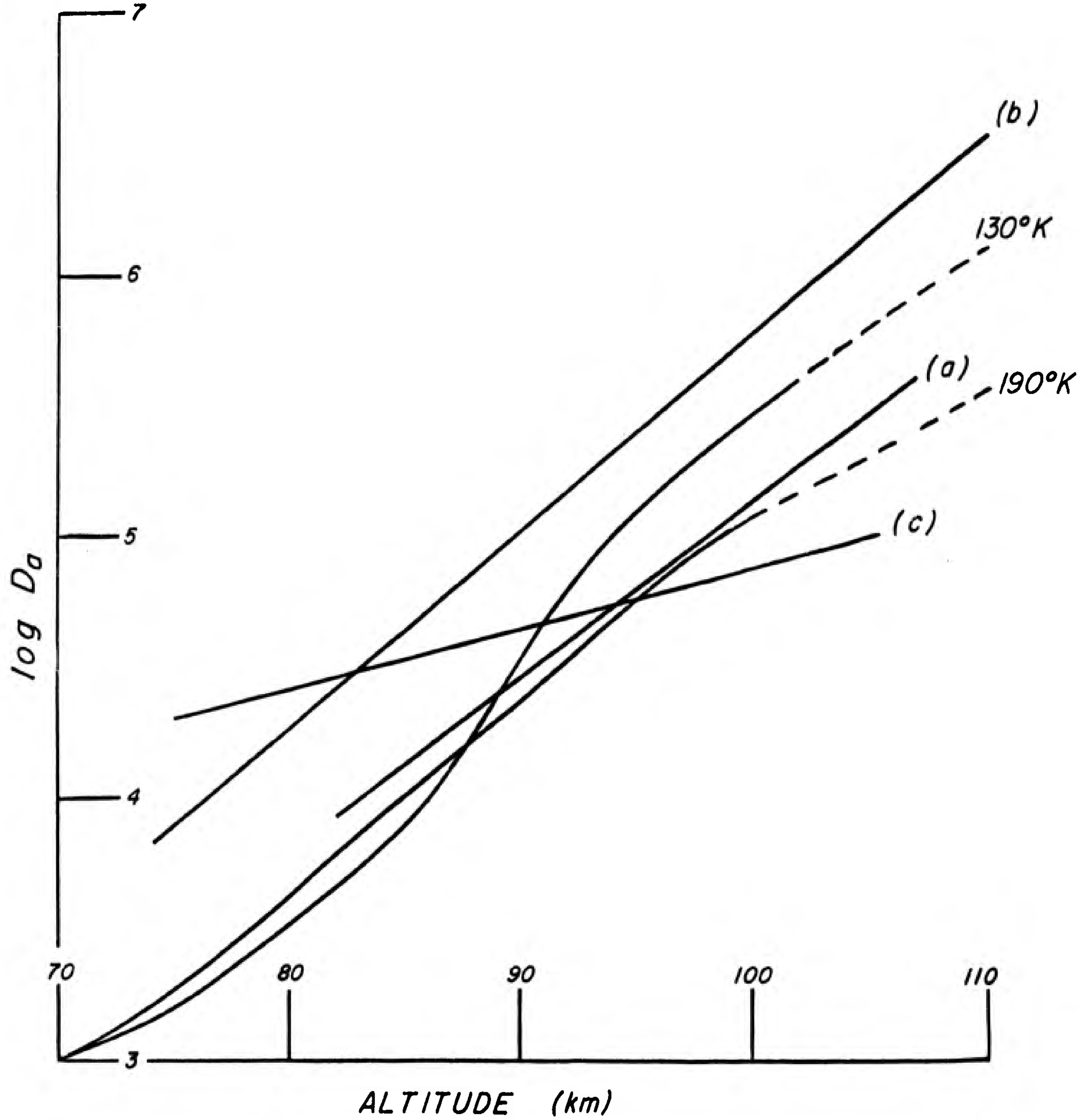

Figure 5. Ambipolar Diffusion

3.1 The Ions of O_2^+, N_2^+ and O^+

Formula (A30) is not appropriate to an ion moving in its parent gas because of quantal symmetry effects occurring in the collision of similar particles. Thus (A37) and (A38) do not apply to O_2^+ and N_2^+ and (A39) and (A40) do not apply to O^+. No general formula such as (A30) can be given for an ion moving in its parent gas and each case must be examined individually.

The available data have been reviewed by Dalgarno (1958a). Burch and Geballe (1957) claim that the mobility K_s of O_2^+ in O_2 at 290°K is 1.9 cm^2 volt^{-1} sec^{-1} [*] so that in air $D_i = 1.6 \times 10^{18}/n$ cm^2 sec^{-1} and $\gamma_i = 4.2 \times 10^{-10}$ n sec^{-1}. There are no measurements of K_s for N_2^+ in N_2, the values obtained at low field strengths for ions in N_2 probably referring to N_4^+ (Varney 1953). Faire and Champion (1959) have derived a value of Dap = (220 ± 30) cm^2 sec^{-1} mm Hg at T = 400°K for positive ions in N_2 and Brömer (1959) has derived a value of 124 cm^2 sec^{-1} mm Hg at 313°K. The former corresponds to K_s = 2.8 cm^2 volt^{-1} sec^{-1} and the latter to 2.6 cm^2 volt^{-1} sec^{-1}. These values seem high in view of the measurements in O_2. Assuming that the role of charge transfer for N_2^+ in N_2 is similar to that suggested by measurements on O_2^+ in O_2, Dalgarno has suggested that K_2 for N_2^+ in N_2 at 290°K may be as low as 1.5 cm^2 volt^{-1} sec^{-1}. The discrepancy raises the possibility that the measurements do not refer simply to N_2^+. Accepting Dalgarno's estimate, it follows that in air $D_i = 1.1 \times 10^{18}/n$ cm^2 sec^{-1} and γ_i =

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[*] Somewhat larger values are obtained by Huber (1955) and Maushart (1958).

7.5×10^{-10} n sec^{-1}.

There are no experimental data for O^+ in O but Dalgarno (1958a) has used a semi-empirical method which yields a tentative value for K_s of 1.6 cm^2 $volt^{-1}$ sec^{-1}. The corresponding values of Di and νi are $1.1 \times 10^{18}/n$ cm^2 sec^{-1} and 1.4×10^{-9} n sec^{-1} respectively.

There is little precise information on the temperature dependence of K_s for O^+, O_2^+, and N_2^+ moving in their respective parent gases. A guide is provided by the estimates of Dalgarno (1958a) for O^+ in O, which are reproduced in table A5.

It may be useful to include here data on the motion of H^+ in atomic hydrogen gas. Values of K_s have been computed to high accuracy by Dalgarno (1958a) for ambient temperatures up to 5,000°K. These calculations have since been extended to $10^{5\circ}$K and the values of K_s, Di and νi are given in table A6.

3.2 Negative Ions

For negative ions moving in unlike gases, the previous results are valid and further discussion is required only for O_2^- moving in O_2 and for O^- moving in O. Burch and Geballe (1957) have claimed that the mobility of O_2^- in O_2 is the same as that of O_2^+. The results given earlier for O_2^+ apply therefore to O_2^-. (It should be noted, however, that Dalgarno (1958a) has questioned the interpretation of Burch and Geballe).

TABLE A5

Diffusion Coefficients and Collision Frequencies of O^+ in O

T^oK	300	400	500	700	1000	2000
$(D_i\ n \times 10^{-18})cm^{-1}sec^{-1}$	1.1	1.3	1.5	1.9	2.3	3.2
$(\nu_i \times 10^{-9}/n)cm^3sec^{-1}$	1.4	1.5	1.7	1.9	2.2	3.1

TABLE A6

Mobilities, Diffusion Coefficients and Collision Frequencies of H^+ in H

T^oK	300	400	500	700	1000	2000
K_s $cm^2 volt^{-1} sec^{-1}$	12.4	11.0	10.2	9.1	7.4	5.7
$(D_i\ n \times 10^{18}) cm^{-1} sec^{-1}$	8.6	10.1	11.7	14.6	17.0	26.2
$(\nu_i \times 10^9/n) cm^3 sec^{-1}$	2.8	3.2	3.4	3.8	4.7	6.1

T^oK	5000	10000	20000	50000	80000	100000
K_s $cm^2 volt^{-1} sec^{-1}$	4.0	3.1	2.4	1.7	1.45	1.3
$(D_i\ n \times 10^{18}) cm^{-1} sec^{-1}$	46	71	110	200	270	300
$(\nu_i \times 10^9/n) cm^3 sec^{-1}$	8.4	11	15	21	24	27

No data are available for O^- in O. However, some calculations by Dalgarno and McDowell (1956) on H^- in H suggest that K_s and Di may be several times smaller and γi several times larger than for O^+ in O with a similar temperature variation.

4. THERMAL PROCESSES INVOLVING ELECTRONS

4.1 Radiative Recombination

Radiative recombination must be the most important recombination process in the very high atmosphere. The processes

$$O^{+} + e \rightarrow O' + h\nu \qquad (A44)$$

the prime indicating all the possible final states, has been investigated by Bates, Buckingham, Massey and Unrin (1939) who obtain the rates given in table A7 The accuracy should be high.

The process

$$H^{+} + e \rightarrow H' + h\nu \qquad (A45)$$

has been studied most recently by Burgess (1958) and Seaton (1959) and their results are also included in table A7.

Since the major contribution to the total recombination rate comes from captures into excited states, similar values should apply to the processes

$$N^{+} + e \rightarrow N' + h\nu \qquad (A46)$$

$$O_2^{+} + e \rightarrow O_2' + h\nu \qquad (A47)$$

$$N_2^{+} + e \rightarrow N_2' + h\nu \qquad (A48)$$

$$NO^{+} + e \rightarrow NO' + h\nu \qquad (A49)$$

TABLE A

Rates of Radiative Recombination Processes

($\alpha \times 10^{12}$) $cm^3 sec^{-1}$

T^oK	250	500	1000	2000
$O^+ + e \rightarrow O' + h\nu$	3.4	2.2	1.3	0.8
$H^+ + e \rightarrow H' + h\nu$	4.8	3.1	2.0	1.3

4.2 Dielectronic Recombination

Dielectronic recombination is the process of inverse autoionization (Massey and Bates 1943).

$$X^{+} + e \rightarrow X'' \tag{A50}$$

$$X'' \rightarrow X' + h\nu \tag{A51}$$

the double prime denoting an excited state, the energy of which lies in the continuum. In general, the rate of dielectronic recombination will be small because of the relative inefficiency of (A51) compared to the inverse of (A50). Massey and Bates (1943) and Massey (1952) remark that it is unlikely that the rate coefficient will exceed 10^{-12} $cm^3 sec^{-1}$.

It should be noted, however, that Garton (1958) has concluded that for argon the rate coefficient might be as large as 10^{-10} cm^3 sec^{-1} at $300^{\circ}K$ since there is some evidence that an excited state of argon exists for which (A51) is anomalously rapid. A more recent analysis of the evidence by Garton, Perry and Codling (1960) suggests a rate coefficient of 5×10^{-11} cm^3 sec^{-1}. A measurement of the transition probability of (A51) or its inverse is needed before this value can be accepted.

It cannot be asserted that suitable excited states of O and N do not exist for which (A51) is anomalously rapid but it seems improbable.

4.3 Dissociative Recombination

The recognition that the process of dissociative recombination

$$XY^+ + e \rightarrow X' + Y' \qquad (A52)$$

must occur and might be very rapid is due to Bates (1950) who suggested that in favourable circumstances the rate coefficient might be as large as 10^{-7} cm^3 sec^{-1}. Early measurements using microwave techniques (Biondi and Brown 1949, Holt, Richardson, Howland and McClure 1950, Biodi 1951) provided evidence of large rate coefficients but the significance of the derived values was obscure (cf. Persson and Brown 1955, Gray and Kerr 1960). It is clear, however, that the range of possible values of the rate coefficient of (A52) is large. Thus for A_2^+ it is about 8 x 10^{-7} cm^3 sec^{-1} (Biondi 1951) and for He_2^+ it is probably less than 2 x 10^{-9} cm^3 sec^{-1} (Gray and Kerr 1960).

4.3.1 Nitrogen

There have been several measurements of recombination in molecular nitrogen, the results of which are shown in table A 8. The measurements of Bialecke and Dougal (1958) show a pressure-dependent rate so that their values do not refer simply to

$$N_2^+ + e \rightarrow N' + N'' \qquad (A53)$$

Their values are probably upper bounds to the actual rates. The study by Faire and Champion (1959) represents an improvement of the earlier

TABLE A 3

Rates of Dissociation Recombination in N_2

Reference	Temperature ^{o}K	$cm^3 sec^{-1}$
Biondi and Brown 1949	300	1.4×10^{-6}
Sayers 1956	2500	1.4×10^{-7}
Faire et al. 1958	300	1.4×10^{-6}
Bialecke and Dougal 1958	92 300	6.7×10^{-6} 8.5×10^{-7}
Faire and Champion 1959	400	4.0×10^{-7}

work of Biondi and Brown (1949) and of Faire, Fundingsland, Aden and Champion (1958) but it does not incorporate any means of definitely identifying the ion concerned in the recombination. Some indication is provided by the ambipolar diffusion coefficient derived by Faire and Champion. Their value Dap $= (220 \pm 30)\,cm^2 sec^{-1}$ at $T = 400^{o}K$ corresponds to a mobility $K_s = 2.8\,cm^2\,volt^{-1} sec^{-1}$ in agreement with the experimental mobility measured at 400°K by Tyndall and Pearce (1935). The work of Varney (1953) and of Kovar, Beaty and Varney (1957) suggests that this value of the mobility refers to an ion which may be N_2^+ during part of its life and N_4^+ during the remainder. The mass analysis of Saporoschenko (1958) is of interest in that it shows that the onset potentials of N_2^+ and N_4^+ are equal. The derived mobility is much larger than the value estimated by Dalgarno (1958a) for N_2^+. His estimate is based, however, on a value of $1.9\,cm^2\,volt^{-1} sec^{-1}$ for O_2^+ in O_2, which is not rigorously established.

