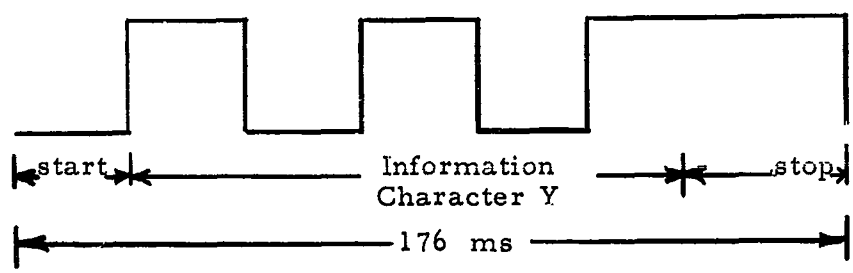
ABSTRACT

Approximately 300 hours of radio teletype data have been analyzed in a manner similar to an analysis performed for telephone channels to determine the applicability of coding theory to radio teletype channels. The results indicate that while correcting codes are impractical for improving the reliability of a teletype channel, error detect and repeat schemes with about 10 per cent redundancy will reduce the probability of error to a negligible amount.

This report is based on error data[1] which was obtained in November, 1959, with a 60-wpm automatic teletype transmission channel from Bermuda to Riverhead, New York. The test equipment, procedure, and terminal facilities for collecting the data are described in detail in the reference so that only a brief description of them will be given here.

Two frequencies, 5220 kcs (night frequency) and 10385 kcs (day frequency), were used. During the test the upper sideband of a single-sideband suppressed-carrier signal was used by A. T. and T. for voice transmission while the lower sideband was used by Lincoln. A single teletype character (the letter Y) was transmitted repeatedly over the lower sideband by frequency shift keying a lower sideband tone sub-carrier of 1600 cps plus or minus 170 cps. Each character was composed of five information bits plus start and stop bits as indicated below. This resulted in an average rate of 28.4 information bits per second. The



received signal was compared with a locally generated signal which was identical to the transmitted character. If errors were noted, the received character together with an elapsed time indication was punched on paper tape for later analysis.

Several quantities, such as the average number of characters in error per minute, distribution of errors within the character, and the distributions of characters in error per minute in error were computed by Greim.[1] The analysis indicated that errors were dependent and highly correlated. Originally, questions of error statistics over sequences of several characters were not investigated in any detail. Since the applicability of coding to a channel depends upon these statistics, the raw data have been re-analyzed in this respect; the results are contained in this report. The cited results assume that synchronization and timing contribute no errors.

Efficient applications of error-correcting and error-detecting codes require coding over many, typically 100 or more, symbols. Code words of this length might be formed in an actual system by using several successive five-bit characters. The effects of noise on code words formed in this way can be obtained from the raw data by combining the appropriate number of consecutive five-bit characters in the data. This was done for code words with 127, 255, and 511 symbols which are typical lengths of Bose-Chaudhuri [2,3] codes. The error statistics for the code words are shown in Figs. 1 through 4 and are tabulated in Table I.

One of the major differences between the teletype and telephone channels is the probability of a code word with errors. The telephone channel data with a CTDS modulation system and 255 symbol code words, to cite a typical example, had an average of one code word with errors every 5700 words[4] compared to one every 70 words for the teletype channel. Although noise affects code words much more often in a teletype channel than with a telephone channel, the distribution of errors and bursts of errors in the words with errors (found in Figs. 1 and 2) are approximately the same for the two channels. The distributions of consecutive words without and with errors are shown in Figs. 3 and 4. Although there are errors in code words on the average of one every 70 words, the channel operates without error for long periods of time. From Fig. 3, 10 per cent of the runs of consecutive words without errors were longer than 200 words, and these long runs contained about 80 per cent of the total number of code words.

The reliability of transmission over a teletype channel can be improved by using redundancy for either error correction, error detection and retransmission, or a combination of these two schemes.

Error correction with the teletype channel is quite costly in terms of the redundancy which is required. For example, Figs. 1 and 2 show that to correct errors in three-fourths of the code words of length 255 which have errors, either a code capable of correcting seven random errors or one capable of correcting error bursts of 86 symbols is required. The former could be done with a Bose-Chaudhuri code with 56 redundancy symbols. A Fire[3,5] code would correct error bursts of 86 or fewer symbols but would require 87 redundancy symbols. Since less than 2 per cent of the transmitted code words are received with errors and since the amount of redundancy mentioned above would correct errors in only three-fourths of the words with errors, it appears that error correction is impractical for teletype channels.

When a feedback channel is used with the telephone circuits, extremely efficient and reliable transmission can be obtained by adding enough redundancy to the transmitted message to detect errors and by repeating a message whenever errors are detected. The teletype channel is also well suited for detect and repeat schemes. As an example, consider a detect and repeat scheme which uses a Bose-Chaudhuri code with 239 information symbols and a total length of 255. This code is capable of detecting all error patterns containing four or fewer errors. The probability that an error pattern with more than four errors is undetected by the code is approximately 2^{-16}. In the 300 hours of teletype data, 961 of the 255 symbol sequences had more than four errors. It is a reasonable simplification to assume that when more than four errors occur, the noise and code words are essentially independent so that each of the 961 sequences has a probability of 2^{-16} of going undetected. From this, the average time between undetected errors can be estimated to be 2 years. Similar results for other codes are tabulated in Table I. If more redundancy is used, the average time between undetected errors is approximately doubled with each additional parity check symbol. Thus, error-detecting codes with about 10 per cent redundancy can reduce the probability of an undetected error to a very negligible amount.

The possibility of combining error correcting with a detect and repeat system has been mentioned. If a few errors occur during the transmission of a code word with such a scheme, they are corrected at the receiver without retransmission; if many errors occur, the message is retransmitted. The feasibility of such a procedure obviously depends on the error statistics of the channel. For this teletype channel, the combined error correction and detection code reduces the average transmission rate by a considerable amount as illustrated by the following example.

Consider a five-error-correcting Bose-Chaudhuri code with 127 symbols including 35 check symbols. Figure 1 shows that this code would correct about two-thirds of the code words with errors. If more than five errors occur, the code is used for error detection. It can be shown that it is reasonable to expect the code to fail in the detection of these errors about once every 100 times or about seven times in the 300-hour sample of data. With a detect and repeat scheme, the feedback signal for a transmitted word is not available at the transmitter immediately after transmitting a code word but is delayed by an amount equal to the two-way propagation time plus the time required for decoding. This amounts to the time equivalent to about two code words. If the transmitter waits to interpret feedback signal for each code word before sending the next word, the forward channel will be idle a large percentage of the time. A more efficient procedure[6] is to transmit the next word or words immediately and retain the previously transmitted words in temporary storage until the feedback signal for them is received and interpreted. Whenever a retransmission is needed, the transmitter "goes back" and retransmits all the words in temporary storage. It follows that with each retransmission a number of extra words, equal to the number of words gone back, are transmitted and these reduce the average transmission rate. For the code under consideration, approximately 700 words, or an average of one every 340 code words, would be repeated. This results in an approximate average information rate of $(92/127) \times (338/340) = 0.72$.

Consider next the average information rate for a detect and repeat scheme which has the same probability of an undetected error as the error-correcting and error-detecting code used in the previous paragraph. It is possible to construct Bose-Chaudhuri code of length 127 with 8 parity checks which will detect all error patterns containing three or fewer errors. The probability of failing to detect the presence of errors if more than three errors are present is about 2^{-8} and this is equivalent to about five undetected errors for the teletype data. An average of one out of every 109 transmitted words has to be repeated so that the effective information rate is $(119/127) \times (107/109) = 0.91$.

The results of this study indicate that error-correcting codes are impractical with the teletype noise data which were used. The results also indicate that error-detect and repeat systems with about 10 per cent redundancy can be expected to operate with the probability of an undetected word error so small it can be neglected. If error correction is combined with error detection and repetition, a high degree of reliability can be obtained, but the same reliability is possible at a much higher rate with a detect and repeat system.