The preliminary measurements of Sayers (1956) do not suffer from those difficulties of identification because his technique includes a mass analysis, and because the formation of N_4^+ at temperatures as high as 2,500°K is much less probable.

Further experimental work is clearly desirable especially since Bromer (1959) has recently presented evidence in support of a value of $10^{-10} cm^3 sec^{-1}$ at $T \cong 313^{o}K$.

4.3.2 Oxygen

Sayers (1960, private communication) has obtained a preliminary value of $(2.0 \pm 1.0) \times 10^{-7}$ cm^3 sec^{-1} at T ~ 2500°K.

4.3.3 Nitric Oxide

Lin (1959) has obtained a value of about 10^{-9} cm^3 sec^{-1} at temperatures of the order of 5000°K.

In no case is the temperature dependence of the rate coefficient known.

Some evidence on the possible rates of dissociative recombination is provided by upper atmosphere data. Thus Bates and Nicolet (1960) remark that the behaviour of the E-layer suggests that perhaps the rate coefficient for the dissociative recombination of NO^+ is about 3×10^{-9} cm^3 sec^{-1} and for the dissociative recombination of O_2^+ is about 3×10^{-8} cm^3 sec^{-1}. Also it is clear from twilight data (Bates 1949, Chamberlain 1958) that dissociative recombination of N_2^+ must proceed faster than that of O_2^+ or NO^+.

4.4 Radiative Attachment

The rate of the radiative attachment process

$$X + e \rightarrow X^- + h\nu \qquad \text{(A54)}$$

may be obtained from the rate of the inverse photo-detachment process

$$X^- + h\nu \rightarrow X + e \qquad \text{(A55)}$$

provided X^- has no discrete excited states. Accordingly from their measurements on

$$O^- + h\nu \rightarrow O + e \qquad (A56)$$

Branscomb, Burch, Smith and Geltman (1958) have obtained a rate coefficient for

$$O + e \rightarrow O^- + h\nu \qquad (A57)$$

which decreases from 1.4 x 10^{-15} cm^3 sec^{-1} at 100°K to 1.2 x 10^{-15} cm^3 sec^{-1} at 2000°K. These values are in harmony with the early calculations by Bates and Massey (1943).

The long wavelength edge of the attachment continuum is 8460A° (1.465 eV) and the photo-detachment rate coefficient appropriate to the atmosphere at noon is

$$\rho(O^-) = 1.4 \text{ per negative ion per second} \qquad (A58)$$

(Smith 1960).

It follows from the theory of Bates and Massey (1946, 1947) that the noon value of the ratio $\lambda(O^-)$ of the density of O^- to that of free electrons is less than 10^{-15} $n(O)$, which is negligible. This argument does not take account of the production of O^- though

$$O_2^- + O \rightarrow O_2 + O^- \qquad (A59)$$

The photo-detachment of O_2^- has also been measured by Burch, Smith and Branscomb (1958), their measurements leading to a noon photo-detachment rate of

$$\rho(O_2^-) = 0.44 \text{ per negative ion per sec} \quad (A\,60)$$

(Smith, Burch and Branscomb 1958). Branscomb (1960, private communication) has estimated the electron affinity to be (0.3 ± 0.1)eV. No value is available for the O_2 attachment coefficient but it is certainly very small.

The photo-detachment of H^- is of some interest. Cross sections have been measured by Branscomb and Smith (1955) and by Smith and Burch (1959) and, calculated by many authors, the most recent being John (1960). Attachment cross sections have been given by Massey (1950).

In an investigation of negative ion formation in ozone, Curran (1961) concludes that the electron affinity of O is greater than or equal to 0.58 eV, whereas Phelps and Pack (1961) obtain a value of 0.46 ± 0.2 eV. It may be possible to resolve the discrepancy by postulating different vibrational distributions in the two experiments.

If the value of the electron affinity measured by Phelps and Pack (1961) is accepted, the photo detachment measurements of Burch, Smith and Branscomb (1950) lead to a rate coefficient for radiative attachment to molecular oxygen at 230°K of 2×10^{-18} cm^3 sec^{-1}.

4.5. Dielectronic Attachment

Dielectronic attachment is the inverse of auto detachment

$$X + e \rightarrow X^{-''} \qquad (A61)$$

$$X^{-''} \rightarrow X^{-'} + h\nu \qquad (A62)$$

It is unlikely that a rate coefficient as large as 10^{-15} cm^{3}sec^{-1} will occur.

4.6 Dissociative Attachment

The attachment measurements of Craggs, Thorburn and Tozer (1957), Tozer, Thorburn and Craggs (1958) and of Chanin, Phelps and Biondi (1959) in molecular oxygen show that the dissociative attachment process

$$O_2 + e \rightarrow O + O^- \qquad (A63)$$

has a threshold of about 3.5 eV and attains its maximum rate of about 4×10^{-11}cm^{3}sec^{-1} at a mean energy of 4.5 eV (Thompson 1959a). The rate decreases rapidly with decreasing electron energy and is negligible at atmospheric temperatures.

4.7. Three-body Recombination

The rate of the three-body recombination process

$$X^+ + M + e \rightarrow X + M \qquad (A64)$$

is given by the classical theory of Thompson (1924) provided the gas pressure is not too high. Formulae for high pressures have been given by Harper (1932), Jaffe (1940) and Wooding (1960). According to Massey

(1952), the rate coefficient may be as large as 2 x 10^{-10} $cm^3 sec^{-1}$, the gas pressure being in mm Hg. Thus three-body recombination if more effective than radiative recombination below an altitude of about 75 km.

The rate of the three-body recombination process

$$X^+ + e + e \rightarrow X + e \qquad (A65)$$

has been investigated for a hydrogen plasma by Bates and Kingston (1961). It is negligible at the electron densities and temperatures of the atmosphere.

4.8 Three-body Attachment

Recent work on attachment of low energy electrons in molecular oxygen (Hurst and Bortner 1959, Chanin, Phelps and Biondi, 1959) at pressures of several mm Hg shows that the dominant mechanism is a three-body process

$$O_2 + M + e \rightarrow O_2^- + M \qquad (A\ 66)$$

According to Chanin et al, the rate coefficient for (A23) has a maximum value of 5 x 10^{-30} $cm^6 sec^{-1}$ for an average electron energy of about 0.09 eV for a gas temperature of 300°K. For electrons of thermal velocities, the rate coefficient has decreased to 2.8 x 10^{-30} $cm^6 sec^{-1}$. There is some evidence that the rate coefficient at thermal electron velocities is smaller at smaller gas temperatures passing through a minimum possibly at about 160°K (Van Lint, 1959).

The mechanism of the attachment process has been discussed most recently by Biondi (1960a).

The attachment process

$$O + e \rightarrow O^- \qquad (A67)$$

has also been investigated by Biondi (1960b) who finds that the rate is about fifty times slower than for (A 63). It follows that the effective rate of electron loss in the atmosphere by three-body attachment is about 100 μ^2 $\sec^{-1}$, so that three-body attachment will be the dominant loss mechanism in the lower ionosphere.

The daytime ratio λ of the number density of O_2^- to that of free electrons (Bates and Massey 1947, 1952) is consequently less than 220 μ^2. The variation of λ with altitude has been illustrated most recently by Nicolet and Aikin (1960).

2.9 Upper Atmosphere Data

Studies of echo decay from meteor trails yield information on the mean time of attachment of electrons to the atmospheric constituents (cf. Kaiser, 1953). Assuming that the loss mechanism is attachment to molecular oxygen, Kaiser (1953) has derived an effective rate coefficient of about 10^{-15} $cm^3 sec^{-1}$ at 80 km and Davis, Greenhow and Hall (1959) have derived a coefficient of $5 \times 10^{-15} cm^3 sec^{-1}$ at 95 km, the value being uncertain by a factor of two. A similar analysis of the data of Booker and Cohen (1956) yields a coefficient of about $5 \times 10^{-15} cm^3 sec^{-1}$ at 90 km.

It is difficult to identify the particular attachment process, but it seems certain that it cannot be radiative attachment to atomic oxygen since the required density is greater than $10^{13} cm^{-3}$.

If the mechanism is the three-body attachment process (A66), the derived values of its rate coefficient (assuming normal atmospheric conditions) lie between $10^{-29} cm^6 sec^{-1}$ and $10^{-27} cm^6 sec^{-1}$ and are much greater than those suggested by the laboratory data. The fact that the values obtained at different heights are not greatly different suggests that a two-body process is responsible but, neither dissociative attachment of O_2 nor radiative attachment to O_2 is attractive.

Information on the attachment rate is also provided by observations of the attenuation of telemetering signals near a nuclear explosion in the upper troposphere. Dubs and Sen (1960) have derived a rate coefficient of 7×10^{-30} cm^{6}sec^{-1} for (A 66). Their approximations are such that the value is probably not reliable to better than an order of magnitude.

Kaiser (1953), has derived an upper limit of 4.5×10^{-12} cm^3 sec^{-1} for the meteoric ion recombination coefficient. This value is in harmony with that expected for radiative recombination (cf. Table A 7) and does not exclude a possible contribution form three-body recombination (A64).

5 THERMAL IONIC PROCESSES

5 1 Symmetrical Resonance Charge Transfer

The cross section for the symmetrical resonance charge transfer process

$$X^+ + X \rightarrow X + X^+ \qquad (A\ 68)$$

increases monotonically as the incident energy E decreases and at thermal velocities usually attains a value considerably greater than gas kinetic. Some typical values have been listed by Dalgarno (1958a).

The behaviour at low energies of resonanace charge transfer of molecular ions in their parent gases

$$XY^+ + XY \rightarrow XY + XY^+ \qquad (A69\ )$$

is less certain but the cross sections are probably finite in the limit of vanishing energy. Mobility data can be interpreted to yield cross sections of about 10^{-14} cm^2 for O_2^+ in O_2 and for N_2^+ in N_2 at thermal velocities (Varney 1953) which may refer partly to resonance charge transfer.

5.2 Non-resonance Charge Transfer

Reviews of non-resonance charge transfer processes have been presented by Massey and Burhop (1952), by Bates and Dalgarno (1956) and by Hasted (1960). Many of the data can be interpreted in terms of the adiabatic hypothesis introduced by Massey (1949). We define an energy, E_0 eV, according to

$$E_0 = 36\,(\Delta E)^2\, M \ell^2 \qquad \text{(A70)}$$

where ΔE is the energy defect of the reaction in eV, M is the mass of the incident particle on the ^{16}O scale and ℓa_0 (a_0 is the Bohr radius) is a measure of the effective range of the interaction. If the incident energy E is much less than E_0, the internal electronic motion is able to adjust to the slowly changing interaction so that the cross section is very small. In this near-adiabatic region the variation of the cross section with energy has roughly the form

$$Q = Q_0 \exp\left(-K \ell\, \Delta E / E^{1/2}\right) \qquad \text{(A71)}$$

where K is a constant. (It may be misleading, however, to use (A71') to extrapolate experimental data to thermal energies). When E is of the order of E_0, Q passes through a maximum and for higher energies decreases rapidly. Thus most non-resonance charge transfer processes are very inefficient at thermal energies.

Exceptions to this qualitative description are not uncommon and arise because ΔE is actually a function of the separation between the colliding particles. A general discussion has been given by Bates and Massey (1954) from considerations of the curve-crossing method which may be applied when ΔE nearly vanishes at some separation. Bates (1960) has shown recently that the curve-crossing method rests upon very doubtful assumptions and the results of quantitative calculations are suspect. The qualitative picture is nevertheless instructive in showing how particular non-resonance processes can be rapid at thermal energies.

3.1 Mutual Neutralization

Mutual neutralization reactions

$$X^+ + Y^- \rightarrow X + Y \qquad \text{(A 72)}$$

do not conform to the adiabatic hypothesis since a crossing of the zero order potential curves associated with the incident and final channels usually occurs. Various examples involving H^- have been considered by Bates and Lewis (1955) and by Bates and Boyd (1956). It appears that at thermal temperatures the rate coefficients may be as large as $10^{-7} cm^3 sec^{-1}$, varying approximately as $T^{-\frac{1}{2}}$. Bates and Massey (1943) (see also Magee, 1952) have investigated the important case

$$O^+ + O^- \rightarrow O + O \qquad \text{(A 73)}$$

Although no definite value can be given for the rate coefficient, it is probably as large as $10^{-8} cm^3 sec^{-1}$ and may be larger. The most probable reaction path is not that corresponding to near resonance but that corresponding to an energy excess of about 1 eV.

Bates and Boyd (1956) have also discussed the rates of dissociative mutual neutralization reactions

$$XY^+ + Z^- \rightarrow X + Y + Z \qquad (A74)$$

and conclude that in favourable uses the coefficients will be as large as those of (A29).

There is some experimental evidence supporting the predicted orders of magnitude, Young (1958) having derived rate coefficients of $1.47 \times 10^{-8} cm^3 sec^{-1}$ and $1.85 \times 10^{-8} cm^3 sec^{-1}$ in iodine and bromine respectively at room temperature.

5.3 Asymmetric Resonance Charge Transfer

Dungey (1955) has drawn attention to the possible importance of the asymmetric (or accidental) resonance charge transfer process

$$H^+ + O \rightleftarrows H + O^+ \qquad (A75)$$

and its influence on the composition of the high atmosphere has been investigated by Johnson (1960). According to the theory of Gurnee and Magee (1957), the reaction is rapid at thermal velocities because of the close resonance. It follows that equilibrium should be attained in the atmosphere so that

$$\frac{n(H^+)}{n(H)} = \frac{9}{8} \frac{n(O^+)}{n(O)} \qquad (A76)$$

(Chamberlain, 1956). Rapp and Ortenburger (1960) have used the theory of Gurnee and Magee to compute the cross sections of

$$O^+ + H \rightarrow O + H^+ \qquad \text{(A75a)}$$

and obtain quite good agreement with the measurements of Stebbings(1960) See Figure 6. , Experimental data refer to impact energies above about 400 eV. The extension of the measurements to lower energies is very desirable for it seems clear from the work of Bates and Lynn (1959) that, in contradiction to the conclusions of Gurnee and Magee (1957), asymmetric resonance processes are quite different in character at low energies from symmetric resonance processes. In particular, they must be comparatively slow at thermal energies. Nevertheless (A75) is probably of significance in determining the proton distribution in the atmosphere.

Another example of asymmetric resonance is provided by

$$O^+(^2D) + N_2(X^1\Sigma_g^+, v=0) \rightarrow O(^3P) + N_2^+(A^2\Pi_n, v=1) \qquad \text{(A77)}$$

which Omholt (1957a) has suggested may explain variations in the relative intensities of the (1.0) and 2,1) bands of the Meinel Negative system (Hunten, 1958, Harrison and Jones, 1959). Again, according to Bates and Lynn, a large thermal cross section is very improbable.

5.4 Collisional Detachment

Collisional detachment is the process

$$X^- + Y \rightarrow X + Y + e \qquad \text{(A78)}$$

Several studies have been made at energies above a few electron volts (cf. Hasted, 1960) but there are no direct measurements at thermal velo-

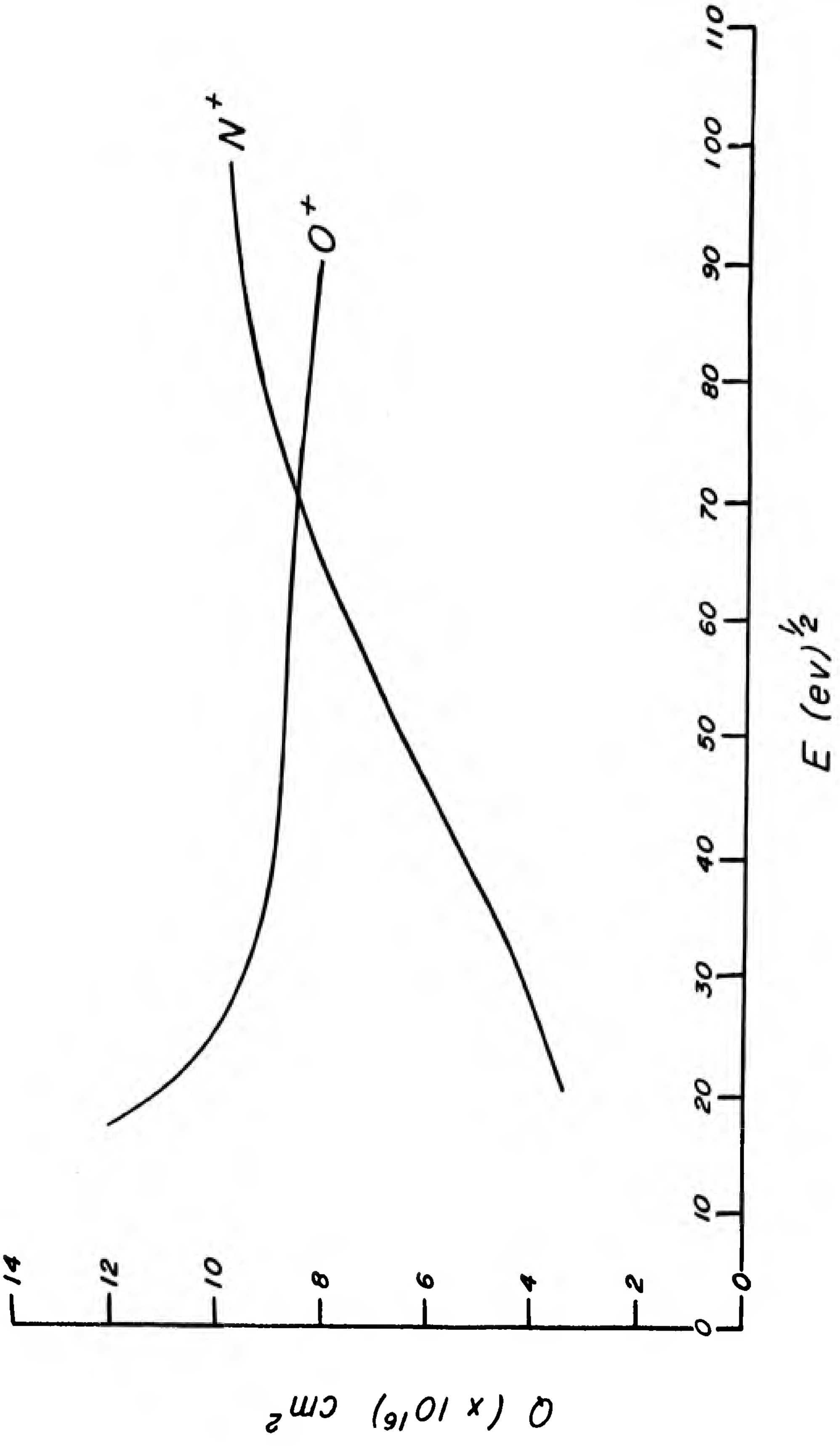

Figure 6. Asymmetric Resonance Charge Transfer

cities.

There is some indirect evidence. Chanin, et al., (1959), believe that they would have observed collisional detachment in their investigations of attachment in O_2 had the rate coefficient for

$$O_2^- + O_2 \rightarrow O_2 + O_2 + e \tag{A79}$$

been greater than $10^{-16} cm^3 sec^{-1}$. Further Bailey (1959) and Bailey and Branscomb (1960, private communication) have analysed observations of polar black-outs in the D region to obtain a rate coefficient of about $2 \times 10^{-17} cm^3 sec^{-1}$ at a mean temperature of 230°K. The specific reaction involved may be (A79) but this has not been established. Phelps and Pack(1961) have recently derived a rate coefficient for (A79) of 4×10^{-20} $cm^3 sec^{-1}$ at 230^oK.

5.5 Ion-atom Interchange

Bates (1955) has pointed out that, although charge transfer is usually extremely inefficient at thermal velocities, ion-atom interchange

$$X^+ + YZ \rightarrow XY^+ + Z \tag{A80}$$

may not be. Ion-atom interchange is similar to an ordinary chemical reaction and can proceed rapidly at thermal energies, provided the activation energy is small, with a rate coefficient of $10^{-9} cm^3 sec^{-1}$.

Similarly, Hultquist (1959) has analysed observations of non-deviative absorption to obtain a value of about 5 for the ratio $\rho(O_2^-)/nk$, k being the rate coefficient for collisional detachment. From (A60) $\rho(O_2^-) = 0.44$ but the mean absorption height and hence, the number density, n, is uncertain. Adopting $k = 2 \times 10^{-17}$ $cm^3 sec^{-1}$ leads to a height of about 50 km which appears to be reasonable (Hultquist and

Ortner, 1959). Values of this order have been measured for ion-atom interchange reactions involving the isotopes of hydrogen (cf. Hasted, 1960).

Those positive ion-atom interchange reactions which may occur in the ionosphere have been listed by Bates and Nicolet (1960). They are

$$O^+ + N_2 \rightarrow NO^+ + N \tag{A81}$$

$$O^+ + O_2 \rightarrow O_2^+ + O \tag{A82}$$

$$O^+ + NO \rightarrow NO^+ + O \tag{A82}$$

$$\rightarrow O_2^+ + N \tag{A84}$$

$$N^+ + O_2 \rightarrow NO^+ + O \tag{A85}$$

$$N^+ + NO \rightarrow NO^+ + N \tag{A86}$$

$$\rightarrow N_2^+ + O \tag{A87}$$

$$N_2^+ + N \rightarrow N^+ + N_2 \tag{A88}$$

$$O_2^+ + N \rightarrow NO^+ + O \tag{A89}$$

Of these, only the rate of (A82) has been measured, Dickinson and Sayers (1960) obtaining a value of about 10^{-11}cm^{3}sec^{-1} for the rate coefficient at T = 300°K.

From observations of the recombination and of the ionic composition of the high atmosphere, Bates and Nicolet (1960) have concluded that

$$k_{81} + 0.1_6\, k_{82} = 1.3 \times 10^{-13} \text{ cm}^3 \text{ sec}^{-1} \qquad \text{(A90)}$$

approximately, the subscripts denoting the equation number of the process to which the rate coefficient k refers. Bates and Nicloet point out that (A90) implies a large amount of steric hindrance or, more probably, a large activation energy, with the consequence that k_{81} and k_{82} are rapidly increasing functions of temperature. The relationship (A90) is therefore in contradiction to the experimental data of Dickinson and Sayers (1960).

Ion-atom interchange processes may effect the negative ion distribution. One possibility is the reaction

$$O_2^- + O \rightarrow O_2 + O^- \qquad \text{(A91)}$$

Its rate is unknown. For the inverse process

$$O_2 + O^- \rightarrow O_2^- + O \qquad \text{(A92)}$$

Thompson (1959b) has concluded that the rate coefficient is less than 2 x 10^{-17} cm^{3}sec^{-1} at a mean energy of 0.15 eV.

5.6 Associative Detachment

The process of associative detachment is

$$X^- + Y \rightarrow XY + e, \qquad (A\ 93)$$

two examples which may occur in the atmosphere being

$$O^- + O \rightarrow O_2 + e \qquad (A\ 94)$$

$$O_2^- + O \rightarrow O_3 + e \qquad (A\ 95)$$

Bearing in mind that auto detachment probably proceeds at a rate about 100 times less than autoionization (Brandsen and Dalgarno, 1956) and using arguments similar to those given by Bates (1950) in estimating the rates of dissociative recombination, it appears that rate coefficients as large as 10^{-10} $cm^3 sec^{-1}$ cannot be excluded.

5.7 Three-body Ionic Recombination

The rates of the three-body ionic recombination process

$$X^+ + Y^- + M \rightarrow X + Y + M \qquad (A\ 96)$$

are given by the theory of Thompson (1924), which is in harmony with the measurements of Mächler (1936), Sayers (1938) and Gardner(1938). Massey (1952) gives for the three-body recombination coefficient in air

$$\alpha = \frac{8 \times 10^{-3}\, p}{T^{5/2}} \quad cm^3\ sec^{-1} \qquad (A97)$$

for pressures p less than 30 mm Hg. Thus adopting a value of 10^{-8} cm^3 sec^{-1} for mutual neutralization (A73), three-body recombination is less efficient at altitudes above about 45 km.

6. ELECTRON CAPTURE, EXCITATION AND IONIZATION BY HEAVY PARTICLE IMPACT

6.1 Resonance Charge Transfer

The process

$$H^+ + H \rightarrow H + H^+ \qquad (A98)$$

is important in connection with the trapping of protons by the magnetic field of the earth. At low energies the cross sections have been calculated by Dalgarno and Yadav (1953) and at high energies by Bates and Dalgarno (1952), by Jackson and Schiff (1953) and by Bessel and Cerjuoy (1960). The accuracy of the predictions in the energy range from 100 eV to 10 keV has been confirmed by measurements of Fite, Brackmann and Snow (1958) and of Fite, Stebbings, Hummer and Brackmann (1960). Recommended values are given in Table A9. Attention is drawn to the rapid decrease of the cross section at high energies, which results from the increasing difficulty of transferring electron momentum. The slowing down of an incident proton beam in atomic hydrogen gas through reaction (A39) has been discussed by Dalgarno and Griffing (1955) and by Dalgarno (1960).

Cross sections for

$$H^+ + H' \rightarrow H' + H^+ \qquad (A99)$$

where H represents a hydrogen atom in the $n = 2$ excited state, have been calculated by Boyd and Dalgarno (1959). They are several times larger than those corresponding to (A98).

Cross sections for the process

$$O^+ + O \rightarrow O + O^+ \qquad (A100)$$

may also be of interest. Values are given in Table A9 but the accuracy is uncertain (Dalgarno 1958a).

6.2 Electron Capture into Excited States

Detailed results on the non-resonance charge transfer process

$$H^+ + H \rightarrow H' + H^+ \tag{A101}$$

where H represents an excited hydrogen atom in any state up to $4f$, have been obtained by Bates and Dalgarno (1953) using the Born approximation. Recent developments (Bates 1958, B ssel and Gerjuoy 1960) should lead to results of greater precision and calculations are in progress (Bates and McCarroll 1960 private communication). The results of Bates and Dalgarno provide a useful indication of the hydrogen emission spectrum arising from an incident proton beam. Of particular importance is the rapid decrease in capture probability as the energy increases so that emission is restricted to a relatively narrow altitude range.

The only quantitative experimental data on specific non-resonance processes involving the atmospheric gases appear to be those of Carleton (1957) and Carleton and Lawrence (1958). For incident protons with energies of a few keV in N_2, they find that the cross section for the production of $H\beta$ by capture is about 6×10^{-19} cm^2, that the cross section for the production of the 0 - 0 band of the first negative system is about 5×10^{-17} cm^2 and that the cross section for the production of the 2 - 0 band of the einel system is about 2×10^{-17}cm^2. The total electron capture cross sections by Carleton and Lawrence appear to be 30% smaller than those measured by Stier and Barnett (1956).