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ABF sc

CODE SIZE			ERROR DATA				
Length (symbols)	Parity Check (symbols)	Minimum Code Words	No. of Code Words	No. of Words With Errors	Est. Av. Time for Undetected Errors (yrs.)	No. of runs of code words with Errors	No. of runs of code words without Errors
125	14	5	238448	2192	0.5	1241	1272
255	16	5	121555	1748	2	991	1024
511	18	5	60122	1359	12	763	795

TABLE I

G25-332

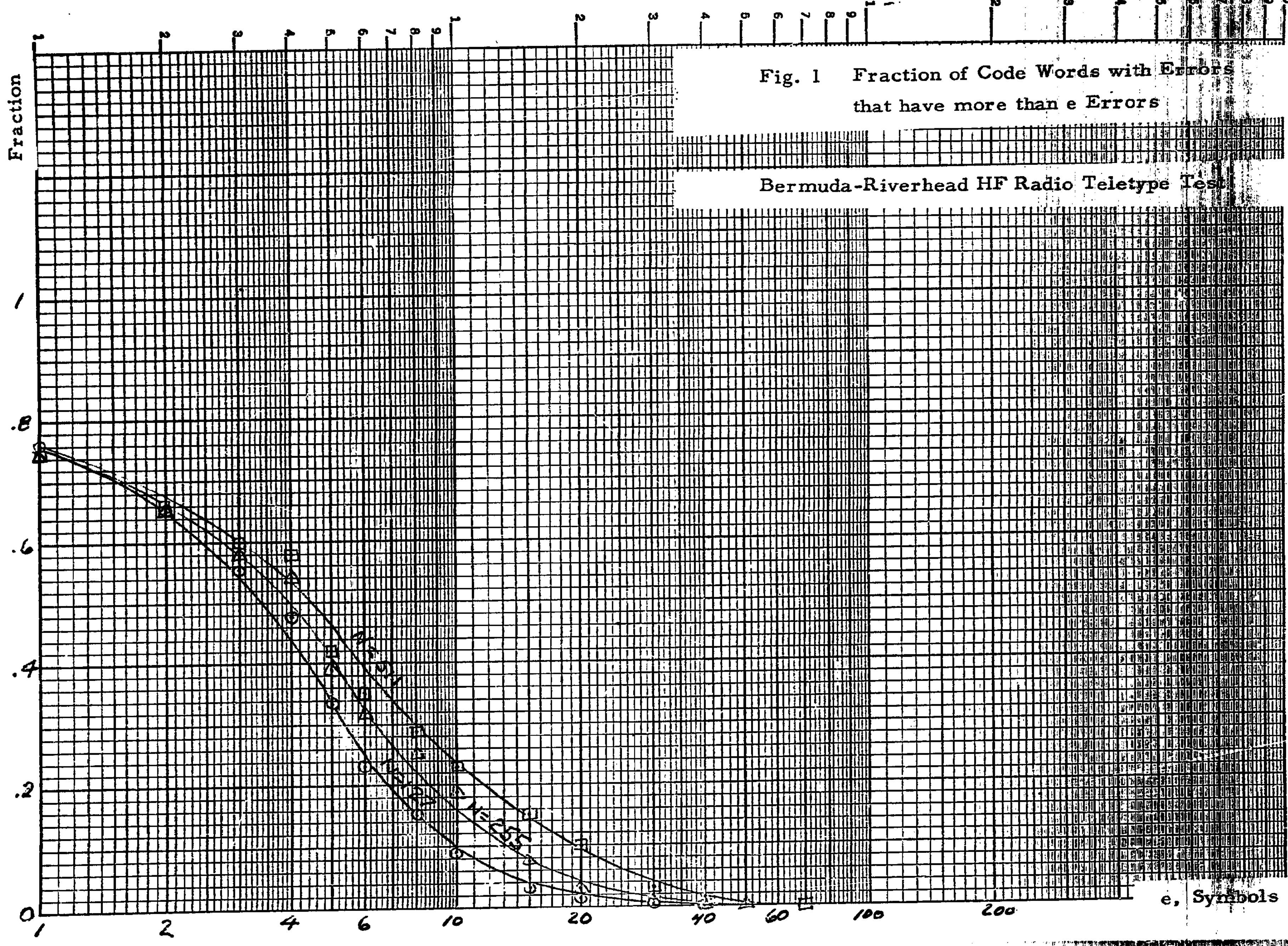

Fig. 1 Fraction of Code Words with Errors that have more than e Errors

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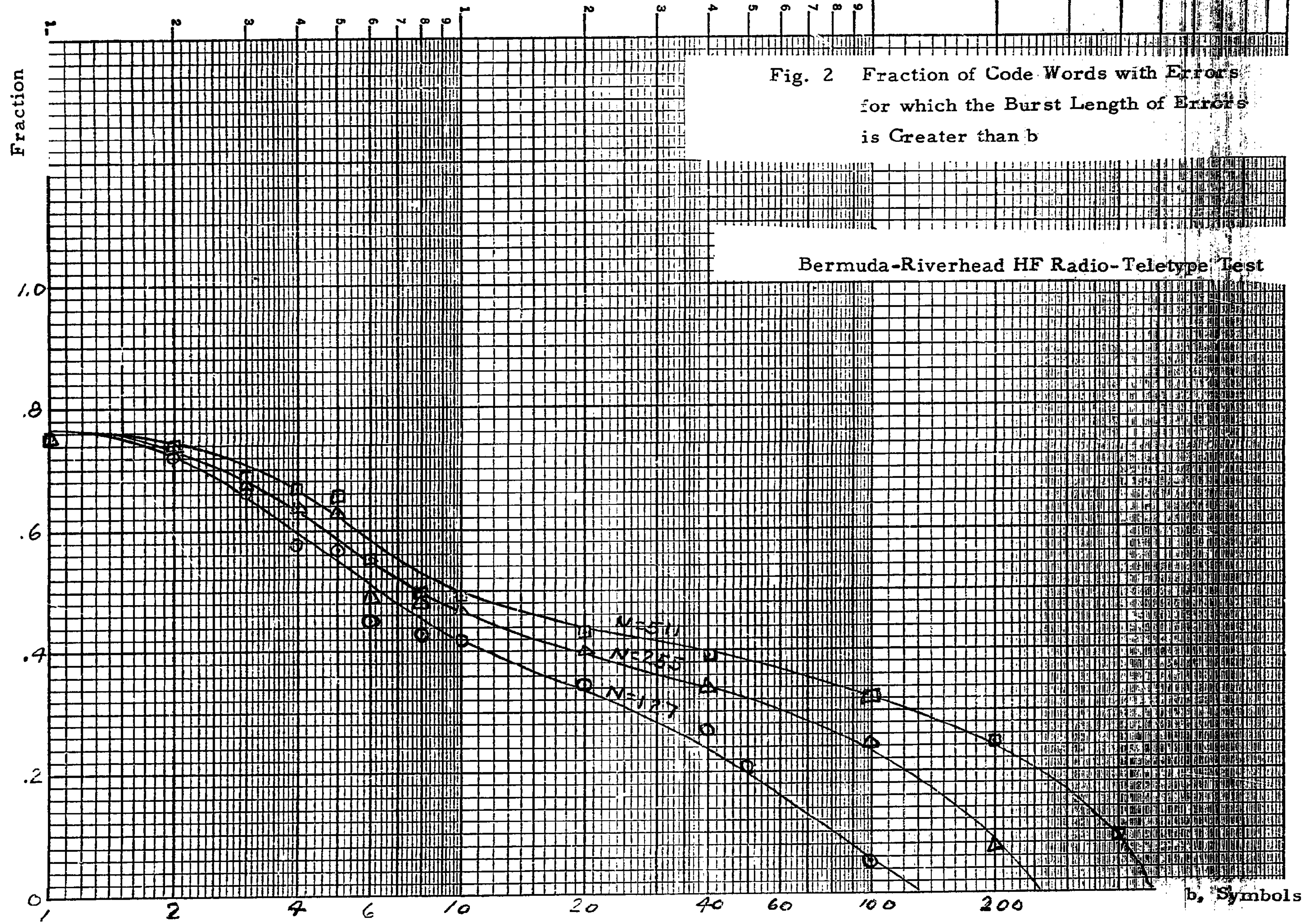

Fig. 2 Fraction of Code Words with Errors for which the Burst Length of Errors is Greater than b