TABLE A 9

Cross Sections for Resonance Charge Transfer (Q x 10^{15}) cm^2

$\log_{10}$	-1	0	1	2	3	4	5
$H^+ + H$	6.0	4.8	3.6	2.6	1.7	0.86	0.10
$O^+ + O$	10.0	8.8	7.8	6.8	5.8	-	-

Sheridan, Oldenberg and Carleton (1961) find that the cross section for the production of H_β in collisions of protons with N_2 has a roughly constant value of 7×10^{-19} cm^2 for energies up to 30 keV.

6.3 Total Electron Capture Cross Sections

Most experimental data refer to the total charge transfer cross sections which take into account all possible end products. Allison (1958) has summarized the data on protons and helium ions with impact energies above 0.2 keV and his values for protons in N_2 and O_2 are reproduced in Table A10. The high energy behaviour in N_2 is significantly different from that in atomic hydrogen. Thus for $E = 100$ keV, $Q(N_2) \sim 8Q(H)$; for $E = 500$ keV, $Q(N_2) \sim 70Q(H)$; for $E = 1000$ keV, $Q(N_2) \sim 150Q(H)$.

6.4 Excitation and Ionization by Proton Impact

Calculations using the Born approximation of the cross sections for excitation

$$H^+ + H \rightarrow H^+ + H' \qquad \text{(A102)}$$

where H' represents any state of atomic hydrogen up to 3d, and for ionization

$$H^+ + H \rightarrow H^+ + H^+ + e \qquad \text{(A103)}$$

have been reported by Bates and Griffing (1953a), who also give the energy distribution of the electrons ejected in reaction (A103). An energy distribution more typical of that expected for ionization of the atmospheric gases is the distribution computed for the ionization of neon (Bates, McDowell and Omholt 1957).

Some interesting observations of the spectra resulting from high energy incident beams in N_2 and O_2 have been made (Fan and Meinel

1953, Branscomb, Shalek and Bonner 1954, Nicholls, Reeves and Bromley 1959, 1960), but there are few quantitative data on the excitation and ionization of the atmospheric gases. Carleton and Lawrence (1958) find that the cross section for ionization of N_2 by protons with energies between 1.5 and 4.5 keV is between 10% and 1.5% of the total capture cross section which implies an ionization cross section of about 10^{-17} cm^2. They also report cross sections between $5 \times 10^{-18} cm^2$ and 6×10^{-18} cm^2 for production of the NI lines at 8188-8216 Å.

At high impact energies the cross sections for ionization and for the excitation of allowed transitions by proton impacts are much the same as those due to collisions of electrons of the same velocity for which some data exist.

Further measurements on the excitation and ionization of N_2 by proton impact have been reported by Sheridan, Oldenberg and Carleton (1961). Their measurements, which cover an energy range from 4 keV to 30 keV, give the total ionization and total charge transfer cross sections and the cross sections for excitation of the 0-0 band of the first negative system, the 2-0 band of the Meinel system and the atomic nitrogen lines near 8210 Å.

In addition, Hooper, McDaniel, Martin and Harmer (1961) have reported values for the total ionization cross sections of N_2 and O_2 by protons with energies varying from 150 keV to 1.10 meV, the data for O_2 being of a preliminary nature. Their results can be expressed in this energy range by the formulae

$$\sigma(N_2) = \frac{14.2}{E^{0.70}} \times 10^{-17} \ cm^2$$

$$\sigma(O_2) = \frac{15.3}{E^{0.75}} \times 10^{-17} \ cm^2$$

where E is the impact energy in meV.

6.5 Excitation and Ionization by Neutral Hydrogen Impact

Calculations using the Born approximation of the cross sections for

$$H + H \rightarrow H\ (\Sigma) + H' \qquad \text{(A104)}$$

and

$$H + H \rightarrow H\ (\Sigma) + H^+ + e \qquad \text{(A105)}$$

where H (Σ) represents all possible states (including the continuum) and H' represents any state up to 3d, have been reported by Bates and Griffing (1954, 1955). They also give the energy distribution of the electrons ejected in reaction (A105). Sheridan, Oldenberg and Carleton (1961) have obtained preliminary evidence that the cross section for the production of the 2-0 and 3-1 bands of the first positive system of N_2 by collisions with neutral hydrogen does not decrease much below 4×10^{-17} cm^2 for energies up to 30 keV. The distribution is given for momenta measured relative to the parent nucleus. If the parent atom is moving rapidly it is necessary in order to compute the secondary ionization to know also the angular distribution of the ejected electrons. This angular distribution has been computed by Dalgarno and Griffing (1958) who show that most of the electrons are ejected in the backward direction.

The only measurement referring to a specific process is that by Carleton and Lawrence (1958) who obtain a cross section of about 4.5×10^{-17} cm^2 for the excitation of the 2 - 0 and 3 - 1 bands of the first positive system of N_2 by collision with neutral hydrogen atoms of a few keV energy. They also estimate that the cross section for the production of $H\beta$ in such collisions is about $8 \times 10^{-19} cm^2$.

6.6 Total Loss Cross Sections

Most of the quantitative experimental data refer to the total loss cross section. Allison (1958) has summarized the data and his values for

$$H + N_2 \rightarrow H^+ + e + N_2(\Sigma) \tag{A106}$$

and

$$H + O_2 \rightarrow H^+ + e + O_2(\Sigma) \tag{A107}$$

are reproduced in Table A10. The high energy behaviour in N_2 is similar to that in atomic hydrogen. Thus for $E = 100$ keV, $Q(N_2) \sim 8Q(H)$; for $E = 500$ keV, $Q(N_2) \sim 2Q(H)$; for $E = 1000$ keV, $Q(N_2) \sim 15Q(H)$.

6.7 Equilibrium Fractions of an H^+ - H Beam

An incident proton beam captures electrons to form neutral hydrogen atoms which then become ionized. An equilibrium state, such

TABLE A 10

Capture Cross Sections for H^+ and Loss Cross Sections for H in N_2 and O_2

Impact Energy	Capture Cross Sections x 10^{17} in cm^2		Loss Cross Sections x 10^{17} in cm^2	
(keV)	N_2	O_2	N_2	O_2
4	118	98	20	17
7	110	90	29	27
15	86	66	38	37
30	57	47	45	39
50	35	28	49	41
70	22	19	48	41
100	10	11	48	41
150	4.4	5.4	42	39
200	1.5	2.1	39	38
300	0.32		30	
500	0.040		20	
700	0.0080		17	
1000	0.0040		14	

that the number of captures per second equals the number of losses, is attained rapidly and the equilibrium fractions for a proton beam in N_2 and O_2 are given in Table A11 (cf. Allison 1958). At energies in the region of 10 keV between 1% and 2% of the beam consists of H^-.

5.8 Stopping Power of Gases

The stopping power of a gas of atomic hydrogen towards an incident beam of protons has been investigated in detail by Dalgarno and Griffing (1955) taking into account capture and loss processes. Over a wide range of energies, the energy dissipated in ionization is about three times that dissipated in excitation and energy loss due to capture into excited states is appreciable below 100 keV.

Experimental data on the stopping power of gases towards protons with energies greater than 20 keV have been reviewed by Allison and Warshaw (1953). Values for N_2 and O_2 have been measured by Philips (1953) by Dunbar, Reynolds, Wenzel and Whaling (1953) and by Chilton, Cooper and Harris (1954) and values for air have been measured by Dunbar, et al. and Weyl (1953). There are discrepancies between the measurements of the various experimenters and the values given in Table A12 have been selected arbitrarily. They are roughly six times those of atomic hydrogen.

The measurements exclude energy loss associated with large angle elastic scattering which manifests itself as an attenuation of the beam. For protons, it is significant only at low impact energies.

For energies outside the measured region, the Bethe formula for the stopping power

TABLE A 11

Equilibrium Fractions of an H^+ - H Beam in N_2 and O_2

Impact Energy	N_2		O_2	
(keV)	$f(H^+)$	f(H)	$f(H^+)$	f(H)
4	0.125	0.871	0.105	0.880
7	0.175	0.815	0.200	0.782
15	0.300	0.688	0.320	0.661
30	0.450	0.544	0.475	0.517
50	0.590	0.410	0.595	0.405
70	0.686	0.314	0.682	0.318
100	0.828	0.172	0.788	0.212
150	0.905	0.095	0.878	0.122
200	0.962	0.038	0.942	0.058
300	0.990	0.010		
500	0.998	0.0020		
700	0.999	0.00077		
1000	1.000	0.00028		

$$-\frac{1}{n}\frac{dE}{dx} = \frac{3.46 \times 10^{-12}}{E} \ln\left(\frac{2.18\,E}{I}\right) \mathrm{eV\ cm^2} \quad \text{(A108)}$$

where E is measured in keV, may be used, I for air being about 94 eV. Expression (A108) has been used to extend Table A12 to 100 MeV.

For calculating the depth of penetration of an incident beam into the atmosphere, it is simpler to use range-energy relationships.

The ranges of protons of various energies E keV in N_2, O_2 and air have been measured by Cook, Jones and Jorgensen (1953) and their results are given in Table A13. The ranges R cm in gases at 1 mm pressure can be approximated by the analytical expressions:

$$N_2 \quad \begin{array}{ll} R = 3.34\,E^{0.71} & E < 62.9 \\ R = 0.0414\,(E + 70)^{1.5} & E > 62.9 \end{array} \quad \text{(A109)}$$

$$O_2 \quad \begin{array}{ll} R = 3.90\,E^{0.71} & E < 79.0 \\ R = 0.0400\,(E + 88)^{1.5} & E > 79.0 \end{array} \quad \text{(A110)}$$

$$\text{Air} \quad \begin{array}{ll} R = 3.26\,E^{0.73} & E < 76.5 \\ R = 0.0393\,(E + 80)^{1.5} & E > 76.5 \end{array} \quad \text{(A111)}$$

The experimental values can be extended higher than those measured by Cook, et al. by using (A108) and the resulting values are given in Table A14. Further values are given by Bethe and Ashkin (1953).

By using these ranges in conjunction with a selected model atmosphere, the depths of penetration of protons into the atmosphere may be calculated (Bates and Griffing 1953b, Bates 1955). In view of the discovery of solar cosmic rays it is desirable to extend the calculations by Bates (1955) to higher energies. In Table A15, the altitudes reached by protons of various energies incident vertically at the top of

TABLE A 12

Stopping Powers of N_2, O_2 and Air for Protons

(Stopping power in keV cm^2) x 10^{15}

Incident Energy (keV)	N_2	O_2	Air
20	30.0	24.0	29.0
30	32.2	27.0	31.0
40	34.2	30.4	33.0
50	35.6	32.8	34.3
60	36.4	33.8	35.4
70	37.0	34.3	35.8
80	37.0	34.3	35.8
90	36.5	34.3	35.6
100	35.8	34.2	35.0
150	32.2	32.3	32.0
200	28.4	29.4	28.4
300	22.4	24.0	25.4
500	16.2	17.6	16.8
1000	9.7	12.0	10.8
5000	3.2	3.7	3.3
10000	1.8	2.1	1.9
50000	0.5	0.5	0.5
100000	0.3	0.3	0.3

TABLE A 13

Range in cm of Protons in N_2, O_2 and Air at 1 mm Hg and 288°K

Impact Energy (keV)	N_2	O_2	Air
6.66	10.3	-	12.5
11.5	-	-	19.4
20	28.0	32.1	28.9
30	36.6	43.9	39.2
40	45.0	54.3	48.1
50	53.2	63.4	57.2
60	62.0	71.5	64.3
70	69.3	80.3	71.9
80	76.9	88.1	80.0
90	85.4	97.0	89.0
100	94.2	-	96.3
125	114	125	115
150	136	149	137
175	159	169	159
200	182	192	182
225	-	217	209
240	219	-	-
250	-	243	232

TABLE A 14

Range in Meters of Fast Protons in Air at 760 mm Hg and 288^{o}K

E (MeV)	Range (m)
0.5	0.0080
1.0	0.023
2.0	0.072
5.0	0.34
10	1.15
50	21
100	71

TABLE A 15

Depths of Penetration of Vertically Incident Protons

E (keV)	Altitude (km)
0.5	140
1.0	125
1.5	115
2.0	110
2.5	100
3.0	90
3.5	75
4.0	65
4.5	45
5.0	30

the atmosphere and traveling in rectilinear paths through it are given. Each altitude is therefore the minimum possible. *

6.9 Mean Energy per Ion Pair

A detailed analysis of the processes contributing to the production of electrons by a proton beam incident upon a gas of atomic hydrogen has been given by Dalgarno and Griffing (1958). At low impact energies, secondary ionization is negligible but in the limit of high energies, constitutes about 40% of the total ionization. The mean energy expended in producing an ion pair is nearly constant at 35.9 eV for energies above 10 keV. It must, however, increase rapidly at lower energies.

The experimental data have been reviewed by Valentine and Curran (1958). There is some evidence that the mean energy per ion pair W in N_2 and O_2 increases as the incident energy decreases but the values of W at any given energy are uncertain by one or two electron volts. For protons with energies greater than 10 keV in the atmosphere it seems best to take W as constant at 35 eV for N_2, at 32 eV for O_2 and at 34 eV for air.

6.10 Ionization [illegible]

The effective cross sections for the production of ionization (primary and secondary) follow immediately on dividing the stopping cross sections of Table A12 by the corresponding mean energy per ion pair. These may be used to construct an ionization-altitude curve for incident protons

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* Bailey (1957) has presented graphs of the depths of penetration of various ions ranging in mass from protons to calcium ions.

of any given energy. Alternatively, the production of ionization by protons in air has been measured by Jentschke (1940). Curves appropriate to the atmosphere have been given by Bates and Griffing (1953b).