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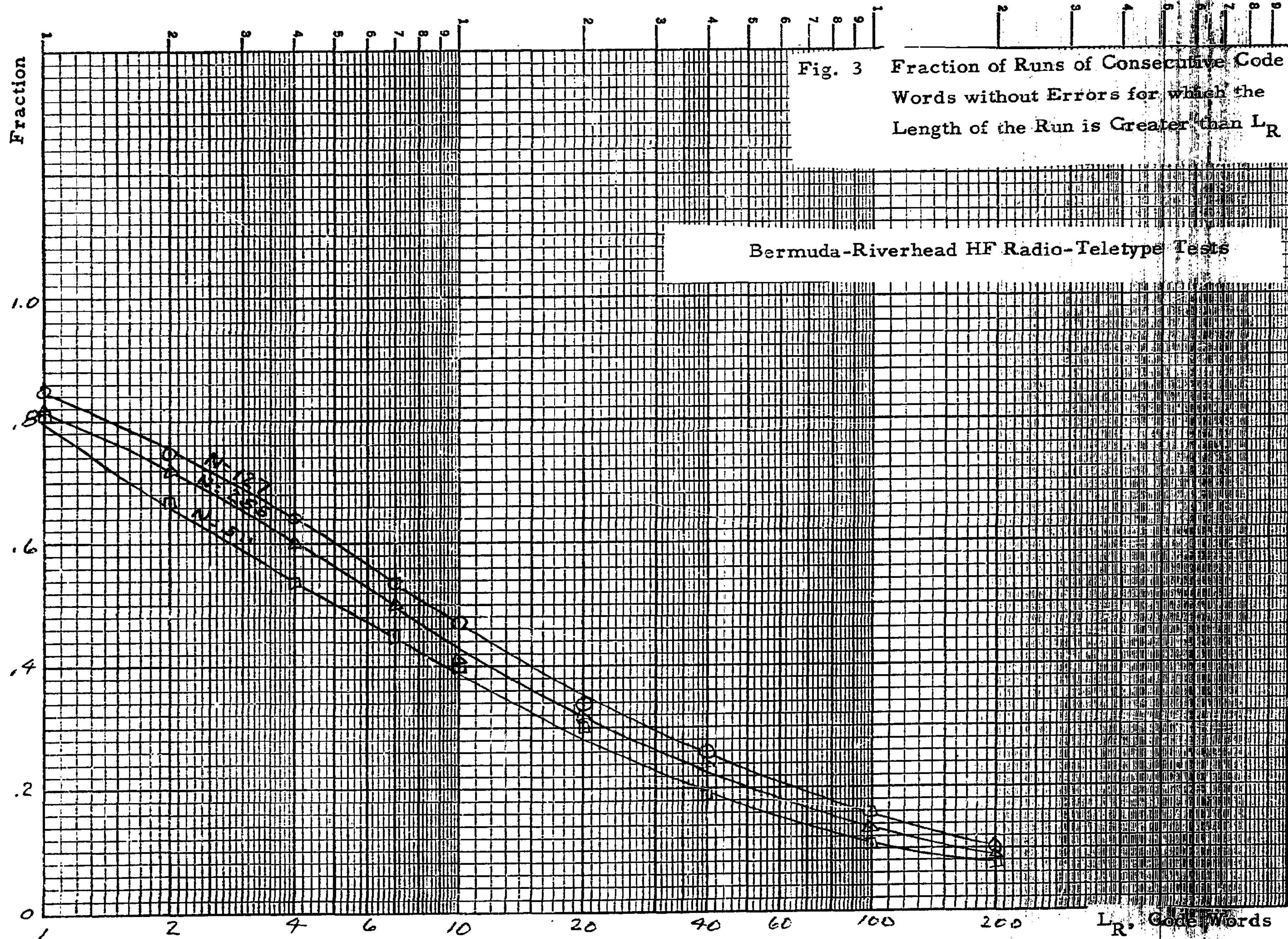

Fig. 3 Fraction of Runs of Consecutive Code Words without Errors for which the Length of the Run is Greater than L_R