7. EXCITATION AND IONIZATION BY ELECTRON IMPACT

7.1 Excitation

Seaton (1953,1956) has computed the cross sections for the excitation processes (See Figure 7).

$$e + O(^3P) \rightarrow e + O(^1D) \qquad \text{(A112)}$$

$$e + O(^3P) \rightarrow e + O(^1S) \qquad \text{(A113)}$$

which are important sources of the red and green lines in auroras. He has also computed the deactivation coefficients for the superelastic collisions

$$O(^1D) + e \rightarrow O(^3P) + e \qquad \text{(A114)}$$

$$O(^1S) + e \rightarrow O(^1D) + e \qquad \text{(A115)}$$

$$O(^1S) + e \rightarrow O(^3P) + e \qquad \text{(A116)}$$

obtaining at $1000^\circ K$, $1.6 \times 10^{-9}\ cm^3 sec^{-1}$ for (A114), $0.6 \times 10^{-9}\ cm^3 sec^{-1}$ for (A115) and $1.2 \times 10^{-9}\ cm^3 sec^{-1}$ for (A116).

The role of (A114) in suppressing the red line has been discussed by Seaton (1954, 1958). Seaton (1953) has also given cross sections for

$$e + N(^4S) \rightarrow e + N(^2D) \qquad \text{(A117)}$$

$$e + N(^4S) \rightarrow e + N(^2P) \qquad \text{(A118)}$$

$$e + N(^2D) \rightarrow e + N(^2P) \qquad \text{(A119)}$$

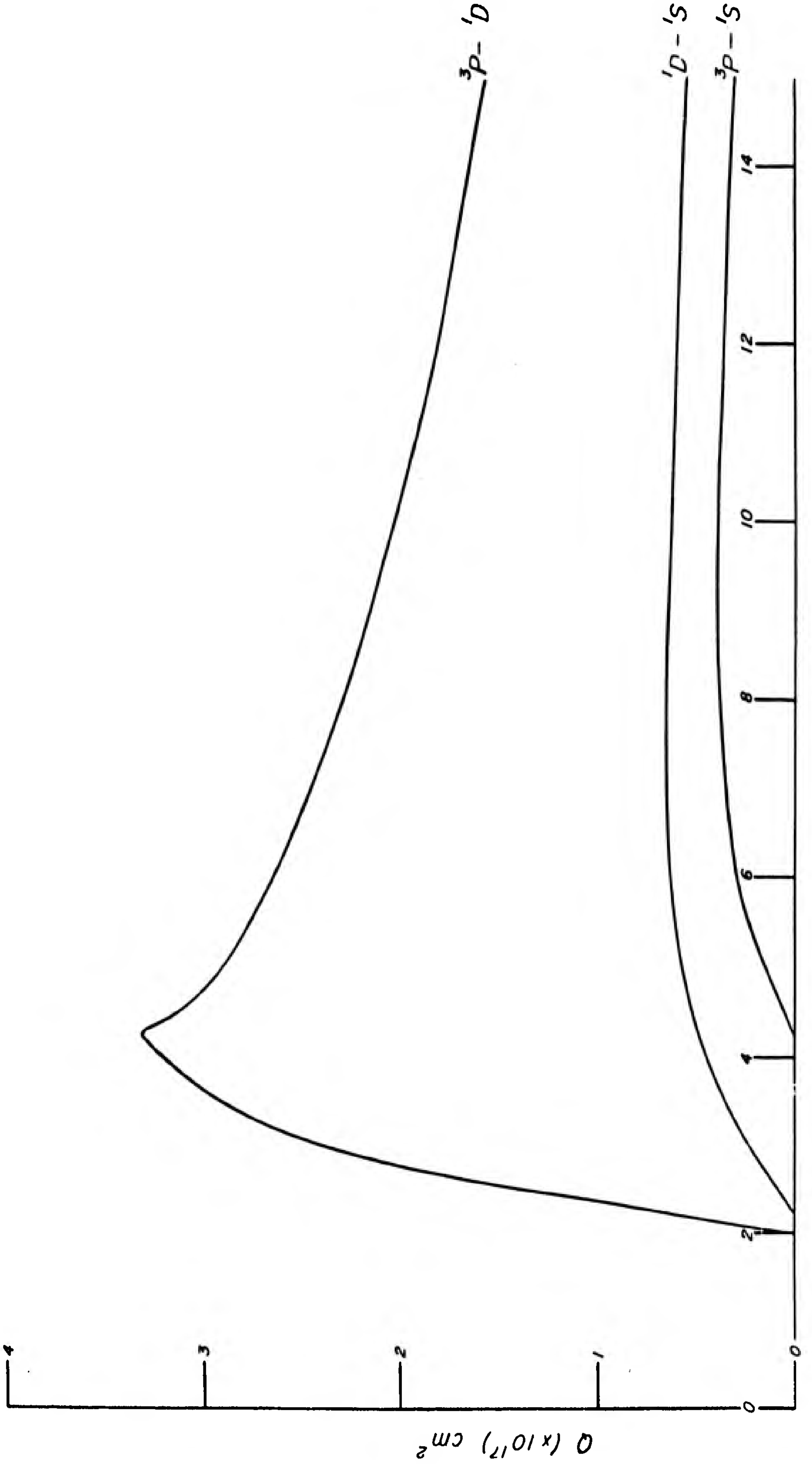

Figure 7. Atomic Oxygen Excitation

TABLE A16

Cross Sections for Excitation and Ionization of Atomic Hydrogen by Electron Impact

	$Q\ (cm^2) \times 10^{17}$			
Process $/\log_{10} E$ (eV)	2	3	4	5
Excitation of n = 2 level	6.2	1.3	0.19	0.031
Excitation of n = 5 level	1.1	0.23	0.30	0.0048
Excitation of all discrete levels	8.3	1.8	0.25	0.040
Ionization	6.4	1.2	0.15	0.022

and his values are reproduced in Figure 8. For the de-activation process

$$e + N(^2D) \rightarrow e + N(^4S) \;, \qquad \text{(A120)}$$

Seaton obtains a rate of about 8×10^{-10} $cm^3 sec^{-1}$ at 1000°K. The only other calculations of excitation of the main atmospheric constituents are those by Perceval (1957) on

$$e + O(^3P) \rightarrow e + O\,(3p\,^3P) \qquad \text{(A121)}$$

$$e + O(^3P) \rightarrow e + O\,(3p\,^5P) \qquad \text{(A122)}$$

the accuracy of which is not high.

There appears to be only one quantitative measurement. Stewart (1956a) has determined the cross sections for excitation of the 0 - 0, 0 - 1 and 0 - 2 bands of the first negative system of nitrogen

$$e + N_2(X^1\Sigma_g^+) \rightarrow e + N_2^+\,(B^2\Sigma_u^+) + e \qquad \text{(A123a)}$$

by fast electrons and Stewart and Gabathuler (1958) have determined the cross sections for excitation of the $v = 0$ level of the second positive system of nitrogen

$$e + N_2(X^1\Sigma_g^+) \rightarrow e\,(C^3\Pi_u) \qquad \text{(A123b)}$$

Their results are shown in Figures 9 and 10. They also give values of the cross sections for the excitation of some of the bands of the second negative bands of oxygen by 100 eV electrons.

Stewart (1956b) has also shown that the maximum excitation of the Meinel bands $N_2^+(A^2\Pi - X^2\Sigma_g^+)$ occurs at 60 eV, some 30 eV lower than the location of the maximum of the first negative bands. He remarks that

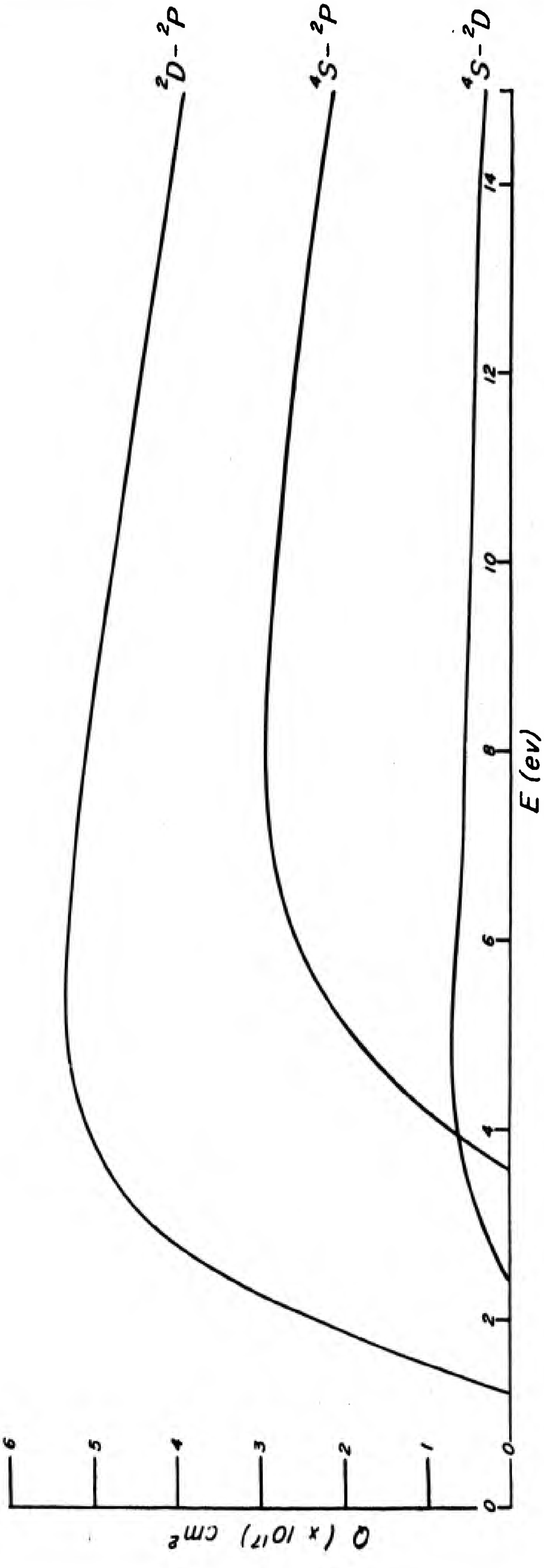

Figure 8. Excitation of Nitrogen

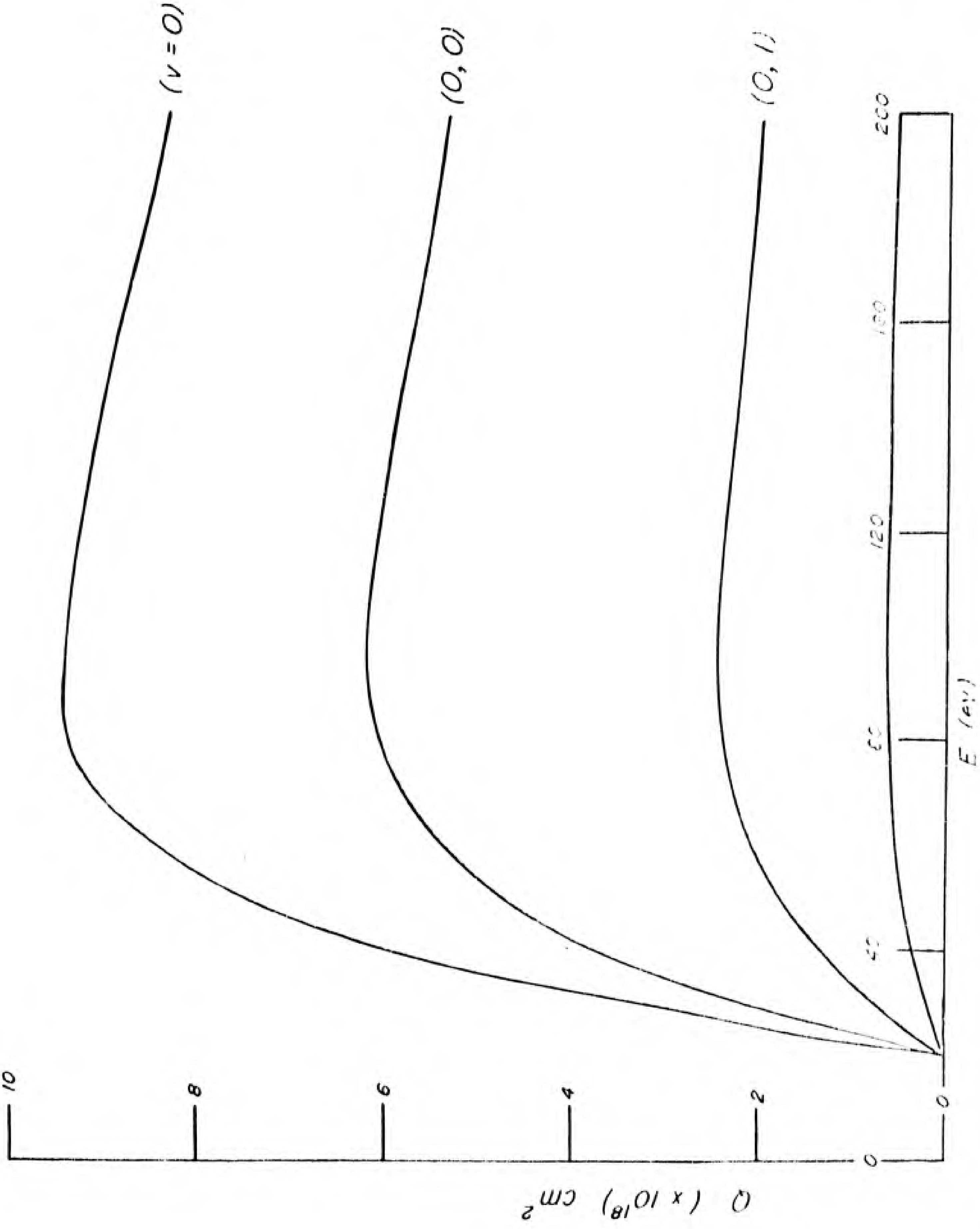

Figure 9. Excitation of Nitrogen

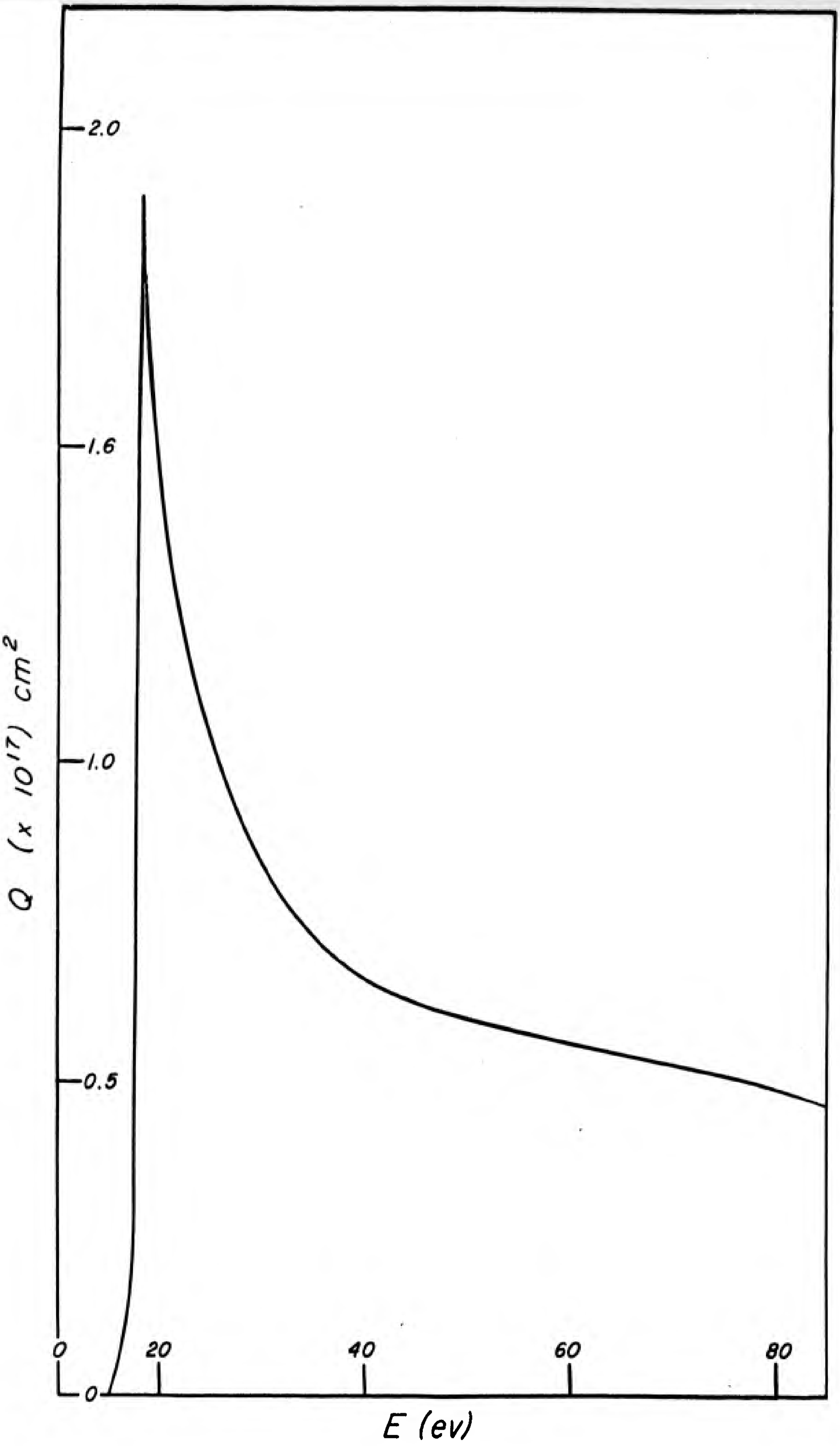

Figure 10. Excitation of Nitrogen

the auroral ratio of intensities of the Meinel bands to the first positive bands suggests a mean energy of 30 eV for the exciting electrons. In addition, Gribbon and Stewart (1956) have obtained quantitative evidence which is in harmony with the suggestion that the auroral infra-red OI lines at 7774 Å and 8446 Å are produced by simultaneous excitation and dissociation of molecular oxygen by electron impact. There are objections to accepting this suggestion (Bates 1960).

7.2 Ionization

Tate and Smith (1932) have measured the total ionization cross sections of N_2 and O_2 by electron impact and Fox and Hickam (1954) have investigated in detail the ionization cross sections of N_2 near the thresholds. More recently, Craggs, Thorburn and Tozer (1957) have obtained cross sections for O_2 which differ from those of Tate and Smith, but relative cross section measurements by Fite and Brackmann (1959) confirm the earlier results. Fite and Brackmann also give cross sections for the specific ionization processes

$$e + O_2 \rightarrow e + O_2^+ + e \qquad \text{(A124)}$$

$$e + O \rightarrow e + O^+ + e \qquad \text{(A125)}$$

measurements of the total ionization of molecular gases include the dissociation ionization process

$$e + O_2 \rightarrow e + O + O^+ + e \quad ; \qquad \text{(A126)}$$

According to Fite and Brackmann, the ratio of atomic ions to

TABLE A17

Stopping Powers of Air for Electrons*

(eV cm^2) x 10^{15}

Incident Energy (eV)	Stopping Power
200	8.5
300	8.2
500	6.8
700	5.8
1000	4.2
2000	3.5
5000	1.5
10000	0.9

*Relativistic effects [not given in equation (A129)] have been taken into account in the calculations.

molecular ions produced by electron impact ionization of O_2 increases rapidly from zero at the threshold 18.5 eV, attains a maximum of about 0.6 at 150 eV and decreases slightly at higher energies. Similar measurements on N_2 would be valuable.

Fite and Brackmann (1958) have also measured the ionization cross sections of atomic hydrogen

$$e + H \longrightarrow e + H^+ + e \qquad \text{(A127)}$$

and their results have been confirmed by Boyd and Boksenberg(1960).

No measurements exist for atomic nitrogen,but Seaton (1959) has used a semi-empirical method together with the atomic oxygen ddta of Fite and Brackmann (1959) to compute cross sections for

$$e + N \longrightarrow e + N^+ + e \qquad \text{(A128)}$$

The cross sections for these ionization processes are shown in Figures 11,12,13.

7.3 Distribution of Excitation and Ionization

The relative amount of primary excitation and ionization produced by electrons of various energies is of interest, and Table A16 contains values of the cross sections of atomic hydrogen - the only case for which detailed results are available.

7.4 Stopping Power and Range of Electrons

An electron of very high energy E passing through the atmosphere will lose energy predominately by bremsstrahlung, the ratio of loss by bremsstrahlung to loss by excitation and ionization being approximately 10^{-5} E E being measured in MeV. The intensity distribution of the bremsstrahlung has been graphed by Bethe and Ashkin (1953). After the

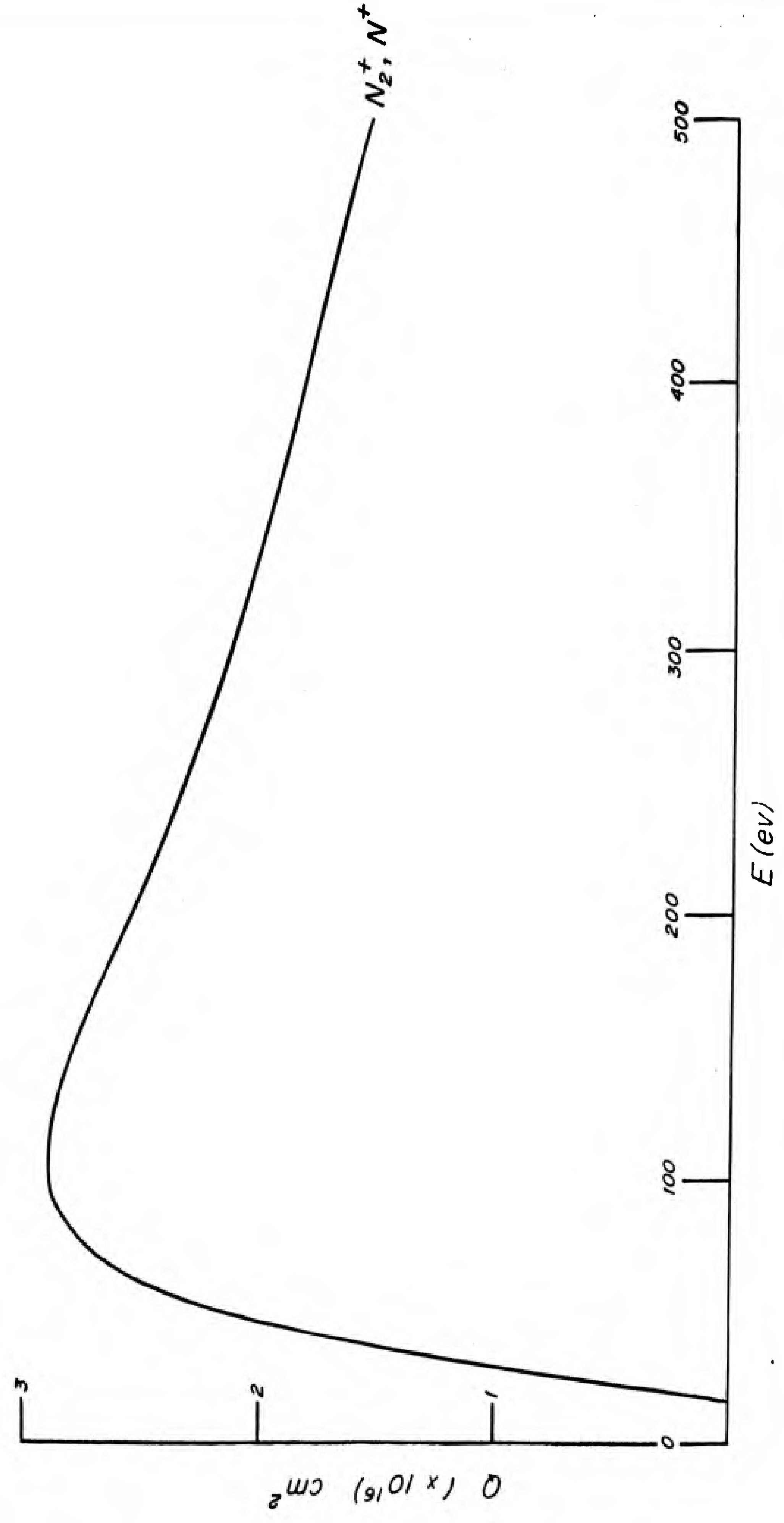

Figure 11a. Cross Section of Ionization Process - Nitrogen

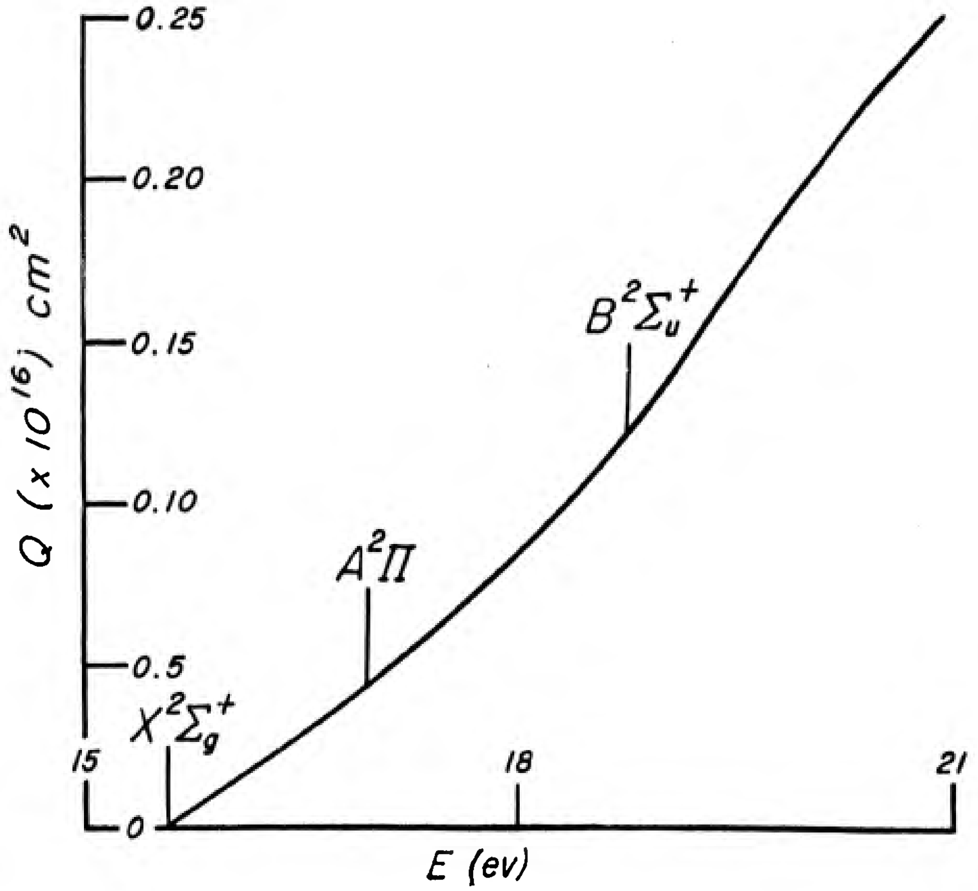

Figure 11b. Nitrogen (Fox and Hickam)