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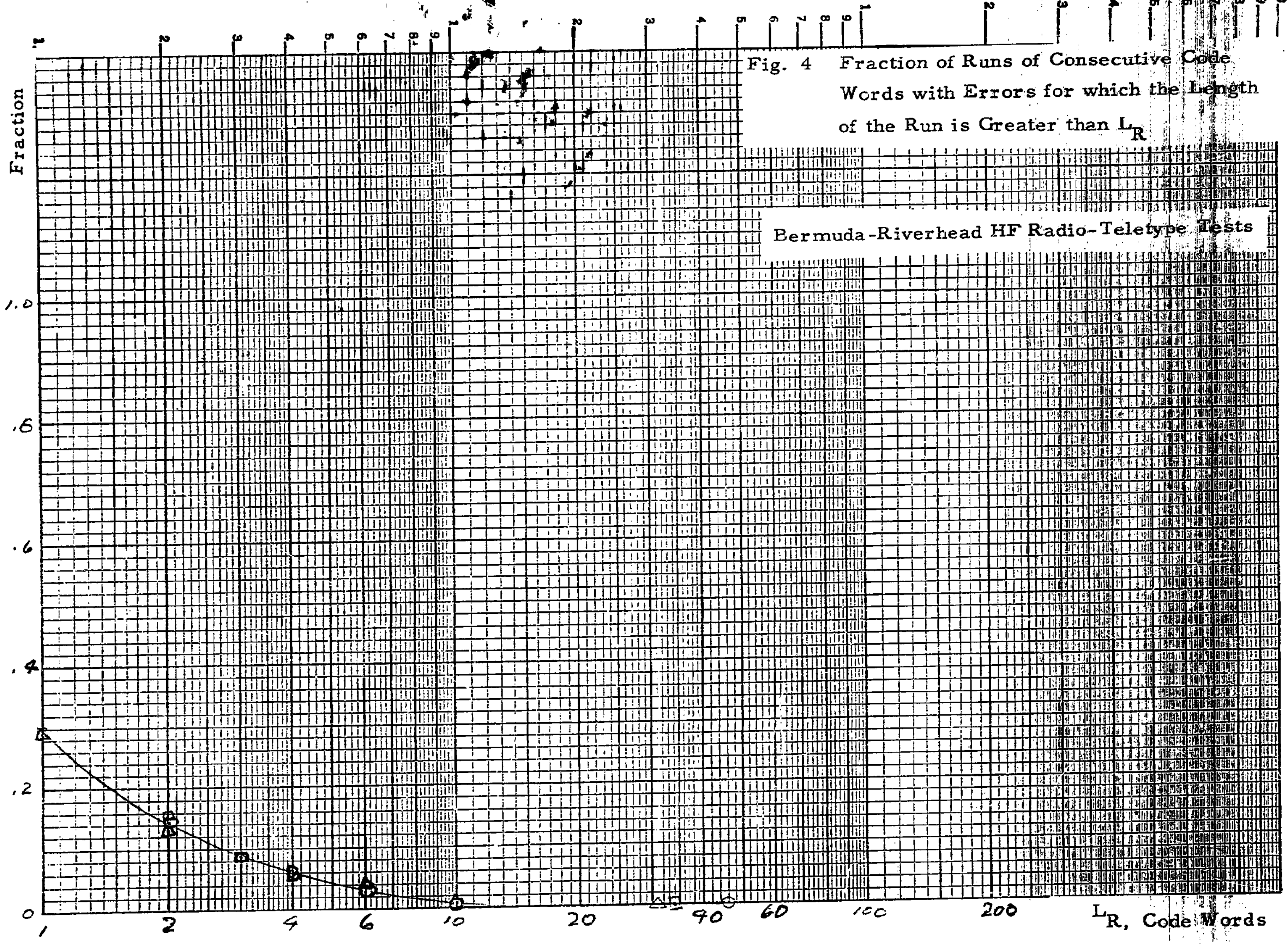
Fig. 4 Fraction of Runs of Consecutive Code Words with Errors for which the Length of the Run is Greater than L_R
Bermuda-Riverhead HF Radio-Teletype Tests
Fraction
1.0
.8
.6
.4
.2
0
L_R, Code Words
1
2
4
6
10
20
40
60
100
200

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