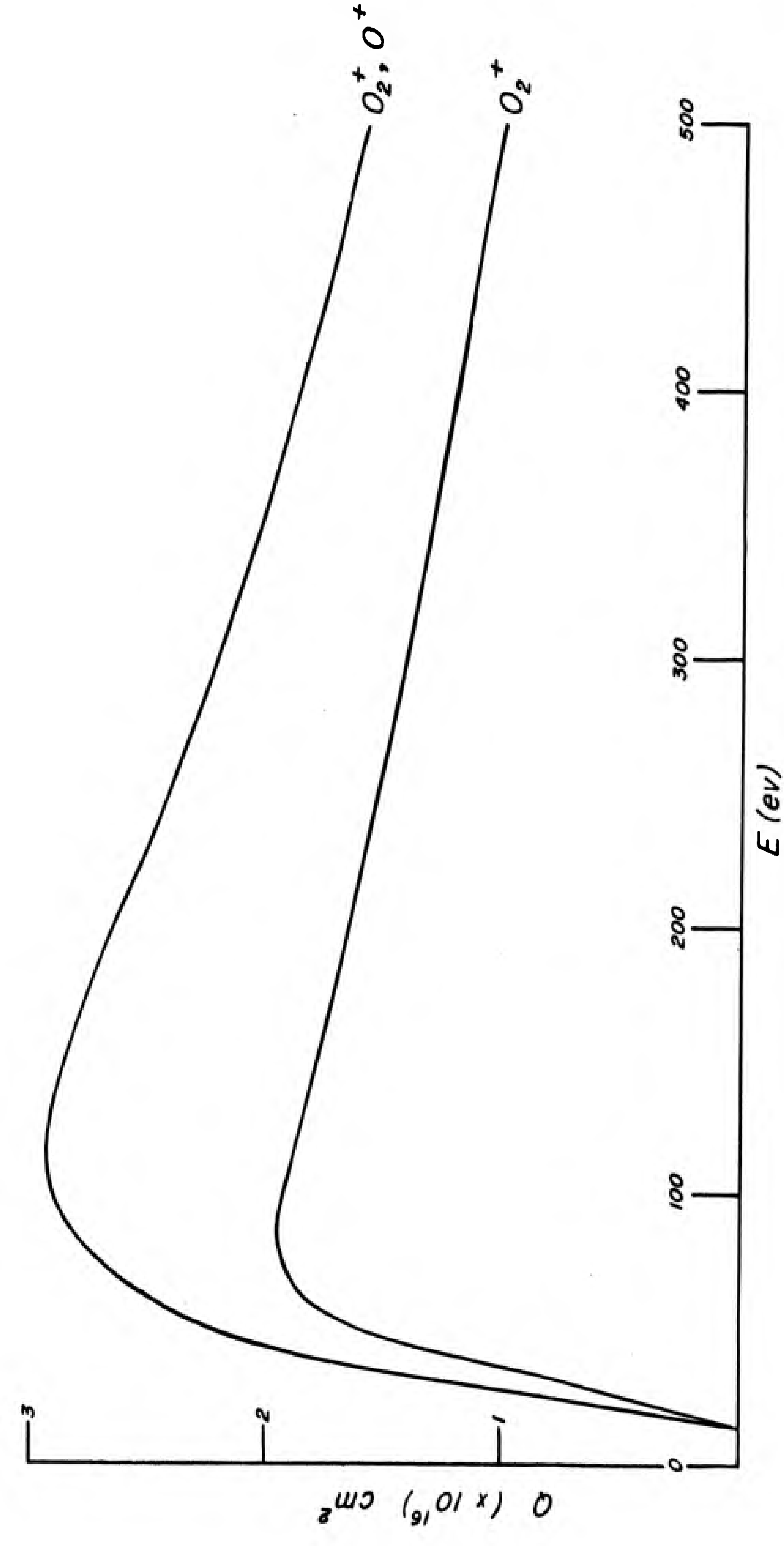

Figure 12. Cross Section of Ionization Process - Oxygen

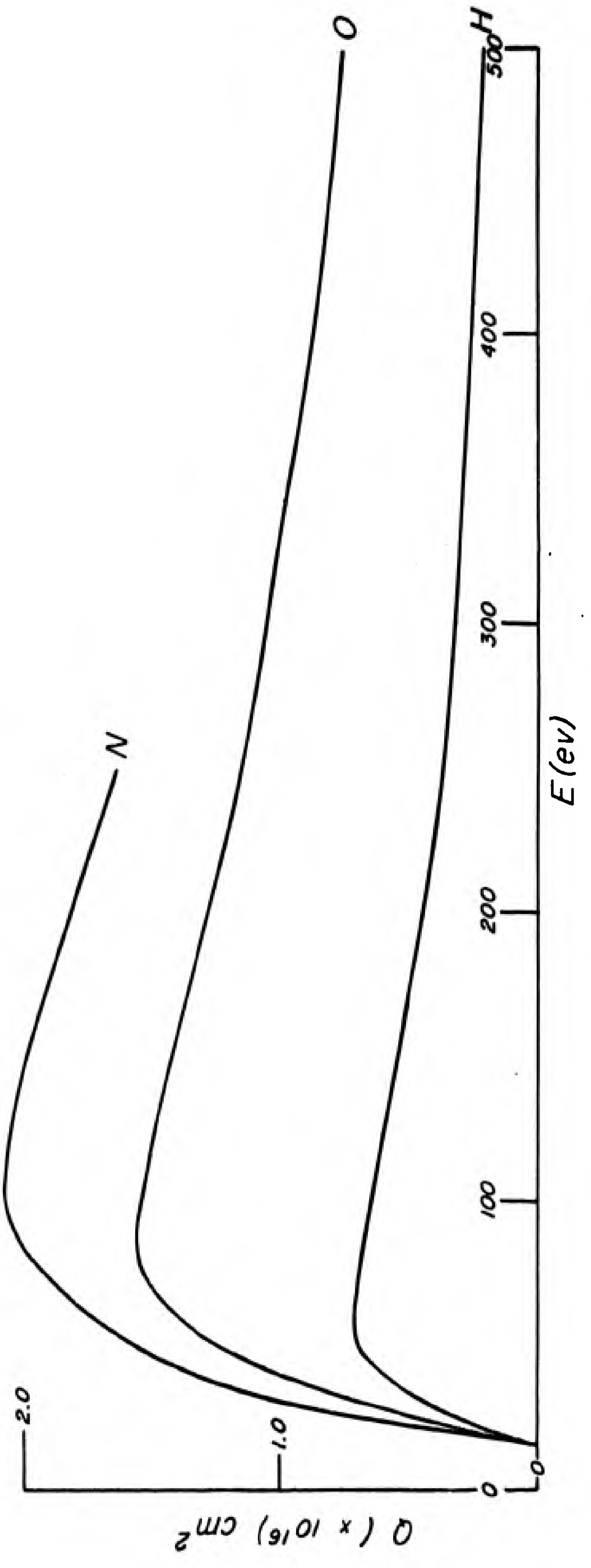

Figure 13. Cross Section of Ionization Process - Atomic Oxygen, Atomic Nitrogen and Atomic Hydrogen

electron is slowed to an energy of about 90 MeV, energy loss occurs mainly through excitation and ionization. This type of energy loss is much the same for protons and electrons of the same velocity.

The rates of energy loss through excitation and ionization for an electron of energy E(eV) are given by the Bethe formula

$$-\frac{dE}{dx} = \frac{1.87 \times 10^{-12}}{E} n \ln(1.17E/I) \text{ eV cm}^{-1} \qquad \text{(A129)}$$

a value of I = 94 eV being a reasonable choice for air. Some values of $-\frac{1}{n}\frac{dE}{dx}$ are given in Table A17.

Values of the ranges of electrons in air have been given by Das Gupta and Ghosh (1946), Heitler (1954), and Spencer (1955), and they may be used to compute the range of electrons in the atmosphere. * Electrons will not travel in rectilinear paths through the atmosphere so that the ranges so obtained do not yield the altitude reached by the electron.

7.5 Mean Energy per Ion Pair

The available data on the mean energy expended in producing an ion pair have been discussed by Valentine and Curran (1958). There are discrepancies between the measurements of different investigators, but it appears that values of the mean energy W expended in producing an ion pair by an electron of more than 250 eV energy can be taken as constant and equal to 34 eV for air, 31 eV for O_2 and 35 eV for N_2 without serious error. W must ultimately increase rapidly for lower energies, and the calculations of Dalgarno and Griffing (1958) on atomic hydrogen are

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* A Table of values has been prepared by Chamberlain (1961).

representative. They show that for evident energies down to 60 eV, W is constant, but for lower energies W increases sharply.

7.6 Ionization Production Rates

The effective cross sections for the production of ionization (primary and secondary) in air follows immediately on dividing the stopping cross sections $-\frac{1}{n}\frac{dE}{dx}$ by 34 eV.

8 COLLISIONAL HEATING

The pressures at altitudes near 200 km measured over Fort Churchill (Horowitz and La Gow, 1958; Horowitz, La Gow and Guilani, 1959) are much greater than those measured over White Sands (Horowitz and La Gow, 1957). The measurements refer to very different periods of the solar cycle, but the suggestion has nevertheless been made that the pressure difference is a permanent feature of the high atmosphere.

The effect of auroral activity on the thermobalance of the atmosphere has been considered by Bates (1960). During a strong arc, the photon flux of the first negative system of nitrogen is about 5×10^{11} cm^{-2}sec^{-1} (Ohmholt, 1954) which, using the cross sections given in section 7, corresponds to an ionization flux of about 10^{13} cm^{-2}sec^{-1}. The associated thermal energy transferred to the atmosphere will be of the order of 10^{14} eV cm^{-2}sec^{-1}. The altitude distribution of the heat source will have much the same shape as the luminosity curve so that the energy will be deposited in the region near 105 km for the majority of auroras. The mean thermal energy supplied by ionizing radiation to the region above the base of the F_1 layer is at least 1×10^{10} eV cm^{-2}sec^{-1} (Bates, 1951). Since visible auroras cover only a small fraction of the earth's surface, it seems that the heating associated with visible auroras cannot be significant within the content of the overall thermobalance of the atmosphere (Bates, 1960).

Appreciable local heating may occur. If we adopt 10 km for the vertical extent of a strong arc, the associated heat source is about 10^8 eV cm^{-3}sec^{-1}, which for a number density of 4×10^{12} cm^{-3} yields a rate of gain of thermal energy per particle of about 2×10^{-5} eV sec^{-1}. The temperature rise is therefore about 5°K per minute. In some auroras the luminosity distribution is greatly extended in altitude (Harang, 1958; Harang and Ohmholt, 1960) and a rapid rise in temperature may occur at the upper levels. The rise will be limited by air motions and by heat conduction.

VanAllen, McIlwain, and Ludwig (1959) and Krassovsky (1960) have drawn attention to the possibility that atmospheric heating may be caused by particles from the Van Allen radiation belts. As Bates (1959) has remarked, the contribution cannot be of major importance to the heat economy of the atmosphere. According to Bates, the required energy flow is 4×10^{18} ergs sec^{-1}, whereas Dessler and Vestine (1960) suggest an upper limit of 6×10^{22} ergs for the energy content of the radiation belts. A very short turnover time would therefore be necessary.

The possibility that the trapped particles cause heating in the auroral zones has been discussed by Jastrow (1959,1960). He supposes that the heat source arising from the particles can be written in the form

$$Q = F \sigma E n \qquad \text{(A130)}$$

where F is the flux of energetic electrons, σ is the inelastic cross section, E is the mean energy transferred per collision and n is the atmospheric number density. For electrons of 10 keV, Jastrow (1960) takes $\sigma = 10^{-18}$ cm^2 and E = 20 eV, corresponding to an effective heat transfer cross section of 2×10^{-17} eV cm^2.

The significance of these values is not entirely clear. The primary mechanism of heat transfer is that of momentum transfer during a collision, and according to equation (A20) the mean energy transferred per collision by a 10 keV electron is about 0.5 eV. The associated cross section is uncertain but is probably of the order of 10^{-18} - 10^{-19} cm^2 giving a primary heat transfer cross section of between 5×10^{-19} eV cm^2. The primary contribution is however negligible compared to that arising from secondary electrons. Thus the stopping cross section associated with excitation and ionization is 9×10^{-16} eV cm^2 (cf. Table A17) and a large fraction of the associated energy will ultimately be converted into thermal energy (through such mechanisms as direct elastic collisions of the secondaries and recombination processes). Associated with the heating effect, there will occur ionization and excitation with effective cross sections (including the secondary processes) of about 3×10^{-17} cm^2. The calculation of the distribution of energy deposition in the atmosphere by the trapped electrons presents a very difficult problem because of the importance of multiple scattering of the primary and secondary electrons, but the heating distribution should be similar to the luminosity distribution.

In his computations, Jastrow (1959,1960) has adopted an energy flux of 4000 erg cm^{-2} sec^{-1} of electrons with energies above 10 keV at an altitude of 300 km. He assumes that the flux is inversely proportional to the ambient density up to altitudes ranging from 400 to 600 km and deduces that the temperature in the auroral zone will rise to about 2500°K. However, his assumed flux would also give rise to an electron production rate of 5×10^4 cm^{-3} sec^{-1} and to a photon production rate of at least 10^{12} quanta per cm^2 column per sec.

It may be remarked that heating by fast particle collisions is always associated with the production of ionization and excitation and that in order to obtain heating without luminosity it is necessary to invoke a flux of slowly moving particles.

We note finally that Dessler (1959) has suggested that heating by hydromagnetic waves may be important but that according to Akasofu (1960) this is not the case.

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B. RECOMBINATION COEFFICIENTS

1. INTRODUCTION

In gases of sufficiently low density, electrons and positive atomic ions can recombine only through the process of radiative recombination represented by

$$X^+ + e \rightarrow X + h\nu \tag{B1}$$

where ν is the frequency of the emitted radiation. In the case of molecular ions a more efficient means of removal of electrons is the non-radiative process of dissociative recombination

$$XY^+ + e \rightarrow X + Y \tag{B2}$$

while for gases of higher density, three-body processes

$$X^+ + e + M \rightarrow X + M \tag{B3}$$

may be the controlling reactions. In many physical situations (such as those prevailing in the outer reaches of most of the planetary atmospheres) it is process (B1) which is important.

The process is of particular interest in that it provides (in principle) a means of measuring the electron temperature and the electron concentration by spectroscopic methods. The positive ion X^+ may capture an electron into any one of a large number of excited states of the neutral atom X which may then radiate. Thus radiative recombination gives rise to a line spectrum superimposed upon a continuum, the intensity distribution depending upon the electron temperature and the intensity magnitudes depending upon the electron density.

Because of cascading it will usually be the case that several lines appear in the observationally convenient spectral range. The process is not very efficient however. For simply charged positive ions, the total recombination rate is of the order of 10^{-12} cm^2 sec^{-1}.

2. THEORETICAL SUMMARY

2.1 The Intensity Distribution

Consider the specific recombination process

$$X^{+} + e \rightarrow X_n + h\nu_r \tag{B4}$$

where X_n denotes the nth excited state of X and let α_n be the corresponding rate coefficient in $cm^3\ sec^{-1}$ defined so that

$$a_n = \alpha_n n_e n_+ \qquad cm^{-3}\ sec^{-1} \tag{B5}$$

is the rate of population of X_n, n_e being the electron number density and n_+ the positive ion number density. The total recombination rate coefficient is given by

$$\alpha = \sum_n \alpha_n \tag{B6}$$

summed over all the excited states of X. It is such that the rate of removal of electrons is given by

$$\frac{dn_e}{dt} = -\alpha n_e n_+ \tag{B7}$$

In most situations, $n_e \sim n_+$ and (B7) becomes

$$\frac{dn_e}{dt} = -\alpha n_e^2 \tag{B8}$$

The analysis of the intensity distribution is more complicated. The atom in the excited state may radiate to other states of lower energy. Let A (n-n') be the probability of a transition from X_n to $X_{n'}$. The number of photons emitted per second in the transition (n-n') following direct capture into X_n is given by

$$d(n-n') = \left[\alpha_r \; A(n-n') \Big/ \sum_{n''<n} A(n-n'')\right] n_e n_+ \tag{B9}$$

However X_n may itself be populated by cascading from higher states. The total number of photons emitted per second in the transition (n-n') is

$$p(n-n') = \beta_n \frac{A(n-n')}{\sum\limits_{n''<n} A(n-n')} n_e n_+ \tag{B10}$$

where

$$\beta_n = \alpha_n + \sum_{n'''>n} \alpha_{n'''} \frac{A(n'''-n)}{\sum\limits_{n''<n'''} A(n'''-n'')} \tag{B11}$$

The prediction of the intensity distribution of the discrete spectrum therefore involves the calculation of the transition probabilities in addition to the specific rate coefficients.

The intensity distribution in the continuum associated with capture into X_n may be written in the form[1]

$$I\nu_n \, d\nu_n = n_+ h\nu_n Q_e^n(v) v n_e(v) dv \tag{B12}$$

where $I\nu_n \, d\nu_n$ is the intensity of radiation with frequency between ν_n and $\nu_n + d\nu_n$ emitted per unit volume of a gas containing n_+ ions per unit volume due to the capture into X_n of electrons with a velocity distribution n_e (v) such that n_e (v) dv is the number of electrons per unit volume with velocities between v and v + dv, and Q_c^n (v) is the cross section for the radiative capture into X_n of electrons of velocity v. The capture cross section may be written

$$Q_c^n = \frac{64\pi^4 e^2 \nu_n^2}{3hc^3} |\underset{\sim}{M}|^2 \tag{B13}$$

where e is the electric charge, c is the velocity of light and $\underset{\sim}{M}$ is the transition matrix element.

If ψ_n is the wave function of the state X_n and $\underset{\sim}{r}_i$ is the position vector of the ith electron of X_n, M may be written

$$\underset{\sim}{M} = \int \psi_n^* \sum_i \underset{\sim}{r}_i \cdot \psi_c \, d\underset{\sim}{r} \tag{B14}$$

where ψ_c represents the initial continuum state normalized to represent a stream of electrons such that one electron crosses unit area normally per second.

The cross section Q_c^n is related to the recombination coefficient α_n according to

$$\alpha_n = \overline{v \, n_e(v) \, Q_c^n(v)} \tag{B15}$$

In the case of a Maxwellian distribution of electrons corresponding to an electron temperature T_e,

$$n_e(v) = 4\pi n_e \left(\frac{m}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT_e}\right) \tag{B16}$$

where m is the electron mass and k is Boltzmann's constant.

2.2 The Capture Cross Sections

By using the principle of detailed balancing, the cross section for capture Q_c^n may be related to the cross section Q_p^n for the photo-ionization process

$$X_n + h\nu_n \rightarrow X^+ + e \tag{B17}$$

Explicitly

$$Q_p^n = \frac{2m^2 v^2 c^2}{h^2 \nu_n^2} Q_c^n \tag{B18}$$

Extensive calculations of Q_p^o have been carried out for atoms in the bound state using various approximate representations of the initial and final wave functions ψ_n and ψ_c in the evaluation of the matrix element (B14). The early calculations were based on (B14) only but more recent work has employed the formally equivalent expressions:

$$\underset{\sim}{M} = -\frac{h}{4\pi^2 m\nu} \int \psi_n^* \sum_i \mathrm{grad}_i \, \psi_c \, d\underset{\sim}{r} \tag{B19}$$

$$\underset{\sim}{M} = \frac{1}{4\pi^2 m\nu^2} \int \psi_n^* \sum_i \mathrm{grad}_i \, V \psi_c \, d\underset{\sim}{r} \tag{B20}$$

where V is the potential term of the Hamiltonian of the neutral atom. Expression (B14) is the dipole length form of the transition matrix element, (B19) is the dipole velocity (or momentum) form and (B20) is the dipole acceleration form. If exact wave functions are employed or if central field (Hartree) wave functions are employed, the expressions yield identical results. Otherwise a comparison of the results of the three formulations provides an assessment of the reliability of the calculations. It may be remarked that for the capture of fast electrons, recent work by Kabir and Salpeter[2] and by Dalgarno and Stewart[3] suggests that the dipole acceleration form may be the most reliable.

An assessment of the accuracy may also be obtained from an examination of the amount of cancellation incurred in the evaluation of (B14), (B19) or (B20). If it is large the results are unreliable but if it is small the results can be accepted with some confidence.

In general the accuracy obtained in the past calculations of Q_p^o has been satisfactory when Hartree-Fock wave functions have been employed and reliable values are available for most of the lighter elements. They do not in general extend over a wide range of electron energies and it may be desirable to repeat some of the work. There are exceptions to the general reliability. In particular cancellation is very slow in the case of the alkali metals and there is no satisfactory theoretical treatment of these cases.

A simple improvement could be effected by taking explicit account of the polarizing effect of the active electron. This is hardly necessary however since experimental measurements are available of the photoionization cross sections of the alkali metals, from which the cross sections for capture into the bound states may be obtained by using (B18).

Apart from atomic hydrogen[4] and the two metastable states of helium[5], there have been no calculations of capture or photoionization cross sections dealing with excited states. This omission is less serious than it may appear in that in many cases the excited states may be closely approximated by hydrogen-like wave functions. Thus the calculations on atomic hydrogen frequently provide a reliable guide, at least so far as the contribution to the total recombination rate from captures into excited states is concerned.

To deal with the spectral intensity distribution, a more precise description is probably desirable. This is provided by the Coulomb approximation of Bates and Damgaard[6] which has been extended to include bound-free transitions by Burgess and Seaton.[7]

Extensive tables pertaining to the evaluation of Q_c^n or Q_p^n have been published.[7] The input data required may be obtained directly from the observed binding energies of the neutral atom, and it is a simple matter then to compute the cross section of interest. The number of excited states which must be included is however very large, so that the calculation for any particular system is rather lengthy.

2.3 The Transition Probabilities

The calculation of transition probabilities is very similar to that of photoionization cross sections except that the transition matrix element has the form.

$$\underline{M} = \int \psi_n^* \sum_i \underline{r}_i \, \psi_{n'} \, d\underline{r} \qquad \text{(B21)}$$

where ψ_n and $\psi_{n'}$ both describe discrete states. The only system which has been discussed at all completely is atomic hydrogen for which extensive tables of A (n-n') are available. Many transition probabilities are available in the literature derived either from theoretical calculations or from experimental studies but the coverage is largely haphazard. There is need for an extensive attack on the problem.

The problem of predicting transition probabilities presents no serious difficulties other than those involved in dealing with the very large number that are required. The calculations are in much the same position as those of photoionization cross sections. For transitions into the bound state Hartree-Fock wave functions will in general yield reliable results while for transitions between excited states the Coulomb approximation may again be employed.

The case of atomic hydrogen is fairly typical of what occurs for other systems, and the corresponding matrix elements (B21) for a large number of states have been given by Green, Rush and Chandler[8]. These values provide a useful guide to other systems, especially when dealing with highly excited states.

2.4 Atomic Hydrogen

Because of its importance in the interpretation of the observations of gaseous nebulae, considerable effort has been devoted to the processes of radiative capture and cascading in an assembly of hydrogen atoms.[9] The most recent discussion is that by Burgess.[10] Burgess has computed the rate coefficients for capture into all states of atomic hydrogen with principal quantum number $\underline{n}$ less than or equal to twelve, and he has distinguished between the various sub-levels with different azimuthal quantum number $\underline{l}$. This distinction is important since for other systems, the degeneracy associated with different azimuthal quantum number no longer occurs.

Define a quantity $f_{ne}(T_e)$ according to

$$\alpha_{nl} = \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{8\pi\alpha A_0^2}{3c^2}\right)\left(\frac{kT_e}{m}\right)^{3/2} f_{nl}(T_e) \qquad \text{(B22)}$$

where α is the fine structure constant and $A_0 = 5.29 \times 10^{-9}$ cm. In the table we reproduce the results of Burgess[10] for $T_e = 10{,}000^{\circ}$K and for $T_e = 20{,}000^{\circ}$K.

Two qualitative features of this table apply to other atoms also. Capture is most probable into the low-lying excited states and capture is most probable into states of small to intermediate values of the azimuthal quantum number ℓ. The table provides a valuable indication of which are likely to be the most intense lines of a recombination spectrum.

2.5 Application to Physical Situations

The most important applications of recombination processes has been to gaseous nebulae following the original work of Menzel.[8] In this application the whole configuration of radiative capture and cascade and ionization by an external radiation field is assumed to be in statistical equilibrium.

A rather different approach is necessary in predicting the behavior of, say, an electron cloud of initially high density in a tenuous gas. For this situation, one must solve the equation

$$\frac{dn_e}{dt} \doteq -\alpha n_e^2 \tag{B23}$$

(if diffusion and velocity drift terms can be ignored) to find n_e as a function of time and space. Substituting back into (B15) then gives the intensity distribution as a function of time and space. This simplified picture assumes a constant temperature. Subsidiary equations are necessary to take into account any cooling resulting from impact excitation of the ambient gas or from elastic collisions with the ambient gas.

TABLE B1

RECOMBINATION COEFFICIENTS FOR ATOMIC HYDROGEN

$t^3 f_{n\ell}(t)$ $(t = 10^{-4} T_e)$

n	ℓ	t=1	t=2
1	0	1757	1183
2	0	261	178
	1	600	362
3	0	87.3	59.1
	1	227	138
	2	193	101
4	0	40.5	27.1
	1	108	64.8
	2	121	63.2
	3	61.9	28.6
5	0	22.2	14.7
	1	58.8	34.9
	2	74.5	38.9
	3	55.0	24.9
	4	19.9	8.40
6	0	13.5	8.86
	1	35.2	20.7
	2	47.8	24.9
	3	41.8	18.8
	4	22.8	9.03
	5	6.52	2.62
7	0	8.85	5.73
	1	22.6	13.1
	2	32.1	16.8
	3	31.4	14.2
	4	20.8	8.22
	5	9.16	3.30
	6	2.23	0.854
8	0	6.11	3.92
	1	15.3	8.70
9	4	14.3	6.07
	5	8.88	3.34
	6	4.18	1.43
	7	1.43	0.473
	8	0.289	0.107
10	0	3.27	2.07
	1	7.90	4.39
	2	11.8	6.13
	3	13.4	6.40
	4	11.7	5.05
	5	7.94	3.05
	6	4.27	1.47
	7	1.79	0.586
	8	0.560	0.183
	9	0.111	0.0406
11	0	2.49	1.56
	1	5.95	3.26
	2	8.85	4.60
	3	10.2	5.00
	4	9.42	4.18
	5	6.91	2.74
	6	4.09	1.44
	7	1.97	0.637
	8	0.767	0.240
	9	0.226	0.0710
	10	0.0435	0.0158
12	0	1.94	1.22
	1	4.45	2.49
	2	6.78	3.52
	3	8.05	3.91
	4	7.63	3.43
	5	5.99	2.42
	6	3.80	1.38
	7	2.04	0.666

TABLE B1 (continued)

	2	22.3	11.7
	3	23.3	10.9
	4	17.6	7.16
	5	9.57	3.47
	6	3.62	1.23
	7	0.787	0.298
9	0	4.39	2.79
	1	10.8	6.07
	2	16.0	8.37
	3	17.4	8.30
	8	0.919	0.280
	9	0.339	0.101
	10	0.0936	0.0289
	11	0.0175	0.00632

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